ABSTRACT

Title of Dissertation: EXAMINING TEACHERS' USES OF INTERACTIVE DIAGRAMS IN SECONDARY MATHEMATICS INSTRUCTION

Thomas Coleman, Doctor of Philosophy, 2017

Dissertation Directed By: Dr. Janet Walkoe
Department of Teaching and Learning, Policy and Leadership

This is a study of secondary mathematics teachers' use of a digitally enhanced educational technology called Interactive Diagrams (IDs). This study is concerned with the ways in which mathematics teachers use IDs as they plan and enact classroom activities involving this technology, as well as the factors that affect their ID use.

Qualitative research methods were used to conduct this exploratory study focused on the practice of three teachers within a large urban school district. Using Remillard's (2005) conceptualization of teacher-curriculum interactions allowed participants' instructional practices to be situated within an environment affected by contextual, curricular, teacher, and student factors. Mediating factors within each of these areas were identified for the study participants using inductive coding techniques. Teachers' intended and enacted uses of selected IDs were also analyzed using this research methodology. The results of this analysis are described, as well as implications for future practice and research.

EXAMINING TEACHERS' USES OF INTERACTIVE DIAGRAMS IN SECONDARY MATHEMATICS INSTRUCTION

by

Thomas Coleman

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Dissertation Committee:
Dr. Janet Walkoe, Chair, Department of Teaching and Learning, Policy and Leadership
Dr. Daniel Chazan, Department of Teaching and Learning, Policy and Leadership
Dr. Tamara Clegg, Department of Teaching and Learning, Policy and Leadership
Dr. Patricia Campbell, Department of Teaching and Learning, Policy and Leadership
Dr. Benjamin Bederson, Department of Computer Science

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Chapter 1

Personal Statement

I believe that digital technologies can profoundly impact the teaching and learning of mathematics. Devices such as graphing calculators, Computer Algebra Systems (CAS), interactive and dynamic Geometry software (i.e. GeoGebra), apps, video simulations, and online curricula can display mathematical concepts and allow users to interact with those concepts in ways that are unique to a digital environment.

Digital technologies are tools. Like any other tool, the contexts within which they are used heavily influence their effectiveness. A paintbrush, for example, can be used to create a great masterpiece in the hand of one artist or a childish scribble in the hand of another. In much the same way, I have experienced the use of digital technologies greatly enhancing mathematical learning as well as completely derailing it. This variation existed both in my own classroom and in the classrooms I now frequent in my role as a teacher coach and instructional lead. This makes me wonder about the particular ways in which teachers use digital technologies within their practice. Additionally, I am curious about the factors that influence this interaction. My hope is that the mathematics education community can come to an understanding of these factors. This understanding can then inform the design of educational environments. The present study aimed to contribute to the growing body of literature that investigates teachers' interactions with digital technologies and the factors that affect this interaction.

Rationale for the Study

It is clear that K-12 teachers interact with curriculum in a variety of ways and that the ways in which they interact with curriculum are affected by a number of internal and external factors (Drake & Sherin, 2006; Remillard, 2005; Stein, Remillard, & Smith, 2007). Much investigation has occurred around teachers' interaction with curriculum resulting in theoretical models that describe the salient factors that influence this interaction. Remillard (2005), for example, theorized four categories of influential categories, namely factors originating from the 1) context, 2) teacher, 3) curriculum, and 4) students, which influence the participatory relationship between the teacher and the curriculum. This participatory relationship then results in the planned and, ultimately, the enacted curriculum. Figure 1 (see page 4) depicts this conceptualization.

Previous studies have investigated particular factors found within Remillard's (2005) theoretical model in order to determine the affect they may have on how teachers and curricula interact. Of particular interest to the current conversation is the increasing focus on studying technology-enhanced instructional materials. Investigations of teachers' interactions with specific technologies, as well as the factors that impact these interactions, continue to enrich the mathematics education community's understanding of the participatory relationship between teachers and technology enhanced curricular materials (for example, see Windschitl & Sahl, 2002). The development of the digital curricula and technology enhanced curricular materials far outpaces the rate at which they can be systematically studied, however.

Consider, for example, the focal technology for this study: interactive diagrams (IDs). "An ID is a relatively small and simple software application (applet) built around a pre-constructed example" (Naftaliev & Yerushalmy, 2013, p. 62). IDs are often found within digital interactive mathematics textbooks, but may be used as a standalone curricular resource. IDs may hold great potential for the teaching and learning of mathematics as their design can specifically target different educational purposes by using varied functionalities and representations of mathematical ideas. Yerushalmy and colleagues (Naftaliev & Yerushalmy, 2011, 2012, 2013; Yerushalmy, 2005) have extensively described the design of IDs and are beginning to study how students learn using IDs. Teachers' interactions with IDs are "largely unexplored" (Naftaliev & Yerushalmy, 2009, p. 3), however.

This study aimed to add to the body of literature pertaining to teachers' interactions with digital curriculum by focusing specifically on IDs and the factors that influenced how teachers use them within their practice. IDs were selected as the focus of this study due to their relative novelty and increasing prominence in online textbooks and other web-based curricular resources (Naftaliev & Yerushalmy, 2013). My hope is that the study of this particular technology will add to and enrich the larger body of literature pertaining to the teaching and learning of mathematics using digital technologies and curriculum more generally.

Theoretical Perspective

The current investigation aimed to examine teachers' instructional interactions with IDs, as well as the factors that influence these interactions. Similar to Remillard (2005), Stein and her colleagues (Henningsen & Stein, 1997; Stein, Engle, Smith, & Hughes, 2008; Stein, Grover, & Henningsen, 1996) and others, this study asserts that various factors originating from myriad sources affect the ways in which curricula is enacted in classrooms; various aspects of the curriculum, teacher, classroom environment, students, school, district, and other contextual elements may influence how curriculum is used within the mathematics classroom. Put another way, teachers' "knowledge [is] distributed among people and their environments, including the objects, artifacts, tools [such as technology], books, and the communities of which they are part" (Greeno, Collins, & Resnick, 1996, p. 17). As such, this study assumes a situative perspective (Greeno, Collins, & Resnick; Putnam & Borko, 2000) where cognition is distributed (Hutchins, 1999; Jenkins, 2007) among individuals, tools, and community. Consequently, teachers' ID uses were viewed within the particular ecosystems where they occurred and not as isolated occurrences.

This study conceptualized IDs as particular examples or instances of curricular activities and, in doing so, aimed to extend the work of those studying teacher-curriculum interactions. Consistent with this, Remillard's (2005, p. 235) conceptual model of the participatory relationship between teachers and curriculum was used to ground the current investigation (see Figure 1). The framework described by Remillard depicts the interactions of teacher, curricular, student, and contextual factors as well as their potential influence on the creation of the planned and enacted curriculum. Embedding teacher-curriculum interactions within a multi-dimensional system highlights the situative (Greeno, Collins, & Resnick, 1996; Putnam & Borko, 2000) nature of the teaching act.

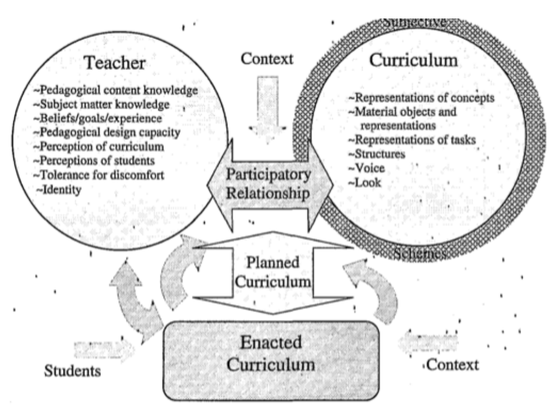


Figure 1- Factors influencing curriculum enactment.

Also consistent with the situative perspective, this study acknowledged the effect that particular aspects of curricula may have on the ways that teachers interact with them. Indeed, the factors named in the "curriculum" circle may influence whether or not, and how, a teacher interacts with a particular curricular resource. For example, IDs may be categorized according to their presentational, orientational and organizational functions (Yerushalmy, 2005) each of which have three, three, and two functions, respectively. This theoretically creates the potential of eighteen distinct types of IDs, each with its own particular design and purpose. The function combinations may play a particular role in the participatory relationship between teachers and IDs. Gaining insight into the role ID characteristics play as teachers use them was a component of the current study. Taken together, this study conceptualized IDs and curriculum more generally as holding agency during their interactions with teachers. For this reason the framework proposed by Remillard (2005) was purposefully selected to draw explicit attention to the bi-directional, participatory relationship that exists between teachers and curriculum. This interaction is depicted in the conceptual framework shown in Figure 1 as the double arrow between the teacher and the curriculum.

Research Questions

Investigating teachers' uses of technology-enhanced curricula is a growing area of research within the mathematics education community. The interest in this area is not enough to keep up with the seemingly exponential rate of development and availability of such educational resources, however. It may not be surprising, then, that upon a review of the literature it was apparent that studies detailing teachers' interactions with IDs were absent even given the explicit call for such research (Yerushalmy, 2005). My particular interest in IDs over other potentially interesting technologies was twofold. First, the literature describing ID design (see Naftaliev & Yerushalmy, 2011, 2012, 2013; Yerushalmy, 2005) was rich with theoretical notions from which this study may be built upon, extend, and challenge. Consequently, pertinent findings from the ID design literature are described in the framing of the current study. Secondly, IDs are increasingly found within curricula that are being used by school systems across our country. The increased use of IDs to teach mathematics warrants further investigation of this technology and its place within the schoolhouse so that its use may be more fully understood.

Further, research suggests that characteristics pertaining to teachers, curriculum, students, and the school context influence enacted classroom instruction. Understanding the relationship between these variables is important so that organizational, professional development, curriculum, and teacher training decisions can be strategically made as we move toward desirable mathematics instruction. This study aimed to investigate the interactions of these factors specifically focused on teachers' use of IDs. In doing so, this exploratory study investigated the following research questions:

1) How do three secondary mathematics teachers use interactive diagrams (IDs) within their instructional practice?
    a. How do teachers create the planned curriculum involving IDs?
    b. How do teachers enact curriculum involving IDs?
2) What factors mediate three teachers' instructional uses of interactive diagrams (IDs)?
    a. How are teachers' uses of IDs affected by the underlying design of the IDs?

b. How do teacher factors, such as their instructional beliefs, affect their use of IDs?
c. How do factors surrounding the school context, including the students within that context, influence teachers' instructional uses of IDs?

Investigating the teacher-curriculum relationship was in response to the body of literature suggesting the significant impact teacher instruction has on student learning (Cohen & Hill, 2001; Gallagher, 2004; Heneman, Milanowski, Kimball, & Odden, 2006).

Significance and Contributions

This study holds a number of potential benefits for mathematics education research and the area of inquiry concerned with the participatory relationship between teachers and curriculum. First, this study aimed to investigate teachers' interactions with IDs and factors that affect this interaction which, in my review of the literature, were unaddressed. Investigating these phenomena is important due to the increasing presence of IDs in mathematics curriculum (Naftaliev & Yerushalmy, 2009) and the well-documented effect teacher-curriculum interaction has on the enacted curriculum (Stein, Grover, & Henningsen, 1996). It was also aligned with Remillard's (2005) recommendation for research to study particular aspects of the participatory relationship between teachers and curriculum, particularly those curricular materials whose format and content are foreign to teachers, such as IDs. Understanding teachers' interactions with IDs, and the factors that affect these interactions, may inform future directions for research as well as practice, including district policy, professional development, and teacher training.

Additionally, studying teachers' use of IDs continued research that examined the use of other technology-enhanced curricular resources. Researchers have discussed pedagogical uses of computers (Cuban, 2001), ubiquitous laptop programs (Windschitl & Sahl, 2002), interactive and dynamic mathematics software (Stols & Kriek, 2011), gaming environments (Squire & Jenkins, 2003), and mobile technologies (Rogers, Price, Randell, Fraser, Weal, & Fitzpatrick, 2005). The present study aimed to extend and enrich this body of literature through the examination of a related technology. Such an investigation was important since "technology…only [becomes] a learning medium when supported by appropriate teacher intervention and tasks" (Kieran, 2007, p. 737).

Lastly, this study aimed to enrich the field's understanding of interactive diagrams. Others have studied their design (Yerushalmy, 2005; Naftaliev & Yerushalmy, 2012) and students' interactions with IDs (Naftaliev & Yerushalmy, 2009). Teachers' use of IDs, however, was not addressed even in the presence of explicit calls for such research (Naftaliev & Yerushalmy, 2012). Thus, the new understandings and questions that the present study raises begin to fill in the third absent piece of the teacher-curriculum-student relationship. My hope was that this would be a first step in coming to an integrated understanding of the relationship between teachers, students, and interactive diagrams.

Overview of the Document

Chapter 2 of this document provides the reader with a context for understanding teachers' interactions with curricular materials, generally, and digital technologies more specifically. Included in this discussion are factors that were identified as particularly pertinent in the enactment of curriculum, as well as a description of IDs. The chapter concludes with a discussion of the conceptual framework used for this study.

Chapter 3 describes the study methodology. This includes a discussion of the context, data collection techniques, study time frame, and a detailed description of the study's data sources. It concludes with an explanation of the data analysis methods employed within the study.

Chapters 4, 5, and 6 detail each participant's ID interactions in turn. Each chapter characterizes how each participant interacted with IDs during the planned and implemented curriculum, as well as the teacher, ID, and contextual factors that affected these interactions. In particular, each participant's underlying goals, the classroom structures they used, students' roles, and their own role as the teacher during each ID activity are described. Each chapter concludes with a summary highlighting the salient components of each participant's participatory relationship with IDs.

Chapter 7 provides a cross case analysis and, consequently, the study findings pertaining to the participatory relationship between teachers and ID. This includes a discussion of those factors that most significantly influenced teachers' ID implementation and the ways in which participants incorporated these factors into their conception of their ID use.

Finally, Chapter 8 discusses the implications of the study findings and presents conclusionary remarks. Limitations and additional directions for future research suggested by the present study are also described.

Chapter 2

This section reviews the research literature relevant to the present study. In particular, the participatory relationship between teachers and curriculum is discussed, including factors that affect this relationship. Then, teachers' use of curriculum is narrowed to digital curricular resources. Lastly, the focal technology for this study, interactive diagrams, is described.

Teacher-Curriculum Interaction

The distinction described by Remillard (2005) between the written, planned, and enacted curriculum points to a number of factors that may alter the content, format, and intent of curricular materials. Figure 1 identifies teacher, curricular, student and contextual factors that may alter curriculum as it moves from the written curriculum to enacted classroom activities. Indeed, a variety of studies demonstrate that teachers seldom implement curriculum as curriculum designers intend (Drake & Sherin, 2006; Stein, Remillard, & Smith, 2007). Stein and colleagues (Henningsen & Stein, 1997; Stein, Engle, Smith, & Hughes, 2008; Stein, Grover, & Henningsen, 1996), for example, examined the factors that mediated the implementation of mathematics tasks. Henningsen and Stein found that the tasks' alignment with students' prior knowledge, teachers' scaffolding, the amount of time dedicated to the task, the teachers' modeling of high-level performance, and the sustained press for explanation and meaning influenced the enacted task. These factors may enrich, maintain, or dilute the intended mathematical rigor of tasks as they move from the written to intended, and ultimately, to the enacted curriculum. Others (Sherin & Drake, 2009) also found that teachers make "significant changes […] in the intended curriculum such as changes in the structure of a lesson, in the activities that comprise the lesson, or in the purpose of the lesson" (p. 30). This included adding, omitting, and modifying particular examples, tasks, and materials in the written curriculum.

Findings such as these have prompted a number of theoretical models for the ways in which curriculum is used (see Remillard, 1999, 2005 [Figure 1 above]; Stein, Grover, & Henningsen, 1996). Each model represents specific underlying assumptions and theoretical perspectives influencing its conception of curriculum use, including following or subverting, drawing on, interpreting, or participating with the text (Remillard, 2005, p. 217). One central theme among these models, though, is the centrality of the teacher. As Remillard (2005) notes,

> Many studies from varied perspectives have pointed to the active and interactive nature of teachers' work when shaping the enacted curriculum, indicating that teaching is a responsive and improvisational activity that cannot be scripted. […] It is actually a highly interactive and multifaceted activity, rather than a straightforward process as may be assumed. (p. 234)

Indeed, written curriculum "[…] are representations of abstract concepts and dynamic activities […] not the activity itself [as they] rely heavily on [teacher] interpretation" (Brown, 2010, p. 21). Simply stated, teachers heavily mediate the enactment of curriculum. Thus, in order to understand how curriculum of any kind is enacted in classrooms one must investigate how teachers interact with that curriculum, and the factors that influence this interaction.

Teacher characteristics influencing curriculum enactment. Teachers play an active role in the translation of written curricular materials into classroom activities; the written curriculum does not transfer directly through the teacher into the classroom unchanged (Brown, 2010; Clandinin, & Connelly, 1992). Instead, teachers " […] alter, adapt, or translate [curriculum] offerings to make them appropriate for their students" (Remillard, 2005, p. 224). What, though, influences how teachers alter curricular materials?

Studies of the relationship between teachers and curricular materials point to a number of factors that affect teacher's use of curricula. One set of factors, namely teacher characteristics, is particularly well documented. First, teachers' knowledge of mathematics content (Lloyd & Wilson, 1998), pedagogy, and student learning (Shulman, 1987) is suggested to mediate teacher use of curriculum. Indeed, "what a teacher knows is [… an] important influence on what is done in classrooms and ultimately on what students learn" (Fennema & Frank, 1992, p. 147). Teachers' professional identities, such as their sense of being a teacher (Drake & Sherin, 2006) or a classroom authority (Wilson & Lloyd, 2000), also play a part in the teacher-curriculum relationship. Additionally, teachers' perceived or actual positionality in relation to curriculum may affect their interaction. For example, Remillard and Bryans (2004) found " […] that teachers had orientations toward using curriculum materials that influenced the way they used them regardless of whether they agree with the mathematical vision within the materials" (p. 352). Teacher perceptions of curriculum may include it as authoritative (Romberg, 1997) or inflexible (Avez-Lopez, 2003). Further, teachers' perceptions of students' capacity may influence their use of curriculum. This is particularly true when teachers consider standards-based curriculum in light of their students' needed areas of growth (Collopy, 2003).

Lastly, it is well established that teacher beliefs heavily influence their practice (Aguirre & Speer, 1999; Calderhead, 1996; Cohen, 1990; Ernest, 1989; Thompson, 1992). This may be in part due to the fact that teachers often treat their beliefs as knowledge (Thompson, 1992). For example, the validity of learning styles frameworks is debated within educational literature (for example, Cuevas, 2015; Willingham, Hughes, & Dobolyi, 2015). Teachers may believe in the importance of attending to students' learning styles, however, and may incorporate this belief as a component of their pedagogical knowledge. Consequently, teachers may attend to students' perceived learning modalities even though such a construct is not confirmed by research nor has any impact of attending to this factor been established. Further, teachers' content knowledge and components of their beliefs are linked (Cooney & Wilson, 1993; Fennema & Franke, 1992). As Lloyd and Wilson (1998) point out, "[…] it appears that many teachers do not separate their [beliefs] about a particular topic from notions about how to teach that topic" (p. 250). Taken together, it is clear that "teachers matter in the curriculum-use equation" (Remillard, 2005, p. 229).

Teacher characteristics influencing digital curriculum enactment. The current study seeks to investigate teacher-curriculum interactions by focusing on a particular type of curriculum, namely those curricula that are enhanced through digital technologies. Consistent with investigations of the teacher-curriculum relationship more generally, teacher, curriculum, contextual, and student factors all influence teachers' interactions with technology-enhanced curriculum (Bate, 2010; Ertmer, 2005). Of note, however, is the addition of two influential factors that are particular to the use of technology-enhanced curriculum: teachers' technological knowledge and the presence of infrastructure/digital equipment.

Examining technological pedagogical content knowledge extends the work of Shulman (1986) who first discussed the intersection of teachers' content and pedagogical understandings. Clearly, digital technologies can significantly impact how content can be represented through digitally enhanced illustrations, examples, demonstrations, and explanations. Thus, "knowledge of technology [and how to use it] becomes an important aspect of overall teacher knowledge" (Mishra & Koehler, 2006) and can significantly impact the interactions teachers have with technologies.

The degree to which digital infrastructure and equipment is present in the educational environment can also impact the interactions teachers have with educational technology. Teachers' interactions with digital technology may be quite limited, for example, if their schools and/or classrooms do not have appropriate equipment available. A similar phenomenon may occur if technology is present, but is unreliable. On the other hand, a teacher may have more numerous interactions with technology if his or her classroom is equipped with abundant and reliable hardware and software. These seemingly intuitive relationships may not exist reliably, however. A particularly enthusiastic teacher may be able to overcome a dearth of technology by acquiring needed equipment and training through creative channels while a well resourced teacher may elect to not utilize their available technology (Cuban, 2003). Cuban's work in this area points to the complex web of factors that may affect teachers' use of technologies.

Taken together, teachers' interactions with digitally enhanced curriculum largely parallel the interactions they have with traditional paper-based curriculum. Teachers' technological pedagogical knowledge and particular aspects of the context in which they teach (i.e. the presence of infrastructure and digital equipment) are additional factors affecting teachers' interactions with digital curriculum, however. Bate (2010, p.58) depicts this relationship using the bridge metaphor depicted in Figure 2.

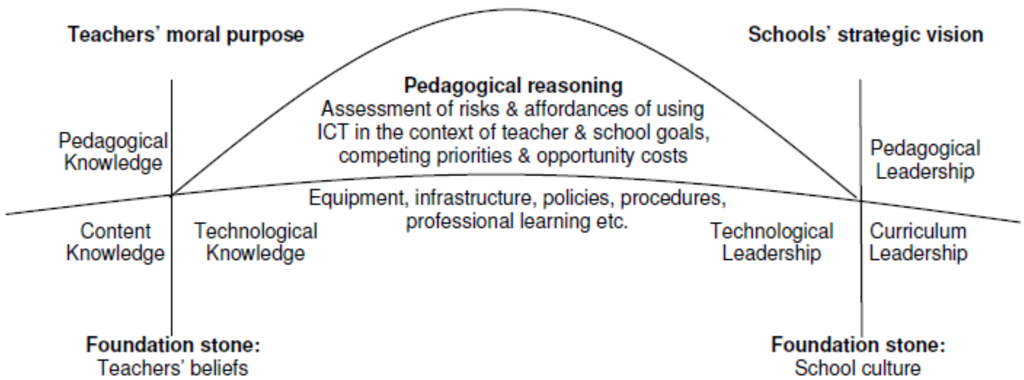


Figure 2- Teacher-ICT (information and communication technologies) interaction expressed as a bridge between teacher and school.

Much like the conceptualization proposed by Remillard (2005), Bate (2010) accounts for teacher, contextual, and technology factors when depicting the participatory relationship between teachers and digital curriculum technologies. In addition, explicit attention is given to teachers' and schools' technological knowledge and leadership, respectively, as well as the availability of digital equipment and infrastructure.

Technology Enhanced Curricular Materials

Digital educational resources have become increasingly present in the teaching and learning of mathematics. Graphing calculators, Computer Algebra Systems (CAS), interactive and dynamic Geometry software (i.e. GeoGebra), apps, video simulations, and online curricula are not uncommon in today's mathematics classrooms. This is likely due, on the one hand, to the increased affordability and widespread availability of these resources and, on the other hand, to the national (National Council of Teachers of

Mathematics, 2000) and regional (Maryland State Department of Education, 2002) calls for the increased integration of technologies appropriate for 21st century learning. Consistent with these recommendations, the Common Core State Standards (NGACBP, 2012) emphasizes the importance of "use[ing] appropriate tools […such as] a calculator, […] a computer algebra system, […] or dynamic geometry software" (p.7) in the teaching and learning of mathematics. It is paramount that the use of these increasingly sophisticated and prevalent technological tools is understood so that their potential benefits can be effectively leveraged to further mathematics achievement.

Technology enhanced curricular materials are professed to have a number of potential benefits in the teaching and learning of mathematics. One such benefit lies in the theory of distributed cognition from the field of cognitive science. Hutchins (1999) describes how an environment consisting of an individual and tools may be thought of as a cognitive system. The tools within this system may then supplement and extend the cognitive capabilities of the individual, distributing the cognitive load between the individual and tools. Put another way, "work in distributed cognition focuses on forms of reasoning which would not be possible without the presence of artifacts or information appliances [such as technologies], which expand and augment human's cognitive capacities" (Jenkins, 2007, p. 106).

Through the lens of distributed cognition one can see how students may not be able to cognitively focus on and/or process important underlying mathematical concepts present in a classroom activity if their attention were occupied on performing more procedural steps necessary to complete the task. While exploring how slope is represented in the table, graph, and equation of a given linear function, for example, the instantaneous production of these representations within a technology rich environment frees the user of the cognitive burden of creating the representations themselves. The technological tool allows students' cognitive energies to focus on the intent of the exploration- understanding the concept of slope and the variety of ways that it can be represented. Computer Algebra Systems (CAS) provide a similar function by eliminating the cognitive work required by symbolic manipulations. This can be helpful, for example, when students are investigating different methods for algebraically solving a given equation for a particular term. Hahkioneimi and Leppaaho (2012) emphasize this point, stating that it "[…] makes it possible to try different kinds of solution methods which would be too inconvenient with paper and pencil. Thus, […] students [are encouraged] to try out multiple ideas as well as to make conjectures to test them" (p. 26). Quickly making and testing conjectures in this example could be too inconvenient and susceptible to algebraic errors without the assistance of CAS.

It is also suggested that the instant feedback enabled by many technologies encourages "play" in the problem-solving process (Jenkins, 2007). Consider, for example, that a student is using a graphing calculator or dynamic mathematics software to determine the effect of changing a quadratic equation's coefficients. The student may elect to "play" with the function by repeatedly substituting larger and more inconvenient coefficients (in terms of one's ease of handling the coefficients in a traditional educational environment) and observe the resulting graphical representation. "Part of what makes play valuable as a mode of problem-solving and learning is that it lowers the emotional stakes of failing: players are encouraged […] to take risks and learn through trial and error" (Jenkins, 2007, p. 100). Using trial and error encourages participants to explore conjectures in a low risk technological environment. Further, the use of technology to interactively link the multiple representations of functions, such as in the previous example, has been suggested to increase functional understandings (Havelková, 2013; Pierce, Stacey, Wander, & Ball, 2011; Swartz & Yerushalmy, 1992). Dynamically linked representations are so beneficial that some (Schwartz & Yerushalmy) have recommended that students always learn about functions in such an environment.

Interactive Diagrams

Interactive diagrams were the focal technology for the present study. "An ID is a relatively small and simple software application (app) built around a pre-constructed example" (Naftaliev & Yerushalmy, 2013). IDs provide representations that students find meaningful as physical objects even though those objects are being viewed and manipulated on a computer screen (Yerushalmy, 2005). It is suggested that such computer representations are more beneficial than their physical counterparts because they are more manageable, flexible, available, and free of potentially distracting features (Brown, McNeil & Glenberg, 2009; Sarama & Clements, 2009; Naftaliev & Yerushalmy, 2011).

IDs were selected as the focus of this study due to their increasing prominence in online textbooks and other web-based curricular resources (Naftaliev & Yerushalmy, 2013), which are also becoming ever more abundant in mathematics education. IDs' increased presence, relative novelty, and relative complexity to paper diagrams (Naftaliev & Yerushalmy, 2012) warrant their purposeful study and the investigation of how they are used in the teaching and learning of mathematics. Others (Naftaliev & Yerushalmy, 2011, 2012, 2013; Yerushalmy, 2005) have discussed the design of IDs and student uses of this digital technology. This study aims to add to the field's understandings of IDs by exploring teachers' instructional use of this technology and the factors that affect this use, an area previously unaddressed in the literature.

Interactive diagrams have a variety of designs and potential uses, which can be classified by their presentational, orientational and organizational functions (Yerushalmy, 2005). Table 1 briefly describes this categorization of IDs' three functions.

Table 1
Functions of IDs

Organizational Function	**Presentational Function**	**Orientational Function**
Degree of interdependence of the text and ID	What example is being illustrated and how it is being illustrated	The tone used to communication between the ID/text and the user, "sketchiness" being an important aspect
Illustrating/Complementing – ID provides a complementary representation of an example introduced or presented in the text, ID complements text Narrating – ID acts as the primary representation of a mathematical example and highlights important features and ideas, text complements ID Elaborating – ID designed to motivate critical thinking and meta-level reflection moving from specific examples to generalization of mathematical ideas, ID extends text	Random – ID produces random or pseudo-random examples within given parameters Generic – ID illustrates the general context of the example found in the text, but can be manipulated to create other examples with similar features Specific – ID presents exact data (often found in the text) as part of the example	Neat diagram- ID emphasizes quantitative features of the example Sketchy- ID emphasizes qualitative features of the example, qualitative features enable "suggestive subtlety"

adapted from Yerushalmy (2005).

Each of the different functions found in Table 1 may influence a user's interaction and conception of a particular ID. Combinations of these functions, for example, may make the ID more or less "open" (Yerushalmy, 2005) as the user views an example or a number of examples, modifies the example(s), or authors their own example(s). A discussion of these functions follows.

Organizational function of interactive diagrams. The organizational function of an ID refers to the connectedness that exists between the components of the text and those of the ID. Put another way, IDs can be classified according to the intended participatory relationship between the text and the diagram itself. In an illustrating diagram, for example, the diagram is meant to illustrate or provide a representation of the example set forth from the text (Yerushalmy, 2005). The illustrative ID intends to visualize objects described in a text by having the user view or manipulate a simple representation of those objects or examples. In doing so, the illustrative ID typically does not include information that is in addition to what is found in the text, but provides another way of interacting with that information (Yerushalmy). Further, the features available to the user in an illustrating diagram are often quite limited since this type of ID is not typically meant for content exploration but "[…] are designed to involve the readers' intuitions in the first stages of reading and to encourage the reader to experience the mathematical actions already defined by the text" (Yerushalmy, p.231).

Narrating interactive diagrams function in quite the opposite manner to illustrative IDs; illustrative IDs complement the activity of the text whereas narrating IDs function as the principle method of communication with the text acting as a support for the ID (Yerushalmy & Naftaliev, 2011). Here, the text may explain the content and/or functions that are present in the diagram, but it does not describe the intended learning outcome or skill embodied in the ID. The intention of a narrating ID is often for the user to interact with the ID through a "support[ed] autonomous guided inquiry […] of open-ended exploration with specific content and [task objectives]" (Yerushalmy, 2005, p. 231). In order to encourage such guided inquiry the narrative ID often includes more numerous linked representations and more available features than are typically present in an illustrating ID.

Finally, elaborating interactive diagrams are, as Yerushalmy (2005) describes, "integrated with the text of the task to such an extent that instructions often direct the reader to use the diagram to reach certain objectives" (p. 233). Elaborative IDs are meant to extend the text's content by providing students opportunities to engage in a number of related examples and fostering meta-level reflection on the processes with which they engaged with the ID. This reflection, then, aims to enable students to create heuristics and generalizations for the explored mathematics content (Yerushalmy, 2005). In order to do this, elaborative IDs often have more representations, related examples, and tools present for the user than narrative and, especially, illustrative diagrams. Students are meant to construct knowledge through actively participating with the elaborative ID. This is important, Foster (2006) explains, since for "students [to] construct meaning [from tools, such as IDs…] this requires more than watching demonstrations; it requires working with tools […] trying them out, and watching what happens. Meaning does not reside in tools; it is constructed by students as they use tools" (p. 1).

Presentational function of interactive diagrams. The presentational function of an interactive diagram "relates to what is being illustrated by the diagram, and how it is being illustrated" (Yerushalmy, 2005, p. 234). The particularity or generality of the example being depicted is a design feature that is important to consider since mathematical visualizations, such as IDs, hold a number of inherent obstacles in their use.

Yerushalmy and Chazan (1990) categorize these as 1) the particularity of the diagram, 2) the challenge of seeing a diagram in different ways, and 3) the perception of standard diagrams as models. As such, the design and use of an ID can be carefully considered in terms of its presentation function.

First, an interactive diagram may function as a random example. Here, a seemingly random example is presented to the user. This example is often created within certain parameters to align with a given task. Random examples IDs are typically used to introduce a concept and are often complementary to the text (i.e. illustrating diagrams) (Yerushalmy, 2005). A feature of a random example is that the "randomization is assumed to create in the reader's mind a sense of an infinite number of cases spanning the entire domain of possibilities" (Yerushalmy, p. 235, p. 235).

An ID may also present a specific example, or an example displaying the precise information present in the text. Here, users may need to input additional information or manipulate the information already present using a set a given tools. As such, specific examples are often found in illustrative and narrative diagrams since "they either illustrate specific data described by the text or are the substance of the task and dominate the text" (Yerushalmy, 2005, p. 235).

Lastly, an interactive diagram may depict a generic example. Generic examples are related to the task so that they are seen as relevant, but do not require particular data or information from the task; "[it] illustrates the context of the text and it [… also] invites the reader to generate other examples that share the common features of the diagram" (Yerushalmy, 2005, p. 236). The user is given an initial example and asked to change it using well-defined parameters. A number of representations are often present when using a generic example so that the consequences of the user's actions may be observed across representations.

Orientational function of interactive diagrams. The last function of interactive diagrams, the orientational function, refers to the tone the text is attempting to use when communicating with the user. Yerushalmy (2005) identifies the degree to which a diagram is "sketchy" or "neat" as a factor that communicates this tone. A sketchy diagram is one that emphasizes the qualitative characteristics of the example. Its' relative generality is meant to highlight important features of the example while reducing unnecessary information that may distract the learner (Naftaliev & Yerushalmy, 2012). A neat diagram, on the contrary, includes more specificity since quantitative features of the example are present, such as measurements, coordinates, or explicit function names. An important note here is that an ID may function as both a sketch and a neat diagram since the functionality of the ID may reveal or repress each quality when it is called upon to do so by the user.

In summary, IDs may be classified according to their presentational, orientational and organizational functions. Using this categorization, one may theorize that eighteen different combinations of these functions are possible (three presentational, three organizational, two orientational). As alluded to earlier, though, particular function combinations are often more harmonious than others. Thus, while eighteen combinations are possible, fewer are prevalent. This study aimed to examine teachers' interactions with those more prevalent combinations.

Conceptual Framework

The research questions for this study addressed the ways in which teachers used IDs within their intended and enacted curriculum, as well as the factors that affect this use. Remillard (2005) (see Figure 1, p. 4) describes teachers' participatory relationship with curriculum and the multiple factors affecting that relationship. As such, Remillard's conceptual framework was selected to guide this study for three primary reasons, which are described in turn below.

First, the conceptual framework described by Remillard (2005) allowed for a structured yet open exploration of teachers' ID use. This study acknowledged that curricula may or may not change as teachers initially plan and then ultimately enact instructional activities within their classrooms. Any number of factors may affect if and how curriculum are altered between these two stages. Remillard provided a general structure to focus the study of teacher ID use by conceptualizing the intended and enacted curriculum as separate but related components. Additionally, this framework organized the factors affecting the intended and enacted curriculum within four major categories (i.e. teacher, student, classroom, and context) and provided examples of salient factors within each. By doing so, Remillard's conception of teacher-curriculum interaction was useful in identifying the broad categories of curriculum and factors present throughout the study. That said, Remillard's framework was also open in that it contained sufficient flexibility to explore each stage of curriculum and the factors that affected the participants' ID use within each of the identified major categories.

Additionally, this study was specifically concerned with investigating teachers' ID use. Other conceptualizations of teacher-curriculum interaction include curriculum as it is designed (Stein, Grover & Henningsen, 1996) and student learning as a result of curriculum enactment (Tarr, Reys, Reys, Chavez, Shih & Osterlind, 2008). While these components are certainly important, they were beyond the scope of the current investigation. Remillard's (2005) framework specifically targeted teachers' interactions with curriculum and, thus, was well aligned with the current study.

Lastly, the framework described by Remillard (2005) emphasized the participatory relationship shared by teachers and curriculum. The current investigation recognizes the bi-directional influence teachers and curriculum share. Indeed, each interacts with the other as the intended and enacted curricula are created. The terminology used within Remillard's framework, that teachers and curriculum share a participatory relationship, emphasizes these interactions. The perspective that this participatory relationship exists within a complex, multifaceted environment was also central to the current study. Thus, Remillard's conceptualization of teacher-curriculum interaction occurring within a system rich with multiple and varied mediating factors was well aligned with the situative nature of teaching (Greeno, Collins, & Resnick, 1996, Putnam & Borko, 2000) assumed within this investigation.

The conceptual framework described by Remillard (2005) was tailored for the current study by focusing on the factors specifically pertinent to teachers' use of technological resources, as described by Bates (2010). Figure 3 depicts this study's conceptual model as a combination of the framework described by Remillard (2005) and the factors noted by Bates (2010). The narrowing of Remillard's framework was consistent with recommendations for further investigation of the teacher-curriculum relationship by investigating specific teacher characteristics and features of curricular materials (Remillard, 2005). While the study's initial conceptual framework populated factors within each of the categories of the framework, it was also open to capturing and analyzing other possible factors within each of these categories, as is desirable in an exploratory study.

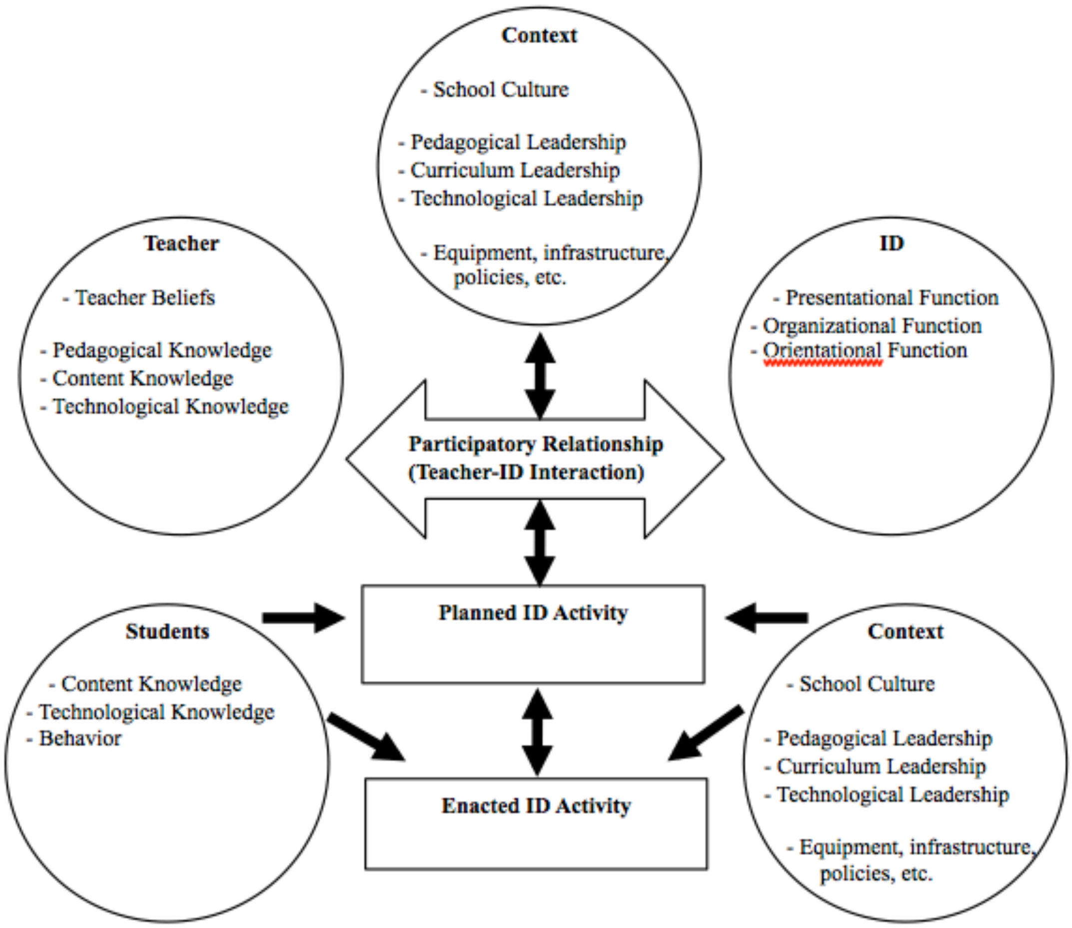


Figure 3- Conceptual framework for study- Teacher uses of IDs and factors affecting it.

Chapter 3

This chapter describes the design of the current study. In doing so, the study setting and the methodology is explained. This includes a description of the data sources and analysis techniques that was used within this study. The chapter concludes by addressing issues of validity and reliability.

Design Setting

This project studied teacher practice within a large urban district in the eastern United States. This school system, which I will refer to using the pseudonym Doylestown School System (DSS), included nearly 200 schools and served approximately 85,000 students. Of those students, approximately 80% were African American and nearly 85% qualified for free and reduced lunch.

This study context was unique in that DSS had fully adopted the use of Agile Mind, an online curriculum that relied heavily on the use of IDs, as its primary curricular resource for secondary mathematics. DSS's use of Agile Mind began during the 2012-2013 school year with the implementation of Agile Mind courses in grades six through Algebra I (i.e. ninth grade). The Agile Mind Geometry and Algebra II courses were added during the 2013-2014 school year as the primary curricular resources for grade 10 and 11 teachers, respectively. Finally, during the 2014-2015 school year, the Agile Mind Statistics course was made available for optional use by teachers. The use of Agile Mind was mandated by DSS for all secondary mathematics teachers for grades six through Algebra II and optional for teachers of statistics. Consequently, the DSS provided a unique context in which teachers' uses of IDs and the factors that affected their uses could be studied. Data collection occurred during the 2015-2016 school year.

Agile Mind. Agile Mind is an online mathematics curriculum with courses ranging from grades six through Calculus AB. This curriculum was created through a collaborative effort between the Charles A. Dana Center at The University of Texas at Austin and the Agile Mind Company that began in 2001. The Agile Mind curriculum is currently being used in school systems across the U.S., particularly those in urban centers.

Numerous curricular materials and resources are provided to teachers and students within Agile Mind's online platform, including content slides, tasks and activities, IDs, teacher planning materials, scope and sequences, standard alignments, assessments, training videos, and research justifying the curriculum. Remillard (2005) reminds us that both the totality of these materials and the particular components within the larger whole may be thought of as curriculum. For the purpose of this study, the finer grain-sized view of curriculum was used to conceptualize particular IDs as instances of curriculum.

Study participants. Focusing on IDs pertaining to equations and functions narrowed the present investigation of teachers' IDs use. This content was selected due to its central importance in the study of mathematics (Dubinsky & Harrel, 1992), mathematical complexity (Chazan & Yerushalmy, 2003), and emphasis in the CCSS-M (NGACBP, 2012). By doing so, the range of possible study participants was limited to those instructors who teach the equation and function concepts most predominantly, namely 7th grade, 8th grade, Algebra I, and Algebra II teachers.

Participation in this study was solicited from approximately 50 instructors teaching the aforementioned courses within approximately 15 DSS schools. I had previously worked with the mathematics teachers within these 15 schools as an instructional coach and district mathematics representative. Thus, I had established relationships with the potential participant pool, which afforded immediate trust and transparency during the data collection process. Importantly, though, participation was only solicited from teachers with whom I no longer in their schools in order to lesson the presence of coercion.

It was the study's intent to strategically select participants that demonstrated a variety of ID uses. This could then facilitate the collection of data useful in comparing and contrasting the research cases and gaining an understanding of the factors that surround teachers' varied interactions with the focal technology. Thus, my previous understandings of the 8 volunteers' ID use were considered when Ms. Edelman, Mr. Clark, and Ms. Allen were selected to participate in the current study. Namely, these three individuals had previously demonstrated ID use that differed from each other. Further, Ms. Edelman, Mr. Clark, and Ms. Allen were selected to represent the range of demographic characteristics and teaching experiences (see Table 2) present in the study context.

Table 2

Participant Demographic Information and Teaching Experience

Participant Pseudonym	Gender	Race/ Ethnicity	Certification Track	Courses Taught During Study	Total Years Teaching IA	Total Years Teaching with Agile Mind	Total Years of Teaching Experience
Adam Clark	Male	White/ Caucasian	Alternative, Alumnus of Program	IA	4	4	4
Fatima Allen	Female	African American	Traditional	IA, Honors Algebra I	1	4	17
Natalie Edelman	Female	White/ Caucasian	Alternative, Currently in Program	IA, Algebra II	2	2	2

This purposeful selection of study participants was conducted so that the diverse characteristics present across DSS's teachers were included within the study population, allowing for greater generalizability of the study findings. Of note was that all study participants taught a course called Intensified Algebra (IA). This occurrence was used as an opportunity to control the course variable in the participants' interactions with IDs. Thus, data collected during this study focused specifically on teachers' ID use pertaining to the IA course. Data pertaining to other courses participants taught was not excluded when it arose, however, since this occurred in only a very limited number of instances.

IA is a ninth grade Algebra I course specifically designed by Agile Mind to provide academic support and remediation for students who have struggled to achieve in mathematics prior to ninth grade. Consequently, IA engages students in CCSS-M aligned Algebra I content while also providing targeted remediation reaching as far back as sixth grade content standards. IA was the predominate course taught in ninth grade classrooms within DSS due to the historically low achievement of its students.

Methodology

This study was interested in understanding teachers' uses of IDs and the factors that affected their ID use. To my knowledge, this was a previously un-examined phenomenon. Teachers' use of other technologies has been documented, but their use of IDs specifically has not been studied explicitly. Consequently, this study used an exploratory research design that aimed to understand how teachers interacted with this technology and how teacher, ID, and contextual factors affected these interactions. The ways in which teachers negotiated these interactions, as well as the processes they utilized in their negotiation, were of particular interest. A qualitative methodology was appropriate for such an examination (Maxwell, 2013). Data was collected and analyzed that would enable a rich description and deep understanding (Merriam, 2009) of the phenomenon in question while conducting this qualitative, multiple-case study. It was my hope that the understandings gained in this study would contribute nuance to the field's understanding of the processes, tensions, and opportunities found within teachers' ID use, as well as the factors that affected this use. Of note is that this study's methodology as described below was designed to align with an exogenous (Stevens, 2010) investigation of teachers' practice, that is, the analysis takes place from *outside* the teachers' perspective. The potential benefits of utilizing an endogenous (Stevens) perspective and accompanying methodologies is discussed in chapter seven and eight.

Data Collection

Data collection for this study occurred over a significant part of one full school year. First, each participant engaged in an initial meeting consisting of a pedagogical beliefs questionnaire and interview in order to ascertain information pertinent to the factors that influence their interactions with IDs. The majority of data collection then occurred during targeted lesson cycles where the participants engaged with specific IDs. Data collection consisted of two such data collection cycles for each of the three teachers. Lastly, a final meeting was conducted to gain further insight into teachers' interactions with IDs. The data collection timeline and accompanying data sources for the study are found in Table 3.

Table 3
Data Collection Timeline

Phase of Data Collection	**Time Frame**	**Data Collection Tool**
Initial Meeting	October	Teacher Belief Questionnaire (TBQ) (appendix A) Initial Interview (appendix B)
Lesson Cycle 1	November/December	Pre-observation Interview (appendix C) Observation Protocol (appendix D) Post-observation Interview (appendix E)
Lesson Cycle 2	December/January	Pre-observation Interview Observation Protocol Post-observation Interview
Final Meeting	March	Final Interview (appendix F)

Each of the four phases of data collection is described in turn.

Initial meeting. The intent of the first meeting with the participants was to gain an understanding of their beliefs, the overall context in which they work, and their general interaction with IDs. To begin, participants completed the TBQ, which will be described more fully in the upcoming data sources section. This questionnaire was specifically designed to collect quantitative data pertaining to teachers' beliefs around what mathematics is, as well as how it should be taught and learned. Next, each participant engaged in an initial interview containing questions meant to collect qualitative data surrounding the general context he or she taught within and their general use of IDs. The data collected in the initial meeting was meant to gain large-grain information pertaining to how each participant used IDs and the factors that influenced their ID use.

Lesson cycles 1 and 2. Each of the lesson cycles were meant to collect data that would add specificity to the information gained during the initial meeting. Here, data was collected while participants were engaged with specific IDs of their choosing; each lesson cycle collected data before, during, and after the participants enacted an ID activity within their classroom. The data collection during the two lesson cycles was meant to gain insight into:

- teacher factors, including beliefs and knowledge, that affected teacher-ID interaction
- the specific context in which participants work
- ID factors that affected teacher-ID interaction
- student factors that affected teacher-ID interaction
- teacher use of specific IDs resulting in the planned curriculum, and
- teacher use of specific IDs during the enacted curriculum

A pre-observation interview occurred with each participant shortly before he or she implemented the ID activity. This interview was focused on how the participant planned to enact the ID in their classroom with their students, as well as the teacher, ID, contextual, and student factors that influenced his or her plan. Next, an observation protocol was used to collect classroom data during the actual enactment of the ID. The observation was predetermined by the participants and researcher and was focused on how the ID was actually enacted and the real-time factors that influenced that enactment. Lastly, a post-observation interview occurred where participants reflected on their ID use throughout the lesson cycle and the factors that influenced this use. The participants self-selected the focal ID during each of the two lesson cycles.

Final meeting. The final meeting was designed to accomplish three goals. First, this meeting was used as an opportunity to probe participants about interesting, unclear, and seemingly contradictory data that was collected throughout the study. Such a measure was important in increasing the validity of the study findings. Second, data were collected surrounding participants' thoughts on a wider range of IDs. The two lesson cycles provided in-depth information pertaining to teachers' uses of two particular IDs. In contrast, participants discussed, compared and contrasted a number of IDs during the final meeting. These IDs were all included in the IA course and were known to the participants. The purposefully selected IDs was meant to elicit information on a boarder range of ID designs, as described by Yerushalmy (2005). Participants were shown the IDs used by the other two study participants and asked questions pertaining to their own uses of those IDs. This procedure was used to facilitate a cross-cases analysis of participants' thoughts on the same IDs.

Data Sources

Six distinct data sources were used to collect data during the present study. Four of these data sources (i.e. the pre-observation interview transcript, observation protocol, the post-observation interview transcript, and the TBQ) were collected in two iterations. Including the initial and final interviews, this totals ten rich pieces of data that were collected and analyzed for each participant. Table 4 shows the alignment between these data sources and the specific research questions that were investigated in this study.

Table 4

Alignment Between Research Questions and Data Sources

Research Question	Data Source
1a. How do teachers interact with IDs as they create the planned curriculum?	Transcript of Audio Recording: Pre-observation interviews Transcript of Audio Recording: Initial and final interviews
1b. How do teachers interact with IDs during the implementation of the enacted curriculum?	Observation Protocol Transcript of Audio Recording: Initial and final interviews Transcript of Audio Recording: Post-observation interviews
2a. How are teachers' interactions with IDs affected by the underlying design and intended purposes of the IDs?	Transcript of Audio Recording: Initial and final interviews Transcript of Audio Recording: Pre-observation interviews Observation Protocol Transcript of Audio Recording: Post-observation interviews
2b. How do teacher factors, such as instructional beliefs, affect their interactions with IDs?	Teacher Beliefs Questionnaire Transcript of Audio Recording: Initial and final interviews Transcript of Audio Recording: Pre-observation interviews Observation Protocol Transcript of Audio Recording: Post-observation interviews
2c. How do factors surrounding the school context, including students, influence teachers' instructional interaction with IDs?	Transcript of Audio Recording: Initial and final interviews Transcript of Audio Recording: Pre-observation interviews Observation Protocol Transcript of Audio Recording: Post-observation interviews

Teacher beliefs questionnaire. The purpose of the TBQ (Appendix A) was to measure participants' beliefs pertaining to content and pedagogy, a particular teacher factor found within this study's conceptual framework (see Figure 3). The TBQ was administered during the initial and final meeting. The data collected from the TBQ was then used to create a profile of each participants' belief structures using what Jonassen, Peck and Wilson (1999) identify as the five attributes of meaningful learning- authentic, active, constructive, cooperative and intentional learning. This profile was then used within the analysis of participants' ID uses. If, for example, a participant believed that mathematics should be taught in a highly constructive manner, did this imply that they would use IDs in a more exploratory manner? Questions such as this were considered during data analysis.

The TBQ used in this study was adopted from the questionnaire used by Bate (2010) to categorize teachers' pedagogical beliefs. Using previous work in measuring teacher beliefs (Frid, 2000; Goos & Bennison, 2002, 2007), Bate created a questionnaire containing 40 statements to which participants indicated their level of agreement on a 5-point Likert scale. A portion of the 40 statements was worded negatively in terms of the particular attribute they measured. After piloting the initial survey and analyzing the results using factor analysis, it was found that five items did not load properly, having a load of less than 0.4 (Bate, 2010). The remaining 35 statements loaded to the five categories shown in Table 5.

Table 5
Questionnaire Categories

Scale (learning is…)	**Items**
Active	1-5
Cooperative	6-11
Constructive	12-23
Authentic	24-31
Intentional	32-35

Analysis of Bates' participants' responses to the final questionnaire resulted in a satisfactory level of reliability (Cronbach Alpha of 0.724 and 0.717 for year 1 and year 2, respectively).

Initial interview protocol. The initial meeting with each participant also included an interview using the protocol found in Appendix B, which was audio taped and transcribed. This initial interview was meant to collect data useful in understanding the participants' general uses of IDs, as well as the teacher and contextual factors that affect these uses. This interview protocol was created by Bate (2010) using the work of Goos (2005). Similar to the teacher beliefs questionnaire, this instrument was adapted for the present study by utilizing language focused specifically on IDs.

