ABSTRACT

Title of dissertation: DECISION ANALYSIS IN CONSTRUCTION CLAIMS

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Claims in construction projects are inevitable and can result in costly litigation. Construction contract ambiguity, overly restrictive terms, and unfairly allocated risks are among the factors increasing the likelihood of conflict between parties in construction claims. The source of conflict is a gap between parties' beliefs over specifics of a claim. This research introduces a settlement negotiation model that provides methods for disagreeing parties to understand the gaps in their beliefs and possibly to come to an agreement before litigation. The quantitative decision analysis approach identifies a range for the optimal settlement amount in the claim process.

Each party holds private information regarding its belief over the specifics of a claim. The specifics of a claim are classified into Liability, the likelihood of the defendant being found liable at a trial, and Damages, unanticipated expenditures plaintiff incurred due to the defendant's alleged fault. A Bayesian Network model quantifies parties' beliefs over Liability and Damages. This model represents parties' legal arguments and their respective strengths and credibility. These beliefs become inputs to a non-cooperative game theory model. Non-cooperative game theory analyzes interactions between the claim parties at each stage of the claim. The asymmetric information game considers each party's actions and strategy based on its belief over the expected outcome from litigation, and its belief over the opponent's expected outcome from litigation. The analysis results in equilibriums that help parties decide how to resolve the claim and avoid costly and timely litigation. The resulting approach reveals predictive outcomes in construction claims using economic theory to analyze construction disputes.

Key Words:

Construction Claims, Game Theory, Settlement Negotiation, Bayesian Networks and the Law, Economics of Law, Non-cooperative Games, Bayesian games, Alternative Dispute Resolution, Decision Analysis, Risk Analysis, Litigation Games, Legal Argumentation, Artificial Intelligence, Bargaining Model, Bayesian Belief Network

DECISION ANALYSIS IN CONSTRUCTION CLAIMS

By

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Dedication

To my father who passed away during preparation of this dissertation.

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Definitions and Notations

Liability: The likelihood of the defendant being found liable at a trial, or the portion of the damages that the defendant will be held liable at the trial.

***Damages*:** unanticipated expenditures plaintiff may have incurred due to the defendant's alleged fault.

Judge: officials appointed to form an opinion or conclusion about cases. Judge includes court officials, jury, board of appeal, arbitrators, mediators, or other types of adjudicator.

Verdict: the monetary value decided by the judge to be transferred from the defendant to the contractor.

Discovery: Discovery is a pre-trial procedure in a lawsuit in which Judge can obtain all pieces of evidence related to the issue both parties.

Claim: means a demand or assertion by one of the Parties seeking, as a matter of right, adjustment or interpretation of the terms of any agreement between the Parties in connection with the Project, for payment of money, extension of time or other relief.

Law: means all laws, statutes, ordinances, building codes, orders, rules and regulations of any Government applicable to the Project.

Design-Build: Design-Build is a project delivery system where the design and construction services are contracted by a single entity to fully design and build a project for a project owner

Services means the services, labor, material and equipment used or incorporated in the design and construction of the Project.

Contract Deliverables means those documents and materials to be prepared by the contract party for delivery to other party under their Agreement.

Preliminary Design Documents means all conceptual design drawings, outline specifications and other documents necessary to comply fully with the Owner requirements, including the design of the size, quality and character of the Project, its architectural, structural, mechanical and electrical systems, and the materials and such other elements of the Project.

Proposal means the complete set of materials the Contractor intends to submit to the Owner, including design drawings, outline specifications, cost estimates, time schedules, models and other graphic and written materials.

Construction Schedule means the schedule for the design and construction of the Project as established by Contractor.

Design Deliverables means the list of the dates for completion of certain selected design services critical to maintaining the Construction Schedule.

Construction Budget means the Contractor's price proposal for Project design and construction.

Construction Documents mean the documents, consisting of drawings and specifications, to be prepared or assembled by Architect/Engineer.

1. Introduction

This dissertation provides a model to predict outcomes in construction claims. It uses economic theory to analyze construction disputes in an attempt to ascertain the outcome prior to settlement or litigation. The model also shows how divergent expectations, or belief gaps, impact settlement negotiations. The questions this research aims to answer are how to define construction disputes in quantifiable elements, how to categorize the elements into measurable variables given complexities and interrelationships, and how the variables determine a each party's decision making process, based on their beliefs over the specifics of the dispute.

This research defines construction claims, and the elements of a claim. Next, it classifies the elements of a claim into variables, or causes, for claim. Next, this research examines the variables of the claim, applying the Bayesian Network to depict the dependencies among variables. The significance of such dependencies in the decision making process is measured by parties beliefs over specifics of a claim. The beliefs become inputs to a game theoretic negotiation model. The game theoretic negotiation model establishes a bargaining tool that identifies the best actions or strategies that each party can take in settlement negotiations, given their information and beliefs at a certain time about a claim.

The model provides a way to identify discrete points of conflict between the parties. It provides a way to ascertain each party's expected outcome, leading to an acceptable range for settlement.

The method proposed in this research is designed to address the following:

- Best action each party should take at each stage of a claim
- Best strategy that a claim party can take to achieve a desirable outcome
- Parameters for selecting the best strategy based on available information
- Incorporation of the strategic parameters into a decision making process
- Impact of a party's belief on their expectation toward litigation outcomes
- Quantitative approach to measure parties belief regarding argument strength
- Optimal amount of settlement offers or demands
- Thresholds of each party in accepting or rejecting settlement offers
- Parties motive to settle or peruse the case to the court
- Impact of attorneys, experts, judges and other parties on forming beliefs

The following are brief discerptions on the content of each chapter:

Chapter 2 - Literature on Construction Conflicts "Analysis of Construction Contracts" discusses the literature review of construction claims. This chapter focuses on construction conflicts, defines claims, and categorizes the causes of claims. It also provides a review of different analytical models used to calculate different aspects of the construction claims, and introduces decision tree models to analyze trial outcomes and Bayesian Networks to predict probability of potential disputes.

Chapter 3 - Literature on Game Theory is a literature review of game theory and Bayesian games used in dispute resolution.

Chapter 4 - Modeling Settlement Negotiation introduces all key variables in context of game theoretic analysis of construction conflicts.

Chapter 5 - Legal Reasoning with Bayesian Networks applies the Bayesian Network to the variables in assessment and modeling the parties' beliefs. It is discussed how Bayesian Network is applied in the game theoretic model of Chapter 4.

Chapter 6 - Case Analysis & Result Discussions analyzes two-real world construction claims and compares parties' beliefs over claim variables.

Chapter 7 - Limitations and Future Research discusses the limitations of this research and future research opportunities with citation to the most recent sources in each field.

Chapter 8 - concludes the dissertation and provides a brief summary of findings.

2. Literature on Construction Conflicts

Construction contracts face uncertainty resulting from imperfect contract terms and a myriad of supplemental documents. In order to mitigate project risk and uncertainty, contract parties needed to collaborate conscientiously. Any flaw in collaboration may create or advance a conflict. Conflicts arising from failure to collaborate may include incomplete or defective plans and specifications, contracts with ambiguity, overly restrictive, or unfairly allocated particularly burdensome risk to one party alone (Rubin, Fairweather, Guy, & Maevis, 1992).

If a conflict is not resolved between the parties, either party may seek clarification and relief from the judicial system in the form of a claim. Claim is defined as a demand asserted by one party on another party relating to services or products specifies in the contract (Barnard, 2005). Construction claims generally are over the four main elements of construction project management: cost, time, quality, and safety. All of these elements eventually boil down to monetary compensation or time relief sought by the contractor.

A claim can be analyzed from different aspects such as engineering, legal issues, relationships, and project constraints. However, regardless of the issue the analysis of the claims are fairly similar. Common elements that become essential in claim considerations are monetary values, liable party for damages, causation and reasoning, and applicable laws and contract terms or conditions.

2.1. Stakeholders in Construction Disputes

The stakeholders for construction claims are not always the same as project stakeholders. In construction claims, there are additional parties that may be involved in the claim processes. In addition to the claimants (i.e. contractors, owners, Architect/Engineers) other stakeholders of a claims includes attorneys, judges or jury, construction experts, and consultants. Each of these stakeholders may have different viewpoints and as a result different interests in the claim. For example, contractor's viewpoint is typically maximizing its interest, and judge's viewpoint is proper application of law to the case. The following provides more detail regarding each stakeholder's viewpoint on a claim.

2.1.1. Claims from contractor's perspective

Contractors in construction projects face a multitude of risks. Inflation, inclement weather, labor problems, material shortages, accidents, and unforeseen conditions are some examples of these risks. Such risks have monetary consequences that may harm contractor's profitability. Contractors tend to be inveterate optimists, believing that the risk is either contractually imposed upon them, or will not occur to them; or even if the risk occurs the contract clause will not be enforced (Rubin et al., 1992). Therefore it is important to consider contractors as risk takers in the calculations. In general, contractors may make a claim about changes to the work, project schedule, or work means and methods.

2.1.2. Claims from owner's perspective

Owners usually bear the risk that the project will not finish on-time, on-budget, or be of expected quality. Less common perils include environmental or regulatory issues or public protests. In general, owners may have concerns about contractor's failure to perform the work as specified in the contract, which includes performance (time), quality, and safety on top of costs. Examples where owners file a claim against contractor include failure to pay subcontractors, completion or repairs of defective work, on time project completion, costs incurred by contractor's suspension or failure to perform the work. In most cases contractors are bonded, therefore any substantial damages owners will seek after bond and surety companies. Therefore, further elaboration of this topic can be contractually analyzed by adjusting contract language. This topic is beyond the scope of this research and needs to be analyzed further by scholars who have contextual interests in contract law.

2.1.3. Claims from Architect/Engineer's perspective

Architect/Engineers (A/Es) usually take the risk of error and omissions in the design. Depending on type of contract and delivery method, if the design does not meet the minimum requirements mentioned in the contract, the A/E is at risk of being sued for design negligence. Contractibility issues or the products malfunctioning post-construction can also be the main reasons for this type of claim. The case study chapter, below, describes how a contractor seeks damages from the A/E due to insufficient designs in the planning phase of the project.

2.1.4. Claims from Attorneys and Expert's Perspective

Attorneys and Experts are typically hired separately by each party to provide support in presenting and defending a case. Experts are typically in charge of finding and stating the facts through a series of reports. Attorneys are in charge of linking those facts to the contract and defining credibility of the arguments. Attorneys and Experts in construction claims typically charge their clients based on an hourly rate fee. One may argue that these stakeholders make more profit by attempting to extend the duration of the claim and avoid settlement. However, due to nature of this business, in the long-term, the Attorneys and Experts that attempt to reach the best outcomes for their client tend to attract more customers and gain more profit by gaining reputation over time.

2.1.5. Claims from Judges Perspective

Judge, jury, board of appeals (or appellate courts), arbitrators, mediators, or any type of adjudicator is the stakeholder who has a slightly different perspective than the others on claims. Judges typically attempt to find the best application of the law or the contract on the merits of the case. The judge's goal is to implement justice between the parties based on the contract and facts presented. This stakeholder perspective introduces a new challenge to the game theoretic approach, which is limited to an economic-based analysis. Section 4.2 provides details regarding how to overcome this challenge.

2.2. Types and Causes for Construction Claim

The construction claim process begins with a dispute between the parties involved in the contract (Construction Industry Institude, 1990). The study conducted by the Construction Industry Institute suggests that each party has limited knowledge and understanding of the claim process. The knowledge includes an interpretation of the facts surrounding the dispute, the contract, and the applicable law. Parties' knowledge on the origin of dispute and type of claims available will affect their decision to pursue a claim.

Therefore, it is important to define and analyze different types of claims from both plaintiff's and defendant's perspective.

Unresolved disputes by either party may climb up the dispute ladder to become a claim. The root cause of the claims are often unclear since there are many parameters that may lead to a claim. The competitive bidding scheme and tight economic positions often force contractors to find profit via post-contract execution negotiations, change orders, and disputes. This method is often referred to as Opportunistic Bidding. Opportunistic Bidding, or bidding at an amount below the contractor's acceptable profit margin in hopes of winning the contract and subsequently increasing the total contract price. A proper categorization of the claims provides assistance to discover the root causes to complicated claims where elements such as Opportunistic Bidding exist.

Barnard (2005) categorized all types of claims based on typical contract terms and provisions. Understanding the types of claims helps parties realize potential disputes and prevent claims by providing adequate documentation or notification, and focuses the parties on the most relevant portions of the contract. The following includes different types of claims based on the contract language:

- Delay
- Directed change
- Constructive change
- Acceleration and constructive acceleration
- Differing site conditions
- Defective and deficient contract documents

- ❖ Owner-furnished items
- ❖ Impossibility of performance
- ❖ Interference with performance
- ❖ Defective inspection
- ❖ Misinterpretation of the contract
- ❖ Superior knowledge
- ❖ Misrepresentation
- ❖ Strikes
- ❖ Weather
- ❖ Suspension
- ❖ Default or nonpayment
- ❖ Termination
- ❖ Warranty

Another categorization is based on root causes. The main root causes for the claims include: risk and uncertainty, collaborative conflict, contract incompleteness, inconsistency, deficiency and defectiveness, relationship factor, and affective conflict. The following provides a comprehensive list of root causes categorized based on the nature of causes:

- ❖ Risk and uncertainty
  - ➢ Inclement weather
  - ➢ Change of government policy
  - ➢ Strike
  - ➢ Fluctuations in material price or in labor cost

- Shortage of materials or labor
- Uncertain ground condition

- Contract incompleteness
  - Ambiguity pf contract
  - The scope of work is unclear
  - The specification is unclear
  - The rules to evaluate the work rate are unclear
  - Measurements are unclear

- Collaborative conflict
  - Contractors employed directly by the client delays in works
  - Nominated subcontractor or supplier delays in works
  - Architect fails to issue instruction within time
  - Engineer fails to provide adequate site investigation details
  - Consultant fails to give information within due time
  - Client requests acceleration unreasonably
  - Client requests change unreasonably

- Inconsistency
  - The quantity of the same items in the contract bills are substantially different
  - Some items are missing from the contract bills
  - The drawings contradict with the specification

- Relationship factor
  - Opportunistic behavior
  - Contractor fails to notify omission of items in the contract bills of quantity
  - Contractor purposely works below the specified standard of care
  - Contractor purposely fails to notify the substantial difference in quantity between contract bills of quantity and actual quantity
  - Client rejects outright extension of time claim submitted by the contractor

- Client rejects outright monetary claim submitted by the contractor
- Contractor excessive costs for progress acceleration
- Contractor purposely fails to disclose the specification of the materials used
- Contractor purposely does not provide invoice for the materials used
- Client orders extra without providing proper cost reimbursement
- Client orders extra without granting justified extension of time

❖ Affective conflict

- Psychological distress such as fear, anger, and guilt project team member(s)
- Emotions such as dominance, assertion, bullying, and forcefulness are displayed
- Intellectually curious, behaviorally flexible, and liberal in their attitudes and values are qualities displayed by project team member(s)
- Hostility, callousness, and cynicism are manifested by project team member(s)
- Excessively neat or overly exact attributes are displayed by team member(s)
- Certain member(s) of the project team are nervous, upset or agitated, irritable or overreacting, impatient, or find it difficult to relax

2.3. Literature on Statistical Analyses over Construction Conflicts

This section provides a literature review of methods for analyzing construction claims.

Aibinu et al. (2011) developed a theoretical model to demonstrate the influence of organizational justice on conflict intensity and contractors' dispute tendencies. They use a structural equation modeling technique with partial least-squares estimation. The main constructs of organizational justice are identified as outcome favorability, decisions outcome fairness, procedural fairness, quality of treatment (the way people are treated), and quality of decision-making process (the way claims are administered).

Barough et al. (2012) applied a game theory approach to develop a decision making framework for conflict in construction projects. They discussed Prisoner's Dilemma and Chicken Game to analyze two specific situations in construction conflicts. The application of game theory in construction conflicts is useful due to the existence of multi-agent decision analysis. The model introduced in their research is a basic format of the game theory, complete information zero-sum games. Parties' information, optimism, uncertainties and litigation fees are disregarded in their model.

Cakmak (2014) used the analytical hierarchy process (AHP) to determine the relative importance of the main causes for construction disputes. He identified 28 main causes of disputes and categorized them based on the responsible party that caused them. The proposed ranking measure helps to conduct a pair-wise comparison. Contract related issues were found to be the most common disputes in the construction industry.

El-adaway (2008) tried various multi-agent simulation and Risk Management models for construction dispute mitigation. He identified change order factors and used them to develop a logical induction algorithm with case-based approaches on the dispute process. The influence of identified factors in the proposed algorithm is simulated by Distributed Artificial Intelligence and Monte Carlo. The simulations resulted in an algorithmic framework to analyze a claim and estimated the mean amount of the settlement based on specific situations.

Ho and Liu (2004) analyzed the relationship between construction claims Opportunistic Bidding. They proposed a game theoretic based model to study people's behavior in various types of claims. The model is based on Subgame-Perfect Nash Equilibrium on sequential offers, where extensive form games are drawn based on all possible scenarios of claims. This study contains advanced application of game theory in construction claims; however, a fundamental assumptions in the study was that the game was modeled by ignoring parameters such as uncertainties and optimistic beliefs by each party. In addition, their research proposes number of possible scenarios that may occur in specific situations rather than providing a generic model. As a result, the model is good for analyzing closed cases and will not provide useful inputs for decision-making before the case is either settled or the court renders an order.

Love et al. (2011) developed a causal diagram with the factors that influence construction disputes. This research analyzed latent conditions inherent within organizational and project related processes, which is referred to as pathogens. Love's method involved analyzing similarities between various social phenomena to determine a casual chain for disputes. The analysis showed a strong association of pathogens with circumstance, practice, and task performance accounted for many disputes. The main contributors to construction disputes were found to be use of traditional lump sum contracting, resistance to altering old policies and procedures, failing to detect errors, and misinterpretation of contract terms and conditions.

Zhang et al. (2015) addressed hidden transaction costs in project dispute resolutions. They designed questionnaires to identify a comprehensive list of claim transaction cost variables and their relative importance in the dispute process. The variables were ranked by a scoring method on questionnaires. Classified factors were also analyzed including reputation, cooperation and trust, emotion, time, and execution of judgments. The results show the most important factors to the contractor are a lack of future cooperation and reputation damage, while project delay is the most critical variable for project owners.

Omoto et al. (2002) analyzed the dispute resolution processes as a two-sided bargaining model with arbitration as an alternative option. This research provided a theoretical analysis of construction claims based on the bargaining model introduced first by Rubenstein (1982). Major limitations of this research are lack of empirical analysis and using a single type of claim procedure (FIDIC).

Yiu and Cheung (2006) used a catastrophe-theory-based analysis on three variables of construction conflict, level of tension, and the amount of behavioral flexibility. Their empirical analysis shows behavioral states can respond dynamically as the magnitude of a conflict increases. In a later paper (T. W. Yiu & Cheung, 2007) they applied Moderated Multiple Regression (MMR) to the mentioned three variable system. Their more recent analysis showed the interactions between behavioral flexibility and the conflict-tension relationship can change radically. Their later model could identify thresholds for flexible individuals that are willing to avoid or resolve construction conflicts.

Yiu et al. (2015) applied a fuzzy fault tree analysis (FFTA) approach to conceptualize the root causes of construction dispute negotiation failure. Inadequate preparation, inappropriate behavior, and contract governance were found to have the highest occurrence likelihood in construction dispute failures.

Jelodar et al. (2015) used a three-stage approach to identify sources of dispute and explore the quality of relationship changes during a dispute event. Causes of conflict are classified into three main categories: project uncertainties, contract and processes, and people behavior. The methodology consists of collecting massive data to assess the construction relationship quality through literature review, review of court cases, and expert interviews. The results show that three factors of contract provision, evidence, and reasoning are essential in success of dispute prevention or handling a claim.

Cheung and Pang (2013) established that claims can be organized into two general categories types: contractual disputes, and speculative disputes. A complementary study (Cheung & Pang, 2014) provided diagnostic approaches to identify construction disputes. The result of this study is a comprehensive list of causes for construction disputes with their respective occurrence likelihood. The ranking of these causes has been determined by designing questionnaires to ask expert opinions on the causes. Results show that construction disputes can be either contractual or speculative. Contract incompleteness and people factors have been identified as main drivers of construction disputes.

2.4. Decision Models for filing claims

One of the most relevant and sound approaches to analyzing claims is by using decision trees. There are numerous research papers that introduce the application of the decision trees to model decision makings in claims. An important decisions in the dispute process is determining whether to pursue a claim at all. For example, Clemen (2014) uses a decision tree model to analyze a famous court case between Taxaco versus Pennzoil. In this study Clement shows how to use the decision tree in a structured problem to find preferred alternative strategies. However, decision trees cannot analyze the interaction between the parties, and may only be valuable for the party who considers the claim from its own point of view.

In decision tree models, the decisions are identified by rectangles, chance nodes are identified by circles, and triangles represent an outcome. Figure 2-1 identifies each party's decisions and chance nodes in parallel to each other. As shown below, each player goes through a sequence of decision and chance nodes. The chance node of one party is equivalent to its opponent's decision node. For example, defendant's decision on whether to offer settlement or litigate impacts Plaintiff's decision on the following step. Therefore, parties have to make decisions in a sequential form and each decision impacts the opponent's actions.

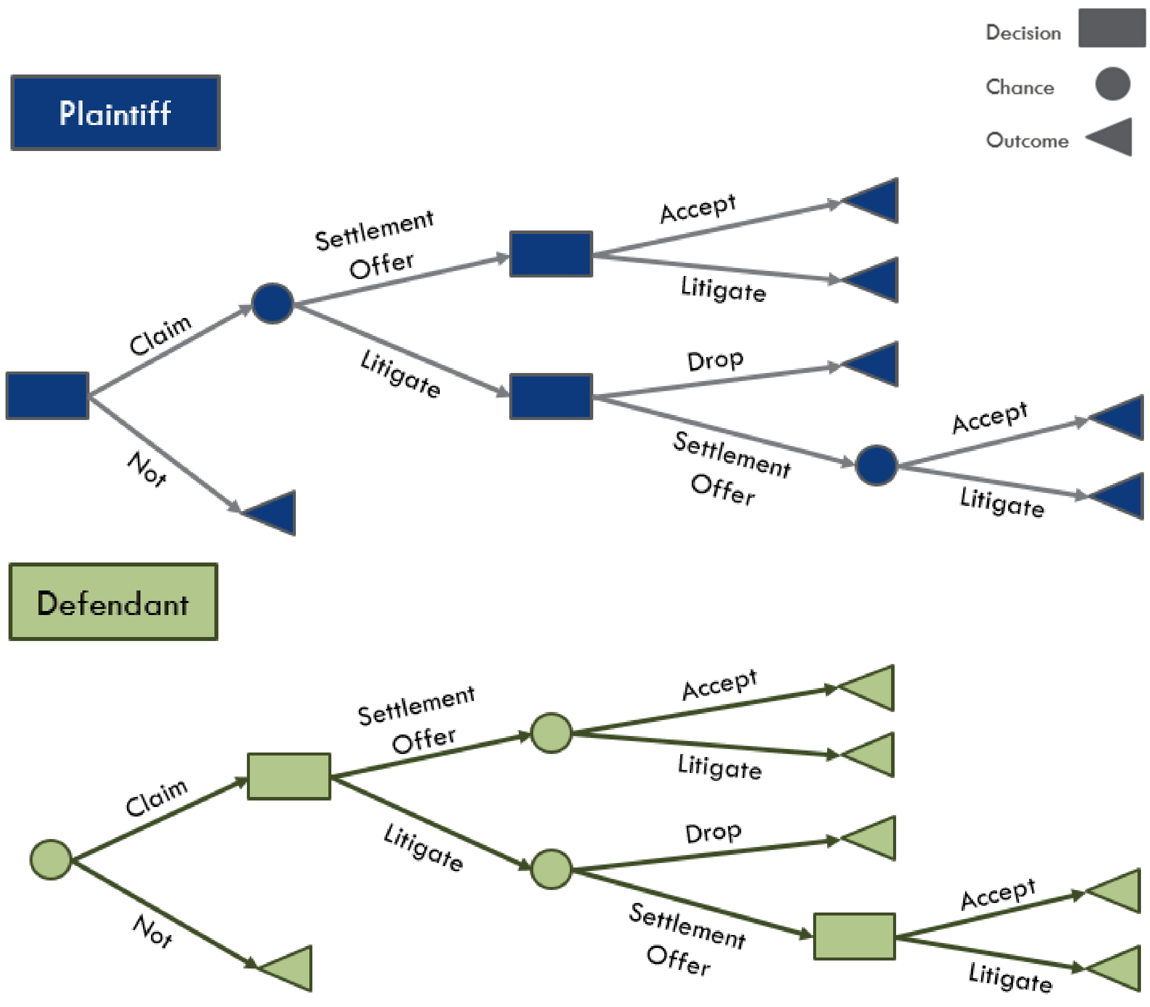

Figure 2-1 Decision Tree Model for Claims

Figure 2-2 shows the merged decision tree for both Plaintiff and Defendant, where decisions for each player impact other player's decision. The right handside column provides both player's payoffs depending on their strategies. This figure provides a representation of the claim process and the extend of the impact for each party's decisions and actions.

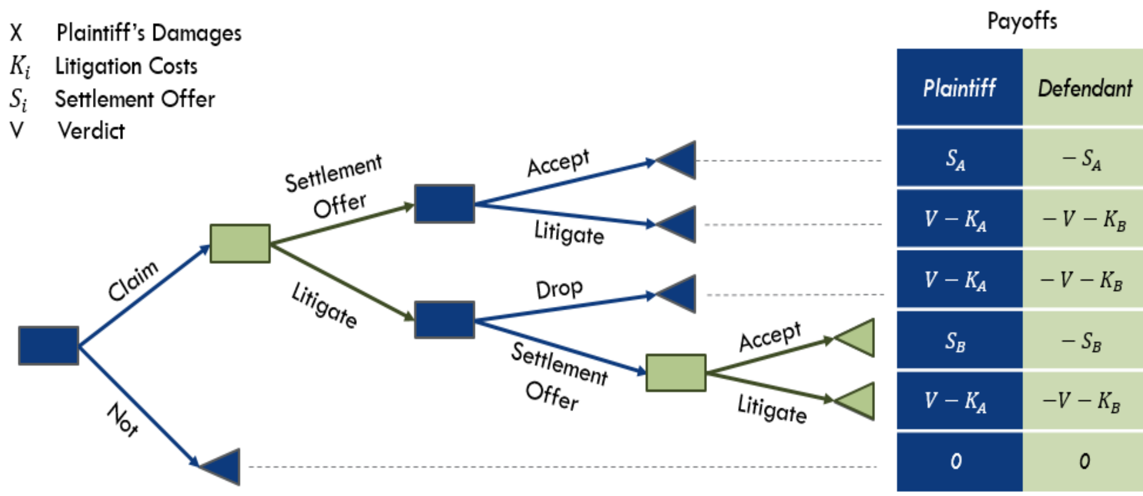

Figure 2-2 Decision Tree Model for both parties' action

2.5. Probability of Facing Disputes or Claims

Multiple organizations provided extensive research on reducing the risk of claims. For example, Project Management Institute dedicates an entire chapter to claim management in its construction extension to the Project Management Book of Knowledge (Project Management Institute, 2007). The topic of claim management is large and covers multiple disciplines such as project management, risk management, psychology, and cognitive sciences. The intent of this research is to limit the deliberations to the analytical studies on construction claim.

Diekmann et al. (1994) introduced a model that quantifies the risk of having claims in construction projects. This model determines the relationship between project characteristics and the likelihood of contract disputes. The calculations result in a measure to anticipate the likelihood of disputes in a construction called Dispute Potential Index (DPI). The DPI is developed based on correlation between project variables and dispute vulnerability. The project variables that are related to the disputes are categorized in three independent groups: People, Process and Project.

In a previous paper Lessani (2016) suggests using Bayesian Networks rather than regression analysis to advance the DPI index. Bayesian Network is a type of statistical model that represents a set of random variables and their conditional dependencies (N. E. Fenton & Neil, 2012). I my previous paper I modeled the main causes of claims introduced by Dikemann using Bayesian Networks to capture the interrelationships between the root causes. This model shows each cause can impact the probability of disputes or claims as a whole. Figure 2-3 shows the three main causes for disputes in a Bayesian Network format.

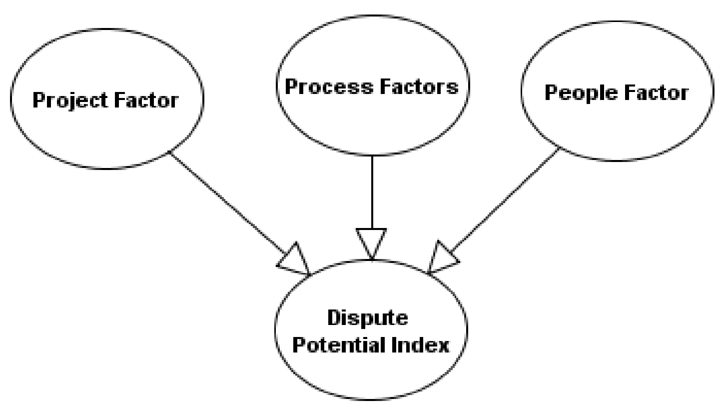

Figure 2-3 Main Causes for Dispute

The following elaborates on each aspect of the potential dispute risk factors:

2.5.1. People Factor

Risk factors involving people may affect organizational relationships, roles and responsibilities, and individual's expectations. The people factor is usually considered to be a main source for claims. For example, claims may not be submitted to maintain a working- relationship in hopes for future projects. Although it is hard to quantify people factors in monetary terms, they are highly probable and can highly influence parties' decision in pursuing the claim. The following causal model suggests dependencies between parameters of people factor.

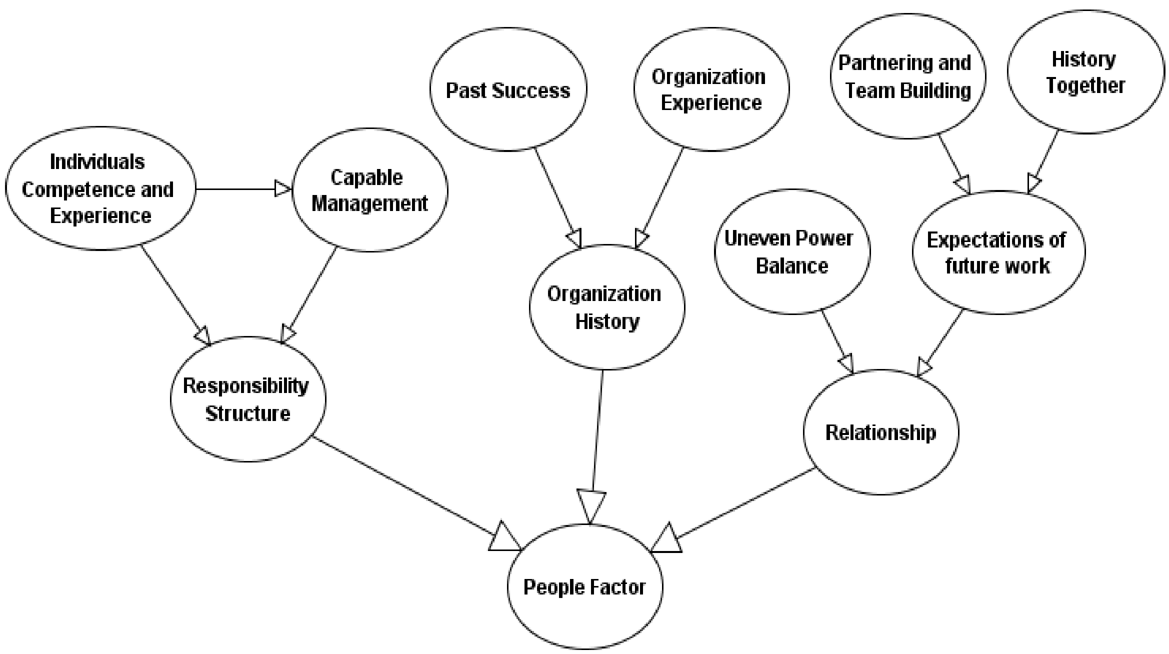

Figure 2-4 Bayesian Network Model for People Factor

2.5.2. Process Factor

Process risk factors include all project management activities throughout the project lifecycle. Typically, process problems lead to clear responsibility of a party in a claim.

Process factors include contractual language, risk allocation, scope definition, communication, and dispute resolution. Figure 2-5 is a causal model for process factor.

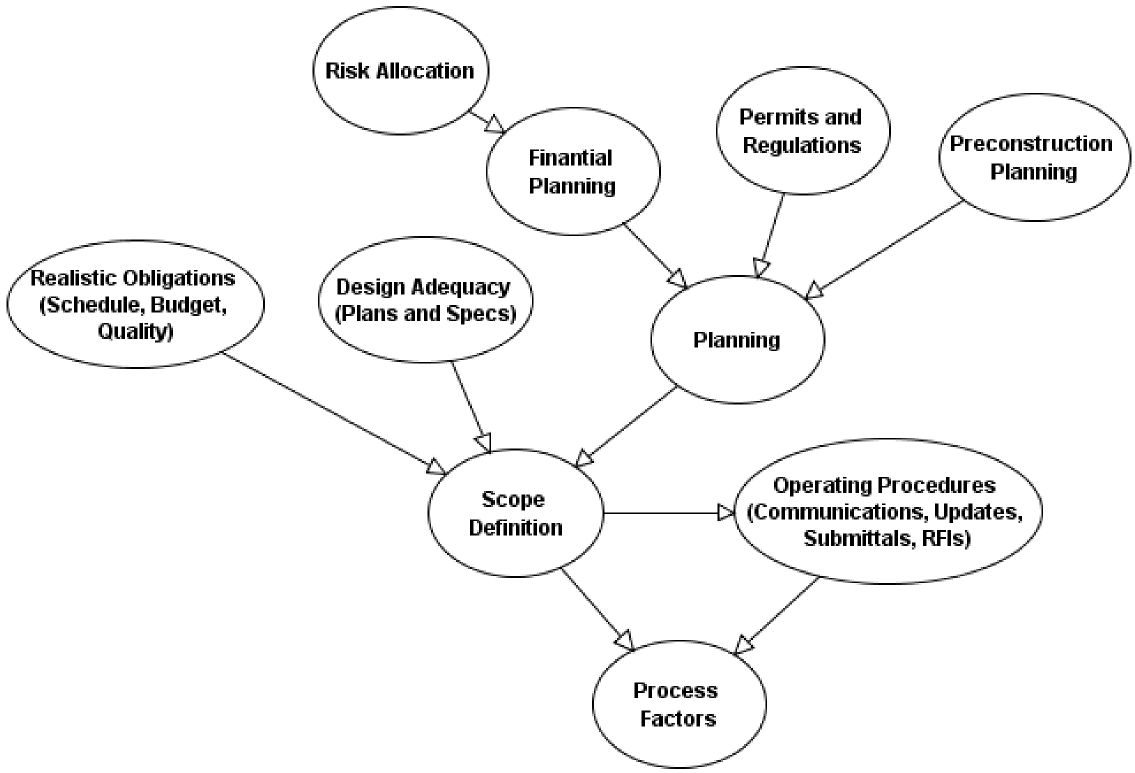

Figure 2-5 Bayesian Network Model for Process Factor

2.5.3. Project Factor

Project risk factors define technical issues with the nature of the project. Project factors are usually associated with monetary values or potential damages for a claim. Since each project has its own unique characteristics, the cause-effect relationships between the nodes may vary from project to project. However, the proposed model is subject to change based on unique characteristics of a project. Figure 2-6 is a causal model for people factor

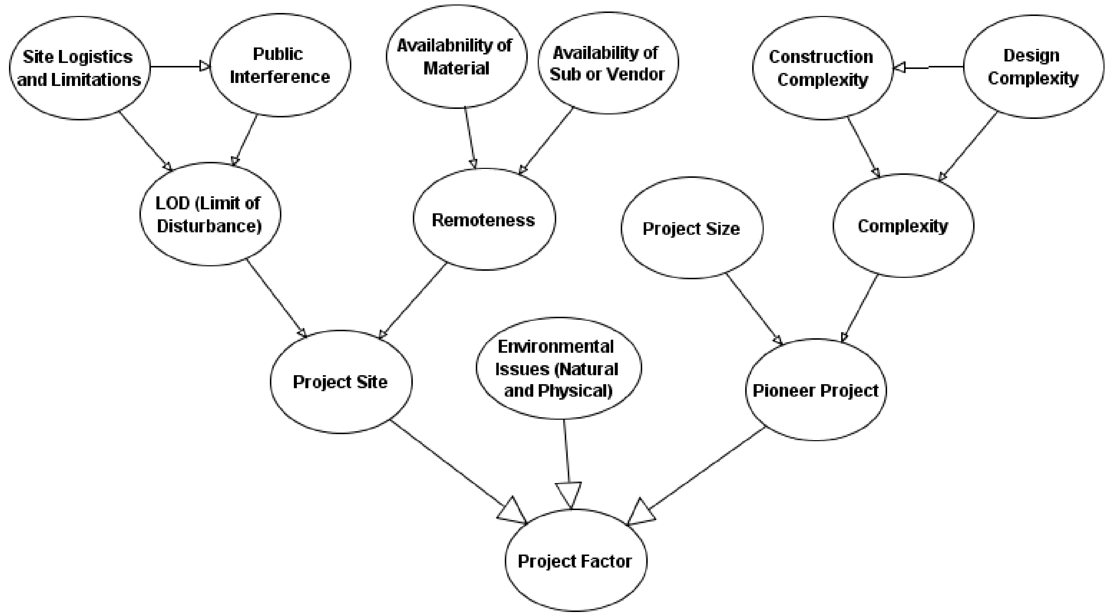

Figure 2-6 Bayesian Network Model for Project Factor

The BN models provided in Figure 2-6 illustrate conditional dependecies between all project risk factors. In order to complete the model, it is necessary to define the probability tables for the nodes, to ultimatley define the mathemathical relationship between the risk factors. There are various methods to define the probability tables. Previous research suggests to gather historical data from projects that resulted in claims. In cases where there is limited historical data, various expert elicitation methods can define the probability tables for the rest of the nodes. Expert elicitation methods are beyond the scope of this research.