The initial interview protocol followed a semi-structured format (Bogdan & Biklen, 2007). As Cohen and Manion (1994) note, this format allows the researcher to stay focused on the topic of interest while having a certain level of flexibility to ask participants follow-up questions when it seems beneficial. The questions asked during the initial interview were open-ended, allowing the participants to elaborate on their responses. Each of the interviews described below followed a similar format.

Pre-observation interview protocol. Participants were interviewed using the protocol found in Appendix C to begin each of the two lesson cycles. These interviews were audio taped and transcribed. The focus of the pre-observation interview was to probe the participants' intended instructional use of the ID and the factors that influenced their planning. Participants' understanding of the ID was discussed. Additionally, participants were asked to predict how the ID would be enacted within the classroom.

Observation protocol. Classroom observations were included to collect classroom data pertinent to how teachers used IDs during the enacted curriculum, as well as the teacher, ID, and contextual factors that affect this use. As such, Judson's (2006) Focus on Integrated Technology: Classroom Observation Measurement (FIT:COM) observation tool was adapted for use in this study (see Appendix D). The FIT:COM is a 25-item instrument meant to capture descriptions of technology use across five categories: design of technology integration, class dynamics, meaning and purpose, content and knowledge, and technology as tools. The participants' ID uses were described in each of these categories. Note that the FIT:COM was adapted by altering the language of the instrument to specifically focus on IDs and not technology generally as it was originally designed.

The FIT:COM instrument was used to collect data during each of the six observations. These observations were prearranged with the study participants. The FIT:COM was used to collect qualitative data across the tool's five areas measuring technology integration and the factors that affect this integration. Results from each observation were recorded on the adapted FIT:COM observation protocol instrument.

Post-observation interview protocol. To conclude each of the two lesson cycles, participants were interviewed using the protocol found in Appendix E. This interview was audio taped and transcribed. The focus of the post-observation interview was for participants to reflect upon the enacted ID and the factors that influenced that enactment. In particular, specific observed moments of interest were articulated, and the participant was asked to discuss what occurred during these instances and why it occurred in that way. Additionally, participants were asked to consider how they might implement the ID differently if they were to use it in the future.

Final interview protocol. A final interview concluded data collection for each participant. This interview protocol consisted of five distinct sections (see Appendix F). Section A asked each participant to complete the TBQ again. Section B probed participants' beliefs pertaining to how mathematics should be taught and the teacher's and students' role in mathematics instruction. Section C asked participants to consider a number of IDs, to order those IDs in terms of how much they preferred each for instructional purposes, and to answer questions pertaining to their ordering, the IDs' design and utility, whether or not they used these particular IDs within their classrooms this year, and what that use looked like. The list of IDs and their underlying design functions (Yerushalmy, 2005) are found in Table 6.

Table 6
IDs Used During Final Interview

ID name	**Participant**	**Organizational Function**	**Presentational Function**	**Orientational Function**
Skateboarder ID	NE	Narrating	Specific	Neat
Perpendicular Lines ID	NE	Narrating	Generic	Neat
Slope ID	AC	Illustrating/ Narrating	Generic	Sketchy and Neat aspects
Line of Best Fit ID.	AC	Illustrating	Specific	Neat
Parallel Lines ID	AC	Narrating/ Elaborating	Generic	Neat
Friendship Problem ID	FA	Illustrating	Specific	Sketchy
Paint Mixing ID	FA	Narrating	Specific	Neat

Section D asked participants to categorize how they typically used IDs within their classroom and, if it differed, how they would enact IDs in their ideal classroom. This was done across four dimensions-- activity structure, teacher role, student role, and goal of activity. Lastly, within Section E, participants were asked to discuss the degree to which various teacher, student, ID, and contextual factors influenced their ID use. A semi-structured format was used throughout the final interview allowing the researcher to probe the participants when it was applicable.

Data Analysis

The collected data were analyzed using the study's conceptual framework. Generally, the qualitative data were deductively coded into the following six major categories –Intended use of ID, Enacted use of ID, ID factor, Teacher factor, Student factor, and Contextual factor. The data collected in each major category were then inductively coded to ascertain emergent themes. The quantitative data from the TBQ were used to supplement the themes that emerged pertaining to teacher beliefs. Lastly, the themes found within the factor categories were connected to those detailing teachers' intended and enacted ID use. A more specific description of the data analysis process for each of the types of data sources is found below.

Analysis of quantitative data: Questionnaire. The data collected from the teacher beliefs questionnaire were used to create a beliefs profile for each participant. To begin, participants' responses to statements worded negatively were inverted on the 5-point Likert scale. The mean score for each of the five categories found on the questionnaire were calculated for each participant, which ranged from 0 to 5. The resulting five means indicated the degree to which each participant's beliefs aligned with the five attributes of meaningful learning, as described by Jonassen, Peck and Willson (1999). The small sample size of this study did not allow meaningful statistical analysis across the study's cases. Instead, the data collected and analyzed using the TBQ were meant to supplement and enrich the qualitative data collected throughout the study.

Analysis of qualitative data: Interviews and observation. Qualitative data analysis comprised the vast majority of this study's analytic methodology. All audio-recorded data collected during participants' interviews were transcribed using a word processor. These transcripts, as well as the data collected using the observation protocol, were imported into the Dedoose qualitative analysis software (SocioCultural Research Consultants, 2016).

Using the Dedoose software, the data were coded using the theoretical model described by Remillard (2005). To accomplish this, the data was first deductively coded into six major categories- Planned use of ID, Actual use of ID, ID factor, Teacher factor, Student factor, and Contextual factor. Then, the data within each of the major categories were inductively coded for emergent themes. In doing so, I aimed to be sensitive to the themes that emerged in this second round of coding. The code *interesting data* was used throughout the coding process to capture data that seemed relevant to the current study, but didn't seem to fit into the study's theoretical framing. Particularly powerful or salient data were coded using an *important data* code. The resulting coding scheme is depicted within Appendix G.

Cross case analysis. A cross-case analysis (Merriam, 2009) was conducted once each of the three participants' questionnaire, interview, and observational data were analyzed. The purpose of this final analysis was to explore the similarities and differences that arose across participants' ID uses and the factors that mediated those uses. It was my hope that the cross-case analysis would add further detail and clarification to the themes that emerged within the individual participant cases, as well as to the larger body of literature.

Validity and Reliability

As the researcher in this study, I acknowledge the potential for subjectivity that may be present in interpretive research. Merriam (2009), for example, points out the significant role the researcher has in both data collection and analysis. This leaves the study susceptible to bias or faulty researcher interpretation. These swayed interpretations may be either intentional or unintended. I combated this phenomenon in a number of ways. First, whenever possible I enlisted my colleagues in examining coded sections of data in order to increase inter-rater reliability. Second, I documented my analytic decisions. This created an audit trail ensuring that my descriptions and interpretations can be verified. Lastly, I used member checking (Lincoln & Guba, 1985) during the final meeting and once analysis was complete as a way for participants to modify or verify my interpretations.

Chapter 4

This chapter describes the case of Adam Clark, a fourth-year teacher in DSS. Mr. Clark's planning and enactment of ID activities were mediated by a variety of factors surrounding his instructional practice. Notably, some contextual and student factors that he did not perceive as impactful influenced how he enacted ID activities. Below, Mr. Clark's instructional use of IDs is described as it occurred during this study. This description begins by detailing the three focal IDs used by Mr. Clark, followed by how he created the intended and enacted curriculum involving these IDs. The ways in which various ID, teacher, student, and contextual factors surrounding his practice mediated his use of IDs are also examined. Note that the participants' descriptions are used whenever possible throughout this and the following two chapters. Instances where participants' descriptions are inconsistent with study observational data are noted. This approach was taken so that each participants' ID use can be, as much as possible, understood from their perspective. An alternate approach will be described in Chapter 7.

Implemented IDs

Mr. Clark utilized two interactive diagrams within his first lesson. These IDs were found within the same lesson in the Agile Mind topic Understandings Slope and Intercepts within the IA course. The first ID (see Figure 4) will be referred to as the *Slope ID*. This ID allowed the user to vertically drag the slider on the left of the screen between the values of -3 to 3. The selected value on the slider then became the slope value for the equation and graph on the right of the screen. The dynamic equation and graph were colored blue to distinguish them from the static equation $y = 1x$ that remained on the screen. In Figure 4 the dynamic graph and the static graph overlap. The text above the ID directed the user to manipulate the slider and observe the resulting shifts on the graph.

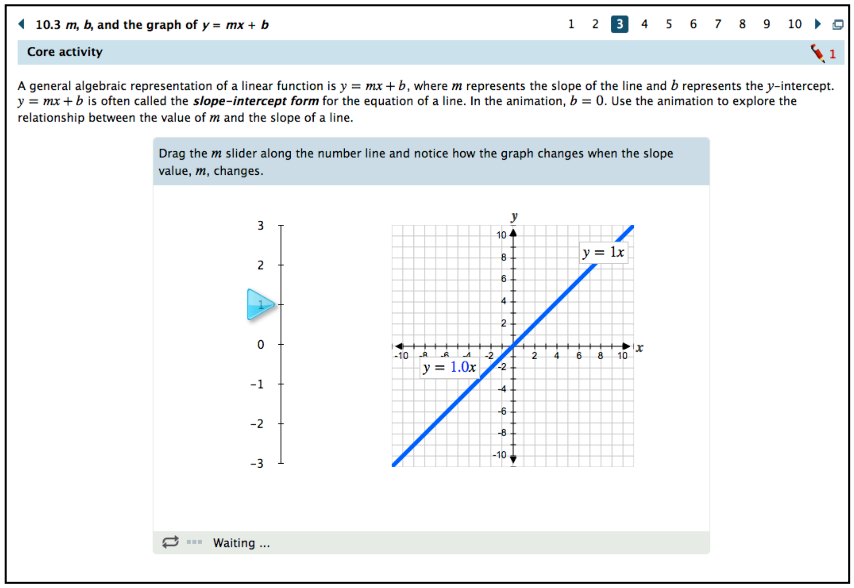


Figure 4. Slope ID. Reprinted from Intensified Algebra Course, topic 10, lesson 3, slide 3 in *Agile Mind*. n.d. Retrieved April 9, 2016 from http://www.agilemind.com. Copyright 2016 by Agile Mind.

The second ID used during that same lesson is referred to as the *Parallel Lines ID* (see Figure 5). Like the *Slope ID,* this ID also allowed the user to manipulate the given equations and graphs using sliders. Here, though, there were two sliders for each of the two different linear functions. The sliders on the left of the screen controlled the slope and *y*-intercept, respectively, of the red linear function. The sliders on the right controlled the slope and *y*-intercept values, respectively, of the blue linear function. The text on the top of the screen instructed the user to utilize the sliders to make the red and blue functions parallel.

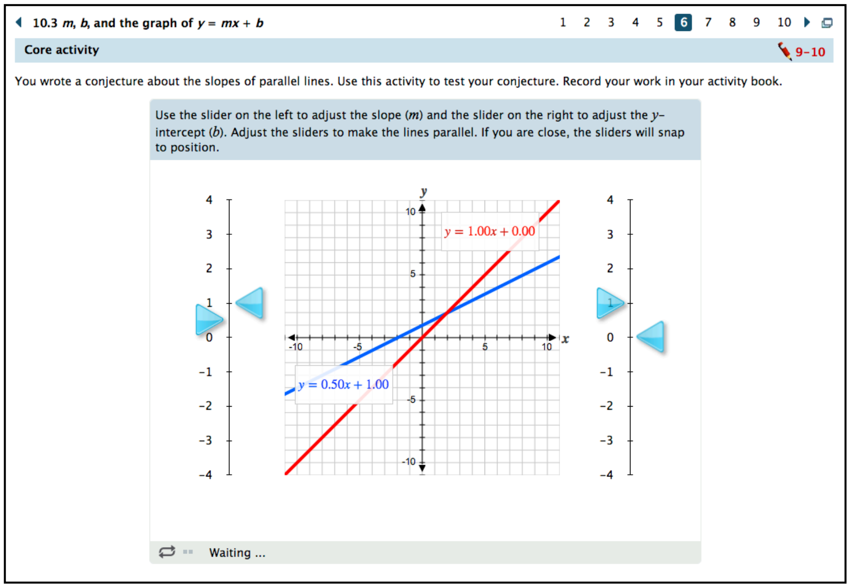


Figure 5. Parallel Lines ID. Reprinted from Intensified Algebra Course, topic 10, lesson 3, slide 6 in *Agile Mind*. n.d. Retrieved April 9, 2016 from http://www.agilemind.com. Copyright 2016 by Agile Mind.

During Mr. Clark's second observed lesson, he implemented an ID we will call the *Line of Best Fit ID* (see Figure 6). This ID was found within the Scatterplots and Trend Lines topic of the IA course. The text above the ID directed the user to alter the two sliders, and thus the slope and y-intercept of the line, to make a linear function that fits the given scatter plot.

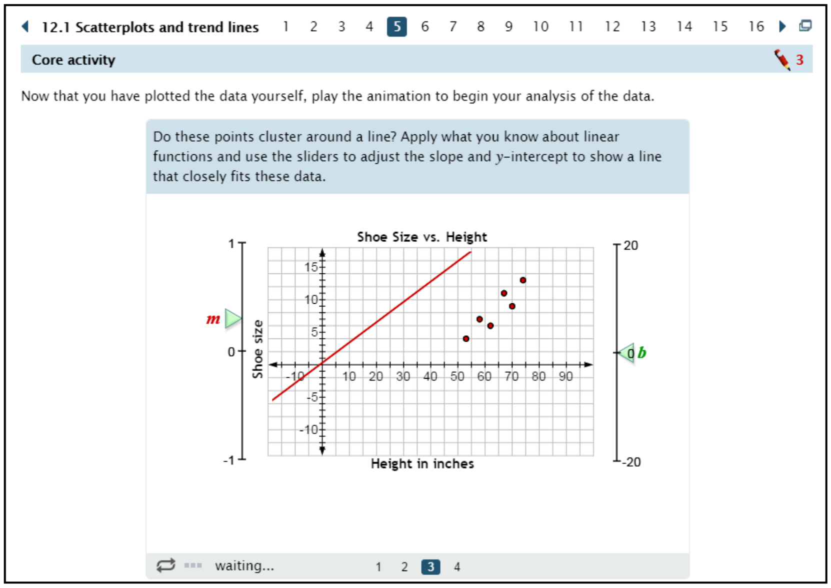


Figure 6. Line of Best Fit ID. Reprinted from Intensified Algebra Course, topic 12, lesson 1, slide 5 in *Agile Mind*. n.d. Retrieved April 9, 2016 from http://www.agilemind.com. Copyright 2016 by Agile Mind.

Intended Curriculum

Mr. Clark expressed a largely consistent vision for planned ID activities. Namely, the goals for ID activities, students' roles and the role he planned for himself were constant throughout the entire study. The planned classroom structures used during ID activities, however, shifted quickly after the initial meeting and then remained constant for the remainder of this investigation. Below each of these four components of Mr. Clark's planning involving ID activities is described in turn.

The intended purpose of using IDs, Mr. Clark explained, was centered on students being active learners and making sense of content. He emphasized, "It's important for students to be actively participating, actively be engaged in whatever the lesson or the material is" (personal communication, February 25, 2016). Thus, he expressed a desire to have students interacting with the ID as much as possible. To this end, Mr. Clark indicated that he planned to "have students play or manipulate the diagram" (personal communications, October 29, 2015) instead of manipulating the ID himself.

Initially, the structure within which Mr. Clark planned to have students interact with IDs varied depending on a number of factors, including the design of the ID and student understandings. He explained,

> I might want individuals to interact with the diagram. Or [they will] work in groups where one person might be able to talk them through what they're thinking [...]. I might also want to do it in a whole group and probe questions and make sure the whole class is getting it at a similar time. (personal communication, February 25, 2016).

Shortly after our initial interview, however, his perception of possible activity structures shifted. He conceded at that point to planning only whole group activities for the majority of the study. This included the ID activities observed during the study's classroom observations.

The intended whole group structure of ID activities followed a consistent format. Mr. Clark planned to project the ID onto the classroom's front board and have a small number of selected students interact with the diagram while the remainder of the class observed these interactions. This general structure was evident in his description of the planned *Line of Best Fit ID* activity.

> So, the plan is similar to how I used the interactive diagrams throughout the year. I'll call on one or two or three students to come up and play with the slope and the

> *y*-intercept and talk out loud, talk the rest of the class through, or think out loud through, exactly what's the relationship between changing those values and the line of best fit on the graph. (personal communication, February 18, 2016)

This description typified Mr. Clark's planning of ID activities. Namely, a few students interacted with a projected ID while the rest of the class observed the ID manipulation and took notes within their workbooks.

All students were expected to respond to the questions posed by Mr. Clark during ID activities. These questions seemed central to his notion of an effectively planned ID activity, as he discussed them frequently and often emphasized their importance. He explained,

> So, one of the things that I think is really important for me to do while it's happening is to ask students what's going on in the diagram to make sure they're on board with what we're interacting with and what we're trying to alter, and also see what they're learning from it. So, I think that the probing questions that I try and ask throughout are some of the key factors in making sure that students understand it and furthering students' knowledge. (personal communication, February 25, 2016)

Mr. Clark attributed significant importance to asking students questions as he seemed to believe that they focused and deepened students' learning with the ID. The combination of students controlling the ID and the teacher asking students questions was meant to guide students' exploration of the ID while not explicitly telling them what they should be learning. The role of a guide for students during learning was consistent with what Mr. Clark planned for himself. Indeed, he intended to "just kind of facilitate learning" (personal communications, October 29, 2015). This view was apparent as Mr. Clark described his planning surrounding the *Line of Best Fit ID* activity.

> They'll play with the diagram and start to generate their own conclusion and then I'll ask them more specific questions and give them more specific suggestions on how to drag the slider or manipulate the interactive diagram. And I'm hoping that [my questions] will help them come to conclusions on what changing the slope will do to a graph of a function specifically. (personal communication, December 1, 2015)

Thus, Mr. Clark's consistent intent during ID activities involved engaging students within a whole group where they actively made sense of the mathematical concepts found within the ID. Purposefully posed teacher questions were a vitally important component during these learning activities.

Enacted Curriculum

Mr. Clark's descriptions of his enacted ID activities largely mirrored his planned curriculum; he described the goal of his enacted activities, their structure, and the roles played by both him and his students as being what he had planned. Inconsistencies were found within these areas, however, once they were examined against classroom observational data. Below Mr. Clark's enacted ID activities are described with an emphasis on contrasting his planned and enacted curriculum as it was observed.

To begin, the ID overall activity structures enacted within Mr. Clark's classroom were consistent with his intended use of IDs. Both observed ID enactments occurred in a whole group setting where an ID was projected onto the front of the classroom for the whole class to see. Two to three selected students controlled the projected ID during the classroom activities while the remainder of the class watched this manipulation. Mr. Clark reflected that the activities went "pretty much according to plan. […] I picked two students to come up and kind of manipulate or play with the interactive diagram and the rest of the class watched" (personal communication, December 2, 2015). The observed activities were quite brief, however, occurring over 12 (observation, December 2, 2015) and 3 minutes (observation, February 19, 2016). Recall that the first classroom observation included the enactment of both the *Line of Best Fit ID* and the *Parallel Lines ID*. Thus, the average observed ID enactment occurred over 5 instructional minutes per ID, representing 5% of Mr. Clark's 100-minute period. The limited duration spent in the classroom using the ID seemed to be at the center of the discrepancies between the intended and enacted ID activities as Mr. Clark described them, and the actual implementation of these activities.

First, there seemed to be shift in the teacher role between what Mr. Clark envisioned for himself and how it was actualized within the classroom. He did ask students a multitude of questions throughout ID activities as he intended. The substance of his enacted questions, however, seemed to deviate substantially from his described intent. Recall that Mr. Clark intended to facilitate learning by asking questions that "help[ed students] come to conclusions" (personal communication, December 1, 2015) themselves as they explored content. The enacted lines of teacher questioning observed during ID activities often led students to particular ways of thinking and specific investigations, however.

This phenomenon occurred during the *Line of Best Fit ID* activity, for example. A student began the activity by manipulating the trend line randomly. At that point, students were beginning to tell the student how to move the trend line to fit the given data points. Mr. Clark quickly focused students' attention specifically on the slope of the line by noting, "Let's stop right there. Should she make the slope bigger or smaller to make it fit the points?" (observation, February 19, 2016). When a number of students incorrectly stated "smaller" he looked at them hesitantly. This gesture, whether intentionally or unintentionally, seemed to communicate to students that they needed to change their answer before the graph's slope was manipulated. Students promptly responded "bigger." Then, once the slope was moved to approximately the correct value, Mr. Clark stopped students' exploration of the slope by advising the class, "Why don't we leave it there and move the y intercept. From where it is now, should we move the y intercept up or down?" (observation). He used questions structured similarly to this, where students were given two options for the next ID manipulation, often during enacted ID activities. Such questions seemed to contrast questions such as "What do you think we should do next?" which was more aligned with his description of the intended curriculum. Then, as the student moved the *y*-intercept of the line up and down, Mr. Clark advised the class when the correct trend line was reached. He prompted the student to end her manipulations by noting, "It looks like we are pretty close" (observation). The activity concluded by Mr. Clark directing students' attention to their workbook. He noted, "Number 5 asks for the full equation. We have the slope [points to the slope on the projected ID] and the *y*-intercept [points to the *y*-intercept on the projected ID]. So, what is our full equation? What is our slope [points]? What is our y intercept [points]?" (observation). He then wrote the equation on the board from students' responses to his latter two questions. Students' responses followed directly from him pointing to the answers of his own questions. This string of interactions seemed to characterize the ways in which Mr. Clark led his students through the observed ID activities. Namely, he asked students how they should interact with the ID using a closed question, then non-verbally indicated if the direction students suggested was correct or not, and finally stopped the exploration when a correct response was found. Lastly, he concluded the ID activity with a brief summary of the activity's findings that included asking leading questions of his students.

The enactment of Mr. Clark's role during ID activities seemed in stark contrast to the less directive role he detailed in the intended curriculum. Indeed, the teacher facilitation described above seemed more teacher-centric than Mr. Clark's description of the intended curriculum would imply. When asked about the enactment of his role and how he guided students' investigation with the *Line of Best Fit ID* he reflected,

> I think it was fairly successful. I think we could have kept going with the slope but I think as long as the y-intercept was off it was going to affect how the slope was portrayed on the graph. (personal communication, February 19, 2016)

Thus, Mr. Clark made the decision to stop students' manipulation of the slope because he felt that this exploration might not result in the desired outcome of the activity. Providing such guidance certainly could have been consistent with the student-centered exploration he described during the intended curriculum if students were encountering significant struggle and frustration during the activity. His decision to focus students' attention on the *y*-intercept occurred quickly, however, which limited students' control of the investigation. He reflected, "I kind of prompted […] the class with questions to get them thinking about changes in slope and the changes in the y-intercept and how that affects the graphic that we worked on" (personal communication). It was the enactment of this "prompting" that caused a significant shift away from the student-centered intent he described in his planned curriculum toward the more teacher-centric activity that actually occurred within his classroom.

The shift in the enacted teacher role had, in turn, an effect on students' enacted roles. As Mr. Clark intended, a few students did manipulate the projected ID while the rest of the class observed their manipulations. Additionally, students were expected to respond to his questions and record particular answers in their workbooks. The types of questions asked of students, however, largely decreased their opportunity to be as in control of their learning, which Mr. Clark indicated was his desire while describing the intended curriculum. Indeed, students' did not so much "come to conclusions" (personal communication, December 1, 2015) themselves, but answered focused, leading questions often with one word answers. Additionally, the answers students provided were frequently embedded within the question he asked. Discussing the *Slope ID*, for example, Mr. Clark asked, "Is this function increasing or decreasing?" (observation, December 2, 2015). Here, he gave students the options of answering either increasing or decreasing within the phrasing of his question. Again, a question more akin to "How can you describe this function?" seemed more consistent with his description of the intended curriculum.

Further, Mr. Clark often fully explained why answers were correct once he obtained a one-word response from the class instead of having students elaborate on their response. His description of the intended curriculum seemed to imply, though, that students would be explaining mathematics themselves as they actively participated in their own learning. Mr. Clark reflected on this when asked about his rationale for explaining mathematical concepts after students answered his questions.

> So, me repeating [math concepts] over and over […], I think it helps [students] to memorize it and just reinforce it in their brain. And because often times I find that when students just write notes a lot of times it doesn't stick. So, me repeating it […] over and over or getting them to repeat it is a way just to help them remember whatever the concept as we are talking about. (personal communication, December 2, 2015)

It seemed from this reflection that Mr. Clark perceived his repeated explanations of mathematics concepts as simply repeating what student had already said. This sentiment was not consistent with the data collected during the observed ID activities, however. Instead, Mr. Clark added additional explanation and justification to the conversation once he received a brief answer from students. He explained further,

> [I'm] modeling the correct use of academic vocabulary and it also reiterates the definition of whatever we're focused on. […] It just depends. If it's new information and we are in exploratory activity, I kind of let them come to that conclusion on their own. (personal communication, February 19, 2016)

Mr. Clark seemed to be limiting students' opportunities to come their own conclusions, however, by the ways in which he enacted what he describes here as modeling mathematics vocabulary and definitions. Consequently, students' roles in ID activities were reduced from their intended role to one that was significantly more limited and passive.

Taken together, Figure 7 summarizes Mr. Clark's intended and enacted curriculum involving IDs, including the shifts between the two that were observed during this study, within a modified version of this study's conceptual framework.

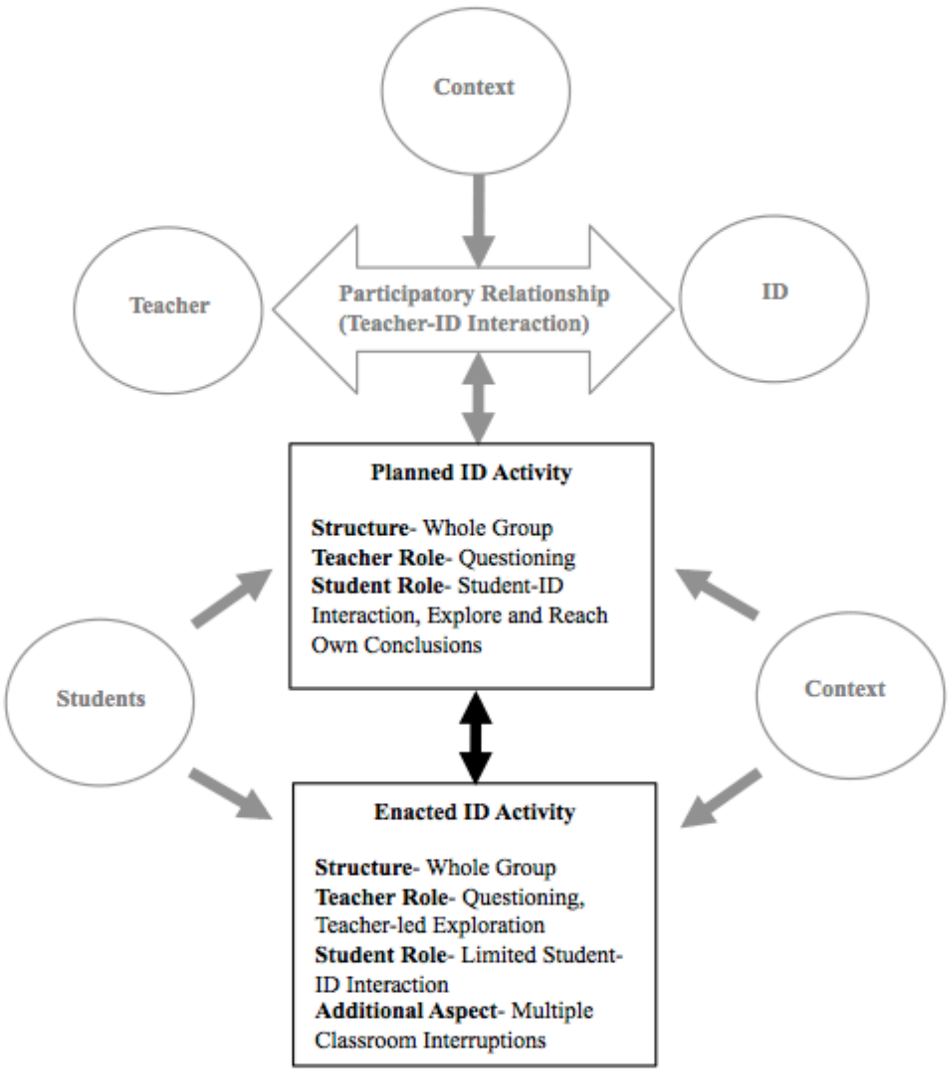


Figure 7. Mr. Clark's planned and enacted curriculum involving IDs.

What, though, influenced the transition from Mr. Clark's plans to the enacted ID activities? The remainder of this chapter attempts to answer this question by detailing the various factors found within the teacher, context, ID, and student circles shown within

Figure 7. Special attention is paid to how these factors affected the intended ID activity, the enacted ID activity, or both.

ID Factors

To begin, factors surrounding IDs themselves seemed to influence the ways in which Mr. Clark created the intended curriculum. Specifically, the mathematical representations used, degree of interactivity, presence of instant feedback, how IDs allowed investigation, their versatility, and the ID context influenced his creation of planned ID activities. These mediating ID factors are described in turn below.

Multiple representations. Mr. Clark noted the benefit of interacting with IDs that incorporated multiple mathematical representations throughout the study. Specifically, he emphasized how dynamically linked mathematical representations help students "see how one [representation] can link to another fairly easily" (personal communication, October 26, 2015). A user is able to alter one representation and observe the consequence of that action in another representation when mathematical representations are dynamically linked. Mr. Clark found this to be "very helpful" (personal communication, October 26, 2015) because students could "actually see what happens when numbers change with the graph and the equation. I think there's often a disconnect between the equations, numbers, and graphing" (personal communication, December 1, 2015). IDs seemed to address his concern.

Mr. Clark's use of the *Slope ID* demonstrated his affinity for IDs that incorporated multiple mathematical representations. Slope, he noted, was a concept his students struggled to understand.

> They'll understand the number and they'll understand how to find it. But I really don't think it really made sense in their heads until they use a diagram like this. This is perfect because it shows us the number- it gets bigger then the line gets steeper, and then as you pass zero going to negative it switches to negative, and then right when the slope is zero it shows that there's a horizontal line. So, there are a lot of things that this one simple diagram shows that are really helpful for students to understand. (pre-observation, December 2, 2015)

Mr. Clark expressed similar thoughts surrounding other IDs. Referring to the *Skateboarder ID* (see Chapter 5, Figure 13) for example, students moved the skateboarder while the ID created a graphical representation of the relationship between elapsed time and position. He found this ID feature to be beneficial. He explained, "I think it makes more sense in your mind as to like, 'Oh, so the up and down [in the graph] isn't a hill, it's actually the speed […].' So I think it helps" (personal communication, October 26, 2015). The ID illustrated how manipulating one representation affected the other representations and, thus, aided students in more fully understanding these mathematical concepts. Mr. Clark desired for his students to gain robust understandings of mathematical concepts such as this. Consequently, the presence of dynamically linked mathematical representations seemed to encourage Mr. Clark's use of IDs.

Interactivity. Mr. Clark also emphasized the importance of the interactivity afforded by dynamic ID features. He noted the ability of a user to manipulate particular components of an ID, for example, explaining, "It's the manipulation piece that I think is really important" (personal communication, October 26, 2015). To him, manipulating IDs assisted students in developing understandings of mathematics content and making otherwise abstract algebra concepts "more approachable for a lot of students" (personal communication, October 26, 2015). Speaking of slope, for example, he explained, "If you're messing with the slope, it can show how the steepness of the line is changing and that helps students to understand the steepness of the line" (personal communication, October 26, 2015). This is particularly important because, as he noted,

> I think [students are] just lost at the way it's presented by a lot of teachers, myself included. Sometimes I'm just like, yeah remember, y_2-y_1 and they're like, 'Okay, I get that but what does that actually mean?' So, if you're interacting with something, I think it really helps them [see] exactly what's going on in the graph and what's changing […]. So, I think with slope [interactivity] is really helpful. (personal communication, October 26, 2015)

Mr. Clark asserted that students' ability to dynamically alter the slope of the lines present in each of the three IDs he used during this study helped them to understand the concept of slope more easily. He desired that IDs include interactive features for this reason.

Conversely, Mr. Clark found IDs that did not include interactive features less useful and was less likely to use them as extensively as those that did. When reflecting on his interactions with the *Mixing Paint ID*, for example, he noted, "It wasn't one that was like super-engaging, super-interactive" (personal communication, February 25, 2016). He rated the *Mixing Paint ID* as his second to least favorite ID during the final interview due to its limited interactivity. Similarly, the *Friendship Problem ID*, which Mr. Clark rated as his least favorite ID, included even less interactivity in his opinion. The IDs he liked the least "just show [mathematics concepts] more as movies then like actually letting kids interact with them" (personal communication, December 1, 2015). Thus, the degree to which an ID allowed students to interact with mathematics seemed positively correlated with how much Mr. Clark desired to use them in his classroom. Thus, the interactivity of a given ID mediated his use of that ID while he created the intended curriculum.

Instant feedback. IDs are able to provide instant feedback to their users as a direct consequence of interactive design features and their use of multiple representations. When, for example, a user modified the slope of an equation in the *Parallel Lines ID* he or she instantly saw the consequence of that action on the graph of that equation's line. Mr. Clark favored instant feedback aided by technology over traditional methods for manipulating mathematical objects. He explained,

> I think [IDs are] helpful because they give immediate feedback. When things change, [students] can see instantly how it happens rather than working for 5 or 10 minutes on a graph and then seeing it […] – and then starting over and drawing another graph in 5 or 10 minutes. I feel like it's instant feedback where [students are] like, "All right, let's move it up, I think it's high." You can instantly see that. (personal communication, February 18, 2016)

In this scenario, IDs eliminated the need for students to create graphs by hand because they were instantly created for them. This was particularly important to Mr. Clark because students took significant amounts of time and effort, and often struggled, to create accurate graphs by hand. Thus, IDs ability to provide instant feedback from students' algebraic manipulations on the graph allowed the investigation to happen more accurately and efficiently.

Mr. Clark emphasized that the instant feedback afforded by IDs was beneficial in students' learning of mathematics concepts. Instant feedback was "super helpful for students [because] they were figuring out what's happening exactly as they were doing it" (personal communication, February 25, 2016). During the *Parallel Lines ID* activity this meant that students could try any manipulation of a slope and quickly see the relationship between that action, the slope represented on the graph, and the degree to which the manipulation was helpful in making the two lines parallel. Then, students could adjust their next manipulation of the slope in light of this feedback. Mr. Clark found IDs very "helpful in that respect" (personal communication, February 19, 2016) as it created an environment were students could investigate mathematical concepts in a more autonomous way, a desired characteristic of learning activities in his classroom. Thus, an ID's ability to provide students instant feedback increased Mr. Clark's desire to incorporate it into his planned curriculum.

Allows investigation. Mr. Clark ultimately combined the three previously discussed ID factors, namely multiple representations, interactivity, and instant feedback, into a broader factor pertaining to an ID's ability to encourage and/or facilitate investigation. Overall, he wanted students to be able to "manipulate or play with the interactive diagram" (personal communication, December 2, 2015). By creating intended curriculum where students played with IDs, Mr. Clark attempted to create "more so hands-on learning, [which] allows the whole class to be more engaged" (personal communication, February 19, 2016). Consider, for example, the *Line of Best Fit ID* activity. Here, students manipulated the line present on the ID to fit the given scatter plot. Mr. Clark described students' participation, "It seemed like a lot of them were […] calling out hoping to see the changes in the slope and the y-intercept and how that affected the graph" (personal communication, February 19, 2016). Students had the opportunity to investigate the line of best fit because of the ID's ability to provide instant feedback while they manipulate the provided mathematical objects. Mr. Clark seemed to appreciate the *Line of Best Fit ID's* ability to facilitate student investigation and looked for IDs that could provide similar opportunities for his students. Thus, the degree to which an ID encouraged and/or facilitated investigation affected Mr. Clark's use of that ID as he created the intended curriculum.

Versatility. An ID's versatility, specifically in how users interact with it, was also a factor Mr. Clark seemed to consider when creating the intended curriculum. In particular, he wanted the user to have significant control when manipulating mathematical objects found within IDs both in terms of the specificity of input values that could be entered and in the freedom to select any value the user wished. While he noted that this factor didn't "necessarily impact how often I use the IDs, because I think I still use them" (personal communication, February 25, 2016), it seems prudent to include his comments pertaining to this factor as it is possible that it implicitly impacted how Mr. Clark used IDs. This is particularly true given the number of times he discussed IDs' versatility.

Mr. Clark expressed that IDs should afford their users significant control over the input values they could enter. Specifically, he thought it was important to have "the ability to input specific numbers into an equation" (personal communication, February 18, 2016). For IDs like the *Slope ID* and the *Parallel Lines ID*, for example, the ability to enter specific parameter values would be in addition to using a slider. Then, the user could input a specific slope value into the algebraic representation. This would decrease restrictions on possible input values by eliminating the pre-determined intervals between possible input values inherent with the sliders.

Mr. Clark also wished to eliminate pre-determined restrictions on the range of possible input values. Recall that the range of slope values was from negative three to three, negative four to four, and negative one to one in the *Slope ID, Parallel Lines ID* and the *Line of Best Fit ID*, respectively. Mr. Clark noted that these intervals restricted possible content explorations and wished these ranges were extended. Discussing the *Line of Best Fit ID*, for example, he explained, "the slope stays between one and negative one […] but, [what] if the slope changes from 5 to 30, what do you notice goes on?" (personal communication, February 18, 2016). Extending the range of the current sliders or allowing the user freedom to input a greater range of values would allow students to investigate this question. Thus, it seemed that the level of control afforded by an ID's design might have implicitly influenced Mr. Clark's creation of intended curriculum. He seemed to favor IDs whose design allowed the user greater control over the input values that could be entered. This factor was not significant enough for Mr. Clark to override the other factors described within this section, however.

Context of ID. The last ID factor Mr. Clark identified as influencing his creation of planned ID activities was the context within which an ID was embedded. Some IDs, like the *Slope ID* and the *Parallel Lines ID*, were not contextualized within a real world setting. Others, like the *Line of Best Fit* or *Skateboarder ID* were embedded within a real-life context. Mr. Clark emphasized the value of the latter. He noted,

> I think the [IDs] that […] I'm using are intentional because they're trying to relate [mathematics] to a real life use of whatever the skill is that we're teaching. So, I

> think in using [the mathematics] in somewhat of a real life situation, […] that's what really makes [IDs] so useful and such a powerful tool. (personal communication, October 26, 2015).

He echoed this sentiment numerous times throughout the study, summarized his thoughts by noting, "Application to real life […] is an overarching theme in my class. It's something that I try and push" (personal communication, February 25, 2016). Thus, he indicated that he was more inclined to use IDs that explicitly made real-world connections to mathematics concepts. Consequently, an ID being embedded within a real-world context seemed to influence how Mr. Clark used that ID.

Mr. Clark emphasized, though, that the real-world connections found within IDs should be meaningful and engaging to students. He described two situations where the contexts of IDs did not meet this criteria. The first pertained to the *Skateboarder ID*. He rated this ID favorably during the final interview, noting that it "was more of a real life application. So, I thought the usefulness in that also was very high" (personal communication, February 25, 2016). He did not rank it as highly as might be expected given this emphasis on the real world applications of mathematics, however. He explained this discrepancy,

> The reason I didn't put it at the top is because Agile Mind uses the same exact skateboard diagram for sixth, seventh, eighth and ninth grade math. So, the repetitiveness of it make students go like, "Oh yeah. We've seen this like three or four times before," or like, "We've done this same lesson before." (personal communication, February 25, 2016)

Repeatedly using the same context decreased Mr. Clark's students' interest in the *Skateboarder ID*. Thus, he "didn't spend as much time on that" (personal communication) ID before moving on in his lesson. The repetition of the skateboarder context and the effect this had on students' engagement directly mediated his use of this ID.

Mr. Clark also explained how a real world application needed to be meaningful to his specific students. He highlighted the *Friendship Problem ID* (see Chapter 6, Figure 21) as an instance where his use of an ID was hindered by a context that his students didn't fully understand. This ID used a scenario were increasing numbers of friends were calling each other as part of a phone tree. Users were charged with using the functionality of the ID to determine the function rule for the relationship between the number of callers and the number of possible calls. Mr. Clark rated the *Friendship Problem ID* as his least favorite ID during the final interview despite this real world context. He explained this ranking,

> The idea of a phone chain is something that [students] don't really use or understand. […] I think that led to confusion right off the bat. That kind of continued throughout the lesson, because they were like, "why wouldn't I just send an email or send a text to all these people. Like, I can send 50 texts at once." […] So, although it was a real life application, it wasn't something they ever encounter. (personal communication, February 25, 2016)

Students' unfamiliarity with the *Friendship Problem ID*'s context caused confusion. This, in turn, required that Mr. Clark discuss the context in more detail than intended so that his students could ultimately engage with the mathematics concepts within the ID.

Unfortunately, "things were more rushed [as a result of explaining the context]. So, I didn't get to call on as many students as I had hoped to or asked as many probing questions as I had hoped to" (personal communication, February 25, 2016). Similar to the *Skateboarder ID* activity, this was an instance where an ID context influenced Mr. Clark's enactment of an ID.

Thus, the context of an ID seemed to mediate the ways in which Mr. Clark created both the intended and enacted curriculum. While he certainly preferred when IDs highlighted real-world applications of mathematics concepts, he noted that an ID "doesn't have to have a real life example, but I do think that can help to deepen [students'] understanding" (personal communication, October 26, 2015). When a real life context was included, though, the number of times that context was presented and students' familiarity with the context seemed to influence both the intended and enacted curriculum.

Taken together, numerous ID factors influenced how Mr. Clark created curriculum using IDs (see Figure 8).

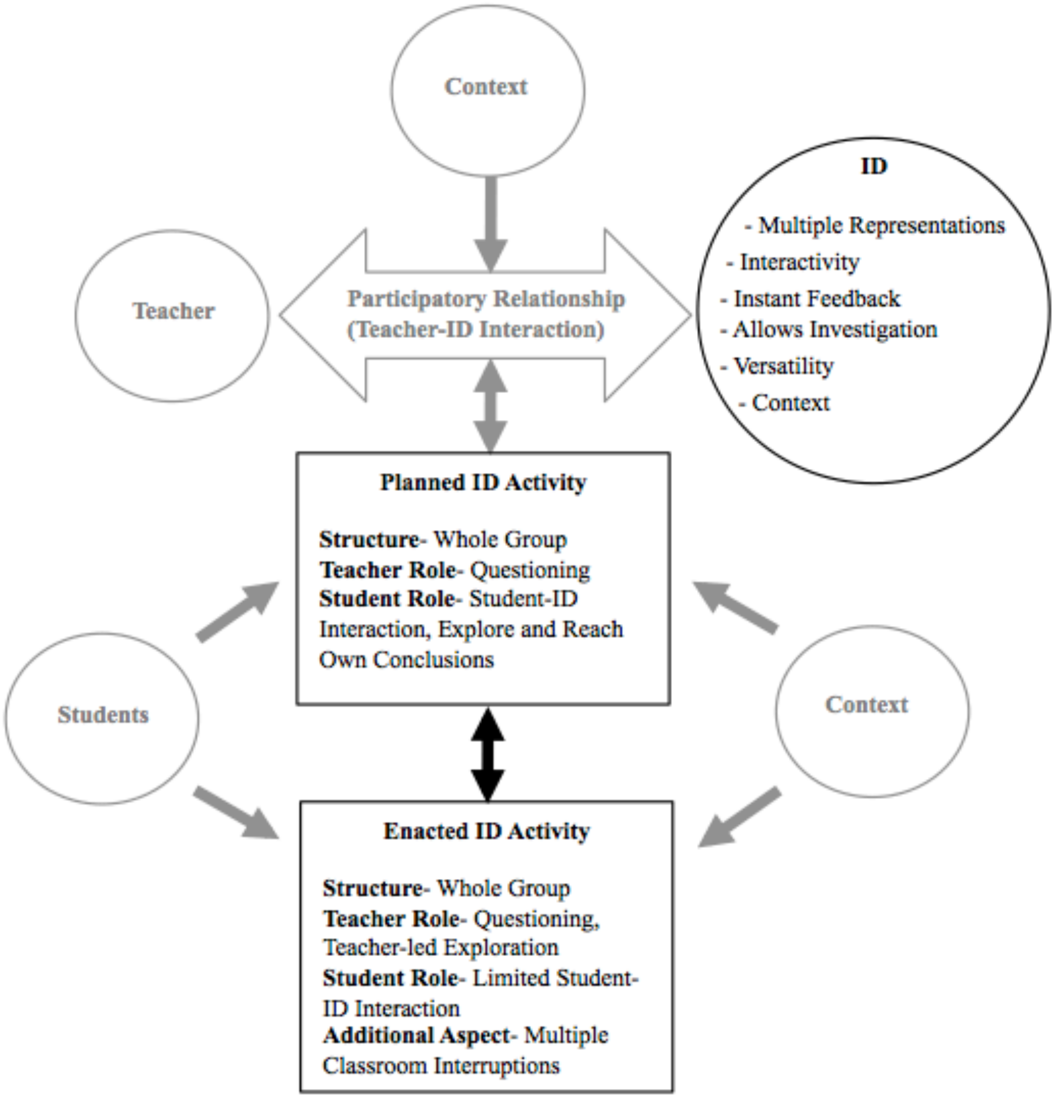


Figure 8. ID factors affecting Mr. Clark's use of IDs.

The presence of multiple mathematical representations, high levels of interactivity, instant feedback, and an ID's ability to facilitate investigations seemed to increase Mr. Clark's likelihood of using an ID. Limited versatility in how a user was able to input data into an ID negatively impacted his ID use, however. Lastly, Mr. Clark viewed an ID being embedded within a real-world context as a generally positive factor, though the repetition and meaningfulness of that context was considered.

Teacher Factors

A number of teacher factors also seemed to influence Mr. Clark's planning and enactment of ID activities. In particular, his beliefs surrounding the teaching and learning of mathematics, teacher knowledge, and experience with particular IDs were influential factors. These factors are discussed below in turn.

Teacher beliefs. Mr. Clark seemed to express a number of beliefs concerning how mathematics should be learned and, as a result, how instruction should be planned and enacted. His results to the TBQ are shown in Table 7.

Table 7

Pedagogical Beliefs Questionnaire Results- Mr. Clark

	Active	Cooperative	Constructive	Authentic	Intentional	Average
Initial meeting	3.80	4.67	4.09	4.13	4.50	4.24
Final meeting	3.60	4.50	4.04	4.25	4.75	4.23
Delta	-0.20	-0.17	-0.05	0.13	0.25	-0.01
Average	3.70	4.59	4.07	4.19	4.63	4.24

The overall average score of 4.24 suggests that Mr. Clark held beliefs consistent with meaningful mathematics learning (Bate, 2010; Jonassen et. al, 1999). The TBQ categories related to active, cooperative, constructive, and authentic learning are described below as these categories emerged as having the most significant influence on Mr. Clark's ID use.

Mathematics learning should be active. Mr. Clark seemed to express the belief that learning mathematics should be active for students throughout the study. Indeed, he emphasized, "I think that learning anything should be active for students. So, specifically, I think learning math should be an active process for students" (personal communication, February 25, 2016). In order to actively engage students, Mr. Clark planned ID activities that involved "students interacting with the diagram more than just the teacher interacting with the diagram [which] helps facilitate and make that learning takes place" (personal communications, October 29, 2015). Students, Mr. Clark seemed to believe, should actively grapple with mathematics content when interacting with IDs. Consequently, the teacher should plan for and allow time and space for students to engage with mathematics. He explained,

> I think it's important for students to struggle and to grapple with complex issues in order to truly understand them [and] I don't think students should be told exactly what they need to know. […] Teachers should allow students to get frustrated when learning math [because] I think that students do learn from failure. […And] I think it's more important and more helpful for students to generate their own conclusions while interacting with the diagrams. […] So, I think it's important for students to just be actively – actively participate, actively be engaged in whatever the lesson or the material is. (personal communication, February 25, 2016)

Planning and enacting ID activities that allowed active learning was important to Mr. Clark. This was consistent with his TBQ score of 3.70 in the active learning category. Indeed, both Mr. Clark's TBQ result and statements indicate a belief that mathematics learning should be active.

Interestingly, though, was that Mr. Clark recorded the lowest TBQ result in the active category. While his score did indicate a belief in active learning, this relatively lower score seems worth noting. While the reason for this phenomenon was not known as a result of this study, I shall propose some possible explanations. Recall that Mr. Clark's questioning patterns and ID exploration facilitation often overly scaffolded and led students' learning. These actions seem inconsistent with the belief discussed here as they decreased the amount of active learning expected of his students. Thus, he may be inclined to project the belief that learning should be active as a result of external factors, such as the pedagogical goal of his district or school (see contextual factors section below). Alternatively, he may not have the teacher knowledge necessary to create instruction more aligned with that belief. Whatever the reason, the seeming discrepancy between Mr. Clark's expressed belief and his classroom actions demonstrate the complexity present when studying classroom practice.

What did seem clear, however, was the mediating effect the beliefs Mr. Clark described about active learning had on his planning of ID activities. He consistently expressed the intent for students to actively engage with mathematics content as they used IDs. This included planning student ID explorations that could last "35 or 40 minutes of a 90-minute period" (personal communication, February 25, 2016) where students have to "grapple with [content], come to their own conclusions, and try to get through that" (personal communication). This was important, he emphasized, since "they're not going to be able to succeed in a lot of higher level math courses" (personal communication) if they don't actively grapple with mathematics in early courses like his.

Mathematics learning should be cooperative. Mr. Clark's TBQ results and statements surrounding cooperative learning also seemed to provide contradictory evidence. The average category score of 4.59, his second highest result, indicated a strong belief that mathematics learning should be cooperative. Thus, one would expect substantial group work within Mr. Clark's classroom. This was not the case, however, as only whole group and individual student work was evident throughout this study. This illustrated a gap between his belief and use of IDs.

> So, in my mindset, in grouping students and how that affects my use of IDs, I don't think the fact that they should or shouldn't work in groups impacts it in anyway. I think that students should work in groups, but I don't […] use them in a group setting. So, that doesn't impact the way I use IDs. (personal communication, February 25, 2016)

So, on the one hand, Mr. Clark does "think that students should work in groups" when learning. His belief did not apply directly to students working with IDs, however. This tension may have been caused by a combination of two factors that mediated his enactment of the belief that learning should be collaborative. First, Mr. Clark had limited access to technology, which prevented students from interacting with IDs in small groups. Additionally, off-task student behaviors interfered with the enactment of ID activities. These off-task behaviors could be most easily controlled, it seemed, during whole group activities. The mediating power of these two factors, which are discussed in the contextual and student factors sections respectfully, seemed to overcome the influence of his belief that mathematics should be learned collaboratively. Consequently, this belief did not influence his planning or enactment of ID activities as significantly as his TBQ result would imply.

Mathematics learning should be constructive. One belief that Mr. Clark consistently expressed in both his actions and his statements was that mathematics learning should be constructive. He seemed to continually articulate that new mathematics learning should be built upon students' previous understandings. As he explained, "Skills are constantly building upon older skills or prior knowledge or older material. […] So, I think it's important to continuously touch on those [and] build upon it" (personal communication, February 25, 2016). His average category score of 4.07 on the TBQ constructive section was aligned with his statements emphasizing the importance of building new learning from what students already understood. Indeed, evidence of Mr. Clark incorporating previous material into his curricula was evidenced in both the *Parallel Lines ID* and the *Line of Best Fit ID* activities. For example, he continually emphasized the concept of slope, a previously addressed mathematical understanding, through his questioning during each activity.

In addition to creating activities that supported students in constructing understandings from their previous knowledge, Mr. Clark also discussed how mathematics curricula should connect conceptual knowledge with procedural understandings, the latter of which he perceived as more prevalent in mathematics instruction. He noted,

> I think a lot of teaching [is] the drill and kill method. My understanding of why we're using more interactive diagrams […] is to develop a deeper understanding and a real understanding [of mathematics content] besides "I have to do this step or this process rather than to really understand the whole purpose of the skill that's being taught." (personal communication, October 26, 2016)

Here, Mr. Clark expressed that students should understand why mathematics makes sense as opposed to only being able to execute procedures. Mr. Clark seemed to believe it was important to address both the conceptual knowledge and procedural skills by creating planned curriculum that facilitated students constructing the connections between them. Indeed, he noted that students "should be able to understand both ideas" (personal communication, February 25, 2016). Consequently, Mr. Clark's belief that mathematics learning should be constructive seemed to influence his creation of intended ID activities by including opportunities to highlight connections between prior and new learning, as well as among conceptual and procedural understandings.

Mathematics learning should be authentic. Additionally, Mr. Clark emphasized the notion that students should be engaged in learning mathematics through the real-world application of mathematics content. By doing so, he asserted, students could learn mathematics in a more authentic way. He explained,

> A lot of teachers in general – maybe not even just math teachers – will teach something so the students know it [procedurally]. Almost like teaching for a test, but students lack the real understanding or the real purpose for whatever the skill is. I think using the interactive diagrams that I'm using now makes [mathematics content] more applicable to real life. So, I think that having these diagrams makes it a little more applicable than just teaching it on the chalkboard. So, personally I think it improved my teaching. (personal communication, October 26, 2016)

Some IDs facilitated Mr. Clark's ability to create curricula emphasizing connections between mathematics content and the applications of that content in the real world. His desire to create such curricula was consistent with the 4.19 average score in the authentic learning section of the TBQ, which indicated a strong believe that mathematics learning should be authentic. Further, recall his emphasis that "application to real life […] is an overarching theme in my class" (personal communication, February 25, 2016). It seemed, then, that his belief that students should learn mathematics authentically mediated his use of IDs by prioritizing those IDs highlighting applications of mathematics content.