3. Literature on Game Theory

This chapter provides literature review on statistical analysis of construction claims, with an overall focus on game theoretic analysis. The material provided in this chapter become a foundation for the models developed in Chapter 4.

3.1. Game theory

Game theory is a mathematical tool used to analyze interactive decision making for multiple agents (also called parties or players). Each agent takes one action within a set of available choices. The decision, or action, that one agent made, potentially influences the other agent's decisions or actions. A set of actions that a player is intended to take in a game forms that player's strategy. Game theory helps to model behaviors of all players and provide suggestions regarding decisions or strategies that lead to best possible outcomes. Modeling the interactivity between players distinguishes this tool from traditional decision theory (Maschler, Solan, & Zamir, 2013).

This research applies game theory models to analyze the problems in construction claims or disputes. Disputes are defined as conflicting interactions between two contract parties, for example between project owners and construction contractors. In construction disputes, players' desires include maximizing their payoff by considering their opponent's strategy. Players may or may not have complete information about all the details of the game, especially in cases of construction claims.

Without complete information, each player has limited knowledge about the parameters of game. This is the reason that construction claims fall into the category of incomplete information games. The lack of complete information results in parties' uncertainties regarding specifics of the claim. Since parties form their beliefs based on available information, the uncertainties result in creation of different beliefs on the specifics of the game. Incomplete information game is the main theme of this research and is discussed throughout this chapter.

3.2. Settlement Negotiation Features

This section introduces the set of features and their associated variables form a basis to analyze settlement negotiation games in construction claims. The following sub-sections introduce the key features of the game used in this research, which includes: players, actions and strategies, outcomes and payoffs, timing, information, and prediction.

3.2.1. Players

The primary players, also known as parties or litigants, include the plaintiff and the defendant. General Contractors, Project Owners, Architect/Engineering firms, or subcontractors are the key construction parties that may participate as either plaintiff or defendant in construction claims. This research considers only two litigants, construction parties, in the game. In more sophisticated models other players such as judge or Jury, attorneys, experts, and other contracting agents (Consultants, Commissioning Agents, etc.) can be considered in calculations.

3.2.2. Actions

An action is player's move out of all of the available options at each stage of the game. The set of actions each player takes during the game is called player's strategy. For example, Contractor's action can be submitting a proposal, filing for claim, or requesting settlement. Project owner's actions could be approving the proposal or offer, rejecting, or responding to counter-proposals. Some models allow for multiple rounds of actions or proposals and some other only consider the final action (take-it-or-leave-it offers).

Depending on the rules of the game, there are limited allowable actions at each decision opportunity of players. Moreover, actions taken at one stage of the game may limit player's future actions. Actions in construction claims usually carry some information to the opponent both explicitly and implicitly. Therefore, players need to carefully evaluate the case before taking any action. Players choose their strategy considering observable actions taken by their opponent, actions taken by the player himself in the past, and the current information player possesses.

3.2.3. Outcomes and Payoffs

The result of all actions played by parties is defined as the outcome. There are broad ranges of outcomes, from the contractor not pursuing the claim and no amount transfer between the parties, or the judge ruling full amount of damages to be transferred from owner to contractor. In general, the outcome is a list of relevant final attributes for each player (Daughety & Reinganum, 2008).

The numerical value of the outcome associated to each party is called the payoff, modeled as either dollar amounts or utility functions. Expected decisions by the judge are different than final payoffs because players incur other costs other than the awarded amount. Additional expenses that each party realizes within the litigation process include court costs, attorney and expert costs, and case preparation overhead. As a result, payoffs include judge's award minus all expenditures each party has associated with the litigation. For the purpose of this research, calculations of expected payoffs are all in dollar values.

3.2.4. Timing

The early settlement models initiated by Nash in 1950, called axiomatic models, were developed based on general theoretical models of bargaining processes. After developments and improvements of these models, a strategic approach was suggested to capture more details of the settlement negotiations, including timing features.

The sequence of play and duration of the claim are the two topics of interest for the timing feature. In the strategic approach timing features, such as sequential versus simultaneous offers, play an essential role in the analysis. In the sequential model, each party may offer and wait for the other player's response. In simultaneous offers, actions from either party cannot be observed by their opponents, or it may not have influence on the opponent's decision for players' strategy.

Duration also can affect the settlement analysis of the claim. Disputes and claims have a finite length of time. Either party may withdraw the claim before the court date, the parties could settle, or the statute of limitations could legally prohibit a claim from being made at all (due to not filing the suit in a timely fashion). As a result, the literature considers multiple phases for pretrial negotiations. This research assumes players have one last opportunity to negotiate in the final stage, after which case proceeds to trial. This assumption requires parties carefully follow the time limits specified in contract clauses and standards or laws applicable in a specific region.

3.2.5. Information

Information is defined as the knowledge that each player has to evaluate and select from the possible actions, strategies, and predictions. Each player's information identifies which player knows what information, and at what stage of the claim. Examples of player information include factual evidence, opponent's beliefs over verdict, or strength of the arguments. Each player information may vary at each stage of the game due to different sources of information, difference in assessment of Damages or Liability, attaining new updates on disputes, or parties may have private information[1] on one or more aspects of a game.

Different informational structures form varying strategic models for settlement bargaining including perfect versus imperfect information games, symmetric and asymmetric information games, and consistent prior versus inconsistent prior information.

In incomplete information games, players do not have full information about their opponents' belief. In these games each player, in addition to its own beliefs over the case, considers its opponents' beliefs. For example, Player A considers Player B's belief before taking any action. Player A also consider player B's belief over Player A's belief on the case, and so on. This concept is known as hierarchy of beliefs in context of incomplete information games.

Various techniques are applied to measure the uncertainty about in the hierarchy of beliefs. The Bayesian approach is recognized as the one of the most widely accepted

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[1] Private information may refer to (1) probability of one side winning the trial, (2) the extend of the injury, or (3) parties' attitude toward risk (Pauwels & Kort, 2009).

statistical decision making approach for modeling this type of games (Maschler et al., 2013). In this method, players have probability distribution over parameters that are unknown to them. Actions taken by each player are based on their beliefs defined in a probability distribution format. Players also have beliefs about each other's probability distribution functions, which update their prior belief on the subject matter. As a result, an infinite number hierarchies of belief will be formed between the players. The challenge of the theory is to incorporate all beliefs into the model.

In the incomplete information game based on players' information regarding the issue, players one of these three stages: Ex-post, Interim, or Ex-ante. Ex-post stage represents the players that know both their own types and their opponent's type. Interim stage represents players that know about their own types, but are not sure about their opponent's type. Ex-ante stage is when players that do not know about anyone's type including their own and is more complicated to model due to high level of uncertainties in player's beliefs. Although there are numerous studies done to analyze ex-ante games, there are disagreements between researchers about the best technique to be used for modeling the games. The following section describes one of the major differences between the two approaches developed in the current literature.

Perfect versus Imperfect information

If players are exactly sure about the outcome (the verdict at trial), the game is called perfect information. In construction claim cases, it is nearly impossible for the players to precisely predict the verdict. Therefore, model introduced in this research is in the imperfect information category.

Symmetric and Asymmetric Information

In imperfect information games, the information may be transferred from one party to another. Timing can also be a source of imperfection. If actions are taken simultaneously, game is analyzed in a symmetric fashion. On the other hand, if actions are taken sequentially, then one player's choice impacts the other player's strategy, and the game needs to be analyzed in an asymmetric fashion. If the information is shared, knowledge between contractor and owner the analysis of the game is symmetric. In cases that each party obtains private information game is considered to be asymmetric. Asymmetric games can be one sided, where only one party has private information, or two sided. Another term used for this concept is complete versus incomplete information games, but this research refrains using such terms to avoid confusion with perfect and imperfect games.

Symmetric information settlement assumes litigants have exactly the same beliefs about the facts and trial outcomes for the case. There is vast literature on analyzing settlements with bargaining games under symmetric information. Dispute cases with symmetric information usually either settle out of court for a positive amount or being dropped by plaintiff to avoid future costs. The assumption of both parties having the same information and belief about the facts and judge's award is too strong and far from reality. This assumption can be relaxed by changing the structure of the game and considering asymmetric information games.

Asymmetric information games provide a greater accuracy in modeling construction claims because they account for the differences in player beliefs, or to be more accurate, players assessment on variables based on private information they possess during the bargaining process. The information that emerges during settlement process, either privately or publicly to parties, may affect their expected payoff from trial. In addition, parties generally have a better understanding of the credibility of the supporting documents and evidence for a case. Furthermore, risk aversion and discount rates of parties, quality and work ethic of the lawyers, are private pieces of information to each player. For example, in construction claims, contractors have private information about the level of damages incurred due to the issues expressed in their claim. On the other hand, project owners tend to know their degree of involvement or level of responsibility in a subject matter (Mitchell & Shavell, 2005).

Consistent versus Inconsistent Prior Information

The Prior Belief is defined as the information that a party has before he learns about specifics of a case. A game has consistent prior information if a player's conditional probability distribution over the other players' information (type) comes from the same overall probability model. This definition requires parties to honestly share their assessment on their opponent's type. In reality, parties' assessment over each other's type are in conflict with rationality. For example, how can both parties think their opponent is highly responsible on a certain subject where responsibility is not evenly distributed between the parties? Some argue that the difference in assessments reflects differences in private information, not differences in parties' views. The differences may also arise from optimistic approach of each party, or in broader terms, irrational behavior by parties.

Most of the current settlement negotiation models use the assumption of consistent prior beliefs. However, this method may not be the actual representation of the claims, but

Many scholars such as Waldfogel believe that the asymmetric information modeling is consistent with cases that settle before the court discovery process (Waldfogel, 1998).

An alternative approach is Divergent Expectation, also known as inconsistent priors, first introduced by Priest and Klein (Priest & Klein, 1984). In Divergent Expectation models parties engage in negotiations while having optimistic assessments over the outcomes. Divergent expectation assumes parties PDF on opponent's private information are not a shared knowledge. However, the information received during the negotiation process is identical to both parties.

The Inconsistent prior approach has been used in a number of empirical studies especially in medical claim cases (Yildiz, 2003) (Watanabe, 2006). In medical claims both parties may receive identical information (i.e. test report) during the claim and update their belief. Divergent expectation describes the issue to be more about how prior belief arise, not the asymmetric information.

3.2.6. Prediction

The main purpose of settlement models is to make a prediction about the outcome of bargaining. In recent literature, the notion of equilibrium has been used for predictions. The two main categories of equilibrium applied to settlement bargaining predictions are cooperative and non-Cooperative games. In cooperative games, players bind themselves to ensure the game results in an efficient solution (there is no money wasted in the process). Non-Cooperative games do not assume any contractual agreement over efficiency[2].

Most of the simplified models, and earlier works in the literature, use the concept of cooperative game theory, where the solution to the game is efficient (no money is wasted in the process). Nash Bargaining Solution (NBS) is an example of axiomatic solution that applies to cooperative games.

Claims and disputes, on the other hand, are non-cooperative conflicts between the parties. In the strategic format of non-cooperative games, players predict the payoffs conditional to the opponent's belief. When there is uncertainty about the information, as in incomplete information games, each player considers their opponents' knowledge in addition to their own knowledge on the parameters of the game. Players may also consider their opponent's knowledge about their own knowledge, and so on. This concept is defined as hierarchies of beliefs in context of incomplete information games.

In this context, the Bayesian approach has been recognized as the most widely acceptable statistical decision making approach for games with incomplete information (Maschler et al., 2013). In this method, players have a probability distribution over parameters that are unknown to them. Actions taken by each player are based on their

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[2] ***Efficiency:*** Bargaining methods are inefficient if litigants are asymmetrically informed. One may think of Nash bargaining model as an alternative to such analysis, but due to the incomplete information these games are not efficient. As a result, lack of efficiency of the incomplete games NBS (Nash Bargaining Solution) or other cooperative solutions will not address the asymmetric aspect of such games (Daughety & Reinganum, 2008).

beliefs defined in the distributions. Players also have prior beliefs about each other's probability distributions. As they receive new information from their opponents, they revise their assessment to form posterior probability distributions on variables. As a result, an infinite number of hierarchies of beliefs form between players. The challenge of the theory is to incorporate the hierarchies of beliefs into a model.

3.3. Generic Game Theory Model for Claims

Models with imperfect information involve parameters of the problem associated with probability distributions. If the probability distributions are common knowledge between the parties, then the information of the players is symmetric. Construction claims typically fall under the category of asymmetric information, since each party has its own knowledge about the matter. In such asymmetric information games, parties have different probability assessments over relevant uncertain aspects of the game. For example, imagine a plaintiff who incurred damages (X) and files a claim against the defendant. Plaintiff may know the true amount of X, but defendant can only estimate damages within a certain interval [X_L, X_H].

The private information of a party in game theory is referred to as the player's type. Different type comes from asymmetric information that results in different estimates for each player. Once this difference exists, each player predicts what their opponent will do based on their type (their available information). They may also analyze the situation from their opponent's viewpoint on their own viewpoint. Parties may transfer their information to each other by different means of communication including the claim itself. The information transfer may happen strategically to manipulate the opponent's belief on key variables of the game. The following section includes questions that show what parameters and features play a crucial role in claims.

3.4. Game Theoretic Approach to Settlement Negotiations

Game theory can be used to analyze interaction between two contract parties in pretrial settlement bargaining, where the goal for each agent is to maximize its own payoff given available information. The following are key questions that show how a game can evolve for conflict negotiations:

- Which player obtains private information about which aspect of the game?
- Parties are risk neutral or risk averse?
- How litigation expenses are shared between the two parties?
- Lawyers fee are fixed or contingent?
- Which player proposes a settlement?
- Why some lawsuits resolved out of court and some go to trial?
- What is the confidence level of judge or jury to award one party in the trial?
- Who pays the legal expenses?
- How to restrict the options for parties to hold lawsuits against each other?

Samuelson et al. (2014) summarizes out-of-court negotiated settlements in a chapter of his book called A Game-Theoretic Approach to Legal Settlements. This chapter introduces Bayesian game theoretic approach on one-sided asymmetric information games with brief examples. The two-sided information games are analyzed by direct revelation game (DRG) in which each side reports its private information truthfully to determine equilibrium outcomes. In this type of analysis there is a payoff-equivalent revelation mechanism that has an equilibrium when the players truthfully report their types. This type of analysis does not exactly resemble the actual disputes because the assumption of having such rational players is strong.

There has been tremendous progress on litigation decision models, where theoretical models are developed based on pure economic outcomes. Although the economic outcomes are considered to be a key driver to the disputes, there are other parameters that might play a crucial role in litigation decision making.

Fenn and Rickman (2013) conducted an empirical analysis on medical claims to determine the relationship between the duration of negotiations and information about case strength. They analyzed data gathered from a group of English hospitals including resolution methods and timing for disputes, evolution of expert assessments of case strength, and the timing of external expert's opinion that affected litigation outcomes. This research defines that of five stages, defendant's liability is at two stages, the initial and the final liability estimates. As time elapses and more information is revealed to each party, parties' decisions may change. Conditional probabilities of different types of claim resolutions are estimated by the cause-specific regression method. There are two major findings in this research. First, over time, the assessed strength of the case diminishes, which increases the probability of case dropping or settling rather than being litigated. Second, the cases that have relatively little uncertainly about liability tend to be resolved over cases with unclear liability.

Sullivan (2011) analyzed settlement delays in asymmetric information over the expected trial verdict. He conducted an empirical analysis of the data and observed that the asymmetric information on the expected payoff from trial may cause up to 95% delay, comparing to symmetric information situations. He also observed that policy changes to mitigate the settlement delays are not strongly effective in reducing the bargaining costs and delays.

3.5. Litigation Decisions

The decision to litigate depends on many parameters that a plaintiff may consider before taking any action. One of the most important decisions involves the private cost and benefit from pursuing the case. For example, a rational party will pursue a case if his expected gross return exceeds the expected costs of litigation. The gross return could be the amount either judge verdicts in case of trial, or parties agreed on in case of settlement or Alternative Dispute Resolution (ADR) processes. The expected costs of the dispute includes, but not limited to, attorney fees, plaintiff's personal costs of effort, opportunity costs, business reputation and future relationships between parties. Considering all these factors, the plaintiff will decide to pursue the litigation only if:

$$\text{Expected gross return} \geq \text{Costs incurred from litigation process}$$

Plaintiffs consider the time and effort to invest in the lawsuit before making their decisions. Aside from business decisions, they also consider their beliefs about underlying facts of the case, contract language, and defendant's possible reaction to their dispute. These factors determine how eager each party is to either pursue the claim or to settle before trial. If the plaintiff wins the trial, the expected payoffs for the parties will be as follow:

- **Plaintiff:** expected judgment at trial – Plaintiff's Litigation Costs *(gain)*
- **Defendant**: expected judgment at trial + Defendant's Litigation Costs *(loss)*

It is assumed that each party pays its own litigation costs regardless of the trial outcome. Litigation costs paid by both parties for the trial process is known as "deadweight loss". This cost can be avoided if parties can agree to settle before the trial. Deadweight loss is not always an element to convince parties to settle. Factors such as the amount of damages, length of time for settlement, strategic environment of claims, and information and beliefs of the two litigants are the main constraints that may affect party's decision.

The settlement analysis consists of a model with multiple rounds of offers between litigants. In each round of the repeated game, the litigants alternate between making settlements offers or litigate. Either party can stop the loop by selecting litigation option, which ends the game with a trial decision and its associated costs. This bargaining process is known as ultimatum game. The ultimate game can be solved with backward induction method. In this model, factors to be considered can be timing of the settlement offers, allocation of the bargaining surplus, and the first/last party who makes the offer can change the dynamic of the equilibrium (Mitchell & Shavell, 2005).

3.6. Extensive-Form Game

The extensive-form game is a graphical tool used to describe the games in context of game theory, allowing explicit representation of the sequencing of players' possible actions. These actions include players' choices at every decision point, and the information each player has about the other player's moves at the decision point, and the player's payoffs for all possible game outcomes.

Figure 3-1 is the extensive form game for a hypothetical construction claim brought by contractors. Whenever and the dispute resolution techniques are not resolving the issues from contractor's point of view, they may make a decision between submitting a claim, or not submitting it (compromising). Factors that may be considered to submit a claim is beliefs about likelihood to win the case, litigation costs, future relationship, bargaining opportunities, and the amount of the disagreement. In response to contractor's claim, the defendant has an opportunity to negotiate (or bargain) the amount of the claim with the contractor, otherwise the case will be litigated and judge or a third party will make the final decision.

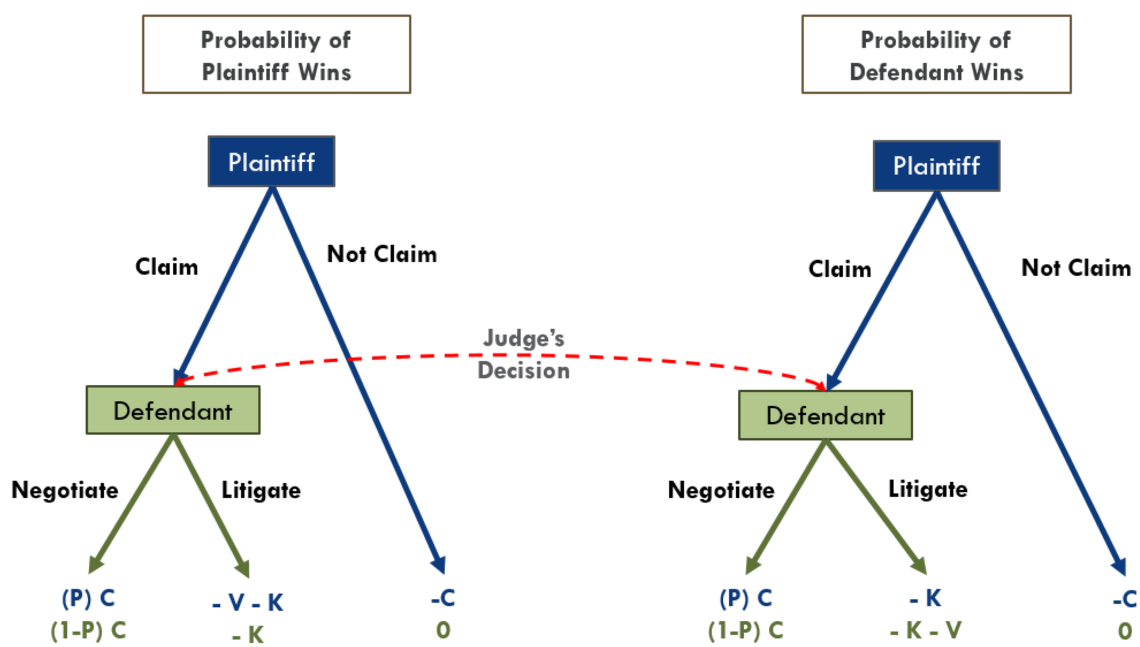

Figure 3-1 extensive form game for construction claim

The following are the notations for the letters used in the extensive-form game:

K: Litigation Costs

V: Verdict

P: Portion of the Bargained amount during Settlement

Extensive-form games can also demonstrate games with asymmetric information. There are two major models developed by Bebchuk (1984) and Reinganum and Wilde (1986) for one-sided asymmetric information games. In addition, P'ng (1983) and Nalebuff (1987) are among the other researchers that contributed to the foundation of this type of game theoretic analysis. In these models, one player makes a single take-it-or-leave-it settlement offer before trial. Most of the current analyses are attempts for generalization of one of these two models.

4. Modeling Settlement Negotiation

This Chapter provides methods to analyze construction claims from an economic standpoint. The Chapter contains six settlement negotiation models for various situations based on which party holds private information about the case, and which party provides the settlement offer. The 6 models are categorized as follows:

- Screening Games - One-sided asymmetric information
    - Defendant proposes a settlement offer
    - Plaintiff proposes a settlement offer
- Signaling Games - One-sided asymmetric information
    - Defendant proposes a settlement offer
    - Plaintiff proposes a settlement offer
- Two-sided asymmetric information games
    - Defendant proposes a settlement offer
    - Plaintiff proposes a settlement offer

4.1. Modeling Claim Procedures

The models introduced in this chapter consider two players (A and B) involved in a construction claim process. Player A represents the Plaintiff, and player B represents the Defendant. Depending on type of the game, parties (players A and B) have private information specific to the claim. Plaintiff is typically privately informed about the level of damages incurred (*Damages*), and the Defendant is typically privately informed about the level of Liability for the Damages incurred by the plaintiff (*Liability*). Damages are defined as additional expenditures plaintiff may have incurred due to the Defendant's alleged fault. Liability is defined as either the likelihood of the Defendant being found liable at the trial, or the portion of the damages that the Defendant will be held liable for at the trial. Since both parties have private information about different elements of the case, the analysis is called a two-sided asymmetric information game.

Parties act in sequence. The process typically starts with the Plaintiff analyzing the situation using a decision tree model to submit a claim after failing to settle in pre-claim negotiations with the Defendant. At this point, one of the parties may offer or demand a final settlement. The party who asks for settlement may update other party's information. Parties strategically offer or demand settlement amounts to increase the likelihood of being accepted. Rejection of the offer results in a trial and the final amount to be transferred from the Defendant to the Plaintiff will be determined by the Judge (the term Judge used for court, jury, board of appeals, arbitrators, or mediators). The parameters determined by the Judge are named *True Damages* and *True Liability*. The parameters that are known to one party or either party are called *Actual Damages* and *Actual Liability*. This model is suitable for any type of civil litigations.

The analysis of the game is provided in stages and sub-stages. Stages show what actions taken by who at what stage of the game. Once each action is taken, the game moves to its following stage. Within each stage, parties may consider strategies within the sub-stages. In the sub-stages players update their beliefs based on the provided information or analyzing their opponent's hypothetical actions or reactions.

4.2. Assumptions

Development of this model requires multiple assumptions. The following are common assumptions for all models throughout this Chapter. Additional assumptions for individual models are discussed if applicable. Chapter 7, Limitations and Future Research, provides details on relaxing these assumptions.

4.2.1. Discovery Process

It is assumed the Judge determines the true values for Damages and Liability. The true values for Damages and Liability are assumed to be fully discoverable to the Judge during discovery. Typically, if there is insufficient information available for a case, parties seek the assistance of the court by filing a motion to compel discovery, or a court order sent to the non-complying party to produce the documentation or information requested in a proper and sufficient manner.

During discovery, parties are obligated to reveal all of their information to the Judge. Therefore, it is assumed that the Judge has complete information about the case to determine the true values of Damages and Liability. This assumption is fairly realistic for civil cases because the Judge can hold his or her decision against the parties who do not reveal their private information.

4.2.2. Determination of Judge's Verdict

The Judge delivers the verdict based on the parties' Liability and Damages. The Liability and Damages are typically determined during litigation. Claimants predict the verdict "V" by estimating the probability of being held liable at trial, and the damages that the Judge determines at trial. Therefore, the expected verdict is a product of Damages and Liability denoted, x_i and y_i. The following is the equation for expected value of the verdict:

$$E(V) = x_i \times y_i$$

Chapter 7 provides further details on various methods used to predict Judge's verdict.

4.2.3. Prior Beliefs over Damages and Liability

Each party forms a belief about the opposing party's private information. The initial assessment of each party about its opponent's private information is defined as the prior probability distribution over the variables. During pre-claim negotiations, parties exchange their prior beliefs before the official claim is submitted. Prior beliefs are assumed to be shared knowledge between both parties. This concept is often referred as C*ommon Prior* in literature. Depending on shared knowledge of the parties, plaintiff (Player A) is assumed to know the probability that the defendant assigns to the Damages before submitting a complaint, or defendant (Player B) is assumed to know the probability that the plaintiff assigns to the Liability before the claim is filed.

In construction claims, plaintiff and defendant are typically involved in pre-claim negotiations, such as change order discussions, or a dispute process. During these processes, the parties express their beliefs through letters, email, meetings, or other types of communications in hope of reaching a resolution. Although the pre-claim negotiations may not fully reveal all private information from one party to another, they form a basis for parties to calculate the opponent's prior belief based on their available information.

4.2.4. Parties' Beliefs Updates

Each party forms a belief about the opposing party's private information during pre-claim negotiations. The information transferred between the parties in this process updates their beliefs on other party's private information. It is assumed that the parties update their Probability Distribution Function after receiving new information from the opponent according to Bayes Rule. The following notation has been selected for prior, first, and second updates.

Table 4-1 Parties Beliefs and Updating Notations

Players	**A**		**B**	
Prior Beliefs	$A_Y^o(y)$	A's prior PDF	$B_X^o(x)$	B's prior PDF
Prior Beliefs	$A_Y'(y)$	A's first updated PDF	$B_X'(x)$	B's first updated PDF over x
Posterior Beliefs	$A_Y''(y)$	A's second updated PDF	$B_X''(x)$	B's second updated PDF

4.2.5. Litigation Costs

Litigation costs are any expenses each party incurs to pursue a claim. Litigation costs include, but are not limited to, attorney fees, expert fees, courts costs, filing fees, parties' office overhead, and other costs parties incurred during the claim process. These costs may vary based on the contract, magnitude of the claim, duration of the claim, and complexity of the issues. Litigation costs may or may not be known to either party. This research assigns functions for litigation costs. Costs for players A and B are denoted by k_A and k_B. Total litigation costs are:

$$K = k_A + k_B$$

The one-sided private information models introduced later in this Chapter assume the value of the litigation costs are known to both players. The two-sided asymmetric information model assumes litigation costs as a function.

4.2.6. Allocating Trial Cost

This research employs the *American Rule* for allocation of litigation costs. The *American Rule* requires that each party to pay its own litigation costs regardless of the outcome from the trial. In contrast, the *British Rule* obligates the losing party to pay all litigation costs including the prevailing party's attorney and expert fees. Depending on the contract agreement between the two parties, either rule may apply. The litigation costs play an important rule while parties are making decisions whether or not to pursue the case to the trial. Section 7.2 discusses researchers who advanced the application of *British Rule* in settlement negotiations.

4.2.7. Final Settlement Offer/Demand

All settlement offers are considered to be final offers. This is also known as take-it-or-leave-it offer. This concept does not allow parties to argue back and forth in unlimited sequences.

4.2.8. Risk Preference

Parties are assumed to be risk neutral meaning that party's decisions are not impacted by their degree of uncertainty in a set of outcomes. A risk neutral party is indifferent between choices with equal expected payoffs, even if one choice is riskier. This assumption can be easily relaxed by substituting utility for money in the mathematical analysis.

4.2.9. Consistent versus Inconsistent Prior Information

The models provided in this research assume parties with consistent priors over certain elements of the claim, and inconsistent priors over the private information. Parties' private information may cause divergent expectations between the parties. It is assumed that parties may not fully communicate the prior beliefs to each other through actions. The level of revealing information impacts parties' beliefs update over Liability or Damages. As the claim proceeds, parties are allowed to update their own beliefs without communicating their private information to their opponent. This process appears to be consistent with civil litigation processes especially construction claim negotiations.

4.3. One-Sided Private Information - Screening Models

One-sided private information games are games where only one player has a private information about the factual elements of the case. The player who holds the private information is called the *Informed Player*, and the opponent player without that specific private information is called the *Uninformed Player*. Both players form beliefs based on their prior assessments and all observable information including their opponent's information. Parties make decisions on taking actions based on their beliefs at each stage of the game. The sequence of actions either player takes updates the belief of the opponent party.

This section introduces the *Screening Model (Sorting Model),* where the *uninformed player* makes the first settlement offer and the *informed player* choses to respond. The reverse version is called *Signaling model,* where the informed player makes the first settlement offer, which will be discussed in Section 4.4. Private information of one party about the factual issues of the dispute result in a different assessment of the trial's expected outcome. Therefore, parties' information about the outcome of the trial is considered to be asymmetric, or one-sided.

Bebchuk's model (1984) on pretrial negotiation is believed to be a major foundation of screening models in pre-trial negotiations. The one-sided private information model introduced in this Section is essentially an advanced extension of his model. In this model, none of the parties know the true value for *Liability* or *Damages*; however, the party with the private information can estimate that value with more confidence than the other party.

4.3.1. Defendant's Decision on making a Settlement Offer - Screening Model

In this model the plaintiff holds private information about Damages and the defendant screens plaintiff's type by offering a settlement. Plaintiff (Player A) is assumed to have private information on Damages X. The private information is denoted as the player's type such that a plaintiff with Damages x is called Plaintiff type x. Plaintiff forms a Probability Distribution Function (*PDF)* over x (its own type) denoted $a_X(x)$. Defendant (Player B) does not have the private information about Damages, but it estimates Damages to be within $[x_L, x_H]$ interval. Defendant's prior *PDF* over Damages (A's type) is denoted $b_X^o(x)$. It is assumed that both parties agree on the true value of *Liability*. The assumptions on parties' information are summarized as follows:

Table 4-2 Player's Information

A (Plaintiff)		**B (Defendant)**	
$\boldsymbol{a_X(x)}$	*A*'s PDF on Damages		
y	*B*'s Liability	y	*B*'s Liability
$\boldsymbol{b_X^o(x)}$	*B*'s prior PDF over x	$b_X^o(x)$	*B*'s prior PDF over x
$\boldsymbol{b'_X(x)}$	*B's* updated PDF over x	$b'_X(x)$	*B*'s updated PDF over x
$\boldsymbol{(x_L, x_H)}$	*A*'s limits of Damages	(x_L, x_H)	*B*'s limits of Damages
$\boldsymbol{k_A}$, $\boldsymbol{k_B}$	Litigation Costs	k_A , k_B	Litigation Costs

The following provides details for the main stages of the game:

Stage 0 – Formation of Prior Beliefs

Stage 1 – Information Exchange between the parties

Stage 2 – Defendant's Decision Analysis

*Table 4-3 Screening Model for Defendant Screens with Settlement Offer**

stage	Plaintiff (Player A)	Defendant (Player B)
0	A forms a PDF $a_X(x)$	B forms a prior PDF $b_X^o(x)$
1.1	Submit a claim C_A	
1.2		B updates its PDF $b_X'(x)$
2.1		B's Decision Analysis $\begin{cases} Pursue\ Litigation \\ \text{Offer Settlement } S_B \end{cases}$
2.2	A Decides based on its interim payoff, u_A $\begin{cases} \text{If } S_B \geq u_A \Rightarrow \text{A would accept} \\ \text{If } S_B < u_A \Rightarrow \text{A would reject} \end{cases}$	⇐ B's belief over A's belief
2.3		B estimates A's interim outcome, $(\hat{u}_A)$ $\hat{u}_A\ (b_X'(x), \text{y}, k_A) = E[b_X'(x)]\ \text{y} - k_A$
2.4		B thinks A would only accept S_B if: $S_B \geq E[b_X'(x)]\ \text{y} - k_A$ $E[b_X'(x)] \leq \frac{S_B + k_A}{y}$ A's critical type: $x^c = \frac{S_B + k_A}{y}$
2.5	A's Strategy $\begin{cases} If\ E[a_X(x)] \leq x^c \Rightarrow A\ accepts \\ If\ E[a_X(x)] > x^c \Rightarrow A\ rejects \end{cases}$	⇐ B's belief over A's belief

*Blue with the white text represents the plaintiff
*Green with the black text represents the defendant

Stage 0 Players A and B form their prior beliefs as follows:

i. Plaintiff forms a belief over Damages, $a_X(x)$

ii. Defendant forms a prior belief over Damages, $b_X^o(x)$

Stage 1.1 The plaintiff submit a claim C_A to the defendant. It is assumed that the plaintiff files a claim only when the minimum awarded in the verdict exceeds the Litigation Costs.

$$x_L\ y - k_A > 0$$

Stage 1.2 The Defendant updates its beliefs over Damages given Plaintiff's claim C_A; denoted by $b'_X(x)$.

Stage 2.1 The Defendant makes a decision based on the following options:

i. Reject the claim C_A and pursue litigation.

If the Defendant rejects Plaintiff's claim, the case will be automatically pursued in litigation. In the litigation process the Judge will define the True Damages, damages to be transferred from the Defendant to the Plaintiff.

ii. Responds to the claim with a settlement offer.

If the Defendant decides to offer a settlement, since the defendant does not have the private information (uninformed player), the offer will screen Plaintiff's private information with the settlement offer. The model assumes any settlement offer is a final offer.

Stage 2.2 Defendant put itself in Plaintiff's situation to see how Plaintiff would react to Defendant's settlement offer. Defendant determines that the Plaintiff would not updates its belief over Damages given the settlement offer since it already has a superior knowledge over Damages.

Defendant analyzes Plaintiff's decision before making its final offer. Defendant's offer screens the Plaintiff for its two available options: acceptance and rejection. Defendant thinks Plaintiff would only accept the settlement offer if the amount offered is higher than Plaintiff's expected outcome from the trial minus litigation costs. Plaintiff decides based on its interim payoff u_A:

$$\begin{cases} \text{If } S_B \geq u_A \quad => \quad \text{Plaintiff would accept} \\ \text{If } S_B < u_A \quad => \quad \text{Plaintiff would reject} \end{cases}$$

Stage 2.3 Defendant estimates Plaintiff's interim expected payoff, $\hat{u}_A$, from litigation using its own beliefs $b'_X(x)$. $\hat{u}_A$ is Plaintiff's estimate of Defendant's expected payoff given the settlement offer S_B.

$$\hat{u}_A\left(b'_X(x), \mathrm{y}, k_B\right) = E\left[b'_X(x)\right]\mathrm{y} - k_A$$

Stage 2.4 Defendant thinks Plaintiff would only accept settlement offers that are equal or more than Plaintiff's expected outcome from litigation.

$$S_B \geq E[b'_X(x)]\,\mathrm{y} - k_A$$

Rearranging the above equation results in:

$$E[b'_X(x)] \leq \frac{S_B + k_A}{y}$$

This analysis results in determination of the critical type (x^c) for Plaintiff. Plaintiff's critical type is defined as the threshold that Plaintiff accepts the offer up to that amount. Plaintiff only accepts offer S_B if the proposed damages are less than its expected awarded amount of the verdict at trial.

$$x^c = \frac{S_B + k_A}{y}$$

Stage 2.5 The Defendant considers Plaintiff's critical type to determine its influence on Plaintiff's decision in acceptance or rejection of the settlement offer S_B. From Defendant's view, Plaintiff's Strategy would be:

$$\begin{cases} If\ E[a_X(x)] \ \le \ x^c \quad => \text{ Plaintiff } accepts \\ If\ E[a_X(x)] \ > \ x^c \quad => \text{ Plaintiff } rejects \end{cases}$$

Defendant's Strategic Analyses

Based on Defendant's settlement offer S_B,

$$\begin{cases} Plaintiff\ will\ accept\ if \qquad x \le x^c \\ Plaintiff\ will\ reject\ if \qquad x > x^c \end{cases}$$

Probability of Plaintiff accepting or rejecting Defendant's offer will be

$$\begin{cases} Pr_A\{acceptance\} = B'_X(x^c) \\ Pr_A\{rejection\} = 1 - B'_X(x^c) \end{cases}$$

Defendant's payoff given Plaintiff's action will be

$$\begin{cases} S_B \qquad\qquad if\ Plaintiff\ accepts \\ E[b'_X(x)]\,\mathrm{y} + k_B \qquad if\ Plaintiff\ rejects \end{cases}$$

Therefore Defendant's expected value from its own settlement offer S_B will be

$$U_B\ (S_B, b'_X(x), \mathrm{y}, k_B) = B'_X(x^c)\ S_B + \{1 - B'_X(x^c)\}\ \{\ E[b'_X(x)]\ y\ + k_B\ \}$$

Solving the equation above results in Defendant's optimal settlement amount, denoted by S_B^*.

The equation above can be rewritten as

$$U_B\ (S_B, b'_X(x), x^c, \mathrm{y}, k_B) = \ B'_X[x^c]\ S_B + \{1 - B'_X[x^c]\}\ \left\{ \frac{\int_{x_L}^{x^c} x\, b(x)\, dx}{B'_X[x^c]}\ y + \ k_B \right\}$$

To maximize Plaintiff's expected outcome, the right hand side of the equation above is differentiated respective to S_B. The solution to the differentiated equation identifies Plaintiff's critical type x^c. This value specifies Defendant's optimized settlement offer S_B^*. The optimal settlement amount needs to be within Plaintiff's lowest and highest expected outcome.

$$x_L\, y - \ k_A \ \leq \ S_B^* \ \ \leq \ \ x_H\, y - \ k_A$$

This equation eliminates the opportunity of having a *strictly dominated strategies* for the plaintiff, meaning that the settlement offer will not be accepted or rejected regardless of plaintiff's beliefs over Damages.

Conclusions

- The likelihood of settlement is identified by the of Plaintiff's probability of acceptance based on plaintiff's critical type:

$$\Pr\{\text{Settlement}\} \ = \ A_X(x^{c*})$$

- An increase in Defendant's updated belief over Damages $b'_X(x)$, increases the settlement offer and the likelihood of a settlement.
- An increase in plaintiff's litigation costs, decreases the settlement offer
- An increase in plaintiff's litigation costs, increases the likelihood of settlement.
- A plaintiff who is more confident about its Damages to be determined at higher amounts in litigation, will more likely reject the lower settlement offers.

4.3.2. Plaintiff's Decision on Settlement Offers - Screening Model

In this model, the Defendant holds private information about Liability and the Plaintiff screens defendant's type by offering a settlement. Defendant (Player B) is assumed to have private information on Liability Y. The private information is denoted as the player's type such that a defendant with Liability y is called Defendant type y. Defendant forms a *PDF* over y (its own type) denoted $b_Y(y)$. Plaintiff (*Player A*) does not have the private information about Liability, but it estimates Liability to be within $[y_L, y_H]$ interval. Plaintiff's prior *PDF* over Liability (B's type) is denoted $a_Y^o(y)$. It is assumed that both parties agree on the true level of *Damages*. The assumptions on parties' information are summarized as follows:

Table 4-4 Player's Information

A (Plaintiff)		**B (Defendant)**	
		$b_Y(y)$	A's PDF on Liability
x	A's Damages	x	A's Damages
$\boldsymbol{a_Y^o(y)}$	A's prior PDF over y	$a_Y^o(y)$	A's prior PDF over y
$\boldsymbol{(y_L, y_H)}$	A's limits of Damages	(y_L, y_H)	A's limits of Damages
$\boldsymbol{k_A, k_B}$	Litigation Costs	k_A, k_B	Litigation Costs

The following provides details for the main stages of the game:

Stage 0 – Formation of Prior Beliefs

Stage 1 – Information Exchange between the parties

Stage 2 – Plaintiff's Decision Analysis

Table 4-5 Screening Model for Defendant Screens with Settlement Offer

stage	PlayerA (Plaintiff)	Player B (Defendant)
0	A forms a PDF $a_Y^o(y)$	B forms a prior PDF $b_Y(y)$
1	Submit a claim C_A	
1.1		B does not receive any new information to update its belief over Liability
2.1	A's Decision Analysis $\begin{cases} Pursue \text{ Litigation} \\ \text{Offer Settlement } S_A \end{cases}$	
2.2	A's belief over B's belief =>	B decides based on its interim payoff, u_B $\begin{cases} \text{If } S_A \leq u_B \quad => \quad \text{B would accept} \\ \text{If } S_B > u_B \quad => \quad \text{B would reject} \end{cases}$
2.3	A estimates B's interim payoff, $(\hat{u}_B)$ $\hat{u}_B\ (\text{x}, a_Y^0(y), k_B) = x\ E[a_Y^o(y)] + k_B$	
2.4	B thinks A would only accept S_A if: $S_A \leq x\ E[a_Y^o(y)] + k_B$ $E[a_Y^o(y)] \geq \frac{S_A - k_B}{x}$ B's critical type: $y^c = \frac{S_A - k_B}{x}$	
2.5	A's belief over B's belief =>	B's Strategy $\begin{cases} If\ E[b_Y(y)] \geq y^c \quad => \quad B\ accepts \\ If\ E[b_Y(y)] < y^c \quad => \quad B\ rejects \end{cases}$

*Blue with the white text represents the plaintiff
*Green with the black text represents the defendant

Stage 0 Players A and B form their prior beliefs as follows:

i. Plaintiff forms a prior belief over Liability, a_Y^o(y)

ii. Defendant forms a belief over Damages, $b_Y(y)$

Stage 1.1 The Plaintiff submit its claim C_A to the Defendant. It is assumed that the Plaintiff files a claim only when the minimum expected outcome exceeds the Litigation Costs.

$$x\ y_H - k_A > 0$$

Stage 1.2 The Defendant has superior information regarding Liability; therefore, an information update does not occur in this phase unlike the previous screening model.

Stage 2.1 The Plaintiff makes a decision based on the following options:

i. Avoid Negotiations and Pursue Litigation

If the Plaintiff avoids settlement negotiations and pursues the litigation process, the Judge will define the true level of Liability, to the extent the Defendant is found liable.

ii. Send a Settlement Offer

If the Plaintiff decides to offer a settlement, since the defendant does not have the private information (uninformed player), the offer will screen Plaintiff's private information with the settlement offer. The model assumes any settlement offer is a final offer.

Stage 2.2 Plaintiff put himself in Defendant's situation to see how the Defendant would react to the settlement offer. Plaintiff determines it would not update its belief over Liability given the settlement offer since it already has a superior knowledge over Liability.

Plaintiff analyzes Defendant's decision before making its final offer. Plaintiff's offer screens the Defendant for its two available options: acceptance and rejection. Plaintiff thinks Defendant would only accept the settlement offer if the amount offered is lower than Defendant's expected outcome (payments) from the trial plus litigation costs. Defendant's expected outcome is defined as total amount that it realize as out of pocket expenditures. Defendant would make a decision based on its interim payoff u_B:

$$\begin{cases} \text{If } S_A \leq u_B & \Rightarrow \text{Defendant would accept} \\ \text{If } S_A > u_B & \Rightarrow \text{Defendant would reject} \end{cases}$$

Stage 2.3 Plaintiff estimates Defendant's interim expected payoff, $\hat{u}_B$, from litigation using its own beliefs $a_Y^0(y)$. $\hat{u}_B$ is Plaintiff's estimate on Defendant's expected payoff given the settlement offer S_B.

$$\hat{u}_B\ (\mathrm{x}, a_Y^0(y), k_B) = x\ E[a_Y^o(y)] + k_B$$

Stage 2.4 Plaintiff thinks Defendant would only accept settlement offers that are less or more than Defendant's expected outcome from litigation.

$$S_A \leq x\ E[a_Y^o(y)] + k_B$$

Rearranging the above equation results in:

$$E[a_Y^o(y)] \geq \frac{S_A - k_B}{x}$$

This analysis results in determination of the critical type (y^c)for Defendant. Defendant's critical type is the threshold for the Defendant who does not accept the offer if its Liability is less than that amount. Defendant only accepts offer S_A if the proposed Liability is less than its expected Liability at trial. Plaintiff's critical type will be:

$$y^c = \frac{S_A - k_B}{x}$$

Stage 2.5 The Plaintiff considers Defendant's critical type, which impacts defendant's decision in acceptance or rejection of the settlement offer S_A. From Plaintiff's view Defendant's Strategy would be:

$$\begin{cases} If\ E[b_Y(y)] \geq y^c \quad => \quad B\ accepts \\ If\ E[b_Y(y)] < y^c \quad => \quad B\ rejects \end{cases}$$

Plaintiff's Strategic Analyses

Based on Plaintiff's settlement offer S_B,

$$\begin{cases} Defendant\ will\ accept\ if & y \geq y^c \\ Defendant\ will\ reject\ if & y < y^c \end{cases}$$

Probability of Defendant accepting or rejecting Plaintiff's offer will be

$$\begin{cases} Pr_B\{acceptance\} = 1 - A_Y^o(y^c) \\ Pr_B\{rejection\} = A_Y^o(y^c) \end{cases}$$

Plaintiff's payoff given Defendant's action will be

$$\begin{cases} S_A & if\ Defendant\ accepts \\ x\ E[a_Y^0(y)] - k_A & if\ Defendant\ rejects \end{cases}$$

Therefore Defendant's expected value from the settlement offer S_A will be

$$U_A(S_A, x, y^c, x, k_A) = \{1 - A_Y^o(y^c)\}\ S_A \quad + A_Y^o(y^c)\ \{x\ E[a_Y^0(y)] - k_A\}$$

Solving this equation results in Defendant's optimal settlement amount, denoted by S_A^*.

$$U_A(S_A, x, y^c, k_A) = \{1 - A_Y^o(y^c)\}\ S_A + A_Y^o(y^c) \left\{ x \ \frac{\int_{y^c}^{y_H} y\, a(y)\, dy}{A(y^c)} - k_A \right\}$$

To maximize Plaintiff's expected outcome, the right hand side of the equation above is differentiated respective to S_A. The solution to the differentiated equation identifies Defendant's critical type y^c. This value specifies Plaintiff's optimized settlement offer S_A^*. The optimal settlement amount needs to be within Defendant's lowest and highest expected outcome.

$$x\, y_L + k_B \leq S_A^* \leq x\, y_H + k_B$$

This equation eliminates the opportunity of having a *strictly dominated strategies* for the plaintiff, meaning that the settlement offer will not be accepted or rejected regardless of plaintiff's beliefs over Damages.

Conclusions

- The likelihood of settlement is identified by the optimal amount of settlement and as a result optimal amount of critical type as follows:

$$\Pr\{\text{Settlement}\} = 1 - B_Y(y^{c*})$$

- An increase in Plaintiff's belief over Liability $a_Y^o(y)$, increases the settlement offer, and decreases the likelihood of settlement.
- Increase in Defendant's litigation costs, increases the settlement offer
- Increase in Defendant's litigation costs, increases the likelihood of settlement.
- A defendant who is more confident about its Liability to be determined as lower amounts at trial will more likely reject the higher settlement offers.

4.4. One-Sided Private Information - Signaling game

Signaling game is a type of one-sided private information game where the informed player (the party who holds private information) makes the settlement offer. The informed player may reveal its private information to its opponent through the settlement offer. Literature categorize the signaling games based on the level of revealed information to identify the equilibrium of the game. The equilibriums include *Revealing Equilibrium*, *Pooling Equilibrium, and Semi-Pooling Equilibrium*.

Revealing Equilibrium applies to cases where full information is transferred through the settlement offer. *Pooling Equilibrium (Separating Equilibrium)* is used when the proposer does not transfer any private information to its opponent. *Hybrid* or *Semi-Pooling Equilibrium* is the cases that the proposer partially transfer its private information to its opponent through the settlement offer. This Section assumes that the game is the Revealing type; however, Section 5.6.6 shows how hybrid games can be modeled using Bayesian Networks.

4.4.1. Defendant Signals with Settlement Offer

In this model, the defendant holds private information about Liability and signals plaintiff through a settlement offer. Defendant (Player B) is assumed to have private information on Liability Y. The private information is denoted as the player's type such that a defendant with Liability y is called Defendant type y. Defendant forms a Probability Distribution Function (*PDF)* over y (its own type) denoted $b_Y(y)$. Plaintiff (Player A) does not have the private information about the Liability, but it estimates Liability to be within $[y_L, y_H]$ interval. Plaintiff's prior *PDF* over Liability (B's type) is denoted $a_Y^o(y)$. It is assumed that both parties agree on the true value of *Damages*. The assumptions on parties' information are summarized as follows:

Table 4-6 Player's Information

A (Plaintiff)		B (Defendant)	
		$b_Y(y)$	A's PDF on Liability
x	A's Damages	x	A's Damages
$\boldsymbol{a_Y^o(y)}$	A's prior PDF over y	$a_Y^o(y)$	A's prior PDF over y
$\boldsymbol{a_Y'(y)}$	A's updated PDF over y	$a_Y'(y)$	A's updated PDF over y
$\boldsymbol{(y_L, y_H)}$	A's limits of Damages	(y_L, y_H)	A's limits of Damages
$\boldsymbol{k_A, k_B}$	Litigation Costs	k_A, k_B	Litigation Costs

The following provides details for the main stages of the game:

Stage 0 – Formation of Prior Beliefs

Stage 1 – Information Exchange between the parties

Stage 2 – Defendant's Decision Analysis

Table 4-7 Screening Model for Defendant Screens with Settlement Offer

stage	Plaintiff (Player A)	Defendant (Player B)
0	A forms a PDF $a_Y^o(y)$	B forms a prior PDF $b_Y(y)$
1	Submit a complaint C_A	
1.1		B does not receive any new information to update its belief over Liability
2.1		B's Decision Analysis $\begin{cases} Pursue\ Litigation \\ \text{Offer Settlement } S_B \end{cases}$
2.2	A updated its belief over Liability $a'_Y(y)$	B's belief over A's belief
2.3	A Decides based on its interim payoff, u_A $\begin{cases} \text{If } S_B \geq u_A \Rightarrow \text{A would accept} \\ \text{If } S_B < u_A \Rightarrow \text{A would reject} \end{cases}$	
2.4		B estimates A's interim outcome, $(\hat{u}_A)$ $\hat{u}_A\ (x, a'_Y(y), k_A) = x\ E[a'_Y(y)] - k_A$
2.5		B thinks A would only accept S_B if: $S_B \geq \text{x}\ E[a'_Y(y)] - k_A$ $E[a'_Y(y)] \leq \frac{S_B + k_A}{x}$ A's critical type: $y^c = \frac{S_B + k_A}{x}$
2.6	A's Strategy $\begin{cases} If\ E[a'_Y(y)] \leq y^c \Rightarrow A\ accepts \\ If\ E[a'_Y(y)] > y^c \Rightarrow A\ rejects \end{cases}$	B's belief over A's belief

*Blue with the white text represents the plaintiff
*Green with the black text represents the defendant

Stage 0 Players A and B form their prior beliefs as follows:

i. Plaintiff forms a belief over Liability, $a_Y^o(y)$

ii. Defendant forms a prior belief over Damages, $b_Y(y)$

Stage 1.1 The plaintiff submit its claim C_A to the Defendant. It is assumed that the Plaintiff files a claim only when the minimum expected outcome from litigation exceeds the Litigation Costs.

$$x \; y_H - \; k_A > \; 0$$

Stage 1.2 The Defendant does not updates its beliefs since it has superior knowledge over Liability.

Stage 2.1 The Defendant makes a decision based on the following options:

i. Avoid Negotiations and Pursue Litigation

If the Defendant avoids settlement negotiations and pursue the litigation process, the Judge will define the true level of Liability, to the Defendant is found liable.

ii. Send a Settlement Offer S_B

If the Defendant decides to offer a settlement, the offer will signals its private information to the Plaintiff.

Stage 2.2 Defendant put itself in Plaintiff's situation to see how the Plaintiff would react to the settlement offer. The Plaintiff updates its beliefs over Liability given the settlement offer as follows:

$$a_Y(y|\, S_B) \quad or \quad a'_Y(y)$$

Stage 2.3 Defendant analyzes Plaintiff's decision before making its final offer. Defendant thinks Plaintiff would only accept the settlement offer if the amount offered is more than Plaintiff's expected outcome from the trial minus litigation costs. Plaintiff would make a decision based on its interim payoff u_A:

$$\begin{cases} \text{If } S_A \geq u_A \quad => \quad \text{Plaintiff would accept} \\ \text{If } S_A < u_A \quad => \quad \text{Plaintiff would reject} \end{cases}$$

Stage 2.4 Defendant estimates Plaintiff's interim expected payoff, $\hat{u}_B$, from litigation using Plaintiff's updated beliefs $a'_Y(y)$. $\hat{u}_A$ is Defendant's estimate on Plaintiff's expected payoff given the settlement offer S_B.

$$\hat{u}_A\,(x, a'_Y(y), k_A) = x\, E[a'_Y(y)] - k_A$$

Stage 2.5 Defendant thinks Plaintiff would only accept settlement offers that are equal or greater than Plaintiff's expected outcome from litigation.

$$S_B \quad \geq \quad x\, E[a'_Y(y)] - k_A$$

Rearranging the above equation results in:

$$E[a'_Y(y)] \leq \frac{S_B + k_A}{x}$$

This analysis results in determination of the critical type for Plaintiff. Plaintiff only accepts offer S_B if the proposed Liability are more than its expected Liability at trial. Plaintiff's critical type will be:

$$y^c = \frac{S_B + k_A}{x}$$

Stage 2.6 The Defendant considers Plaintiff's critical type, which impacts plaintiff decision in acceptance or rejection of the settlement offer S_B. From Defendant's view Plaintiff's Strategy would be:

$$\begin{cases} If\ E[a'_Y(y)] \quad \leq \quad y^c \quad => \quad A\ accepts \\ If\ E[a'_Y(y)] \quad > \quad y^c \quad => \quad A\ rejects \end{cases}$$

Defendant's Strategic Analyses

Based on Plaintiff's settlement offer S_B,

$$\begin{cases} Plaintiff\ will\ accept\ if \qquad y \leq y^c \\ Plaintiff\ will\ reject\ if \qquad y > y^c \end{cases}$$

Probability of Plaintiff accepting or rejecting Defendant's offer will be

$$\begin{cases} Pr_A\{acceptance\} = \qquad A'_Y(y^c) \\ Pr_A\{rejection\} = \qquad 1 - A'_Y(y^c) \end{cases}$$

Defendant's payoff given Plaintiff's action will be

$$\begin{cases} S_B \qquad \qquad if\ Defendant\ accepts \\ x\ E[b_Y(y)] + k_B \qquad if\ Defendant\ rejects \end{cases}$$

Therefore B's expected value of its own offer S_B is:

$$U_B\ (S_B, \text{x}, a'_Y(y), b_Y(y), k_B) = \quad A'_Y(y^c)\ \ S_B + \{1 - A'_Y(y^c)\}\{\, x\ \ E[b_Y(y)] + k_B\ \}$$

Solving this equation results in Defendant's optimal settlement amount, denoted by S_A^*.

$$U_B\ (S_B, \text{x}, a'_Y(y), b_Y(y), k_B) = \ A'_Y(y^c)\, S_B + \{1 - A'_Y(y^c)\}\left\{ x\ \frac{\int_{y^c}^{y_H} y\ b_Y(y)\ dy}{B_Y(y^c)} + k_B \right\}$$

To maximize Defendant's expected outcome, the right hand side of the equation above is differentiated respective to S_B. The solution to the differentiated equation identifies Plaintiff's critical type y^c. This value specifies Plaintiff's optimized settlement offer S_B^*. The optimal settlement amount needs to be within Plaintiff's lowest and highest expected outcome.

$$x\ y_L + k_B \leq S_B^* \leq x\ y_H + k_B$$

This equation eliminates the opportunity of having a *strictly dominated strategies* for the Plaintiff, meaning that the settlement offer will not be accepted or rejected regardless of Defendant's beliefs over Damages.

Conclusions

- The likelihood of settlement is identified by the optimal amount of settlement and as a result optimal amount of critical type as follow:

$$\Pr\{\text{Settlement}\} = 1 - B_Y(y^{c*})$$

- An increase in Plaintiff's updated belief over Liability $a_Y'(y)$ will increase the settlement amount and decrease the likelihood of a settlement.
- An increase in Plaintiff's litigation costs will decrease the settlement offer
- An increase in Plaintiff's litigation costs will increase the likelihood of settlement.
- A plaintiff who is more confident about the Liability to be determined as higher amounts at trial will more likely reject the lower settlement offers.

4.4.2. Plaintiff Signals with Settlement Demand

In this model, the plaintiff holds private information about Damages and signals defendant through a settlement offer. Plaintiff (Player A) is assumed to have private information on Damages X. The private information is denoted as the player's type such that a plaintiff with Damages x is called Plaintiff type x. Plaintiff forms a Probability Distribution Function (*PDF)* over x (its own type) denoted $a_X(x)$. Defendant (Player B) does not have the private information about Damages, but it estimates Damages to be within $[x_L, x_H]$ interval. Defendant's prior *PDF* over Damages (A's type) is denoted $b_X^o(x)$. It is assumed that both parties agree on the true value of *Liability*. The assumptions on parties' information are summarized as follows:

Table 4-8 Player's Information

A (Plaintiff)		B (Defendant)	
$\boldsymbol{a_X(x)}$	A's PDF on Damages		
y	B's Liability	y	B's Liability
$\boldsymbol{b_X^o(x)}$	B's prior PDF over x	$b_X^o(x)$	B's prior PDF over x
$\boldsymbol{b_X'(x)}$	B's first update over x	$b_X'(x)$	B's first update over x
$\boldsymbol{b_X''(x)}$	B's updated belief over x given the settlement offer	$b_X''(x)$	B's updated belief over x after the settlement offer
$\boldsymbol{(x_L, x_H)}$	A's limits of Damages	(x_L, x_H)	A's limits of Damages
$\boldsymbol{k_A}$, $\boldsymbol{k_B}$	Litigation Costs	k_A , k_B	Litigation Costs

The following provides details for the main stages of the game:

Stage 0 – Formation of Prior Beliefs

Stage 1 – Information Exchange between the parties

Stage 2 – Defendant's Decision Analysis

Table 4-9 Screening Model for Defendant Screens with Settlement Offer

stage	PlayerA (Plaintiff)	Player B (Defendant)
0	A forms a PDF $a_X(x)$	B forms a prior PDF $b_X^0(x)$
1	Submit a claim C_A	
1.1		B updates its belief over Damages $b'_X(x)$
2.1	A's Decision Analysis $\begin{cases} Pursue \text{ Litigation} \\ \text{Offer Settlement } S_A \end{cases}$	
2.2		B updates PDF over x given S_A, $b''_X(x)$ B decides based on its interim payoff, u_B $\begin{cases} \text{If } S_A \leq u_B \quad => \quad \text{B would accept} \\ \text{If } S_B > u_B \quad => \quad \text{B would reject} \end{cases}$
2.3	A's belief over B's belief	
2.3	A estimates B's interim payoff, ($\hat{u}_B$) $\hat{u}_B\ (\ b''_X(x), y, k_B) = E[\ b''_X(x)]\ y + \ k_B$	
2.4	B thinks A would only accept S_A if: $S_A \quad \leq \ E[\ b''_X(x)]\ y + \ k_B$ $E[\ b''_X(x)] \ \geq \ \frac{S_A - k_B}{y}$ B's critical type: $x^c = \frac{S_A - k_B}{y}$	
2.5	A's belief over B's belief	B's Strategy $\begin{cases} If\ E[\ b''_X(x)] \ \geq \ x^c \quad => \quad B\ accepts \\ If\ E[\ b''_X(x)] \ < \ x^c \quad => \quad B\ rejects \end{cases}$

*Blue with the white text represents the plaintiff
*Green with the black text represents the defendant

Stage 0 Players A and B form their prior beliefs as follows:

i. Plaintiff forms a belief over Liability, $a_X(x)$

ii. Defendant forms a prior belief over Damages, $b_X^o(x)$

Stage 1.1 The Plaintiff submits its claim C_A to the Defendant. It is assumed the Plaintiff files a claim only when the minimum expected outcome from litigation exceeds the Litigation Costs.

$$x_H \text{ y} - k_A > 0$$

Stage 1.2 The Defendant updates its beliefs over damages given Plaintiff's claim as follows:

$$b_X(x|C_A) \quad or \quad b_X'(x)$$

Stage 2.1 The Plaintiff makes a decision based on the following options:

iii. Avoid Negotiations and Pursue Litigation

If the plaintiff avoids settlement negotiations and pursue the litigation process, the Judge will define the true level of Damages.

iv. Send a Settlement Offer S_A.

If the Plaintiff decides to offer a settlement, the offer will signals its private information to the Defendant.

Stage 2.2 Plaintiff put himself in Defendant's situation to see how the defendant would react to the settlement offer. The Defendant updates its beliefs over damages given Plaintiff's settlement offer as follows:

$$b_X(x|C_A, S_A) \quad or \quad b''_X(x)$$

Stage 2.3 Plaintiff analyzes Defendant's decision before making its final offer. Plaintiff thinks Defendant would only accept the settlement offer if the amount offered is lower than Defendant's expected outcome (payments) from the trial plus litigation costs. Defendant's expected outcome is defined as total amount that it realize as out of pocket expenditures. Defendant would make a decision based on its interim payoff u_B:

$$\begin{cases} \text{If } S_A \leq u_B \quad => \quad \text{Defendant would accept} \\ \text{If } S_A > u_B \quad => \quad \text{Defendant would reject} \end{cases}$$

Stage 2.4 Plaintiff estimates Defendant's interim expected payoff, $\hat{u}_B$, from litigation using Defendant's updated beliefs $b''_X(x)$. $\hat{u}_B$ is Plaintiff's estimate on Defendant's expected payoff given the settlement offer S_A.

$$\hat{u}_B\,(b''_X(x), y, k_B) = E[\,b''_X(x)]\,y + \; k_B$$

Stage 2.5 Plaintiff thinks Defendant would only accept settlement offers that are equal or less than Defendant's expected outcome from litigation.

$$S_A \;\; \leq \;\; E[\,b''_X(x)]\,y + \; k_B$$

Rearranging the above equation results in:

$$E[\,b''_X(x)] \;\geq\; \frac{S_A - k_B}{y}$$

This analysis results in determination of the critical type for Defendant. Defendant only accepts offer S_A if the proposed damages are less than its expected Damages at trial. Defendant's critical type will be:

$$x^c = \frac{S_A - k_B}{y}$$

Stage 2.6 The Plaintiff considers Defendant's critical type, which impacts defendant's decision in acceptance or rejection of the settlement offer S_A. From Plaintiff's view Defendant's Strategy would be:

$$\begin{cases} If\ E[\, b_X''(x)] \geq x^c & => Defendant\ accepts \\ If\ E[\, b_X''(x)] < x^c & => Defendant\ rejects \end{cases}$$

Plaintiff's Strategic Analyses

Based on Plaintiff's settlement offer S_B,

$$\begin{cases} Defendant\ will\ accept\ if & x \geq x^c \\ Defendant\ will\ reject\ if & x < x^c \end{cases}$$

Probability of Defendant accepting or rejecting Plaintiff's offer will be

$$\begin{cases} Pr_B\{acceptance\} = 1 - B_X''(x^c) \\ Pr_B\{rejection\} = B_X''(x^c) \end{cases}$$

Plaintiff's payoff given Defendant's action will be

$$\begin{cases} S_A & if\ Defendant\ accepts \\ E[b_X''(x)]\ y - k_A & if\ Defendant\ rejects \end{cases}$$

Therefore Defendant's expected value from the settlement offer S_A will be

$$U_A(S_A, x, x^c, y, k_A) = \{1 - B_X''(x^c)\}\ S_A + B_X''(x^c)\,\{\, E[b_X''(x)]\ y - k_A\}$$

Solving this equation results in Defendant's optimal settlement amount, denoted by S_A^*.

$$U_A(S_A, x, x^c, y, k_A) = \{1 - B_X''(x^c)\}\ S_A + B_X''(x^c) \left\{ x\ \frac{\int_{x^c}^{x_H} x\, b_X''(x^c)\, dx}{B_X''(x^c)} - k_A \right\}$$

To maximize Plaintiff's expected outcome, the right hand side of the equation above is differentiated respective to S_A. The solution to the differentiated equation identifies Defendant's critical type x^c. This value specifies Plaintiff's optimized settlement offer S_A^*. The optimal settlement amount needs to be within Defendant's lowest and highest expected outcome.

$$x_L y + k_B \leq S_A^* \leq x_H y + k_B$$

This equation eliminates the opportunity of having a *strictly dominated strategies* for the Defendant, meaning that the settlement offer will not be accepted or rejected regardless of Defendant's beliefs over Damages.

Conclusions

- The likelihood of settlement is identified by the optimal amount of settlement and as a result optimal amount of critical type as follows:

$$\Pr\{\text{Settlement}\} = A_X(x^{c*})$$

- Increase in Defendant's belief over Damages $b_X(x)$, increases the settlement offer and decrease the likelihood of a settlement
- Increase in Defendant's litigation costs, increases the settlement demand
- Increase in Defendant's litigation costs, increases the likelihood of settlement.
- A Defendant who is confident about the true damages to be determined at lower amounts at trial will more likely reject the higher settlement demands

4.5. Two-Sided Private Information Model

This model consolidates both Screening and Signaling games into one model. The two-sided asymmetric information game considers both parties (players A and B) have private information on the specifics of the claim. Each party has private information on *Damages* and *Liability*. The analysis varies depending on which party submits the final settlement offer. The following describes information that each party holds regarding the claim.

Each player is assumed to have private information on Damages X and Liability Y. The private information is denoted as the player's type. For example, Plaintiff with Damages x is called Plaintiff type x. Plaintiff forms a prior Probability Distribution Function (*PDF***)** on Damages denoted $a_X^o(x)$, and Liability denoted $a_Y^o(y)$. Defendant forms a prior *PDF* on Damages denoted $b_X^o(x)$, and Liability denoted $b_Y^o(y)$.

Each player is assumed to form a PDF over its estimated litigation costs and its opponent's litigation costs. Plaintiff's beliefs over its own litigation costs are denoted $a_K(k_A)$, and Plaintiff's beliefs over Defendant's litigation costs are denoted $a_K(k_B)$. Defendant's beliefs over its own litigation costs are denoted $b_K(k_B)$, and Defendant's beliefs over Plaintiff's litigation costs are denoted $b_K(k_A)$. The assumptions on parties' information are summarized as follows:

Table 4-10 Player's Information

A (Plaintiff)		B (Defendant)	
$\boldsymbol{a_X^o(x)}$	A's prior PDF over x	$b_X^o(x)$	B's prior PDF over x
$\boldsymbol{a_Y^o(\mathrm{y})}$	A's prior PDF over y	$a_Y^o(\mathrm{y})$	B's prior PDF over y
$\boldsymbol{a_K(k_A)}$	A's Litigation Costs	$b_K(k_A)$	A's Litigation Costs
$\boldsymbol{a_K(k_B)}$	B's Litigation Costs	$b_K(k_B)$	B's Litigation Costs

4.5.1. Defendant Signals with Settlement offer

In this model, the defendant signals plaintiff about its private information by submitting a settlement offer. The following provides details for the main stages of the game:

Stage 0 – Formation of Prior Beliefs

Stage 1 – Information Exchange between the parties

Stage 2 – Defendant's Decision Analysis

Stage 0 Players A and B form their prior beliefs as follows:

i. Plaintiff forms prior beliefs over Damages $a_X^o(x)$, and Liability $a_Y^o(y)$

ii. Defendant forms prior beliefs over Damages $b_X^o(x)$, and Liability $b_Y^o(y)$

Stage 1.1 The Plaintiff submit its claim C_A to the Defendant. It is assumed the Plaintiff files a claim only when the minimum expected outcome from litigation exceeds the Litigation Costs.

$$\min(\mathrm{X}) \times \min(Y) - \max(k_A) > 0$$

Table 4-11 Two-sided private information – Defendant Offers Settlement

stage	PlayerA (Plaintiff)	Player B (Defendant)
0	A forms prior PDFs $a_X^0(x)$ and $a_Y^0(y)$	B forms prior PDFs $b_X^0(x)$ and $b_Y^0((y)$
1	Submit a claim C_A	
1.1		B updates its belief over Damages $b'_X(x)$ and Liability $b'_Y(y)$
2.1		B's Decision Analysis $\begin{cases} Pursue\ Litigation \\ \text{Offer Settlement } S_B \end{cases}$
2.2	A updated its belief over	
2.3	Damages $a'_X(x)$, and Liability $a'_Y(y)$ A Decides based on its interim payoff, u_A $\begin{cases} \text{If } S_B \geq u_A \implies \text{A would accept} \\ \text{If } S_B < u_A \implies \text{A would reject} \end{cases}$	B's belief over A's belief
2.4		B estimates A's interim outcome, $(\hat{u}_A)$ $\hat{u}_A\ (a'_X(x), a'_Y(y), b_K(k_A)) =$ $E[a'_X(x)\ a'_Y(y)] - E[b_K(k_A)]$
2.5		B thinks A would only accept S_B if: $S_B \geq \hat{u}_A$ $E[a'_X(x)\ a'_Y(y)] \leq S_B + E[b_K(k_A)]$ A's critical type: $A^c = S_B + E[b_K(k_A)]$
2.6	A's Strategy $\begin{cases} If\ E[a'_X(x)\ a'_Y(y)] \leq A^c\ accept \\ If\ E[a'_X(x)\ a'_Y(y)] > A^c\ reject \end{cases}$	B's belief over A's belief

*Blue with the white text represents the plaintiff
*Green with the black text represents the defendant

Stage 1.2 The Defendant updates its beliefs over both Damages and Liability.

$$b_X(x|C_A) \quad or \quad b'_X(x)$$

$$b_Y(y|C_A) \quad or \quad b'_Y(y)$$

Stage 2.1 The Defendant makes a decision based on the following options:

v. Avoid Negotiations and pursue litigation.

If the Defendant avoids settlement negotiations and pursue the litigation process, the Judge will define the true Liability and true Damages.

vi. Send a settlement offer S_B.

If the Defendant decides to offer a settlement, the offer will signals its private information to the Plaintiff.

Stage 2.2 Defendant put himself in Plaintiff's situation to see how the Plaintiff would react to the settlement offer. The Plaintiff updates its beliefs over Damages and Liability given the settlement offer as follows:

$$a_X(x|S_B) \quad or \quad a'_X(x)$$

$$a_Y(y|S_B) \quad or \quad a'_Y(y)$$

Stage 2.3 Defendant analyzes Plaintiff's decision before making its final offer. Defendant thinks Plaintiff would only accept the settlement offer if the amount offered is more than Plaintiff's expected outcome from litigation minus litigation costs. Plaintiff would make a decision based on its interim payoff u_A:

$$\begin{cases} \text{If } S_B \geq u_A & => \quad \text{Plaintiff would accept} \\ \text{If } S_B < u_A & => \quad \text{Plaintiff would reject} \end{cases}$$

Stage 2.4 Defendant estimates Plaintiff's interim expected payoff, $\hat{u}_B$, from litigation using Plaintiff's updated beliefs. $\hat{u}_A$ is Defendant's estimate on Plaintiff's expected payoff given the settlement offer S_B.

$$\hat{u}_A \left(a'_X(x), a'_Y(y), b_K(k_A)\right) = E[a'_X(x) \;\; a'_Y(y)] - E[b_K(k_A)]$$

Stage 2.5 Defendant thinks Plaintiff would only accept settlement offers that are equal or more than Plaintiff's expected outcome from litigation.

$$S_B \geq E[a'_X(x) \;\; a'_Y(y)] - E[b_K(k_A)]$$

Rearranging the above equation results in:

$$E[a'_X(x) \;\; a'_Y(y)] \leq S_B + E[b_K(k_A)]$$

This analysis results in determination of the critical type for Plaintiff. Plaintiff only accepts offer S_B if the proposed damages and liability are more than its expected outcome at trial. Plaintiff's critical type will be:

$$A^c = S_B + E[b_K(k_A)]$$

Stage 2.6 The Defendant considers Plaintiff's critical type, which impacts plaintiff decision in acceptance or rejection of the settlement offer S_B. From Defendant's view Plaintiff's Strategy would be:

$$\begin{cases} If \; E\left[a'_X(x) \;\; \boldsymbol{a}'_{\boldsymbol{Y}}(\boldsymbol{y})\right] \leq A^c & => \; A \; accepts \\ If \; E\left[a'_X(x) \; \boldsymbol{a}'_{\boldsymbol{Y}}(\boldsymbol{y})\right] > A^c & => \; A \; rejects \end{cases}$$

Defendant's Strategic Analyses

Based on Defendant's settlement offer S_B,

$$\begin{cases} Plaintiff\ will\ accept\ if & xy \leq A^c \\ Plaintiff\ will\ reject\ if & xy > A^c \end{cases}$$

Probability of Plaintiff accepting or rejecting Defendant's offer will be

$$\begin{cases} Pr_A\{acceptance\} = & A'_{XY}(A^c) \\ Pr_A\{rejection\} = & 1 - A'_{XY}(A^c) \end{cases}$$

Defendant's payoff given Plaintiff's action will be

$$\begin{cases} S_B & if\ Defendant\ accepts \\ E[b'_X(x)\ b'_Y(y)] + E[b_K(k_B)] & if\ Defendant\ rejects \end{cases}$$

Therefore B's expected value of its own offer S_B is:

$$U_B\left(S_B, a'_X(x), a'_Y(y), b'_X(x), b'_Y(y), b_K(k_A), b_K(k_B)\right) =$$

$$A'_{XY}(A^c)\quad S_B \quad + \quad \{1 - A'_{XY}(A^c)\} \quad \{E[b'_X(x)\ b'_Y(y)] + E[b_K(k_B)]\}$$

To maximize Defendant's expected outcome, the equation above can be differentiated respective to S_B. The solution to the differentiated equation identifies Plaintiff's critical type A^c. This value specifies Plaintiff's optimized settlement offer S_B^*.

$$\min(X)\min(Y) + \min(K_B) \leq \quad S_B^* \quad \leq \max(X)\max(Y) + \max(K_B)$$

The equation above ensures that the settlement offer will not be rejected or accepted no matter what the plaintiff's beliefs are on the case. This equation eliminates opportunity of having a *strictly dominant strategy* for the plaintiff.