Knowledge. Mr. Clark insisted that his grasp of mathematics content facilitated his interactions with IDs. Specifically, his understanding of mathematics concepts allowed him to understand the mathematics IDs addressed and the ways in which they attempted to engage students with that content. He explained, "I understand my content. So, I understand the use of a lot of the IDs, and I think that's one of the reasons that I use [IDs] a lot" (personal communication, February 25, 2016). If, on the other hand, he did not understand the content IDs were addressing he may be less inclined to use those IDs in his curriculum. Mr. Clark reflected on the relationship between a teacher's content knowledge, their potential uses of IDs, and his or her resulting effectiveness teaching with IDs when he noted,

> I think the IDs have to go along with a teacher that understands the math and knows what they're trying to get from the lesson. I don't think you could just throw out an idea and students would generate the same conclusions if that teacher wasn't asking probing questions. (personal communication, February 25, 2016)

Thus, Mr. Clark asserted that his content knowledge (Shulman, 1987) mediated his ID use by allowing him to create learning targets and aligned probing questions. This was in addition to understanding the mathematics content found in the ID and the method the ID used to engage students in that content.

Experience with course. Lastly, Mr. Clark's experience with the IA course seemed to influence how he utilized IDs within his planned and enacted curriculum. He was familiar with the IDs throughout the course and was able to reflect on his previous implementations of ID activities since he'd "taught the same curriculum for three years now. So, I've thought about how these specific [ID activities] went and whether students were able to master the objective and fully understand the information" (personal communication, February 25, 2016). Three years of experience using the same IDs within the IA course allowed Mr. Clark the opportunity to consider upcoming ID activities in light of his past uses of those IDs. He was particularly concerned with reflecting on and refining the questions he posed to students as he facilitated their learning. He explained,

> I think the more I use [IDs], I think I'm able to script better questions. […] So, I think, in using them, I think it just allows me to think of better ways of asking questions and questions that will get them to use high order thinking or deepen their understanding. I don't think I've changed a lot in how I'm going to use them but rather the questions and the probing that I'm going to do. (personal communication, February 25, 2016)

Mr. Clark's focus on questioning was apparent throughout the study. Indeed, he consistently emphasized his planning of teacher questions while describing the intended curriculum and reflecting upon enacted ID activities. His experience with the IA course and the specific set of IDs within it seemed to increase Mr. Clark's perceived ability to effectively reflect on and continually improve the probing questions he posed to his students. In this way, Mr. Clark believed that his experience with IA and resulting curricular knowledge (Shulman, 1987) mediated his use of the course IDs.

Taken together, various teacher factors seemed to influence the ways in which Mr. Clark created planned and enacted ID activities, as is depicted in Figure 9. Mr.

Clark's expressed beliefs that mathematics learning should be active, cooperative, constructive, and authentic mediated the IDs he used, as he favored IDs that facilitated learning aligned with these beliefs. The influence of Mr. Clark's beliefs on his ID interactions was mediated by other factors, however, creating a nuanced and tension-filled web of factor interactions. Mr. Clark's content knowledge and experience with the IA course were additional teacher factors that mediated his ID use.

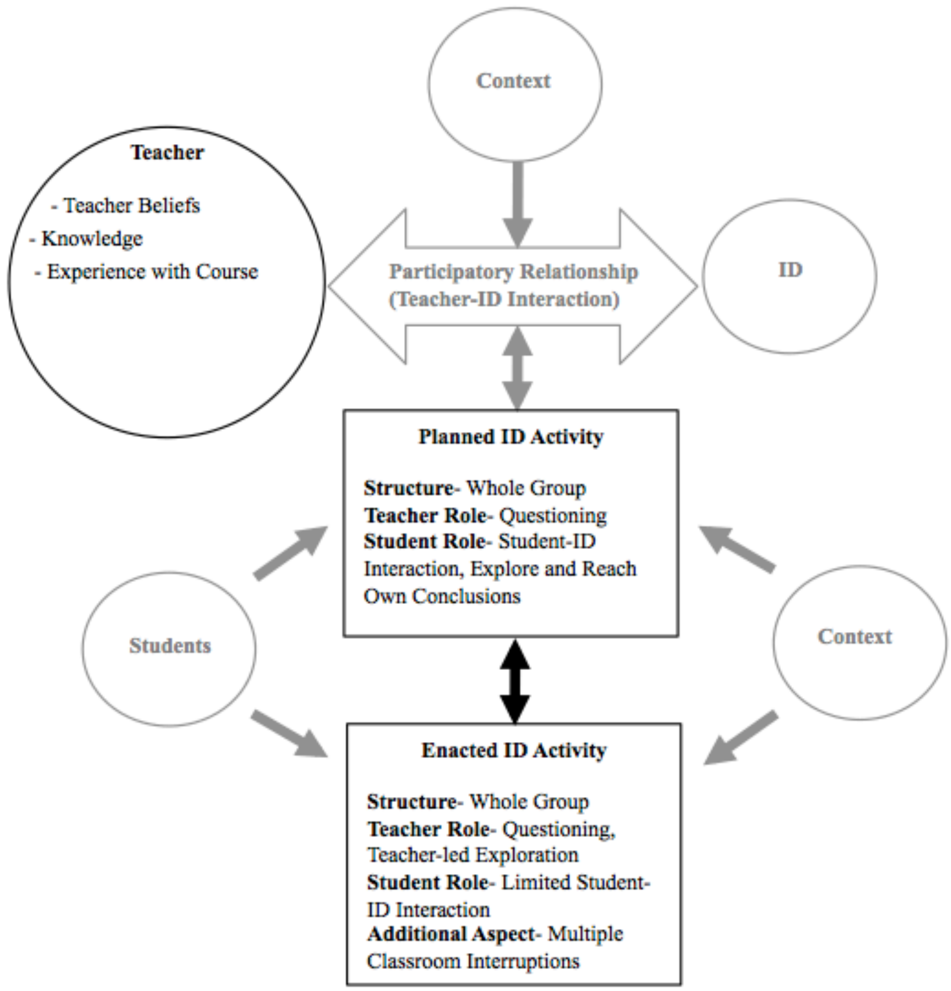


Figure 9. Teacher factors affecting Mr. Clark's use of IDs.

Student Factors

Factors related to students seemed to mediate Mr. Clark's interactions with IDs as well. Below the influence of students' content knowledge, levels of engagement, perceived learning styles, and behaviors on Mr. Clark's ID use is described as he perceived it and as was observed during ID enactments.

Content knowledge. Mr. Clark explicitly spoke of factors pertaining to students infrequently throughout the study. He did point out, however, that students' content knowledge was a factor that affected his use of IDs. The results of district-wide assessments indicated that Mr. Clark's students performed at a mathematics level well below their current grade. The IDs throughout the IA course, though, were designed to engage students in current grade-level material. When asked if and how the gap between students' content knowledge and grade-level content affected his use of IDs, Mr. Clark noted,

> I think IDs are particularly helpful for students that aren't at grade level […]. I think the IDs give them a chance to access the material when they might not see it without interacting with the diagram. […] just saying slope or just saying some math term, they might not connect with it until they see it, until they interact with the diagram. (personal communication, February 25, 2016)

Mr. Clark viewed IDs as beneficial tools in bridging the gap between students' content knowledge and grade level. Indeed, he indicated many of the ID design features discussed in the ID factor section above as supporting students in accessing grade-level content even when they lacked important pre-requisite skills. Using IDs combined with "giving out things like calculators […] so that some of the skills they might not have mastered can be done with a calculator" (personal communication, October 26, 2015) allowed Mr. Clark to address Algebra I content within his classes. This was desirable and aligned with his statements expressing that students who are behind in their mastery of mathematics content can learn grade level concepts (personal communication, February 25, 2016). Thus, IDs' ability to provide access to Algebra I content for his students encouraged Mr. Clark's use of this technology.

Further, Mr. Clark used his understanding of students' content knowledge to determine which students directly interacted with IDs. Recall that enacted ID activities occurred exclusively in whole group with a small number of students manipulating the ID using the teacher's computer. Mr. Clark explained, "I often picked lower performing students to interact with the diagram" (personal communication, February 25, 2016). It seemed that he strategically selected students with lower content knowledge to interact directly with the ID in order to provide them greater access to the content. He explained this thinking while discussing a particular student he selected to interact with an ID,

> I was trying to engage her and get her […] hands-on and on the interactive diagram. Specifically for her, so she could see and like understand how she interacted with the diagram, what's happening. Because I think some of the higher performing students catch on to […] what's going on and they can see it visually, understand it. But she seemed to struggle, at least, like, a disconnect with her specifically. So, I chose her to interact with the diagram, hoping that it would further her understanding specifically. (personal communication, February 19, 2016)

Mr. Clark used the more "hands-on" learning afforded by controlling the ID to facilitate learning with his lower performing students. Higher performing students, he seemed to think, didn't require as much direct interaction with the ID to learn grade level concepts, as viewing the manipulated ID was sufficient. Thus, students' content knowledge mediated his use of IDs within the planned and enacted curriculum by influencing which IDs he used and who he selected to interact with the ID during the classroom activity.

Learning styles. Additionally, Mr. Clark's ID use seemed to be mediated by his perception of students' learning styles and preferences[1]. He explained,

> Students' learning styles are something I try and keep in mind with using [IDs]. Like, the fact that if they're visual or kinesthetic, or get more out of the manipulation of things. I think it's like definitely one of the most important reasons to why I use [IDs]. (personal communication, February 25, 2016)

Mr. Clark viewed ID activities as a "way to make sure that the visual and kinesthetic learners are actively engaged in the learning process" (personal communication, February 25, 2016). Specifically, he seemed to appreciate how the multiple representations and interactivity afforded by IDs engaged visual and kinesthetic learners, respectfully. Mr. Clark reflected on how attending to a student's learning style affected his enactment of the *Line of Best Fit ID*, for example, when he noted,

> Yeah, so, she's more like a hands-on learner, […] just seeing things, she doesn't pick up on in the same as most of the students in the class. So, I felt like she was a

[1] As noted in chapter two, learning style frameworks are debated in educational literature (for example, Cuevas, 2015; Willingham, Hughes, & Dobolyi, 2015). That said, the term "learning styles" will be used throughout chapters three through six, as this was how participants spoke of this aspect of their instructional practice.

> good person to pick up and have her to interact with the diagram. (personal communication, February 19, 2016)

Here, a student who Mr. Clark perceived as favoring kinesthetic learning was selected to directly interact with the ID while other students who were more visually oriented were left to observe those interactions. By enacting the ID activity in this way, Mr. Clark attempted to "hit certain learning styles, [… since] this is a way to make sure that the visual and kinesthetic learners are actively engaged in the learning process" (personal communication, February 25, 2016). Thus, the alignment between students' learning styles and an ID's ability to facilitate that type of learning seemed to influence Mr. Clark's creation of both planned and enacted ID activities.

Engagement. Student engagement surfaced as a significant student factor Mr. Clark considered while using IDs. He expressed that, generally, the interactivity of IDs increased student engagement. He noted, for example, that enacting an ID activity was "more engaging for the students than me creating a table by hand and moving to a graph by hand or whatever the situation might be" (personal communication, October 26, 2015). Indeed, "keeping students on track and managing behaviors […] just comes with using the diagrams" (personal communication, February 25, 2016) because they were so engaging for Mr. Clark's students. During the observation of the *Parallel Lines ID* activity, for example, "the kids that were selected [to use the ID] were excited about using it" (personal communication, December 2, 2015).

Mr. Clark leveraged the student engagement fostered by IDs to facilitate learning in his classroom. He noted, however, that this engagement was not essential in his use of IDs "because […] students can still get a lot out of the IDs even if they dislike them" (personal communication, February 25, 2016). That said, students were observed being overwhelming excited about using IDs. They demonstrating their engagement by stating "I want to go next!" (observation, December 2, 2015) and other such comments in reference to using the ID. Students clearly enjoyed interacting with IDs.

Only a small group of select students had the opportunity to directly manipulate IDs within Mr. Clark's classroom, however, due to the limited technology he had access to. This, he explained, could easily "leave the rest of the class out to dry or like they don't really understand what's going on" (personal communication, February 19, 2016). Consequently, Mr. Clark created ID activities that intentionally engaged the remainder of the class using teacher-lead questioning. He explained his thinking around student engagement while discussing the *Line of Best Fit ID* activity.

> So, my hope is to ask [questions] to someone who's not the person interacting with the diagram. It's a way to make sure that the students are also paying attention with what's happening with the diagram while the person up there is interacting with the diagram. […] I don't know if engagement is the word, but they're paying better attention when they know that they're going to be asked questions in the audiences and it's not just the one person, like, an interaction between me and that one person. (personal communication, February 18, 2016)

Thus, Mr. Clark planned and enacted ID activities that were particularly concerned with engaging students who were not directly interacting with the ID. He worked to engage the entire class by asking questions to everyone about what was occurring in the ID, what steps should be taken next, and what they are learning about the focal content. The specific attention Mr. Clark paid to student engagement in this way influenced the ways in which he used IDs in his planned and enacted curriculum.

Behaviors. While IDs seemed to naturally engage students in his class, Mr. Clark still experienced challenges surrounding student behaviors. He seemed to recognize that, "there's always times where student behavior can interrupt a lesson or cause an issue with the lesson" (personal communication, December 1, 2015). While he identified student behaviors within the school as heavily influencing his use of IDs (see contextual factors section) Mr. Clark expressed that challenging student behaviors within his own classroom did not have an effect on his ID use. This was interesting as a number of off-task and otherwise distracting behaviors were recorded during both observed lessons. The student behaviors included students playing basketball with balled up papers and the trashcan, using inappropriate language, being confrontational toward each other, picking up and moving classroom furniture, and making sexually explicit comments during the ID activity (observations, December 2, 2015 & February 19, 2016). Mr. Clark explained that the behaviors recorded during the *Parallel Lines ID* activity "carried over from [other school programs] where some students were on their phones and students were just talking and came in and were a little giddier, excited and talkative" (personal communication, December 2, 2015) (See contextual factors section for more information on school programming). Mr. Clark addressed behavior challenges with general success. Nonetheless, these behaviors, as well as Mr. Clark's efforts to address them, distracted students from the ID activity and decreased the amount of time available to enact his planned curriculum. Thus, students' behaviors mediated the enacted curriculum involving

ID activities in ways that Mr. Clark could not predict. He also didn't seem to fully recognize the degree with which student behaviors affected his ID enactment.

Taken together, student factors significantly influenced the ways in which Mr. Clark used IDs within both his planned and enacted curriculum. These factors are depicted in Figure 10 using the study's conceptual framework.

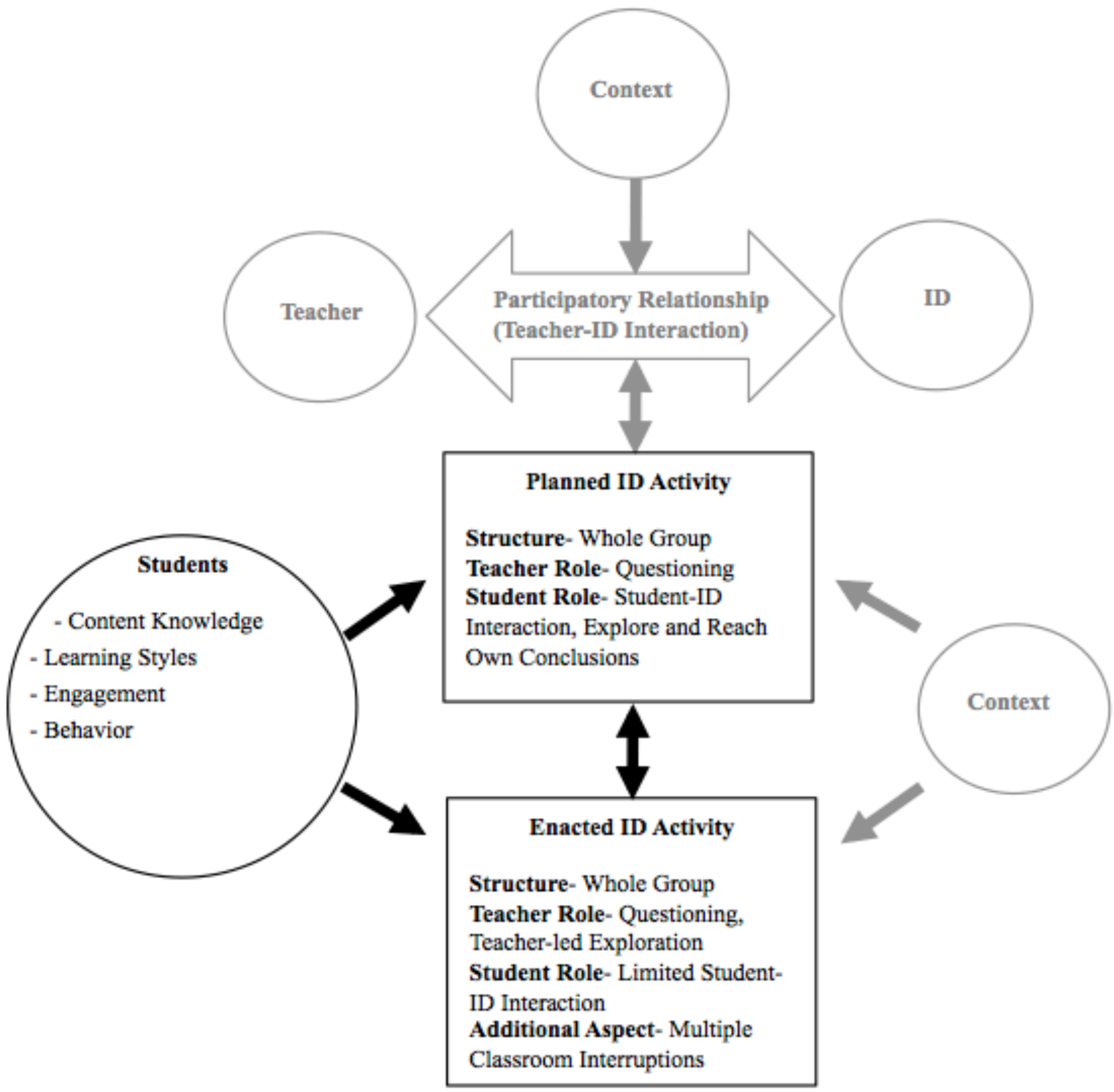


Figure 10. Student factors affecting Mr. Clark's use of IDs.

First, students' content knowledge encouraged his use of ID activities, as Mr. Clark believed that they increased students' access to grade level content. Individual student's content knowledge also influenced which students used IDs during the enacted activity. Additionally, Mr. Clark considered the alignment between students' learning styles and the design of particular IDs. Lastly, the inherent ability of IDs to increase student engagement encouraged his ID use while challenges surrounding student behavior mediated how planned ID activities were enacted within the classroom.

Contextual Factors

Lastly, contextual factors also had a significant impact on both how Mr. Clark created the planned curriculum and how that planned curriculum was enacted during IDs activities. Of note was the mediating effect of the technology available to him and his school's culture and climate. These are described below, as well as how other district and school-level contextual factors affected Mr. Clark's ID use.

Mandated curriculum. As mentioned previously, Agile Mind was the mandated curriculum for secondary mathematics in DSS. This factor seemed to be a consideration for Mr. Clark's instructional use of IDs. As he explained, "The curriculum we use is full of [IDs]. So, I try and use them and follow somewhat close to the curriculum" (personal communication, October 26, 2015). Thus, the fact that IDs were heavy integrated throughout Agile Mind and, consequently, widely available to Mr. Clark facilitated his use of IDs while creating the planned curriculum.

The accountability measures associated with using the district-mandated curriculum also seemed to influence his planning of ID activities. He explained,

> If someone's checking in to make sure you're using it, the fact that it's mandated […] If district representatives are checking in on you and ensuring that you're using it and scolding if you're not, or congratulating you if you are. I think that's something that impacts it as well. (personal communication, February 25, 2016)

The feedback Mr. Clark received from district personnel about his use of Agile Mind and the included IDs incentivized him to use them frequently. This feedback mechanism was not limited to district representations, he noted, as it also applied to the school administrations' focus on using the mandated curriculum.

> I also think the [school] administration's push to use a mandated curriculum is also something that definitely impacts it. […] I think our administration was really on top of making sure that our usage data was up, which relates to the IDs as well. (personal communication, February 25, 2016)

Thus, DSS's decision to adopt Agile Mind, an ID rich curricular resource, as mandated curriculum for secondary mathematics seemed to increase the frequency with which Mr. Clark incorporated IDs into his planned curriculum.

Further, Mr. Clark indicated that he used the teacher resources provided by Agile Mind as guides for how he should plan ID activities. Of note were the suggested questions included on the Student Activity Sheets and Advice for Instruction planning materials provided to teachers by Agile Mind. He affirmed that it was typical for him to structure ID activities using the "specific questions, specific parts of the workbook that go along with their diagram" (personal communication, December 2, 2015).

> I think a lot of the questions that are built into Agile Mind and into the Student Activity Sheets get students to think, and thinking differently ways about the material that they are studying, whatever we are looking at. So, I ask a lot of the question orally that are on there. (personal communication, February 19, 2016)

Thus, Mr. Clark's planning of ID activities was affected by the supplementary resources embedded within the larger curricular resource provided to him by the DSS. This was particularly true for the questions he planned to ask his students while they engaged with IDs in his classroom.

School climate and culture. Another contextual factor influencing Mr. Clark's interactions with IDs was the climate and culture of his school. This factor didn't seem to directly influence his planning of ID activities, but did significantly affect the enactment of these activities. He explained,

> The whole year so far has been fairly rough with behavior school-wide. […] The [student] behavior in the hallway definitely affects the classroom, you know, whether it's fire alarms being pulled or fights happening or students running down the hallway and yelling that a fight happened or whatever is going on and then the students are getting totally distracted and it takes some time to bring them back in. But yeah, it definitely affects the classroom. (personal communication, February 18, 2016)

Climate and culture concerns mediated the enactment of ID activities by distracting both his and his students' attention away from the activity at hand. This decreased the instructional time available to enact IDs in the ways that they were planned. Mr. Clark described a specific instance that illustrated how the school's culture and climate affected his ID implementation as follows-

> So, today I was supposed to teach a lesson [using an ID] and, unfortunately, […] there was a fight and I had to break up the fight and this is while I was supposed to be teaching. So, I'm breaking up a fight in another room and obviously I didn't get through the lesson I had hoped to get through. So, I came back and kind of gave remedial information because there was only so much time left in the period after that. […] So, [school climate] is something that definitely impacts whether I get to use interactive diagrams. (personal communication, October 26, 2015)

The actual fight he described here was only the start of the distraction for Mr. Clark's instruction, though, as he also pointed out that when "there was a fight the period before I have my kids, it's really hard to get them settled back down" (personal communication, October 26, 2015). The time spent settling his students down continued to decrease the amount of instructional time available for enacting planned ID activities.

While the school's climate and culture may have been challenging, Mr. Clark was quick to emphasize that this did not affect whether or not he planned activities using IDs. It did, however, affect the implementation of any classroom activity, including those of interest to this study. In other words, he saw that the school's climate and culture didn't "directly attribute to whether I use [IDs]. It just impacts the usefulness of the ID. I can project the ID, but if someone continues to bang on my door every two minutes… I'm kind of using it but I'm not getting out of the ID what I'd hoped" (personal communication, February 25, 2016). Thus, the behavioral concerns outside of his classroom seemed to be a significant mediating factor affecting primarily his enacted curriculum.

School staffing. Other school-level factors also influenced Mr. Clark's ability to implement ID activities. Of note was the impact that the absence of highly qualified and effective teachers had on his ability to remain in his own classroom. Exiting his classroom, in turn, mediated the amount of instructional time he had available to enact ID activities. Speaking about the presence of substitute teachers in the building, for example, he explained, "there's things […] that pulled me from the classroom that made me give lessons to a substitute or a hall monitor or […] cover a class" (personal communication, October 26, 2015). Mr. Clark was pulled from his classroom to stabilize classrooms where substitute teachers were present, for example. Thus, he wasn't able to enact ID activities in his own classroom during these instances. Managing behaviors in classrooms where substitute teachers were present was not the only situation where Mr. Clark was pulled from his own classroom. He explained,

> Last year […] unfortunately we had a lot of long-term subs and we had some teachers that aren't officially certified. […] Some of the state testing you had to be a certified teaching to administer the test. So, last year I was pulled for a period that felt like once or twice a week for like two months […] to give one part for English, part for math, which is the state test. (personal communication, October 26, 2015)

Taken together, Mr. Clark was pulled from his regular classroom instruction where he might enact IDs due to covering classrooms where substitutes were present and to administer standardized assessments. These occurrences prevented the enactment of ID activities.

School staffing challenges also affected Mr. Clark within the four walls of his own classroom. During an observed lesson, for example, he began introducing an ID activity by stating, "before we do this, we started talking about this yesterday, but we got interrupted by the art teacher" (observation, December 2, 2015). During that same lesson an adult knocked on Mr. Clark's classroom door. He answered the door and spoke with the adult, pausing the classroom activity. When asked about these moments, Mr. Clark shared how the visual arts teacher across the hall often interrupted his classroom instruction.

> So, there's an art room across the hall that has a teacher who is […] struggling with the classroom management. […] Kids just kind of enter and exit his room at will. And so one of the things that I was trying to do with the kids from his classroom was get them sent over here for misbehaving. […] I think he had brought over a student or something that I allowed to sit at the door and then as time went on more and more students just came over and were opening the door, coming in and kind of talking to students. So, distracting students. (personal communication, December 2, 2015)

Interruptions such as these occurred so frequently, in fact, that Mr. Clark anticipated similar episodes while planning ID activities. He noted, "often times students are sent into my room or the teacher will pull me into like reprimand or to pull a student out" (personal communication, December 1, 2015). Distractions originating from the classroom across the hall negatively impacted Mr. Clark's ability to enact IDs in the ways that he planned because they decreased his and his students' focus, as well as the instructional time available for the activity.

School programming. The presence of extracurricular programs within Mr. Clark's school also mediated his enactment of planned ID activities. He explained, "School programs interrupt or cancel class periods. So, we have the mentoring [program] PGC. We also have assemblies and shortened days that split classes or class coverage. That definitely impacts [ID implementation]" (personal communication, February 25, 2016). Mr. Clark pointed out that many of these programs, like assemblies and shortened days, were unexpected. As such, these events mediated the enactment of planned curricular activities much like the climate and culture challenges described above, namely by disrupting or eliminating class time.

School programs also impacted ID activity planning. PGC, which was briefly mentioned above, was a peer-mentoring group present in Mr. Clark's school. This group met regularly and, consequently, the school's faculty and staff typically knew when these meetings would occur with advanced notice. Thus, PGC meetings could be accounted for in Mr. Clark's planning. The meetings affected his planned curriculum in that they occurred during class periods when he could otherwise enact an ID activity. This is illustrated in the following conversation.

> Researcher: Does PGC, does that take place on a period […] when you would normally be teaching?
>
> Mr. Clark: Yes. So, I didn't teach my middle class, this is the last class in the day. My 3/4 period was my middle period of the day. I did not get to teach the same lesson.

> Researcher: Okay. So, then that period wouldn't have been able to interact with the ID?
>
> Mr. Clark: Exactly. (personal communication, December 2, 2015)

Other aspects of school programming also mediated Mr. Clark's planning of ID activities, including being excused from teaching classes to attend meetings. He described such an instance, "I was pulled for an SST 504 meeting […] and so there was another teacher covering my class. So, I didn't have the chance to teach the same lesson [involving an ID] to the period prior" (personal communication, February 19, 2016). Taken together, PGC and other meetings removed both Mr. Clark and his students from the mathematics classroom during particular class periods throughout the day. This altered how IDs were enacted, or more accurately not enacted, for that particular class period.

In addition to affecting his planned curriculum by canceling class periods, Mr. Clark explained how the PGC meetings mediated the enacted curriculum in ways that were less predictable.

> And typically the class period after [PGC] there's like a carryover of kids being hyper excited. So, it just typically takes longer to get students to engage or refocus back to the typical routines in the day because [PGC] kind of pulls them out of their normal routines that they are used to. And again, there's not an official teacher there. So, sometimes some of the PGC groups are a little wilder. So, some of the students come and they have just been throwing things or playing. […] So, it takes a little longer to get them to refocus.

The nature of PGC meetings resulted in disruptive student behaviors during that class period and the next. During an observed lesson that followed a PGC meeting, for example, Mr. Clark pleaded with his students to focus on the class content by saying, "Come on ladies and gentlemen, I'm not going to fight with you all period. I know GSC was last period. Let's go" (observation, December 2, 2015).

Thus, it seemed that school programming had a mediating effect on both Mr. Clark's planning and enactment of ID activities. A primary effect of school programming, including PGC meetings, SST 504 meetings, assemblies, and irregular school schedules, was that Mr. Clark or his students would be pulled away from the classroom, preventing ID enactment. Secondarily, these programs negatively affected students' behaviors, which, in turn, mediated his enacted ID activities.

Technology availability. The functionality and availability of technology seemed to have the largest impact of any contextual factor on Mr. Clark's interactions with IDs. He had a teacher "laptop and projector hooked up, which are what [he] mainly [used to] access the interactive diagrams" (personal communication, October 26, 2015). A laptop cart was also always present in his room. Mr. Clark described the presence of this cart as not necessarily beneficial, however, due to its poor functionality. He explained, "We physically have [laptops]. I have a laptop cart there that has three working computers on it for an entire class. So, that obviously doesn't help much."

There were other technological resources within the school that Mr. Clark had access to, including "a computer lab which is occasionally available" (personal communication, October 26, 2015). This computer lab, though, "may only have 20 working computers and you may have 28 kids in your class. So, that brings up a little issue there." There is also a Mac lab "which is nice, [but] doesn't have enough computers for the sizes of the class." Mr. Clark repeatedly expressed concern surrounding the low number of available, reliably functioning computers.

The availability of mostly unreliable laptops and desktop computers significantly influenced the ways in which Mr. Clark created the planned curriculum involving IDs. He explained,

> I've been caught like a couple of times where I'm like, 'Awesome, I have this great lesson where we're all going to get on computers' […]. And then we get there and there's only 10 working computers […] and it just becomes a nightmare of trying to get three kids all on one computer. So, I think it definitely is discouraging at times to have things like that happen and then you're like, 'Well, I'm not going to go to the lab for a while.' (personal communication, October 26, 2015)

Mr. Clark was less likely to structure a classroom activity where students were interacting with IDs on individual computers because he wanted to avoid the situation where those computers failed to operate correctly. Not surprisingly then, Mr. Clark's typical ID enactment involved solely the use of his one teacher computer and a projector.

Mr. Clark emphasized that his typical enactment of ID activities was incongruent with what he envisioned as his ideal enactment. He noted, "I wish there's a better way to get all students on a computer [so that] that they can manipulate it themselves" (personal communications, October 29, 2015). Instead, most students watched the ID being manipulated by someone else. The difference between Mr. Clark's desired and actual ID enactment was highlighted when he was asked to describe his ideal classroom.

> Mr. Clark: If everything was perfect and my kids either all had laptops or tablets or something they could all interact with [the ID], I'd just be able to give more students the opportunity to play with the slope or whatever's on the interactive diagram and generate their own conclusions because you can only do so much on a projector with one or two students coming up and playing or messing with […] the diagram.
>
> Researcher: So would you say you have enough technology to [enact IDs] how you want to?
>
> Mr. Clark: No, I do not, unfortunately. (personal communication, February 18, 2016)

Mr. Clark's limited access to technology seemed to explain why his typical ID enactment overwhelmingly involved whole group activities instead of individual or small groups.

The state of the school's Internet seemed to be an additional technological factor that affected Mr. Clark's enactment of ID activities. He explained, "The Internet has been somewhat of an issue. Sometimes it would work, sometimes it won't" (personal communication, October 26, 2015). This was an obstacle because all of the IDs Mr. Clark intended to use were found online. Consequently, he couldn't access IDs when the Internet was malfunctioning. Faulty Internet prevented the enactment of any planned ID activity.

Lastly, the school's climate and culture also had an impact on the technology Mr. Clark had access to. He explained,

> Students have been breaking into classrooms and stealing things before and after school. And during that process my projector was stolen. […] I took one for the team and gave the remaining projectors to the other teachers. (personal communication, December 1, 2015)

Mr. Clark no longer had a projector to display any ID he might want to enact during the latter portion of the study. Without a projector he was mostly "having to chalk out graphs and tables and things that you can develop through […] interactive diagrams" (personal communication, December 1, 2015) on his classroom chalkboard. He also pointed out that there were a number of IDs within Agile Mind that he skipped because he could not secure a projector to display them. Thus, the lack of access to a projector significantly mediated the ways in which he created planned curriculum by eliminating the possibility of using any ID in his classroom.

Technology was clearly a factor that heavily mediated Mr. Clark's ID use. The absence of a class set of functioning laptops or desktop computers persuaded him to create planned ID activities absent of small group or individual work despite his desire for such classroom structures. The enactment of ID activities was also influenced by the inconsistency of his school's Internet. Further, the theft of his projector later in the school year ceased the creation of planned curricula involving IDs at all. Taken together, Mr. Clark emphasized, "Access to technology or the lack of technology is definitely something that impacts the way […] I use [IDs], and when and how I'm able to use it" (personal communication, February 25, 2016).

School administration. Technological leadership at the school level did not seem to be a significant factor directly influencing Mr. Clark's interactions with IDs.

Indirectly, however, the instructional freedom given to Mr. Clark by his school administration did seem to affect his ability to utilize IDs how he desired within his practice. Mr. Clark expressed, for example, that his principal has "been nothing but supportive" (personal communication, December 1, 2015) of his use of IDs. He continued that it was "a blessing this year that [administration was] opened to experimentation. They trust the teachers and they trust that whatever the teacher is doing is right for the students" (personal communication, February 18, 2016). Mr. Clark was neither encouraged nor discouraged to utilize IDs as long as the school administration viewed his instruction as beneficial for students. He elaborated on this idea by noting, "I think [IDs do] increase engagement. So, I think if [administration] came in and students were manipulating some type of interactive diagram and discussing the concept, I think they would be like 'your T3 [student] engagement was great today'" (personal communication, October 26, 2015). Administration, it seemed, was more focused on the teacher evaluation framework when discussing Mr. Clark's practice than on his use of IDs specifically, as indicated by Mr. Clark highlighting T3 student engagement (a particular component of this framework). IDs seemed to foster instruction aligned with school administration's ideas of quality instruction. Thus, they were amenable to their use. Administration did not explicitly support or encourage the enactment of IDs, however.

Taken together, there seemed to be a number of contextual factors that influenced both Mr. Clark's intended and enacted curriculum involving IDs, as shown in Figure 11.

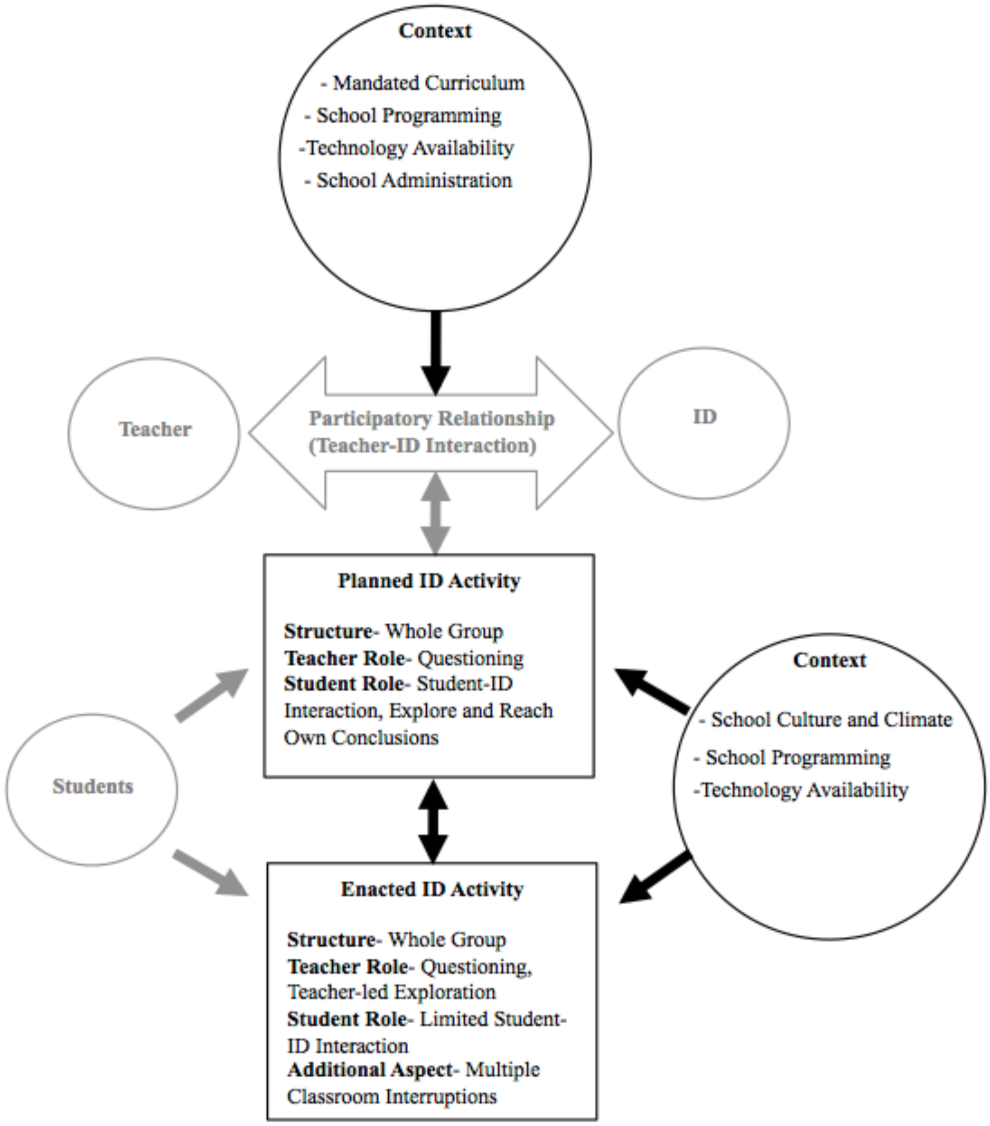


Figure 11. Contextual factors affecting Mr. Clark's use of IDs.

These factors included the district mandate that Agile Mind was to be used to teach the IA course, school administration's flexibility in their technology leadership, programming occurring within Mr. Clark's school, the availability of technology needed to use IDs, and the climate and culture of the school. The latter three of these factors seemed to have the most significant influence on Mr. Clark's ID use.

Summary

Mr. Clark planned and enacted ID activities within a complex system filled trade-offs, tensions and nuance within his practice. Figure 12 captures this system as it was observed throughout the current investigation utilizing the study's conceptual framework. Here, we see that numerous ID, teacher, student, and contextual factors combined to mediate both the planned and enacted curriculum involving IDs.

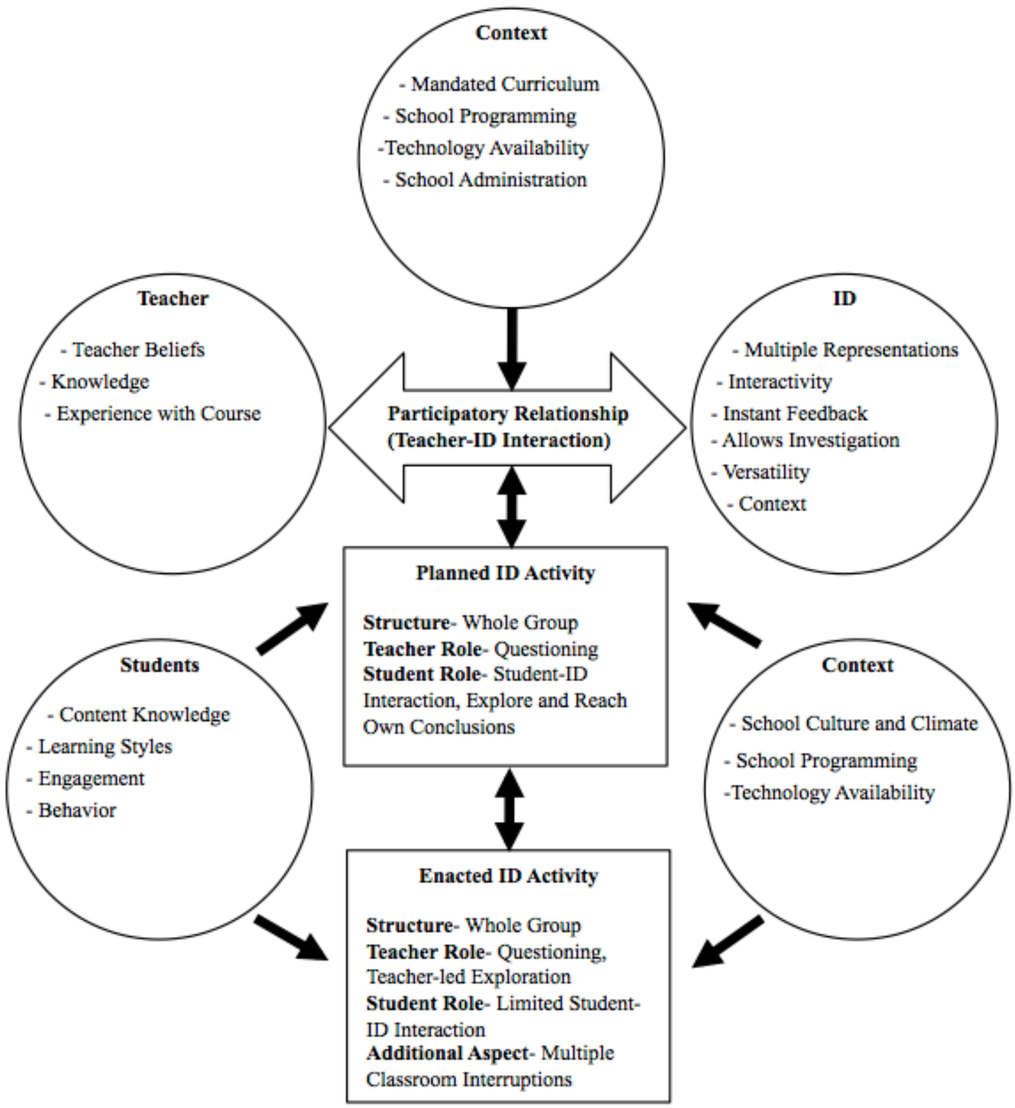


Figure 12. Mr. Clark's ID use and the factors affecting it.

Focusing on ID themselves, Mr. Clark described a number of factors that affected his creation of the planned curriculum. He favored, for example, IDs that facilitated content exploration by incorporating multiple mathematical representations, a high level of interactivity between the ID and its user, and instant feedback resulting from the user-ID interaction. Additionally, he emphasized the importance of an ID's context demonstrating real-world applications of mathematical concepts within contexts that were meaningful and engaging for his students. Consequently, IDs that included these design features were more likely to be used by Mr. Clark within his planned curriculum.

Student factor also mediated Mr. Clark's instructional use of IDs. Deficiencies within students' content knowledge, for example, encouraged his use of IDs and affected how he used them in his classroom. This was due to the fact that IDs enabled access to grade-level content by providing support for overcoming students' gaps in necessary pre-requisite skills. Additionally, Mr. Clark identified many of his students as either visual or kinesthetic learners. This type of learning could be well achieved, he insisted, using ID design features and was, consequently, something he attempted to facilitate during ID activities. Lastly, instances of students' off-task behavior affected the enactment of ID activities by decreasing both the teacher's and students' focus, as well as the instructional time needed to engage with the ID as he intended. Mr. Clark largely overlooked this latter factor's significant mediating affect while discussing the enactment of ID activities, however.

Factors associated with Mr. Clark seemed to influence how he used IDs as well. He identified both his content and curricular knowledge as factors that mediated how he used IDs. The former enabled him to understand how IDs approached content and, thus, use them appropriately while the latter enabled him to reflect of past enactment of familiar IDs and refine his ID use. Lastly, Mr. Clark seemed to hold a number of beliefs affected his use of IDs. These included the beliefs that mathematics learning should be meaningfully active, cooperative, constructive, and authentic.

Lastly, Mr. Clark identified contextual factors as having the most significant affect on the ways in which he used IDs. When asked explicitly about the effect of all factors surrounding his instructional use of IDs, Mr. Clark explained, "the presence of students in the hallway, the presence of substitute teachers, the state testing are things that on a day-to-day basis may impact it. […] *But whether you have technology will affect you every single day*" (personal communication, February 25, 2016, emphasis added). Mr. Clark elaborated on how the availability of technology was the paramount factor affecting ID enactment,

> I mean, if I don't have the technology or a projector or a laptop there's no way I'm ever going to be able to use the IDs. Period. So, I figured that was the most impactful [factor]. And again, the functionality or the usefulness of the technology- if I have a broken projector again, I'm not going to be able to use it. I thought those were the most important two, because I could entirely predict whether I'm going to use it or not.

Thus, both the presence and the functionality of any available technology were of paramount influence in creating planned ID activities and in the enactment of those activities within Mr. Clark's classroom.

Chapter 5

This chapter describes the case of Natalie Edelman, a second-year teacher in DSS. Ms. Edelman described numerous factors that seemed to affect her instructional uses of IDs, including but not limited to those surrounding her beliefs about the ways in which mathematics should be taught and learned, the degree to which ID design features allowed students to engage with mathematics content, and students' behaviors. The focal IDs Ms. Edelman used during this study, her intended and enacted curriculum involving these IDs, and the various ID, teacher, student, and contextual factors that influenced her interactions with IDs are described below.

Implemented IDs

The ID used by Ms. Edelman during her first lesson was found within the topic Exploring Rate of Change in Motion Problems within the IA course. This ID will be referred to as the *Skateboarder ID* (Figure 13). In this ID, the user initially positions the skateboarder figure on the bottom of the screen. The skateboarder's position is measured relative to the motion sensor to the left of the screen by the number line present just below the skateboarder. Then, as the user manipulates the skateboarder, a graph of time versus position is produced on the coordinate plane at the top of the screen. The goal for the user is to match the provided green graph with the graph produced when he or she manipulates the skateboarder. The ID can be reset as many times as the user wishes, allowing him or her to explore how to match the graph over a number of attempts.

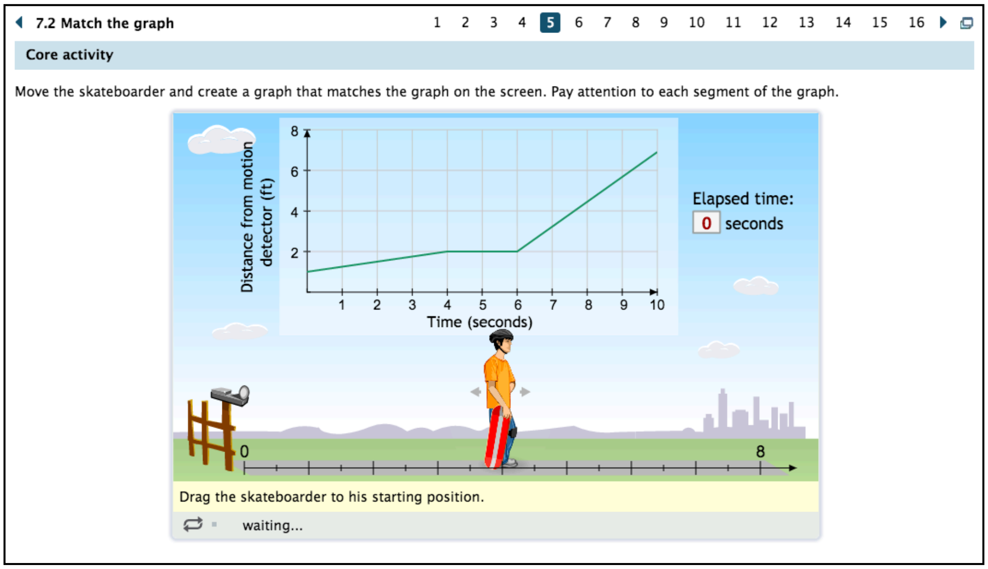


Figure 13. Skateboarder ID. Reprinted from Intensified Algebra Course, topic 7, lesson 4, slide 5 in *Agile Mind*. n.d. Retrieved April 9, 2016 from http://www.agilemind.com. Copyright 2016 by Agile Mind.

The ID used by Ms. Edelman in the second observation, which we will refer to as the *Perpendicular Lines ID* (Figure 14), was found within the Understandings Slopes and Intercepts topic of the IA course. Here, the user encounters two graphed linear functions and their coordinating algebraic representations, one in blue and one in red. The sliders on the left of the screen allow the user to alter the blue function's slope and *y*-intercept. Similarly, the sliders on the right allow the user to manipulate the red function's slope and *y*-intercept. The algebraic and graphical representations of each function change dynamically to reflect how the user alters the values of the slope and y-intercept. The text on the top of the screen invites the user to manipulate the two linear functions as they assess the validity of a previously created conjecture about the relationship between the slopes of perpendicular lines.

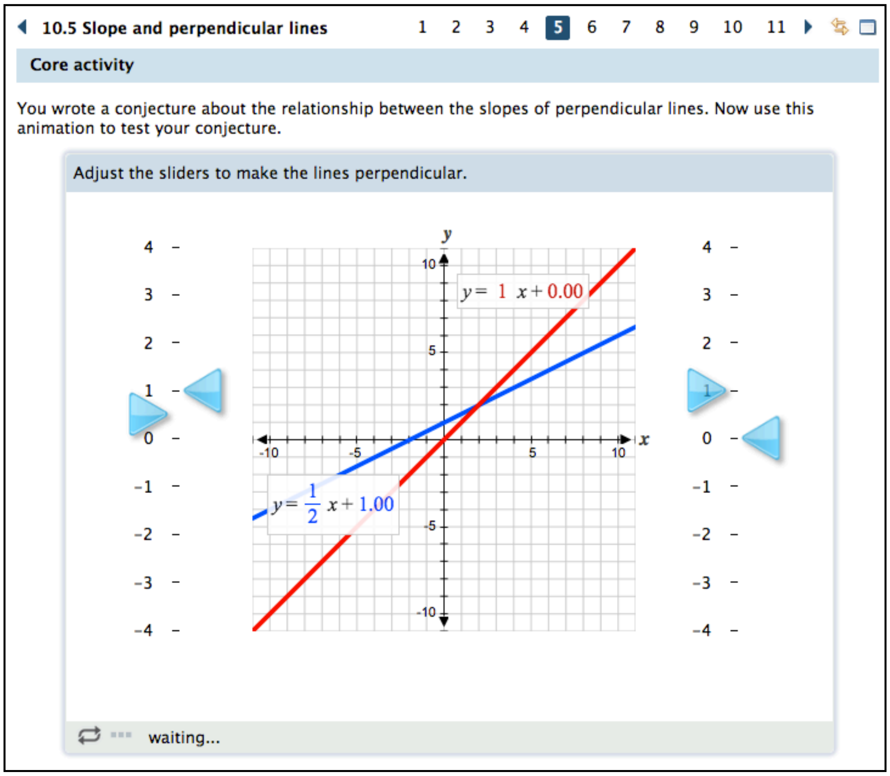


Figure 14. Perpendicular Lines ID. Reprinted from Intensified Algebra Course, topic 10, lesson 5, slide 5 in *Agile Mind*. n.d. Retrieved April 9, 2016 from http://www.agilemind.com. Copyright 2016 by Agile Mind.

Intended Curriculum

Ms. Edelman's planned ID activities were largely consistent throughout the study in terms of their underlying goals, the classroom structure used to achieve these goals, her perception of students' roles, and her planned role as the teacher. Each is described in turn below.

To begin, Ms. Edelman continually expressed that an important goal in using IDs was for students to make sense of mathematics content through inquiry, conversation, and productive struggle. She explained,

> I think it's [important] to facilitate problem-solving skills. Even if [students] never used [a particular mathematics concept or skill] in their job or real life, they have developed skills to look at problems, break them apart, figure out how to solve them and developed a bunch of tools they can use to apply to a diverse amount of problems that they might run into. (personal communication, March 10, 2016)

Her emphasis on engaging students in problem solving was apparent in how she spoke about planning IDs activities. Ms. Edelman utilized classroom activities where students "messed with" IDs in order to discover particular mathematical relationships, such as those that exist between the distance, time, rate, and slope of a linear function.

The intended classroom structure used to facilitate ID activities remained consistent in both Ms. Edelman's reflections and her planned classroom activities. Namely, students would work in small groups or pairs using a common laptop computer. Within these groups, students used "the guided notes in the Agile Mind workbook [to answer] questions about what's going on [in the ID]" (personal communication, December 15, 2015) and lead them toward the desired mathematical concepts embodied in the ID. A whole class debrief of the activity was planned "just to make sure that everybody is on the same page […] especially if I see there are common misunderstandings in the class." During this debrief, Ms. Edelman often planned to have particular students "share out what they learned" to the entire class.