Conclusions

- The likelihood of settlement is identified by probability of defendant offers above plaintiff's critical type, and plaintiff acceptance of that settlement offer:

$$\Pr\{\text{Settlement}\} = A'_{XY}(A^c)\ \{1 - B'_{XY}(A^{c*})\}$$

- An increase in plaintiff's updated belief over Damages $a'_X(x)$ or Liability $a'_Y(y)$ will increase the settlement amount and decrease the likelihood of a settlement.
- An increase in defendant's updated belief over Damages $a'_X(x)$ or Liability $a'_Y(y)$ will increase the settlement amount and increase the likelihood of a settlement.
- An increase in Plaintiff's litigation costs will decrease the settlement offer and increase the likelihood of settlement.
- An increase in Defendant's litigation costs will increase the settlement offer and increase the likelihood of settlement.
- A Plaintiff who is more confident about his Damages to be determined as higher amounts at trial will more likely reject the lower settlement offers.
- A Defendant who is more confident that the Liability will be determined as higher amounts at trial will more likely willing to offer higher settlement amounts

4.5.2. Plaintiff Signals with Settlement Demand

In this model, the Plaintiff signals defendant about its private information by submitting a settlement offer. The following provides details for the main stages of the game:

Stage 0 – Formation of Prior Beliefs

Stage 1 – Information Exchange between the parties

Stage 2 – Defendant's Decision Analysis

Table 4-12 Two-sided private information – Defendant Offers Settlement

stage	Player A (Plaintiff)	Player B (Defendant)
0	A forms prior PDFs $a_X^0(x)$ and $a_Y^0(y)$	B forms prior PDFs $b_X^0(x)$ and $b_Y^0((y)$
1	Submit a claim C_A	
1.1		B updates its belief over Damages $b'_X(x)$ and Liability $b'_Y(y)$
2.1	A's Decision Analysis $\begin{cases} Pursue \text{ Litigation} \\ \text{Offer Settlement } S_A \end{cases}$	
2.2		B updates its belief give S_A Damages $b''_X(x)$, and Liability $b''_Y(y)$
2.3	A's belief over B's belief →	B decides based on its interim payoff, u_B $\begin{cases} \text{If } S_A \le u_B \quad => \quad \text{B would accept} \\ \text{If } S_B > u_B \quad => \quad \text{B would reject} \end{cases}$
2.3	A estimates B's interim payoff, ($\hat{u}_B$) $\hat{u}_B\ (b''_X(x), b''_X(y), a_K(k_B)) =$ $E[\, b''_X(x)\, b''_Y(y)\,] \ + \ E[\, a_K(k_B)\,]$	
2.4	A thinks B would only accept S_A if: $S_A \ \le \ \hat{u}_B$ $E[\, b''_X(x)\, b''_Y(\mathbf{y})\,] \ \ge \ S_A - E[\, a_K(k_B)\,]$ B's critical type: $B^c = S_A - E[\, a_K(k_B)\,]$	
2.5	A's belief over B's belief →	B's Strategy $\begin{cases} If \ \ E[\, b''_X(x)\, b''_Y(\mathbf{y})\,] \ge B^c & accept \\ If \ \ E[\, b''_X(x)\, b''_Y(\mathbf{y})\,] < B^c & reject \end{cases}$

*Blue with the white text represents the plaintiff
*Green with the black text represents the defendant

Stage 0 Players A and B form their prior beliefs as follows:

iii. Plaintiff forms prior beliefs over Damages $a_X^o(x)$, and Liability $a_Y^o(y)$

iv. Defendant forms prior beliefs over Damages $b_X^o(x)$, and Liability $b_Y^o(y)$

Stage 1.1 The Plaintiff submit its claim C_A to the Defendant. It is assumed that the Plaintiff files a claim only when the minimum expected outcome from litigation exceeds the Litigation Costs.

$$\min(\mathrm{X}) \times \min(Y) - \max(k_A) > 0$$

Stage 1.2 The Defendant updates its beliefs over both Damages and Liability.

$$b_X(x|C_A) \quad or \quad b'_X(x)$$

$$b_Y(y|C_A) \quad or \quad b'_Y(y)$$

Stage 2.1 The Plaintiff makes a decision based on the following options:

vii. Avoid Negotiations and Pursue Litigation

If the Plaintiff avoids settlement negotiations and pursue the litigation process, the Judge will define the true Liability and true Damages.

viii. Send a settlement offer S_A

If the Plaintiff decides to offer a settlement, the offer will signals its private information to the Defendant.

Stage 2.2 Plaintiff put himself in Defendant's situation to see how the Defendant would react to the settlement offer. The Defendant updates its beliefs on damages and Liability given the settlement offer as follows:

$$b_X(x|C_A, S_B) \quad or \quad b_X''(x)$$

$$b_Y(y|C_A, S_B) \quad or \quad b_Y''(y)$$

Stage 2.3 Plaintiff analyzes Defendant's decision before making its final offer. Plaintiff thinks Defendant would only accept the settlement offer if the amount offered is equal or less than Defendant's expected outcome from litigation plus litigation costs. Defendant would make a decision based on its interim payoff u_B:

$$\begin{cases} \text{If } S_A \leq u_B & => \quad \text{Defendant would accept} \\ \text{If } S_A > u_B & => \quad \text{Defendant would reject} \end{cases}$$

Stage 2.4 Plaintiff estimates Defendant's interim expected payoff, $\hat{u}_B$, from litigation using Defendant's updated beliefs. $\hat{u}_B$ is Plaintiff's estimate on Defendant's expected payoff given the settlement offer S_A.

$$\hat{u}_B\ (b_X''(x), b_X''(y), a_K(k_B)) = E[\, b_X''(x)\, b_Y''(y)\,] \ + \ E[\, a_K(k_B)\,]$$

Stage 2.5 Plaintiff thinks Defendant would only accept settlement offers that are equal or less than Defendant's expected payoff from litigation (defendant's payoffs are out of pocket expenditures at trial).

$$S_A \quad \leq \quad E[\, b_X''(x)\, b_Y''(y)\,] \ + \ E[\, a_K(k_B)\,]$$

Rearranging the above equation results in:

$$E[\, b_X''(x)\, b_Y''(y)\,] \quad \geq \quad S_A - \ E[\, a_K(k_B)]$$

This analysis results in determination of the critical type for Defendant. Defendant only accepts offer S_B if the proposed damages and liability are less than its expected outcome at trial. Defendant's critical type will be:

$$B^c = S_A - E[\, a_K(k_B)]$$

Stage 2.6 The Plaintiff considers Defendant's critical type, which impacts defendant's decision in acceptance or rejection of the settlement offer S_A. From Plaintiff's view Defendant's Strategy would be:

$$\begin{cases} If\ E\left[b''_X(x)\ \ b'_Y(y)\right] \ \geq \ B^c \ => \ B\ accepts \\ If\ E\left[b''_X(x)\ b'_Y(y)\right] \ < \ B^c \ => \ B\ rejects \end{cases}$$

Defendant's Strategic Analyses

Based on Plaintiff's settlement offer S_A,

$$\begin{cases} Defendant\ will\ accept\ if & xy \ \geq \ B^c \\ Defendant\ will\ reject\ if & xy \ < \ B^c \end{cases}$$

Probability of Defendant accepting or rejecting Plaintiff's offer will be

$$\begin{cases} Pr_B\{acceptance\} \ = & 1 - B'_{XY}(A^c) \\ Pr_B\{rejection\} \ = & B'_{XY}(A^c) \end{cases}$$

Plaintiff's payoff given Defendant's action will be

$$\begin{cases} S_A & if\ Plaintiff\ accepts \\ E[a'_X(x)\ \ a'_Y(y)] - E[a_K(k_A)] & if\ Plaintiff\ rejects \end{cases}$$

Therefore B's expected value of its own offer S_B is:

$$U_A\left(S_B, a'_X(x), a'_Y(y), b''_X(x), b''_Y(y), a_K(k_A), a_K(k_B)\right) =$$

$$\{1 - B'_{XY}(A^c)\} \quad S_A \quad + \quad B'_{XY}(A^c) \quad \{E[a'_X(x)\ a'_Y(y)] - E[a_K(k_A)]\}$$

To maximize Defendant's expected outcome, the equation above can be differentiated respective to S_B. The solution to the differentiated equation identifies Plaintiff's critical type A^c. This value specifies Plaintiff's optimized settlement offer S_B^*.

$$\min(X)\min(Y) - \max(K_A) \leq \quad S_A^* \quad \leq \max(X)\max(Y) - \min(K_A)$$

The equation above ensures that the settlement offer will not be rejected or accepted no matter what the Plaintiff's beliefs are on the case. This equation eliminates opportunity of having a *strictly dominant strategy* for the Plaintiff.

Conclusions

- The likelihood of settlement is identified by the optimal amount of settlement and as a result optimal amount of critical type as follow:

$$\Pr\{\text{Settlement}\} = \{1 - A'_{XY}(A^c)\} \ B'_{XY}(A^c)$$

- Increase in Defendant's belief over Damages $b_X(x)$, will increase the settlement demand and decrease the likelihood of a settlement.
- Increase in Defendant's litigation costs will increase the settlement demand and/or increase the likelihood of settlement.

A Defendant who is confident about its Damages to be determined as lower amounts at trial will more likely reject the higher settlement demands

- An increase in Plaintiff's updated belief over Damages $a_X'(x)$ or Liability $a_Y'(y)$ will increase the settlement amount and decrease the likelihood of a settlement.
- An increase in Defendant's updated belief over Damages $a_X'(x)$ or Liability $a_Y'(y)$ will increase the settlement amount and increase the likelihood of a settlement
- An increase in Plaintiff's litigation costs will decrease the settlement offer and increase the likelihood of settlement
- An increase in Defendant's litigation costs will increase the settlement offer and increase the likelihood of settlement.
- A Plaintiff who is more confident about its Damages to be as higher amounts at trial will more likely offer lower settlement amounts.
- A Defendant who is more confident that the Liability to be determined as higher amounts at trial will more likely willing to accept higher settlement amounts

4.6. Refinements to Sequential Equilibrium

There are multiple equilibriums defined for the non-cooperative asymmetric information game. These equilibriums refine the

4.6.1. Interiority of Equilibrium

Interiority of the equilibrium is an equilibrium defined in non-cooperative sequential games. This equilibrium provides limits to the range for the settlement offers each party may make. Plaintiff's maximum possible outcome should exceed defendant's minimum possible payment, and vise versa.

for defendant's settlement demand $y_H\ (x_H - x_L\ )\ \geq\ k_A\ +\ k_B$

for contractor's settlement demand $x_H\ (y_H - y_L\ )\ \geq\ k_A\ +\ k_B$

4.6.2. Intuitive and Divinity Refinements

Bayesian Nash Equilibrium and sequential equilibrium has provided multiple restrictions to the outcome of this game. In addition, Intuitive Criteria and Divinity Criteria (D_1) are widely accepted refinements that provide additional restrictions to the range of possible outcomes. Based on refinements of Divinity Equilibrium, all pure and semi-pooling equilibria are eliminated. Therefore, optimal settlement offer by B, S_B^* , to maximize A's expected payoff needs to satisfy the following criteria:

$\mathrm{E}[a'_X(x)] = x$ or $\mathrm{E}[b'_X(x)] = x$

$\mathrm{E}[a'_Y(y)] = y$ or $\mathrm{E}[b'_Y(y)] = y$

As a result, B's optimal settlement offer should update A's beliefs in a way that A's posterior belief equals to B's beliefs over true level of Damages and Liability. This would requires full reveal of the private information by both parties

5. Legal Reasoning with Bayesian Networks

This Chapter provides a framework to model parties' legal arguments for claims as presented in the previous Chapter. Bayesian Networks are used to measure parties' beliefs over the dispute elements, Liability, defined as the likelihood of the defendant being found liable at the trial, or the portion of the damages that the defendant will be held liable at the trial and Damages, or the unanticipated expenditures plaintiff may have incurred due to the defendant's alleged fault. Disputed elements are classified into measurable variable and the model then delivers structured patterns to define dependencies among the variables. Parties' arguments or counterarguments are formed based on assembling the structure patterns together. The Bayesian Networks update parties' beliefs over Liability and Damages after observing new information. The models in this chapter become inputs to the game theoretic model discussed in Chapter 4 - Modeling Settlement Negotiation.

5.1. Opposition to Statistical Analysis in Legal Practice

Although scientific research has made progress on quantifying legal reasoning, the statistical analysis often faces objections by the legal community. Despite significant literature on the application of Bayesian theory in quantifying legal arguments,[3] especially in criminal cases, legal communities are resistant to using statistical analyses for legal arguments is because it is considered to be overreliance on the use of the likelihood ratios and odds as probability measures. In addition, there are complicated underlying mathematical calculations, or engaging probabilities,[4] in calculating the verdict.

The concept of probability and prediction are often misinterpreted in the legal community. For example, one may argue: "a defendant is either liable or not liable; there is no such verdict that says the defendant is liable with probability of 70%." To respond to this concern, Liability is defined as a party's prediction on the Judge's decision over this element in future (at the time of trial). This should not be misinterpreted as the partial liability of the party. In fact, Liability is the likelihood that the judge rules the verdict. Once the Judge makes its decision on Liability (i.e., guilty, or not guilty) there is no proportionality on Liability of a party.

Subjective beliefs as prior probability is another objection of the legal community in applications of the Bayesian theory in the law. This research addresses the problem with subjectivity of beliefs by using ranges of probabilities for the variables (i.e., probability

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[3] *Legal argument refers to any rational discussion presented as part of support for legal cases.*

[4] *Probability is a statement expressing an uncertainty about an event happening. Probability can be expressed as percentages. In the law literature it is common to express probability by odds. Odds are defined as a ratio of a chance that an event not happening divided by the chance of the event happening. For the purpose of this research all probability expressions are converted to percentages.*

distribution functions with continuous variables). Details regarding using continuous variables in game theoretic analysis is provided in Chapter 4.

5.2. Bayesian Networks

The Bayesian Network (BN) is a graphical tool to calculate causal dependencies among a set of variables. The BN graph is in a form of nodes and edges. The nodes correspond to variables, and the edges correspond to the direction of the causal dependencies between the nodes. Each node has an underlying probability distribution called Node Probability Table (NPT). The NPT defines conditional dependencies between a variable node and its parent node(s)[5].

Bayesian Networks contain two main components, graphs and NPTs. The BN graphs represent the variables in the model as nodes and show the direction of the edges between the variables. NPTs are sets of conditional probability tables (or probability distributions) that define the interrelationship between the nodes. For example, the application of NPTs in this research outlines the underlying interrelationships between the variables of a claim so that each NPT define the impact of each cause to its parent node(s).

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[5] *The direction of the edge in Bayesian Networks is from the parent node to its child node*

5.3. Steps in Building a BN Model for Claims

This section illustrates building a Bayesian Network for claims. The first step is to identify the sets of variables (nodes), second, to identify the states for the variables. The third step is to define the edges between the nodes as the casual relationships among these variables. Last, the NPTs for each variable form dependencies between the parent and child nodes. The following describes these steps in detail.

5.3.1. Step One: Identifying the Set of Variables

There are two different groups of variables, inputs and outputs. The input variables for construction claims include: relevant contract languages, factual evidence, and strength or credibility of the arguments. The output variables for construction claims are Liability and Damages. The output variables are considered to be independent, meaning changing Liability does not impact Damages, and changing Damages does not impact Liability.

5.3.2. Step Two: Identify Set of States for Each Variable

Each variable can be either discrete or continuous. The number of states for discrete variables varies depending on the measurement accuracies. The Bayesian Network allows continuous variables as long as they are discretized[6]. Depending on the accuracy of the data, the number of states can be selected for each node.

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[6] *Discretization is a process of defining continuous variables into discrete intervals. This process makes the continuous variables suitable for numerical calculations in computerized BN modeling.*

5.3.3. Step Three: Identify the Direction of the Edges between Variables

Edges represent relationships between each two sets of variables (nodes). This direction clarifies the direct dependencies between the parent and child nodes. Mathematically, the direction of the edge between cause and effect does not change the final results as long as the NPTs are adjusted accordingly. In general, if there is a clear cause and effect relationship between two node, the direction would be from the cause to the effect. However, if the prior probability of the effect is more readily available than the cause, the direction can be set from the effect to the cause.

5.3.4. Step Four: Specify NPTs for Each Node

NPTs are sets of conditional probability tables (or probability distributions) that define the interrelationship among variables. The NPTs structure shows how the inferences and learning occur in the BN model. Considerations of defining proper NPTs are discussed individually for each structure pattern.

5.4. Bayesian Reasoning for Civil Cases

Bayesian Networks can improve understanding of legal arguments by quantifying its underlying rational. The model provided in this research defines the causal relationship between the variables of a legal arguments. The underlying calculations of this casual model allows claim parties to form a beliefs over the claim variables, and update those beliefs after observing new information.

The application of the Bayesian Network to the law has grown in recent decades. However, as noted, the literature is mostly focused on modeling criminal cases. The following provides details on transitioning the advanced models from criminal cases to a model for criminal cases. The differences between civil and criminal cases are briefed below in Table 5-1.

Table 5-1 Comparison between Criminal Cases and Civil Cases

Comparison	Criminal Cases	Civil Cases
Players	Judge vs Suspect	Plaintiff vs Defendant
Hypothesis	Suspect is innocent	Opponent is at fault
Evidence	Evidence to prove guilty	Evidence to prove faulty action
Constitution	Law	Contract Agreement

Defining similarities between the civil and criminal cases helps to use the similar concepts developed primarily for criminal cases. The following provides details regarding the contrast between criminal and civil cases:

- In criminal cases the Judge institutes legal proceedings against the suspect. In civil cases the plaintiff files a claim against the defendant.
- The Judge's hypothesis in a criminal case is the exact opposite of parties' hypothesis in civil cases. For example, the plaintiff starts with a hypothesis that the opponent is at fault and finds evidence in support of his claim. On the other hand, in criminal justice the Judge first assumes the defendant is innocent until proven guilty (*Presumption of Innocence)*. The Presumption of Innocence is a hypothesis developed by the prosecutor over the innocence of the suspect. The prosecutor then finds factual evidence in support of the case, or in substantiation of the accusations.

- In civil cases, litigants make assumptions about their opponent's guilt (or their own innocence). For example, a plaintiff typically assumes that the defendant is at fault and then tries to find evidence to support this hypothesis. A defendant, on the other hand, assumed no Liability for himself for the complaint prepared by the plaintiff. These hypotheses form the prior beliefs for both parties over their Liability and Damages.
- In criminal cases, the prosecutor considers evidence to define the innocence or guilt of the suspect by applying an appropriate law to the case. In contrast, in civil cases the prosecutor applies the contract agreement to the case given factual evidence.

5.5. Probabilistic Reasoning of Legal Evidence

The process of defining legal reasoning contains a hypothesis and evidence as a support to truthfulness of the hypothesis. Each party starts with a hypothesis ***H*, (*opponent's failure to fulfill a contract requirement*)**, and for the hypothesis there is evidence, ***E*,** defined as the factual event that supports parity's failure to follow that specific contract requirement.. **Evidence** updates parties' beliefs over the hypothesis. Figure 5-1 shows the legal reasoning as a simple Bayesian Network model.

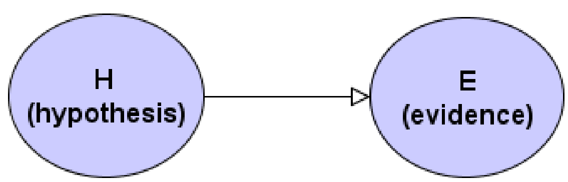


Figure 5-1 Hypothesis Evidence Relationship

The direction of the causal structure indicates that if the Hypothesis ***H*** is true, the probability Evidence ***E*** being true increases. If Evidence ***E*** is proven to be true, player's belief in Hypothesis $\boldsymbol{H}$ being true increases. Therefore, the Bayesian inference between $\boldsymbol{H}$ and $\boldsymbol{E}$ can go in both directions.

5.5.1. Plaintiff's Reasoning

The legal reasoning described above is consistent with Bayesian inference. The Plaintiff starts with $\boldsymbol{H}$ and prior probability $P(\boldsymbol{H})$ for the hypothesis (***defendant is at fault)***. The probability of observing $\boldsymbol{E}$ given $\boldsymbol{H}$ is true denotes $\boldsymbol{P}(\boldsymbol{E}|\boldsymbol{H})$. Therefore, observing $\boldsymbol{E}$ updates plaintiff's prior probability over $\boldsymbol{H}$ by backward propagation. Propagation is the updating process in a Bayesian Network model.

In claim proceedings, both parties observe new evidence on the specifics of a case. Observing new evidence includes finding new information found during document investigation, or revealing information during parties' communication and information transfer between them. Taking the new evidence into the equation may change parties' beliefs on Liability and Damages. In the Bayesian Network, model observations are used as inputs to update the marginal probabilities of all the unobserved variables.

5.5.2. Defendant's Reasoning

Defendant starts with a complementary prior probability for the hypothesis *"defendant is not at fault."* A defendant consider itself at no fault until proven guilty. Once the plaintiff submits its complaint, the defendant updates his beliefs over Liability. Plaintiff's complaint is defined as the document that plaintiff submits to the Judge as his statement of claim. This document typically contains legal reasoning, argumentation, and evidence to support parties' entitlement to Liability or Damages.

5.5.3. Modeling a Legal Reasoning

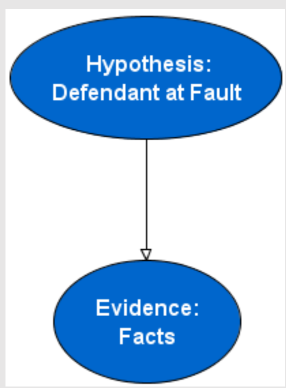
The Plaintiff starts with the Hypothesis H "defendant at fault," and prior probability of $a^o(H)$. The Defendant, on the other hand starts with the reverse Hypothesis "defendant at no fault" and prior probability of $b^o(H)$. Then, parties connect their available evidence E to their hypothesis. The Bayesian Network provides the following model to calculate updated beliefs over H, which is the conditional probability of E given H.

NPTs | A - Plaintiff | B - Defendant

Parties' Prior Beliefs[7]

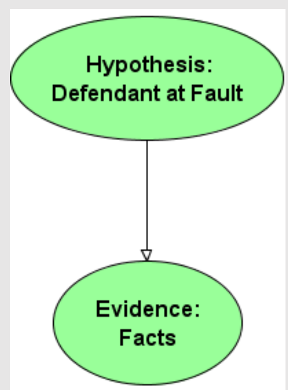
A - Plaintiff:

$a^o(H)$	
False	0.01
True	0.99



B - Defendant:

$b^o(H)$	
False	0.99
True	0.01



NPTs for the Evidence Nodes

A - Plaintiff: $a(E|H)$ or $a'(H)$

Hypothesis: Defendant is at Fault	False	True
False	0.99	0.01
True	0.01	.99

B - Defendant: $b(E|H)$ or $b'(H)$

Hypothesis: Defendant is at Fault	False	True
False	0.99	0.01
True	0.01	0.99

Figure 5-2. Parties' prior beliefs and NPTs

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[7] *It is assumed that patties always leave a small probability for their hypothesis to be wrong. This idea is represented by allocating 0.99 probability in the NPTs rather than 1.0.*

The basis of legal reasoning includes inference from Evidence E to Hypothesis H. This reasoning is a perfect match for Bayesian inference where the prior assumption over H and the likelihood of the evidence E is captured formally by the node probability tables.

Bayes' theorem provides the following equation for updating parties' prior beliefs about H in light of observing E to arrive at a posterior belief over H, $P(H|E)$:

$$P(H|E) = \frac{P(E|H)\,P(H)}{P(E)}$$

5.6. Building Legal Arguments Using Bayesian Network Model

This Chapter provides a systematic approach for modeling legal arguments with Bayesian Networks. The approach contains structure patterns that define repeatable arrangements for a set of nodes. The structure patterns are built based on previous models of both Hepler and Fenton. Hepler introduces an object oriented approach (Hepler & Dawid, 2007) and Fenton uses a notion of idioms[8] for development of legal arguments (Lagnado, Fenton, & Neil, 2013). These two methods are focused on developing legal

[8] *Idioms are defined as single cause-effect BN structures that are a part of a bigger BN model. Combination of multiple Idioms can form a complete BN structure for a case. In this research I use the term Structure Pattern in lieu of the word Idiom.*

arguments for criminal cases. The following illustrates how those concepts are converted to a model for civil cases using the comparison analyses discussed in the previous sections.

Legal arguments often involve multiple variables with complicated interdependencies. This section articulates complex legal issues into a simplified model. The dependencies are defined in repeatable structure patterns, single cause-effect BN structures which are similar to the concept of Idiom by Fenton. The structure patterns breakdown the node interdependencies into distinguishable variables. The structure patterns connect together to form a larger structure that defines parties' beliefs over Liability and Damages. This section provides examples of structure patterns specific to legal arguments. The repeatable structure patterns reflect the way human mind analyzes complex cases to develop legal reasoning and sound arguments.

5.6.1. Evidence Pattern

The Evidence Pattern is the cause-consequence structure that models the uncertainty based on observable evidence. This pattern resembles legal reasoning that creates connection between the Hypothesis and the Evidence in a civil case:

- The Hypothesis is the opponent's failure to fulfill a contract requirement. The plaintiff, for example, believes that the defendant failed to perform the work as specified in the contract.
- The Evidence is the factual event that supports party's failure to follow that specific contract requirement. Evidence is typically investigated by claim analysts or experts.

The purpose is to attribute a value to Liability[9], the probability of the defendant being held liable at the trial, or the portion of the damages that the defendant will be held liable for at the trial. Figure 5-3 demonstrates the BN structure for a cause-effect relationship between the Hypothesis and the Evidence.

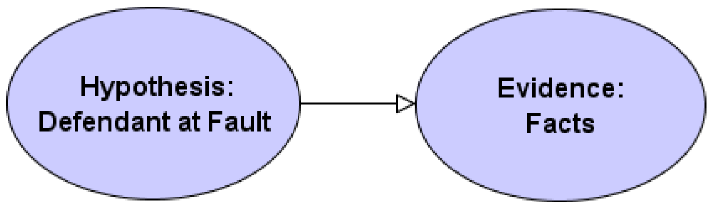

Figure 5-3 Evidence Pattern

Each hypothesis may contain more than one piece of Evidence. Not every Evidence is in support of the hypothesis. There may be contradicting Evidence that declines argument strength and eventually declines the Hypothesis. One piece of Evidence may support that the "defendant is at fault," and the other Evidence supports that the "defendant is at no fault." Therefore, there are two groups of structure patterns for Evidence, corroboration pattern and conflict pattern.

- *Corroboration pattern* is the situation where two pieces of evidence that both support one side of the argument
- *Conflict pattern* is the situation where two pieces of evidence are in conflict with each other, and support different sides of the argument

[9] Alternatively, Liability can be defined as a variable belonging to the plaintiff.

Figure 5-4 shows the evidence structure with multiple supporting facts including corroboration and conflict patterns.

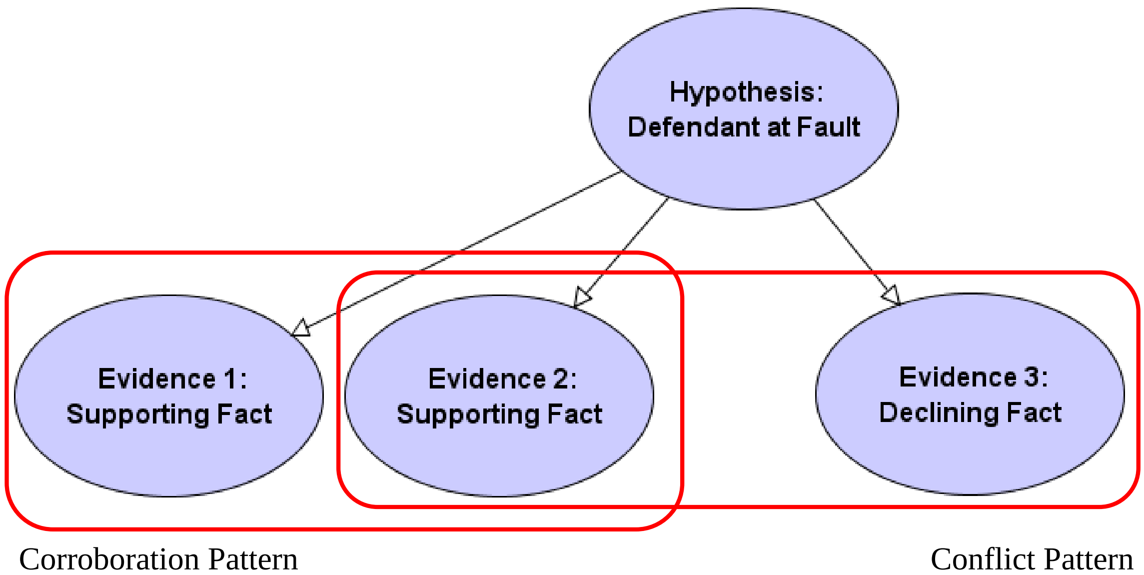

Figure 5-4 Corroboration / Conflict Patterns

Although the supporting and declining facts look identical in the model, the underlying Node Probability Table (NPT) over the Evidence distinguishes between the two factual parameters. One method that plaintiffs often use is to include all possible supporting Evidence. Defendants in return try to disqualify the supporting Evidence by adding the declining facts. Detailed discussions over interaction between the claim parties are provided toward the end of this chapter. The following is a hypothetical example that is used throughout this section to provide a detailed explanation of the application of structure patterns.

Table 5-2 provides the Node Probability Table for the Evidence Pattern that can be used for all evidence nodes with single parent node.

Table 5-2 Node Probability Table for Evidence Pattern

	H: Hypothesis	False	True
E: Evidence	False	0.99	0.0
	True	0.01	1.0

The Lakehouse Example:

*The Lakehouse Example is a hypothetical example to explain the application of Structure Patterns in a claim through the section. Imagine a Contractor and an Owner signed an agreement to build a house next to a lake. As part of their contract agreement, the contractor was obligated to perform soil testing in the design phase to design appropriate foundations.. The Contractor did perform the soil testing and then designed and built the house. A few months after the house was built, the walls started to crack due to unbalanced settlement (**Evidence 1**). The Owner hired an expert to inspect the foundation. The expert identified that the unusual settlement is due to inadequate design for the footings (**Evidence 2**). The Owner then filed a claim against the Contractor for failing to test the existing soil conditions as part of the contract requirement (**Hypothesis**). Contractor received the statement of claim from the Owner. Contractor argued that he did an adequate number of borings as a* *standard practice for soil testing for this size of project, but the borings did not show the poor soil conditions (**Evidence 3**).*[10]

Figure 5-5 provides the model of this hypothetical scenario. (The blue with white text represents the Contractor and green with black text represents the Owner's Evidence.)

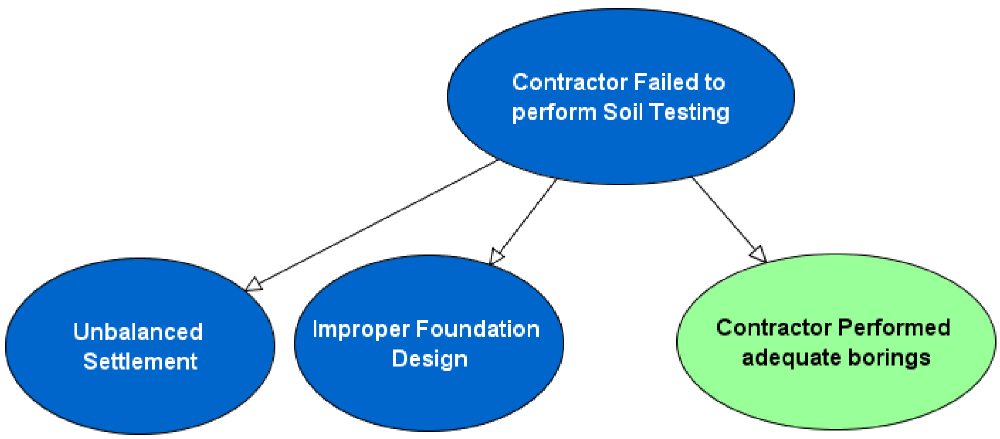

Figure 5-5 - Example for Evidence pattern

5.6.2. Soundness Pattern

There are two main types of arguments in legal disputes, deductive and inductive arguments. A ***deductive argument*** is an argument that a party claims is the truth of its statement (premises or assumptions) and guarantees the truth of its evidence. Inductive argument will be discussed in the Validity Pattern, Section 5.6.3.

[10] *This case will continue in the following sub-sections*

In a deductive argument, the hypothesis provides a full support to the evidence, such that if the hypothesis is true, then it is impossible for the evidence to be false. An argument in which the hypothesis successfully guarantees the evidence is called a ***valid argument***. If a valid argument has true hypothesis, the argument is said to be a ***sound argument***. Figure 5-6 provides the structure pattern for a sound argument.

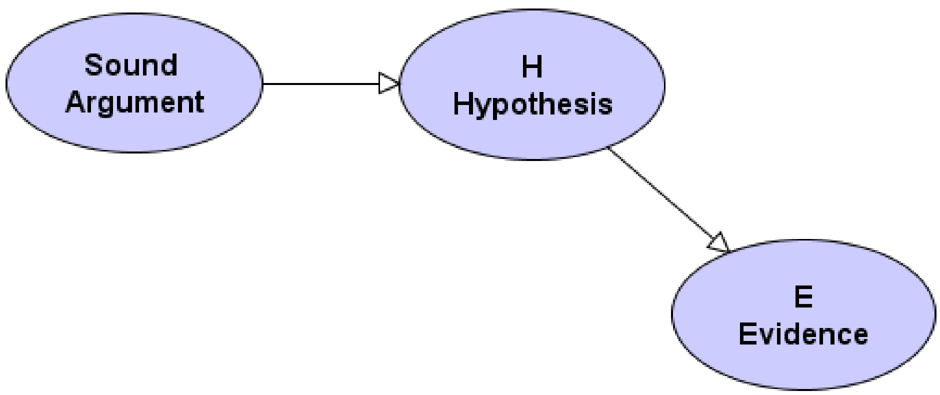

Figure 5-6 Soundness Patter (for Deductive Arguments)

Table 5-3 provides the Node Probability Table for the Soundness Pattern which can be used for all Hypothesis nodes if deductive argument applies.

Table 5-3 Node Probability Table for Soundness Pattern

	H: Hypothesis	False	True
E: Evidence	False	0.99	0.01
	True	0.01	0.99

The Lakehouse Example (Cont.)

Continuing with the Lakehouse example: the Owner argues that the Contractor failed to provide warranty of his design and construction and it resulted in failure of the product. The Contractor responded *that he did his due diligence to design and construct per industry standard, similar to what any other reasonable Contractor would do. Figure 5-5 provides the model of this hypothetical scenario. (blue with white text represents the Contractor and green with black text represents the Owner's evidence.)*

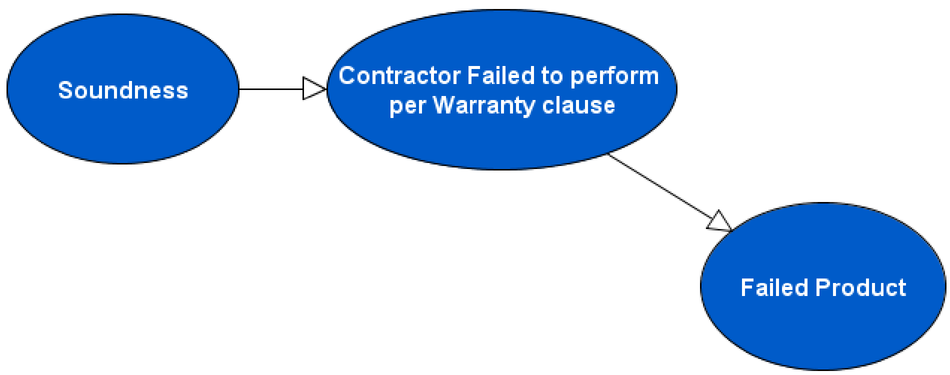

Figure 5-7 Example for Soundness Pattern

5.6.3. Validity Pattern

Typically in legal arguments, the validity (also called legitimacy and accuracy) of each party's argument is directly related to the strength of the link between the Hypothesis and the Evidence. Parties only accept the presentation of an evidence as a proof to the hypothesis if they are strongly connected. Therefore, the conditional probability between the Hypothesis and the Evidence defines the accuracy of an argument.

An ***inductive argument*** is an argument where a party (or party's attorney) attempts to establish or increase the probability of the evidence. In an inductive argument, parties try to increase the likelihood of the hypothesis by providing factual evidence. Proving the truthfulness of a piece of evidence or increasing the number of true pieces of evidence may increase the probability that a hypothesis is true. In this type of argument, the argument strength is conditioned on the overall accuracy or reliability of an evidence node. This accuracy is called the ***Validity*** node, which is measured by a parent node to each Evidence node. Figure 5-8 shows the BN structure of the accuracy pattern.

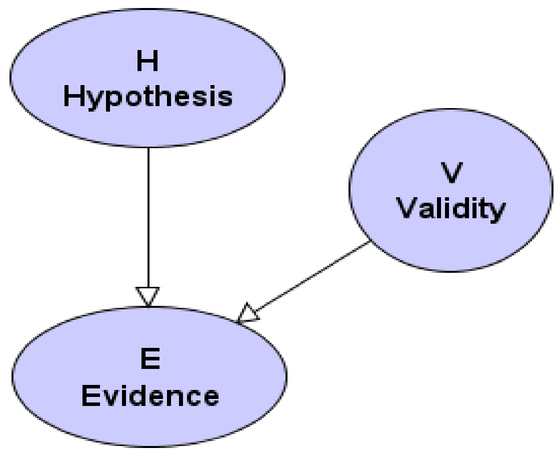

Figure 5-8 Validity for Inductive Argument

The NPT for the explaining away Pattern is provided in Table 5-4.

Table 5-4 Node Probability Table for Validity Pattern

	V: Validity Node	False		True	
	H: Plaintiff's Hypothesis	False	True	False	True
C Constraint	False	0.5	0.5	0.99	0.01
	True	0.5	0.5	0.01	0.99

The validity is designed in a way that if the argument is not valid, then the judge will stay neutral about defendant's liability. If the argument is valid, then the judge will weight that argument in his calculations.

The Lakehouse Example (Cont.)

In the example provided above (Section 5.6.1), the Defendant's failure to perform adequate soil testing gave rise to an improper design of the foundations that then lead to unbalanced settlement of the house. This argument contains a number of assumptions. First, the design of the foundation was truly inadequate. Second, the Contractor did not have any impact on the design process[11]*. Third, there is no other reason besides lack of soil testing that could have resulted in inadequate design and the house settlement. Fourth, the Plaintiff provides the evidence in the entirety (i.e., all facts are presented). In fact, responding to any of these assumptions changes parties' beliefs about the argument. The Validity Pattern takes all of these four considerations into account as shown in Figure 5-9.*

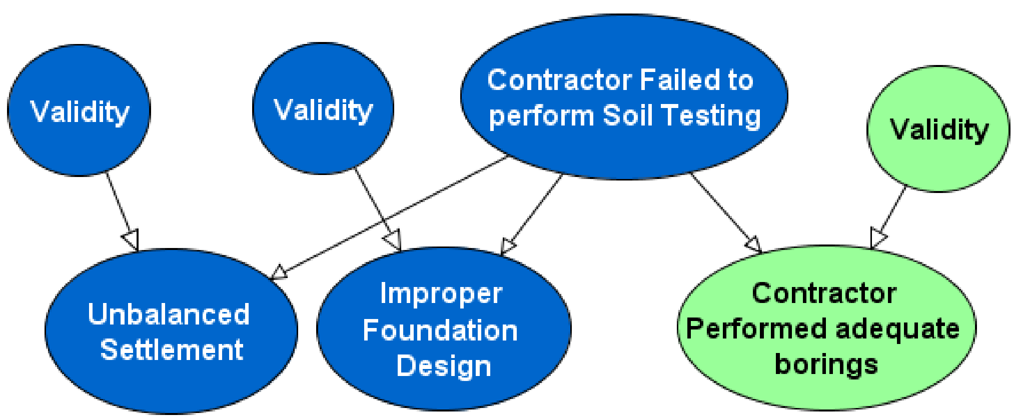

Figure 5-9 Example for Validity Pattern

[11] *This fact is called contributory negligence, when one party contributes to another party's faulty action.*

Validity of an argument impacts the strength of the argument by influencing the evidence. This aspect of the evidence can be lumped into one node in the Bayesian Network model. Validity is described as the underlying assumption about the accuracy or reliability of a piece of evidence. The structure pattern to model the strength of an argument measures the level of truth for the hypothesis. The more accurate level of evidence results in a closer value of the evidence to the true value of the hypothesis.[12]

The Validity node is an input to the model and represents subjective beliefs of the attorney(s). This pattern clarifies what inferences should be drawn from a piece of evidence. In practice, this concept is important becuase calculating the strength of the case is done by subjective judgements of attorneys and experts on the argument.

This research uses a binary variable for the Validity node. The prior beliefs come from the attorney's subjective judgement on the strength of each evidence to its hypothesis. Updating the node occurs by using the true/false scenario for the node in BN software. If there is almost no uncertainty about the relationship between the evidence and the hypotheses, the scenario is true. If the evidence is either inaccurate or unrelated to the hypothesis, the scenario is false.

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[12] *The validity node introduced here is similar to the accuracy node mentioned by Fenton (N. Fenton, Neil, & Lagnado, 2013) which is used for criminal cases, and also similar to the reliability node introduced by Boven and Hartmann (Bovens & Hartmann, 2003) used for reliability of measurement instruments.*

Although inductive arguments are more common in legal arguments, the deductive argument also exists. However, deductive arguments are not useful in this model as they usually result in summary judgement, dismissal, or settlement of the case.

5.6.4. Liability Pattern

The Evidence Pattern (Section 5.6.1) shows the relationship between a single contractual failure and its associated Evidence. In construction claims, it is common to argue a party's failure to meet multiple contractual obligations. Therefore, each claim contains multiple Evidence structures. The Liability Pattern measures the overall Liability of the defendant based on parties' beliefs over the claim elements. In this structure, all hypothesis nodes are linked to the Liability to form a conditional probability based on the NPTs. Figure 5-10 – Liability Pattern shows a general structure model as follows:

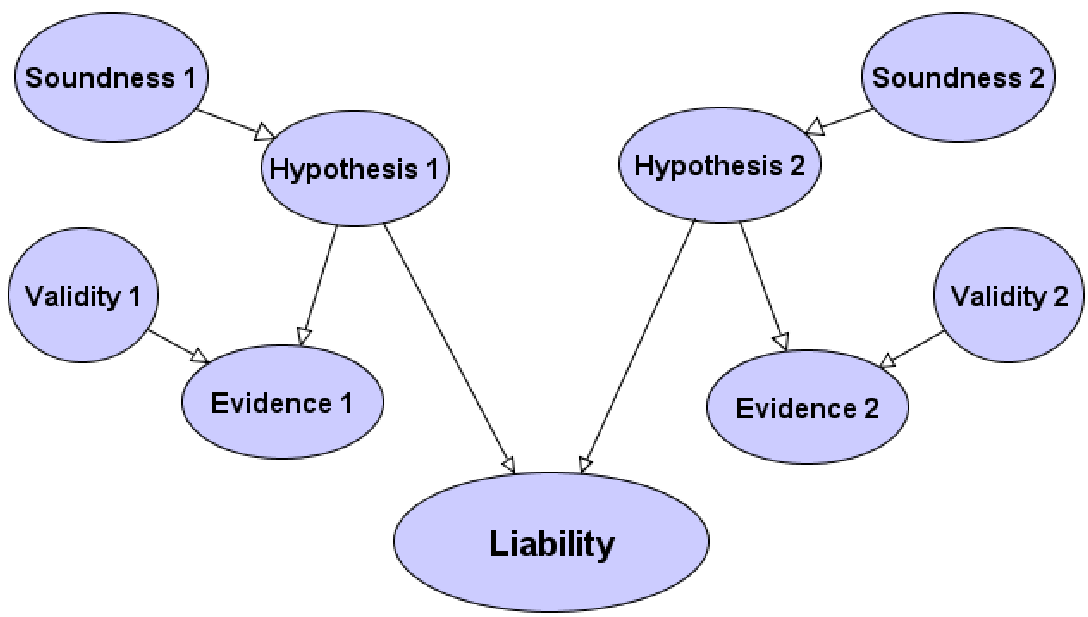

Figure 5-10 – Liability Pattern

The Node Probability Table for Liability may vary based on the number of hypotheses and comparative strength of each hypothesis over the liability of a party. This study suggests continuous node type for the Liability for accuracy. In this case, conditional probabilities are defined using probability functions. This model uses Truncated Normal Distribution[13] to define the end limits for the Liability node. Table 5-5 is an example for NPT of the Liability node.

Table 5-5 Node Probability Table for Liability Pattern

H1	False		True	
H2	False	True	False	True
Expressions	TNormal(0,0.1,0,1)	TNormal(0.4,0.3,0,1)	TNormal(0.6,0.3,0,1)	TNormal(1,0.1,0,1)

The Lakehouse Example (Cont.)

Once both the Owner and Contractor exchange their arguments as shown in the previous sections, they can use the Liability Pattern to form their beliefs over Liability (the probability that the Contractor (defendant) will be held liable at the court). The parties do not necessarily use both Validity and Soundness Patterns at the same time. Figure 5-11 provides Owner's assessment of its Liability.

[13] *Truncated Normal Distribution is the probability distribution of a normally distributed random variable whose value is bounded on below and above limits.*

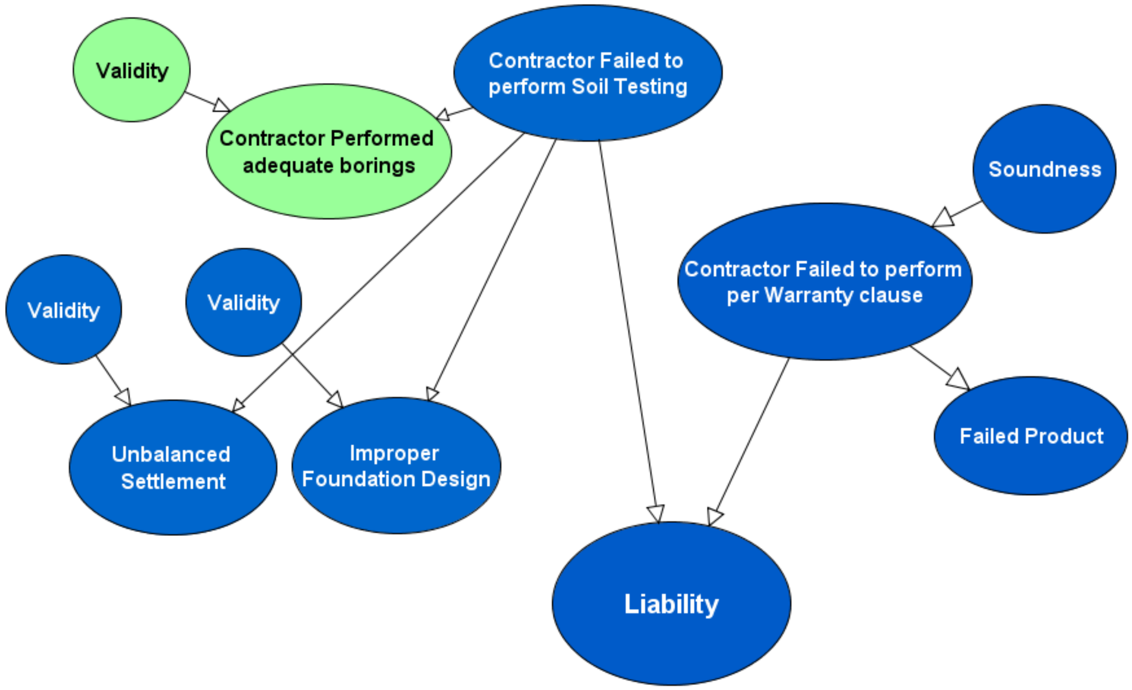

Figure 5-11 Example for Liability Pattern

5.6.5. Explaining Away Pattern

Plaintiff and Defendant, each as players, attempt to disqualify their opponent's argument using a variety of methods. One of the common disqualification approaches is to disconnect the Evidence nodes from their parent node, Hypothesis. For this purpose, a player may introduce a contradictory Hypothesis with the original supporting Hypotheses to argue that the evidence is originated by a different root cause. Since the Hypothesis nodes eventually will be connected to the Liability node, this can directly impact parties estimate on the Liability.

The Explaining Away Pattern includes a new constraint node that has three states, one for each causal path from the hypothesis to the constraint, and an additional state called the *Impossible State*. The NPT for this node is defined such that the Impossible State is only true when both causes are true, or both causes are false as follows[14]

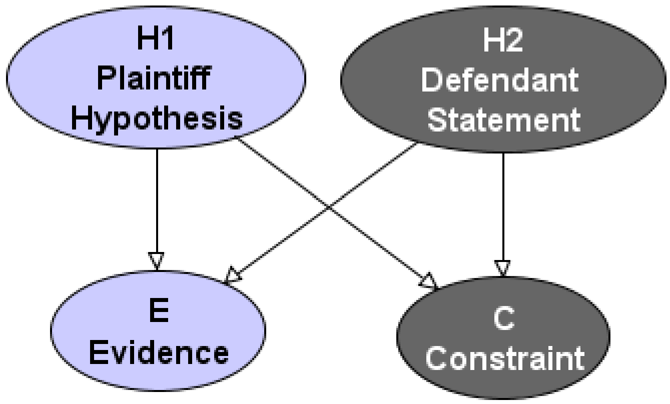

Figure 5-12 Explaining Away Pattern

Table 5-6 Node Probability Table for Explaining Away Pattern

	H1: Plaintiff's Hypothesis	False		True	
	H2: Defendant's Hypothesis	False	True	False	True
C Constraint	False	0	0.99	0.01	0
	True	0	0.01	0.99	0
	Impossible	1	0	0	1

[14] *In order to ensure the impossible node is excluded from the model the constraint node is set as a soft evidence in AgenaRisk™. See (N. E. Fenton & Neil, 2012) for more details*

The Lakehouse Example (Cont.)

In the Lakehouse example, the Contractor argues that the improper foundation design is the result of an unforeseen condition of the soil (Constraint). The Contractor argues that based on the Contract the unforeseen conditions, such as differing site conditions are the Owner's risk, and not the Contractor (Defendant's Statement). Figure 5-13 shows the example of the explaining away pattern.

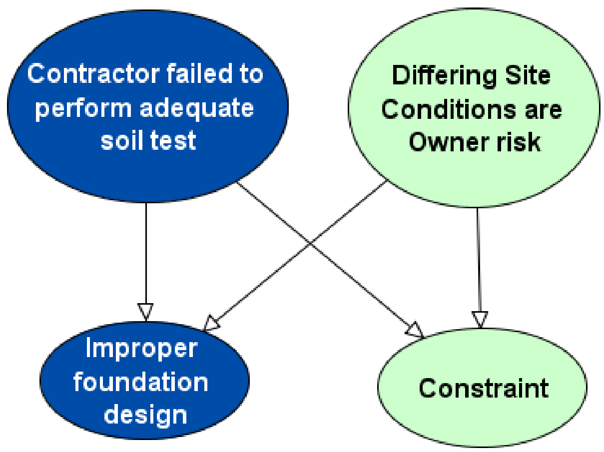

Figure 5-13 Example for Explaining Away Pattern

The Explaining Away Pattern is a tool to use for quantifying attorney's opinion on evidence that can be connected to two contradicting clauses of the contract. This Pattern should only be used if it is impossible to define values for the Validity Nodes.

5.6.6. Damages Pattern

This section provides a structure pattern that defines parties' updated beliefs over Damages using the Bayesian Predictive Model. The structure pattern models parties' updated beliefs by calculating the weighted average of the new information and prior beliefs. The formula for the weighted average is conditioned to the quality of the new information. The quality of the new information is measured by the amount or level of revealed private information[15]. The Revealing node identifies the accuracy or reliability of the new information that as supportive evidence for the true amount of damages.

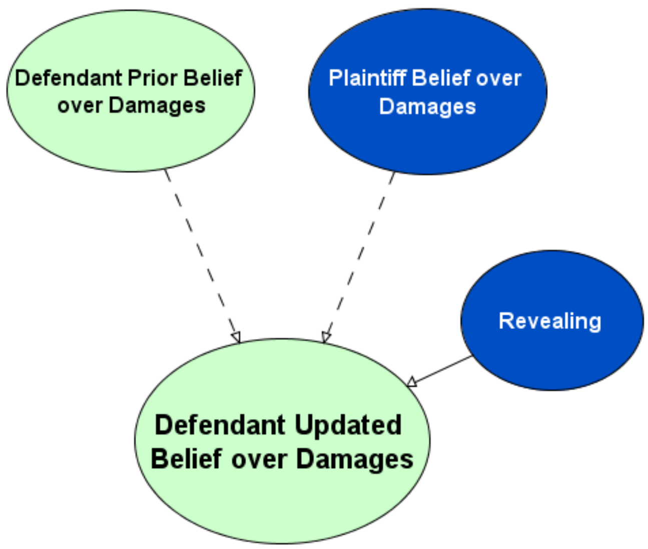

Figure 5-14 BN Structure for Damages Pattern

The dashed line identifies no probabilistic relationship between the nodes. The Revealing node is defined as a ranked node to measure Plaintiff's degree of revealing information to the Defendant. Following Table 5-7 is the NPT for the Updated Belief over damages conditioned to the Revealing node:

---

[15] *The player who holds the private information is called informed player (See chapter 4).*

Table 5-7 Node Probability Table for Explaining Away Pattern

Revealing	**Very Low**	**Low**	**Medium**	**High**	**Very High**
Expressions	$\frac{4\, a_X^o(x) + b_X^o(x)}{5}$	$\frac{3\, a_X^o(x) + 2\, b_X^o(x)}{5}$	$\frac{a_X^o(x) + b_X^o(x)}{2}$	$\frac{2\, a_X^o(x) + 3\, b_X^o(x)}{5}$	$\frac{a_X^o(x) + 4\, b_X^o(x)}{5}$

The Lakehouse Example (Cont.)

In the Lakehouse example, the Owner believes the damages incurred are approximately \$3M. The Contractor estimates Damages to be between \$2M to \$4M with the best estimate of \$2.5M. The Contractor updates its beliefs over Damages once he receives the official claim from the Owner. The Contractor verifies the degree of information revealed regarding the actual damages incurred in the Owner's statement of Claim is low. The Contractor's belief is identified using applying this scenario as follows. (The blue nodes with white text represents the Contractor and light green with black text represents the Owner's evidence.)

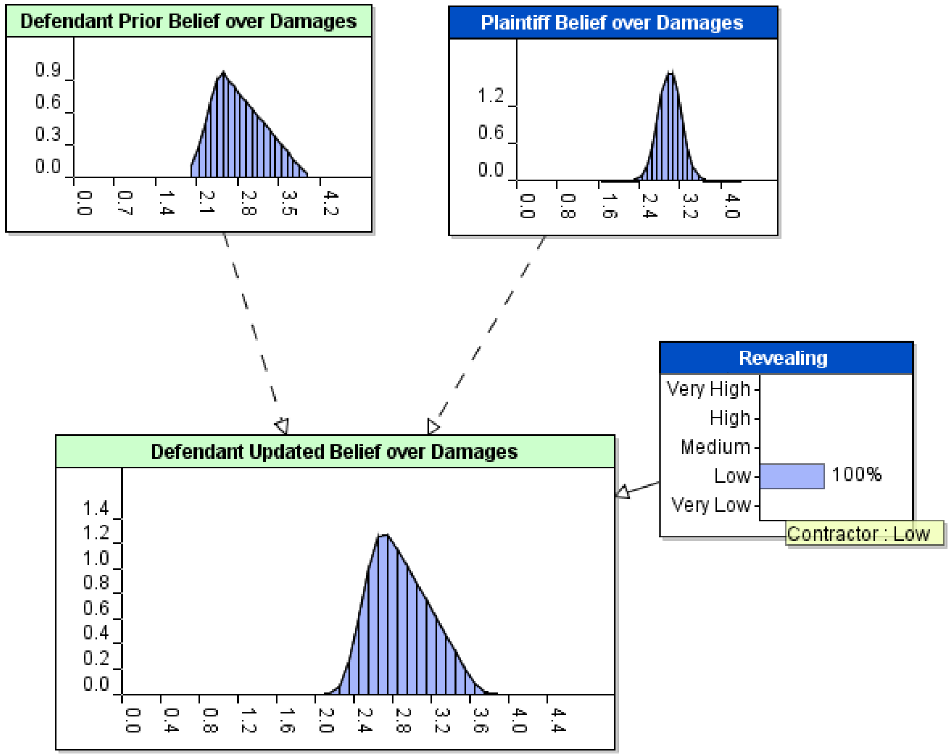

Figure 5-15 Example for Damages Pattern

5.7. Legal Arguments in Bayesian Networks

Legal arguments are a series of statements is typically used to persuade a conclusion that can be reached through logical reasoning on hypotheses. Legal arguments include debates and negotiations to reach a mutually acceptable conclusion. It also encompasses a debate in which victory over an opponent is the primary goal. This art and science is often the means by which parties protect their beliefs or self-interests in rational dialogue in the claims proceedings. Legal arguments are a tool that a party (or party's attorney) presents to the Judge, in testing the validity of certain kinds of evidence.

Legal arguments usually involve a number of hypotheses and are supported by evidence. Hypothesis typically start with a belief that one party has breached at least one of the contract clauses. The hypothesis usually needs to be reinforced by Evidence. Evidence is supporting facts of the case to prove the truthfulness of the assumptions made in the hypothesis. A group of such hypotheses with their underlying premises, assumptions, and evidence forms the liability of a party in failing to comply with contract obligations.

The Conflict Pattern (Section 5.6.1) and Explaining Away Pattern (Section 5.6.4) are examples of legal arguments using Bayesian Networks. By using the Soundness Patterns (Section 5.6.2) parties' focuses on lowering the accuracy and reliability of their opponent's Hypothesis. By using the Validity Pattern (5.6.3) parties try to distort the direct relationship between Evidence and its Hypothesis.

5.8. Application of Software

Hybrid Bayesian Network is a BN structure that incorporates both discrete and continuous nodes. AgenaRisk™ uses a new iterative algorithm that efficiently combines dynamic discretization with propagation algorithms to perform inference in hybrid BNs. AgenaRisk™ models continuous by numerical approximations using static discretization. Discretization allows approximate inference in a hybrid BN without limitations on relationships among continuous and discrete variables. The implementations require defining a uniform discretization of the states in pre-defined intervals. The more intervals defined achieves the more accuracy, but at a heavy cost of computational complexity and excessive time to run the model.

To keep this model simple, all nodes (except Liability) are considered binary. The accuracy of the model can be increased by defining ranked or continuous nodes. Evidence nodes require expert's inputs. Validity and Soundness nodes require attorneys' inputs.

Hypothesis nodes are outputs of the model which predicts Judge’s decision on each contract clause.

Once the NPT is defined, the Contractor’s belief regarding the truthfulness of the evidence itself can be inserted to the model using scenario function in Agenarisk™. The scenario function allows the structure to insert an observation to a node and calculate updates throughout the structure. The scenario is “Yes” for a pieces of evidence that is believed to be the result of its parent Hypothesis.

6. Case Analysis & Result Discussions

This chapter provides two case studies to show how the game theoretic models introduced in the Chapter 4 and Bayesian Network model introduced in the Chapter 5 can be used together in analyzing settlement negotiations in real-world construction claims.

Researches in the literature of modeling settlement negotiations and in general non-cooperative games often face limited access to the information of real world cases. Construction claims and disputes are usually solely discussed between top level managers and attorneys of the claimant parties. In addition, detailed information and communication between attorneys and their clients or even construction experts are typically privileged and confidential. However, all court documents are typically available to public, those information are often focused on the final verdicts. The information regarding cases that settle before the final trial, or those cases that are never filed in court system would not be publicly available. Since this research models decision making during the dispute process, historical data for individual claims are necessary to model the parties' thought process.

The cases analyzed in this chapter are based on data acquired from real-world construction claims. In addition to acquire access to confidential information for performing case studies, the theories behind the process have been discussed with construction attorneys and attorneys. Due to sensitivity of the information and nondisclosure agreements the case studies are redacted and simplified.

The cases include each party's proposal and claims over the span of the claim process. This include all written documentation regarding initial proposals, plaintiff's claims, defendant's counterclaims, offers, settlements, and verdicts. The people involved in the claims were interviewed to assess their views about the process as follows:

1. Conduct initial interviews and discussions with attorneys,
2. Sign non-disclosure agreements to get access the claim documents,
3. Review, assemble, index, and organize documents,
4. Perform project familiarization including reviews of contract and agreements, project plans, specifications, correspondences, meeting minutes, and other related documents,
5. Identify and analyze specific project issues related to the claim,
6. Conduct interviews with project personnel,
7. Conduct interviews with attorneys and experts,
8. Review expert reports,
9. Prepare summary of parties' claim and defense referencing the contract language,
10. Review Judges' Verdict or Settlement Agreement documents,
11. Analyze the case in the model.

The models were analyzed using AgenaRisk™, a Bayesian Network software application for modeling risk and making predictions about uncertain events. Advanced features of AgenaRisk™ such as ranked nodes, simulation nodes, partitioned expressions, and continuous variable are used to create a predictive model. The Hybrid BN Model[16] provided reflects uncertainties for both discrete and continuous variables.

[16] Models containing mixture of continuous and discrete distributions are called Hybrid Bayesian Network Models.

6.1. Case I – Claim between Contractor and A/E

This section studies a Design-Build project where a Construction Management firm ("Contractor") hired an Architecture/Engineering firm ("A/E") to design a new roadway project for a local government ("Owner"). Contractor encountered cost overruns during the construction phase, after which it investigated the issue and noticed significant differences between A/E's Preliminary Design and Final Design. Contractor priced and bid the project based on a deficient Preliminary Design which lead to Contractor's low estimates and low bid. Later in Final Design, the A/E made modifications to its Preliminary Design which resulted in Contractor's cost overrun. Contractor submitted a claim against the A/E to recover cost overruns due to A/E's negligence in preparing the Preliminary Design.

6.1.1. Project background

The roadway project included construction of a 32-mile highway. The project included construction of reversible tolled express lanes, the addition of general purpose lanes, and installation of toll stations. This Project was developed in cooperation with local and regional stakeholders to relieve traffic congestion. It was also the first project of a multi-phased program that was implemented through a series of contracts.

In this project, Contractor in two separate phases signed agreements ("Contract") with the A/E to provide design engineering services under two separate agreements:

> *1. The Preliminary Design Agreement required A/E to prepare and furnish preliminary design documents ("Preliminary Design") to contractor for use in preparing the proposal submission to the Owner.*

2. The Final Design Agreement required A/E to advance the Preliminary Design documents to prepare Issued-For-Construction design documents ("Final Design").

The Owner specified the scope of work and all related requirements for design-build Contractor to build the project in a format of Request for Proposal ("RFP"). Based on the agreement between Contractor and A/E, the A/E had a contractual obligation with the Contractor to prepare the Preliminary Design which complies with the RFP such that Contractor could rely on that design and perform detailed cost estimates. Contractor then used the detailed cost estimates to price the proposal.

6.1.2. Expert Analysis

A roadway design engineer ("Expert") was hired to perform claims analysis of the project. Expert reviewed project documents including all contracts and agreements, project requirements, proposal and Final Design documents, correspondence, and other related materials. Expert also conducted multiple interviews with project personnel to identify and examine the issues that gave rise to the claim.

Expert reports concluded that Contractor relied on the Preliminary Design to perform detailed cost estimates and a proposal for the Project. Owner subsequently awarded the Project to the Contractor and, in turn, the Contractor commissioned the A/E to provide further design services under the Final Design Contract. A/E while progressing the Preliminary Design to Final Design corrected major portions of its Preliminary Design. Some of these corrections were required to bring the Final Design in compliance with the

Owner's RFP. Other corrections were required to comply with safety codes and roadway standards, and other portions related to constructability issues. Those corrections gave rise to scope growth, large variations in estimated quantities, and additional Project costs.

The scope growth and cost overruns included different elements of the highway such as roadway geometry, pavement, structures, retaining walls, and drainage.

6.1.3. Game Theory Model for Parties Interactions

Chapter 4 provided game theoretic models to analyze construction claims from the economic standpoint. The intent of these models is to identify the optimal settlement amount by predicting the game strategies. The analysis of the game will identify how parties should make decisions based on their beliefs over Damages and Liability. Parties consider their opponent's beliefs over these two variables before taking their actions. The model shows how parties' actions updates their opponent's belief over the private information of their opponent.

The game theoretic model contains three main stages as follows:

Stage 0 – *Formation of Prior Beliefs*

On this stage Contractor emailed the A/E about the potential issues and claims. At this stage both parties gather information before plaintiff files a claim.

Stage 1 – *Information Exchange between the Contractor and the A/E*

At this stage parties formally exchange their beliefs on the specifics of the issues through attorneys. The information exchanges results in updated beliefs for both parties over Liability and Damages.

Stage 2 – *Contractor's Decision Analysis on Settlement*

Table 6-1 – Game Theoretic Interaction between the Claimants

stage	A - Contractor	B - A/E
0	Contractor forms its prior beliefs over Liability $a_Y^o(y)$ and Damages $a_X^o(x)$	A/E forms its prior beliefs over Liability $b_Y^o(y)$ and Damages $b_X^o(x)$
1.1	Contractor submits its claim C_A	
1.2		A/E updates its belief over Liability $b'_Y(y)$ and Damages $b'_X(x)$ A/E responds with defense D_B
1.4	Contractor updated its belief over Liability $a'_Y(y)$ and Damages $a'_X(x)$	
2.1	Contractor's Decision Analysis $\begin{cases} \mathit{Pursue\ Trial} \\ \text{Demand Settlement } S_A \end{cases}$	
2.2	Contractor's belief over A/E's belief	If Contractor Demands S_A A/E updates its belief over Damages $b''_X(x)$ and Liability $b''_Y(y)$
2.3		A/E decides based on its payoff, u_B $\begin{cases} \text{If } S_A \le u_B \implies \text{A would accept} \\ \text{If } S_B > u_B \implies \text{A would reject} \end{cases}$
2.4	Contractor estimates A/E's interim payoff, $\hat{u}_B\left(b''_X(x), a'_Y(y), a_K(k_B)\right) = E[b''_X(x)]\,E[a'_Y(y)] + E[a_K(k_B)]$	
2.5	Contractor believes A/E only accept S_A if: $S_A \le E[b''_X(x)]\,E[a'_Y(y)] + E[a_K(k_B)]$	

6.1.4. Stage 0 – Contractor's Prior Beliefs

At this stage parties have negotiated and exchanged some information in an effort to reach settlement. The settlement has failed, and parties formed their prior beliefs such that they both believe their expected outcome from the trial exceeds their litigation costs. The parties' beliefs over the claim variables are as follows:

Contractor argued that the A/E failed to prepare and submit to Contractor a Preliminary Design that fully complied with the requirements of the Owner's RFP. The RFP stated that the Preliminary Design needed to be sufficient for the Contractor such that it could accurately perform detailed cost estimates to bid the project. Contractor argues that, after the proposal phase, A/E made numerous corrections to its Preliminary Design to produce a Final Design that were fully compliant with the requirements of the Owner's RFP. Many of the design corrections that A/E made to its Preliminary Design to bring the Final Design into compliance with the Owner's RFP resulted in scope growth and cost increases on the Project. A/E's failure to meet its contractual obligation to Contractor was a breach of contract.

In the Preliminary Design Contract, A/E warranted that the Preliminary Design that would fully comply with the requirements of the Owner's RFP such that Contractor could price the Preliminary Design with confidence that the work shown there would comply with the Owner's RFP. The Preliminary Design, in many respects, did not depict the actual scope of work and actual work quantities necessary for Contractor to construct the Project in accordance with the requirements of the Owner's RFP and the Final Design. A/E's failure to deliver on its promise to prepare and submit to Contractor a Preliminary Design that fully complied with the requirements of the Owner's RFP is a breach of warranty.

Contractor also believed that A/E failed to perform all services under its Design Contracts with Contractor in a manner consistent with the care and skill ordinarily exercised by members of the same profession currently practicing on projects of similar size and complexity. This act is considered a violation of the professional standard of care.

Contractor stated that the damages incurred due to changes in the Final Design are approximately $8 million dollars. Contractor also estimates its litigation costs would be approximately $800 thousand dollars.

a. Contractor's Argument 1 at Stage 0

Contractor argues that A/E was negligent in preparing its Preliminary Design. The Standard of Care for the A/E is defined in the contract as follow:

> ***Article IV.E: STANDARD OF CARE***. *The standard of care for all professional Services provided by the A/E pursuant to this Design Agreement shall be the care and skill ordinarily exercised by members of the same profession currently practicing in United States, on projects of similar size and complexity at the time the Services are performed.*

Contractor hired an independent third party designer to review the case and determine whether the A/E followed the applicable professional Standard of Care. The Expert report determined that the other designers in the area with the same professional license would perform differently to avoid the issues caused in this project. Therefore, the A/E is subject to violation of Standard of Care.

- **Contractor's Hypothesis 1:** A/E Violated the Standard of Care based on the contract Article IV.E.
- **Evidence 1:** The evidence to prove violation of Standard of Care is expert testimony provided by another licensed engineering in the same field. In this case the expert provided affidavit to prove this violation. Figure 6-1 provides is the BN model for this argument.

For this argument, Standard of Care, the Contractor provides a deductive argument from legal standpoint. Contractor shows Expert Affidavit as Evidence to A/E's negligence. Evidence Pattern defines the structure of Contractor's reasoning as discussed in Section 5.6.1. The proper way to show the strength of this evidence is to show the truthfulness of the affidavit report via Soundness node. As discussed in Section 5.6.2, the Soundness Pattern is designed to model the accuracy of this argument. Figure 6-1 shows how the Evidence and Soundness Patterns forms Contractor's prior belief over A/E's Liability.

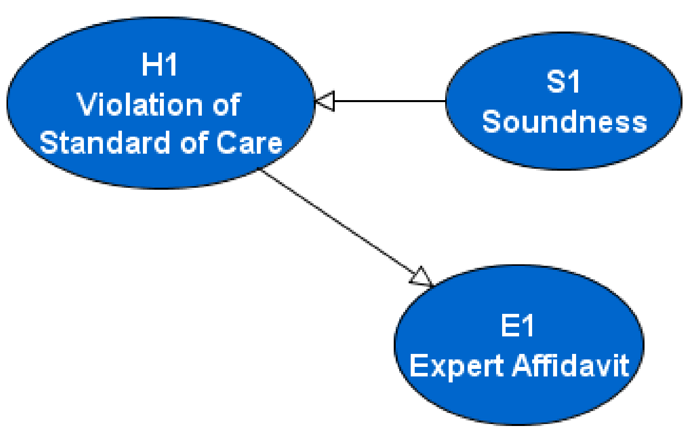

Figure 6-1 Contractor's prior belief over A/E's Liability

The argument 1 is a key argument in Contractor's Claim. Contractor knows that if this argument is proven to be wrong, the entire claim will be questionable. If the Contractor cannot prove that the A/E was negligent and didn't fail to fulfill the requirements of the professional Standard of Care, then Contractor will not be entitled to any of the Damages. Providing NPTs discussed in Section 5.6.4 and adding the Liability pattern results in Contractor's prior belief over Liability s follows:

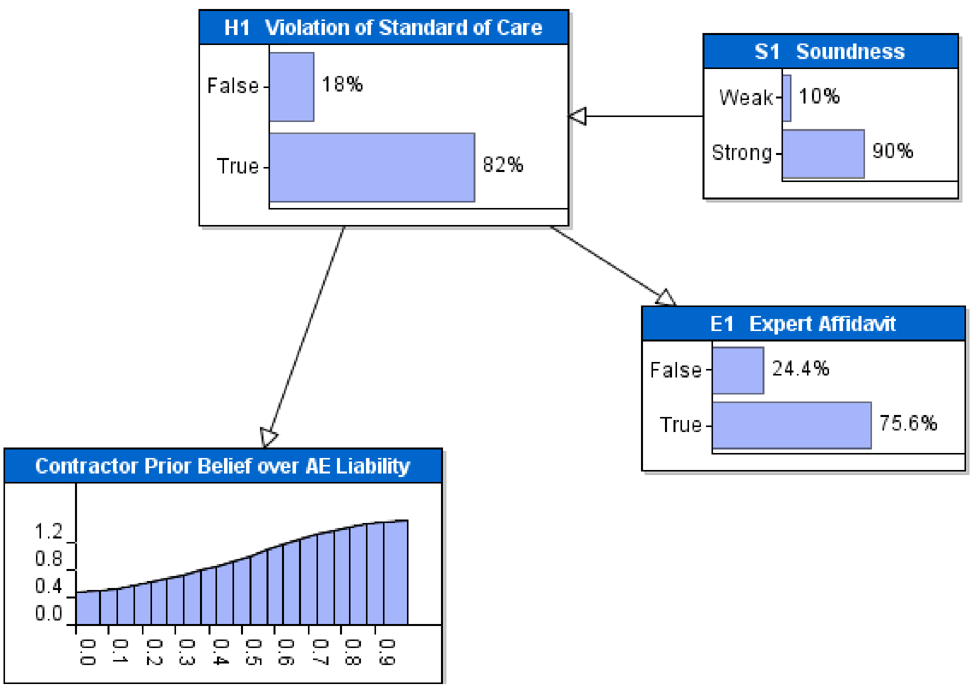

Figure 6-2 – Contractor's Prior Belief over Liability, $a_Y^o(y)$

Contractor's Prior beliefs over Damages are defined by the following truncated normal distribution:

$$a_X^o(\text{x}) = \text{TNormal (8,0.2,0.0,12)}$$

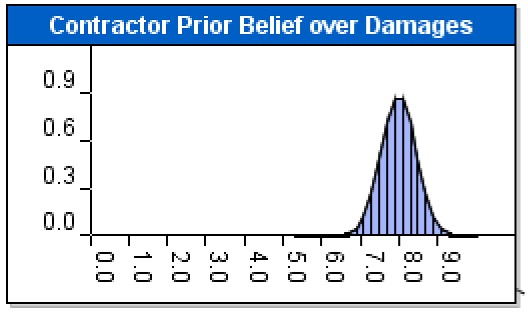

Figure 6-3 – Contractor's Prior Belief over Damages, $a_X^0(x)$

6.1.5. Stage 0 - A/E's Prior Beliefs

A/E on the other hand believed that the Preliminary Design was intended to be a work product which establishes the basis for the Project designs. A/E further stated that the Preliminary Design was not supposed to include all details and full information for the Contractor to build the project. In the standard practice the Preliminary Design will be advanced and refined so as to become the final, "for construction" Project designs.

A/E further argued that the Contractor was involved in all the design decisions that were made during the proposal design. The A/E stated that the Contractor is subject to Contributory Negligence where the Contractor provided input and insights during the design process to the A/E, and A/E's negligence is result of those inputs from the Contractor. Therefore, the Contractor is barred from recovering for damages proximately caused by its Contributory Negligence.

A/E's Prior beliefs over its own Liability is formed based on its disagreement on the soundness of Contractor's argument. This function is defined by applying scenario "Weak" to the soundness node as follows:

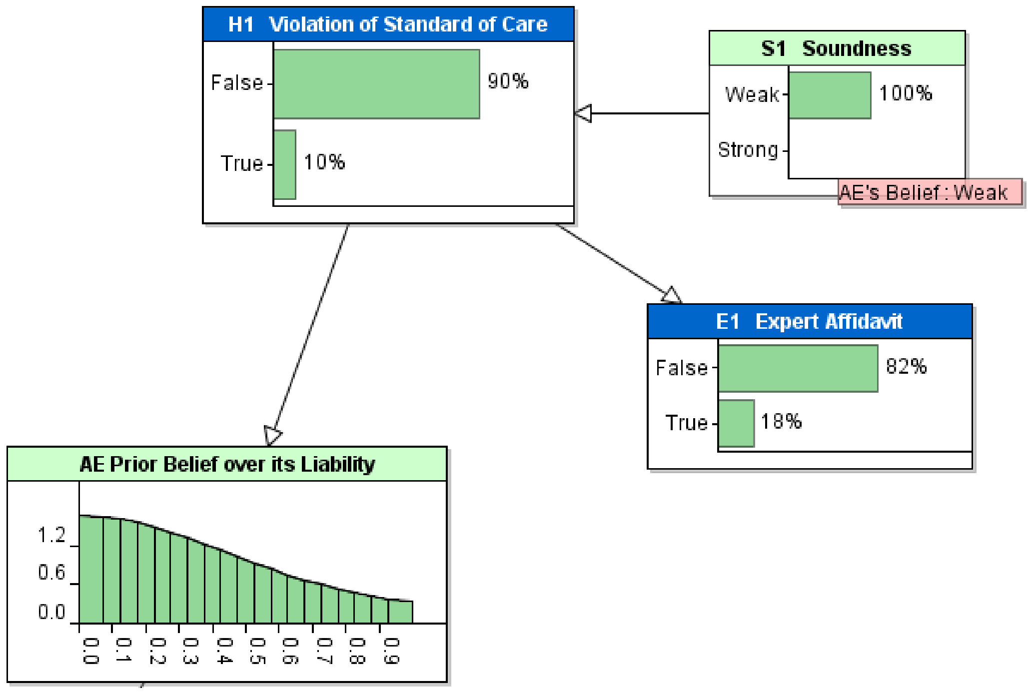

Figure 6-4 – A/E's Prior Belief over Liability, $b_Y^0(y)$

A/E received estimates from its attorneys and experts and concluded that the total litigation costs will be approximately $600 thousand dollars. A/E does not have accurate information regarding the damages, but it estimates the damages to be from $4 million to $7 million dollars with the best estimated of $5 million dollars.