Ms. Edelman used the structure described above with the goal of putting students in control of their own learning. She explained, "I'm hoping to get the kids to take some ownership and do some stuff themselves using this ID" (personal communication, December 15, 2015) because she saw students' role as an active one during mathematics learning, generally, and ID activities, specifically. Consequently, she would often "give the kids significant time to play with [the ID] and see what happens" (personal communication, October 29, 2015). Note that Ms. Edelman's intent was to engage students in recognizing mathematical patterns within an inquiry-based learning approach. Students were charged with discovering mathematics concepts by discussing the patterns they noticed in the ID with their peers. Speaking of the *Perpendicular Lines ID*, for example, Ms. Edelman described how she planned for this type of active learning to occur in her classroom.

> Everybody has an opportunity to mess with [the ID] and they have time to talk to each other about what they're learning. […] They get to play with [the ID]. Other kids get to see while you and your partner plays with it or somebody at the table does. They're going to be able to discuss with each other, what they noticed about what's going on in the graph or in the ID. (personal communication, December 15, 2015)

In short, Ms. Edelman's students were to "mess with stuff [in the ID], answer some questions and have a discussion about it" (personal communication, October 21, 2015) as they acquired mathematical understandings. She was quick to point out the importance of student perseverance during this type of learning activity. She emphasized, "I guess the student's role is […] to persevere, like not shut down when something becomes challenging or they don't understand something or it's confusing and to work through that. It's problem solving. Preserving through problems" (personal communication, March 10, 2016).

The teacher role, Ms. Edelman described, was that of a facilitator of the student-centered learning she hoped to create in her classroom. She planned to "give [students] instructions and then let them do it and then come back maybe a minute later […] and check to see where they are with it. And then have the kids explain […] what's going on" (personal communication, October 29, 2015). She planned to introduce an ID activity by explaining only what was minimally required for students to begin the task, usually the basic functions of the ID and what, generally, she wanted students to figure out. Then, she planned to circulate the room to "make sure that everybody is on task [… and reinforce] instructions about what they're supposed to do as far as writing down their findings in the workbook" (personal communication, December 15, 2015). Ms. Edelman noted that this approach allowed students the "space to figure out [patterns and mathematical concepts] once they're given a problem or something to figure out and allow them to integrate that information or knowledge" (personal communication, March 10, 2016). Most importantly, though, was that she engaged students with questions which probed what they were learning from the ID and assessed their emergent understandings. In doing so, she aimed to help students,

> Trust their own intuition about what they see [in the ID]. Because I think a lot of kids over think it when I ask them questions like, 'What did you see here? What's going on in this?' They're like, 'I don't know what you mean.' (personal communication, October 21, 2015).

Thus, Ms. Edelman seemed to view her role as developing students into more confident and active mathematics learners by engaging them in tasks rooted in problem solving and discovery learning.

In summary, Ms. Edelman desired a classroom where students were working in small groups or pairs to investigate mathematics concepts autonomously. She viewed her role as that of a facilitator who engaged her students through purposeful questions aimed at guiding students in their discovery of mathematics understandings. Ms. Edelman's emphasis on planning student-centered group activity was epitomized when she stated, "So, I think that with the interactive diagrams, like giving the kids autonomy and trusting them with that autonomy is like a big piece of it" (personal communication, October 21, 2015).

Enacted Curriculum

Ms. Edelman's enacted curriculum involving IDs was generally consistent with her intentions described above; overall, the observed instruction and teacher-student interactions communicated and facilitated the underlying goals, classroom structures, students' roles, and teacher role that Ms. Edelman planned for ID activities. The implementation of ID activities was altered, however, when one looks at the details of the enacted curriculum. Ms. Edelman's enacted curriculum involving IDs is described and contrasted with her planned ID activities below.

The intended goal of engaging students in ID activities focused on inquiry learning, peer collaboration, and productive struggle was consistently evidenced within enacted ID activities. Indeed, student groups were charged with exploring the mathematical concepts present in both the *Skateboarder ID* and the *Perpendicular Lines ID* with little initial explanation from Ms. Edelman. When introducing the latter, for example, she introduced the ID by simply asking students to "move the sliders around […] talk with your group […] and tell me what happens. What do you notice? Then, in the space on your paper, tell me" (observation, December 16, 2015). Students were asked to note the patterns and relationships they observed as they interacted with the ID in their workbooks. Their explorations were facilitated by the probing questions Ms. Edelman asked, such as "When we move the slider on the left, what happens?", "What did you have to do to get [the graphs] lined up?", and "How do we know that?" (observation, November 3, 2015). Her questions focused on what students were noticing, how those observations were related to each other and other content they had learned previously, and other mathematical questions they may want to investigate with the ID. Ms. Edelman also allowed students to struggle when the answers to her questions were not immediately apparent. This enactment was, in large part, aligned to what Ms. Edelman had intended for ID activities.

The enacted activity structure contained some inconsistencies with Ms. Edelman's planned curriculum, however. First, most students interacted with the ID while working in groups of approximately four. During both observations, however, there was a student seated by himself in the back of the classroom. This student was inconsistently interacting with each ID on an individual basis instead of working collaboratively with his peers, as Ms. Edelman desired. Additionally, student groups interacted with the ID using a ratio at or above three students to one piece of technology (observation, November 3, 2015). This was contrary to Ms. Edelman's intent to "have a computer per pair, but there were some pairs that went without it and I made them use the smart board or something else. Or I made everyone at a table [of four] use a computer. (personal communication, December 16, 2015). Lastly, the *Perpendicular Lines ID* activity concluded with a whole group debrief discussing students' findings as she intended. The debrief that occurred during the *Skateboarder ID* activity was abbreviated, however, as Ms. Edelman noted,

> I wanted to share those kids' findings with some of the other kids who were struggling because I was feeling like we were running out of time. I was like, "We need to make this connection." And I felt like I had to force it for all the other kids who may have not made those connections. (personal communication, December 16, 2015)

Thus, due to time constraints, the activity debrief was not enacted in the manner she planned. Taken together, the enacted activity structure included instances of students working individually, large student groups with limited technology, and a reduced activity debrief, all of which were not part of the planned curriculum Ms. Edelman created.

The aforementioned modifications to the intended ID activity structures impacted students' roles during the enacted curriculum. Recall, for example, that Ms. Edelman wanted students interacting with IDs. As she noted, "The students' role is to engage with the ID and try to use the ID to help understand the content" (personal communication, March 10, 2016). Aligned with this thinking, Ms. Edelman's instructions to the class and individual students throughout the activity consistently emphasized that all students should "now play with this [ID]" (observation, November 3, 2015). Numerous students did not directly interact with the ID during the classroom observations, however, as they never controlled the laptop or SmartBoard within their group. Ms. Edelman recognized this when she noted, "Students watched someone else use the ID because I think that's what most kids are doing because I rarely have computers to be able to use in the classroom" (personal communication, March 10, 2016). Many students were not directly interacting with the ID, but were indirectly observing the direct interactions of a small number of students who controlled the ID on the group's laptop or the class's SmartBoard.

Further, Ms. Edelman planned for students to discuss their emergent understandings with their peers as they interacted with the activity IDs. This occurred during the *Skateboarder ID* activity, for example, when students helped each other match the graphs they created by moving the skateboarder to the given graph by telling their peers to "go faster, the line is steep" and "stop!" (observation, November 3, 2015). Ms. Edelman highlighted students' discussions as she reflected on the activity. She noted, "I had a lot of kids explain to each other what was going on" (personal communication, November 3, 2015). She also encouraged student discussions during the *Perpendicular Lines ID* activity through her facilitation of the activity, such as when she asked a student, "You get it right? Can you explain it to [another student]" (observation, December 16, 2015). Ms. Edelman did not intend for there to be such frequent peer-to-peer interactions focused on non-academic matters, however. Students were observed holding frequent off-task conversations. These conversations will not be detailed here as they are discussed at length in the student factor section below. It is important to note, though, that students' non-academic conversions distracted from their discussions surrounding content. Ms. Edelman had not intended this when she created planned ID activities.

Ms. Edelman's role during the enacted ID activities also evidenced modifications from what she described in her intended curriculum. Recall that Ms. Edelman desired to facilitate students' learning through purposeful questioning and not simply tell students what they needed to know. Before using the *Perpendicular Lines ID*, though, Ms. Edelman "went over what perpendicular means and that the two intersecting lines have to be at a right angle [...]. And then after we've established that, we used the ID" (personal communication, December 16, 2015). Similarly, when she enacted the *Skateboarder ID*, Ms. Edelman spent time ensuring that students knew "what the *x* and *y* axes are and what's labeled" (personal communication, November 3, 2015) in the graph before allow them to interact with ID. In both cases, Ms. Edelman used direct instruction to teach and/or reinforce pre-requisite skills required to investigate the grade-level content contained in the ID. She explained,

> I gave them whatever background I think they need in order to successfully understand the concepts that are being presented in the ID. [... I made] sure the kids understand what the different [pre-requisite] parts are. And then I let them [...] use the IDs to figure out the [grade-level] concepts. (personal communication, March 10, 2016)

It seemed, then, that Ms. Edelman used direct instruction during the enactment of ID activities in preparation for the exploratory, student-centered component she desired. She summarized this practice, "I think I do teach content a lot. I feel like there's a lot of remediation and content that needs to be covered in order for [students] to even access the information presented in the ID" (personal communication, March 10, 2016). It seemed, then, that Ms. Edelman scaffolded her instruction in this way in order to provide students access to ID activities.

Some of the lines of questioning Ms. Edelman used during enacted ID activities were more scaffolded than she seemed to intend. Note, for example, the following interaction she had with students during the *Perpendicular Lines ID* activity.

> Ms. Edelman- "How can we use all of this information to see if two lines are perpendicular if we don't have a protractor?"
>
> Students silent while Ms. Edelman implements wait time. There is no student response.
>
> Ms. Edelman- "What if we look at the numbers we found for the slope? […] Do you think the slopes have something to do with this?"
>
> Student- "Like, they are flipped."
>
> Ms. Edelman- "What do you mean?"
>
> Student- "Like, one goes one over and four down. The other goes over four and then down one."
>
> Ms. Edelman- "What about the sign of the number?"
>
> Student- "They are opposite."
>
> Ms. Edelman- "Do you think that is a rule? What would it be?" (observation, December 16, 2015, italics added)

This interaction began with an open-ended question, which seemed to be what Ms. Edelman wanted to engage her students with during ID activities. When this question was met with no reply, however, Ms. Edelman asked a much more guided question that directed students' attention specifically toward the slopes of the lines. Then, her questioning also directed students to the sign of the slopes. When asked to reflect on this specific interaction, Ms. Edelman explained, "I was trying to get them to make the connections but a lot of my questioning became more and more specific and guided, even though I wanted them to figure it out themselves" (personal communication, December 16, 2015). It seemed, then, that not only did Ms. Edelman recognize that she was asking students "more specific and guided" questions, but also that these types of questions were contrary to students learning in a more exploratory manner.

Lastly, Ms. Edelman spent noticeable time and energy on non-academic matters during ID activities, classroom management being the most notable. Numerous instances of Ms. Edelman addressing student behaviors were noted during the study's two classroom observations. As she noted, "I manage students because there's lots of behaviors that are happening in my class all the time" (personal communication, March 10, 2016). Ms. Edelman also spent considerable time attending to the technology she utilized in her classroom. During the *Perpendicular Lines ID* activity for example, a student explained, "This computer is not working" (observation, December 16, 2015) even after Ms. Edelman had already spent time during the class working on it. She told the student to "just put it to the side [since] some of these computers have connection issues," which she was attempting to fix during the time she had planned to be engaging students with the ID.

A number of classroom interruptions originating from outside of Ms. Edelman's classroom also occupied her time during this same ID activity. Specifically, she addressed a visiting parent, intercom communications, and students entering her classroom while she planned to facilitate the ID activity. Ms. Edelman's attention to these three areas, namely student behaviors, technology challenges, and classroom interruptions, was a departure from her intended role during ID activities, as described within her planned curriculum.

Taken together, Ms. Edelman's enacted ID activities overwhelmingly followed the spirit of her planned curriculum, but were altered in a number of ways. Figure 15 depicts this using the study's conceptual framework.

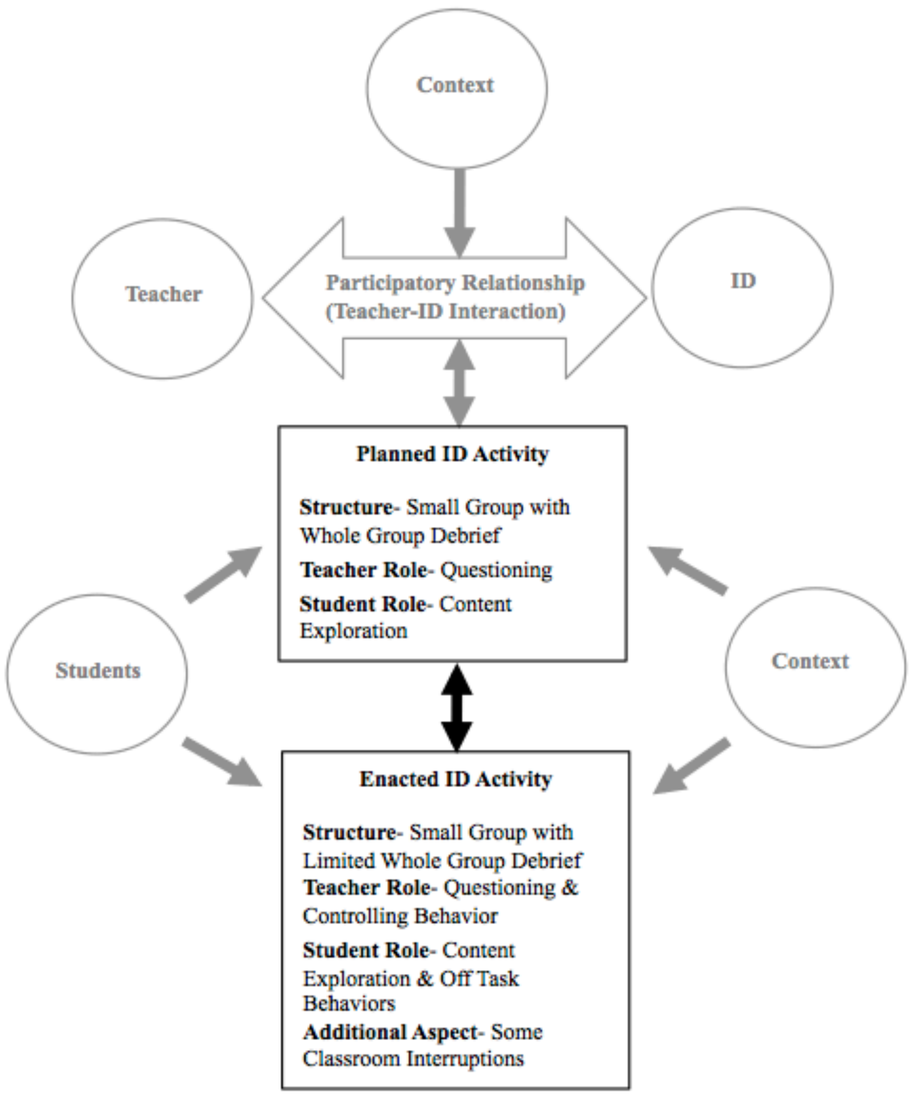


Figure 15. Ms. Edelman's planned and enacted curriculum involving IDs

The remainder of this chapter details the various ID, teacher, student, and contextual factors that mediated Ms. Edelman's creation of the planned ID activities and the enactment those activities within her classroom.

ID Factors

Factors imbedded within the design of particular IDs seemed to have a significant effect on Ms. Edelman's planning of ID activities. She considered these factors when deciding which IDs to incorporate into her classroom and as she planned ID activities for her students. The particular ID factors that mediated Ms. Edelman's curricular decisions are described below.

Context of ID. To begin, Ms. Edelman discussed the context embedded within particular IDs as a factor influencing her intended use of IDs. Specifically, she discussed how using an ID set within a real-world context that students were interested in could have a positive effect on her students' engagement. Conversely, an ID utilizing a context that was unfamiliar or uninteresting to her students may decrease student engagement. When discussing the *Skateboarder ID*, for example, she noted that students might have been more engaged if the context of the ID was altered. She noted, "I don't know, maybe the kids don't like skateboards, maybe they would like to see a fast car or something […] yeah, maybe a different context" (personal communication, October 29, 2015). Consequently, she may be more likely to create activities using an ID with a fast car than a skateboard in the future.

Additionally, Ms. Edelman seemed to value when an ID's context illuminated how mathematical concepts applied to real world phenomenon. She explained,

> I think [IDs] should kind of have a real context. I think it gives students – it kind of brings math out of the abstract for them. Because you always get, even from adults and teachers, when are you ever going to use this in real life? (personal communication, March 10, 2016)

It seemed that Ms. Edelman desired to maximize students' engagement by using ID contexts that aligned with students' interests and/or demonstrated how mathematics was applicable in the real world. Consequently, IDs with contexts consistent with this desire were utilized more frequently.

Multiple representations. The multiple representations often found within IDs (i.e. descriptions of real world scenarios, graphs, tables, and equations) seemed to be an even more significant mediating factor in Ms. Edelman's ID use. She noted, "I like it when [IDs] use multiple representations [… and] visuals. I think that's kind of the point of an ID" (personal communication, March 10, 2016). She further emphasized,

> So, I think my biggest concern when I choose an ID to present or to teach is whether or not the ID makes abstract concepts more concrete, [that] it gives a visualization of the abstract concept. [The ID] may not be nuts and bolts concrete, but to see how an algebraic function or something […] is represented on a graph. That seems to make something more concrete. Kids can see something happening. (personal communication, March 10, 2016)

Ms. Edelman seemed to appreciate IDs' ability to dynamically link multiple representations, such as a function's graph and algebraic equation, because it provided her students with a way to visualize abstract mathematical concepts. When speaking of the *Skateboard ID*, for example, she explained, "It'd be very difficult to relate distance and time if you don't have some sort of visual" (personal communication, October 29, 2015). So, she appreciated how this ID provided "a really nice visualization for the kids, a nice way to visualize the concepts" (personal communication, October 29, 2015).

Ms. Edelman also highlighted how the dynamically linked representations found in the *Perpendicular Lines ID* provided students with a means to visualize the abstract concepts of slope and *y*-intercept. She explained,

> When we're given […] these function rules sometimes it can be difficult to relate them to what's going on in other forms. Like, I know that kids do struggle with identifying or matching the different representations. So, in this [ID] you use one slider to manipulate two different representations where you have the function rule and the graph. And I think that this will help kids make the connection stronger between those two representations and what's going on with both of them and how you can graph based on the information in a function rule or get a function rule based on a graph. (personal communication, December 15, 2015)

Ms. Edelman consistently noted how valuable IDs' ability to connect multiple representations was in teaching her students. She viewed interactively connecting the more visual graph of a mathematical relationship with the less visual and more abstract algebraic representation as a significant benefit. This was so important to Ms. Edelman, in fact, that she sought out other educational technologies that also connected different mathematical representations when an ID was not available for a particular topic within her curriculum.

Allows investigation. Ms. Edelman's emphasis on utilizing linked mathematical representations also seemed aligned with her desire to use IDs that facilitated student investigations. This was, in turn, connected with the student-centered, active, and inquiry-based learning she hoped to foster in her classroom. She explained, "a really important factor [when deciding whether or not to plan an activity around an ID] is how much students can play with it" (personal communication, March 10, 2016) because IDs aren't "just like crunching numbers or deriving formulas or something. They're able to use [the ID] to develop an intuitive understanding of that concept" (personal communication, October 29, 2015). Ms. Edelman consistently emphasized the importance of students "messing" or "playing" with the ID as they explored mathematical concepts for this reason, namely to develop a deeper understanding of the concepts. While discussing the *Perpendicular Line ID*, for example, she noted,

> I think this ID is a really great way to get kids to explore how the slope and how the y-intercept relate to the graph and also how it relates to the function rule because […] it has an interactive component. It's not just sitting there passively watching something. Them being able to manipulate something on screen and see how it reacts, […] see how it affects the graph and also see how it affects the function rule. (personal communication, December 15, 2015)

Engaging her students in active learning, as described here, was always at the forefront of Ms. Edelman's thinking. Thus, she planned and implemented activities where students could interact with mathematics concepts by adjusting an aspect of the ID and observing the subsequent change.

The importance Ms. Edelman attributed to using IDs that actively engaged her students was solidified when she spoke of other types of IDs. She noted that some IDs functioned more as a presentation of information than as a means to investigate mathematics. While she occasionally used these presentation IDs, she preferred not to as she viewed them as less effective in her classroom. She explained,

> [Students are] not interested in watching [presentation IDs]. But I noticed that [IDs] where we have to input the information and check it, that seemed to be a lot more effective because the kids will collaborate and argue with each other about what is correct, […] As soon as [presentation IDs] start playing it's not interesting to them and then they'll get off task. I guess it really depends on the [ID]. But for the most part [presentation IDs] haven't been as effective. (personal communication, December 15, 2015)

We see here how IDs that presented information instead of allowing active student interaction had a negative effect on students' engagement. Thus, Ms. Edelman planned and enacted ID activities where "everybody has an opportunity to mess with [the ID]" (personal communication, December 15, 2015). It seemed, then, that the degree to which an ID allowed student investigation mediated how Ms. Edelman incorporated it into her planned curriculum.

Instant feedback. Another important ID factor, Ms. Edelman noted, was that students received instant feedback from IDs while conducting their investigations. When investigating the effect of altering the slope and y-intercept of a linear equation with IDs similar to the *Perpendicular Lines ID*, for example, she emphasized the value of students being able to see the graph immediately move as a consequence of changing the algebraic representation. She explained,

> Instant feedback, so, that they could move the slider and actually see what's going on [in the graph] rather than me giving them an equation and them graphing it

> which is the old-school way of doing it. […] You could graph that a million times and maybe still not pick up on that [without the ID]. (personal communication, March 10, 2016)

By contrasting the more time consuming, "old-school way" of graphing individual graphs by hand with the instantaneous graphs produced by IDs, Ms. Edelman seemed to appreciate how the instant feedback received by students from the ID facilitated student exploration of mathematical concepts and their ability to efficiently discern patterns in their investigations. She emphasized this same point while discussing the *Paint Mixing ID*. "If [students] made a mistake [… the ID] would actually show that it's not the same color. So, I thought that was a very good way to help correct kids whenever there were misunderstandings." Ms. Edelman appreciated how instant feedback could facilitate both the creation and the refinement of student conjectures during ID activities. Thus, an ID's ability to provide students with instant feedback seemed to increase the likelihood that Ms. Edelman used it, as she desired the type of student-centered, exploratory learning facilitated by this ID feature.

Supports students. Lastly, Ms. Edelman expressed that IDs had the ability to support students engaging in grade-level concepts even when they struggled with required pre-requisite skills. As discussed more extensively in the student factor section, the students Ms. Edelman taught had significant gaps in their content knowledge. She noted, though, that IDs had the potential to support students in bridging those gaps so that they could engage in Algebra I content. She explained this notion while discussing graphing functions.

> I think that when they sit down [with] a blank sheet of graph paper and I tell them to draw axis and label the axis and they're just so overwhelmed with that. […] So, maybe if they're able to, for example, conceptualize how to scale better because the [ID] is doing it. If I'm able to really point that out, 'Hey, look at the scale, these are the […] *x* values and *y* values. This is what the [ID] is determining as an appropriate scale for that.' […] I think they find [graphing functions] tedious or something like they always say that that's too much work. (personal communication, October 21, 2015)

Students described graphing as "tedious" and resisted doing it because they struggled with exactly how to create an appropriate graph. Consequently, students' learning of grade-level content was negatively impacted when not using educational technologies that could automatically create graphs. Ms. Edelman also seemed to view IDs' ability to automatically create appropriate graphs as a tool to teach students this skill. Thus, by students using this ID function and being exposed to how IDs scale graphs, Ms. Edelman hoped that students "eventually […] could be more comfortable doing it by hand. But I think it would be a good intermediate step to working towards that."

Taken together, the factors surrounding the design of IDs seemed to have an important influential effect on Ms. Edelman's use of this technology, as is depicted in Figure 16.

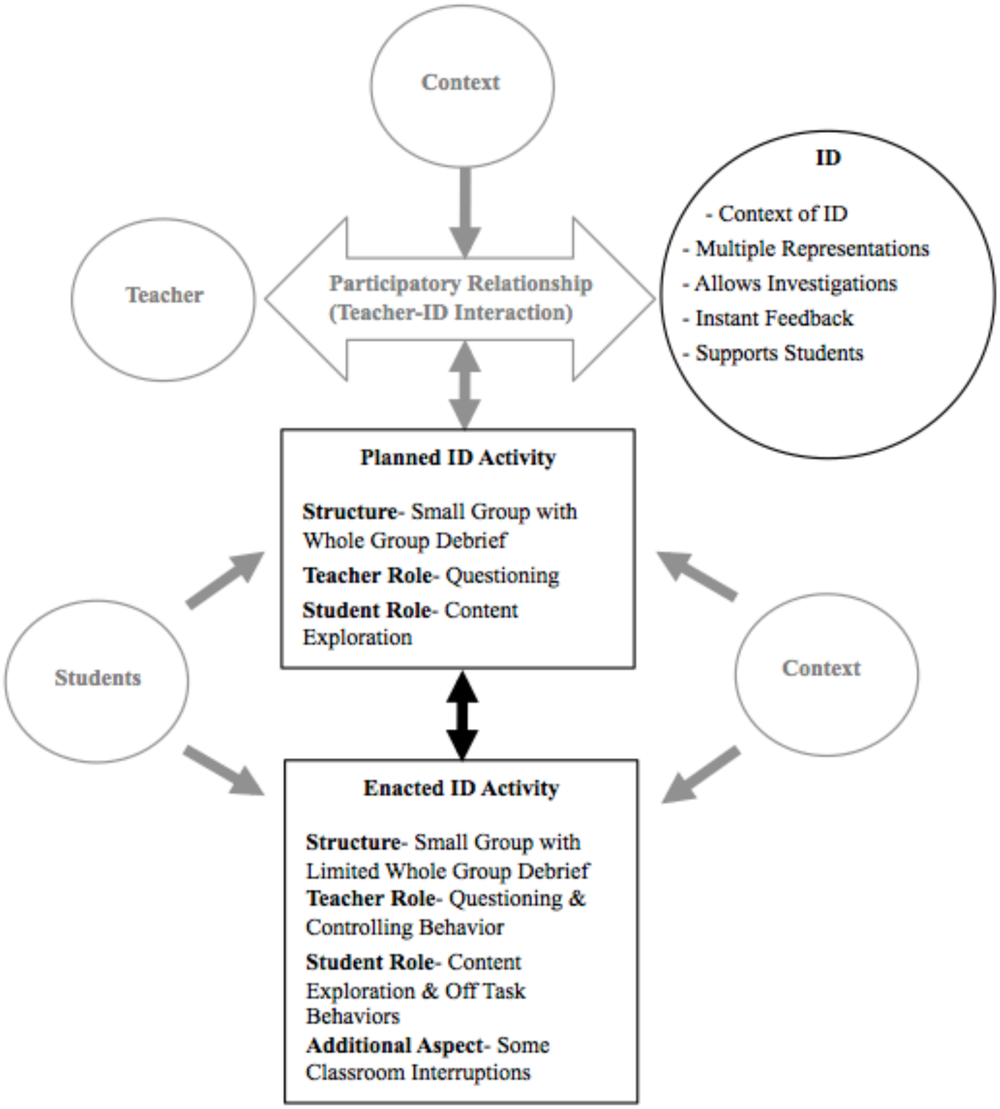


Figure 16. ID factors affecting Ms. Edelman' use of IDs

She desired IDs that used contexts which increased student engagement, utilized dynamically linked multiple representations, encouraged student-centered investigation, provided instant feedback to facilitate those investigations, and supported students engaging with grade-level content by augmenting their ability to accurately perform pre-requisite skills.

Teacher Factors

Factors associated with Ms. Edelman herself seemed to influence the ways in which she used IDs as well. These factors included her knowledge, classroom management, and beliefs. These factors are described in turn below.

Knowledge. Ms. Edelman was a second-year teacher and, as such, was at the beginning of her development as a mathematics instructor. While her mathematics content knowledge seemed sufficient to teach the IA course, Ms. Edelman identified and demonstrated a need to increase her pedagogical knowledge (Shulman, 1987). She understood that she needed to continually develop the knowledge and skills required to effectively engage students in mathematics content and classroom activities, including those involving IDs. Her current lack of this knowledge, however, impacted her ability to plan and enact the teaching and learning she desired. When reflecting on how she facilitated inquiry learning, for example, Ms. Edelman noted, "I try not to guide them too much. I'm still learning" (personal communication, November 3, 2015). Without a deep understanding about how to effectively enact exploratory activities, though, Ms. Edelman thought that students "required my presence and attention far too often. I wish they were a little bit more independent [during the activity]. So, I'm trying to figure out a way to teach them how to do that in my class" (personal communication, November 3, 2015).

The manifestation of Ms. Edelman's developing pedagogical knowledge was evident in the planning and enactment of the *Perpendicular Lines ID*. Ms. Edelman noted that there were a number of things she "would do differently" (personal communication, December 16, 2015) the next time she used this ID. The activity wasn't "structured enough for students to feel like they couldn't go off task." Ms. Edelman indicated that she would use "different guided notes [with] more specific questions than were offered in the worksheet [because they were] just a little too ambiguous." Students got off task during the activity because Ms. Edelman didn't anticipate these aspects of the ID activity. One can conjecture, then, that the planned curriculum would have been enacted differently if Ms. Edelman had the pedagogical knowledge required to anticipate and mediated such concerns.

Additionally, Ms. Edelman pointed out that gaps in her pedagogical content knowledge (Shulman, 1987) resulted in not effectively planning for student misconceptions that arose while her students engaged with IDs. She explained,

> There are a lot of misunderstandings the kids are coming up with that I never anticipated. I'm still learning how to anticipate those mathematical misunderstandings that happen in my class so that I can make sure [IDs] present content that they feel successful with. (personal communication, March 10, 2016)

Gaps in Ms. Edelman's pedagogical content knowledge resulted in not pre-planning strategies effective in helping students reconcile common content misconceptions. She thought this was a "huge factor" in how she interacted with IDs because she missed important aspects of the content that she would have otherwise caught if she had a more thorough understanding of mathematics teaching.

Thankfully, as Ms. Edelman pointed out, the Agile Mind curriculum supported filling the gaps in her knowledge to some degree. "Agile Mind [… gives] you a lot of information about how to use the ID. So, that's really good. And that's why, as a new teacher, that has been invaluable for me to figure out how to present the content to the kids" (personal communication, March 10, 2016). Nonetheless, Ms. Edelman's current pedagogical content knowledge seemed to impact both the planning and enactment of ID activities.

Classroom management. A factor stemming from Ms. Edelman's emerging teacher knowledge was her ability to effectively manage her classroom. Specifically, gaps in her understanding of classroom management strategies, processes, and protocols resulted in inconsistencies while managing students' behaviors and classroom activities. As a first-year teacher she admitted to having difficulty with students "being on their cell phones or having side conversations [let alone] just trying to get them to be in their seat" (personal communication, October 21, 2015). She was quick to point out, though, "I have a little better handle on classroom management this year." Contrasting her first and current years of teaching, for example, Ms. Edelman desired to,

> Give [students] a lot more autonomy. I think that's not something that I gave enough of last year. But again, that's because the classroom management peaks. But I think this year I feel more comfortable giving the kids autonomy to figure things out on their own and actually being able to let them do that without having to correct their behavior every 2 seconds […]. Like last year, I couldn't get kids to stop calling each other dummy all the time and this year nobody is calling anybody a dummy. So, it's a huge difference in the classroom.

Thus, last year's classroom management challenges significantly affected Ms. Edelman's ability to enact the type of learning activities she desired. This year, however, she felt that her classroom management had improved enough to allow for such activities.

While her classroom management may have improved from her first year of teaching, Ms. Edelman still experienced classroom management challenges during the course of this study that influenced her planning of ID activities. This was evidenced when she discussed small group investigations.

> I can imagine [small group investigations] really going well in one of my classes because they're amazing and then the other class I'm not sure. It really depends on who's there that day [because of their] behaviors. But I have one class that I completely trust to work autonomously on a lot of stuff. (personal communication, October 21, 2015).

Thus, Ms. Edelman's planning of ID activities was influenced by her anticipation of particular students' behaviors. In addition to concerns managing small groups, Ms. Edelman noted, "having whole class discussions, that's been a challenge for me" (personal communication, December 15, 2015) and that "we do individual stuff once in a while, but it's kind of rare because it's really hard to manage" (personal communication, March 10, 2016). Thus, all classroom structures, namely whole group, small group/pairs, and individual work, resulted in management concerns for Ms. Edelman. When asked to reflect on the tension between her ability to manage the classroom and the classroom structures she planned for her students she noted, "I just like for them to work in smaller groups and that way they get a lot more attention from me and I don't have to fight with them about being quiet while I'm giving them the instructions on what's going on" (personal communication, November 3, 2015). Recall that Ms. Edelman's most desired classroom structure was to have students in small groups. From her comments here, it seemed that her tendency to plan ID activities using small groups was at least partially influenced by classroom management concerns.

Additionally, Ms. Edelman's difficulty in managing student behaviors mediated the enactment of IDs. Varied and consistent evidence of student off-task behaviors was collected throughout the course of this study. This confirmed Ms. Edelman's comments about experiencing classroom management challenges. (This evidence is discussed at length in the student factor section below). Her ability to effectively address student misbehaviors and successfully get students engaged in the ID activity at hand was inconsistent, often resulting in students returning to the misbehaviors she originally addressed. As she noted, "they require my presence and attention far too often [for behaviors]" (personal communication, November 3, 2015). Consequently, Ms. Edelman's opportunities to facilitate students' content exploration were limited since student behaviors occupied her time and efforts; classroom management concerns significantly influenced Ms. Edelman's enactment of ID activities. As she concluded, "I'm still struggling with classroom management" (personal communication, March 10, 2016).

Beliefs. Ms. Edelman seemed to express a number of beliefs concerning how mathematics should be learned and, as a result, how mathematics instruction should be planned and enacted by an instructor. Her results from the TBQ are shown in Table 8.

Table 8
Pedagogical Beliefs Questionnaire Results- Ms. Edelman

	Active	Cooperative	Constructive	Authentic	Intentional	Average
Initial meeting	3.60	3.83	3.42	3.50	3.75	3.62
Final meeting	3.20	3.83	3.58	3.88	3.75	3.65
Delta	-0.40	0.00	0.17	0.38	0.00	0.03
Average	3.40	3.83	3.50	3.69	3.75	3.63

Ms. Edelman's overall average score of 3.63 indicated a tendency toward beliefs consistent with meaningful mathematics learning (Bate, 2010; Jonassen et. al, 1999). The

TBQ categories related to active, cooperative, constructive, and authentic learning are discussed below, as these categories emerged as the most cogent in Ms. Edelman's ID use.

Mathematics learning should be active. Ms. Edelman consistently expressed the belief that learning should be an active process for students. In fact, she described her ideal classroom as one where "students should be able to figure stuff out and [the teacher] should just facilitate it" (personal communication, March 10, 2016). Students, she indicated should be given opportunities to actively engage in their learning. Thus, she planned ID activities where students could be "more active and it's not just me lecturing. [This includes allowing] students to get frustrated when learning math. That's where learning happens. It's in that area of confusion and them trying to find your way out of that." It seemed, then, that Ms. Edelman believed students should be active in figuring out the content they were to learn. This belief influenced her to plan activities using IDs that encouraged and facilitated active learning.

Interesting, though, was that Ms. Edelman recorded the lowest result in the active category of the TBQ, a 3.40 average. This score may have been a consequence of the tension she felt surrounding the district mandated course pacing. She described this tension,

> With [mandated] assessment schedules […] it has, for me, been a struggle to balance giving the children a chance to kind of explore mathematics with, okay, well, let me just tell you what this is because we got to move on at some point. That way, they can integrate the knowledge and they're not just taking my word

> for everything, but everything has an internal logic and they can figure it out on their own. (personal communication, March 10, 2016)

Thus, it seemed, the pressure of covering all of the content detailed in the course scope and sequence interacted with her belief that students should actively learn content. The result was that students initially engaged with content in an active manner and then they were more passively told what they were to know if they did not learn the content in a certain period of time.

Mathematics learning should be cooperative. Ms. Edelman consistently emphasized that learning should be cooperative. As she noted, "Peer collaboration is really key, especially in my class" (personal communication, March 10, 2016). She continued by explaining how students used their peers' understandings during learning activities to collaboratively come to the knowledge she intended for them to learn. As she explained, "One kid will get a part of it; another kid will get a different part of it. If you put it together, then it creates a whole picture. So, I try to facilitate that as much as possible" (personal communication, March 10, 2016). This allowed students a "better opportunity to make sense of mathematical concepts because they are able to […] have a discussion about it" (personal communication, October 21, 2015). Ms. Edelman's emphasis on peer-to-peer learning was consistent with her overall mean score of 3.83 on the TBQ cooperative belief category. This was her highest category score, indicating a belief in student collaborative learning, and was aligned with Ms. Edelman's desire to use small groups throughout the study.

Mathematics learning should be constructive. The belief that students should learn mathematics constructively also seemed apparent in Ms. Edelman's comments and practice. Her TBQ constructive category overall mean of 3.50 indicated beliefs consistent with constructive learning. This was evidenced by her emphasis on exploratory learning. She explained,

> In an ideal classroom, […] math should be way more exploratory and it should be way more time-involved. […] I wish it was just more like, 'Hey, let's just talk about circles today. What can we come up with? What's [the] connections?' And I think in that way, the students would be able to get a deeper understanding. (personal communication, March 10, 2016)

Ms. Edelman indicated that students should have the time and opportunity to engage with content and construct their own understandings, as this type of learning would result in deeper understandings. Constructive learning was of particular importance, she explained, the first time students were learning about a new concept because "it's a good idea for them to kind of own it, like feel like they came up with it themselves" (personal communication, November 3, 2015).

Mathematics learning should be authentic. Ms. Edelman also seemed to think that mathematics learning should, at least to some degree, be facilitated within a context that naturally prompted the intended learning. Authentic learning may be facilitated within a real-world scenario or simulation, both of which can be found in the context of an ID. She explained,

> It feels more like a physics class, I think, than a math class when you start to include the IDs and the kids can think of [… how] it relates to the real world rather than some concepts that somebody came up with. So, I think that it becomes more like an experiment in a lab […] rather than just getting information

> from a teacher and memorizing stuff. (personal communication, October 21, 2015)

Thus, Ms. Edelman seemed more likely to select an ID with a context that facilitated authentic learning. This was consistent with her comments pertaining to ID contexts and her above average score of 3.69 on the authentic section of the TBQ.

Taken together, Ms. Edelman's comments and TBQ scores were consistent with beliefs that mathematics learning should be active, cooperative, constructive, and authentic. Consequently, Ms. Edelman planned ID activities featuring design features useful in facilitating learning aligned with these beliefs and attempted to enact them as such.

In summary, factors associated with Ms. Edelman seemed to mediate the creation of both the planned and the enacted curriculum involving IDs, as shown in Figure 17.

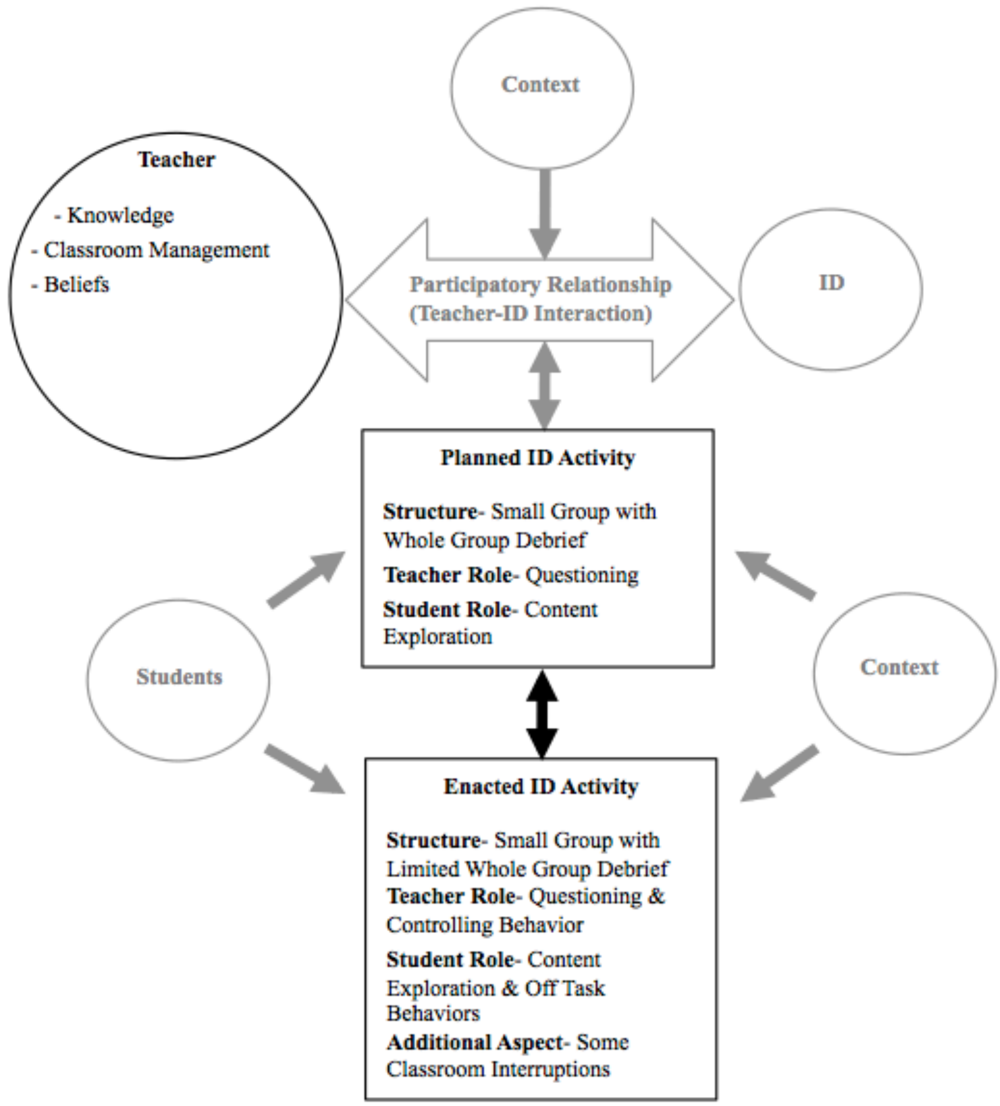


Figure 17. Teacher factors affecting Ms. Edelman's use of IDs

First, gaps in her knowledge of pedagogy and classroom management, which were related to her inexperience as an instructor, seemed to influence the effectiveness of ID activities. Second, the beliefs Ms. Edelman expressed favoring meaningful mathematics learning seemed to mediate the IDs she selected to enact, the structure within which they were enacted, and the teachers moves she attempted within that enactment.

Student Factors

Factors surrounding Ms. Edelman's students affected both the planning and enactment of ID activities as well. The student factors that affected ID activities throughout the study are described below, including students' content knowledge, learning styles, engagement, and behaviors.

Content knowledge. Ms. Edelman's understanding of her students' mathematical content knowledge seemed to be a consideration when planning ID activities. Historically, her students did not achieve success in mathematics courses, as indicated by past and present student achievement data. This resulted in gaps in students' knowledge of mathematics concepts and skills. As Ms. Edelman noted, "I think that this year with a lot of the data that I received through diagnostic testing […], I'm trying to figure out ways to incorporate […] remediation. That's kind of been my focus in classes right now" (personal communication, October 21, 2015). For example, "students struggle with graphing a lot […] just the concept of graphing, a lot of them don't understand." In order to support her students with this skill, Ms. Edelman incorporated graphing review into classroom activities through her use of IDs. As discussed in the ID factors section, some IDs produce graphs automatically. Ms. Edelman saw this feature as "a really good way to bring graphing into the lesson." Using IDs that produced graphs for students, then, was desirable because she explicitly sought opportunities to remediate students' deficient skills.

Ms. Edelman pointed out that IDs also had the potential to engage students in grade level content while they were in the process of learning deficient pre-requisite skills. She explained,

> Whether or not they understand the content, everybody loves to slide the sliders on the IDs. So at least, on some level, they're engaged with the [grade-level] content and hopefully they're able to make the connections even if they don't necessarily have the background [pre-requisite skills]. (personal communication, March 10, 2016)

Thus, the degree to which an ID supported pre-requisite skill remediation and students engaging in grade-level content seemed to influence Ms. Edelman's creation of the planned curriculum.

Students' mathematical content knowledge also affected the enacted curriculum involving IDs. This occurred when students unexpectedly struggled with a pre-requisite skill to such a degree that it affected the pacing and/or focus of the planned ID activity. Speaking of the implementation of the *Perpendicular Lines ID*, for example, Ms. Edelman reflected,

> The kids needed a lot more direction than I think that I anticipated. So, I had to break it down into smaller chunks and I had to go back to some basics since they were struggling. […] They didn't know how to make [a slope triangle]. So, I was like, "Oh my gosh, I have to teach them slope triangles before I can do this [ID activity]." I basically had to show them how to […] construct the slope triangle and how the slope triangle is used to find the slope. (personal communication, December 16, 2015)

In this instance, the gap in students' content knowledge required that instructional time be used to remediate unexpected pre-requisite skills. The planned ID activity was then abbreviated to accommodate the reduced instructional time, significantly altering the planned ID activity. Taken together, gaps in students' mathematical understandings mediated both Ms. Edelman's planning and enactment of ID activities.

Learning styles. Ms. Edelman cited her understanding of students' preferred mode of learning as a factor influencing her interactions with IDs. She emphasized that she had "a lot of visual leaner's students and [that …] they like playing with things, hands-on" (personal communication, October 21, 2015). When asked how this influenced her instructional use of IDs, she noted, "I think the IDs are really good for [visual learners] because I think [IDs] address a lot of students' needs" (personal communication, March 10, 2016). It seemed that Ms. Edelman attended to the alignment between students' preferred learning style and the design features of particular IDs. Thus, IDs including visual representations and opportunities for students to actively interact with something that was "hands on" seemed to be desirable to Ms. Edelman. The relationship between students being visual learners and the presence of graphical representations in an ID was evidenced during the implementation of the *Perpendicular Lines ID*. During this activity, students were asking inquisitive, content focused questions while interacting with the ID. When asked about this, Ms. Edelman responded,

> I don't know if it's atypical or not but I definitely think that when they're working with the IDs those questions come up more often. […] I think the kids who are visual learners are the ones that are going to be more engaged with this ID and more likely to ask these questions where they probably wouldn't have otherwise, if we weren't using it. (personal communication, December 16, 2015)

Since Ms. Edelman was so highly concerned with engaging students in active content exploration, it seemed apparent that she would want to plan ID activities that prompted students to ask and investigate mathematical questions. Thus, Ms. Edelman's curricular planning involving ID activities seemed to be influenced by her attention to students' learning styles.

Engagement. Ms. Edelman's attention to students' content knowledge and learning styles was connected to an underlying concern surrounding student engagement. Ms. Edelman seemed to explicitly seek out pedagogical techniques and resources that increased students' willingness and interest to interact with mathematics content. This engagement seemed to come from a variety of places including ID activities that made students feel supported in their mathematics learning, those that aligned with students' interests, and activities that accounted for students' preferred learning style. Each of these areas was discussed above, but it is important to note their underlying interaction with student engagement and the attention Ms. Edelman gave to this relationship. As she emphasized,

> I think that the […] biggest benefit to using IDs is student engagement. They just get the kids engaged on a much deeper level than when you're trying to […] teach them in other, some teacher-lead way. Or even when they're doing independent work, like, trying to work out a problem, there might be some misconceptions or gaps in understanding that IDs can clarify for them. (personal communication, March 10, 2016)

The "deeper level" of engagement Ms. Edelman refers to here seemed present during the implementation of the *Skateboarder ID* when students were exclaiming "awesome!" "look at mine!" and "Oh! I've got it. Look!" (observation, November 3, 2015) as they interacted with the ID. Upon reflecting on her ID use throughout the study, Ms. Edelman concluded, "I definitely use IDs to engage the students in the class. If it's [an ID] the students are just going to be more engaged, going to be more interested in the content" (personal communication, March 10, 2016). Thus, an ID's perceived ability to increase student engagement in mathematics content seemed to be a significant factor mediating Ms. Edelman's creation of the planned curriculum.

Behaviors. Speaking now of enacted ID activities, student behaviors seemed to have a significant impact on the ways in which planned ID activities were implemented in Ms. Edelman's classroom. Student off-task behaviors were a constant challenge for Ms. Edelman. Classroom observations yielded evidence of varied, frequent, and sometimes escalated student behaviors that did not meet Ms. Edelman's classroom expectations. These behaviors included off-task conversations, negative and confrontational student-to-student interactions, eating and drinking in the classroom, the use of cellular telephones and other electronic devices, and students entering and exiting the classroom without permission. Ms. Edelman was cognizant of these behaviors and addressed them with varying levels of success, which was discussed in the teacher factor section.

Student misbehavior affected the enactment of ID activities by shifting teacher and student focus away from content and reducing the instructional time available for teaching and learning. Ms. Edelman summarized this phenomenon when asked to identify the biggest factor affecting ID activity implementation.

> For most, it's student behaviors. That's a huge factor about whether or not [students are] going to be using the IDs at all, whether they're going to be using them effectively, whether they're going to talk about basketball or whatever

> instead of the lesson, and just getting them to be on task while they're using the ID. (personal communication, March 10, 2016)

Here, Ms. Edelman identified student behaviors as the most significant factor affecting her implementation of ID activities. She elaborated specifically on how students' behaviors can affect the implemented curriculum while reflecting on the *Perpendicular Lines ID* activity. There was no whole group debrief conversation following students' direct interaction with the ID as she had intended there to be. This was because, as she explained, "It was just really hard to get them to be silent enough for us to have a full class discussion, […] it really depends on the students. […] I mean, sometimes the kids are calmer and sometimes they're not. It is challenging" (personal communication, December 15, 2015). Student behaviors directly affected the implemented lesson by eliminating the whole class debrief, a component of the planned activity.

Ms. Edelman continued to describe how her facilitation of a planned ID activity might change due to student behaviors. She explained how "If there are more behaviorally challenging students in the classroom then I might spend more time on […] giving them direct instruction" (personal communication, March 10, 2016) instead of facilitating the more inquiry-based, exploratory learning she wanted to implement. It seemed, in fact, that Ms. Edelman implemented ID activities that were increasingly less consistent with the planned curriculum as student behaviors increased. Said a slightly different way, she "would have been a lot more hands-off [in students' explorations] if there were less behaviors in my classroom" (personal communication, December 16, 2015). Taken together, students' behaviors seemed to be a strongly influential factor mediating Ms. Edelman's enacted curriculum.

In spite of students' misbehaviors, Ms. Edelman never vilified or blamed students for the impact their behaviors had on the enacted curriculum. Instead, she framed her statements about student behaviors either as facts, that these student behaviors occurred, or in terms of her own classroom management, that she did not yet have the teacher skills to effectively manage her classroom. The latter was discussed in the teacher factors section.

In summary, student factors seemed to significantly mediated Ms. Edelman's planning and enactment of ID activities (see Figure 18).

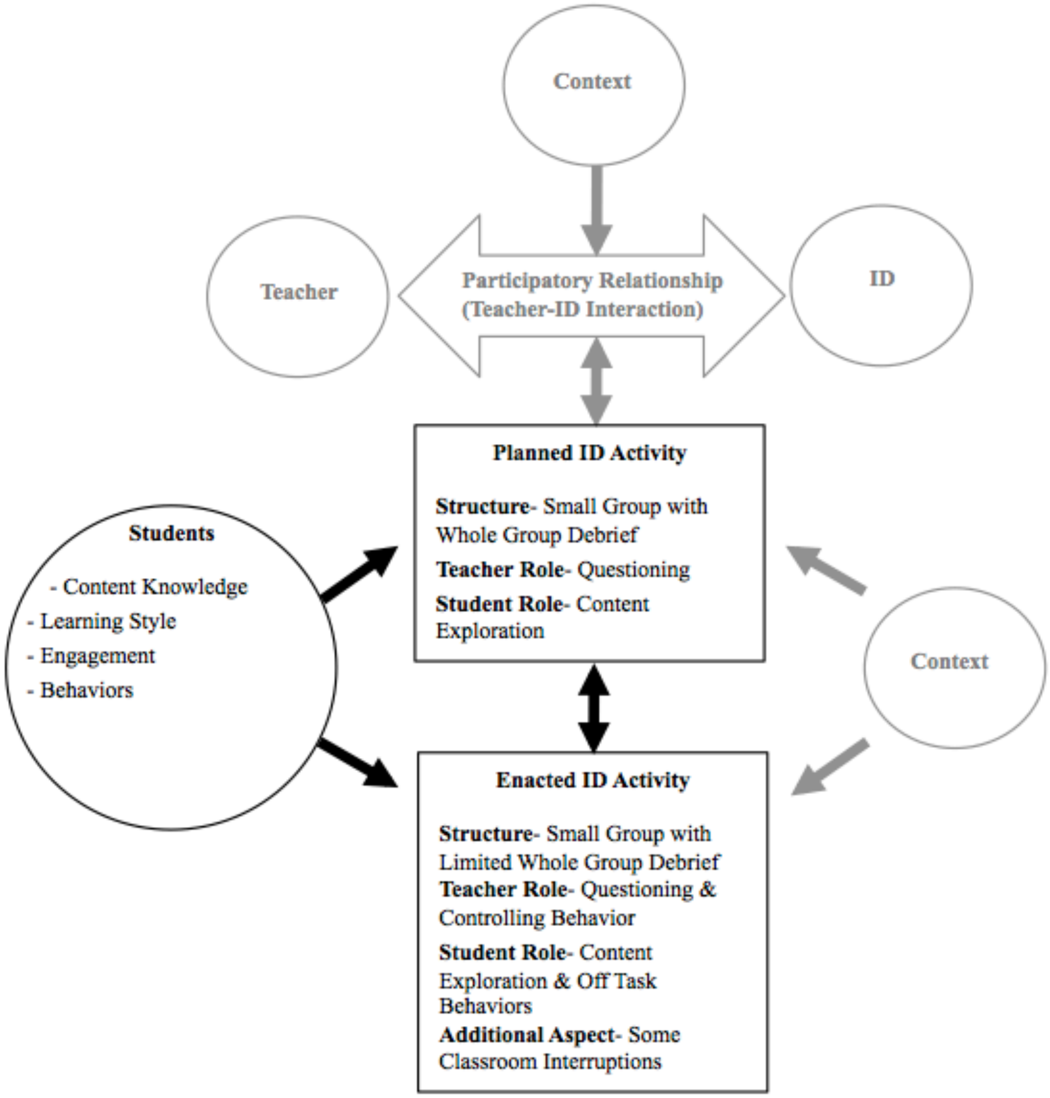


Figure 18. Student factors affecting Ms. Edelman's use of IDs

Students' content knowledge and learning style, as well as how these affected student engagement, seemed to influence Ms. Edelman's planning of ID activities. Student behaviors, on the other hand, had a significant impact on the enactment of those ID activities.