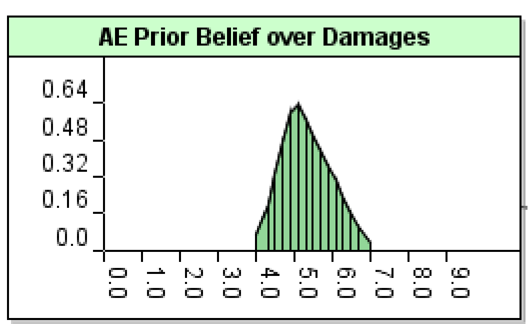

Figure 6-5 – A/E's Prior Belief over Damages, $b_X^0(x)$

6.1.6. Stage 1.1 – Contractor's Claim

Contractor makes a decision whether to pursue the case in litigation. Contractor use attorneys and experts to develop legal arguments including hypotheses of A/E's failure to follow the contract language and evidence(s) to support the hypotheses. In addition to Argument 1 discussed in Section 6.1.4.a, the following are other arguments that Contractor forms against the A/E.

a. Contractor's Argument 2 – A/E's Inadequate Soil Investigation

The contract agreement between the Contractor and the A/E states that the A/E is obligated to perform site investigation to determine soil and subsurface conditions as follow:

> ***Article IV.C.8:*** *Architect/Engineer shall furnish Services of geotechnical engineers and other consultants for determining site, subsoil, subsurface, air and water conditions. Architect/Engineer shall select and pay said geotechnical architect/engineers or other consultants. Such Services shall include, as required, applicable test borings, test pits, soil bearing values, percolation tests, air and water pollution tests, and other necessary operations for determining site, subsoil, subsurface, air and water conditions, with reports and appropriate professional recommendations. Notwithstanding, unless Architect/Engineer fails to perform Services in accordance with the*

standard of care set forth in Article IV.E., Architect/Engineer assumes no liability for unanticipated subsurface conditions, including the locating of underground utility lines.

Contractor believed that A/E failed to consider the existing soil conditions in the vicinity of a road interchange in its Preliminary Design. The interchange involved a series of new bridge structures and ramps located near a railroad. During the proposal phase, A/E had access to all historic data of the soil including borings for the railroad project. The railroad borings indicated the presence of poor soils in the area. In addition, the soil test reports showed high amounts of sulfate in the area. Soil with high sulfate can have negative impact on foundations need to be designed accordingly. A/E prepared and submitted to Contractor a Preliminary Design for the Interchange that did not adequately consider the poor soil conditions and high amount of sulfate. A/E later in Final Design corrected its deficient Proposal Design by increasing the length of bridges and using deeper foundations and drilled shafts to accommodate the existing soil conditions. These modifications also necessitated changes to the roadway (ramps), drainage, and maintenance of traffic design at the Interchange. This variation between Proposal Design and Final Design increased Contractor's scope, work quantities, and costs for the Project.

The Contractor believes that a substantial portion of the scope growth, variations in the estimated quantities, and additional costs that arose on the Project were the direct result of A/E's failure to prepare satisfactory Preliminary Design. Contractor identified the number of individual design errors and/or omissions contained in the Proposal is gross negligence. Contractor believed that the A/E committed breach of contract, breach of warranty, and violations of the professional standard of care.

- **Contractor's Hypothesis 2:** A/E breached the contract Article IV.C.8. The contract clearly states that it is A/E's Liability to perform all the soil investigations and design accordingly. However, the A/E failed to perform adequate site borings and failed to investigate the historic data regarding the subsurface condition.
- **Evidence 2.1:** The length of the bridges were increased significantly from Preliminary Design to the Final design.
- **Evidence 2.1:** The concrete foundations and drilled shafts were bigger and longer in the Final Design comparing to the Preliminary Design to suffice geotechnical requirements for the high level of sulfate and poor soil conditions.

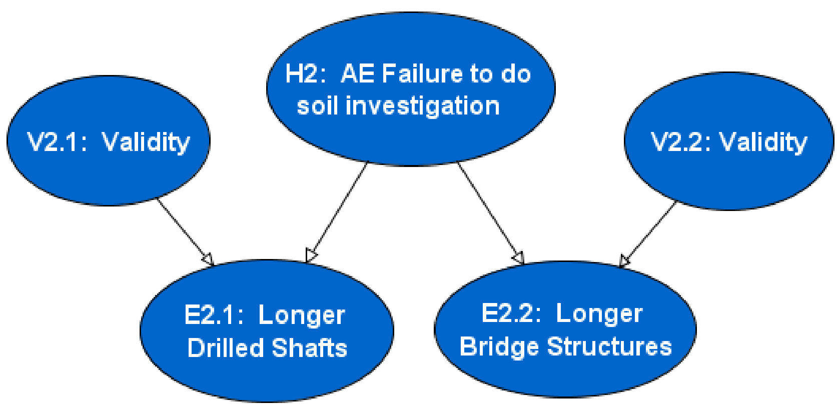

Figure 6-6 Contractor's Argument Regarding A/E's Failure to perform soil investigation

Based on the expert reports as part of Contractor's claims, he believes his argument is accurate and he is confident that the argument will hold the A/E liable for its failures. The reflection of attorneys' belief is defined by the Node Probability Tables for the Evidence Node as described in Section 5.6.1.

H2 ...	Low		High	
V2.1 Validity	Weak	Strong	Weak	Strong
False	0.5	0.9	0.5	0.1
True	0.5	0.1	0.5	0.9

Figure 6-7 NPT for Evidence Nodes

As described in Section 5.8, the scenario function from Agenarisk™ allows the structure to insert an observation to a node and calculate updates throughout the structure. The scenario for all pieces of evidence are "Yes". This means the Contractor believes that the event actually occurred.

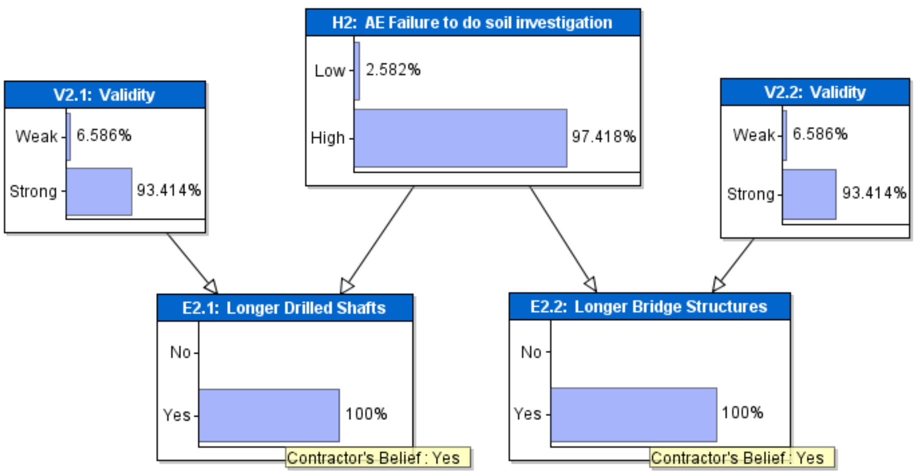

Figure 6-8 - Contractor's Prior Belief for Argument 2

The model incorporates Validity Pattern as discussed in Section 5.6.3. The Validity node measures the strength of the link between the hypothesis and a piece of evidence. The NPT of the Validity node depends on attorney's judgement on the credibility of the argument.

b. Contractor's Argument 3 – Express Lanes at Entrance Ramps

Contractor believed that the A/E failed to provide its Preliminary Design in compliance with the Owner's RFP. Specifically,

> ***Article IV.C.3:*** *The design contained in the Preliminary Design Documents shall substantively respond to, and be in compliance with, the Owner's RFP and all applicable standards, laws, statutes, ordinances, building codes, orders, rules, and regulations.*
>
> ***Article IV.C.12:*** *The A/E shall provide Services reasonably required to* ***fully comply with the requirements of the Owner's RFP****, including the design of the size, quality and character of the Project, its architectural, structural, mechanical and electrical systems, and the materials and such other elements of the Project to* ***permit Contractor to do the cost estimating and scheduling****.*

The express lane included multiple entrance ramps along the length of the highway. There is a toll station at each entrance ramp to the express lane. A/E's Preliminary Design for toll stations failed to comply with the Owner's RFP requirements. The Owner's RFP required the Preliminary Design to include a minimum of 400 linear feet (LF) of entrance ramp with concrete barrier on each side of the toll station. The purpose of express lane is to let traffic go either direction based on the peak hours of traffic.[17] Contrary to this requirement, the Preliminary Design showed less than 200 LF of concrete traffic barrier

[17] The expressed lanes are designed to give additional lanes between the northbound and southbound. Vehicles that are traveling toward the city may use these additional lanes in the morning peak hours. The managed lane changes direction in the afternoon to let vehicles travel out of the city during the afternoon peak hours.

and on only one side of the toll stations throughout the project. A/E later discovered this error was corrected in the Final Design to meet the RFP requirements. To correct its Preliminary Design, A/E increased the length and width of several express lane entrance ramps. This change had a profound impact on the horizontal and vertical roadway geometry at the toll stations and gave rise to scope growth related to numerous Project elements including structures, retaining walls, roadway, and drainage system.

This issue required modification in the design of the interchange egress/ingress ramps and the geometry of retaining walls. The change of retaining walls is evidenced in the Final Design documents which show those retaining walls to be significantly larger than the walls shown in the Preliminary Design. In other instances, A/E had to increase the width of the entire roadway at the toll station in the Final Design to be able to extend the length of the entrance ramps. The widening in turn increased work quantities, and costs associated with roadway work and retaining walls.

A/E's deficient Preliminary Design also occurred at multiple bridges. A/E designed the express lane entrance ramps to begin in the span of the bridge. A/E corrected the Preliminary Design by lengthening the entrance ramps in the Final Design which, in turn, necessitated the realignment of the entire roadway. Consequently, the Final Design included larger concrete decks, concrete piers, and concrete girders for bridges.

The first step in creating the model is to define defendant's Liability using the Bayesian Network model, Chapter 5. Contractor's Argument 3 includes:

- **Contractor's Hypothesis 3:** A/E failed to comply with Owner's RFP as specified in the contract Article IV.C.12 and Article IV.C.12.

- **Evidence 3.1:** A/E designed short entrance ramps for the express lanes.
- **Evidence 3.2:** A/E's failure to incorporate DIS to its Preliminary Design and therefore, missing entrance ramps that are specified in the DIS.

Per Evidence Pattern introduced in Section 5.6.1 the Argument is modeled as follows:

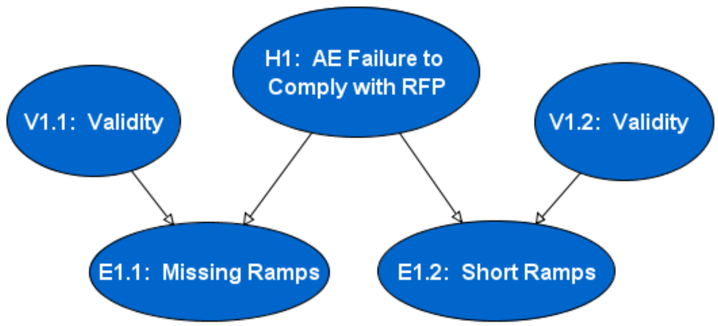

Figure 6-9 - Contractor's Argument Regarding A/E's Failure to Comply with Owner's RFP

Based on the expert reports as part of Contractor's claims, he believes his argument is accurate and he is confident that the argument will hold the A/E liable for its failures. The reflection of attorneys' belief is defined by the Node Probability Tables for the Evidence Node as described in Section 5.6.1. This NPT shows

AE Failure to Comply with RFP	Low		High	
Validity	Weak	Strong	Weak	Strong
Low	0.5	0.9	0.5	0.1
High	0.5	0.1	0.5	0.9

Figure 6-10 NPT for Evidence Nodes

The scenario for all pieces of evidence are "Yes". This means the Contractor believes that the event actually occurred as shown in Figure 6-11.

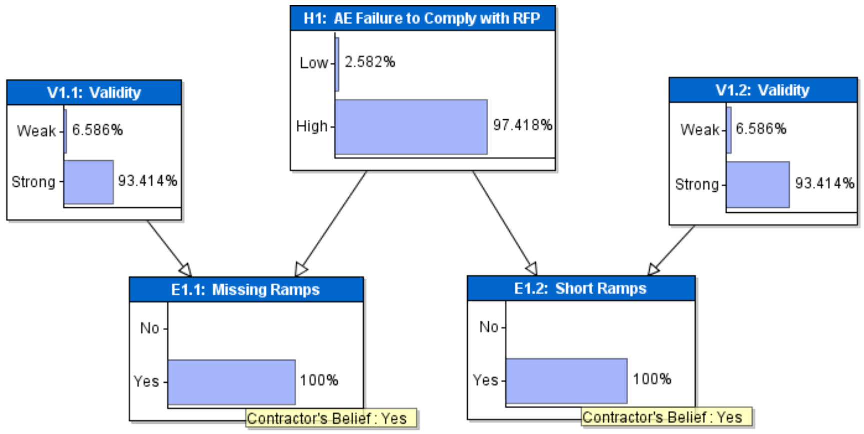

Figure 6-11 - Contractor's (Attorney and Expert) Prior Belief for Argument 1

c. Contractor's updated Belief over Liability

Connecting the hypotheses 2 and 3 to the Liability node results in Contractor's updated belief over Liability as follows.

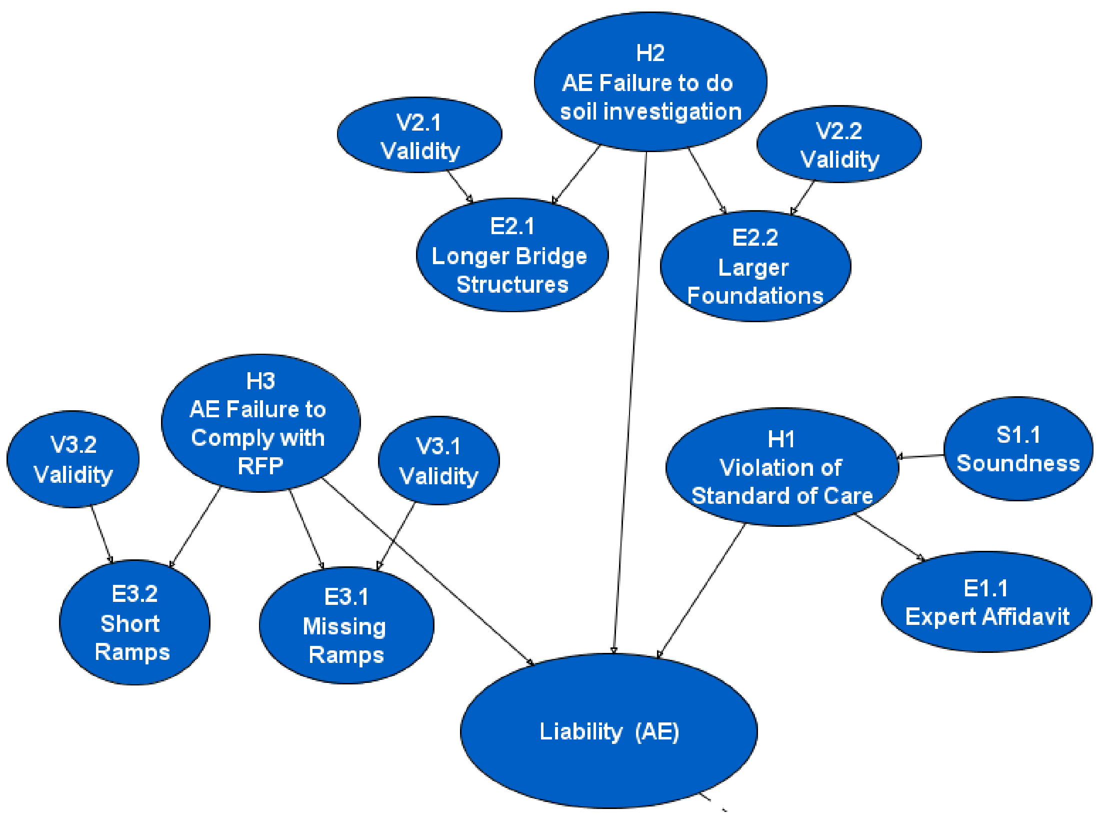

Figure 6-12 BN structure for Contractor's Belief on Liability, $a'_Y(y)$

The next step is to use the Liability Pattern which measures the Liability (for Defendant) that each party assume. The Liability node is a continuous node, which is then discretized using 0.1 intervals. The NPT for the Liability is conditioned based on partitioned expressions using Truncated Normal (TNormal) distributions. The strategy in defining the NPT is to consider Hypothesis 3 (H3), Standard of Care, the main driver of the Liability. If it's proven that the A/E violated the standard of care, the Liability will be significantly higher for other contract failures.

H1: AE Failure to Comply with RFP	Low				High			
H2: AE Failure to do soil investigation	Low		High		Low		High	
H3: AE Violation of Standard of Care	False	True	False	True	False	True	False	True
Expressions	TNormal(0,0.1,0.0,1.0)	TNormal(0.1,0.2,0.0,1.0)	TNormal(0.3,0.3,0.0,1.0)	TNormal(0.8,0.2,0.0,1.0)	TNormal(0.3,0.3,0.0,1.0)	TNormal(0.8,0.2,0.0,1.0)	TNormal(0.5,0.5,0.0,1.0)	TNormal(1,0.1,0.0,1.0)

Figure 6-13 – Node Probability Table for Liability Node

The Argument 2 and 3 in this case are both related to one location of the project. As a result the total damages from both arguments are included into one node. If the arguments are for separate locations of the project, their Liability and Damages need to be calculated separately.

In addition, it is preferred to include the litigation costs in the calculations as a separate node that is deducted from the outcome for Contractor as the plaintiff. Due to software limitations the costs are incorporates into damages.

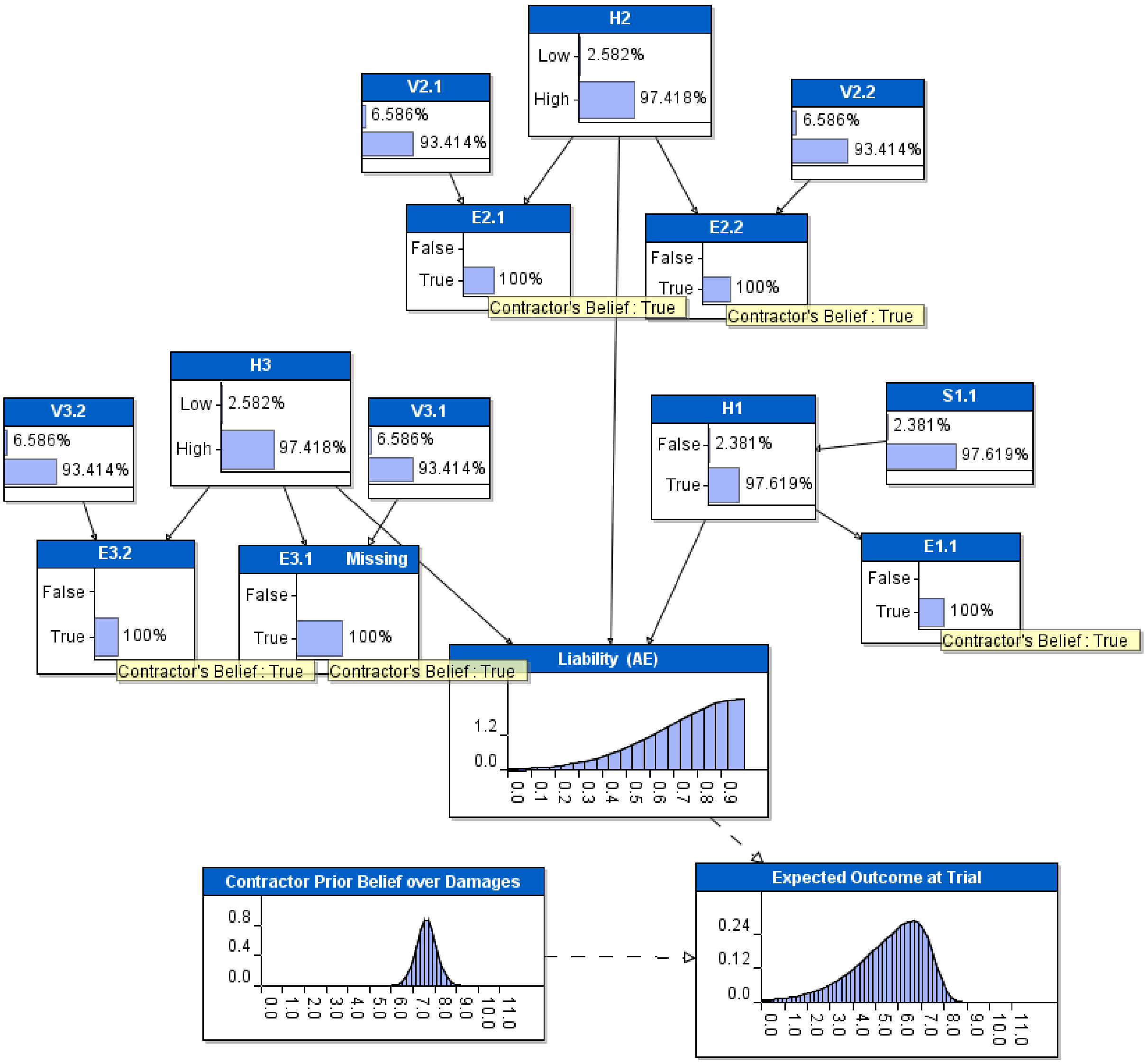

Figure 6-14 Contractor's updated belief over expected outcome

6.1.7. Stage 2.2 – Contractor's Estimate on A/E's Updated Beliefs

Once A/E receives the claim from the Contractor, it makes an effort to disqualify the arguments by associating new reasoning or counterarguments to the original claims. Following are the counterarguments that A/E presents as a defense to Contractor's Claim.

a. A/E's Counterargument 1 – Insufficient Design Time

A/E argued that rarely for such large highway projects a design-build contractor choose to invest substantial funds in pre-proposal preliminary plans. Rather, at the pre-proposal stage the design-build contractor will usually opt for Preliminary Design with little details or dimensioning. The contract agreement provides the following statement

> ***Article IV.A.8:*** *The Parties acknowledge that the Project* ***quantity estimates shall be based upon partial design development****, the RFP documents, publically available reference documents and any studies and tests performed during Proposal preparation. Prior to submittal of the Proposal, the Parties will make a mutual determination regarding quantity contingencies, additional studies and testing required for design development, and probability of substantial changes in estimated quantities.*

A/E believes it followed the industry norm and did not fully designed the entire highway will exhaustive details. The Preliminary Design were not to be used for construction as is standard engineering practice in design-build Preliminary Design. It is the contractor's Liability to determine the financial risks and allocate adequate contingencies to the unknown elements to avoid project cost overruns.

In this counterargument the A/E tries to prove that it did not violate the Standard of Care. A/E does not recognize Contractor's argument soundness and believes it followed the contract agreement Article IV.A.8 and also it is a common practice in the industry to produce Preliminary Design that is subject to minor changes later in the Final Design.

b. A/E's Counterargument 2 – Lack of Design Review Comments

A/E's used contract Article IV.A.5 to argue that the Contractor failed to provide design review comments and corrections as specified in the contract. A/E stated that however the Preliminary Design was not intended to be finalized, the Contractor was obligated to review and raise its concern during the proposal phase. Contractor's failure to review the drawings would not hold the designer liable for deficiencies or omissions during the rough Preliminary Design.

> ***Article V.D.1:*** *Contractor shall* ***meet with the Architect/Engineer at appropriate intervals*** *to review the progress of the Architect/Engineer's Phase I Services, exchange information and* ***provide design review comments*** *for completion of the Proposal and to discuss any other matters requiring Contractor's decisions.*
>
> *This argument applies to all issues including the soil investigation issue.*

c. A/E’s Counterargument 3 – Insufficient Design Time

A/E also counter argued that the Contractor did not allow the A/E sufficient time to review the final proposal submitted to the Owner. Contractor did the quantity take-offs solely on its own and shared the detailed quantity take-offs used for project estimates with the A/E a day before proposal. One day is not considered sufficient time for the A/E to review and comment on the proposal. Therefore, Contractor’s claim regarding the quantities has not merit.

> ***Article III.B.11:*** *The A/E shall prepare and submit to Contractor,* ***Preliminary Design Documents*** *which are compliant with the requirements of the Owner's RFP based on the approach to design and construction of the Project selected by Contractor in* ***sufficient time for A/E*** *to incorporate them into the Preliminary Design and meet the Proposal Deliverables and Schedule.”*
>
> ***Article IV.A.5:*** *The Architect/Engineer and Contractor shall prepare a Phase I Deliverables and Schedule, setting forth the dates for completion of Architect/Engineer's Phase I Services sufficient to* ***allow Contractor and the Architect/Engineer to meet the RFP's requirements and deadlines*** *for completion and submission of the Proposal.*

d. *A/E's Updated Belief over Liability*

According to the counterarguments 1, 2 and 3, as describe above, the A/E would updates its belief over liability as follows:

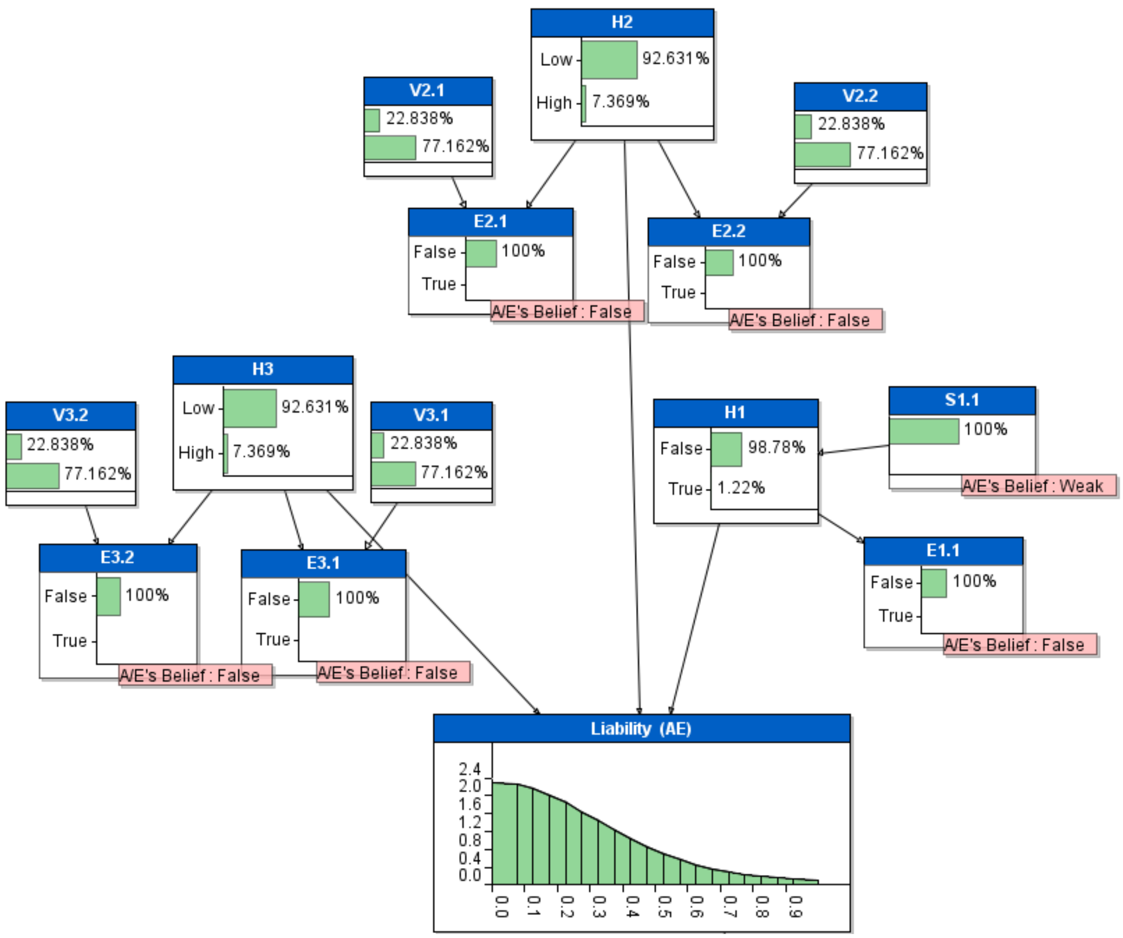

Figure 6-15 A/E's Updated Belief over Liability, $b'_Y(y)$

e. *A/E's updated Belief over Damages*

A/E's updated belief over Damages is calculated using Damages Pattern. The underlying calculation is conditioned to Contractor's revealing of the private information.

Following is the NPT for the Updated Belief over damages conditioned to the revealing node:

Revealing ...	Very Low	Low	Medium	High	Very High
Expressions	Arithmetic((CD_1...	Arithmetic((3*CD...	Arithmetic((5*CD...	Arithmetic((7*CD...	Arithmetic((9*CD...

Figure 6-16 – NPT for Revealing Private Information

A/E updates its belief over Damages given its belief on Contractor's degree of revealing private information as follows.

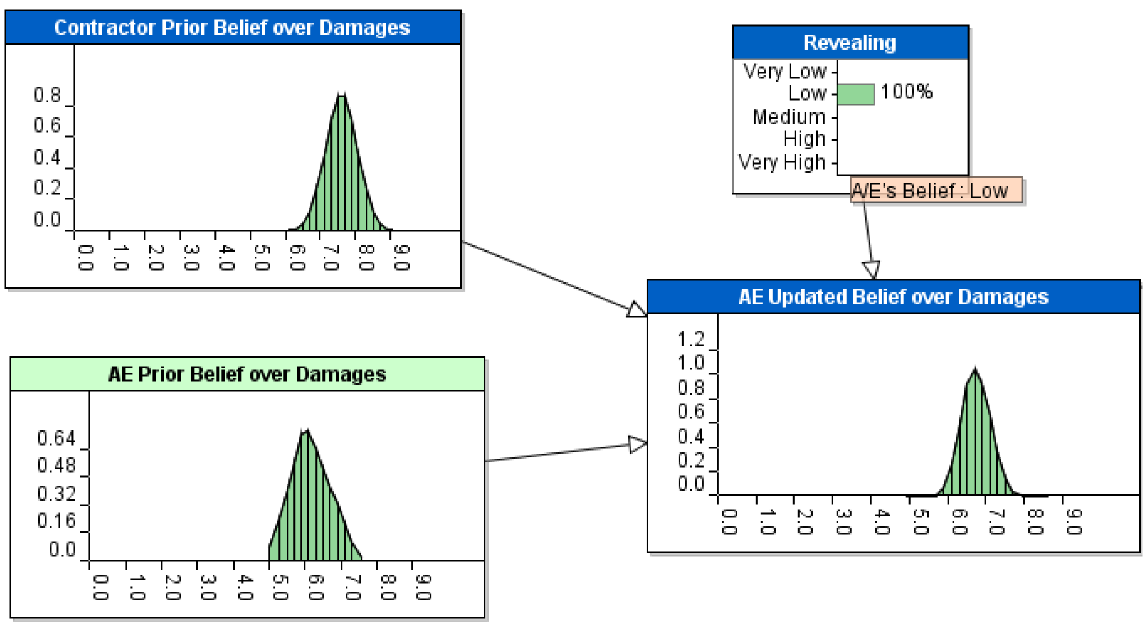

Figure 6-17 – A/E's updated Belief over Damage, $b'_X(x)$

6.1.8. Stage 2.3 – Contractor's Settlement Analysis

Contractor considers its belief over expected outcome (blue distributions) and compares it to A/E's beliefs (green distributions) as shown.

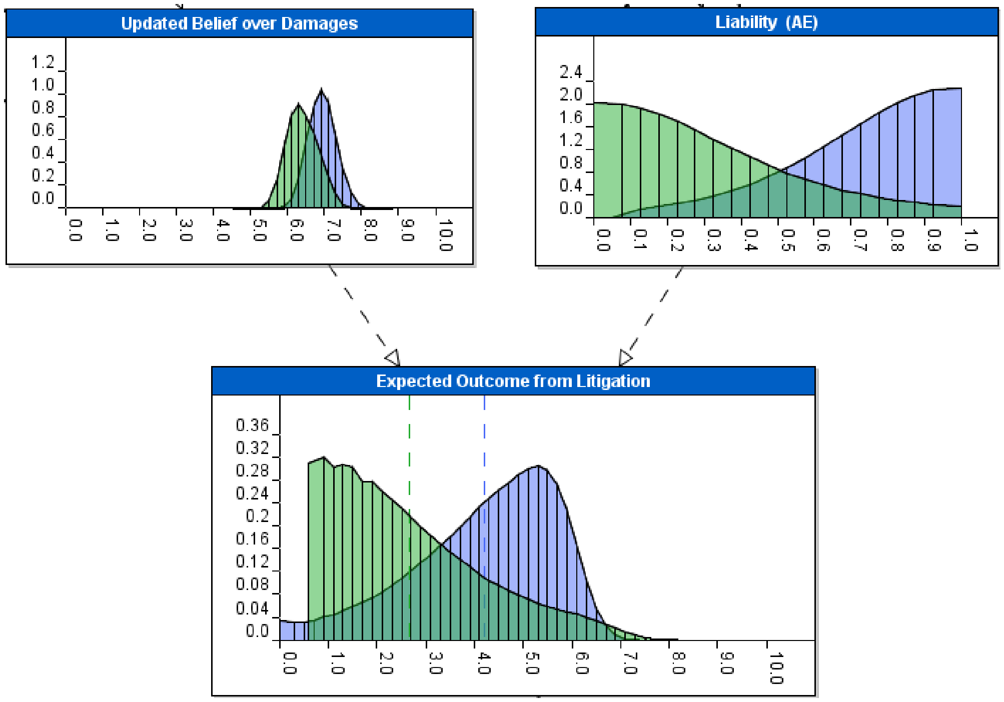

Figure 6-18 – Parties' Belief Regarding Expected Payoffs

6.1.9. Stage 2.4 – Contractor's Settlement Analysis

As discussed in Section 4.4.1, the probability of settlement is directly related to the expected outcome of the party who receives the settlement offer. In this case the Contractor is analyzing to send a settlement demand to A/E. Therefore, A/E's expected payoff from trial defines the likelihood of settlement. This likelihood is defined in form of Commulative Distribution Function (CDF) as follows.

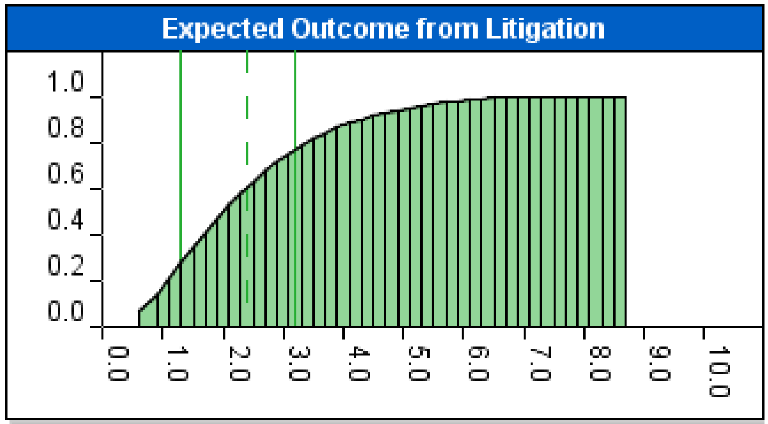

Figure 6-19 – A/E's Expected Outcome from Litigation

Summary Statistics for A/E	
Mean	2.41
Median	2.10
Variance	1.92
Standard Deviation	1.39
Lower Percentile [25.0]	1.31
Upper Percentile [75.0]	3.19

The Contractor's expected payoff from trial defines his decision on demanding the settlement amount as follows.

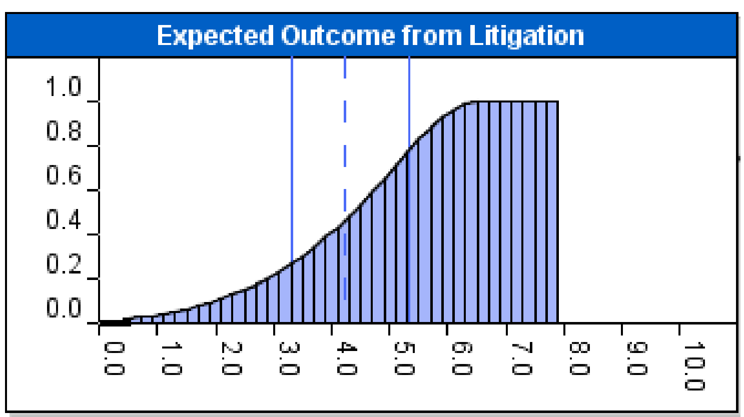

Figure 6-20 – Contractor's Expected Outcome from Litigation

Summary Statistics for Contractor	
Mean	4.21
Median	4.47
Variance	2.13
Standard Deviation	1.46
Lower Percentile [25.0]	3.31
Upper Percentile [75.0]	5.34

Parties may evaluate their expected litigation costs if they decide to proceed with trial and then make a decision for a settlement offer. Using the mix strategy introduced in the Game theoretic section provides the following graph regarding Contractor's belief over its expected value given its settlement offer.