Contextual Factors

Various contextual factors also influenced Ms. Edelman's ID use during both the planned and enacted curriculum. These factors are described below beginning with those at the district-level and moving to those within Ms. Edelman's school.

Mandated curriculum. The district-level decision to adopt Agile Mind as the required curricular resource was a mediating factor in Ms. Edelman's ID use. Agile Mind contains a large number of IDs. Consequently, Ms. Edelman used these IDs frequently as she progressed through the district-mandated curriculum. She was quick to point out that this was a positive aspect of her practice. She noted, "I like how [IDs are] kind of built into the district curriculum, the Agile Mind" (personal communication, March 10, 2016). The fact that Ms. Edelman was a novice teacher added context to this statement, which she explained, "Since I'm a new teacher I don't really know how to teach otherwise [without IDs]. I mean, that's just what the curriculum tells me to do, to try it out and see" (personal communication, October 21, 2015). Perhaps, as she seemed to allude here, Ms. Edelman's lack of experience and the district-required curriculum combined to result in her high degree of compliance with the district mandate. What seemed clear, was that Ms. Edelman would likely interacted with IDs less frequently or not at all if the district mandated an alternative curriculum void of IDs. She pointed this out by stating, "Agile Mind is the district-mandated curriculum. Yeah, that definitely affects me because I wouldn't even have access to these IDs if this wasn't our curriculum. So, this is what I have to use. So, I use it" (personal communication, March 10, 2016).

Additionally, DSS published a mandated scope and sequence that outlined the sequence of course topics and pacing of each topic for Agile Mind courses. Secondary mathematics teachers were required to follow the course scope and sequence as they progressed through their respective courses. Each IA topic had associated assessments at their conclusion that were tracked at the district level. Ms. Edelman described this pacing mandate as having a negative effect on her classroom practice.

> That's the biggest barrier to me using the curriculum the way that I would want to. I think that math should be way more exploratory and it should be way more time-involved. And, unfortunately, with assessment schedules […] that's not always a possibility. So, it has, for me, been a struggle to balance giving the children a chance to kind of explore mathematics with, okay, well, let me just tell you what this is because we got to move on at some point. (personal communication, March 10, 2016)

It seemed that the pacing within the district scope and sequence was contrary to what Ms. Edelman viewed as the time required to engage students the investigative activities, including those involving IDs. Indeed, the existence of the district scope and sequence seemed to reduce the frequency and/or duration of exploratory ID activities in her classroom.

Thus, two district-level factors influenced Ms. Edelman's planned ID use. These included the district requirement that Agile Mind be used as the primary curricular resource for secondary mathematics in DSS and the presence of the associated course-pacing schedule.

School administration focus. At the school level, it seemed that Ms. Edelman was not encouraged nor deterred from using IDs. When asked about her school's administrative directives, Ms. Edelman explained, "I'm pretty much on my own at the school. I brought up some ideas about how […] to structure my classroom to the administration at the school and they're like, 'You're a professional, do what you need to do'" (personal communication, October 21, 2015). As a new teacher, Ms. Edelman found the lack of instructional feedback she received problematic, stating that it would "be better if I got feedback because I don't get any" (personal communication, March 10, 2016).

That said, the school administration seemed to look for particular instructional practices when observing classrooms. Ms. Edelman noted, "they seem to really push group work" (personal communication, November 3, 2015) where students are "having meaningful dialogue with each other [… using] academic talk" (personal communication, October 21, 2015). Productive student collaboration was an area identified within the district-wide teacher evaluation framework. Thus, it seemed unsurprising that school administration would attend to this aspect of their teachers' classrooms. Similarly, administration were "looking for more student-led activities" (personal communication, December 15, 2015), another area explicitly addressed in the teacher evaluation framework. Ms. Edelman highly valued student collaboration and students taking an active role during mathematics learning, in general, and in ID activities specifically. Thus, the school administration's focus on group work and student-led learning were aligned with what Ms. Edelman already attended to in her planning and enactment of ID activities.

School climate and culture. Ms. Edelman's school's climate and culture influenced her enactment of ID activities in a manner that seemed to parallel that of student behaviors. Namely, school climate and culture challenges interrupted her classroom, distracted both students and Ms. Edelman, and decreased the instructional time available for planned ID activities. Speaking of students in the hallway during class periods, for example, Ms. Edelman stated,

> We have hall walkers […], they are just all over the place. If they check the door, if they see an open door they'll walk in. […] I try to maintain control of the door just to make sure that we don't have disruptions from people coming in and out that aren't supposed to be here. (personal communication, November 3, 2015)

Numerous such instances of students entering Ms. Edelman's classroom were noted during the study's observations. At times, these instances required only that Ms. Edelman quickly go to the door and reclose it, as to prevent students in the hallway from further yelling into her classroom. Other instances, such as convincing a bullish student who had entered her classroom and refused to leave, required Ms. Edelman's attention for a longer period of time. No matter how long these distractions lasted, though, they consumed instructional time that would otherwise be used to enact the ID activity in the manner that Ms. Edelman intended. Ms. Edelman summarized this point while reflecting on her ID use,

> The presence of students in the hallway is a huge factor in my school of just interrupting the flow of the lesson. So, [it is] definitely affecting my use of IDs or just use of anything, because it affects the flow of my lessons." (personal communication, March 10, 2016)

Thus, the school climate and culture seemed to mediate the ways in which planned ID activities were enacted in Ms. Edelman's classroom.

School staffing. Ms. Edelman identified the inexperience of her schools' faculty as a contributing factor to the culture and climate challenges she encountered. She explained,

> I would say that more than half our staff is first year teachers right now. […] So, if we had more effective teachers in the school a lot of the other factors wouldn't be so great, like students in the hallway, for instance. (personal communication, March 10, 2016)

Here, she suggested that the large quantity of inexperienced staff at her school directly contributed to the challenges she experienced with students from the hallway. This was due to the staff's inability to get students from the hallway into classrooms, keep them there, and generally manage students' behaviors. No matter the reason, the result was that school staffing mediated ID implementation through the climate and culture challenges it created.

School programming. Additionally, aspects of school programming seemed to affect how Ms. Edelman used IDs. For example, students were pulled from Ms. Edelman's class during particular times of the year preventing the enactment of ID activities with those students. She explained,

> Standardized assessments interrupting class periods was another thing where kids are taken out of my class. And I usually know ahead of time, maybe, that they're going to be taken out of my class for these tests, but not always. And also, some of my classes are mixed-grade levels. So, that affects testing as well. Some kids might be taken out, some not. (personal communication, March 10, 2016)

It seemed, then, that students being pulled from Ms. Edelman's classes to take standardized assessments affected her ID implementation in two related, yet different ways. First, Ms. Edelman may not plan an ID activity as she would otherwise had, if at all, when she knew that students were being pulled out of her class. Additionally, the implementation of an ID activity may be altered when students are unexpectedly pulled from her class. In either case, school programming surrounding standardized testing affected the ways in which Ms. Edelman planned and implemented ID activities.

Daily school scheduling also affected both Ms. Edelman's planning and implementation of IDs. A lunch period was scheduled in the middle of one of Ms. Edelman's mathematics classes. Thus, students in that particular class engaged with mathematics in Ms. Edelman class for 45 minutes. They then went to lunch for approximately 30 minutes and returned for the latter 45 minutes of the class. This schedule had a significant negative impact on the instructional time available for that class to engage with IDs since student off task behavior increased immediately before and after lunch. Ms. Edelman recognized this behavior during her lesson by acclaiming "its 11:12. I didn't dismiss you yet!" (observation, December 16, 2015) when students began packing their belongings 3 minutes before it was time to leave the classroom. After lunch, the majority of students weren't present until approximately 6 minutes after the bell rang. Adding these intervals together, 9 minutes or 10% of the class period was lost due to the lunch transition. This was time that Ms. Edelman's other classes engaged in the ID activity.

School events also mediated how the planned curriculum was enacted in Ms. Edelman's classroom. At times, school-wide events would occur without teachers having prior knowledge of the event. These events canceled classes, preventing the enactment of any planned ID activity. Ms. Edelman explained,

> School programs interrupt and cancel class periods. That does affect me greatly because I can't really plan for events or programs that happen in the school because they kind of are sprung on the teachers, like, 'Oh yeah, we're having an assembly today, by the way. Oh okay.' So that definitely affects me using IDs. I may not even be able to teach that day and I didn't know that until that morning [...] or maybe the day before. (personal communication, March 10, 2016)

In other instances, school events resulted in classes that were shortened, ended early, or begun late. The instructional time available to enact ID activities as Ms. Edelman intended was affected in all of these cases. During the study, for example, Ms. Edelman altered an ID activity due to a school assembly. She noted, "We had the assembly yesterday. So, we are going to do this assignment today" (observation, November 3, 2015). Thus, it seemed that factors related to school programming, namely how standardized assessments, scheduling, and school events were managed, affected Ms. Edelman's ID use.

Technology availability. Ms. Edelman expressed a consistent desire for enough technology to enact activities where students could interact with IDs individually or in pairs. As she explained, "In an ideal classroom, I would have enough computers for every student, and they would have an opportunity to sit there and mess with IDs [...] or do some group work or independent work" (personal communication, March 10, 2016). Technological devices with the capability to enact IDs activities seemed available to Ms. Edelman. She had a classroom SmartBoard, a teacher laptop, and access to a number of laptop carts and a computer lab. This technology was not reliably functional, however. When discussing a cart of touch-screen laptop/tablet hybrid computers, for example, she pointed out that they were "really fancy [but she hasn't] used them at all because they don't connect to the Internet" (personal communication, October 21, 2015). Additionally, "None of the desktops in the computer lab work at all" (personal communication, October 21, 2015). Ms. Edelman made similar comments about other available technologies at various points throughout the study.

The availability of functional technology had a significant influence on how Ms. Edelman planned and enacted ID activities. When asked how she planned the structure of an ID activity, she explained, "It really depends on what technology I have available to me. I usually do small groups, […but] if I only have the SmartBoard to use then [it'll be whole group]" (personal communication, March 10, 2016). Thus, the structure of an ID activity could shift from a more desirable structure (i.e. small groups) to a lesser desirable structure (i.e. whole group) depending on the amount of functional technology available to Ms. Edelman. When planning the *Perpendicular Lines ID* activity, for example, she hoped for students to work in pairs but there weren't enough functioning computers to facilitate this. Thus, she planned to have "maybe one [laptop] for every two to four people. So, there'd be at least one computer for every group" (personal communication, December 15, 2015). During the enacted activity, however, the number of working computers was reduced from 15 to 6 due to technical issues. These technological challenges resulted in some students using the classroom SmartBoard instead of a laptop and much larger student groups than Ms. Edelman had intended.

Scenarios such as the one just described did not seem abnormal in Ms. Edelman's classroom. She often noted that she anticipated technical issues during ID activities, even going as far to say, "I probably will be going around troubleshooting most of the period" (personal communication, November 3, 2015) instead of interacting with students around content. This comment was noteworthy because it demonstrated one way that malfunctioning technology mediated the enacted curriculum. Ms. Edelman was not able to facilitate the activity in the ways that she planned because she was preoccupied addressing technological concerns. Another effect of technical difficulties was that it increased student group sizes. Larger group sizes then affected the productivity of each student group since student behaviors impairing effective collaboration often increased. Ms. Edelman recognized this relationship,

> I wish I had a couple of more computers so that everybody would have a computer to work in a pairs. […] I think two people per laptop would have worked a little bit better and there would have been a lot more collaboration between the two students. (personal communication, December 16, 2015)

Decreasing the amount of working technology, it seemed to Ms. Edelman, had a direct relationship on students' ability to collaborate and actively engage in the ID activity.

The availability of functioning technology was further limited due to standardized assessment administration within Ms. Edelman's school. All school laptops and tablets were collected for testing during the administration of standardized assessments. As she described, "because we're doing testing right now I probably shouldn't have them [laptops] in the first place, but I told Mr. Richards I need them for a lesson. So he's like, 'You could keep about eight'" (personal communication, December 15, 2015). Thus, it seemed, any technology that was functioning properly was unavailable during school-wide test administrations unless a teacher fell into the favor of an administrator. Even so, only a small number of laptops were made available to Ms. Edelman.

While technology was sparse during testing times specifically, it should be noted that there was no instance during the course of the study where enough functional technology was available for Ms. Edelman's students to work in pairs. This lack of available functional technology significantly mediated how Ms. Edelman created the planned and enacted curriculum due to its effect on the ID activity's structure, students' level of collaboration, and how Ms. Edelman spent her time during the enacted ID activity.

In summary, there were a number of contextual factors that mediated Ms. Edelman's ID use during both the planned and enacted curriculum. These factors are depicted in Figure 19.

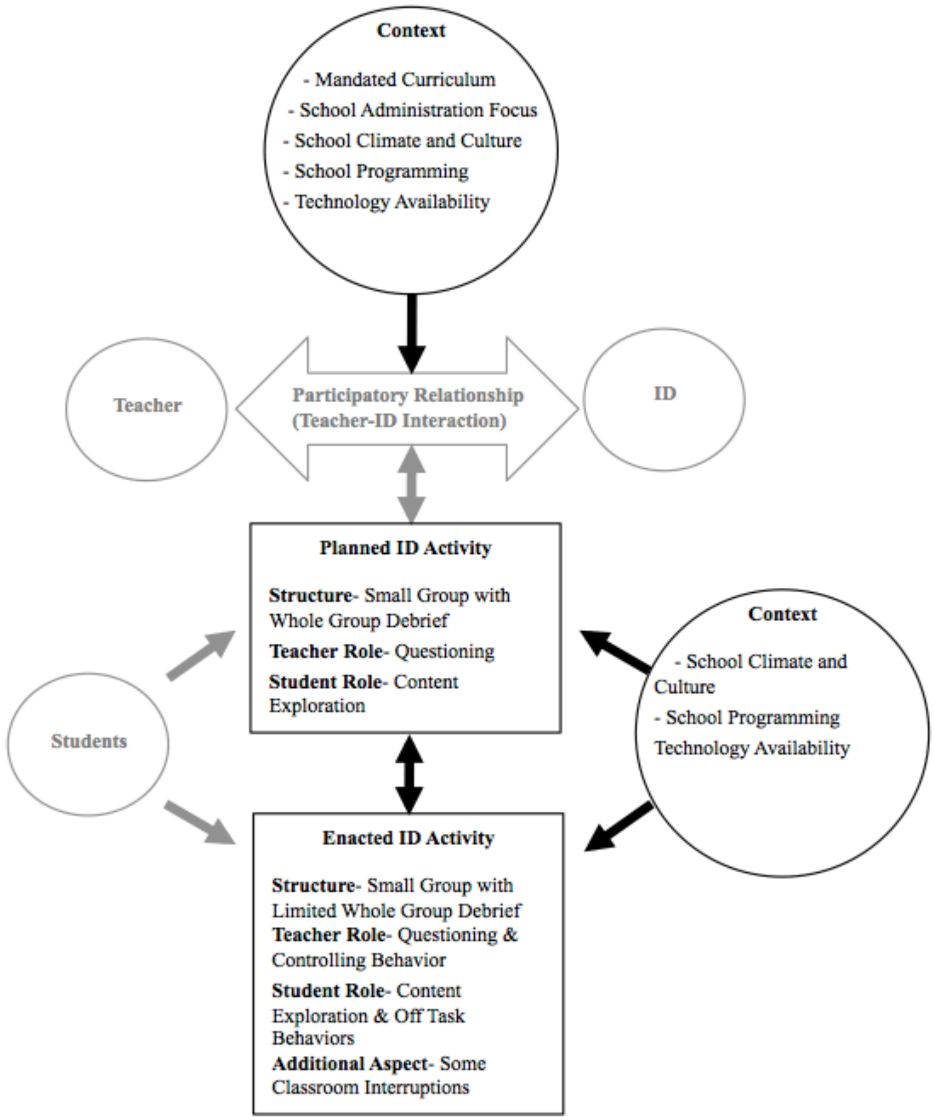


Figure 19. Contextual factors affecting Ms. Edelman's use of IDs

Here, we see the impact the district-mandated curriculum, school administration's focus, climate and culture, school programming, and technology availability had on the participatory relationship between Ms. Edelman and IDs. The latter three factors specifically mediated the planned ID activities moving to the enacted curriculum.

Summary

Ms. Edelman's ID use can be conceptualized within a multifaceted system where ID, teacher, student, and contextual factors influenced both her planned and enacted curriculum. Figure 20 captures this system utilizing this study's conceptual framework.

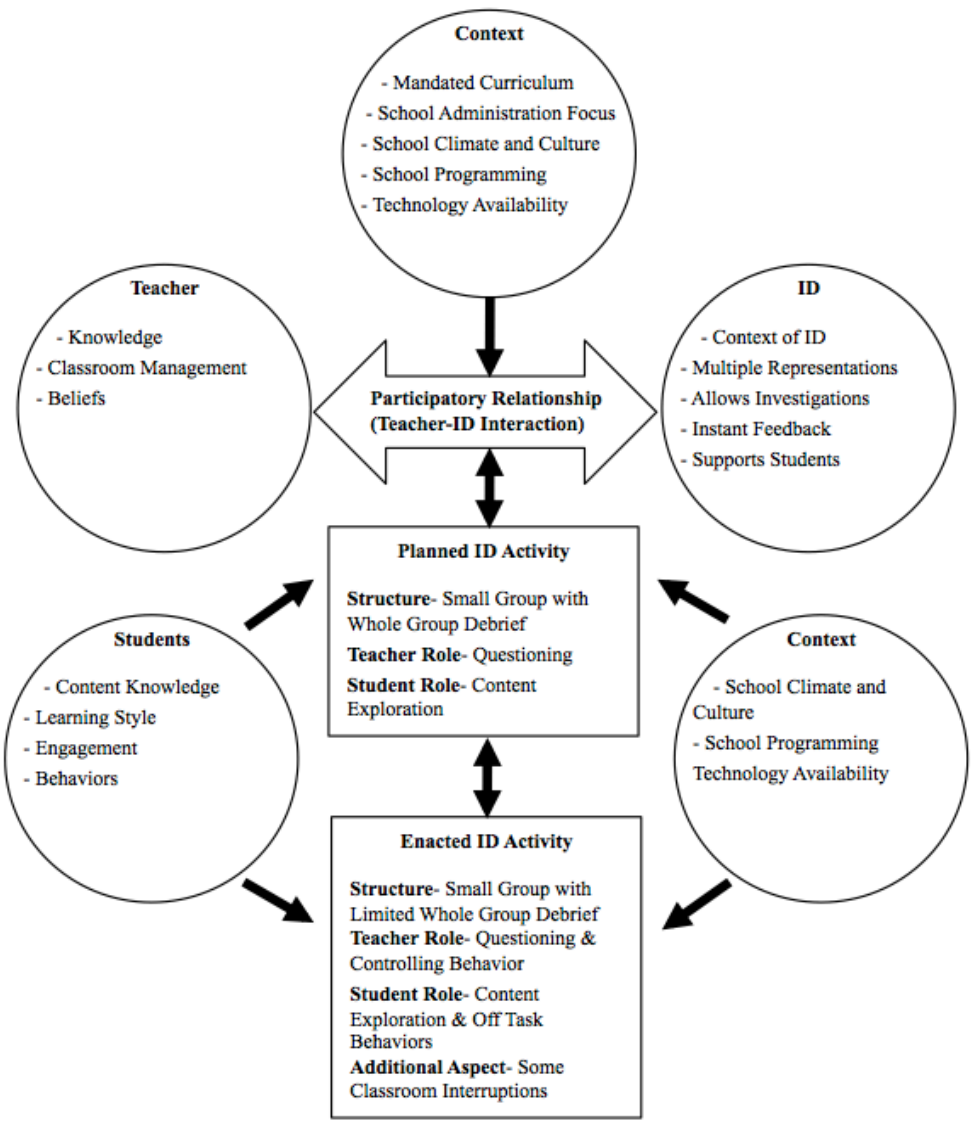


Figure 20- Ms. Edelman's ID use and the factors affecting it

Figure 20 depicts how various teacher, contextual, and ID factors mediated the Ms. Edelman's creation of planned ID activities through the participatory relationship that exists between the teacher and the ID. Specifically, challenges stemming from Ms. Edelman's pedagogical content knowledge and classroom management were seemingly in tension with her beliefs that mathematics should be learned in a meaningful way. She balanced this tension within a context that often inhibited her use of IDs. Indeed, various school factors, such as programming, climate and culture, and technology availability, seemed to inhibit or prevent the types of learning activities Ms. Edelman preferred. Additionally, the district mandates surrounding IDs both encouraged and created barriers for the ways in which Ms. Edelman interacted with IDs, while the school administration's focal void allowed Ms. Edelman curricular freedom to make decisions surrounding IDs as she wished. ID design features seemed to play a significant role in creating the planned curriculum as well. Specifically, Ms. Edelman seemed more likely to design an activity utilizing an ID that was engaging to students and facilitated student-centered, inquiry learning. She sought IDs with contexts students were interested in and revealed real-world applications of mathematical concepts, used multiple representations, allowed students to investigate mathematics, provide students instant feedback during those investigations, and supported students with pre-requisite skills so that they could engage with grade-level content.

Ms. Edelman negotiated this conglomerate of factors with competing and complimentary forces as she created planned ID activities focused on inquiry-based learning. She predominately intended for students to work in small groups as they interacted directly with the ID, discussed the patterns they observed with their peers, and discovered the mathematical concepts embedded in the ID. A whole group debrief was planned to solidify these understandings. Ms. Edelman planned to facilitate student learning through intentional teacher questions and providing an activity conducive for exploratory learning.

The enactment of Ms. Edelman's planned ID activities was also mediated by numerous factors. To begin, the teacher and contextual factors just described continued to both facilitate and discourage the enactment of the small group, exploratory learning activities planned by Ms. Edelman. In addition, students' often deficient content knowledge, predominately visual learning styles, varied levels of engagement, and challenging behaviors mediated the enacted curriculum.

The result of the various mediating factors surrounding Ms. Edelman's practice was that ID implementation embodied the spirit of what she intended, but that it was altered in a number of ways. First, students did work in groups where they controlled the ID, but these groups were larger than intended due to a lack of technology. Students were encouraged to explore mathematics concepts in their groups through use of the ID and discussion with their peers. This occurred inconsistently, however, as a number of factors distracted students from the ID activity. Further, Ms. Edelman consistently attempted to facilitate students' content inquiry through questioning, but was frequently distracted from probing students' understandings by technology concerns, student behavior, and climate and culture concerns. Lastly, the time allotted for planned whole group debrief was either abbreviated or eliminated due to the small group component of the activity lasted longer than Ms. Edelman intended.

Chapter 6

This chapter describes the case of Fatima Allen, an educator in DSS with seventeen years of teaching experience. The year of the data collection was Ms. Allen's first year teaching the IA course. The IDs Ms. Allen used during this study, her intended and enacted curriculum involving these IDs, and the ways in which various ID, teacher, student, and contextual factors influenced her ID interactions are described below.

Implemented IDs

The ID used during Ms. Allen's first lesson cycle was found within topic 5 of the IA course, which focused on students learning about problem solving and metacognition. This ID will be referred to as the *Friendship Problem ID* (Figure 21).

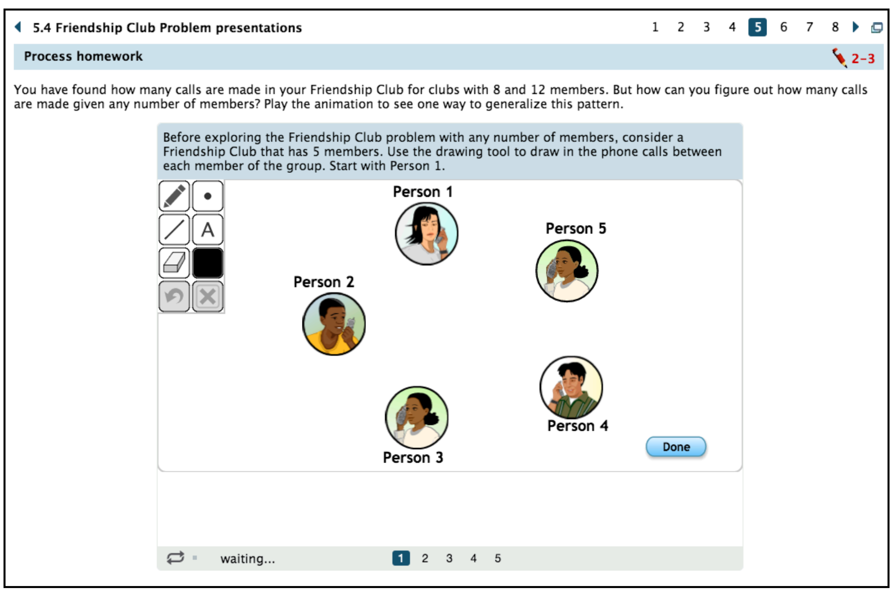


Figure 21. Friendship Problem ID. Reprinted from Intensified Algebra Course, topic 5, lesson 4, slide 5 in *Agile Mind*. n.d. Retrieved April 9, 2016 from http://www.agilemind.com. Copyright 2016 by Agile Mind.

It included eight different tools at the top left of the screen for the user to use while interacting with the *Friendship Problem ID*. These tools were the pencil tool, point plotter, line tool, text tool, eraser tool, color palette, undo and clear all. Each tool's name denoted its functionality. The text at the top of the screen invited the user to use the provided tools to draw the phones calls that occurred between the five people depicted in the ID.

The ID used during Ms. Allen's second lesson cycle was found within topic 9 of the IA course. This topic focused on developing proportional reasoning. The ID will be referred to as the *Paint Mixing ID* (Figure 22) since students were charged with mixing quantities of blue and yellow paint.

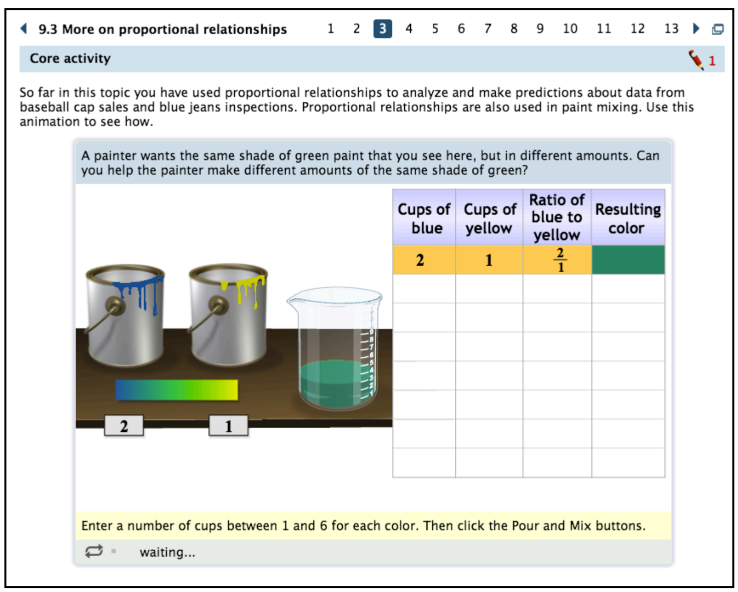


Figure 22. Paint Mixing ID. Reprinted from Intensified Algebra Course, topic 9, lesson 3, slide 3 in *Agile Mind*. n.d. Retrieved April 9, 2016 from http://www.agilemind.com. Copyright 2016 by Agile Mind.

The user's goal when interacting with this ID was to combine the blue and yellow paints in quantities that resulted in the same shade of green that was given in the first row of the provided table. The particular shade of green was made from a 2:1 ratio of blue to yellow. To accomplish this, the user inputted his or her desired quantities of blue and yellow paint in the grey boxes directly below the paint cans. These quantities were restricted to values between 1 and 6. Once quantities were entered the "pour" and then "mix" buttons appeared, which the user selected in turn to see their resulting shade of green. Information describing the new paint color was recorded in the table present in the ID. A check mark appeared when a shade of green exactly matched the given color, indicating that they were the same. Lastly, the user could reset the table at any time.

Intended Curriculum

Ms. Allen created the planned curriculum in a relatively consistent manner throughout this study. The intended structure of ID activities varied between two different models, but her underlying goal for the activity, the role of students, and the teacher role in the activities remained constant. These activity structures, her goal, the role of students, and the role of teachers are discussed below.

The ID activities planned by Ms. Allen featured two different classroom structures. The first, as illustrated by the planned *Friendship Problem ID* activity, involved students working in a whole group with the ID projected on the board from the teacher computer. Using this structure, Ms. Allen intended to "let the kids come up and they do it" (personal communication, November 9, 2015). In other words, she planned for students to take turns coming to the teacher computer and manipulating the ID. The rest of the class would observe as individual students controlled the ID on Ms. Allen's computer.

The second activity structure Ms. Allen used was to group students in small groups as they engaged with an ID on laptop computers. Then, students would debrief the small group work as an entire class. Ms. Allen explained that in this structure she planned to have "three students to a computer and have them go through the [ID] themselves and answer the questions and […] then have a whole group discussion" (personal communication, October 26, 2015). Students would interact with the ID directly on laptop computers within their small group for "approximately 10 minutes" (personal communication, January 14, 2016). Next, during the whole group discussion that followed, the ID would be projected from the teacher computer onto the board as students "share with the whole class what they found" (personal communication, October 26, 2015). Ms. Allen noted that this structure "puts more on the kids as opposed to them waiting for information from me. It puts it on their lap and has them do the research or the finding of the answers" (personal communication, October 26, 2015). The *Mixing Paint ID* activity was planned using this structure.

While Ms. Allen planned to engage students with IDs in both small and large groups, she seemed to prefer the former. Aligned with this, she explained that small groups would work better than large groups for the *Mixing Paint ID* activity.

> I think there will be more trial and error with them trying to get the paint color […] correct. So, I think the smaller groups will […] have a better outcome than having a larger group. Whereas one person will say [something in the whole group], there's only so many people that may or may not understand. […] If the one person says it [in a small group], he's only saying it to his partner. Trying to

> get a better understanding of how to mix the paint and what does it mean. (personal communication, January 14, 2016)

Ms. Allen seemed to think that small group or partner work afforded more opportunities for students to engage both with the content and with each other as they made sense of the task at hand. Thus, small group ID activities seemed to be prioritized over whole group activities within Ms. Allen's planned curriculum.

No matter which structure was planned, though, Ms. Allen described a consistent underlying goal for ID activities. Overall, she intended for "students to understand the concept, not just memorize the process" (personal communication, March 8, 2016). She expressed the desire for students to understand *why* mathematics worked and not just *how* to do it. To accomplish this, she aimed to create activities where students "use IDs to make conjectures, make conjectures and test them, and learn something new" (personal communication, March 8, 2016). When Ms. Allen planned the *Friendship Problem ID* activity, for example, she intended for students to explore the mathematical relationship embedded within the ID between the number of callers and the number of calls that could be made. Then, she planned for students to test their hypothesized patterns using the ID. Ms. Allen intended for students to ultimately create an equation representing the problem situation through this iterative process of creating and testing conjectures.

Whole group collaboration was consistently used within Ms. Allen's intended curriculum. She emphasized the importance of students sharing their ideas and questions within the whole group so that students could learn from each other. To that end, Ms. Allen's planning of the *Mixing Paint ID* activity included a time for students to "come back as a [whole] group and […] discuss what they came up with [in small groups]" (personal communication, January 14, 2016). Ms. Allen emphasized the importance of students sharing their ideas even when the entire ID activity was set in a whole group setting. Speaking of the *Friendship Problem ID* activity, for example, she noted, "this whole [plan] was for them to present, you know, it's like a presentation" (personal communication, November 9, 2015) to their peers about the understandings they are gaining from using the ID. Thus, Ms. Allen included opportunities for students to share their thoughts within a whole group setting as an essential component of her planned ID activities.

Ms. Allen also planned instruction before engaging students with the actual ID. In the planning of the *Friendship Problem ID* activity, for example, Ms. Allen preceded students interacting with the ID by working on a similar problem. This, she indicated, was intended to have students "get the idea of what's happening with the phone calls" (personal communication, November 9, 2015). Thus, by introducing, discussing, and beginning to find solutions to the handshake problem, Ms. Allen planned to support students' ability to interact with and understand the *Friendship Problem ID*. Similarly, Ms. Allen planned for students to review concepts related to proportional relationships before they engaged with the *Mixing Paint ID*. This, she explained, gave students "more of the information in the discussion, and they can have it answered" (personal communication, January 14, 2016). In other words, Ms. Allen planned for students to understand the mathematical terms and concepts they would then use in the whole group discussion following the small group component of the ID activity. In both of these cases, students engaged in preliminary work that "could be 15 minutes to 20 minutes before they are working together […] on their [ID] activity" (personal communication, March 8, 2016).

Ms. Allen consistently articulated that the pre-ID work she planned for students was meant to support their learning during the ID activity. She explained, "The reason why I'm doing it that way is because I want them to get the idea of what we're getting ready to do [with the ID], to have a better understanding of it" (personal communication, November 9, 2015). She seemed to think, then, that students needed to obtain particular pieces of content knowledge before they could successfully interact with the ID, answer the activity questions, or discuss the ID with their peers. In conclusion, Ms. Allen planned pre-ID instruction since "that's just like a piece to help them understand [the ID]" (personal communication, November 9, 2015).

Students' roles during ID activities remained constant in Ms. Allen's description of her intended curriculum. First, she desired students to "take notes [in their] notebooks" (personal communication, March 8, 2016) during the pre-ID work. Their notes, she explained, would help students when they were interacting with the ID. Then, she planned for students to be "working together […] with their peers to solve problems" (personal communication, March 8, 2016) during the times in which they were directly interacting with the ID. During the *Mixing Paint ID* activity, for example, she included an opportunity "where the kids actually have to play around with this [ID] and come up with the proportions themselves" (personal communication, January 14, 2016). This particular opportunity was in small groups, but it could also occur within a whole group like during the *Friendship Problem ID* activity. Lastly, she planned for students to be "active participants in conversations" (personal communication, March 8, 2016) within both the small and whole group components of ID activities. Here, Ms. Allen intended for students to share their suggestions on how to interact with the ID, questions, and emergent understandings with their peers.

The teacher's intended role, then, was to guide students toward desired learning outcomes as they actively participated in ID activities. Ms. Allen explained that she planned to act as a "facilitator and guide them to have conversations with each other and work together – for them to work together to increase their knowledge about whatever is going on [in the ID]" (personal communication, March 8, 2016). She intended, then, to function as a facilitator of learning that directed students as they collaboratively constructed mathematical understandings. Ms. Allen planned to utilize pedagogical strategies such as "ask[ing] students about what is happening in their ID, […] ask[ing] students about what they are learning, and assess[ing] student understanding" while guiding student learning.

Ms. Allen also viewed keeping students focused on content during the ID activity as a component of her role during ID activities. Speaking of the *Mixing Paint ID* activity, for example, she emphasized, "When they're doing the small group, my role is to just circulate [in order to] make sure everybody is on task" (personal communication, January 14, 2016). It seemed that Ms. Allen planned to explicitly attend to whether or not students were engaged in the activity while she circulated around the room. Indeed, during the small group component of the *Mixing Paint ID* activity she planned to be "Just circling to make sure that they're on the right page […], make sure they're answering, listening to their questions and make sure that they're talking about what they're supposed to be talking about." The student actions she planned to focus on were primarily those relating to students being on task.

Taken together, Ms. Allen planned IDs activities that were composed of consistent underlying goals, student roles, and teacher roles. Namely, students were to learn mathematical concepts and procedures by actively engaging with IDs and their peers. Student interaction with IDs could occur in either a small or whole group setting, but was always preceded by some introductory learning and followed with a whole class debrief conversation. Ms. Allen planned to function as the facilitator of student learning in addition to keeping students focused on the task at hand.

Enacted Curriculum

The overall structure of Ms. Allen's enacted ID activities largely followed what she planned. Indeed, both the *Friendship Problem ID* and the *Mixing Paint ID* activities began with pre-ID work aligned with the content present in those IDs. Then, students directly engaged with the IDs in large and small groups, respectively. This occurred for approximately 18 minutes during each activity while Ms. Allen circulated the room as she intended. Lastly, a whole group debrief conversation occurred to discuss the IDs. In this way, the enacted ID activity structures resembled Ms. Allen's planned curriculum.

That said, the enactment of these structures demonstrated shifts in the teacher role, students' roles, and the underlying goal of Ms. Allen's intended curriculum. Consider, for example, the pre-ID work component of the planned ID activities. Ms. Allen had students revisit a problem they previously solved and answer review questions pertaining to that problem prior to engaging with the *Mixing Paint ID* (see Figure *23*).

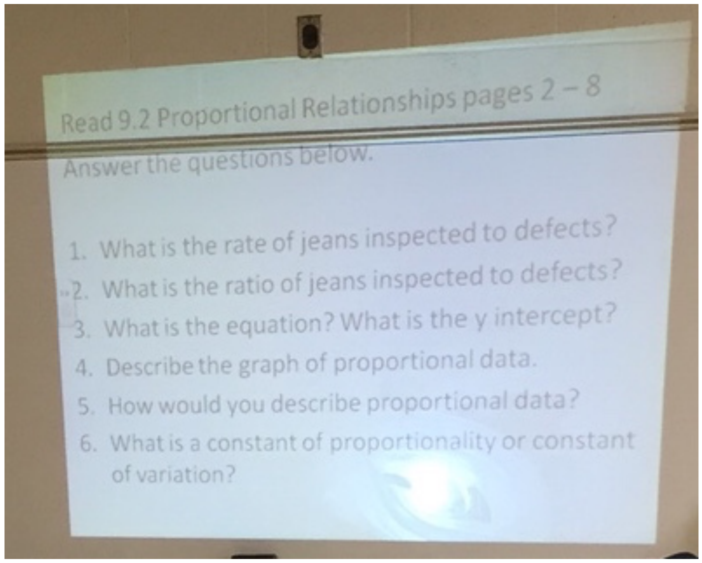


Figure 23. Ms. Allen's pre-ID work, *Paint Mixing ID* activity.

The problem depicted in Figure 23 involved a proportional relationship between the number of jeans inspected at a factory and the number of jeans found to be defective. The questions Ms. Allen posed for this problem exactly paralleled the work students would then engage in while interacting with the *Mixing Paint ID*.

Similarly, Ms. Allen preceded the *Friendship Problem ID* with a problem we will refer to as the handshake problem. The handshake problem included the exact same mathematical patterns, concepts, and solution as the *Friendship Problem ID*. The only difference between the two problems was the context each utilized- one involved people shaking hands whereas the other involved making phone calls. Ms. Allen acknowledged this to a degree when she noted, "the purpose of the shaking hands problem was for [students] to start thinking and *basically model the phone call situation*" (personal communication, November 10, 2015, emphasis added).

Taken together, Ms. Allen seemed to enact pre-ID work that closely paralleled the IDs she used in her classroom. Pre-ID problems were discussed as a whole group where students were expected to record the solution in their notes. Thus, students concluded the pre-ID work with a recorded solution or a significant start toward one that could be applied directly to the ID itself. By introducing, discussing, and solving the pre-ID problems Ms. Allen was actually having students solve the problems found within the IDs. Further, she drew students' attention to this fact, as was apparent in the following interaction.

> Ms. Allen: So, the Friendship problem asks you to find out how many phone calls there would be. Is the handshake problem the same?
>
> Students: No
>
> Ms. Allen: But can you link the two?
>
> Students: Yes. (observation, November 10, 2015)

The problems appeared to not be the same due to their different contexts, but the mathematics found within these problems was equivalent.

This enactment of pre-ID work seemed contrary to the goal Ms. Allen described for her planned ID activities. Recall that her intent during ID activities was to actively engage students as they "use IDs to make conjectures, make conjectures and test them, and learn something new" (personal communication, March 8, 2016). By enacting pre-ID work in the ways she did, students did not engage in exploratory learning as she intended. Instead, students directly applied the pre-ID, whole group conversation and their recorded notes to the ID component of the activity. This was particularly apparent when Ms. Allen reflected on the implementation of the *Friendship Problem ID* activity, stating that students "need more prepping with stuff" (personal communication, November 10, 2015) than she included to be successful during the activity. It seemed, then, that Ms. Allen's goal during the enactment of ID pre-work was to guide students through a problem parallel to the ID and, thus, heavily influence their thinking surrounding the ID. Students, in turn, were to apply the pre-ID work to their use of the ID.

The enacted teacher role seemed to shift as a consequence of the enacted pre-ID work. Specifically, Ms. Allen used the time before engaging students with the ID to "clarifying information [… and] model what I wanted to them to do before they broke back off into their small groups [with the ID]" (personal communication, March 8, 2016). She used pre-ID instruction to model how she wanted students to think about content, represent that thinking, and create their solution. This occurred during the *Friendship Problem ID* activity when Ms. Allen drew a picture representing the handshakes that could occur within the pre-ID handshake problem. Then, she created a table to represent the problem (observation, November 10, 2015). Ms. Allen predisposed students to particular ways of creating and representing their solution by emphasizing these two representations before students used the ID. This, again, decreased the degree to which students could engage in the inquiry learning Ms. Allen expressed a desire for while creating planned ID activities.

Ms. Allen did realize the two intended teacher roles during the time where students were directly engaging with the IDs, however. First, her role of keeping students on task did occur as she planned. She consistently circulated the room while students were interacting with IDs, assessed if they were focused on the task at hand and addressed students when their focus waned. This included telling students to go to the correct screen on their computer, record answers in their workbooks, and discuss mathematics with their peers in response to a number of student off-task behaviors (observations, November 10, 2015 & January 15, 2016).

Ms. Allen also worked to guide students' learning as they interacted with the IDs. During the *Mixing Paint ID* activity, for example, Ms. Allen directed students' attention to the slope of the line representing the proportional relationship between blue and yellow paint by asking, "How would you describe that red line?" (observation, January 15, 2016). This question and others like it were useful in guiding students as they interacted with the ID. Once the students correctly answered that the line "has a constant rate," Ms. Allen proceeded in explaining the content students were meant to reach through their interactions with the ID.

> Yesterday when you had to draw your lines some of you said they were going up at a constant rate and some said that it was going through the origin. This is what we need to pay attention to here. […] It goes through the origin and has a constant rate. (observation, January 15, 2016)

Ms. Allen used leading questions during this activity to direct students' attention. During the *Mixing Paint ID* activity she asked the class, "When you divide each of those what do you get?" (observation, January 15, 2016). Here, she referred to dividing each of the correct ratios of blue to yellow paint student found to reveal the constant of proportionality. Once a student correctly answered 2/1 she explained, "That's right, they are all the same. Write this down" (observation, January 15, 2016).

Ms. Allen directed students' attention by explicitly stating the content they should learn. She frequently explained concepts students were charged with exploring in both small and whole group settings. Consequently, Ms. Allen's role of facilitating students' learning became largely teacher-centered. There were certainly instances where she refrained from explaining content to students. In one such instance a student complained that she hadn't gotten a correct answer yet on the *Mixed Paint* ID. In response, Ms. Allen encouraged her to "keep going until you get it" (observation, January 15, 2016). Instances like this were much less frequent than those where she would instruct the student on how to proceed, however. Thus, Ms. Allen's enacted teacher role was in contrast with her intent to "facilitate the conversation with the students and, for like, a student to lead" (personal communication, March 8, 2016). Ms. Allen alluded to this discrepancy while reflecting on her enactment of IDs, but did not seem to be fully cognizant of how frequently she explained mathematics in her classroom. She noted, "some [students] just want to see the teacher do it but that's not, you know, that's not really what I want" (personal communication, March 8, 2016). Whether Ms. Allen preferred or intended it, her students often watched their teacher do mathematics.

There were some instances where Ms. Allen enacted opportunities for her students to take a more prominent role in explaining mathematical concepts. During the *Friendship Problem ID* activity, for example, two students were called to the board to present their solutions to the problem found in the ID. These presentations were given in turn to the entire class and included slightly different ways to approach the problem. Ms. Allen had each student share their solution while interjecting questions or statements meant to direct students' attention to components of the students' solutions. At the beginning of the second student presentation, for example, she noted, "Notice that she is starting in a different spot" (observation, November 10, 2015). Further, she used the students' solutions as an impetus for getting students to discuss their thinking, as demonstrated in the following interaction.

> Ms. Allen: Does anyone notice a pattern [in this solution]?
>
> Student: Each time you add a student the number of handshakes increases by 4"
>
> Ms. Allen: Everyone look at the table, is that true?
>
> Students: No.
>
> Ms. Allen: So, everyone look at the table and help him fix it.
>
> Student 2: It goes up by 2 then 3 then 4.
>
> Ms. Allen: So, if there were 6 people how many handshakes would there be? (observation, November 10, 2015)

It seemed that Ms. Allen "wanted [students] to come up and actually show" (personal communication, November 10, 2015) their thinking while she facilitated the conversation surrounding this thinking. Thus, the student presentation component of the *Friendship Problem ID* activity was enacted in a more student-centered manner than during the ID pre-work or while students were directly interacting with the ID. This instance was not typical, though, as the enactment of ID activities was largely teacher led.

The influence of factors surrounding Ms. Allen's practice seemed to result in alterations to ID activities as she negotiated the enactment of her planned curriculum. Aspects of the planned and enacted ID activities observed during this study are depicted in Figure 24.

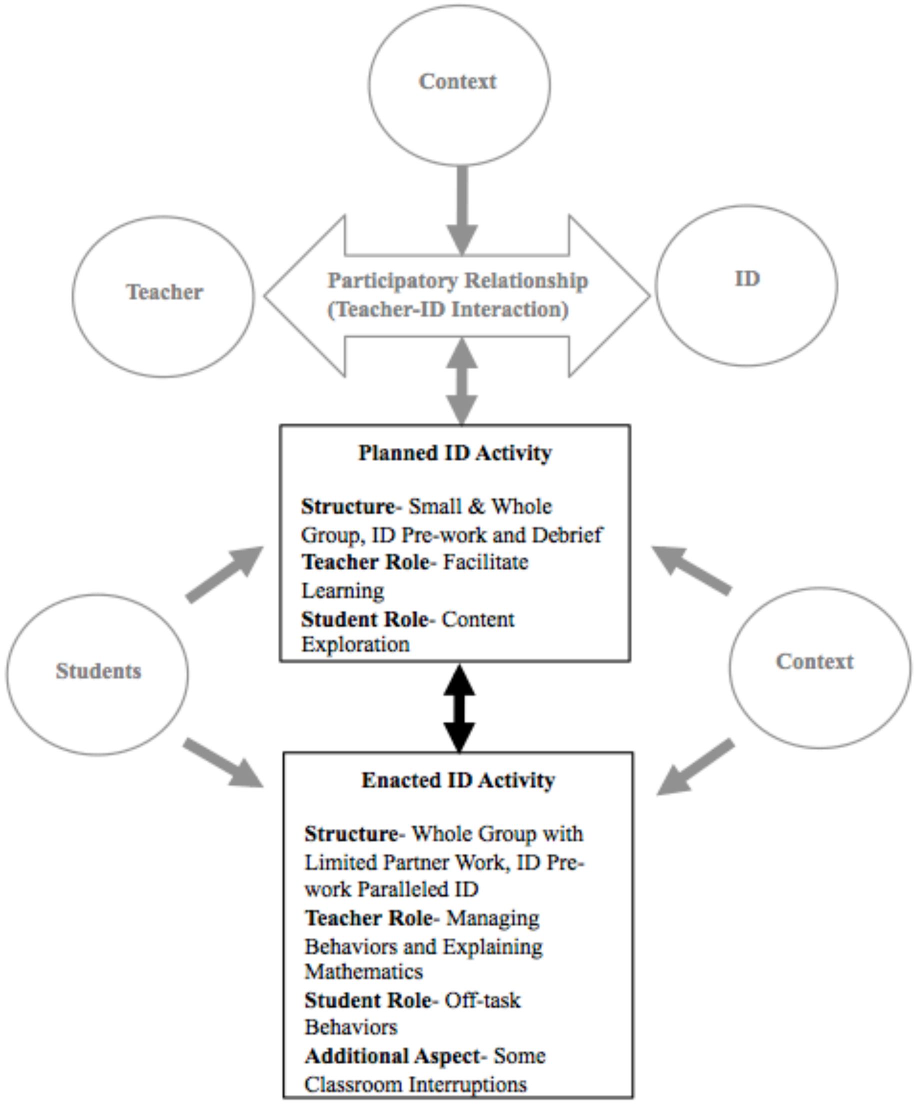


Figure 24. Ms. Allen's planned and enacted curriculum involving IDs.

Next, the factors related to IDs, the teacher, students, and the context within which Ms. Allen taught are discussed.

ID Factors

Factors surrounding ID design seemed to influence how Ms. Allen created the intended curriculum. Specifically, she considered an ID's context, versatility, degree to which it could support students, if it provided instant feedback, embedded mathematical representations, and interactive functionality when considering if and how to plan an ID activity. These mediating factors are described below.

Context of ID. Ms. Allen noted various aspects of the context embedded within an ID as being important in her creation of the planned activities. First, she pointed out, "Of course, it's a good idea to use a real life situation" (personal communication, January 14, 2016). She seemed to desire IDs that used real-world contexts. She explained why this was important to her when discussing the *Paint Mixing ID*.

> Researcher: Why do you like it for teaching that particular content?
>
> Ms. Allen: I think just because it shows how […] the proportion [of paint] has to be right in order for it to be proportional. It's kind of like directly related to the world. […] Like say every time it's proportional I know it just means equal fractions. You know, like the definition just means equal fractions, but what does that mean in reference to something in real life. (personal communication, January 15, 2016)

Ms. Allen seemed to want students to see how mathematical concepts relate to the real world; she wanted students to see that understanding proportions was important because it applied very concretely to a situation they may encounter in their lives.

Further, Ms. Allen seemed to appreciate IDs' ability to engage students in real world contexts that were inaccessible without technology. When discussing an ID involving elevators, for example, she explained, "[…] especially if you can't get the concrete [object]-- we can't go and measure the elevator. The [ID] allows them to see what's going on" (personal communication, October 26, 2015). Here, the ID allowed students to explore the mathematical concept of linear motion within a context they would not otherwise have access to, namely elevators moving at varied rates and directions within a building.

Ms. Allen even preferred IDs at times when a context was available but was not desirable. This situation arose when Ms. Allen spoke of the *Mixing Paint ID*. She noted,

> Of course, we're not using paint in the classroom. [She laughs] […] No, I'm not buying paint, and I wouldn't want – uh-uh. Not unless we was like, okay let's do an art collaboration with the art teacher, but no. I wouldn't have the kids using paint. They're too foolish. (personal communication, January 14, 2016)

Using actual paint was not something Ms. Allen was willing to incorporate into her classroom. Students were still able to explore paint mixing, though, through the ID. Taken together, it seemed that an ID's context, specifically that it was set within a real world context and provided students access to real-life situation that was otherwise inaccessible or undesirable, mediated Ms. Allen's creation of planned ID activities.

Versatility of ID. Ms. Allen also considered the versatility of particular IDs when creating the planned and enacted curriculum. Reflecting on the *Paint Mixing ID* activity, for example, she noted that there was no ID corresponding to the context of "making purple with red and blue" (personal communication, January 14, 2016). Ms. Allen wished that the *Paint Mixing ID* had the capability to show both how proportions of yellow and blue make different shades of green and how proportions of red and blue make various tones of purple. Similarly, she wished to extend the capability of the *Friendship ID* to show the situation with "six students and seven students and eight students" (personal communication, November 10, 2015). If this were possible, she explained, she would have had students work in small groups with the ID to discern patterns they saw up to n = 5, "have them present out […] and ask them to make the prediction if it was six […] seven and […] eight and then use the interactive diagram" (personal communication, November 10, 2015). Thus, her planning of the *Mixing Paint* and *Friendship ID* activities were mediated by their limited versatility. Conversely, Ms. Allen noted that she "really liked the skateboarder [ID] because it did give several opportunities for the kids to interact with that skateboarder in the [different] graphs" (personal communication, March 8, 2016).

The versatility built into an ID's design was something Ms. Allen considered when thinking about planning ID activities. When asked about how she created her intended curricula, she reflected on "how useful [an ID] is in multiple ways. […] Is it just good for one question or can I use it several times?" (personal communication, March 8, 2016). She preferred the latter, that IDs by useful over several examples or problems. Thus, the versatility of an ID seemed to influence Ms. Allen's creation of the intended curriculum as she was inclined to use IDs with increased built in versatility for longer durations than IDs that were more limited.

Supports students. Another factor that seemed to influence Ms. Allen's ID use was how they could support her students. As will be discussed further in the student factor section, Ms. Allen's students had gaps in their mathematics content knowledge. Thus, deficient pre-requisite skills could prevent students from accessing grade level content. IDs, Ms. Allen noted, could eliminate this barrier. She explained this when asked about the *Paint Mixing ID* and students' understandings of fractions. "If a kid struggles, it may be something with [the ratio] two divided by one, which they don't have to […] reduce it because the third column shows the ratio. So, they don't really have to reduce anything" (personal communication, January 14, 2016). The ID displayed a reduced ratio when students input quantities of blue and yellow paint, alleviating any concern over students being able to correctly simplify fractions. Ms. Allen expressed appreciation for this aspect of the ID.

Also speaking of the *Mixing Paint ID*, Ms. Allen discussed how it supported students' understandings of fractions as a concept. This concept, she noted, was something her students continued to struggle with.

> So, if you say a 2:1 ratio, they don't necessarily really, really, really know what that means. […] So, with the ID they can actually see that [ratio. …] They can see the mix happening. […] They wouldn't get the true effect [of the ratio] if they didn't have the ID. (personal communication, January 14, 2016)

Here, Ms. Allen described how this particular ID supported students' understanding of what a two to one ratio meant, a concept that should have been mastered by her students in previous years, while also engaging them in content closer to grade level (i.e. proportionality). Thus, an ID's ability to support the gaps present in her students' pre-requisite knowledge seemed to mediate Ms. Allen's ID use.

Instant feedback. Ms. Allen also discussed how the *Mixing Paint ID* provided students with feedback as to whether the quantities of paint they entered were correct or incorrect. She explained,

> If they don't come up with their correct proportion, their color is not going to come out correctly. So, they're actually using the colors for them to see that. This is the green that they want and […] if their proportion is off, they're not going to get the correct green. (personal communication, January 14, 2016)

The ID's ability to visually create the green that resulted from the entered ratio of blue to yellow paint allowed students to assess their understandings after each input. Ms. Allen appreciated the instant feedback students received. This feature, combined with a check mark that appeared when the proportion was correct, also seemed to resonate with students. During the *Mixing Paint ID* activity, for example, students exclaiming, "I think I've got it!" (observation, January 15, 2016) after using the feedback functions of the ID to obtain a correct answer. Ms. Allen confirmed this, stating that they "got it because [the ID] has a check" and encouraged the student to find more equivalent ratios using this feedback mechanism (observation, January 15, 2016). It seemed that Ms. Allen appreciated when IDs provided instant feedback to students and encouraged them to use this feature. Thus, the degree to which an ID provided students instant feedback influenced Ms. Allen's planning of this and other ID activities.

Multiple representations. Also apparent in Ms. Allen's intended curriculum was the importance she attributed to the multiple representations often found within IDs. Her comments related to the *Mixing Paint ID*, for example, stressed the value of including both the tabular representation and the more concrete real-world depiction of paint mixing. Ms. Allen made similar comments throughout the study, emphasizing the importance of multiple representations in mathematics learning in general and with IDs specifically. Her description of IDs involving the manipulation of an equation and graph of a general function by altering the coefficients *a*, *h*, and *k* highlight this.

> The equation changed [when] you move the little slider and it tells you what the equation is. […] So, when you look at the equation and you saw that *k* makes [the graph] go up and down, well, they get to see how to the value *k* increases and so does the graph. So, they get to see what you say. (personal communication, October 26, 2015)

Ms. Allen valued that "you get the visual with the interactive diagram" (personal communication, November 10, 2015) so much so that she identified this as the most important mediating ID factor in her creation of ID activities. The visual representation being dynamically linked to other representations was particularly important to her "because it's another way of helping kids make connections" (personal communication, November 9, 2015). Thus, multiple representations being present and dynamically linked seemed to influence Ms. Allen's instructional use of ID.

Allows active student interaction. The degree to which ID design allowed and encouraged students to actively interact with it also seemed to mediate how Ms. Allen created the planned curriculum. She constantly expressed a desire for students to "do something" or "play" with IDs. She enjoyed implementing the *Skateboarder ID* activity, for example, because "the kids were involved in it. So, it wasn't just me doing something" (personal communication, March 8, 2016). Ms. Allen seemed to prefer having students actively interact with IDs, as opposed to activities that were more teacher-centric. She described IDs similar to the *Skateboarder ID* as "simulations" (personal communication, November 9, 2015) and "investigations" (personal communication, January 15, 2016) at times throughout the study. She focused on students' active, hand-on interaction with the content found within IDs, though, and not specifically on what these terms might otherwise define.

A "hands-on" ID even appeared, at times, to be more appealing than an ID that used multiple representations. Ms. Allen explained this relationship while reflecting on her interactions with different IDs,

> I think that [specific ID] might just be a visual. It's not anything that [students] have to do. […] The ones […] where the kids can come do it. They definitely get involved. They pay attention more to what's going on. So, those are definitely the ones that I definitely use. (personal communication, October 26, 2015)

Thus, an ID that got students actively doing something was more desirable than an ID where students passively watched it, even when visual representations were present. Ms. Allen even attributed students' success on a recent assessment to students actively interacting with IDs. As she highlighted, "because they were able to put their hands on something and work with it [in IDs], they were able to have a better outcome on the scores" (personal communication, October 26, 2015). She concluded, "the more they play with [a concept], the better understanding they get" (personal communication, March 8, 2016).

Taken together, various ID factors seemed to influence how Ms. Allen created planned ID activities (see Figure 25).

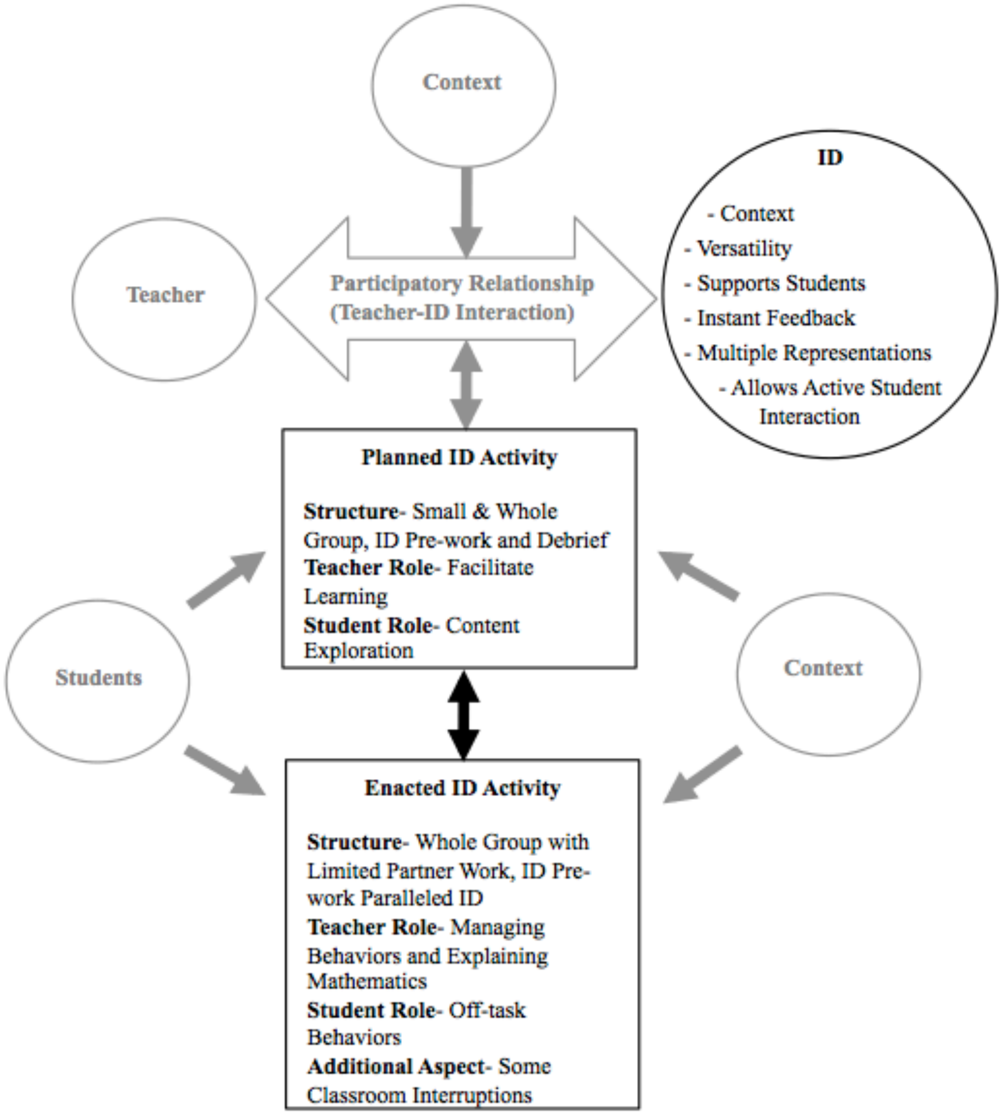


Figure 25. ID factors affecting Ms. Allen's use of IDs.

Of note were the context embedded within the ID, its versatility, how it supported students accessing the content to be explored, and whether or not the ID offered instant feedback to the user. Having multiple mathematical representations present in the ID also mediated Ms. Allen interactions with ID, as did how actively students could interact with an ID.

Teacher Factors

A number of factors originating from Ms. Allen also seemed to affect her planned and enacted ID activities. In particular, her teacher knowledge, beliefs surrounding the ways in which mathematics should be learned, and classroom management were influential factors in her ID use. These are discussed below in turn.

Knowledge. Ms. Allen's knowledge emerged as a factor that affected the creation of the intended curriculum. Recall that Ms. Allen had 17 years of experience as a secondary mathematics teacher. Her experience seemed to afford her a thorough understanding of Algebra I content. While reflecting on how she selected activities to include in the planned curriculum, for example, she noted,

> I have to admit, there have been times [in the past] when I would not have shown something that I wasn't clear on [… because students] give you a different view of looking at things. I was learning just like they are learning […] but it really doesn't happen now. (personal communication, March 8, 2016)

Ms. Allen's past concern about not understanding one of the varied student perspectives, misconceptions, and viewpoints on content that might emerge in mathematics instruction would prevent her from using particular curricula. This was no longer a concern, she emphasized, because she was now knowledgeable enough to anticipate and react appropriately to students' perspectives. Her content and pedagogical content knowledge (Shulman, 1987), then, influenced the creation of the intended curriculum by enabling her to use any ID she encountered.

That said, there were instances where gaps in Ms. Allen's pedagogical knowledge seemed to influence the planned curriculum. In the beginning of the study, for example, she reflected that she "never thought about" (personal communication, March 8, 2016) asking students what they learned during ID activities nor planning small group activities where students interacted directly with an ID by using a laptop computer. Ms. Allen used her participation in this study as a way to challenge these aspects of her pedagogical knowledge. Speaking of the differences in her planning of the *Friendship Problem ID* and the *Mixing Paint ID*, for example, she explained,

> Last time we just used the one laptop and projected it. But after talking to you, I'm like, well, *I never thought to say, let's do small groups [with group laptops]* to see how it goes. And honestly, that's why I like to have discussions with you. (personal communication, January 14, 2016, emphasis added)

Ms. Allen had not considered having students interact with an ID on laptops computers while in small groups until this structure was mentioned in this study's interview protocol. This and other gaps in her pedagogical knowledge mediated the creation of the intended curriculum in that she was not aware of, and consequently did not utilize, certain instructional strategies in her planning of ID activities.

Beliefs. Ms. Allen seemed to express a number of beliefs concerning how mathematics should be learned and, as a result, how instruction should be planned and enacted. Her results for the TBQ are shown in Table 9. Ms. Allen's overall average score of 3.99 suggested beliefs consistent with meaningful mathematics learning (Bate, 2010; Jonassen et. al, 1999).

Table 9
Pedagogical Beliefs Survey Results- Ms. Allen

	Active	Cooperative	Constructive	Authentic	Intentional	Average
Initial meeting	3.40	4.00	4.00	3.88	4.50	3.96
Final meeting	2.80	4.33	4.17	4.00	4.75	4.01
Delta	-0.60	0.33	0.17	0.13	0.25	0.06
Average	3.10	4.17	4.09	3.94	4.63	3.99

The TBQ categories related to active, cooperative, constructive, and intentional learning are discussed below, as these categories emerged as the most cogent to Ms. Allen's ID use.

Mathematics learning should be active. Ms. Allen consistently made statements aligned with the belief that learning mathematics should be an active process for students. She noted, for example, "Learning mathematics should be active for students […]. So, I do believe that they should be more engaged with it not just sitting in the seat and having conversation, but where they actually would be doing something" (personal communication, March 8, 2016). Consistent with this statement, she continued, "Mathematics should be taught more like science where it has labs [… and] hands-on activities" (personal communication, March 8, 2016) for students to create and test conjectures.

Ms. Allen's description of hands-on, active learning seemed consistent with her description of the underlying goal of the *Mixing Paint ID* activity. She noted the desire for students to "play with the animation until they figured out the correct color of green that they were trying to get" (personal communication, January 15, 2016). In other words, Ms. Allen wanted students actively engaged with mathematical content as they learned about proportional relationships through their use of the ID.

Interestingly, Ms. Allen's lowest category score on the TBQ was a 3.10 average in the active category. While this score indicated a tendency toward active learning, it also seemed low given her emphasis on active student learning. One possible explanation for this was that there seemed to be a tension between Ms. Allen's desire for students to actively learn mathematics and her need to manage the classroom. Recall, for example, that she elected to use the *Paint Mixing ID* instead of having students actually mix real paint because "they're too foolish" (personal communication, January 14, 2016). Thus, it seemed that Ms. Allen might have been willing to sacrifice the active learning possible with labs involving real paint in favor of an ID simulation due to classroom management concerns.

Mathematics learning should be cooperative. Ms. Allen continually expressed the belief that student cooperation was important when learning mathematics. In particular, she emphasized the importance of students working in small groups, noting that classroom activities should begin with students in small groups and then move to a whole group structure. She explained why the progression from small group to whole group was so important in her classroom,

> It's better if you ask the students to talk about [content and] work together, because they're not necessarily sure about what they're doing all the time. […] So, normally the kids would want to work with somebody first. […] So, when they don't agree, they have these small group conversations. […] You get more student engagement […] when they've had the opportunity to discuss questions with their partner […] because students that are not sure don't always want to raise their hand or don't always want to participate [in whole group]. So, it's

> really good to have them do the small groups. (personal communication, January 14, 2016)

Small groups seemed to allow her students opportunities to discuss content they were still grappling with or didn't understand in a setting they found more comfortable. Thus, students engaged more actively in learning content, became more comfortable in discussing that content and their learning of it, and were more willing to participate in the whole group conversation that followed.

IDs were beneficial tools in fostering the small group conversations and content learning Ms. Allen desired. She explained,

> [Students are] supposed to work collaboratively. So, I think that [an ID] helps with them having discussions. […] and actually have something for them to use as justifications to support their answers. So, yeah, I think [the ID] definitely helps. It helps them with their learning. (personal communication, October 26, 2015)

Ms. Allen used IDs that enabled and encouraged collaborative mathematical discussions frequently. Her desire for cooperative learning was aligned with her description of the ideal classroom where students were working in pairs and small groups, as well as her high average score of 4.17 on the cooperative category of the TBQ.

It seemed, though, that Ms. Allen's belief pertaining to cooperative learning interacted with her attention to classroom management. When discussing her thoughts about cooperative learning she noted, "Students should work in groups while learning mathematics because *I think that helps them like keep on track*, it helps them to collaborate" (personal communication, March 8, 2016, emphasis added). Ms. Allen seemed to be thinking about how to "keep students on track," or engaged in work, so that behavior management did not become a challenge. Thus, Ms. Allen's belief that learning should be cooperative, like her belief that mathematics learning should be active, seemed to interact with her attention to classroom management and student behaviors.

Mathematics learning should be constructive. Ms. Allen also seemed to express the belief that mathematics should be constructive. Her average score of 4.09 on the constructive category of the TBQ supported her desire for students to construct their understandings as opposed to being told what they need to know. She noted the importance of students creating "both procedure and conceptual knowledge […] since it helps them make connection as they go further in other areas" (personal communication, March 8, 2016). She seemed to want students to view problems from varied perspectives and create different methods for reaching a solution. Ms. Allen enacted this belief during the *Friendship Problem ID* activity by encouraging students who were struggling with the problem to "keep going until you get it" (observation, November 10, 2015). Additionally, Ms. Allen had students present their solutions to the class. Her selection of students was purposeful in that one student had not yet completed the entire solution and was struggling with how to move forward. The other solution was more complete, but used a different solution method than that which was used by the majority of students within the class. By selecting these two students to share their solutions, Ms. Allen attempted to get students thinking about the focal problem from different viewpoints; she seemed to want students considering different perspectives and constructing their own understandings from the cooperative discussion. She noted that frustration was "just a natural process of human learning" (personal communication, March 8, 2016) and welcomed it during her classes since she knew "that they can all get it" (personal communication, November 10, 2015).

Mathematics learning should be intentional. Lastly, Ms. Allen seemed to express the belief that mathematics learning should be intentional. To begin, her average score of 4.63 on the intentional category of the TBQ indicated a strong belief that learning should be explicitly monitored and reflected upon by students. This belief was aligned with the planning and enactment of the *Friendship Problem ID* activity in that the primary focus of this activity was for students to learn about and engage in metacognition. Ms. Allen emphasized the problem solving process and students' being cognizant of their own thinking by asking them questions explicitly targeting these areas, such as "What is the first step […] of the math problem solving process?", "How did you plan [how you would approach the problem]?", "Did you look back when you were [creating your solution]?", and "Did you notice their strategy?" (observation, November 10, 2015).

Additionally, Ms. Allen enactment of the *Friendship Problem ID* emphasized how valuable creating drawings can be when solving a problem. She reflected,

> They didn't think to draw a picture [when solving the problem]. So, I think that's another way of helping them see this is how you could solve a problem, keep track of [your solution], or get the visual with the interactive diagram. (personal communication, November 10, 2015)

By explicitly attending to the strategy of drawing a picture during problem solving, Ms. Allen seemed to provide students a means for intentionally illuminating, keeping track of, and later reflecting on their thinking surrounding the problem. This was consistent with the belief that mathematics learning should be done intentionally.

Taken together, Ms. Allen's TBQ scores and comments indicated that she held beliefs aligned mathematics learning that was active, cooperative, constructive, and intentional. Consequently, she seemed to plan and attempt to enact activities utilizing IDs that she viewed as beneficial in facilitating learning consistent with these beliefs. Ms. Allen's beliefs, then, mediated the creation of the intended and implemented ID activities.

Classroom management. Classroom management also emerged as a factor affecting Ms. Allen's instructional use of IDs. To begin, she leveraged a number of the design features of IDs, which were described in the ID factor section above, for positive behavior management. She explained,

> If I felt like there was someone who, for instance, somebody that doesn't necessarily like to do their work, that will be probably one of them […] I would choose to come up here [to the computer with the ID] because it's hands-on. […] That person will probably be the person that's like, 'Oh, let me do it, let me move this [ID] around' and get involved and learn that way. So, I would definitely use it with my most […] challenging student. (personal communication, October 26, 2015)

An ID being "hands-on" seemed to motivate even Ms. Allen's more resistant students to engage with the course content. She recognized this and purposefully called on these students while using IDs in class. Thus, Ms. Allen seemed to plan and enact IDs as, at least to some degree, an engagement strategy for behaviorally challenging students.

Behavior management challenges during classroom activities also had a mediating affect on the enactment of IDs. Numerous instances of student misbehavior were recorded during the implementation of both the *Friendship Problem ID* and the *Mixing Paint ID*. These behaviors are detailed below in the student factor section. Ms. Allen addressed students' misbehavior with varied success. She quickly and directly spoke to students about their misbehavior, which, in most instances was effective (observation, November 10, 2015 & January 15, 2016). There were instances, however, where she was ineffective in eliminating students' persistent misbehaviors. During the *Friendship Problem ID* activity, for example, a student's behaviors were deemed so disruptive to learning that Ms. Allen directed the student to leave the classroom to "cool off" (observation, November 10, 2015). This student's behaviors and Ms. Allen's continued interactions with him distracted other students' and her own attention from the ID activity. Thus, classroom management concerns stemming from both the desire to engage students in class activities and reacting to student behaviors that did not meet her expectations seemed to mediate Ms. Allen's implementation of IDs.

Taken together, various teacher factors seemed to influence Ms. Allen's planning and implementation of ID activities, as shown in Figure 26.

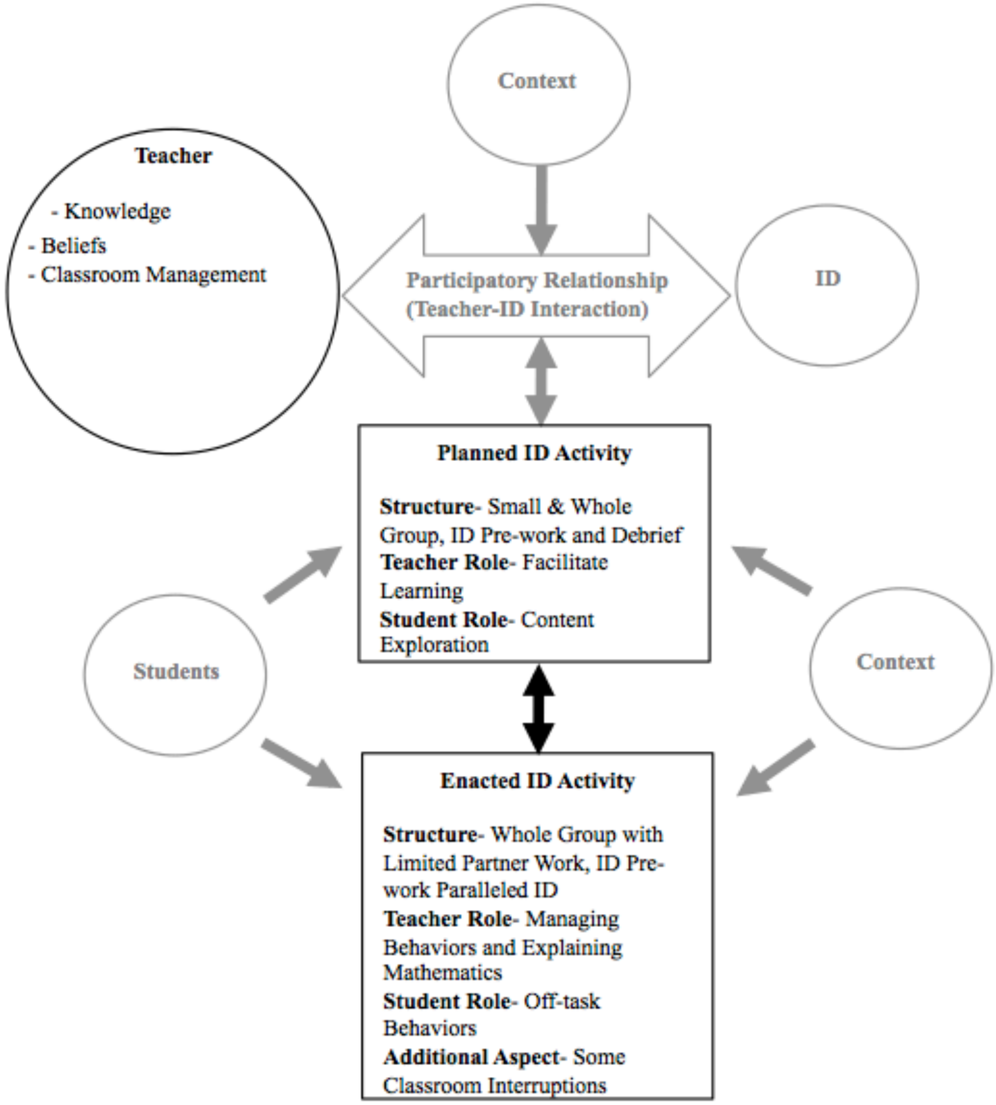


Figure 26. Teacher factors affecting Ms. Allen's use of IDs.

First, the strengths and areas of growth within Ms. Allen's pedagogical and content knowledge influenced the creation of the intended curriculum. Her beliefs about the ways in which mathematics should be learned also influenced the planned curriculum, as she preferred IDs conducive to creating learning activities aligned with these beliefs. Lastly, classroom management concerns influenced the enacted curriculum as Ms. Allen aimed to engage students in the ID activity and addressed student misbehaviors.

Student Factors

Factors related to Ms. Allen's students also mediated her use of IDs. Of note was the influence of students' perceived learning styles, content knowledge, behavior, and engagement. Each is discussed below in turn.

Learning styles. Ms. Allen perceived that her students had particular learning style preferences and aligned her instruction with those preferences. Reflecting on her planning and implementation of the *Friendship Problem ID* activity, for example, she asserted that students needed hands-on learning. Recall that before using the *Friendship ID*, Ms. Allen engaged her students in a parallel problem involving groups of individuals shaking hands. Students physically demonstrated how the handshakes would occur in a whole group setting. Students then engaged with the ID only once the handshake problem was fully enacted and discussed. Ms. Allen explained why she planned the *Friendship Problem ID* activity as such,

> Everybody doesn't understand everything the same way. Like with this group of kids, I feel like they're very much hands-on. We're talking about a group of hands-on learners, even not just visual but actually doing. So, yes, I think they needed both. (personal communication, November 10, 2015)

Ms. Allen emphasized the importance of both the physical movement found in her implementation of the handshake problem and the visual representation present within the *Friendship Problem ID*. She thought that the hands-on engagement and the visual representation were necessary in her students' learning. Of note was that Ms. Allen delineated the interactivity designed within the *Friendship Problem ID* from the hands-on learning she wished to provide for her students. Thus, part of why she used the handshake problem was because she thought the *Friendship Activity ID* did not provide hands-on learning. It seemed, then, that Ms. Allen's perceptions of her students' learning styles mediated her instructional use of IDs.

Content knowledge. More significant, however, was the impact of students' content knowledge on the ways in which Ms. Allen planned and implemented ID activities. She consistently described her students as "below level" and "very low level functioning" (personal communication, November 10, 2015). While she did teach one class designated as honors, she noted that overall all but a few of her students struggled in mathematics. "Their math levels are low. Their cognitive levels are low. Everything is just low" (personal communication, November 9, 2015).

Consequently, Ms. Allen seemed to have doubts about students' ability to successfully interact with IDs without scaffolded instruction preceding the ID. For example, she noted that students would likely have difficulty deriving the function rule for the problem posed in the *Friendship Problem ID.* She explained, "These kids just struggle with coming up with [function rules] when you're looking at input-output table. So, I didn't want to try to jump to the rule [… because] I didn't feel like all of them really understood" (personal communication, November 10, 2015). Consequently, Ms. Allen introduced the handshake problem before the *Friendship Problem ID* so that students were introduced to the content present in the ID before actually engaging with it. She explained, "That's the whole purpose why I'm doing the handshake ahead of time. We're going to see if there's any patterns and what happens" (personal communication, November 9, 2015). Any observed patterns in the handshake problem would also be present in the *Friendship Problem ID* since they were parallel problems. By asking students to solve the handshake problem, Ms. Allen was essentially asking them to solve the friendship problem posed in the ID. Even so, she still had reservations about students' ability to successfully complete the *Friendship Problem ID* activity.

> I don't want to say I'm thinking too low of my students, because I feel that they may be able to do this once they do the handshake. But then and again, it's like, I know my students and I really don't think they can. (personal communication, November 9, 2015)

Thus, Ms. Allen was not surprised when the *Friendship Problem ID* activity took longer than she anticipated. She reflected, "We didn't get as far as we could have, but we got as far as 'we' could have but not as far as what is set up in Agile Mind. We didn't get there at all. […] The students are very, like I said, behind" (personal communication, November 10, 2015). The pacing described in Agile Mind, Ms. Allen seemed to be indicate, was more ambitious than what she thought students could achieve.

Ms. Allen's concerns surrounding students' content knowledge impacted her ID use more generally. First, she noted that the type of activity she would engage students in depended largely on students' mathematical understandings. Contrasting how she used IDs with her regular classes and her honors section, for example, she noted, "For the one [honors] class, I can give it to them as more of an exploratory activity versus this class I have to give them more like guided information" (personal communication, March 8, 2016). Students perceived as having higher-level mathematics skills would interact with IDs during inquiry learning activities, but Ms. Allen indicated that other students needed more teacher guidance.

In addition to the underlying activity approach, students' content knowledge also affected the structure of the ID activities Ms. Allen planned. Speaking once again about the *Friendship Problem ID*, she explained how the misalignment between students' content knowledge and the understandings required to complete the problem affected how she grouped students. Ms. Allen explained, "This topic right here […] is a little above level, or should I say, them and their level. So, for this activity, I wouldn't, say, break them into small groups" (personal communication, November 9, 2015). Utilizing small groups initially with the ID, she seemed to suggest, would not be effective because students would struggle too much. Consequently, she planned the activity using a whole group structure where she could make "clarifying comments [… since] individuals struggle with understanding like how we do it" (personal communication, March 8, 2016). Individual student-ID interactions were never planned, it seemed, because Ms. Allen thought that students' deficient mathematics skills would prevent them from effectively accessing the mathematics embedded within IDs by themselves.

Behaviors. Students' behaviors also emerged as a factor that mediated Ms. Allen's enactment of ID activities. Multiple instances of student misbehavior were observed during the study's observations, which included mostly occurrences of off-task talking, listening to music, sleeping, eating, and other relatively minor misbehaviors (observations, November 10, 2015 & January 15, 2016). Ms. Allen addressed students directly in these instances, which largely eliminated their misbehaviors. The misbehaviors often began again as the class progressed, however. There were two students in particular who exhibited persistent and escalating misbehaviors that were eventually asked to leave the room to complete a reflection (observation, January 15, 2016). Each instant of misbehaviors distracted students' and Ms. Allen's focus from the task at hand and occupied instructional time that was planned for the ID activity.

Overall, Ms. Allen thought that students exhibited "constant behaviors that are not appropriate. So, I have to manage students' behaviors first and then keep the students on task" (personal communication, March 8, 2016). Ms. Allen's efforts were focused primarily on students' behaviors and secondarily on engaging students in content, the latter of which she desired. This desire was made explicitly clear while she described the difference between her actual and ideal classrooms.

> In the ideal world I wouldn't have to worry about behaviors and kids being on task. They would be on task. They would be engaged […] and the focus can be on the lesson. […] The teacher role would definitely change because that way I can move [the factors] manage student behaviors and keep students on task all the way to the bottom [of factors that affect my practice]. At the top I can ask students about what is happening in their ID and then ask students about what they are learning and assess student understanding. So, there would be a shift there. (personal communication, March 8, 2016)

Student behaviors seemed to influence Ms. Allen's role in the classroom so significantly that without it she would completely shift her teacher actions during ID activities. In fact, when asked what student factors mediated her interactions with IDs, Ms. Allen identified the most significant factor as "students' behaviors. That affects everything" (personal communication, March 8, 2016).

Student behavior also seemed to have a mediating influence on the activity structure Ms. Allen planned within the intended curriculum. First, when considering small group work, she only planned for groups of two. "I think that's enough, because […] they're too playful" (personal communication, January 14, 2016). Students acting "playful," it seemed, prevented Ms. Allen from grouping three or four students together during ID activities. When using pairs, she was also careful to,

> limit the time that they have to fool around. […] So, when I put them in pairs to do things, it's like 'Okay, you have two minutes to do something' […] then I check, and we discuss. […] Everything has to be really scaffolding and laid out for them. (personal communication, January 15, 2016)

Student-to-student collaboration occurred within "shorter periods of time […] which was key" (personal communication, March 8, 2016) in preventing student misbehavior. Finally, student behavior even prevented the planning of student groups all together. After the *Mixing Paint ID* activity, for example, Ms. Allen reflected on how she would alter this activity for future classes and noted that she would eliminate the small group portion for some of her students. She explained,

> So, [my other class] of students can actually sit next to each other without all the playing, giggling and laughing. Versus this group, they look at each other and everything is funny. So, that group, I would […] do the ID and give them the computers like that to work in groups, I would. But this group, they're too much of a bother. (personal communication, January 15, 2016)

Students in different classes would then experience different classroom structures during future enactments of the *Paint Mixing ID*. This, in turn, could have a significant impact on students' learning experience.

Taken together, student behaviors had a significant impact on both the planned and enacted curriculum involving IDs. Behaviors mediated the amount of instructional time available for ID activities, the opportunities for students to collaborate while interacting with the ID, and the class structure used during the ID activity.

Engagement. Lastly, students' levels of engagement seemed to influence Ms. Allen's interactions with IDs. Ms. Allen was very concerned with fully engaging her students in classroom activities both so that they were actively learning and to prevent misbehaviors. IDs were useful in this, she noted, "because the kids do like to do stuff. And some of them definitely want to be on the computer. So, I think having them […] drawing graphs and stuff [with an ID], they want to participate" (personal communication, January 14, 2016). Ms. Allen's students seemed to want to interact with the ID so much so that during the *Friendship Problem ID* activity students who were otherwise disengaged stated "Oh! You didn't call on me!" and "Can I do it now?" (observation, November 9, 2015) once the ID was being used. Additionally, there was a noticeable increase in student on-task behavior during the *Paint Mixing ID* activity when students began using the ID on laptop computers (observation, January 15, 2016). Ms. Allen reflected on this increase in student engagement,

> By the time they got to […] the ID, they were definitely more engaged than when I was just asking them to answer the six questions. […] I felt like the kids would be more engaged when they're working with the ID and I think that they actually were more engaged [than I thought]. […] They definitely like to see stuff happening and participate in stuff. (personal communication, January 15, 2016)

Ms. Allen anticipated that the *Mixing Paint ID* would engage students, but was pleasantly surprised when students' engagement exceeded her expectations. Later, she emphasized that she really "liked that ID because it definitely keeps the kids interacted, the kids stay interested with the lesson [because they] can play with the ID" (personal communication, March 8, 2016). Thus, an ID's ability to affect students' levels of engagement seemed to influence the ways in which Ms. Allen created ID activities.

Taken together, Ms. Allen's planned and enacted curriculum involving IDs were mediated by various student factors (see Figure 27).

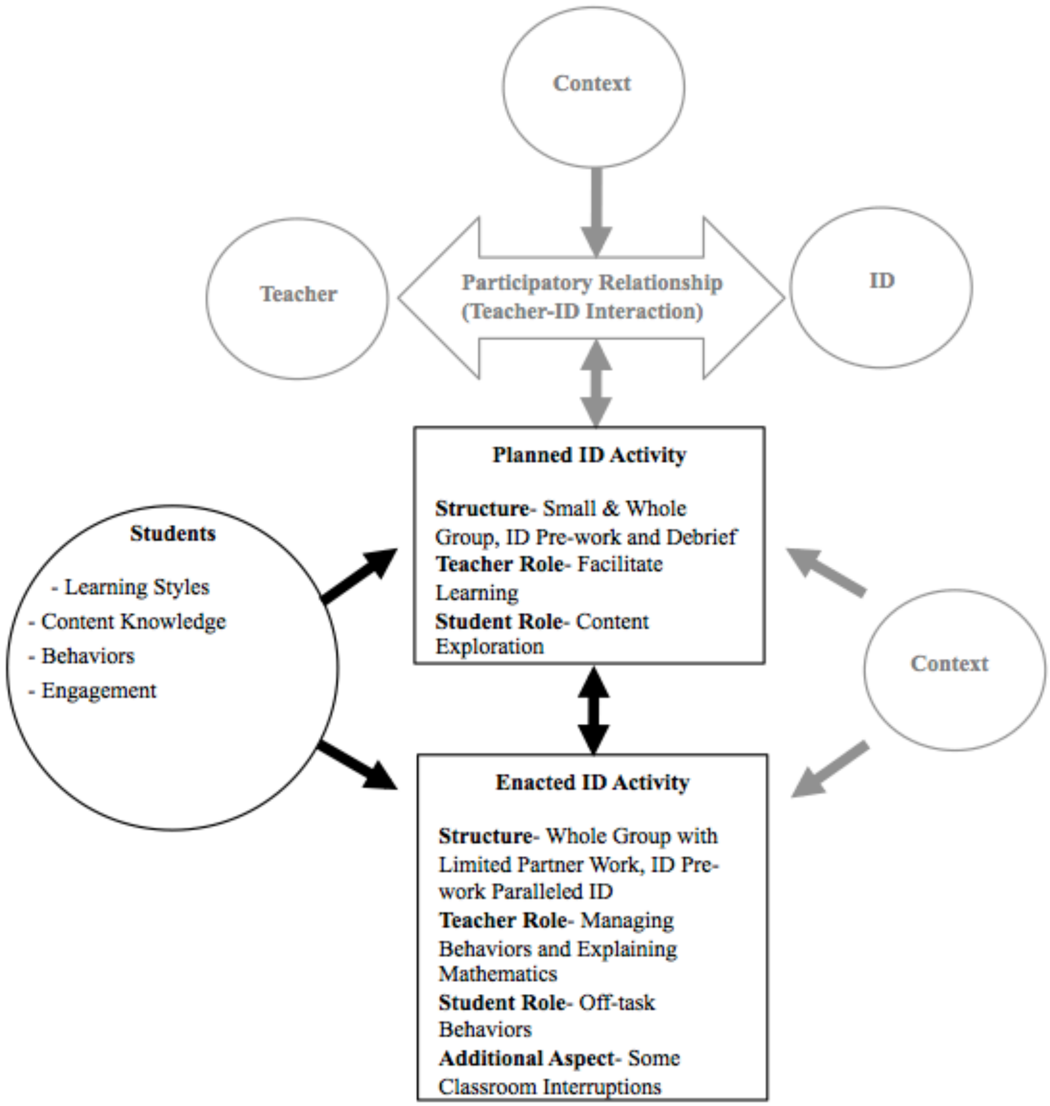


Figure 27. Student factors affecting Ms. Allen's use of IDs.

Namely, students' perceived hands-on learning style, gaps within their content knowledge, instances of off-task and otherwise distracting behavior, and shifts in their engagement all mediated the ways in which Ms. Allen interacted with IDs during the planning and enactment of curriculum.

Contextual Factors

Finally, various contextual factors also mediated Ms. Allen's interactions with IDs. Of note was the influence brought about by the district mandated curriculum, technology availability, and the climate and culture of her school. Each is discussed in turn below, as well as other contextual factors that had a less significant influence on Ms. Allen's ID use.

Mandated curriculum. As noted previously, Agile Mind was the mandated curricular resource for secondary mathematics teachers in DSS. Teachers, Ms. Allen explained, "have to use it and we have to have a certain amount of time showing that we are using it" (personal communication, March 8, 2016). District and school-based leadership assessed teacher usage of Agile Mind and, as she noted, "would be happy that we're using Agile Mind, which is what we're supposed to use" (personal communication, January 14, 2016). Ms. Allen viewed the district's mandate as a positive aspect of her practice, as she thought the Agile Mind curriculum was effective in facilitating student learning. When asked if she would use Agile Mind and the accompanying IDs even if she wasn't being told to do so she responded, "Oh yeah, I think IDs help, it definitely helps" (personal communication, October 26, 2015). Nonetheless, the fact that IDs were present in Agile Mind and, thus, widely available to Ms. Allen seemed to facilitate frequent teacher-ID interactions.

School administration focus. It seemed that Ms. Allen's school administration gave her significant curricular freedom even within the context of the DSS curricular mandate. When asked if her principal visited her room frequently, for example, she responded, "No. Definitely not. No" (personal communication, October 26, 2015). As a result, Ms. Allen was unclear about her school administration's thoughts surrounding IDs because "I've never asked them. I don't think they even know anything about interactive diagrams that go on in [my class]" (personal communication, October 26, 2015).

While the school administration didn't seem to deter nor encourage Ms. Allen's use of IDs specifically, she did note that there were aspects of ID activities that could be aligned with their instructional vision for the school. First, administration desired for students to be actively involved in the class. Thus, when Ms. Allen used IDs that effectively engaged students, "They'll be like, the kids are involved. It's interactive!" (personal communication, November 9, 2015). Also, administration seemed to want students using computer technologies in their learning. Ms. Allen noted, "It's good that [IDs are] on technology. […] We're a technology school and [administrators] want us to use technology. So, they would see it as, you know, thumbs up, they're utilizing the technology" (personal communication, November 9, 2015). Thus, the school administration seemed favorable to the combination of students using technology to access IDs and their ability to actively engage students in learning. It was unclear whether these factors actually encouraged Ms. Allen to use IDs more than she otherwise would have, or if it was a non-factor in her interactions with IDs. What was clear was that it did not deter her ID use.

Technology availability. Ms. Allen indicated that she had access to a sufficient amount technology to implement IDs in the ways she intended. She had a laptop, projector and document camera in her classroom. There was also reliable Internet, three computer labs, and two laptop carts in her school. While she noted that sometimes the functionality of the computer labs and carts did change due to how they were maintained, overall they seemed to be in a consistently functional state. As she explained, "The only thing I would say is having the access to the computers, the computer lab or like the laptops […] is pretty good unless we are testing or if the Internet isn't working, which is usually pretty good" (personal communication, March 8, 2016). Speaking of the Internet, Ms. Allen clarified that when DSS first implemented Agile Mind there would often be "something wrong with the Internet. […] But you know, year two and later things are working […] better." Thus, Ms. Allen's access to technology was only limited in the rare occasions were the Internet was malfunctioning or when standardized testing monopolized the school's laptops and computer labs. Throughout the study, though, she continually indicated that her school had sufficient technology for her to enact IDs as she wished.

Interestingly, though, there were times when Ms. Allen did not access the technology available in her school. While reflecting on her ID interactions throughout the year, she noted that she did not have challenges with,

> The availability of the technology based on the school. […] It's me not thinking on how I should plan the lesson in a certain way where I should go get the laptops and bring a couple down here and use them for the lesson. […] So, I could use it more but this [was] my fault. (personal communication, March 8, 2016)

Thus, Ms. Allen's planning seemed misaligned at times with the classroom structures she described within her ideal classroom, namely that pairs of students interact with IDs directly on laptops. She continued, "I just honestly never thought about doing small group, because we have a laptop cart. I could actually […] bring the cart down here and have kids do small group activities. […] I just never thought of doing it that way" (personal communication, November 9, 2015). Thus, Ms. Allen's use of technology was limited not because it was unavailable. Instead, lapses in teacher pedagogical knowledge and planning prevented more extensive use of the technology available to Ms. Allen.

The functionality of the school's technology did seem to influence Ms. Allen's enactment of ID activities despite her non-attention to this factor or any resulting impact it had on her ID use. Ms. Allen experienced technology challenges during her enactment of the *Paint Mixing ID* activity, for example, while laptops were turning on. It took a significant amount of time for the eight laptops students were assigned to reach the point where they could begin interacting with the ID. She explained this delay, "Every time you go to another computer and you sign in [with] your login it has to initialize, put your stuff on the computer. So, it takes more time. This is why [students were] waiting for so long" (personal communication, January 15, 2016). She continued by reflecting on how this delay could have been eliminated. "Another class had them" (personal communication, January 15, 2016). Another class having the laptop cart before her class prevented Ms. Allen from "getting the cart earlier, and me logging on to the computers already." This would have prevented the delay in the ID activity. Thus, Ms. Allen seemed to have the technology needed to engage students with IDs, but the logistics of when she received that technology did mediate the enactment of her planned ID activity.

The delay in students being able to interact with the ID affected the enacted activity in two ways. First, this delay reduced the instructional time available for students to interact with the ID in the ways that Ms. Allen intended. Additionally, it increased occurrences of student off task behaviors. Evidence of students holding non-academic conversations, dancing, and otherwise not meeting Ms. Allen's expectations was collected while students waited for their laptops to initialize. One teacher-student interaction in particular highlighted the relationship between the functionality of technology and students' behaviors. Here, Ms. Allen addressed a pair's off-task behaviors by telling them, "You are separated. Get another computer." One student, frustrated by her directive, responded, "It's the computer. It is slow" (observation, January 15, 2016). Thus, the functionality of the technology available to Ms. Allen seemed to mediate her enactment of ID activities.

Standardized assessment logistics. Additionally, the administration of standardized assessments within Ms. Allen's school had a multifaceted effect on her enactment of ID activities. To begin, the vast majority of the technology in her school was designated for test administration during the various assessment windows throughout the year. The *Friendship Problem ID* activity occurred during one of these windows and, consequently, was affected.

> Right now, for the last two weeks we've been testing, so instead of us using the cart…I would normally use [computer lab room] 344, but since we've been testing, that limited everybody else that wasn't testing to just the one cart. But normally we have like three available labs and […] two [laptop] carts. (personal communication, November 10, 2015)

Only one of the five class sets, or 20%, of the school's computers were available for instructional purposes during test administration windows. The limited technology available for the *Friendship Problem ID* activity influenced Ms. Allen's decision to plan a whole group activity. Limited technology, she indicated, mediated possible classroom structures (i.e. pairs, small groups, whole group) during ID activities at other times as well.

Testing windows also created unpredictable student attendance within Ms. Allen's classes. Groups and individual students were taken out of her classes, sometimes without prior warning, so they could complete standardized assessments and make-up testing. A number of students were not present during the *Mixing Paint ID* activity, for example, for this reason. Consequently, the class size was reduced from the 23 enrolled to 11 present. Once students were placed in pairs, each pair was given a laptop computer to use during the ID activity. Each pair was able to get their own computer since there were fewer pairs than expected. Thus, students being pulled from Ms. Allen's classroom to test affected the enacted curriculum by allowing all students to interact more directly with the ID within their pair.

School logistics. Other school logistics also influenced Ms. Allen's ID use. In particular, as an instructional leader for her school she attended regular district meetings. Ms. Allen attended these off-site school leadership meetings approximately once a month. The impact of being outside of the building during this time was observed at the beginning of the *Paint Mixing ID* activity when she used the PowerPoint slide shown in Figure 23. She explained one aspect of why she began the activity in this way,

> The reason I did that was because I was out on a meeting yesterday and I was trying to combine two lessons. […] I probably only saw [students] two times or three times this week. [Content] is not sticking. Because I'm testing one day, I have to go to the meeting the next, test them the next day. And so I had them

> trying to squeeze two lessons of information together. (personal communication, January 15, 2016)

The combination of her students testing and her being out of the building created a situation where Ms. Allen felt that she had to combine two lessons, thus mediated the creation of the planned ID activity.

School climate and culture. Ms. Allen identified the school's culture and climate as a non-factor in her interactions with IDs. She assertively denied that school behavior management affected ID enactment in her classroom. Her comments were misaligned with the evidence collected during the lesson observations, however. During the *Friendship Problem ID* activity, for example, there were a number of instances where behaviors originating from outside of Ms. Allen's classroom disrupted the enactment of the ID activity. First, near the beginning of class, there was a notable level of noise coming from the hallway. Ms. Allen stepped into the hallway to address the students making this noise on two occasions. At one point it sounded like there may be a fight beginning. Ms. Allen's students also seemed to think this, as all but four students ran from their seats to the hallway to see what was happening. Consequently, Ms. Allen stopped the ID activity and went to retrieve her students from the hallway (observation, November 10, 2015). This sequence of events posed a significant distraction for the ID activity.

Secondly, students not in Ms. Allen's class entered the classroom, created a disturbance, and, thus, mediated the enactment of the ID activity. Again during the *Friendship Problem ID* activity, a student entered Ms. Allen's classroom, skipped throughout the entire classroom, and then exited into the hallway. A few moments later the same student stood in the doorway of Ms. Allen's classroom, asked non-academic questions to Ms. Allen's students, sang loudly, used profanity, and otherwise directed attention away from the ID activity. Ms. Allen called the main office to request support in addressing this student, but this support was not provided. She described the situation, "I pressed the intercom button. Nobody answered right away and then [the student] started playing with the button. So, something like that takes time away from the class" (personal communication, November 10, 2015). Indeed, occurrences like this mediated the enacted curriculum since they decreased students' focus on the ID activity and the instructional time planned to implement the ID in the way that Ms. Allen had intended.

Ms. Allen didn't seem to fully understand the impact such events had on her implementation of ID activities. She explained,

> Sometimes there might be [distractions from outside the classroom], but the kids aren't necessarily focusing on it. The kids in here aren't necessarily focusing on it. […] I might have to go into the hallway and say something to the students. […] It's not a big distraction. You know, if I hear noise and I go out there, it's not like the biggest thing. (personal communication, January 15, 2016)

This statement was contradictory to the evidence collected during the study observations. It did seem that students were focusing on the noise outside of the classroom, as they noticed when it was escalating and exited the classroom to observe. Also, Ms. Allen going into the hallway to "say something to the students" occupied instructional time when she intended to be facilitating the ID activity. Thus, Ms. Allen's commentary about the impact of the school's climate and culture on her enactment of IDs seemed incongruent with the actual influence that was observed. The school's climate and culture did have a mediating affect on the enactment of ID activities.

Taken together, a number of contextual factors seemed to influence Ms. Allen's planning and implementation of ID activities, as depicted in Figure 28.

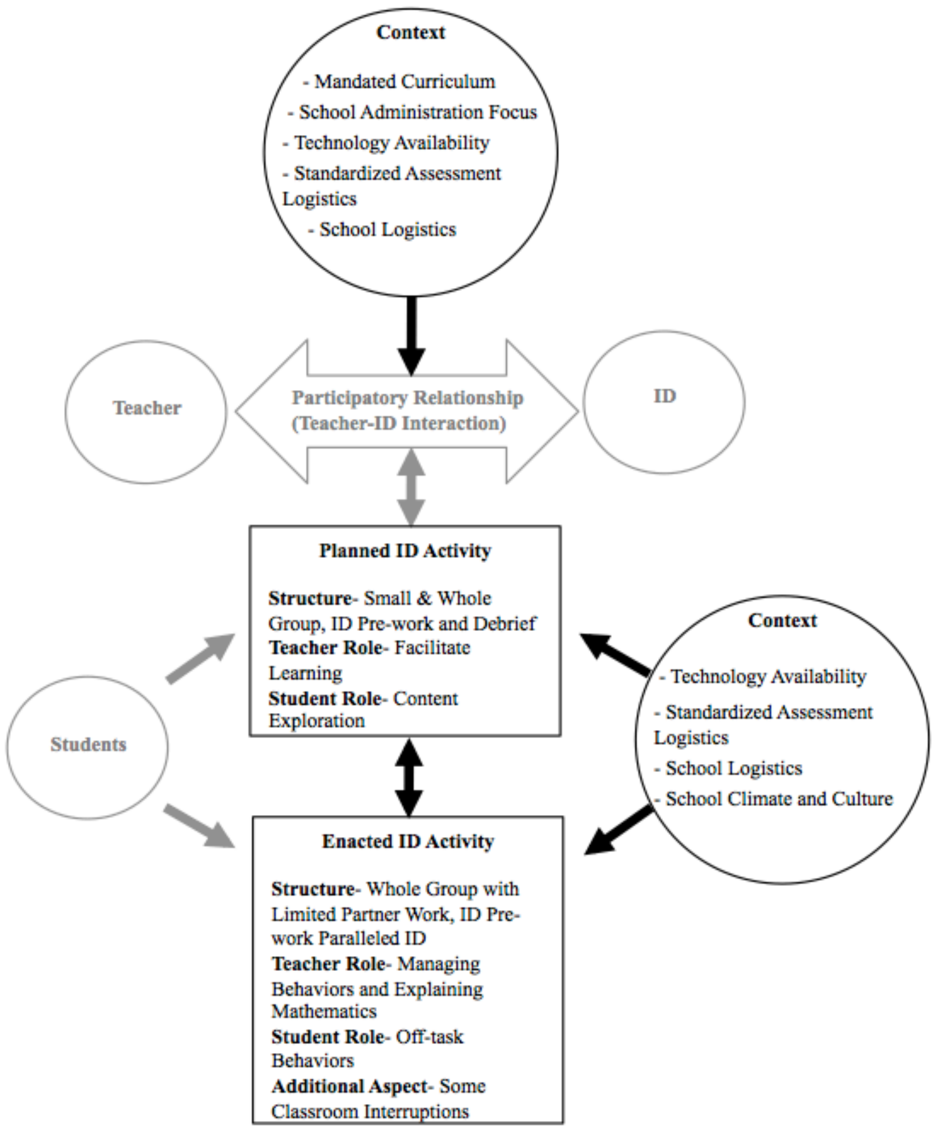


Figure 28. Contextual factors affecting Ms. Allen's use of IDs.

Her planned curriculum was mediated by the DSS mandating the use of Agile Mind and its embedded IDs, as well as the varied availability of technology throughout the school year. The functionality of technology, school logistics, and aspects of her school's climate and culture then mediated the enactment of her intended curriculum.

Summary

ID, teacher, student, and contextual factors mediated Ms. Allen's participatory relationship with IDs within a complex and situated system of interactions. Figure 29 captures this system utilizing this study's conceptual framework.