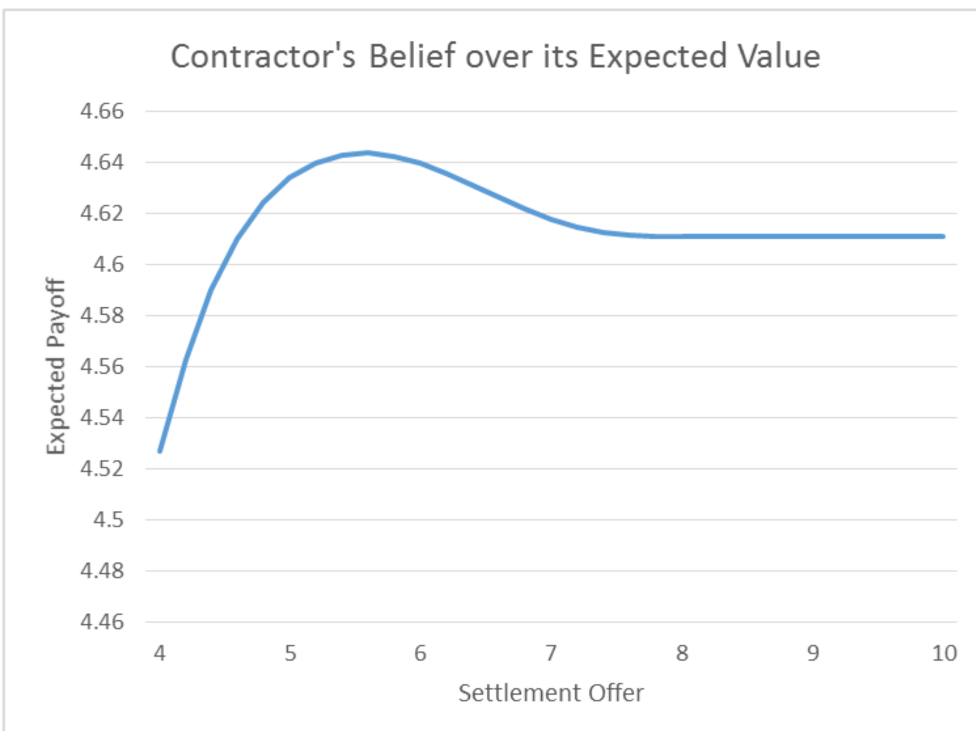

Figure 6-21 – Contractor's Optimized Settlement Offer

6.2. Case II – Schedule and Productivity Delay in a Windfarm Construction Project

This case analyze the *Mountain Wind Energy project* (“Project”), which was a large and complicated project that involved the construction of over fifty wind turbine generators (“Turbine”) along a mountain ridge. The project site access and climate conditions were extremely challenging to the point where the project delivery system was fraught with risk for all parties. Although the project was completed and equipment operate in its full functionality, there were significant schedule delays, costs overrun in the project that resulted in numerous lawsuits. The lawsuits were eventually settled in arbitration process.

Figure 6-22 Installed Turbines on the ridge of the mountain

Site access was a major controlling factor on this project. Construction included a one-way access road over twenty miles of the mountainous terrain. Construction of that access road was impacted by third-party agreements and land leases and environmental issues. In addition, the owner delivered turbine components on large off-road trailers without any dedicated location for storage. The turbine components were roughly 25 pounds and 100 feet long. The Project constrains included access limitations, difficult weather conditions, owner-supplied components, construction schedule maintenance, contractual notice provisions, and assignment of risk for third party obligations.

6.2.1. Project Background

The Mountain Wind Energy Project ("Project") is a 100-megawatt electric generation facility, comprising over fifty turbines, located along twenty mile stretch of the Mountain ridge line. The Contractor agreed to construct a windfarm, to clear, grub and construct a road, construct foundations for the erection of the turbines, offload and erect the components for the wind turbines, and build a substation and interconnect the entire facility. Contractor also provided certain engineering services limited to road design layout, wind turbine collection system design layout and procurement of long lead items.

Access to the site was limited, where the only way to enter or leave the site was through a single point. The road was only eighteen feet wide, one-way in and one-way out. As a result, the progress of the construction was controlled by the capacity of the road. Using a single one-way lane road and traversing mountainous terrain to provide means of egress for all construction traffic were one of the most important constraints of the Project.

Construction of the Project required several operations to be done in a sequence of work orders. The first order of business was for the Owner to obtain full access to the site. Based on the agreement between Contractor and Owner ("Contract"), the Owner was required to acquire Real Property Rights[18] prior to the start of the construction. This included obtaining signed leased agreements from all impacted property owners.

Next, the Owner was obligated to remove the timber from the site and then issue the Clearing Access Notice. The Clearing Access Notice would allow Contractor to mobilize the equipment to the site to begin clearing and grubbing (removal of vegetation, tree stumps, etc.). This equipment included large track-mounted dozers, loaders, and large trucks. Clearing and grubbing was the first phase of Contractor's work on-site and was necessary to begin before any other operations on-site.

As the Project site was cleared, grading (the next step in building the road) followed behind. The grading operation involved large dozers, loaders, and earthmoving equipment. The grading operation involved removing earth from high spots (cut) and placing that material in low spots (fill) in order to create a road surface, which followed the design profile (This operation is known as cut and fill). As earth is placed in fills it had to be compacted by large compaction equipment. The final step in building the road involved placing gravel on the road surface. Placing the gravel involved moving thousands of truck-loads of stone and using large dozers, loaders, trucks and motor graders for placing and leveling.

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[18] *"Real Property Rights" means all rights in or to real property (such as leasehold or other rights to use or access the Project Site), leases, agreements, Permits, easements, including licenses, private rights-of-way, and utility and railroad crossing rights required to be obtained or maintained by Owner in connection with construction of the Project on the Project Site, transmission of electricity to the Grid, performance of the Work, or operation of the Project, etc.*

The Project also involved excavation of turbine foundations, delivering reinforcing steel, delivering concrete, delivering wind turbine components to the site, erecting components, construction of the Operations and Maintenance Building, and construction of the substations. Each subsequent major component of work required thousands of truckloads of material, equipment and personnel. Each turbine foundation required about 1800 truckloads of concrete for the foundation, and 600 truckloads of turbine components. It was recorded that over 20,000 trucks would passed through the single access point to construct the Project.

Working space at each pad site was limited because the pad sites were carved out of a densely wooded forest and the constructed foundation and crane pads occupied a large portion of the cleared area. Therefore, erection of the wind turbines required careful coordination of the deliveries, off-loading of components to specific locations within the pad site, and the crane movement between pad sites. Cranes had to be set-up on stable pads and once the crane was set-up on the pad, it was stationary. The major components had to be strategically placed where they could be reached by the crane in order to begin erection.

To optimize productivity during the erection of the wind turbines, the delivery of major components, off-loading those components and erection of those components had to proceed in a linear sequence from one wind turbine pad site to the next. Off-loading the turbine components from the trailers and erecting those components required three types of cranes: cranes to unload trailers (off-load cranes), cranes to erect the base and middle tower sections, and cranes to erect the top sections, hubs, nacelles and blades (top-out cranes).

6.2.2. Development of Claim

In order to plan this Project properly and ultimately develop a reasonable schedule, the sequences of work described above had to be integrated with the constraints of the site, the limited capacity of the road, severe weather conditions, and the delivery schedule for the turbines. Owner would benefit from a schedule that allowed it to take delivery of all turbines by **December 31, 2010**, and would allow those turbines to be delivered directly to their pad sites. This would allow Owner to avoid the cost of off-site storage and double handling of the turbines.

Because of the severe winter weather, the ideal plan was to begin construction in early spring in order to get majority of weather sensitive work done before the winter. In fact, Contractor in its early planning, prior to the Contract, envisioned a **March 2010** start of construction.

Later the Owner notified the Contractor that construction cannot be started until summer. Contractor proposed the concept of a two-phase schedule, whereby the most weather-sensitive work would be performed before and after winter weather season and less weather-sensitive work would proceed through the winter. This concept would require suspension of most on-site construction activity during the winter months, off-site storage of some turbines, and double handling of those wind turbines stored off-site.

Ultimately, aside from the late (summer) start of construction, Owner decided to proceed with an accelerated schedule that would not require an off-site laydown yard or double handling of the turbine components. That new schedule required all wind turbine pad sites to be complete prior to winter, which would have reduced overall project cost because Owner would avoid the cost of an off-site laydown yard and double handling of wind turbine components.

The key of success for this schedule was maintaining the road traffic capacity. This was to be accomplished in the schedule by minimizing the overlap between the two operations that placed the heaviest demand on the road, the Concrete Operations and the Delivery. In fact the Project Schedule provided that these two operations would overlap by only one week. For example, the delivery would not start until the Concrete Operation was one week from completion.

As-planned, the site work was to begin in June 2010, the other construction operations could not begin until August 2010, the road had to be complete by October 2010, the delivery could only overlap with the Concrete Operation for one week and the delivery had to be carefully coordinated with a complex crane operation, minimizing crane movements between turbine pads. All of this had to occur in a linear manner beginning at the south end of the Project, proceeding in sequence to the north.

Unfortunately this did not happen. Interruptions and major events during the construction caused significant delays in the Project. Complicated contractual obligations and unbalanced risk allocations between the parties, caused disagreements between the parties and resulted in a timely and costly litigation. The project was behind the schedule and over budget. Following is a snapshot of one month comparison between as-planned and as-built for the project schedule.

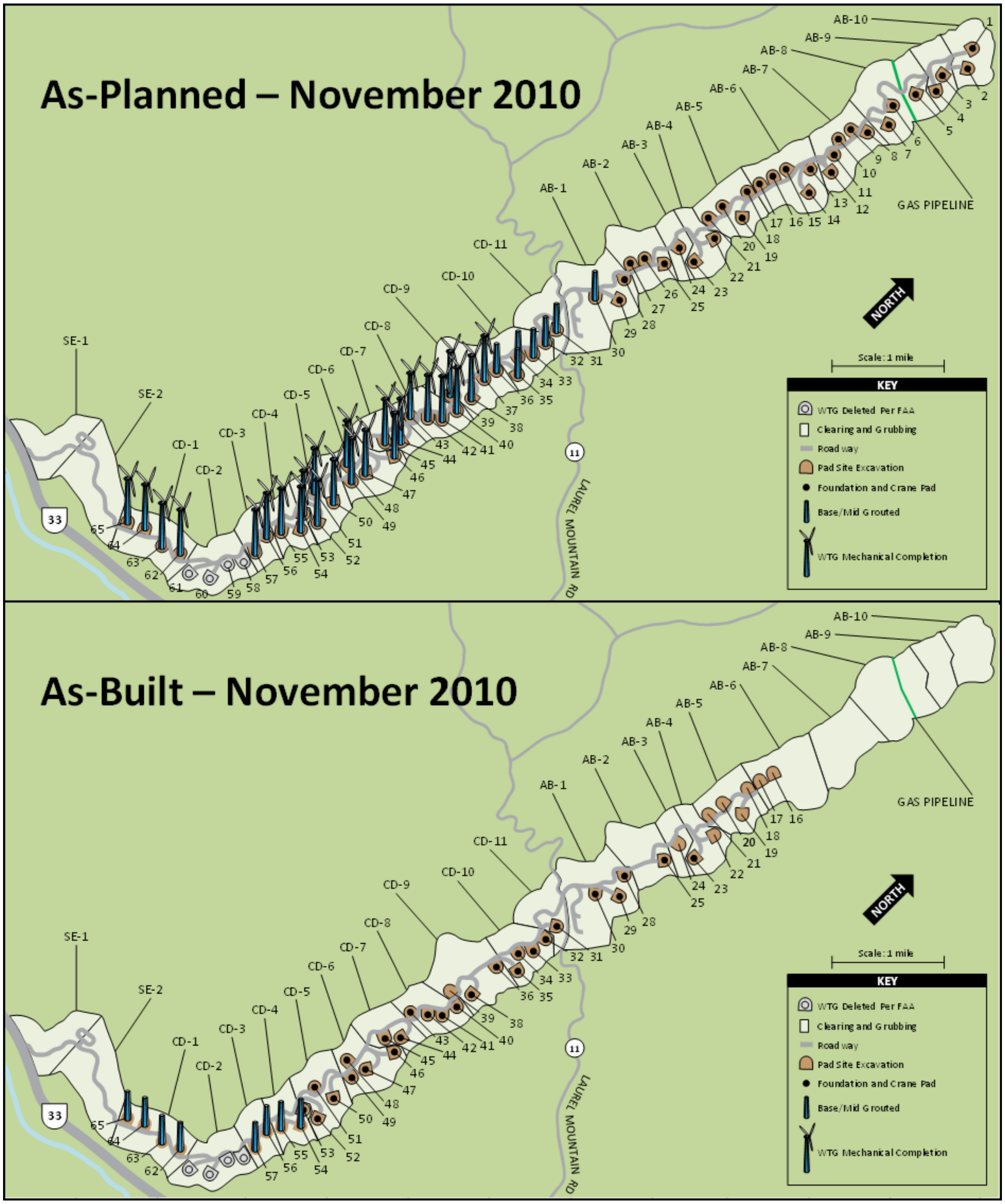

Figure 6-23 As-Built versus As-Planned Schedule

6.2.3. Accelerated Project Schedule

One of the largest risk for all parties was the accelerated project schedule. The Contract Schedule was developed over the contract negotiations between the Owner and Contractor. The Project Schedule was an important part of Owner's financial projections for the Project, particularly with respect to the delivery of turbines and potential off-site storage of turbines. Off-site storage would also require double handling of the components. Owner's desire was to reduce its cost by eliminating off-site storage of turbines. This would require that all turbines be delivered directly to the site by Dec 2010.

In May 2010, Contractor provided three schedule scenarios requested by Owner that were expressly contingent upon several key assumptions, the most critical including no overlap of major tasks, i.e. pouring of concrete foundations and delivery and erection of turbine components. All three schedule scenarios included the concept of a two-phase schedule, whereby the most weather-sensitive would be performed before and after the anticipated winter weather season and less weather-sensitive work would proceed through the winter. The three options were are follows:

- Option 1: Start August 2010, Winter Suspension, Restart in March 2011
- Option 2: Road starts in June 2010, August 2010 Full Notice to Proceed, turbine Erection in Spring 2011
- Option 3: Road starts in June 2010, August 2010 Full Notice to Proceed, Split turbine Erection between Fall 2010 and Spring 2011

Later in May 2010, Owner requested that Contractor reduce its proposal by one million dollar ($1M). Owner told Contractor that without the reduction the Project would not be approved by the executive board. Contractor and its subcontractors reviewed possible schedule scenarios that would yield cost savings. Contractor submitted another cost proposal to Owner that reduced the overall cost of the Project by one million dollars.

The savings would be achieved by accelerating the Project to have all of the turbines erected by mid-January 2011 and receive substantial completion in mid May 2011. The accelerated schedule contained significant risk for all parties given that the schedule required little overlap of major construction activities on a project that was scheduled to take place from fall through winter into spring on a mountain ridgeline.

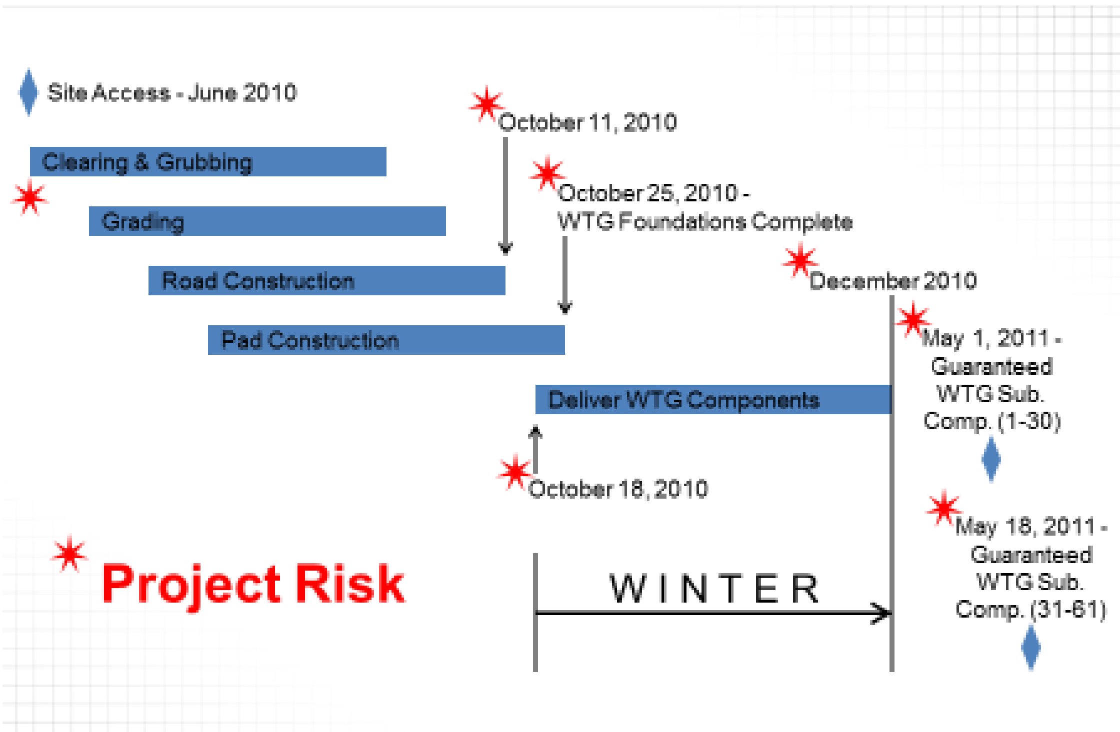

Figure 6-24 Project Schedule Delayed to the winter time

6.2.4. Game Theory Model for Parties Interactions

Table 6-2 provides the mathematical approach for interactions between the claimants. Details regarding such interactions are discussed in Section 4.4.1

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Table 6-2 Game Theoretic Interaction between the Claimants

stage	PlayerA (Plaintiff)	Player B (Defendant)
0	A forms a PDF $a_X^0(x)$ A forms a prior PDF $a_Y^0(y)$	B forms a prior PDF $b_X^0(x)$ B forms a PDF $b_Y^0((y)$
1	Submit a claim C_A	
1.1		B updates its belief over Damages $b'_X(x)$ and Liability $b'_Y(y)$ B Responds with Defense D_B
	A updates its beliefs as follows $a'_X(x)$ or $a_X(x \mid D_A)$ $a'_Y(y)$ or $a_Y(y \mid D_A)$	
2.1		B's Decision Analysis $\begin{cases} Pursue\ Litigation \\ \text{Offer Settlement } S_B \end{cases}$
2.2	A updated its belief over Damages $a'_X(x)$, and Liability $a'_Y(y)$	B's belief over A's belief
2.3	A Decides based on its interim payoff, u_A $\begin{cases} \text{If } S_B \geq u_A \text{ => A would accept} \\ \text{If } S_B < u_A \text{ => A would reject} \end{cases}$	
2.4		B estimates A's interim outcome, $(\hat{u}_A)$ $\hat{u}_A\ (a'_X(x), a'_Y(y), b_K(k_A)) =$ $E[a'_X(x)\ a'_Y(y)] - E[b_K(k_A)]$ B thinks A would only accept S_B if: $S_B \geq \hat{u}_A$ critical type: $A^c = S_B + E[b_K(k_A)]$
2.5	A's Strategy $\begin{cases} If\ E[a'_X(x)\ a'_Y(y)] \leq A^c\ accept \\ If\ E[a'_X(x)\ a'_Y(y)] > A^c\ reject \end{cases}$	B's belief over A's belief

6.2.5. Stages of the game

Before filing a claim, parties typically discuss the issues through several emails or meetings. During such communications the parties exchange some private information with in a hope of resolving the dispute. Before filing a claim parties form their beliefs over specifics of the game. These beliefs are referenced as prior beliefs in the Stage 0 of Table 6-2.

The project described above experienced 5 months of delay in substantial completion. The delay caused damages to both parties in different ways. The Owner particularly suffered from the late completion since the Wind Power Plant was not operable and couldn't generate the estimated revenue and return on investment.

The significant delay resulted in multiple meetings and discussions between the Contractor and the Owner. These meetings raised so many conflicts between the parties regarding who is liable for the delay and how to calculate the damages. During these conversation some information were exchanged that formed parties' prior over these variables. Stage 0 describes the beliefs formed in these communications.

At stage 1, Contractor decided to file a claim against the Owner. The Owner responded with a defense argument. These information exchanges updated parties' beliefs over specifics of the case. At stage 2, Owner is analyzing the case and make determinations of its limits to submit final settlement offer to the Contractor.

6.2.6. Stage 0 - Owner's Prior Beliefs

The Owner believed that the delays were mostly occurred due to Contractor's failure to identify the risks which resulted into failure to meet the substantial completion deadline. Owner pursued compensation using liquidated damages clause in the agreement.

Liquidated damages are identified as damages that the Contractor owes to the Owner if it fails to complete the construction of each Turbine on time as follows:

> ***Article 6.9.1*** *Turbine Delay Liquidated Damages. Owner and Contractor acknowledge and agree that any failure of Contractor to cause the minimum number of Turbines to achieve Turbine Substantial Completion by the applicable Guaranteed Substantial Completion Date will directly cause substantial damage to Owner, which damage cannot be ascertained with reasonable certainty. Thus, if such failure occurs, Contractor shall pay to Owner, as liquidated and agreed damages and not as a penalty, the following amounts: For the first 30 days of delay $800 per day per Turbine, and for Day 31 to date of Turbine Substantial Completion for applicable Turbine $1,200 per Day per Turbine.*

a. *Stage 0 - Owner's Prior Belief over Liability*

Owner forms its prior belief $b_X^0(x)$ such that the Contractor failed to meet the substantial completion date, Dec 31, 2010. The Contractor instead completed the project five months later, May 15, 2011. The following is Owner's hypothesis and evidence used to support its case.

- **Owner's Hypothesis H1:** Contractor's failure to meet contract Article 6.9.1.
- **Owner's Evidence E1:** Contractor did not meet the substantial completion and delayed the completion 5 months.

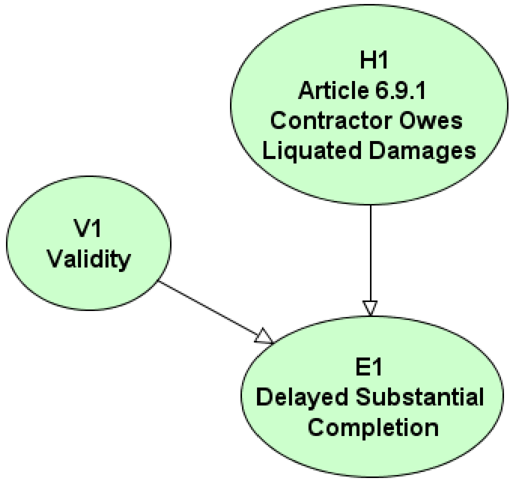

Figure 6-25 BN Structure Pattern for Owner's Prior Belief

The Validity node specifies the strength of this argument as described in Chapter 5.6.3.

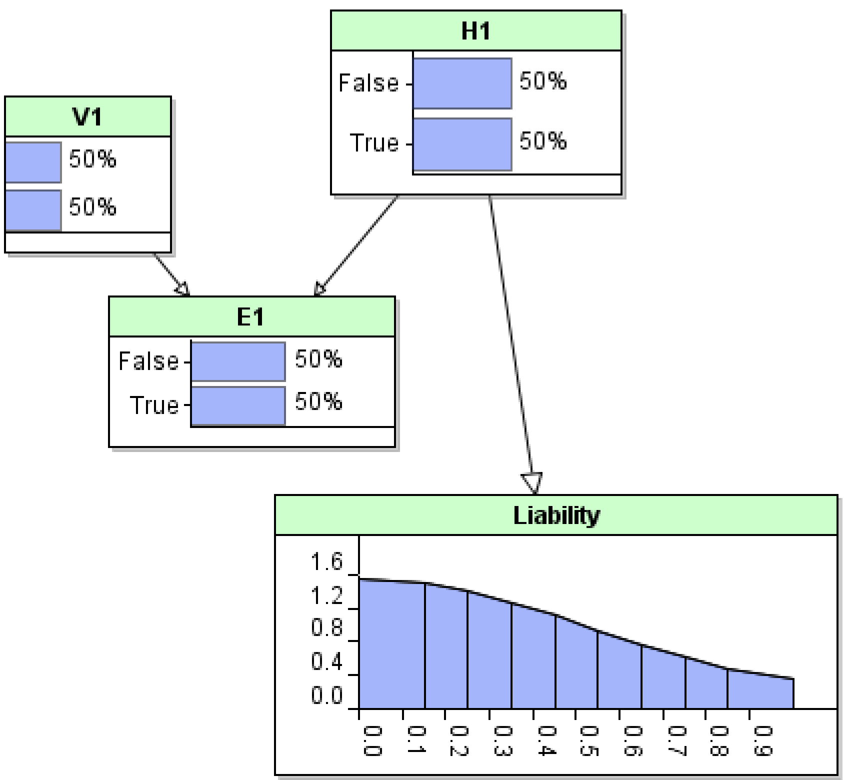

Figure 6-26 Owner's Prior Belief over Liability

b. Stage 0 - Owner's Prior Belief over Damages

The Contractor delayed the substantial completion from Dec. 31, 2010 to May 18, 2011. This durations is equal to 135 days. Per Contractor Article 6.9.1, for the first 14 days of delay $800 per day per Turbine, and for Day 15 to date of Turbine Substantial Completion for applicable Turbine $1,200 per Day per Turbine. The calculations for liquated damages are as follows:

Table 6-3 Owner's calculation of Liquidated Damages

Delay (Days)		Number of Turbines		Liquidated Damages per Turbine per day		Damages
30	×	50	×	$800	=	$1,200,000
108	×	50	×	$1,200	=	$6,300,000
				Total		**$7,500,000**

The Owner's best estimate for the amount that Contractor owes as liquidated damages is $7.5M. Based on other Owner's calculations the total amount that contractor may owe to the Owner is between $2M to $10. Owner forms a triangle distribution over Damages. Owner's PDF over Damages are considered as Contractor's expenditure (negative amounts).

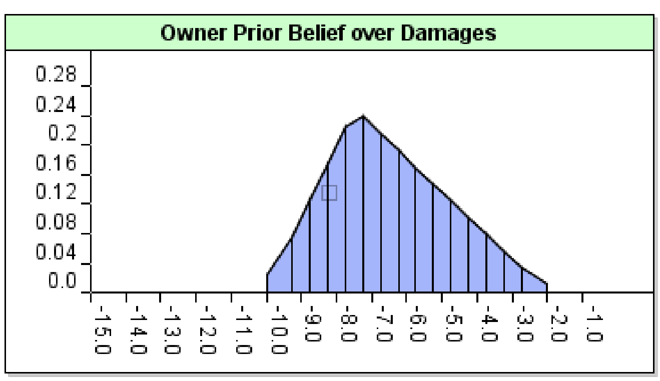

Figure 6-27 Owner's Prior Belief over Damages

6.2.7. Stage 0 - Contractor's Prior Belief

Contractor forms its prior beliefs over Damages $a_X^0(x)$ and Liability $a_Y^0(y)$. Contractor believes that late site access, owner-supplied Turbines, and weather related delays are all attributed to Owner's Liability based on the agreement. Following are the detailed reasoning from contractor regarding the three arguments (Arguments 2 through 4).

a. Argument 2 - Third-Party Controlled Site Access

Contractor believed Owner failed to acquire Real Property Rights prior to the start of construction. In addition, the Contract Agreement required Owner to issue a Clearing Access Notice by June 14, 2010 in order for Contractor to begin clearing operations (clearing and grubbing). The Contract Agreement also required Owner to issue a Limited Notice to Proceed on the same date (by June 14, 2010) in order for Contractor to begin work on the Substation and road. The Clearing Access Notice and Limited Notice to Proceed contained the following language:

> ***Article 2.6.1*** *Access to Project Site; Commencement of Work. Contractor will commence performance of all off-Site Work not previously authorized under the Service Contract promptly after the Effective Date, including engineering work and ordering "long lead time" Equipment. Contractor will not perform any clearing Work on the Project Site until Owner issues to Contractor a written notice allowing the same (the "Clearing Access Notice"). Other than as required for such clearing Work, Contractor will not otherwise mobilize to the Project Site until it receives a Limited Notice to*

Proceed from Owner. When Owner is prepared for Contractor to mobilize to the Site and commence the Work as required in relation to the Substations and the roads, Owner will issue a written notice to Contractor directing the same (the "Notice to Proceed"), and Contractor will promptly thereafter commence such scope of the Work. If Owner has not issued the Clearing Access Notice or Notice to Proceed by June 14, 2010, or an equivalent release under the Service Agreement, then Contractor shall be entitled to a Change Order as provided in Section 9.5.1.

The Contract Agreement indicated that Contractor should have been able to proceed immediately with all clearing work and all work beginning June 14, 2010. Owner failed to acquire all of the property rights and failed to have the properties timbered prior to Contractor and its subcontractors mobilizing to the site. The result of Owner's failures caused Contractor and its subcontractors to perform the work out of sequence and resulted in delays and disruptions to the major elements of work later on in the Project. Dead-zones existed in which no road work could begin until the logging was completed. The total length of these Dead-zones was approximately 1.7 miles, or roughly 12% of the length of the Project.

As a result of delayed access to portions of the Site (Dead Zones), Contractor had less borrow material than anticipated to help build the access road at the southern end of the Project. As a result, Contractor required additional time to complete the work, since the fill material had to be hauled a longer distance than anticipated. Due to the delayed access caused by Owner, Contractor was forced to complete the access road in a chaotic manner, therefore, completion of the roadway took much longer than anticipated and stretched into winter weather. In addition, the roadwork was performed concurrently with the turbine deliveries and turbine erection, which was not anticipated in the contract schedule. This resulted in a major congestion on the road, further obstructing the ability to complete the access road and delaying turbine component deliveries.

Contractor forms the following belief regarding this issue as follow:

- **Contractor's Hypothesis H2:** Owner breached contract Article 2.6.1.
- **Evidence E2.1:** Owner failed to acquire property rights and timer the site.
- **Evidence E2.2:** following the Evidence E2.1 Owner failed to give Notice to Proceed to the contractor for clearing and grubbing the site.

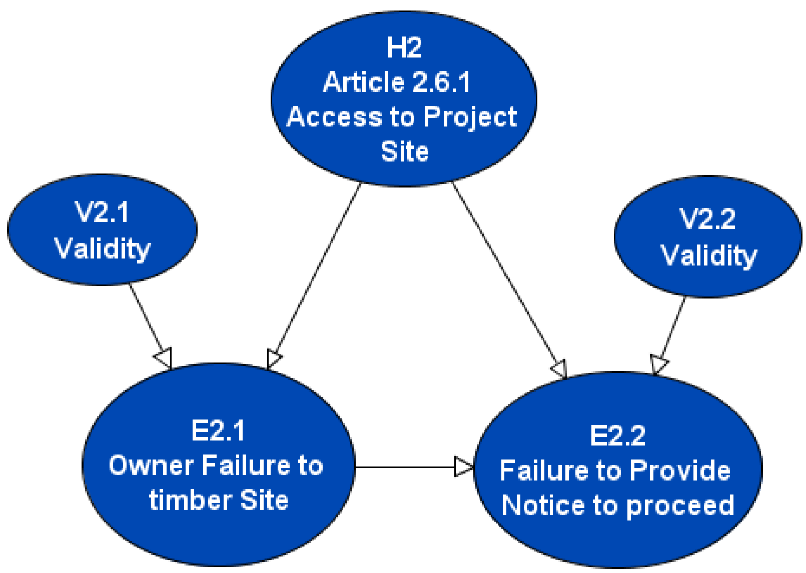

Figure 6-28 BN Structure Pattern for Contract's Prior Belief – Argument 1

b. Argument 3 - Owner-Supplied Turbine Components

Another significant source of risk imposed on Contractor by Owner was the Owner-supplied turbine components. Owner separately contracted with the Turbine Vendor for the Project.

> ***Article 2.3.1****: Owner shall cause Turbine Vendor to deliver the Turbines to the Project Site in accordance with the delivery schedule attached as Exhibit E-2. Delivery shall be to the nearest point to the Unit site pad that is accessible by standard highway configured vehicles used for transportation of wind turbine components…..Exhibit E-2; provided, that if a Turbine is delivered early, Contractor and Owner shall use Commercially Reasonable Efforts to accommodate and accept such early Turbine delivery; provided, further, that if Contractor's accommodation results in acceptance of delivery at a location on the Project Site other than adjacent to the applicable pad site, then the reasonable additional handling cost incurred by Contractor in moving the subject Turbine to the location adjacent to the applicable pad site shall be borne by Owner…. From time to time, Owner shall provide Contractor with updates of the actual anticipated dates of delivery of all Turbine components.*

The turbine delivery schedule indicated eight complete units planned for delivery each week beginning the week of October 18, 2010 and ending the week of December 17, 2010. A complete turbine included one base, one mid, one top, one nacelle, one hub and one set of three blades. Contractor was not required to unload more than thirty truckloads per Business Day.

According to the turbine delivery reports, during the month of November, Owner delivered a mixture of twenty-one bases, twenty-three midsections, twenty-three tops, twenty-one hubs, seventeen nacelles and thirteen sets of blades. Therefore, during the month of November, Owner only delivered thirteen complete units, compared to the twenty-seven complete turbines that were planned to be delivered. By mid-December 2010, Contractor and its concrete subcontractor expected to have received in total to date forty-five complete turbine units. However, Owner/GE had delivered enough pieces for only twenty-four complete turbines. Equipment inefficiencies became a problem, as large cranes were constantly moving back and forth between pad sites to offload the various turbine components that were being delivered in a random and haphazard manner. Ultimately, Owner and turbine vendors completed delivery of the turbine components to the site during the week of March 18, 2011 – twelve weeks later than the agreed upon turbine delivery schedule.

> ***Article 8.3.1*** *Force Majeure; Turbine-Vendor-Caused Delays. Without limiting the definition of Turbine-Vendor-Caused-Delays, notwithstanding anything in this Agreement to the contrary, in any case where this Agreement states that Owner "shall cause" Turbine Vendor to take or not to take a certain action, the Parties agree that if the Owner fails to meet that obligation, such failure shall exclusively constitute a Turbine-Vendor-Caused Delay and shall not constitute an Owner Event of Default, and Contractor's sole and exclusive*

remedies as a result thereof will be as set forth in this Article 8.3 and Sections 9.5.1(e) and 13.7.

Contractor forms the following belief regarding this issue as follow:

- **Contractor's Hypothesis H3:** Owner breached contract Articles 2.3.1 and 8.3.1.
- **Evidence E3.1:** Owner (or its vendor) failed to deliver Turbine components as specified in the delivery schedule.

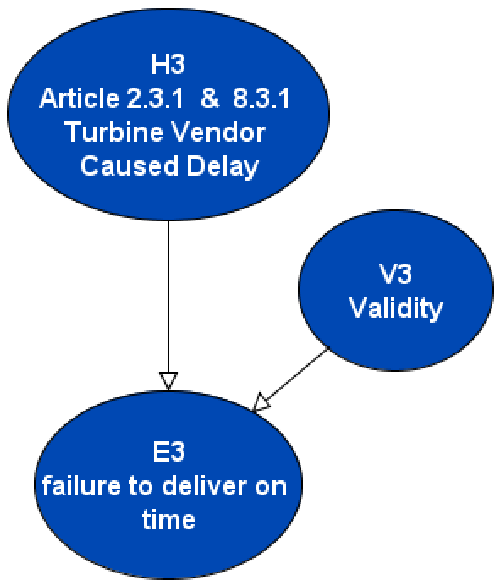

Figure 6-29 BN Structure Pattern for Contract's Prior Belief – Argument 2

c. *Argument 4 - WEATHER RELATED DELAYS*

The Contract Agreement contained several clauses related to weather that allocated risk related to weather entirely to Contractor. In accordance with the Contract Agreement, there were only two ways that Contractor was granted relief from weather related delays:

***Article 8.5.1**. The occurrence of a Weather Delay was determined in increments of half-day periods (7:00AM to 11:59AM was the first* *half-day period and 12:00PM to 5:00 PM was the second half-day period). Weather conditions were evaluated at the start of each half-day period, at 7:00 AM and 12:00 PM. A Weather Delay condition only existed if fog or ice accumulation was present at the start of a half-day period when weather measurements were taken. Wind speeds in excess of 11 m/s during base and mid-section erection or installation or 10 m/s during all other crane operations constituted a Weather Delay.*

A force majeure event was the other form of relief available to Contractor and its subcontractors for weather related delays. A force majeure event was defined as:

Article 8.5.2. *An unavoidable event beyond the control of Contractor such as: acts of God, natural disasters, wildfire, earthquakes, tornadoes, lightning, floods, etc. In addition, severe inclement weather conditions that cannot be considered Weather Delays can also be force majeure events if the severe inclement weather condition exceeds by ten percent (10%) the twenty-five year daily average for such weather condition for the date of the occurrence according to the records of the National Oceanic and Atmospheric Administration for the vicinity of the Project Site.*

Further increasing the risk for Contractor was a contractually stipulated $50,000 cap as the maximum relief that Contractor could receive due to force majeure events.

Article 9.5. *Change Order Due to Weather Delays. Subject to Section 9.5.1 if a Weather Delay has occurred with respect to a Half-Day*

Period, Owner will issue a Change Order to increase the Contract Price by a flat amount of four thousand dollars ($4,000) per day…. Contractor agrees that weather delay charges shall not exceed fifty thousand dollars ($50,000) in total. The Change Order(s) described herein shall be Contractor's sole and exclusive remedy for any delays and increased costs resulting from excessive winds, fog or ice accumulation, and Contractor will not be entitled to any payment, damages or other compensation in connection with any such delays or increased costs.

Contractor forms the following belief regarding this issue as follow:

- **Contractor's Hypothesis H4:** Owner accepts risk per contract Articles 8.5 and 9.5.
- **Evidence E4.1:** Contractor observed severe weather conditions as described above.