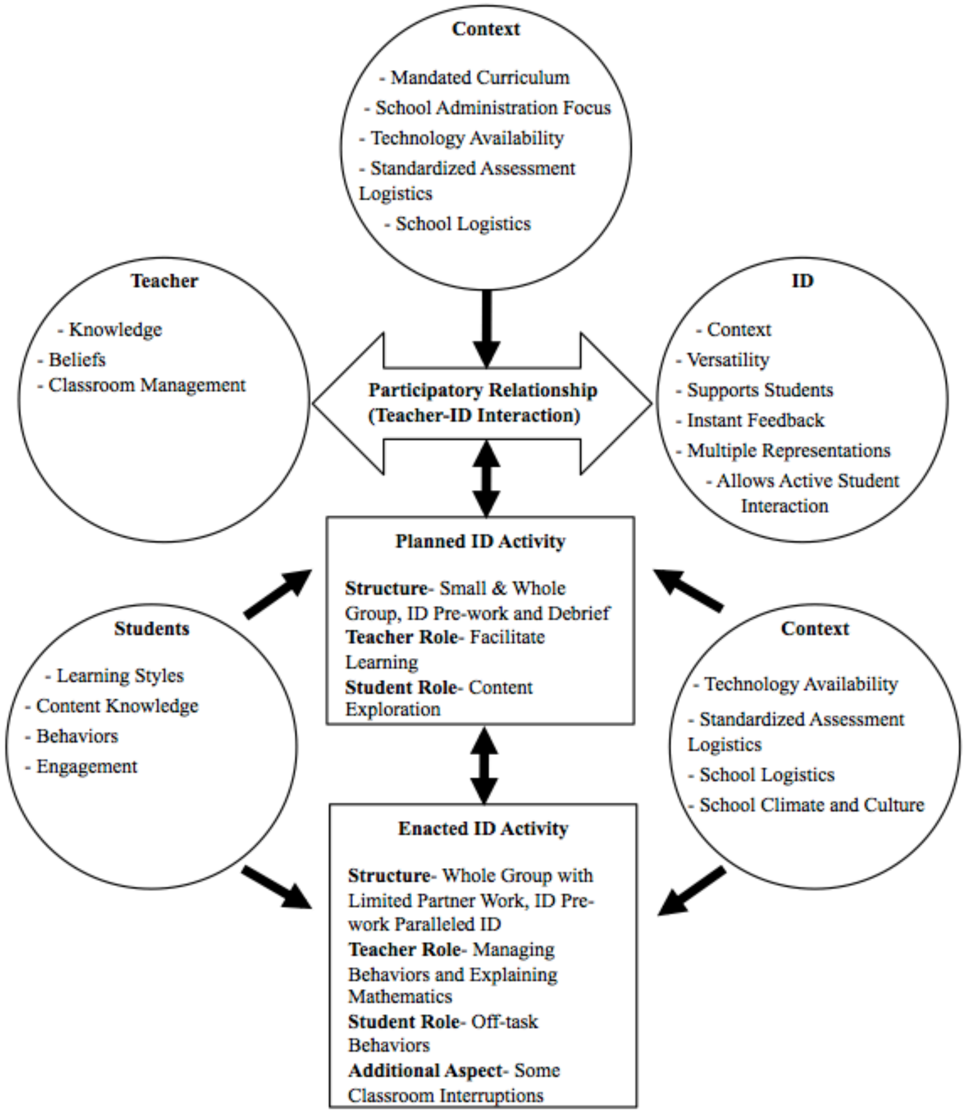


Figure 29. Ms. Allen's ID use and the factors affecting it

The planned curriculum was affected by a number of factors surrounding Ms. Allen's instructional practice. First, ID factors mediated if and how an ID was incorporated into the intended curriculum. Most important was whether an ID included multiple mathematical representations and allowed students to actively interact or "do something" with the ID, both of which she desired. This latter factor, that an ID encourage active interaction, was aligned with Ms. Allen's belief that mathematics learning should be an active process and her perception that students preferred hands-on learning. This alignment was further reinforced the use of IDs including interactive design features. Ms. Allen's experience and content knowledge ensured that she would understand the content embedded in IDs, but gaps in her pedagogical knowledge seemed to exclude planning particular instructional strategies at times. She also heavily attended to students' content knowledge, engagement levels, and behaviors when planning the structure of ID activities. Lastly, the curricular freedom Ms. Allen enjoyed from her school administrators allowed her to make instruction decisions based on the aforementioned factors while still meeting the district mandate of using the Agile Mind curriculum.

Ms. Allen navigated the interactions and tensions between and among these factors as she created the planned curriculum. She created planned ID activities that engaged students in learning mathematical concepts and procedures through active interactions with IDs and their peers. These interactions occurred within both small and whole group structures, but were always preceded by ID pre-work and followed by a whole group debrief conversation. She intended to work as a facilitator of what she viewed as student-centered learning throughout the entire activity.

Numerous factors then mediated the enactment of Ms. Allen's planned ID activities. Most notable were those factors relating to students' behaviors and Ms. Allen's ability to manage the classroom, including the behaviors of students within and outside of her classroom. Misbehaviors of varied severity and frequency distracted from ID activities and decreased the instructional time available to enact the planned activity. Additionally, standardized assessment logistics affected Ms. Allen's enactment of ID activities by decreasing the amount of technology available to access IDs and unpredictably altering her students' attendance. Lastly, Ms. Allen was pulled from her classroom to perform other duties, such as attend school and district meetings. This mediated the ways in which she could enact that day's ID activity as well as the planning for the ID activity on the following day.

The resulting effects of the aforementioned factors was that Ms. Allen's enactment of ID activities often became less focused on students actively making sense of content and more so on managing students' behaviors and significantly scaffolding instruction. Indeed, Ms. Allen's predominant role during the enacted curriculum shifted to explaining the mathematics content she desired students to learn from the ID. She also spent a significant amount of time proactively and reactively keeping students on task. While there were instances where students took more active control of their learning, as she intended while planning ID activities, these instances seemed to be less frequent than Ms. Allen's overall more teacher-centric approach.

Chapter 7

This study of three secondary mathematics teachers sought to answer the following research questions:

1) How do three secondary mathematics teachers use interactive diagrams (IDs) within their instructional practice?
    a. How do teachers create the planned curriculum involving IDs?
    b. How do teachers enact curriculum involving IDs?
2) What factors mediate three teachers' instructional uses of interactive diagrams (IDs)?
    a. How are teachers' uses of IDs affected by the underlying design of the IDs?
    b. How do teacher factors, such as their instructional beliefs, affect their use of IDs?
    c. How do factors surrounding the school context, including the students within that context, influence teachers' instructional uses of IDs?

Chapters 4, 5, and 6 addressed these questions for each of the study participants individually. This chapter addresses these questions through a cross-case analysis.

Each of the study cases was unique. There were particular trends that can be highlighted, however. In this chapter, I will compare and contrast participants' instructional uses of IDs. Then, I will discuss the ID, teacher, student, and contextual factors that influenced their ID use using the studies' conceptual framework. I conclude the chapter by examining each participant's conception of their participatory relationship with IDs with that which was observed during this study. This latter discussion seemingly holds significant implications for future work within the mathematics education community, as is described in Chapter 8.

Instructional Uses of IDs

Each participant illustrated particular ways in which the planned and enacted curricula were created. Figure 30 illustrates the overall features of the planned and enacted ID activities identified for the study participants.

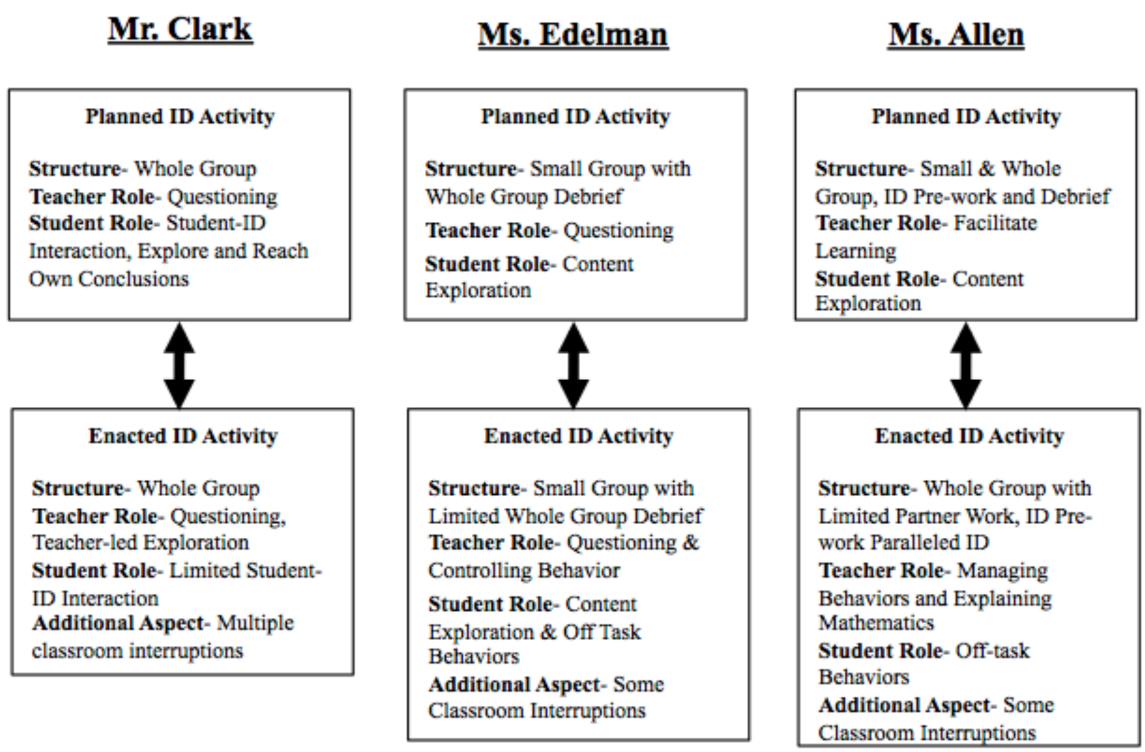


Figure 30. Study participants' planned and enacted ID activities.

Similar to other studies of teacher curriculum use (Henningsen & Stein, 1997; Remillard, 2005; Stein et al., 2008; Stein, Grover, & Henningsen, 1996), all three participants' experienced shifts in aspects of their ID activities as they moved from the planned to the enacted curriculum. This included shifts in the goals that undergirded their

ID activities, the classroom structures used, and the teacher and student roles present during those activities.

All three participants expressed the desire to facilitate student-centered learning through their instructional use of ID. Indeed, each consistently discussed planning ID activities meant to engage students in context exploration. The enactment of this goal was inconsistent across the study cases, however. Ms. Edelman's planned and enacted goal demonstrated the most alignment as she continuously worked to enact the authentic inquiry learning she planned for her classroom. The enactment of content exploration was interrupted by a number of factors, but the overall focus of her enacted ID activities remained focused on students' learning content through exploratory ID use. Mr. Clark and Ms. Allen's enactment of ID activities included shifts away from content exploration. Indeed, Mr. Clark largely directed the content learning during ID activities. While his students did engage with the content found in IDs to some degree, their content exploration was very limited by how Mr. Clark directed the group's attention. Ms. Allen' use of pre-ID tasks eliminated students' opportunities to engage in inquiry learning. Thus, Ms. Allen's enactment of ID activities included the most dramatic shift away from her intended goal.

The classroom structure used during planned and enacted ID activities also varied across the study participants. Mr. Clark's ID use demonstrated the most consistency, as his planned and enacted ID activities incorporated solely a whole group structure. The ID activities found within Ms. Allen and Ms. Edelman's classrooms were not completely enacted with the structures they planned, however. Ms. Edelman planned for students to begin engaging with an ID within small groups or with a partner. Students were often arranged in larger groups than she planned, however. Then, the whole group debrief she intended to follow the small group work was either abbreviated or eliminated due to time constraints. Ms. Allen, on the other hand, consistently planned and enacted a whole group structure before and after the focal ID. She intended to engage students with the ID in both small groups and as a whole group, but overwhelmingly used a whole group structure during the enacted activity. Thus, Mr. Clark and Ms. Allen enacted ID activities largely within a whole group setting while Ms. Edelman incorporated small to medium-sized group activities, the latter two of which was a shift from their planned ID activity.

The teacher roles within participants' ID activities generally followed a similar pattern when moving from the planned to the enacted curriculum. Each participant intended for his or her role to include facilitating student-centered, inquiry learning. This was consistent with their intended goals and often included the purposeful use of teacher questions, particularly for Mr. Clark and Ms. Edelman. Participants' roles shifted in varying degrees, however, toward managing the classroom and controlling the enacted classroom activity. Ms. Edelman spent the most significant amount of time managing the classroom, but Ms. Allen and Mr. Clark's role certainly included this as well. Interestingly, Ms. Edelman also spent the most significant amount of time facilitating learning in ways that were aligned with her intended teacher role. Indeed, she consistently enacted teacher actions focused on facilitating content exploration, such as circulating the room during small group activities asking students probing questions about what they were learning. She also avoided directly teaching students content and, instead, directed them to how content could be learned from the ID or their peers. Mr. Clark also used teacher-questioning techniques as he planned, but his enacted questions were more directive than he seemed to intend and understand. Consequently, Mr. Clark's role during enacted ID activities shifted to that of leading the class's ID interactions through leading questions and explaining content. Further, Ms. Allen's enacted role included an even higher degree of content explanation with more infrequent questions than either of the other two study participants.

The shifts in the aforementioned enacted goals, structures, and teacher roles had parallel effects on the enacted student role during ID activities. Ms. Edelman's students, for example, explored mathematics content as she planned by interacting with IDs on group computers. Students discussed content with their peers during their explorations, but to an admittedly lesser extent than she desired. Ms. Allen and Mr. Clark's students, however, had limited or no opportunities to interact with IDs directly. Thus, students' roles shifted to answering teacher questions in a whole group and recording notes in their workbooks. Taken together, the enactment of ID activities seemed to reduce the meaningful mathematics learning (Bate, 2010; Jonassen et. al, 1999) each participant intended in varying degrees as activity goals, classroom structures, teacher roles, and student roles shifted.

Factors Affecting ID Use

Numerous factors mediated the study participants' planned and enacted ID use. Figure 31 depicts the totality of the contextual, teacher, ID, and student factors that were identified by this study. The first letter of each participant's name is included after factors that were found to mediate their participatory relationship with IDs (i.e. "C" for Clark, "E" for Edelman, and "A" for Allen). Factors that affected all study participants are included first within each factor circle followed by other factors in no particular order.

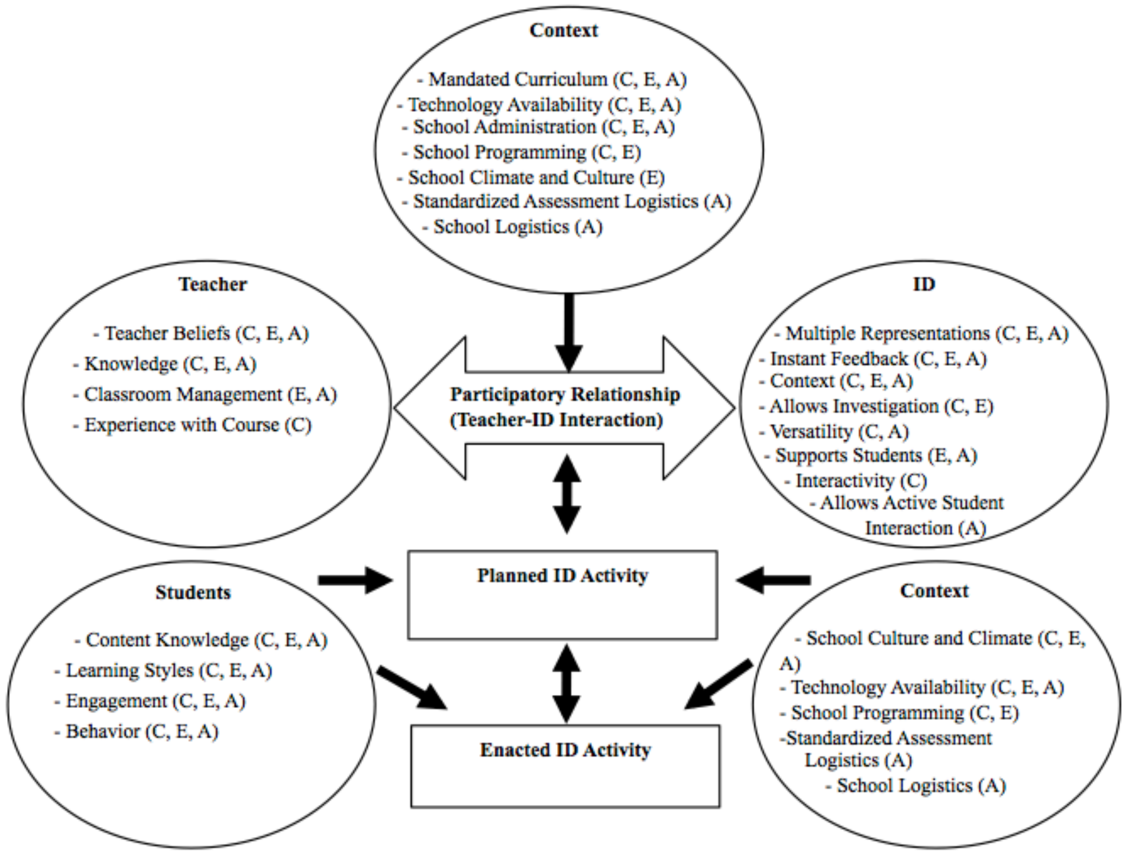


Figure 31. Factors affecting study participants' ID use.

Below the factors identified by this study as affecting teacher-ID interactions across the study participants are discussed. Specific attention is given to whether particular factors affected participants' ID use during planned ID activities, enacted ID activities, or both.

ID factors. Multiple ID factors surfaced as having a mediating affect on the study participants' use of IDs. Two factors, interactivity and allowing student interaction, were identified as affecting the ID use of one participant each. This may be misleading, however, as these design features were tightly connected to other features that were more widely attended to by the study participants. For example, an ID's level of interactivity was related to its ability to allow investigation, support students, and provide instant feedback, which were all identified as having a more widespread affect. That said, Mr. Clark and Ms. Allen specifically called out interactivity as a factor affecting their ID use, while Ms. Edelman discussed this aspect as being more integrated into other ID factors.

An ID's potential role in distributed cognition (Hutchins, 1999; Jenkins, 2007) was a mediating factor for both Ms. Edelman and Ms. Allen. This seemed largely due to their students' content knowledge creating barriers to engaging with grade-level mathematics learning. Mr. Clark alluded to this as well, but focused his comments surrounding how IDs supported student learning more so on how IDs engaged students in learning and provided multiple representations than specifically supporting gaps in their content knowledge. Additionally, Mr. Clark and Ms. Allen spoke specifically of the versatility of IDs. Both desired that IDs have the capability to be used across multiple examples and with a broad range of input values. This speaks to the presentational function of an ID's design in that they seemed to prefer random or generic IDs over those that were designed to be specific (Yerushalmy, 2005). Ms. Edelman did not speak to this aspect of an ID's design affected her use of IDs.

The existence of multiple representations embedded within an ID surfaced as a mediating factor across all three study cases. Indeed, Mr. Clark, Ms. Allen, and Ms. Edelman all desired that the ID activities they created include dynamically linked mathematical representations. Each noted that it was beneficial for students to "actually see what happens when numbers change" (Mr. Clark, personal communication, December 1, 2015) when manipulating the equation of the function represented graphically. This was "because it's another way of helping kids make connections" (Ms. Allen, personal communication, November 9, 2015). Researchers (Havelková, 2013;

Pierce, Stacey, Wander, & Ball, 2011; Swartz & Yerushalmy, 1992) agree that linking multiple mathematical representations in this way can have a positive affect on students' functional understandings. Ms. Edelman put it simply that dynamically linked mathematical representations were "kind of the point of an ID" (personal communication, March 10, 2016).

An ID's ability to provide instant feedback was also identified as an influential factor across all study cases. Participants related this design feature to the presence of multiple mathematical representations in that the instant feedback they often valued pertained specifically to those representations. As Ms. Allen explained, "When things change [in the equation, students] can see instantly how it happens [on the graph] rather than working for 5 or 10 minutes on a graph and then seeing it" (personal communication, February 18, 2016). Participants appreciated that students could "try different kinds of solution methods which would be too inconvenient with paper and pencil" (Hahkioneimi & Leppaaho, 2012, p. 26) due largely to gaps in students' content knowledge. IDs' ability to automatically produce linked representations that accounted for alterations in another representation utilized distributive cognition (Hutchins, 1999; Jenkins, 2007) as a way to "…expand and augment human's cognitive capacities" (Jenkins, p. 106). For example, IDs reduced students' cognitive load by automatically creating graphs while they continually altered the coefficients of a function's algebraic representation. Thus, IDs allowed students to focus on understanding the links between the multiple representations and not solely on creating the representations themselves.

The multiple representations and instant feedback ID factors encouraged students to play (Jenkins, 2007) as they engaged in problem solving, risk taking, and trail and error. These aspects of ID activities embodied the active mathematics learning participants uniformly expressed desire for. IDs' ability to include multiple representations and instant feedback, and thus facilitate investigation through user-technology interaction, highlighted the organizational ID design function (Yerushalmy, 2005). Indeed, participants preferred narrating and elaborating IDs over illustrating IDs because they more "open" and "support[ed] autonomous guided inquiry […] of open-ended exploration with specific content" (Yerushalmy, p. 231). The content exploration described here was consistent with the type of active student engagement participants sought to create within their classrooms.

Lastly, all study participants explicitly attended to the context embedded within an ID as a factor influencing their use of this technology. Mr. Clark, noted, "Application to real life […] is an overarching theme in my class. It's something that I try and push" (personal communication, February 25, 2016). He, Ms. Edelman, and Ms. Allen all noted how an ID's real-world context can "bring math out of the abstract" (Ms. Edelman, personal communication, March 10, 2016) by demonstrating where mathematical concepts are applied more concretely in the real world. Mr. Clark and Ms. Edelman noted that an ID's context had to be meaningful for students, however, so that they would engage with the ID and the mathematical content within it. Ms. Allen did not express concern about this aspect of an ID's context. She alone did note that an ID could provide access to a context that was otherwise undesirable or inaccessible, however. Thus, all of the study participants identified an ID's context as a mediating factor, but this factor's affect was slightly different for Mr. Clark and Ms. Edelman than for Ms. Allen. Interestingly, context was not an explicit component of the design framework described by Yerushalmy (2005). This was surprising given the significance the study participants attributed to this ID factor.

Teacher factors. This study identified four teacher factors that had a mediating effect on the participatory relationship between the study participants and IDs. To begin, Ms. Edelman and Ms. Allen both described experiencing challenges effectively addressing and preventing off task student behaviors. These behaviors, then, mediated the enactment of their planned ID activities. Mr. Clark also experienced classroom management challenges, as evidenced by the study's classroom observations. He attributed classroom management challenges to students' behaviors caused by contextual factors, however, and not as a teacher factor. Additionally, he did not fully acknowledge the degree to which these challenges affected his enacted ID activities. Thus, classroom management was found to be a mediating factor to some degree for all study participants, but was explicitly attended to only in Ms. Edelman and Ms. Allen's use of IDs.

Mr. Clark identified his experience with the IA course as a factor that mediated his ID use. Mr. Clark was in his fourth consecutive year teaching IA during the data collection phase of this study. Ms. Allen and Ms. Edelman, on the other hand, had taught IA for one and two years, respectively. Thus, Mr. Clark had more opportunity to "think about how these specific [ID activities] went [… and] think of better ways of asking questions" (Mr. Clark, personal communication, February 25, 2016). It seemed logical that Mr. Clark would cite his experience as a factor that facilitated his use of IDs. What was perhaps less clear, however, was that Ms. Edelman and Ms. Allen did not note their lack of experience with the particular IDs found within the IA course as an influential factor in their practice.

Evidence that teacher beliefs were a mediating factor in the use of IDs was collected for all student participants. A summary of the pedagogical beliefs survey results is shown in table 10. There was certainly variation in the scores depicted in Table 10. Generally, though, all participants indicated that they held beliefs that tended toward meaningful mathematics learning (Bate, 2010; Jonassen et. al, 1999).

Table 10
Pedagogical Beliefs Survey Results- All Participants

	Active	Cooperative	Constructive	Authentic	Intentional	Average
Mr. Clark	3.70	4.59	4.07	4.19	4.63	4.24
Ms. Edelman	3.40	3.83	3.50	3.69	3.75	3.63
Ms. Allen	3.10	4.17	4.09	3.94	4.63	3.99

Indeed, it seemed that all three participants planned and attempted to enact ID activities that were aligned with their expressed beliefs. Thus, participants' beliefs seemed to have a significant mediating affect on their participatory relationship with IDs. This finding is consistent with researchers' (Aguirre & Speer, 1999; Bate, 2010; Calderhead, 1996; Cohen, 1990; Ernest, 1989; Thompson, 1992) assertion that teachers' beliefs influence teacher instructional practice. That said, it is noteworthy that Ms. Edelman scored the lowest on the PBS but enacted instruction most aligned with meaningful mathematics learning. This occurrence seemed to highlight the complexity found within teachers' participatory relationship with IDs in that no one factor fully explained the enacted curriculum for each participant.

Teacher knowledge also emerged as having a mediating affect on participants' ID use. The type of teacher knowledge (Shulman, 1987) pertinent to each case was important to delineate. Mr. Clark, Ms. Allen and Ms. Edelman all noted that their content knowledge facilitated the planning of ID activities since each understood the content embedded within the IDs throughout their course and, thus, allowed the use of those IDs. Mr. Clark alone noted curricular knowledge as having an effect on his planned curriculum, however. Namely, Mr. Clark's experience with the course IDs allowed him to reflect on past uses of the IA course's IDs and refine his future ID activities. All study participants also indicated that knowledge of their learners, particularly surrounding students' content knowledge, mediated their intended use of IDs. Lastly, the enactments of ID activities were effected by the general pedagogical knowledge and pedagogical content knowledge of all three participants. The latter mediated the ways in which participants were able to facilitate mathematics specific inquiry learning, while the former affected how teachers were able to manage their classrooms. Taken together, the various components of Mr. Clark, Ms. Allen, and Ms. Edelman's teacher knowledge (Shulman) affected both the ways in which ID activities were planned and the enactment of those activities within their respective classrooms.

Student factors. A total of four student factors were identified as having an effect on participants' ID use as a result of this study. Ms. Allen, Mr. Clark, and Ms. Edelman each attended to students' content knowledge, learning styles, engagement, and behaviors within their planned and enacted curriculum. These factors mediated ID use in ways that seemed similar across the study cases. Nuance was found, however, upon close examination of each instructor's intended and implemented ID activities.

To begin, ID use was mediated by all three participants' perception of students' content knowledge. This finding was consistent with other studies investigating teachers' curriculum enactment (Collopy, 2003). Indeed, all three participants indicated that their students had multi-year deficiencies in their content knowledge. Thus, an ID supporting distributed cognition surrounding students' deficient skills, specifically those that were pre-requisite to grade level content, was a consideration when creating planned curriculum. IDs that automatically produced graphs given an equation, for example, were appreciated and used frequently across the study cases. By using IDs with this design feature, participants were able to engage students in grade-level concepts even when graphing by hand was a challenge for students. Ms. Edelman summarized this by noting, "They're engaged with the [grade-level] content […] even if they don't necessarily have the background [pre-requisite skills]" (personal communication, March 10, 2016). Thus, students' content knowledge seemed to mediate the particular IDs each participant incorporated into their classroom activities.

Students' content knowledge also mediated the ways in which ID activities were designed and enacted. The affect of this factor was different across the study participants, however. Ms. Edelman, for example, focused on IDs' ability to support students surrounding prerequisite skills and sought to engage students in grade-level content while using this support. Thus, she relied on the design features of IDs to automatically complete prerequisite tasks that students struggled with, such as graphing, algebraic manipulations, and arithmetic calculations, while she engaged them with grade-level content learning. Ms. Allen, on the other hand, expressed that she did not see how students could engage with particular IDs due to their lack of content understandings. Consequently, she included tasks prior to students' interaction with IDs that largely mirrored the ID activity. This pre-ID instruction focused on students learning the content embedded within the ID since she did not see how students could learn this content through their ID engagement alone. When discussing students' ability to engage with the mathematics content contained in the *Friendship Problem ID*, for example, Ms. Allen expressed, "I know my students and I really don't think they can" (personal communication, November 9, 2015). Taken together, Ms. Edelman and Ms. Allen's ID use was affected by student's content knowledge in quite different manners. Ms. Edelman used the design features of particular IDs to support students' grade-level content learning while engaging with IDs. Ms. Allen, on the other hand, engaged her students in substantial pre-ID instruction she indicated was necessary for students to successfully interact with an ID.

Participants also expressed that students' learning styles impacted their use of IDs. Indeed, all three instructors noted that their students preferred and were more successful when engaged in tactile and visual learning. IDs were often aligned with these learning styles, participants asserted, due to the presence of multiple representations (Yerushalmy, 2005) and the ability to play with virtual objects (Jenkins, 2007). There is debate in the literature surrounding the validity of learning style frameworks (for example, Cuevas, 2015; Willingham, Hughes, & Dobolyi, 2015). Regardless, all three participants perceived that their students' learning styles were important to consider in their instructional decision-making and intentionally designed their curricular activities surrounding this idea. Consequently, students' perceived learning styles mediated the creation of ID activities across the three study cases.

Student engagement also emerged as a universal factor affecting teacher-ID interactions. Indeed, Ms. Allen, Mr. Clark, and Ms. Edelman all noted that IDs increased students' engagement in their respective classes and that they used IDs for this purpose.

Participants' notions of what engagement entailed varied, however. Ms. Allen and Mr. Clark noted how IDs engaged students in such a way that prevented behavior management challenges. As Mr. Clark emphasized, "Keeping students on track and managing behaviors […] just comes with using the diagrams" (personal communication, February 25, 2016). Ms. Edelman also seemed to appreciate the behavior management benefits using IDs provided her, but she desired more from her ID activities. She expressed that with IDs, "The students are just going to be more engaged, *going to be more interested in the content because they get to do something*" (personal communication, March 10, 2016, emphasis added). It was this engagement with mathematics content that Ms. Edelman ultimately worked to foster through her ID use. Ms. Edelman expressed and enacted an earnest desire for students to engage with mathematics during ID activities. Ms. Allen, on the other hand, stated that she wanted students investigating mathematics, but it seemed more so that she wanted students engaged with IDs to prevent off task behaviors. Thus, while student engagement was a mediating factor across the study participants, its influence manifested in different manners for each participant.

Lastly, student behavior affected both the planning and enactment of ID activities across the study participants. Ms. Allen and Ms. Edelman both explicitly attended to this factor as they created the planned curriculum. Ms. Edelman noted that her participation in instruction "would have been a lot more hands-off if there were less behaviors in my classroom" (personal communication, December 16, 2015). This, she continued, would result in a more student-centered, inquiry-based learning environment. Ms. Allen elaborated on a similar idea- "In the ideal world I wouldn't have to worry about behaviors and kids being on task. […] The teacher role would definitely change [to be more academically focused]" (personal communication, March 8, 2016). Thus, both Ms. Edelman and Ms. Allen desired to create ID activities where their role was academically focused. Students' behaviors mediated the planning of their teacher role, however, as they dedicated more significant attention to classroom management than was desirable.

Student behaviors also mediated the enactment of ID activities. Multiple and varied off task student behaviors were observed in Ms. Edelman, Ms. Allen, and Mr. Clark's classrooms. These behaviors mediated the enactment of ID activities by distracting participants' and students' attention away from the ID activities, which ultimately occupied instructional time otherwise intended for those activities. Student behaviors had such a significant impact on ID use, in fact, that Ms. Edelman and Ms. Allen went as far to identify it as the most significant factor affecting their enactment of ID activities. Taken together, student behavior was a mediating factor for the creation of both planned and enacted ID activities.

Contextual factors. Finally, numerous contextual factors surfaced as impacting participants' participatory relationship with IDs. Specific aspects of each participant's school contexts, for example, were identified as having a mediating effect on the individual study cases. School programming, such as assemblies and extracurricular activities, altered and prevented the ways in which Ms. Edelman and Mr. Clark planned and enacted ID activities in both foreseeable and unexpected ways. While Ms. Allen did not identify this factor specifically, she did note that other school logistics had a similar affect of her ID use. In particular, standardized assessment logistics had a parallel effect on Ms. Allen's ID use as PGC for Mr. Clark. Thus, while these particular contextual factors were specific to the individual cases, their impact on each teacher's participatory relationship with IDs was similar.

School climate and culture was found to have a consistent effect on all participants' enactment of ID activities. Namely, climate and culture challenges originating from outside of Mr. Clark, Ms. Edelman, and Ms. Allen's classrooms interrupted their classroom instruction. These interruptions occupied their and their students' attention, decreasing the instructional time available to enact ID activities as they were intended. While the culture and climate challenges seemed to originate from slightly different aspects of each participant's school, such as the school staffing for Mr. Clark and Ms. Edelman, what was clear was the foundational impact (Bate, 2010) school climate and culture had on teacher-ID interactions.

Additionally, school administration emerged as a mediating factor across the study cases. School administration did not explicitly encourage nor discourage the study participants' use of IDs, however. Instead, Ms. Edelman, Ms. Allen and Mr. Clark each noted that their respective administrations viewed their classroom instruction through the lens of the district-wide evaluation rubric. This rubric highlighted the degree to which students were engaged in Mr. Clark and Ms. Allen's classrooms, the presence of student collaboration and group work within Ms. Edelman's classroom, and Ms. Allen's use of technology. Participants' asserted that their use of IDs promoted and facilitated these instructional practices. Consequently, their ID use was aligned with the pedagogical and technological leadership (Bate, 2010) present within their schools' and district's strategic visions. While school administration did not directly focus on teachers' ID use, their overall academic visions seemed to mediate participants' instruction by encouraging practices participants then included within their planned ID activities.

The participatory relationship between the study participants and IDs also seemed to be mediated by the district mandating particular curriculum. Recall that DPS mandated the use of Agile Mind, which was rich with IDs, as the primary curricular resource for secondary mathematics teachers. Consequently, school administrators attended to participants' use of Agile Mind and, inadvertently, IDs. As Mr. Clark noted, "I also think the [school] administration's push to use a mandated curriculum is also something that definitely impacts [my ID use since they were] really on top of making sure that our usage data was up" (personal communication, February 25, 2016). Thus, by following the curricular leadership (Bate, 2010) of the district, participants' respective administrations encouraged ID within each participant's classrooms.

Lastly, the technology available to participants emerged as a significant mediating factor for the enactment of ID activities across the study cases. This finding was consistent with Bate's (2010) conception of technological equipment and infrastructure being a bridge within the teacher-technology interaction framework. Observational data confirmed that technological challenges mediated the ways in which planned ID activities were enacted within the classrooms of all study participants. Technology availability affected each participant's planning of ID activities differently, however. Ms. Allen, for example, expressed that her planning of ID activities was minimally affected by this factor since she had sufficient access to the computers and Internet. Ms. Edelman and Mr. Clark, on the other hand, both described the substantial effect that the lack of technology had on their planning of ID activities. Both participants even went as far to assert that this was the most impactful factor on their participatory relationship with IDs.

The impact of the available technology factor was revealed, however, when examined across the study cases. Ms. Edelman enacted IDs in ways that were largely consistent with her planning even when technology was sparse or dysfunctional. Mr. Clark, on the other hand, did not even attempt to enact IDs as he desired due to his lack of available technology. Finally, Ms. Allen generally had access to sufficient technology yet she largely underutilized it. Thus, while the availability of technology seemed important, there appeared to be other factors that mediated the effect this particular factor had on the participants' ID use. Indeed, the availability of technology alone did not dictate how participants utilized technology in their instructional practice (Cuban, 2003).

Reframing Conceptions of Teachers' Participatory Relationship with IDs

This study identified various aspects of ID use as well as those factors that seemed to mediate ID use across the study's three cases. Up to this point, the narrative surrounding teacher-ID interactions described the participatory relationships between teachers and IDs from an exogenous viewpoint (Stevens, 2010) using the participants' words as frequently as possible. At times within these narratives, however, the perspective of an outside observer was required to note aspects of participants' ID use that seemed inconsistent with their own descriptions. Thus, to this point, I have described what actually occurred surrounding the participants' participatory relationship with IDs as observed from an outsider's perspective. A potentially different narrative may be presented, however, if one were to consider solely each participant's perspective without interjecting an outside observer's view. This latter narrative would focus on endogenously (Stevens, 2010) describing each participant's participatory relationship with IDs as he or she perceives it. Juxtaposing these two narratives, that of each participant's perception and of what was observed throughout the study, yields interesting findings in our study of the participatory relationship between teachers and IDs. This is explored for each participant in turn.

Ms. Edelman's participatory relationship with IDs. Ms. Edelman described her participatory relationship with IDs in a manner that was largely consistent with that which was observed during the study. Namely, her underlying belief about the ways in which mathematics should be taught and learned was a main guiding factor in both her described and observed ID use. She was also cognizant of the influence each observed teacher, ID, contextual, and student factor had on her ID use, as evidenced by her statements throughout the study.

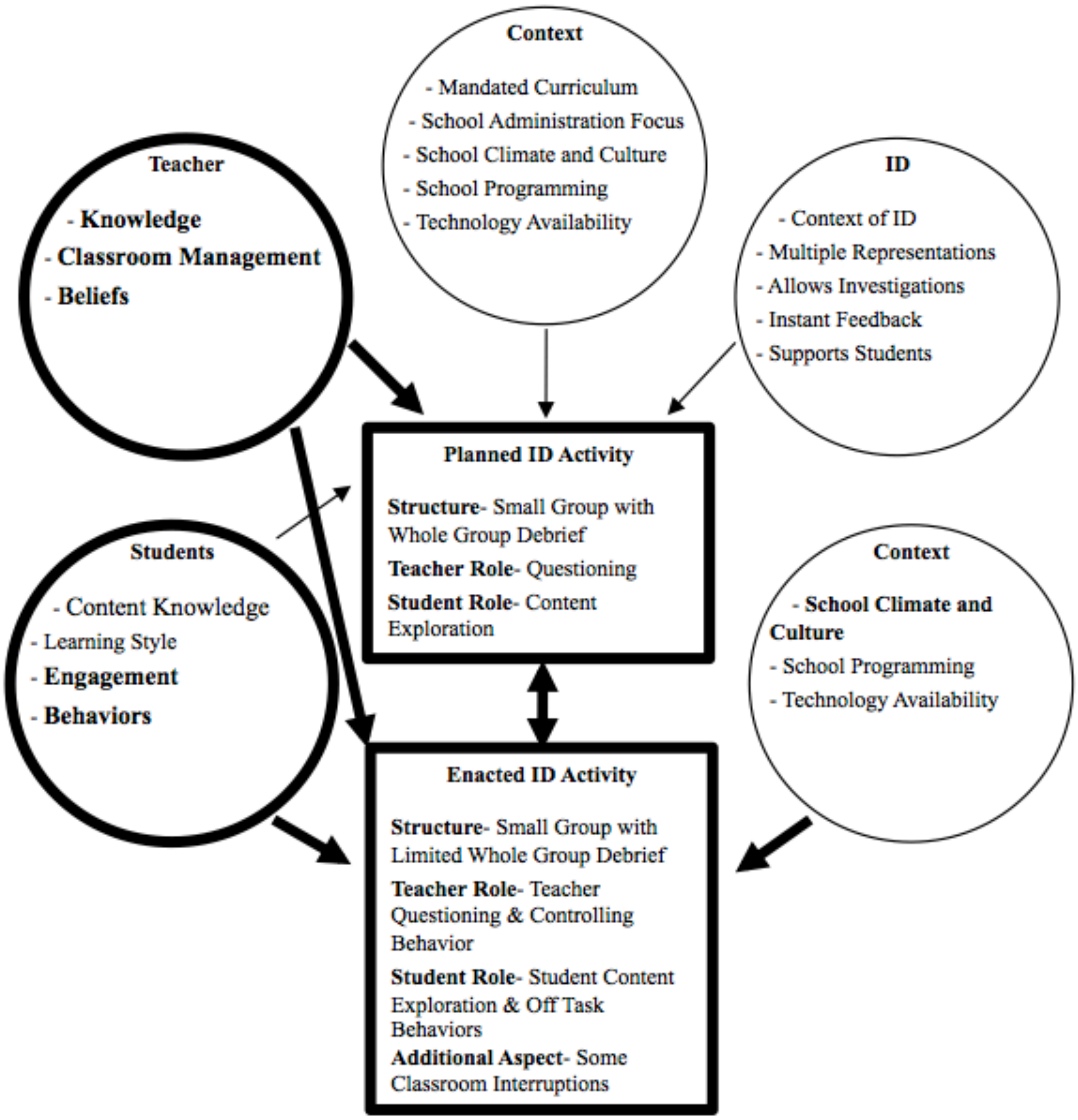


Figure 32. Ms. Edelman's conception of her participatory relationship with IDs.

Specifically, as Figure 32[2] depicts, Ms. Edelman noted the significant impact of students' engagement and behaviors within her classroom and the school as a whole (i.e. school climate and culture) on her ability to enact ID activities as she intended. Ms. Edelman acknowledged that her lack of sufficient pedagogical knowledge, including the

[2] Bolded text, circles, and arrows within the study's conceptual framework found within this and subsequent figures denote those factors and interactions that most significantly impacted the participant's conception of his or her participatory relationship with IDs.

understandings required to manage her classroom, decreased her ability to effectively address students' behaviors and actively engage students with IDs as she desired. Consequently, ID activities were altered by students' behaviors during their enactment in a way that was significant. While other factors where identified throughout the study, Ms. Edelman's conception of her participatory relationship with IDs was most significantly impacted by six factors (i.e. teacher knowledge, classroom management, beliefs, student behaviors, engagement, and school culture and climate). These factors are bolded in Figure 32 to denote their significant influence within Ms. Edelman's conception of her ID use.

Two aspects of Ms. Edelman's participatory relationship with IDs are important to note here. First, Ms. Edelman's descriptions of her ID use and the factors that affected that use were consistent with what was observed throughout the study. She clearly described enacted ID activities in a manner that was consistent with the study's observational data. She also identified and articulated the impact of the factors shown in Figure 32. Taken together, Ms. Edelman was aware of both how the observed IDs activities were enacted and the factors that affected those enactments.

Secondly, Ms. Edelman accredited the inconsistencies between planned and enacted ID activities to her inability to mediate the influence of specific student and contextual factors. She seemed to understand what the realization of meaningful mathematics learning (Bate, 2010; Jonassen et. al, 1999) would look and sound like during an enacted ID activity. Her level of teacher knowledge prevented her from enacting such learning, however, because she did not yet know how to decrease the impact of the factors shown in Figure 32. Ms. Edelman was, as she noted, still working to "build my skills around being able to use it effectively to facilitate students' learning of the content" (personal communication, March 10, 2016). Thus, Ms. Edelman understood what occurred during ID activities and why it occurred. She did not yet have the required knowledge to create her desired enactment of ID activities, however.

Ms. Allen's participatory relationship with IDs. Similar to Ms. Edelman, Ms. Allen expressed a desire to enact student-centered, inquiry learning where students "use IDs to make conjectures, […] test them, and learn something new" (personal communication, March 8, 2016). Then, after conducting a small group exploration with the ID, students would "share with the whole class what they found" (personal communication, October 26, 2015). Ms. Allen noted that this structure "puts more on the kids as opposed to them waiting for information from me. It puts it on their lap and has them do the research or the finding of the answers" (personal communication, October 26, 2015). Indeed, this thinking was consistent with the beliefs she expressed throughout the study and seemed to undergird her conception of the participatory she engaged in with IDs.

The observations conducted of Ms. Allen's enacted ID activities evidenced more teacher-centric and closely directed learning, however. Namely, the pre-ID activities Ms. Allen implemented explained the mathematics embedded within the IDs and created a solution path that directly paralleled what could be applied to the IDs. Thus, the use of pre-ID activities virtually eliminated the need for students to discover mathematics while interacting with the ID and reduced the enacted ID activity to applying the previously explained concept to a new context.

Ms. Allen did not acknowledge the significant impact her use of pre-ID activities had on altering the ID activities she enacted. Indeed, when asked to describe her overall goals of enacted ID activities at the conclusion of this study, she emphasized how she provided students with opportunities for "making conjectures and testing them with the ID. That's giving the students the opportunity to have a conversation and then to see what the ID shows" (personal communication, March 8, 2016). Ms. Allen explicitly contrasted this with activities where students "learn something new from the teacher then use the ID. […] Some people just want the see the teacher do it but that's not […] really what I want. I don't want them to watch me. […] I'd rather them do it" (personal communication, March 8, 2016). Ms. Allen described this latter situation as undesirable, but this was precisely what occurred during the enacted ID activities observed throughout this study. She seemed to be not fully aware of this fact, however.

Taken together, Ms. Allen's conception of her ID enactment (see Figure 33) was inconsistent with the observed enactment of these IDs. The reasons for these inconsistencies could not be determined definitively as a result of this study. I propose two possible explanations, however, as we attempt to understand more fully the participatory relationships between teachers and IDs. In both proposed explanations I employ an endogenous (Stevens, 2010) perspective similar to that of an ethnographer (Hammersley & Atkinson, 2007) by assuming that the descriptions of enacted ID activities Ms. Allen presented were representations of the ways in which she understood her world.

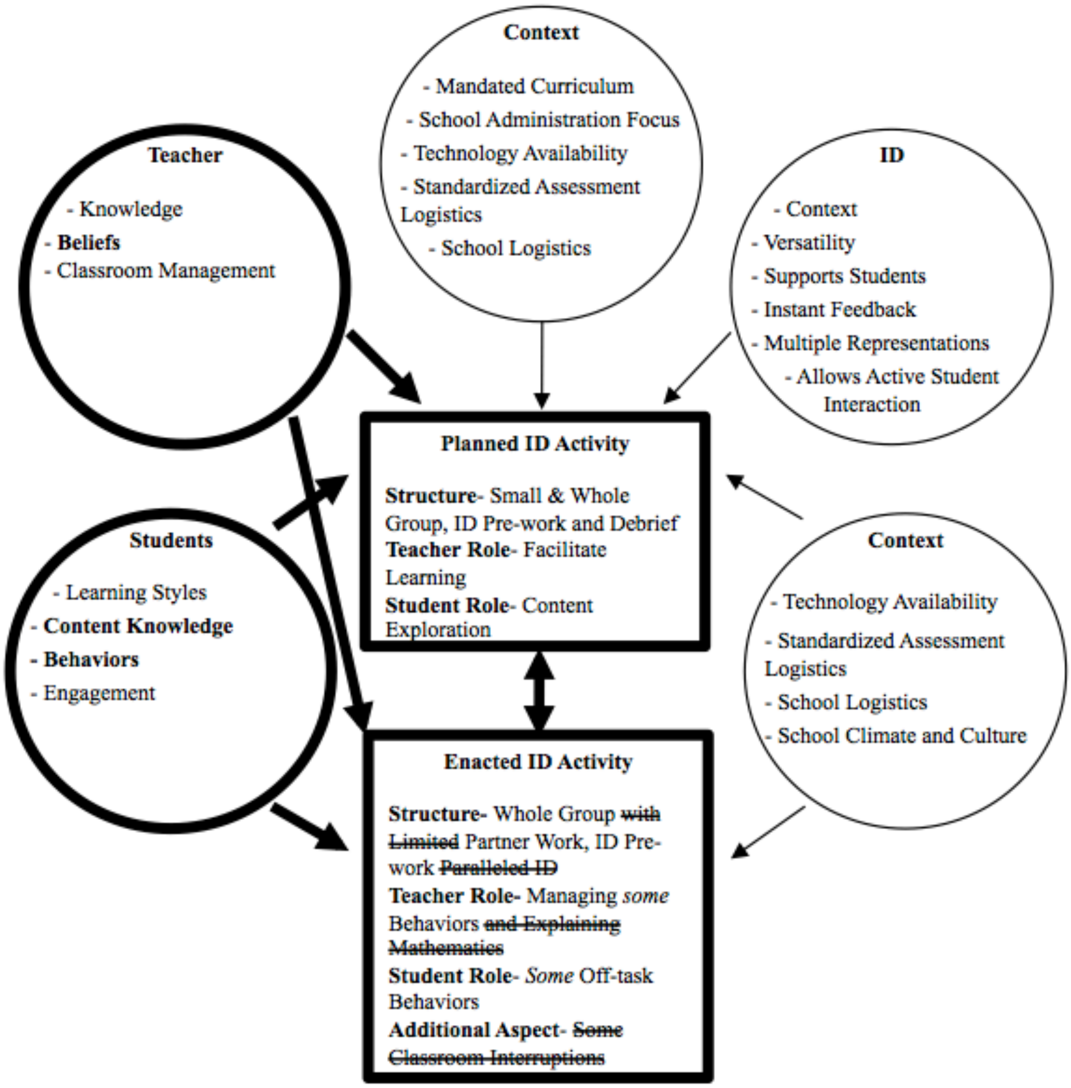


Figure 33. Ms. Allen's conception of her participatory relationship with IDs.

The first possible explanation for the inconsistencies identified between Ms. Allen's conception of her ID enactment and the observed ID enactment is related to her students' content knowledge. Ms. Allen described her students' mathematical understandings as "below level" and cognitive abilities in general as "very low level functioning" (personal communication, November 10, 2015). Consequently, she may have held concerns about students' ability to effectively engage with IDs within an inquiry activity. While discussing the *Friendship Problem ID*, for example, she explained,

> I don't want to say I'm thinking too low of my students, because I feel that they may be able to do this once they do the handshake. But then and again, it's like, I know my students and I really don't think they can. (personal communication, November 9, 2015)

Here, Ms. Allen expressed concern over students' ability to successfully interact with the *Friendship Problem ID* even after completing the parallel, pre-ID handshake problem. Thus, Ms. Allen seemed to view the level of inquiry present within her enacted ID activities as appropriately aligned with her perception of students' low ability levels. Said another way, in her mind the enacted IDs did actively engage students in discovering mathematics in a way that was aligned with their current content knowledge. This possible explanation is denoted in Figure 33 by bolding *content knowledge* within student factors.

A second possible explanation was that Ms. Allen might have viewed student behaviors as being so disruptive to the classroom that her enactment of IDs was, in fact, as student-centered as it could be. Ms. Allen consistently noted that students were "too playful" (personal communication, January 14, 2016) in her classroom and that "students' behaviors […] affect everything" (personal communication, March 8, 2016). Thus, her conception of enacted IDs might have, one the one hand, used the necessary structures and strategies that she perceived as effective in controlling students' behaviors. She alluded to this when asserting that she had "to manage students' behaviors first" (personal communication, March 8, 2016) in the classroom over any other teacher role.

On the other hand, students did interact with the IDs, though it occurred in a limited fashion. This act of interacting with the IDs may have been sufficient for Ms. Allen to perceive that students were active in their learning within the limits determined by their behaviors. This second possible explanation is denoted by bolding the student factor *behaviors* in Figure 33.

While neither explanation presented above could be confirmed as a result of this study, it is important to note two things about Ms. Allen's conception of her participatory relationship with IDs. First, similar to Ms. Edelman, she was cognizant of the student factors that affected her use of IDs. Namely, Ms. Allen expressed that students' content knowledge and behaviors affected her use of IDs. Unlike in Ms. Edelman's case, however, Ms. Allen's conception of the degree to which these factors affected her enactment of IDs was somewhat inconsistent with the observed ID activities. Consequently, her conception of enacted ID activities themselves was also inconsistent with the observational data collected throughout the study. This is depicted in Figure 33 by crossing out the aspects of her ID enactment that were observed during this study but that are incongruent with her conception. Thus, Figure 33 includes only what Ms. Allen described within the enacted ID box.

Mr. Clark's participatory relationship with IDs. Mr. Clark's conception of his participatory relationship with IDs offers further detail surrounding how teachers may perceive their participatory relationship with IDs. Mr. Clark described ID enactment throughout the study as occurring "pretty much according to plan" (personal communication, December 2, 2015). Specifically, he emphasized how "students interact[ed] with the diagram" (personal communications, October 29, 2015) while he asked "probing questions to help them generate their own conclusions" (personal communication, February 25, 2016). In doing so, he asserted that students were pushed to "struggle and to grapple with complex issues [as they] actively engaged in the lesson" (personal communication, February 25, 2016).

Mr. Clark's descriptions of enacted IDs contained significant inconsistencies with the observational data collected during this study, however. Figure 34 depicts the differences between Mr. Clark's conception of enacted ID activities and those that were observed during this study within the enacted ID activity box. Specifically, Mr. Clark described student-ID interactions when only limited interactions were observed. He also noted how students explored content using the ID in order to reach their own conclusions. Observed ID activities were highly scaffolded, however, with the teacher largely directing the use of the ID and content attainment through directed, closed questioning techniques. Lastly, the significant impact of the multiple classroom interruptions occurring during ID activities caused by school climate and culture concerns and students' behaviors were not acknowledged in Mr. Clark's descriptions of enactment ID activities at all. Instead, he focused primarily on how technology availability within his school mediated his participatory relationship with IDs.

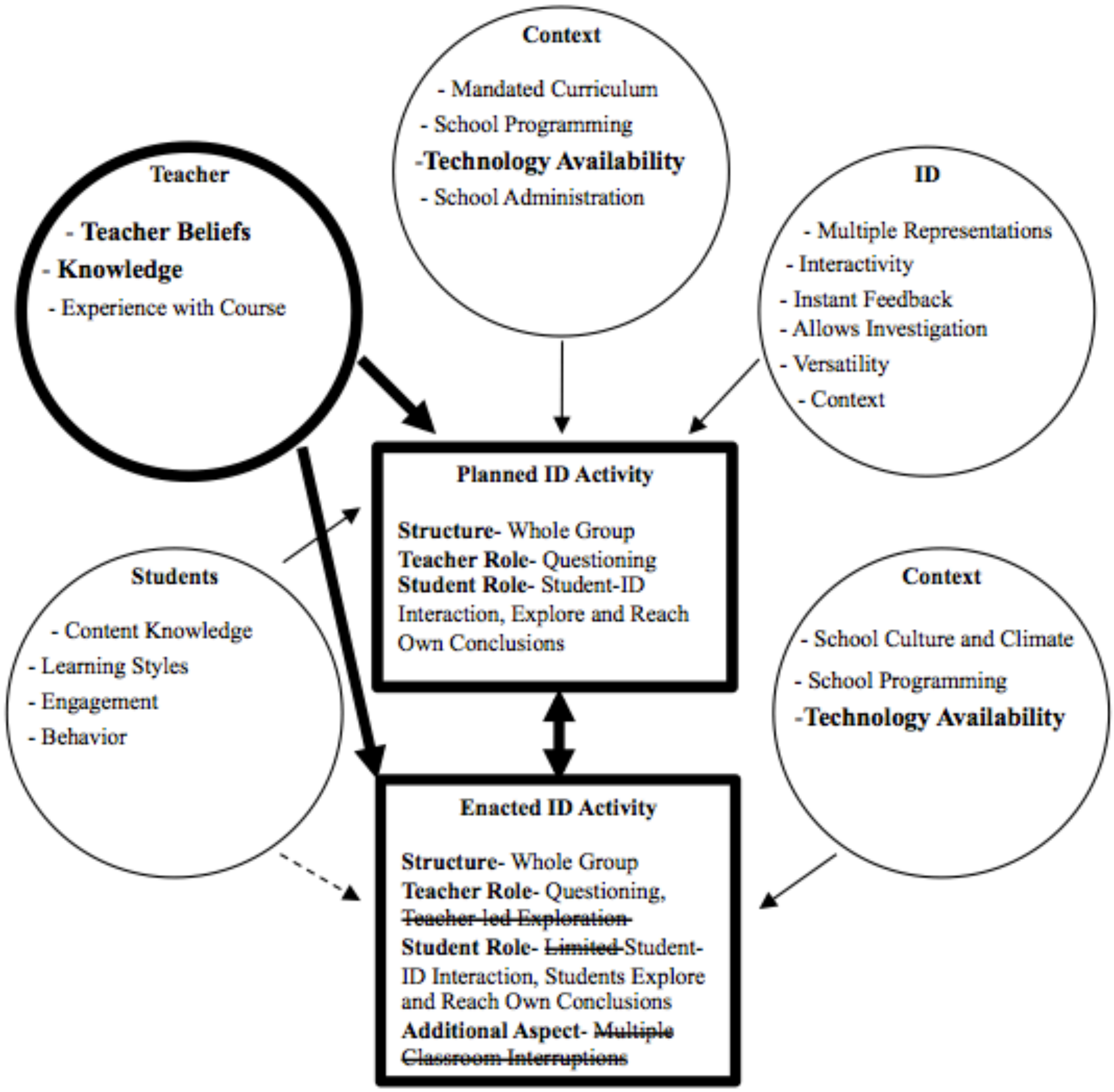


Figure 34. Mr. Clark's conception of his participatory relationship with IDs.

Taken together, Mr. Clark's conception of his ID enactment was largely inconsistent with the observational data collected during this study. Similar to the case of Ms. Allen, the reasons for these inconsistencies could not be determined definitively as a result of this study. Below propose two possible explanations, however. Again, I employ an endogenous (Stevens, 2010) or ethnographic (Hammersley & Atkinson, 2007) perspective here as a means to understand the ways in which Mr. Clark understood his world.

First, it seemed possible that Mr. Clark's pedagogical knowledge (Shulman, 1986) was not aligned with the understandings present within mathematics education research, the latter of which were used when noting inconsistencies between his actual ID enactments and how he described his conception of these enactments. Specifically, Mr. Clark's understandings of the best practices for facilitating student-centered exploration of mathematics content and how probing questions are effectively used in this process may be consistent with what was observed during the enactment of ID activities. If this were the case, he did enact student-centered learning as he understands it. Consequently, from Mr. Clark's perspective, there were no inconsistencies between the intended and enacted ID activities. In this scenario, Mr. Clark's pedagogical knowledge was a significant mediating factor in his conception of his participatory relationship with IDs. This impact is denoted in Figure 34 by bolding *knowledge* within teacher factors.

A second possible explanation pertains to Mr. Clark's beliefs about mathematics teaching and learning. Throughout the study, Mr. Clark contrasted using IDs to teach mathematics with using more traditional or "old school" methods, such as drawing diagrams on a classroom whiteboard. In every instance he favored using an ID since students could "play with or manipulate the diagram" (personal communications, October 29, 2015). Students' ability to interact with mathematics content in this way seemed essential to Mr. Clark's belief about how mathematics should be learned. Indeed, such interactivity can facilitate content exploration and active learning, of which he desired. It was possible, then, that the interactivity embedded within IDs was so tightly aligned with

Mr. Clark's beliefs that simply incorporating IDs within an activity satisfied his criteria for content exploration. Put differently, it may have been possible that Mr. Clark did not define content exploration by how the ID was used, just that it was used at all. Thus, using this perspective, students did explore content because IDs were enacted within his classroom. This would explain why Mr. Clark did not articulate the inconsistencies between his planned and enacted ID activities that were identified using the study's observational data. His beliefs would be a significant mediating factor in this scenario, which is denoted in Figure 34 by bolding *beliefs* within teacher factors.

Although these two explanations seem plausible given Mr. Clark's use of IDs, neither of these proposed explanations could be confirmed as a result of this study. In both cases, however, it was important to note that a factor originating from Mr. Clark (i.e. his beliefs or knowledge) influenced his perception of enacted ID activities in such a way that he did not recognize significant mediating factors, namely students' behaviors. This is depicted in Figure 34 by the dotted arrows drawn from student factors to enacted ID. Note that the arrow between contextual factors and enacted ID is solid because Mr. Clark did emphasize the impact of technology availability within his use of IDs. Lastly, Mr. Clark was not cognizant of the inconsistencies between the planned and enacted curriculum that were noted when observing his practice. Figure 34 depicts this phenomenon by crossing out the aspects of the observed ID enacted that were not included in Mr. Clark's descriptions.

Summary

This chapter sought to more fully understand the participatory relationship between teachers and IDs by highlighting findings across the study cases. In doing so, participants' planned and enacted ID activities were compared using a cross case analysis which highlighted how ID activity structures, goals, teacher roles, and students roles moved from the intended to the enacted curriculum. Of note was that Ms. Allen and Mr. Clark's enactment of planned ID activities seemed to decrease students' opportunities to meaningfully engage with mathematics content through their interactions with IDs. Both Ms. Allen and Mr. Clark were not fully cognizant of this phenomenon, however. Ms. Edelman's enactment of ID activities, on the other hand, generally included the student-centered content exploration she planned.

Cross case analysis was also used to examine the teacher, contextual, ID and student factors that affected ID use across the study cases. This analysis allowed the identification of mediating factors that universally affected the participatory relationship between teachers and IDs (see Figure 31). To begin, using a district-mandated curriculum containing IDs, whether or not functional technology was available, and the school administration's focus were contextual factors that mediated all of the study participants' creation of planned ID activities. Intended curriculum involving IDs was further influenced by teacher beliefs and knowledge as participants considered a number of ID design features (i.e. the presence of multiple mathematical representations, instant feedback to user input, and the context within which the ID was embedded). Each student factor identified within this study mediated the ID use across all of the study cases; student's content knowledge, learning styles, engagement, and behaviors mediated all three participants' enactment of ID activities. An additional contextual factor (i.e. school culture and climate) also had a widespread affect on participants' ID activity enactment.

The degree to which participants were aware of the factors that mediated their ID enactment varied across the study cases (see Figure 10 in chapter 8). Ms. Edelman and Ms. Allen, for example, identified various factors that affected their enactment of IDs that were consistent with observational data collected during this study. Mr. Clark, on the contrary, did not seem aware of the student and contextual factors that altered his enacted ID activities. Further, participants had varying levels of awareness surrounding the degree to which various factors mediated their enactment of ID activities. Ms. Edelman was the most cognizant of the mediating effect of factors surrounding her ID use as evidenced by her descriptions of, among others, the significant impact students' behaviors had on enacted ID activities. While Ms. Allen identified that some factors had an impact on her ID enactment, she did not express that those factors affected her enactment as significantly as was observed during the study. She was somewhat aware of the degree to which mediated factors effected her ID enactment. Mr. Clark, on the other hand, was completely unaware of the effect various factors observed throughout the study had on his ID enactment. Lastly, each participant's descriptions of their enacted ID activities contained varying levels of alignment with the observational data collected during this study. Ms. Edelman's conception of her ID enactment was highly consistent with what was observed. The ID activities observed within Ms. Allen and Mr. Clark's classrooms were misaligned with their respective conceptions of that enactment, however.

Chapter 8

Researchers have done significant work to understand the ways in which teachers interact with curricula, utilize curricula in their instructional practice, and the factors that affect those teacher-curricular interactions. Investigations include examining teachers' interactions with curricula embedded within and enhanced by digital technologies. This study was designed to contribute to this body of work by studying teacher interactions with a previously understudied technology-interactive diagrams. In this chapter, a brief synthesis of the study and its key findings are provided. Implications of the study are then discussed in light of the study findings that may inform how teacher education and training design and facilitate learning surrounding teachers' use of IDs. The chapter concludes by discussing the limitations of the present study and suggesting potential areas of future research.

Synthesis

This study of secondary mathematics teachers' ID use and the factors that mediate their ID use was conducted in order to expand the body of knowledge surrounding teacher-curriculum interactions. Specifically, the study juxtaposed understandings surrounding teachers' use of curriculum more broadly (Drake & Sherin, 2006; Remillard, 2005; Stein, Remillard, & Smith, 2007) and digital technologies specifically (Cuban, 2001; Rogers et al., 2005; Squire & Jenkins, 2003: Stols & Kriek, 2011; Windschitl & Sahl, 2002) to teachers' participatory relationship with a digital technology which was, to this point, "largely unexplored" (Naftaliev & Yerushalmy, 2009, p. 3).

The study used qualitative research methods (i.e. interviews and observations) within an exploratory, case study research design to capture, explore, and identify the ways in which participants used IDs within their planned and enacted curriculum, as well as the factors that affected their use. This qualitative methodology facilitated a thorough examination, rich description, and deep understanding (Maxwell, 2013; Merriam, 2009) of the processes, tensions, and opportunities the study participants negotiated while interacting with the focal technology.

Each study participant taught 9th grade mathematics within the Doylestown School System (DSS), a district predominately composed of African American students within a large urban center. They used the Agile Mind online curricular resource, a district-mandated curricular resource rich with IDs, in their teaching. Data collection occurred during the 2015-2016 school year. Prior to this study, I worked closely with Ms. Edelman, Mr. Clark, and Ms. Allen as an instructional coach and district mathematics representative.

The conceptual framework described by Remillard (2005) grounded this examination of the participatory relationship between teachers and IDs. Findings pertaining specifically to teacher's use of technology (ex. Bate, 2010) were used to focus Remillard's framework more specifically on digitally-enhanced curricular materials. The resulting study framework facilitated data collection and analysis of each participant's planned and enacted ID activities, as well as the teacher, contextual, ID, and student factors mediating those ID activities. Inductive analysis was used within each of the areas identified by the study's conceptual framework to explore the salient features of the participatory relationship between each participant and IDs.

This study then employed a cross-case analysis (Merriam, 2009). The structure, intended goal, students' role, and teacher's role within participants' ID activities were compared and contrasted. Additionally, the teacher, contextual, ID, and student factors salient to each participant were considered across the study cases. This examination surfaced factors that affected teacher-ID interactions more broadly. It also highlighted important detail surrounding how ID use was mediated by particular factors.

Lastly, two different perspectives were presented for understanding each teacher's participatory relationship with IDs as this study attempted to more deeply understand these relationships. Specifically, each participants' lived experience with IDs, in an ethnographic sense, was juxtaposed against their observed uses of IDs. This comparison illuminated varied levels of alignment between participants' conception of their ID use and the observational data collected during this study (see Table 11).

Table 11

Alignment of Participant Conception of ID Enactment and Observational Data

Participant	**Was participant aware that various factors affected their ID enactment?**	**Was participant cognizant of the degree to which factors affected their ID enactment?**	**Was participant's conception of ID enactment aligned with observational data?**
Ms. Edelman	Yes	Yes	Yes
Ms. Allen	Yes	Somewhat	No
Mr. Clark	No	No	No

Implications

Given the findings of this study, a variety of implications within areas related to teachers' participatory relationship with IDs will now be presented. First, implications for how teachers' ID use is conceptualized are discussed. This is followed by implications for how teacher education and training may be informed by the study findings. Lastly, the study findings are applied directly to the instructional context of schools and school systems.

Implications for conceptualizing teachers' curricular use. The framework proposed by Remillard (2005) was used as the foundational bedrock upon which this study conceptualized the participatory relationship between teachers and IDs. Remillard's conceptual model was extremely useful in identifying and examining teachers' instructional use of IDs. In particular, it allowed for the description of planned and enacted curricula both as individual aspects of the study participants' interactions with IDs and as related components within a larger participatory relationship. Similarly, isolated and integrated consideration of the teacher, ID, student, and contextual factors surrounding ID activities were facilitated by Remillard's conceptual framework.

The study findings pertaining to the alignment between teachers' perception of their ID use and their observed ID use was not accounted for in Remillard's (2005) framework, however. Indeed, this framework may be employed to depict either an exogenous or endogenous (Stevens, 2010) view of teachers' ID use, given that these depictions are contrary in some way. Each of these two representations, as well as the relationships between them, may hold value in mathematics education theory and practice and, consequently, should be considered further. Thus, it seems that this study highlighted the need to carefully consider the perspective or view of knowing (Greeno, Collins, & Resnick, 1996) from which one conceptualizes the participatory relationship between teachers and IDs. Further, while the current study focused specifically on a particular curricular resource, it seems notable to apply this phenomenon surrounding teacher-curriculum interaction more generally. Namely, further research surrounding the perspective taken to conceptualize teachers' participatory relationships with varied curricular materials may be needed, including the degree to which varied perspectives are aligned.

Implications for creating desirable teacher curriculum use. This study highlighted the ways in which various teacher factors mediated the creation of ID activities. If we are to assume that particular characteristics of ID activities are desired, such as those expressed by Jonassen, et. al (1999), it stands to reason that teacher education and in-service professional development should focus on developing particular teacher factors consistent with those desired features. This study reinforced, for example, the relationship between teachers' knowledge (Shulman, 1987) and their ability to recognize and enact student-centered, inquiry learning. Thus, the former teacher factors could be explicitly attended to if the latter skill was viewed as desirable. Indeed, the relationships described in this study highlight the importance of teacher education and professional development paying particular attention to teachers' beliefs, knowledge, and classroom management and the ways these teacher factors facilitate desired ID use.

In much the same way, schools and school systems may define desired ID use within the walls of their mathematics classrooms. Thus, it would be useful to understand the environmentally situated (Greeno, Collins, & Resnick, 1996; Putnam & Borko, 2000) relationships between teachers' ID use and factors that may be influenced at the school and school system levels. School and district leaders could then modify contextual factors, such as technology availability, leadership foci (Bate, 2010), school programming and logistics, and climate and culture, in order to alter teachers' instructional uses of IDs. Targeted student and ID factors may also be affected at this level.

Additionally, this study highlights particular ID factors that affected participants' use of this technology. Those individuals who create IDs may attend to the particular design elements which facilitate desired teacher-ID relationships. For example, the context within which an ID is embedded may be designed in such a way to create desired ID use. Additionally, the presentational, orientational and organizational functions (Yerushalmy, 2005) of IDs may be strategically incorportated to facilitate desired teacher ID use.

Taken together, stakeholders surrounding mathematics instruction may be able to influence a number of factors affecting teachers' ID use. This, in turn, may alter how teachers create planned and enacted ID activities. Given stakeholders' indirect affect on teachers' ID use, it seems particularly important that the mathematics community at large understand and agree upon a collective definition of effective and desirable learning using IDs.

This line of reasoning may be expanded to include teachers' use of curricular materials more generally. Indeed, this study may be thought of as a particular example of teacher-curriculum interaction. Thus, mathematics education stakeholders may consider how to impact the factors surrounding teachers' practice as they attempt to create desirable curriculum use.

Limitations

The aforementioned implications of this study must be considered given the limitations of this study, however. These limitations are natural consequences of the research methodology, focus, and context utilized for this examination of teachers' ID use and, thus, should be used to situate this study within in the larger body of research surrounding this topic.

To begin, a qualitative research methodology was used to guide this exploratory study. Qualitative methods were appropriate for such an examination as it allowed rich description of the focal phenomena (Maxwell, 2013; Merriam, 2009). Multiple-case study does not allow one to generalize the findings of this study beyond the examined context, however. Indeed, the findings of this study are beneficial in understanding the study cases' participatory relationship with IDs and in framing that of others within their context, but one must be careful not to directly generalize these findings beyond the study participants and the context in which they work.

Teachers' ID use was also examined within a particular context during this study. Namely, the three participants taught ninth-grade mathematics within a large, urban district that mandated their use of an online curriculum rich in IDs. Consequently, the study is limited by these contextual factors and is unable to directly speak to teacher ID use outside of this context. How, for example, do teachers of middle school, upper high school, or collegiate mathematics use IDs? How is the participatory relationship between teachers and IDs affected by teaching students whose mathematical understandings are on grade-level? How do teachers use IDs when they are not mandated to use an ID rich, digital curriculum? These and other related questions cannot be answered as a result of this study.

Lastly, this study illuminated a relationship relevant to the present investigation, but that was not directly addressed by its research questions. Namely, the alignment between teachers' perception and their observed ID use arose as an interesting phenomenon that varied across the study cases. While this study allowed for a description of this relationship, it unfortunately could not fully answer questions of why, when, and how due to the present focus. Answering such questions would be beneficial in more completely understanding the participatory relationship between teachers and IDs.

Areas of Continued Study

Examining the practice of secondary mathematics teachers' use of IDs was an incredible learning experience both for my own growth as a researcher and the body of research pertaining to this topic. That said, this study seems to reinforce and highlight the need for continued study in both previously identified and novel areas.

First, the participatory relationship between teachers and IDs should be investigated further. Additional qualitative studies could add necessary description (Merriam, 2009) surrounding teachers' ID use and the factors that affect that use while quantitative investigations could speak to scale and generalizability. In particular, studying teacher-ID interactions within various contexts, such as those not mandating curriculum or with students who have demonstrated various levels of content mastery, or with teachers embodying varied teacher factors, such as their beliefs and knowledge, seems prudent. Doing so would continue to acknowledge the situative nature (Greeno, Collins, & Resnick, 1996; Putnam & Borko, 2000) of teacher-curriculum interaction and likely increase the breadth and depth of our understandings within this area of research. Such work would be consistent with recommendations for additional study surrounding the participatory relationship between teachers and digital curriculum (Naftaliev & Yerushalmy, 2009, 2012: Remillard, 2005).

Additionally, clarification surrounding the relationship between teachers' own perspective of their ID use and their actual ID use could be sought in subsequent studies. Such studies could investigate if, how, and why this phenomenon exists more broadly. Do particular factors, for example, consistently correlate with a teachers' level of awareness surrounding their ID use? Is that relationship constant or is it mediated by even further factors? Answering these and related questions could refine and enrich the conceptualization of teachers-ID interactions. Studies of other digital and non-digital curricular materials surrounding this phenomenon also seem valuable.

Lastly, this study is situated within a larger body of research pertaining to IDs. Other researchers (Naftaliev & Yerushalmy, 2011, 2012, 2013; Yerushalmy, 2005) have described design features and student learning facilitated by IDs. How, though, does this study of teachers' ID use interact with the work underway pertaining to other areas relevant to IDs? For example, the study participants within this study attended to the organizational design function of IDs, but did not explicitly discuss the presentational or orientational functions (Yerushalmy, 2005). Studies specifically designed to isolate these design functions and study teachers' uses of IDs across these functions could be beneficial in understanding teachers' participatory relationships with ID in a way that is more tightly aligned with the design framework described by Yerushalmy. Additionally, this study suggests that the context that is embedded within an ID may be an important design consideration. This is currently absent from the ID design literature. These and other connections between how IDs are designed, teachers interact with, and students engage and learn from IDs seem ripe for continued study.

Conclusion

It is my hope that this study is the first step in increasing our understandings of teachers' participatory relationship with IDs. While further research in this area is certainly needed, this study begins the investigation into how teachers use IDs within their mathematics instruction and the factors that affect their use. It seems clear as a result of this study and others (Bate, 2010, Drake & Sherin, 2006; Remillard, 2005; Stein, Remillard, & Smith, 2007) that the curriculum-teacher relationship is complex. Indeed, complimentary and contradictory factors interact in ways that can be nuanced and unpredictable. Teachers' understandings of their own ID use further complicates this picture.

What was clear to me as a result of this study was that the three teachers described throughout this document intended to use IDs with the best intentions surrounding student learning. Indeed, at the core of Ms. Edelman, Ms. Allen, and Mr. Clark's instructional use of IDs was the idea that what they were doing was the best they could do for their students. Each teacher planned and enacted IDs in the most effective ways they understood. IDs, like any other digitally enhanced curricular resource, however, are only an effective "learning medium when supported by appropriate teacher intervention and tasks" (Kieran, 2007, p. 737). Given this, I feel an increased urgency in more fully understand how to best facilitate effective mathematics learning through the use of IDs. Such an understanding could be meaningfully combined with teachers' desires to facilitate highly effective ID activities through professional reflection, learning, and growth. Additionally, a more complete understanding of the factors that mediate creating and enacting effective ID activities could facilitate the creation of contexts conducive for teachers using IDs effectively. In short, I hope that this and future studies continue to enrich our understandings surrounding teacher-ID interactions. Ultimately, though, my hope is for such studies to result in the enactment of effective mathematics learning within classrooms like those visited during this study.

Appendix A: Pedagogical Beliefs Questionnaire

(based on Bate (2010))

Name ______________________________________ **Date** _____________

(All names are treated as confidential and will be coded to protect the identity of the respondent.)

Instructions: For each statement below, circle the number under the response that best describes what you think or feel. There are no right or wrong answers. I am interested in your ideas about mathematics teaching and learning. Your answers and comments will remain confidential.

		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
1)	Learning mostly involves trial and error.	1	2	3	4	5
2)	Learning mostly involves accessing and understanding factual information.	1	2	3	4	5
3)	Students' mistakes are usually caused by a lack of practice.	1	2	3	4	5
4)	Being able to memorize facts and procedures is important for learning.	1	2	3	4	5
5)	It is not appropriate to encourage students to manipulate data.	1	2	3	4	5

Comments

____

		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
6)	Effective teachers plan so that students spend most of their time working independently.	1	2	3	4	5
7)	Encouraging independent learning is more important than encouraging collaborative learning.	1	2	3	4	5
8)	Cooperative group work is an important aspect of effective teaching.	1	2	3	4	5
9)	Effective teachers limit collaboration because it is difficult to determine who is responsible for what.	1	2	3	4	5
10)	Effective teaching involves class discussion in which students share ideas and negotiate meanings.	1	2	3	4	5
11)	Students should be given the opportunity to discuss with each other how to approach tasks.	1	2	3	4	5

Comments

____

		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
12)	Learning mostly involves creative thinking.	1	2	3	4	5
13)	There are often several different ways to interpret something.	1	2	3	4	5
14)	Effective teachers show students the proper procedures to answer questions.	1	2	3	4	5
15)	Students learn by being shown the correct ways to interpret symbols and situations.	1	2	3	4	5
16)	Students should be encouraged to build their own ideas, even if their attempts contain much trail and error.	1	2	3	4	5
17)	Effective teachers provide students with solutions to problems.	1	2	3	4	5
18)	Students' errors often reflect their current understandings of ideas or procedures.	1	2	3	4	5
19)	Learning is about getting to the truth.	1	2	3	4	5
20)	Learning is enhanced if students are encouraged to use their own interpretations of ideas and their own procedures.	1	2	3	4	5
21)	Effective teachers value periods of uncertainty, conflict, confusion or surprise when students are learning.	1	2	3	4	5
22)	Students learn best if they are shown clear, precise procedures for doing things.	1	2	3	4	5
23)	The role for the teacher is to provide students with activities that encourage them to wonder about and explore their subject.	1	2	3	4	5

Comments

____

		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
24)	Problems should be solved in one consistent way.	1	2	3	4	5
25)	Solving a problem usually involves finding a truth, rule or formula that applies.	1	2	3	4	5
26)	Effective teachers regularly devote time to allow students to find their own methods for solving problems.	1	2	3	4	5
27)	Effective teachers show students lots of different ways to look at the same question.	1	2	3	4	5

28)	Effective teachers teach only what is important for assessment.	1	2	3	4	5
29)	Effective lessons progress step-by-step in a planned sequence towards the lesson objectives.	1	2	3	4	5
30)	The use of physical objects and real life examples to introduce ideas is an essential component of learning.	1	2	3	4	5
31)	Engaging with lots of problems is the best way for students to learn.	1	2	3	4	5

Comments

___

		Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
32)	Students should always be encouraged to articulate their own learning goals.	1	2	3	4	5
33)	Regular assessment against curriculum outcomes is an effective way to monitor student progress.	1	2	3	4	5
34)	It is important for students to monitor progress towards their own goals.	1	2	3	4	5
35)	Students should be encouraged to think carefully about how they learn.	1	2	3	4	5

Comments

___

Thank you for participating in the questionnaire!

Appendix B: Initial Interview

(based on Bate (2010))

Researcher will describe IDs and give examples

A- Interactive Diagrams and Teaching, General

Do you use (have you used) interactive diagrams in your teaching? How so?

- What kinds of interactive diagrams? How often?
- What were your reasons for using the ID?

Do you think interactive diagrams make a difference in the way you teach? How so?
What is your role as a teacher when using an interactive diagram?

Were there any constraints or restriction that influenced whether, when, and how you used IDs?

In your ideal classroom, how would you use IDs?

B- IDs and Teaching, Specific

What topics are IDs most helpful or appropriate?
What topics are IDs least helpful or appropriate?
Are there topics or tasks for which you would never use IDs?

C- IDs and Learning

Do you think IDs make a difference to the way students learn?
Does learning with IDs have an effect on the way students work in the classroom?

- Discussion/communication
- Group vs. individual work
- Student centeredness vs. teacher centeredness

Does learning with IDs have an effect on students' understanding of ideas and concepts?

- Does deeper, faster, better learning occur?

D- Facilities and Equipment

Describe the technology infrastructure and equipment that are available in your classroom and school that allow you to use IDs.

Are facilities and equipment challenging in your use of IDs? How so?

E- School/ District Context

What does your administrator think about IDs?
Are you encouraged or deterred from using IDs? How so?

How does the district think about using IDs?

Have you received training or professional development around IDs?

Appendix C: Pre-Observation Interview

A- Intended Use of ID, including ID Factors

What ID will you be using?
Why are you using this ID?

How will you be using the ID? Walk me through the activity step by step.
Why are you using the ID in that way?
- Will the ID be projected?
- Who will control the ID?
- Will students use the ID on individual or group computers?
- What other materials, if any, will be available for students at that time?

How much time will you dedicate to using the ID?

What do you like about this ID? Why?
What do you dislike about this ID? Why?

What content will you teach with this ID/ what will students learn with this ID?
Why is this ID good for teaching/learning this?

B- Contextual Factors

Do you have enough technology to use this ID in the way that you would like to?
If your administrative team walked in during the ID use, what would they think?

C- Teacher Factors

Do you think this ID is the best way to teach this particular content? Why or why not?

D- Student Factors

How do you think students will use the ID? Why do you think that?

Appendix D: Observation Protocol

(based on Judson (2002))

1.Background Information

Teacher: ________________________________ Date: __________________

School: ____

Location: classroom / computer lab / other:___________________________________

Subject: _________________________ Grade level: _________________________

Start time: _______________________ End time: ___________________________

2.Lesson Content and Activities

Description of lesson

Classroom setting (e.g. space, seating arrangements, etc)

Number of students: ____________________

Transcript of dialogue during ID activity:

3.Technology context

Number of computers: ________ Other media: ________________________________

Description of the ID interaction within the lesson including student to media ratio and locus of control in terms of ID.

Amount of ID use (i.e. proportion of the lesson): _________________________________-

Kinds of ID use (e.g. instructional game, drill and practice, presentation, exploration, modeling, etc)

Context for ID use (e.g. independently for students, as a whole group, in small groups)

Sketch of physical lay out of classroom (i.e placement of technology, teacher and students- indicate mobility)

4.Design of ID integration

		Researcher Comments
1	The design of the ID integration allowed students to learn in ways not otherwise possible.	
2	The ID was a means for supporting curricular objectives, as opposed to being a separate curricular focus.	
3	The selection of the ID was appropriate to meet the learning objectives.	
4	This lesson allowed student operation of the ID	
5	The integration of the ID was designed to promote intellectual challenge (students pose questions, direct their own work, and assess their own work).	

5.Class dynamics

		Researcher Comments
6	The teacher and/or user of the ID prompted students toward higher-order thinking.	
7	Students had a voice in the selection of the ID tools and how the ID was being utilized.	
8	Interaction with the ID provided students with a sense of independent control and mastery over an environment.	
9	The teacher provided appropriate assistance to guide student activity.	
10	Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.	

6.Meaning and purpose

		Researcher Comments
11	Connections within the content and to other content disciplines were explored and valued.	
12	Students took pride in new learning and/or work produced with the aid of the ID.	

13	The ID was used to investigate real phenomena and real world situations.	
14	Students developed problem-solving strategies. Where appropriate, the ID tools aided the development of these strategies.	
15	Students used the ID to solve problems and make informed decisions.	

7.Content and knowledge

		Researcher Comments
16	The lesson emphasized fundamental concepts outlined in the curriculum framework.	
17	The integration of the ID into the lesson promoted strong, coherent conceptual understanding.	
18	The teacher had a solid grasp of the subject matter content and the use of the ID.	
19	Students were reflective about their own learning.	
20	Students used the ID to aid the construction of meaningful knowledge.	

8.ID as a tools

		Researcher Comments
21	The use of the ID aided the clarification and communication of ideas.	
22	Students employed the ID to develop strategies for solving problems.	
23	Students used the ID to construct models, increase productivity, and produce creative work.	
24	Students utilized the ID to collect information, process data and report results.	
25	Students used the ID for inquiry and exploration. Students made predications, estimations, and/or hypotheses and devised means for testing them.	

Appendix E: Post-Observation Interview

A- Enacted Use of ID, including ID Factors

How did the classroom use of the ID go?
Why did it go that way?

What did you like about using this ID? Why?
What did you dislike about using this ID? Why?

What content did you teach with this ID/ what did students learn with this ID?
How did the ID work for teaching/learning this?

During the observation, I noticed that (*describe moment of interest from observation*). What happened here? Why do you think this happened?

B- Contextual Factors

Did you have enough technology to use this ID in the way that you wanted to?
If your administrative team had walked in during the ID use, what would they have thought or done? OR When your administrative team walked in during the ID use, what did they think or do?

C- Teacher Factors

Do you think this ID was the best way to teach this particular content? Why or why not?

D- Student Factors

How did your students use the ID? Why did they use it in that way?

Appendix F: Final Interview Protocol

A- Belief survey

Give each participant the belief survey to complete once again. Have each complete it beforehand and turn in if possible.

B- Clarifying teacher's beliefs

Ask each participant the following questions. Then, use their responses as a means to clarify their beliefs when asking about each individual ID in part C.

- How should mathematics be taught?
- What is the teacher's role when teaching mathematics?
- What is the students' role when learning mathematics?
- What is your biggest goal/pedagogical point of view as a teacher of mathematics?

C- Questions for all participants- Specific IDs

Give participants the cutout IDs on the following pages. Tell them that they are all found within the IA course. Ask them to order the IDs in the order that they like them, from most to least.

Then ask participants the following, referring to their ordering:

- What were you considering when you ordered them in this way?
- Why do you like this one/these ones the most?
- Why do you like this one/these ones the least?

Possible probing questions include-

- Did you use this ID this year?
    - If so…
        - Why did you use it?
        - How did you PLAN to use it in the classroom?
            - What were students doing?
            - What were you doing?
        - Did it go as you had planned? How so?
            - If not, What did it ACTUALLY LOOK like in the classroom?
                - What were students doing?
                - What were you doing?
        - Would you use it differently the next time? Why?
    - If not…
        - Why didn't you use it?
- What did you like about using the ID?
- What did you dislike about using the ID?
- Having used it, is there anything you wished it could do, but didn't?

D- Questions for all participants- ID models

For each model component (activity structure, teacher role, student role, and goal of activity):

1) Without showing different options, ask each participant how they usually structure their ID activities in this area. What do they do the MOST and why?
2) Take out the different options cards and have participants sort them in order of most implemented/what they usually do. Ask probing questions-
    a. What are you considering when ordering them in this way?
    b. Why do you do this the most? What do you like about this option? Why?
    c. Why do you do this the least? What do you dislike about this option? Why?
3) Repeat this for all four components.

For each component, ask participants to now consider their ideal classrooms, where everything is how they would want it.

4) Ask participants what is different between their current classroom and their ideal classroom.
    a. If needed, ask them specifically about contextual (i.e. technology, school climate and culture, school logistics and programming) and student factors (i.e. content knowledge, behaviors).
5) Have participants sort the different options for all categories in the order of what they would do in their ideal classroom. Ask probing questions-
    a. Why would you do this the most? What do you like about this option? Why?
    b. Why would you do this the least? What do you dislike about this option?

E- Questions for all participants- Factors affecting ID interactions

- What things affect the way you use IDs? Please name everything that you can think of.
- Which of those things are most influential in affecting your use of IDs in the classroom? Why?

For **each category of factors** affecting ID interactions listed below, have the participant sort the factors in order of how much they influence their use of IDs. Then ask:

- Is there anything missing from this list? If so, where would it go in your ordering?
- Is there anything that is included that does not affect your use of IDs?
- Why are these factors the most influential in use of IDs?
- Why are these factors the least influential in use of IDs?

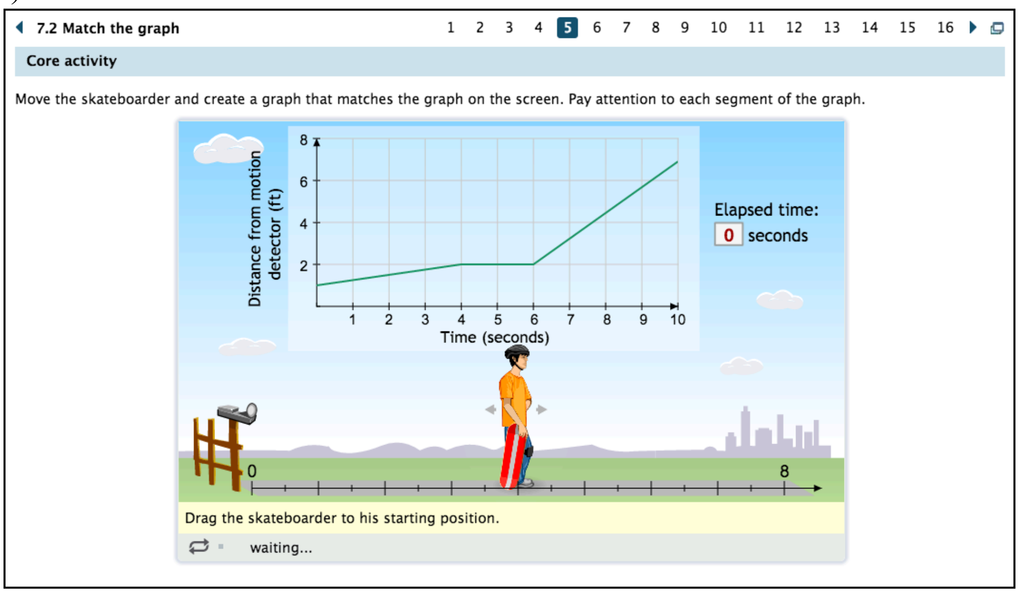
Then, asking participants to consider ALL of the categories, ask the following:

- Overall, what factors are most influential in your use of IDs? Why?
- Overall, what factors are least influential in your use of IDs? Why?

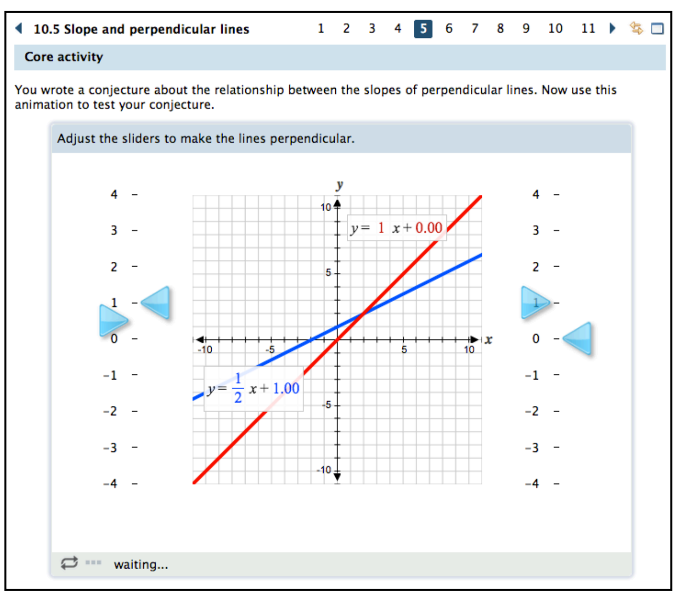
As participants are talking during this section, the interviewer will reference the list of factors found to affect each participant. If/when a participant says something that is inconsistent with these finding the interviewer will ask the participant to clarify.

Part C- Specific IDs

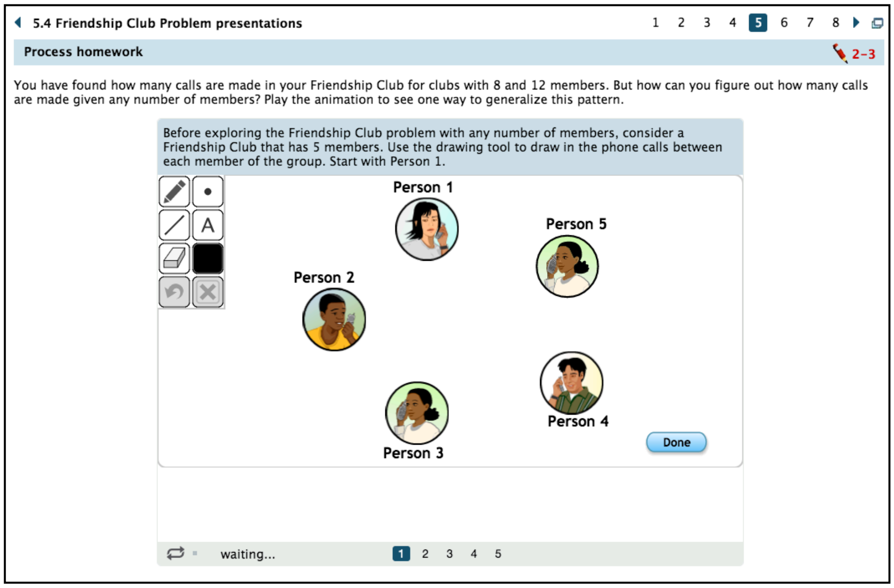
1)



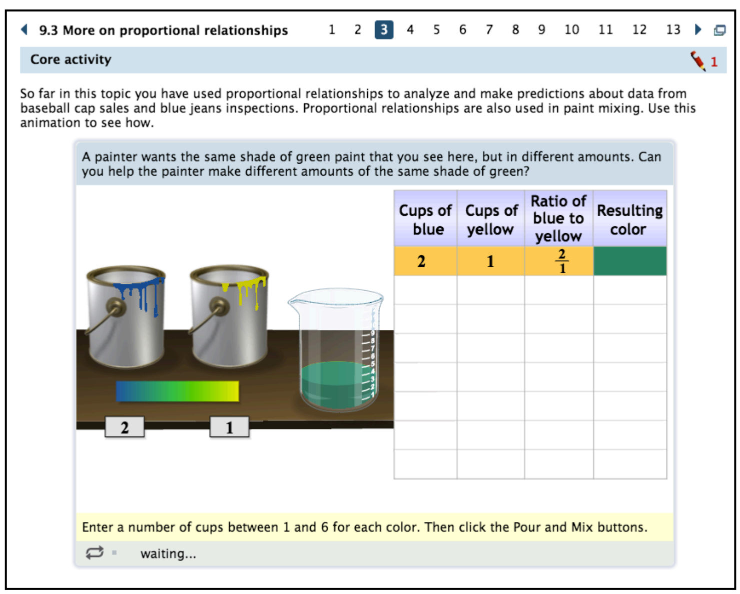
2)



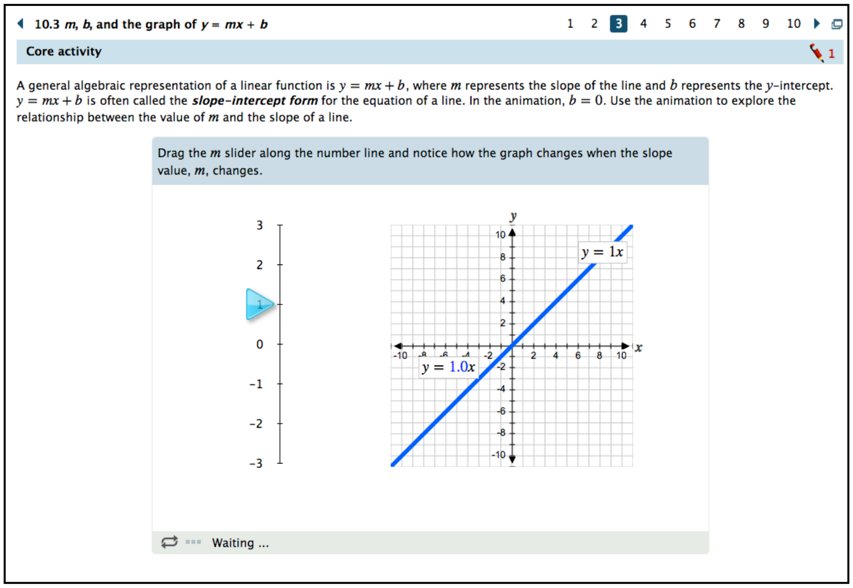
3)


5.4 Friendship Club Problem presentations
1 2 3 4 5 6 7 8
Process homework
2–3
You have found how many calls are made in your Friendship Club for clubs with 8 and 12 members. But how can you figure out how many calls are made given any number of members? Play the animation to see one way to generalize this pattern.
Before exploring the Friendship Club problem with any number of members, consider a Friendship Club that has 5 members. Use the drawing tool to draw in the phone calls between each member of the group. Start with Person 1.
Person 1
Person 5
Person 2
Person 4
Person 3
Done
waiting...
1 2 3 4 5

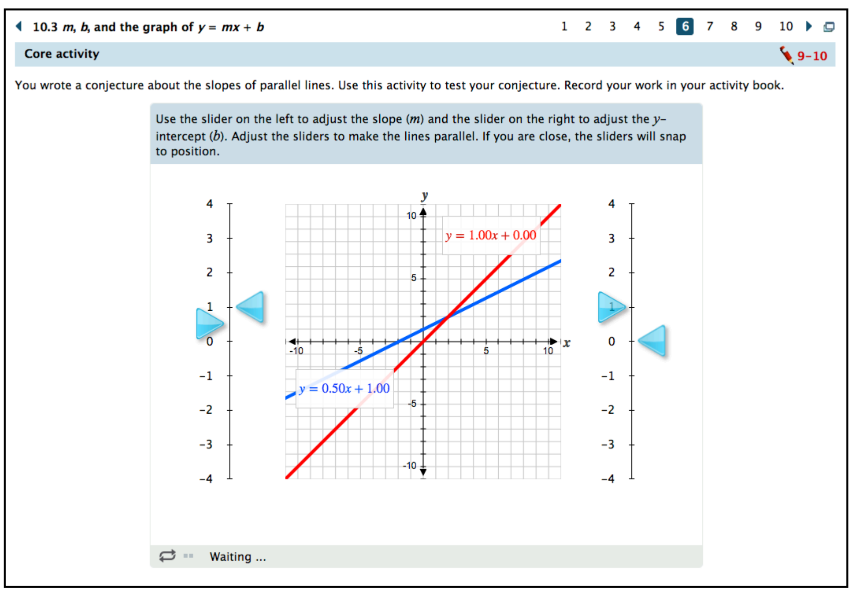
4)


9.3 More on proportional relationships
1 2 3 4 5 6 7 8 9 10 11 12 13
Core activity
1
So far in this topic you have used proportional relationships to analyze and make predictions about data from baseball cap sales and blue jeans inspections. Proportional relationships are also used in paint mixing. Use this animation to see how.
A painter wants the same shade of green paint that you see here, but in different amounts. Can you help the painter make different amounts of the same shade of green?
Cups of blue
Cups of yellow
Ratio of blue to yellow
Resulting color
2
1
2/1
2
1
Enter a number of cups between 1 and 6 for each color. Then click the Pour and Mix buttons.
waiting...

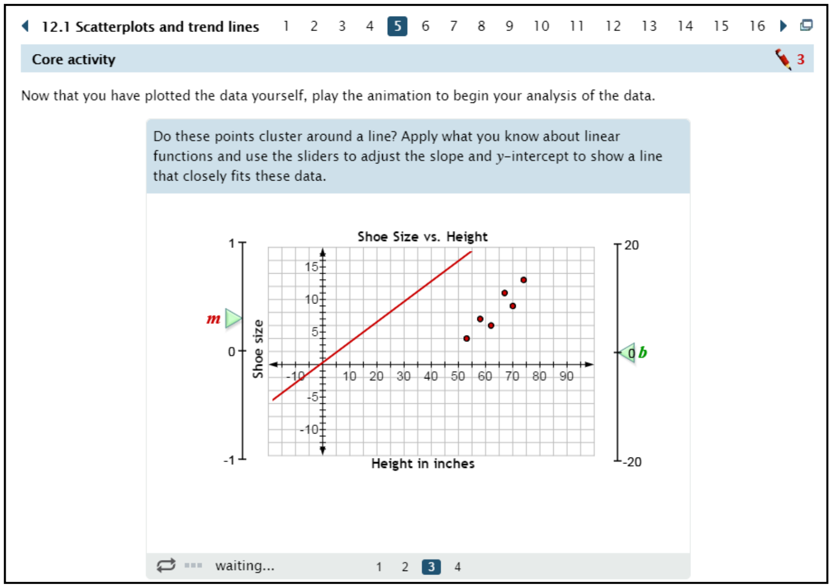
5)


10.3 m, b, and the graph of y = mx + b
1 2 3 4 5 6 7 8 9 10
Core activity
1
A general algebraic representation of a linear function is y = mx + b, where m represents the slope of the line and b represents the y-intercept. y = mx + b is often called the slope-intercept form for the equation of a line. In the animation, b = 0. Use the animation to explore the relationship between the value of m and the slope of a line.
Drag the m slider along the number line and notice how the graph changes when the slope value, m, changes.
y = 1x
y = 1.0x
Waiting ...

6)


10.3 m, b, and the graph of y = mx + b
1 2 3 4 5 6 7 8 9 10
Core activity
9-10
You wrote a conjecture about the slopes of parallel lines. Use this activity to test your conjecture. Record your work in your activity book.
Use the slider on the left to adjust the slope (m) and the slider on the right to adjust the y-intercept (b). Adjust the sliders to make the lines parallel. If you are close, the sliders will snap to position.
y = 1.00x + 0.00
y = 0.50x + 1.00
Waiting ...

7)



(1 and 2 used by NE)
(3 and 4 used by FA)
(5, 6, and 7 used by AC)

Part D- ID models
(cut out pieces)

Activity structure		
Whole group	Small group/pairs	Individuals
Teacher role		
Teach content	Manage student behaviors	Keep students on task
Ask students about what is happening in the ID	Ask students about what they are learning from ID	Assess students' understandings
Student role		
Learn something new from teacher, then use ID	Play with ID	Watch someone else use ID
Learn something new from ID	Talk about what is happening in ID	
Goal of activity		
Make conjectures and test them with ID	Use ID to make conjectures	Use ID to check answer
Learn something new with ID	Review content with ID	Engage students in class by using ID

Part E- Factors affecting ID interactions
(cut out pieces)

Teacher Factors		
Teacher belief- Students should/ should not understand both procedures and conceptual knowledge	Teacher belief- Grade level content can/cannot be learned when students' skill are below grade level	Teacher belief- Learning mathematics should/should not be a active for students
Teacher belief- Students should/should not work in groups while learning mathematics	Teacher belief- Students should/should not be told what they need to know	Teacher belief- The teacher should/should not allow students to get frustrated when learning math
Teacher understands/doesn't deeply understand the content he/she is teaching	Teacher has a lot/a little experience teaching the course	
Contextual Factors		
Availability of technology (laptops, LCD projector, etc)	Functionality of available technology	Presence of substitute teachers in school
Presence of new/novice/ineffective teachers in school	Presence of students in the hallway	School programs interrupt/cancel class periods (ex. mentoring program, assemblies, lunch splitting the class period, etc)
Standardized assessments (HSA, PARCC, etc) interrupt/cancel class periods	School administration feedback/lack of feedback	School administration gives minimal instructional support/direction
Agile Mind is the district-mandated curriculum	Standardized assessment expectations	

Student Factors		
Level of students' math skills/knowledge	Students like/dislike using the computer	Students like/dislike using IDs
Students' behaviors	Students' learning styles (ex. visual learner)	Students' ability to work together
Student interests		
ID Factors		
Uses/doesn't use a real-world context	How much students can "play with it"	Uses/doesn't use visuals
Gives the teacher ideas on how to teach the content	Allows students to investigate mathematics	Uses multiple representations
Makes abstract concepts more concrete	Limitations of ID/ wish it could do something more	

Category of Factors	**Individual Factors**
Teacher Factors	- Belief- Students should/should not understand both procedures and conceptual knowledge - Belief - Grade level content can/cannot be learned when students' skill are below grade level - Belief - Learning mathematics should/should not be a active for students - Belief - Students should/should not work in groups while learning mathematics - Belief - Students should/should not be told what they need to know - Belief - The teacher should/should not allow students to get frustrated when learning math - Teacher understands/doesn't deeply understand the content he/she is teaching - Teacher has a lot/a little experience teaching the course

Contextual Factors	- Availability of technology (laptops, LCD projector, etc)
	- Functionality of available technology
	- Climate and culture of the school
	    o Presence of substitute teachers in school
	    o Presence of new/novice/ineffective teachers in

	school o Presence of students in the hallway - School programs interrupt/cancel class periods (ex. mentoring program, assemblies, lunch splitting the class period, etc) - Standardized assessments (HSA, PARCC, etc) interrupt/cancel class periods - School administration feedback/lack of feedback - School administration gives minimal instructional support/direction - Agile Mind is the district-mandated curriculum - Standardized assessment expectations
Student Factors	- Level of students' math skills/knowledge - Students like/dislike using the computer - Students like/dislike using IDs - Students' behaviors - Students' learning styles (ex. visual learner) - Students' ability to work together - Student interests
ID Factors	- Uses/doesn't use a real-world context - How much students can "play with it" - Uses/doesn't use visuals - Gives the teacher ideas on how to teach the content - Allows students to investigate mathematics - Uses multiple representations - Makes abstract concepts more concrete - Limitations of ID/ wish it could do something more

Appendix G: Coding Scheme

First-Level Codes	**Second-Level Codes**	**Third -Level Codes**
Intended enactment	Activity structure	Whole class work
		Group work
		Individual work
	Student role in activity	Make sense of content/inquiry
		Play with ID
		Presentations/ model for class
		Make mistakes
		Discussions
		Watch ID
		Record in workbooks
	Teacher role in activity	Facilitate student learning
		Ensure students on task
		Assess student understandings
		Questioning
	Goals of activity	
	Supplemental materials	
	Use parallel example	
	Teach content before ID	
	Activity debrief	

First-Level Codes	**Second-Level Codes**	**Third -Level Codes**
Actual enactment	Activity structure	Partner work Individual work Group work Whole class work
	Student role in activity	Make sense of content/inquiry Control ID Presentations Discussions Watch ID Record in workbooks
	Teacher role in activity	Facilitate student learning Ensure students on task Record ID findings Allow student struggle Explaining Questioning Assess student understandings Focus on process/procedure
	Scaffold ID	Use parallel example Teach content before ID
	ID use	Check answer Create conjectures Confirm conjectures Re-engage student
	Introduction to ID Activity Debrief Ran out of time	

First-Level Codes	Second-Level Codes	Third -Level Codes
Contextual factors	School-level factors	Climate and culture
		Logistics/programming
		Instructional time
		Focus of administration
		Technology present and working
	District-level factors	
	Standardized assessment expectations	
	Content Considerations	
ID factors	Interaction with user	Allows investigation/play
		Allows investigation otherwise impossible/undesired
		Presenting information
		Simulation
	Functionality/presentation	Limited functionality
		Drawing function
		Instant feedback
		Multiple representations
		Visual
		Context of ID
		Supports students with deficit skills
	Supports teacher	
	Makes abstract more concrete	
	Already created	
	Connects concepts	

First-Level Codes	Second-Level Codes	Third-Level Codes
	Planning	
	Pedagogy, in general	
	Pedagogical content knowledge	
Teacher factors	Allows student struggle	
	Teacher beliefs	
	Experience with course	
	Teacher frustration	
	Classroom management	
	Student behaviors	
	Student engagement	
	Student knowledge/skills	
Student Factors	Attendance	
	Type of learner	
	ELL	
	Context familiarity	

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