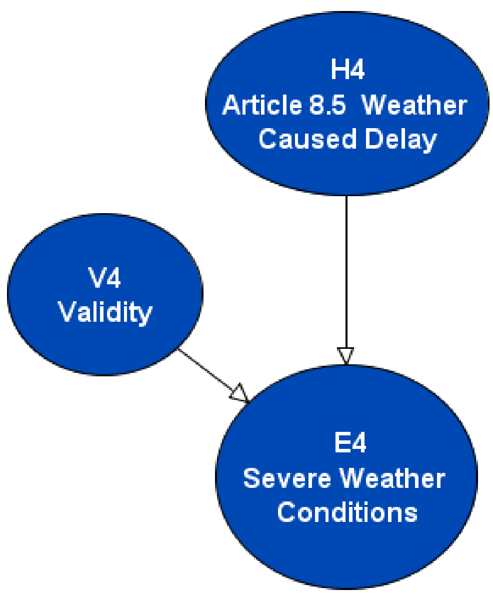

Figure 6-30 BN Structure Pattern for Contract's Prior Belief – Argument 3

a. Contractor's Prior Belief over Liability

Based on the Liability Pattern discussed in Section 5.6.4, the Contractor's Arguments are modeled as follows:

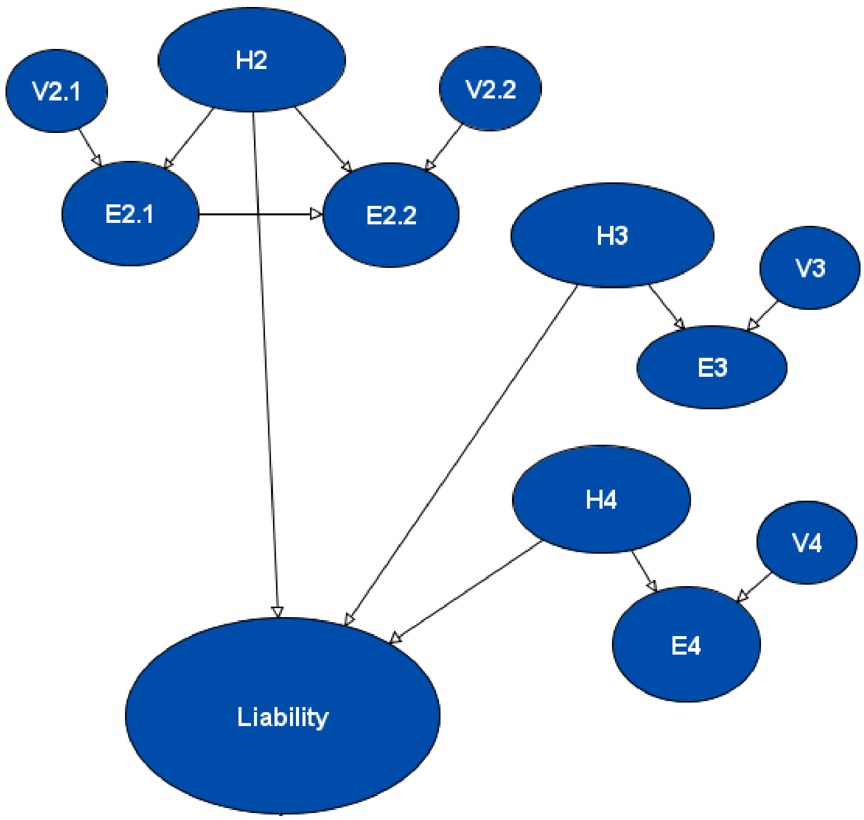

Figure 6-31 Liability Pattern for Contractor's Prior Beliefs

As discussed in Section 5.6.1 the Evidence nodes are inputs from the Experts. The Experts determine the factual evidences and whether the evidence supports the hypothesis. Validity nodes are inputs from the Attorneys. These node determine the strength of each Hypothesis with its associated Evidence(s). For example the Expert determines if the severe weather conditions support the contract Article 8.5, and the attorney determines the strength of that argument is. The Validity nodes are set to be 90% true and 10% false as prior beliefs of the contractor. Providing inputs to the model results as follows:

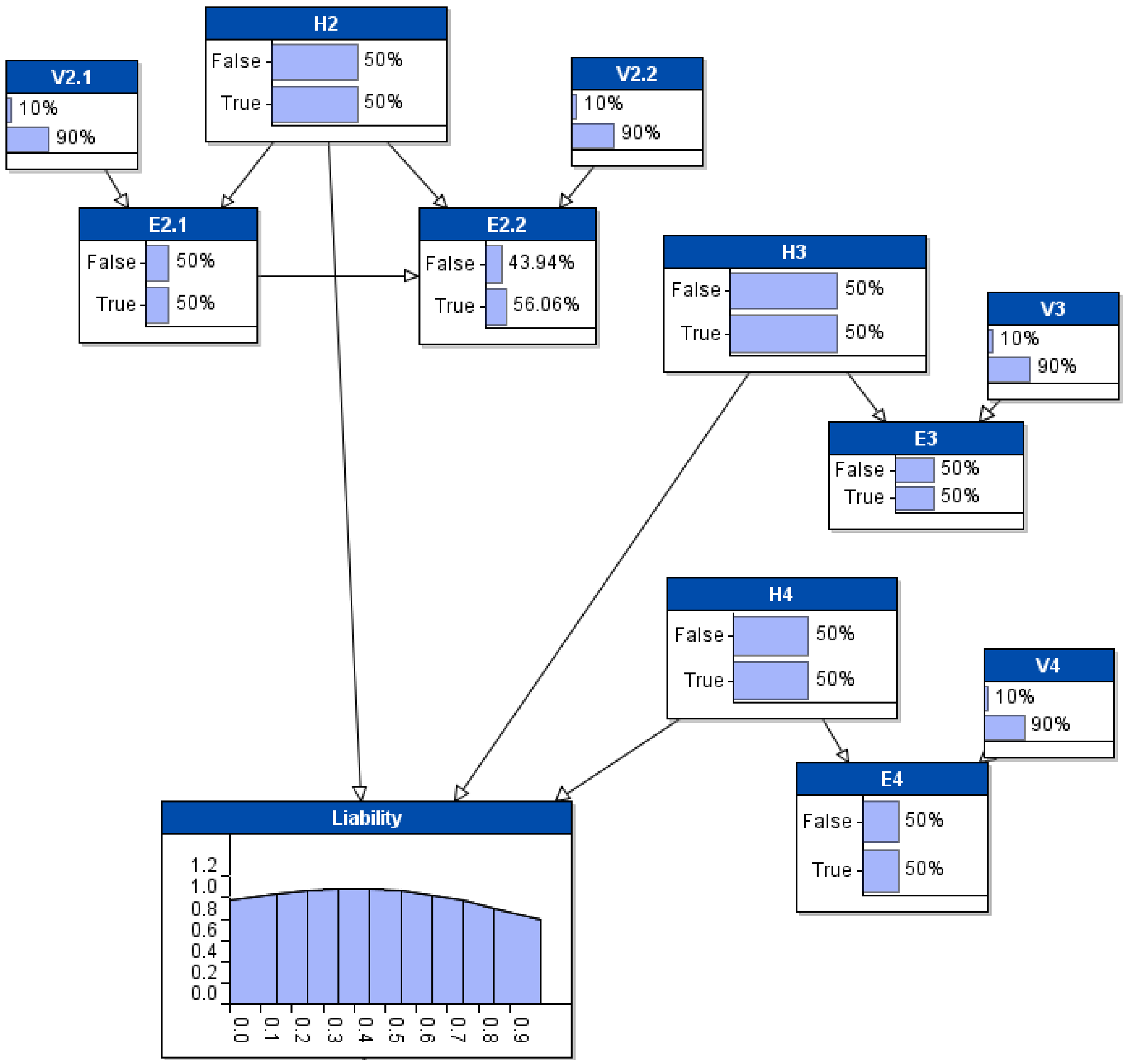

Figure 6-32 Contractor's Prior Beliefs over Liability

a. Contractor's Prior Belief over Damages

The contractor believes that it has incurred approximately $18M in damages including direct and indirect costs. Direct costs included additional equipment and labor on-site, and sub-contractor's extra charges for the additional work, and indirect costs are overhead and management fees due to additional 135 days of involvement in the project. The details for Contractor's damages are eliminated to simplify the case. Contractor forms its PDF over Damages using Truncated Normal Distribution as follows:

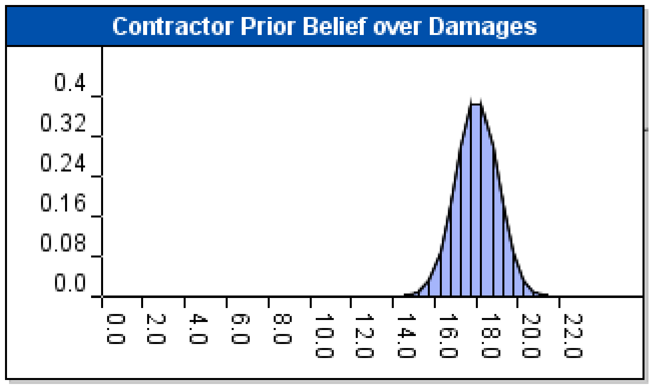

Figure 6-33 Contractor's Prior Beliefs over Damages

The mean for the Truncated Normal function is considered \$18M with \$1M variance. In Contractor in its statement of claim asks for \$21M in damages, which is the highest possible award that the Judge can determine.

6.2.1. Stage 1.1 – Contractor Submits its Claim

Contractor prepares a statement of Claim and submit it to the Owner. The scenarios applied to the evidence nodes (True/False) represent Contractor's belief regarding connection between the evidence and the contract obligation. Contractor forms its claim as follows:

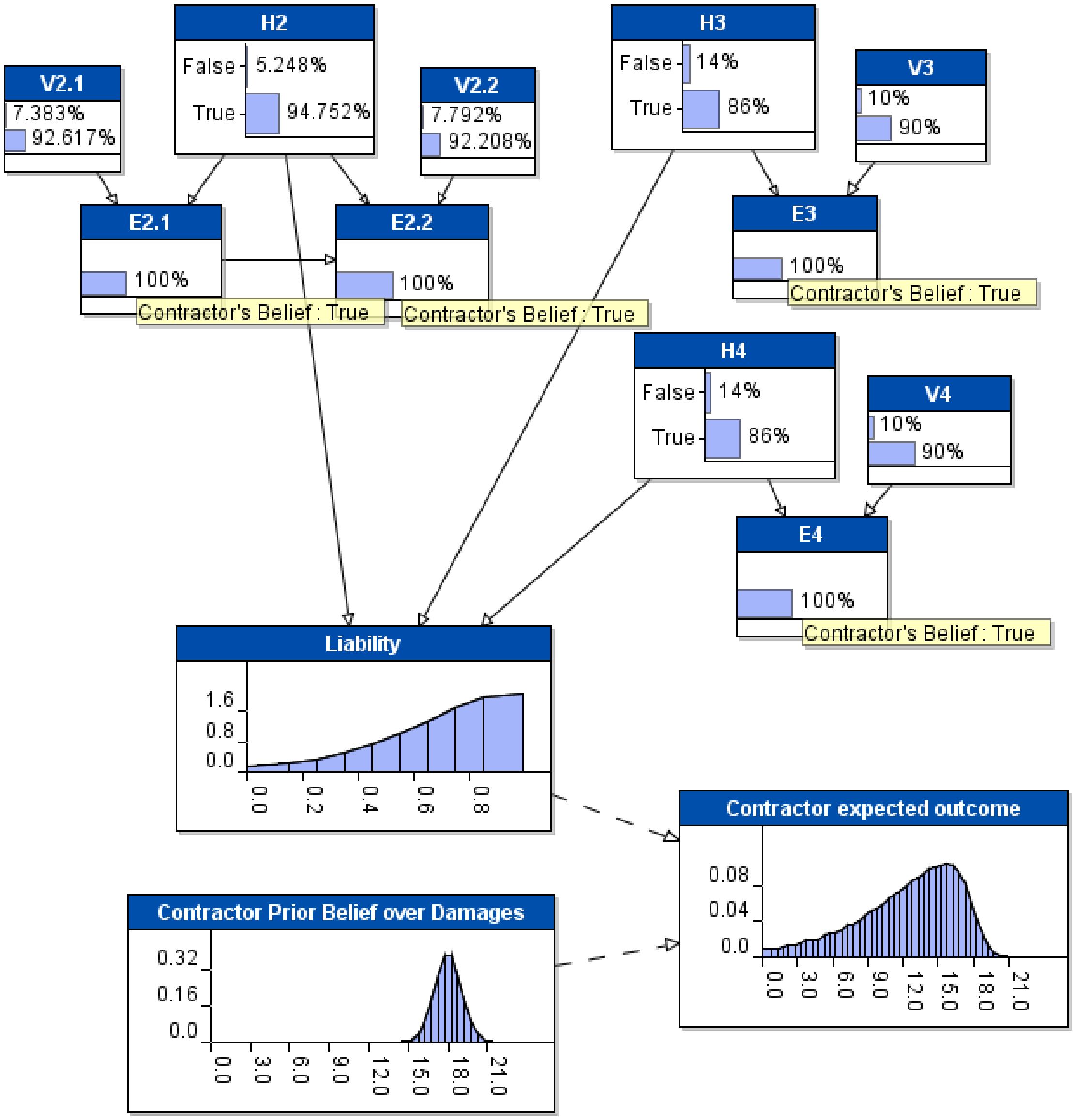

Figure 6-34 Contractor's Claim

Contractor's belief for the credibility of all are identified high. The 90% validity for the evidences reflects attorney's inputs. All the evidences are identified to be related to the hypothesis by experts. The Contractor's Belief scenario "True" for the evidence nodes reflects expert's suggestions in preparing the statement of claim.

6.2.2. Stage 1.2 &1.3 – Owner's updated beliefs and Defense

Owner installs the liquidated damages Clause to the model provided by contractor. Therefore, the model will be updated as follow to form Contractor's belief regarding Liability:

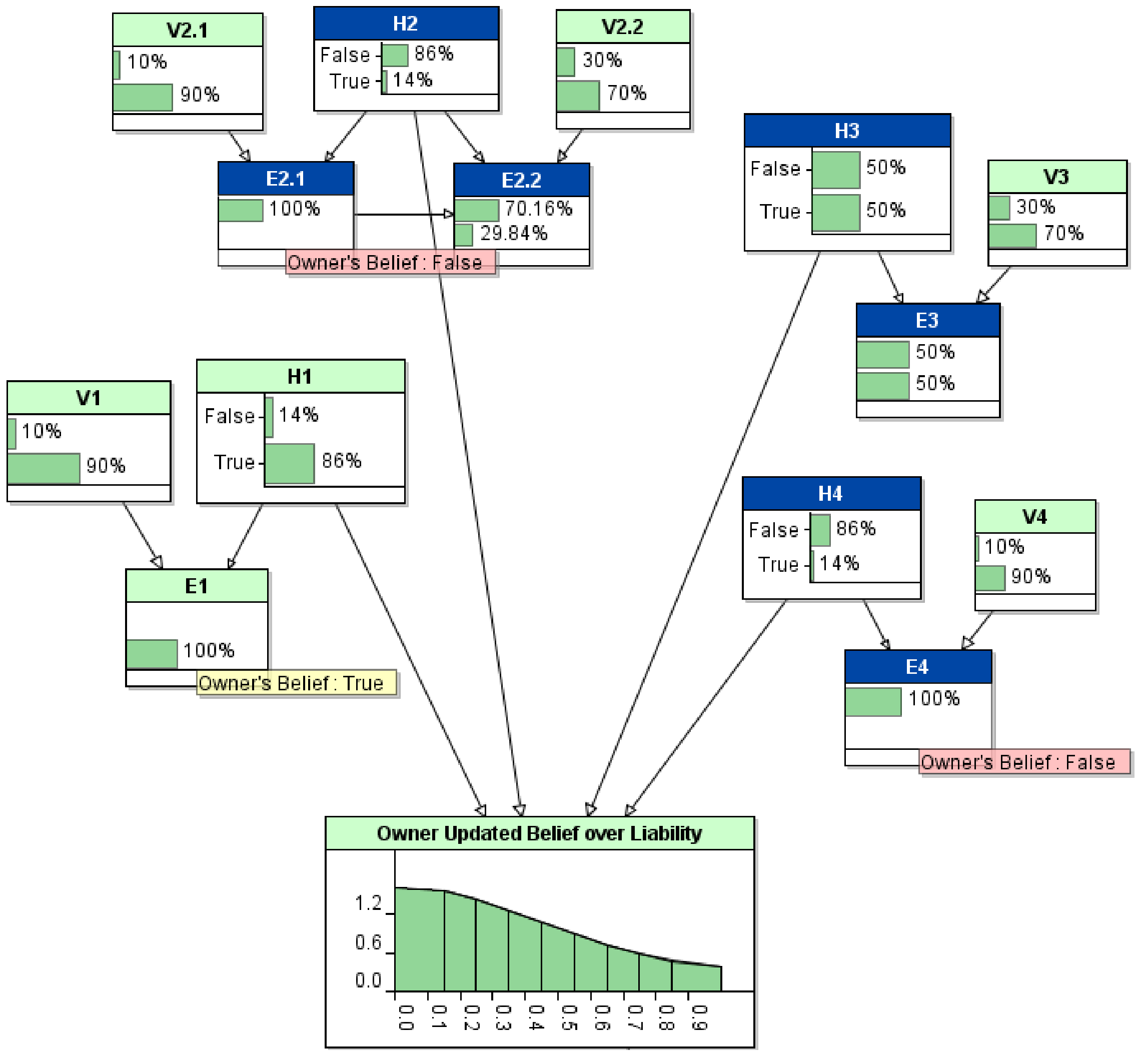

Figure 6-35 – Owner's Updated Belief over Liability in its Defense

Owner responded to the Contractor's Claim with its denfense statement. Owner rejected that the acquiring property site and timerbring the site (Evidence 2.1) can be used as evidences for breach of contract Article 2.6.1 (Hypothesis 2). Owner also does not consider the weather condition severe in this year (Evidence 4) comparing to the previous years; therfore, Owner does not accept the weather caused delay argument (Argument 4). These counterarguments are reflected in the the Owner's Updated Belief by using the scenario "False" for Evidences 2 and 4.

Owner also found failure to provide notice on-time (Evidence 2.2) as a result of late timbering a weak arugment. It also stated that the delay in delivery of the turbines (Evidence 3) is not a totally accurate since the delivery were partially on-time. These two aker counterarguments are reflected by using 70% true for the Validity nodes V2.2 and V3.

Owner received Contractor's argument regarding damages and believes that the amount of supporting documents provided are revealing average amount of information. Based on Damages Pattern provided in Section 5.6.6**Error! Reference source not found.**, the Owner's updated belief over damages is calculated as follows:

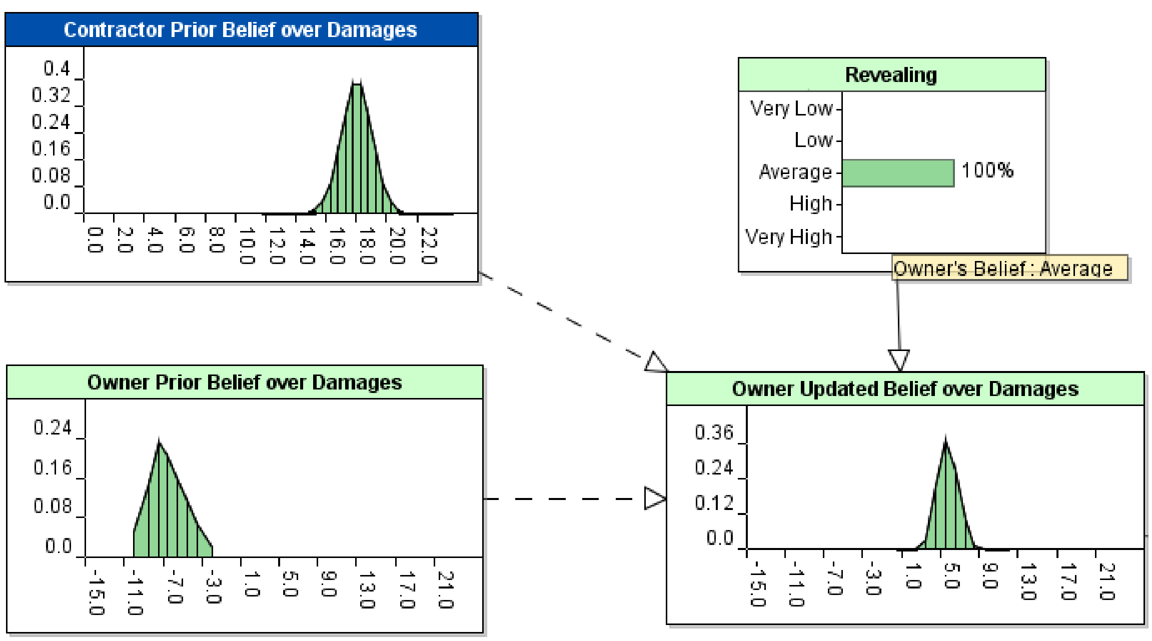

Figure 6-36 Owner's Updated Belief over Damages

6.2.3. Stage 1.4 – Contractor's Updated Beliefs

Contractor updates its beliefs over Liability after receiving the Defense argument as follows:

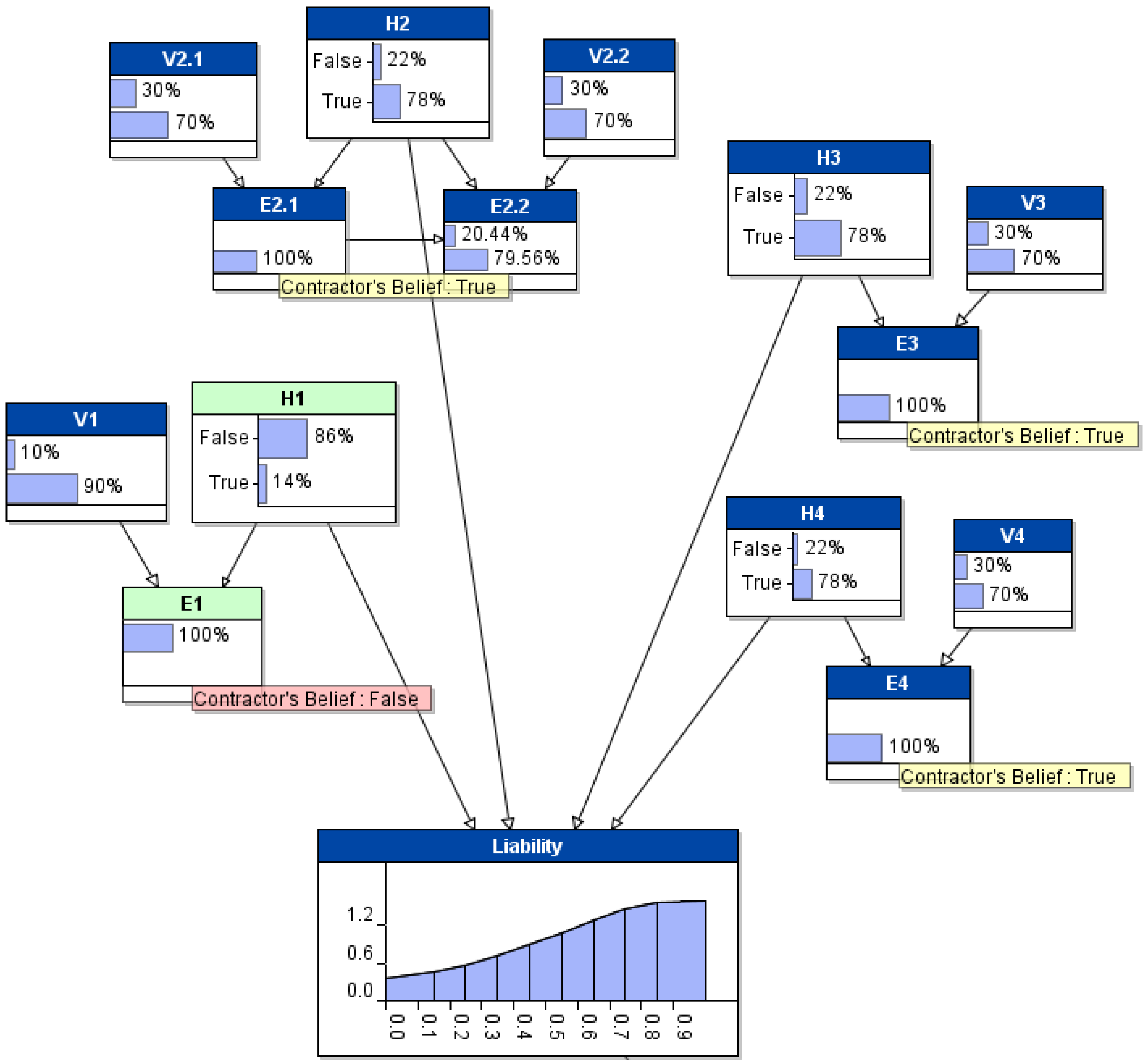

Figure 6-37 Contractor's Updated Belief over Liability

Contractor decides not to change its position on damages. Therefore, the damages are not updated.

6.2.4. Stage 2.1 – Owner's Settlement Analysis

At this stage owner makes its decision on the settlement offer. For the settlement decisions, Owner compares its beliefs with contractor's belief as follows

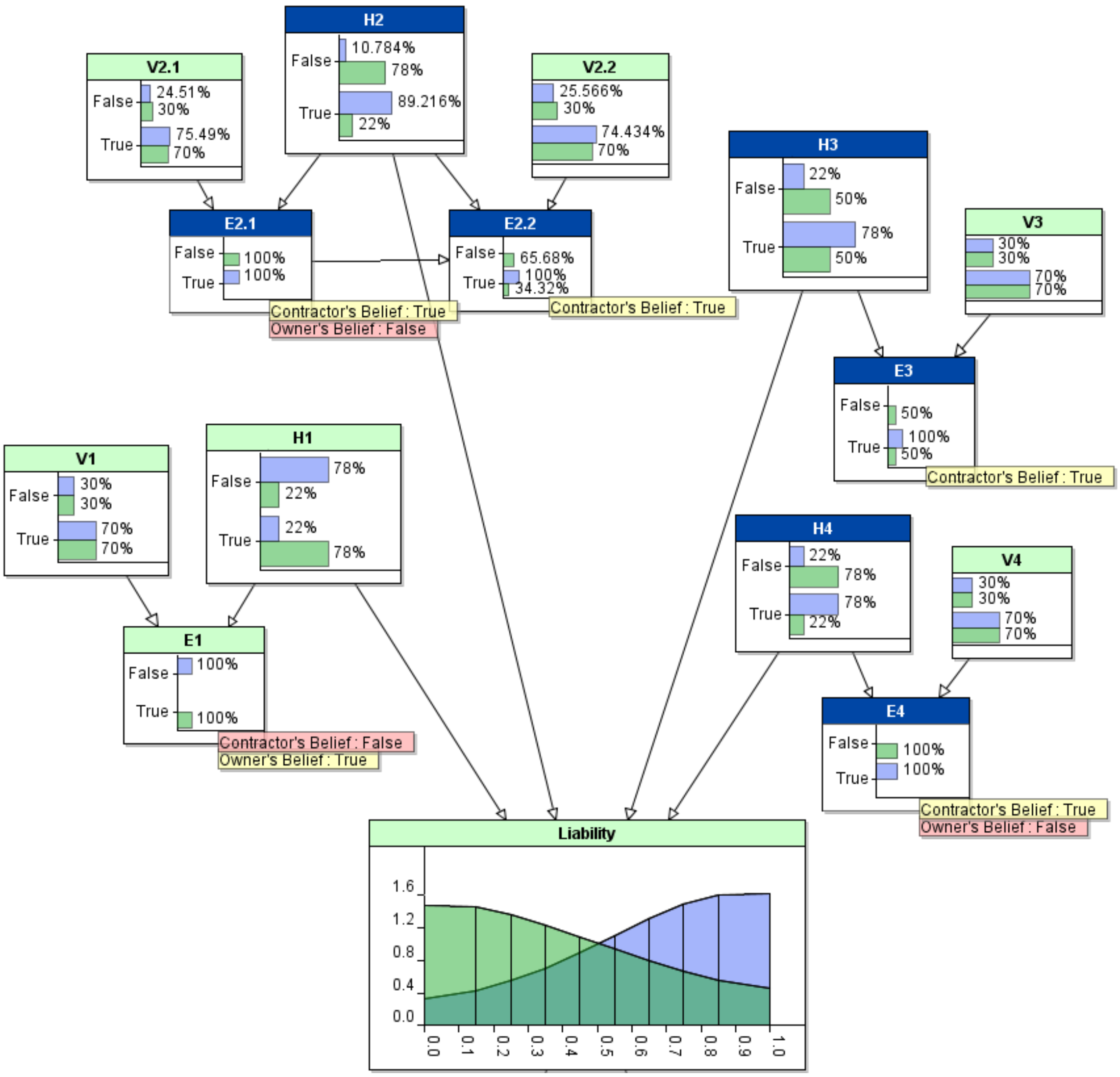

Figure 6-38 Owner's Comparison of its Beliefs with Contractor's Beliefs over Liability

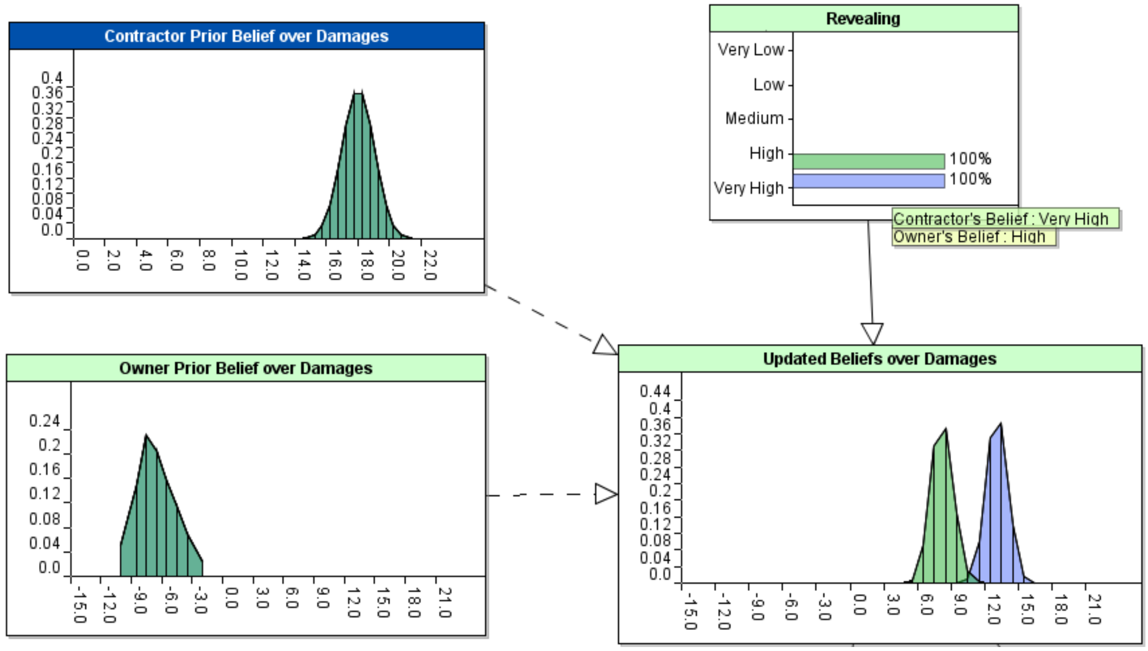

Figure 6-39 Owner's Comparison of its Beliefs with Contractor's Beliefs over Damages

Owner calculates its expected payment from litigation as product of Damages and Liabilities plus its litigation costs.

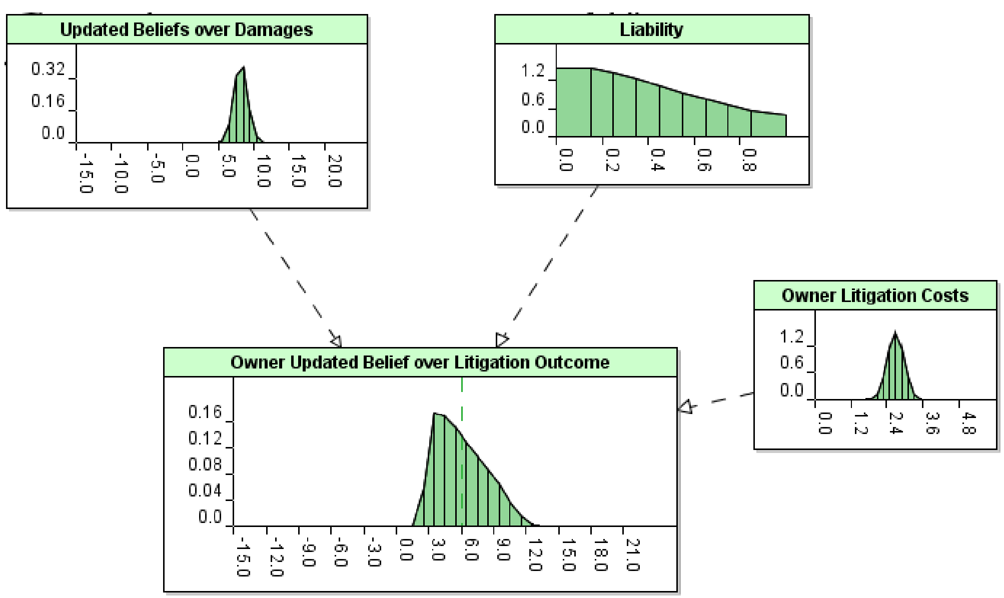

Figure 6-40 Owner's Belief over Litigation Outcome

The Cumulative Distribution Function (CDF) of Owner's belief over litigation outcomes follows:

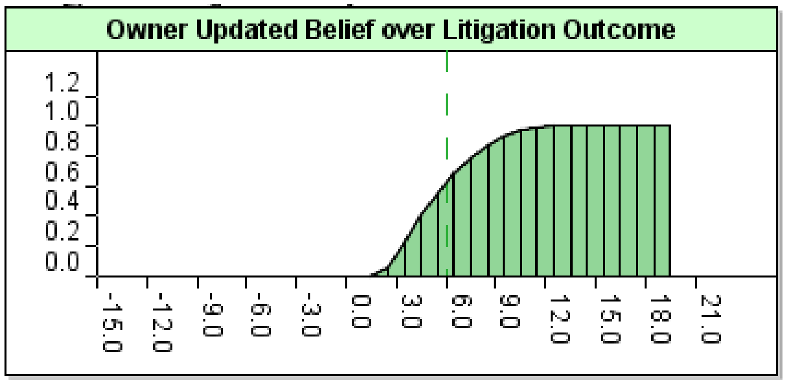

Figure 6-41 Owner's CDF over Litigation Outcome

Owner calculates Contractor's expected outcome from litigation as product of Damages and Liabilities minus Contractor's litigation costs.

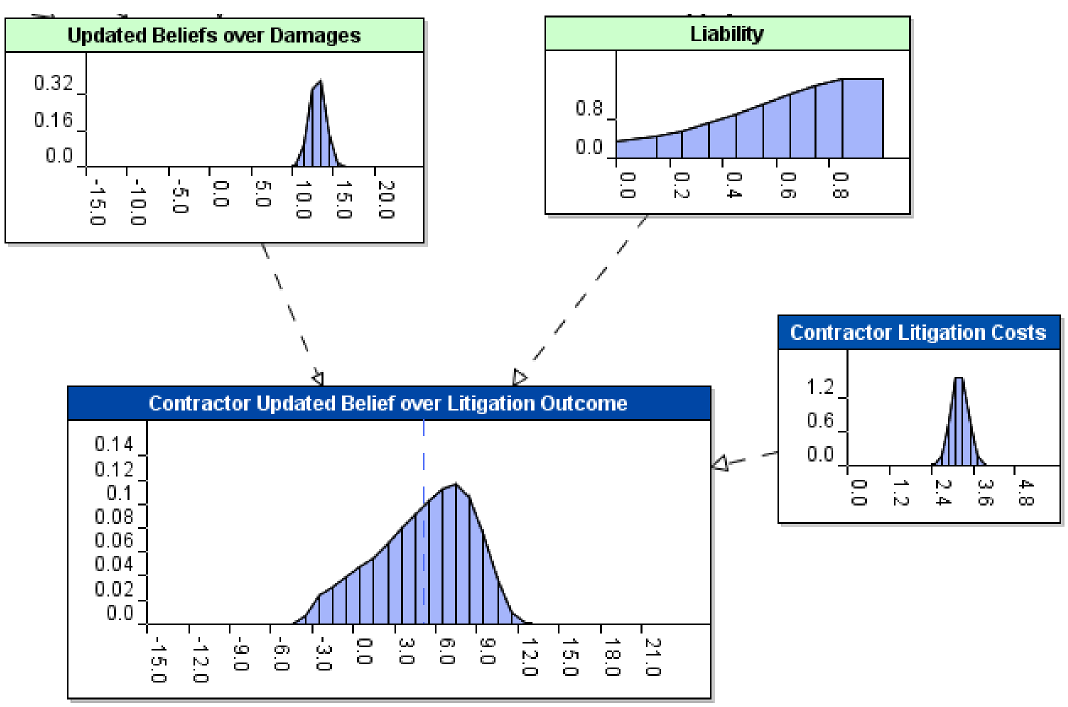

Figure 6-42 Owner's Belief on Contractor's Belief over Litigation Outcome

The Cumulative Distribution Function (CDF) of Owner's belief on Contractor's Belief over litigation outcomes follows:

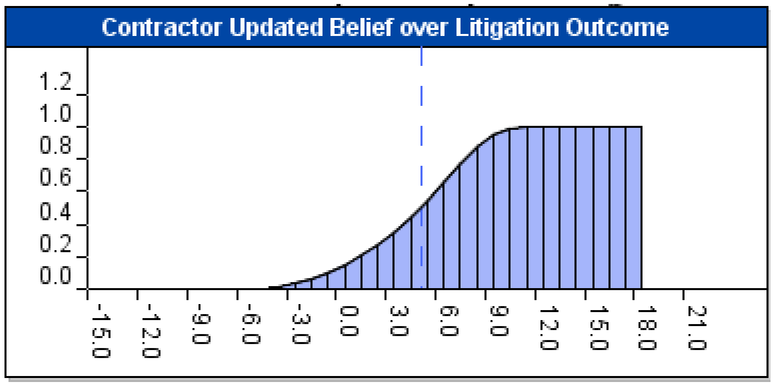

Figure 6-43 Owner's CDF over Litigation Outcome

6.2.5. Case II Summary and Conclusions

Here is the summary conclusion for case study II:

- Owner's expected value of the litigation outcome is $6M
- Contractor's expected value of the litigation outcome is $11M
- Parties changed beliefs significantly during the Claim process.
- 50% chance that the Contractor accept Settlement offer of $11.4M and above

7. Limitations and Future Research

This chapter elaborates on the assumptions used to develop the model and opportunities to relax those assumptions. In general, the use of Bayesian Networks in Game theory has been limited. Consequently, there are numerous opportunities to expand the game theoretic models using Bayesian Networks s in different sciences. The focus area in this research is non-cooperative game theory in settlement negotiation. The following includes different paths to develop the model introduced in this research with reference to current literature.

7.1. Final Offer vs Unlimited Offers

Spier (1992) has extended this framework by considering a sequence of settlement offers before trial. Her work considers the phase of bargaining before trial where parties may sequentially argue over the case. In each period, the plaintiff makes a settlement offer that the defendant can either accept or reject. If agreement is not reached, then the case will go to trial. These two approaches are both used in different settlement negotiation models. However the sequential bargaining model seems to be more in parallel with the actual claims, the high cost and time to prepare an offer in each period is not practical

7.2. Allocating Litigation Costs

As discussed in the modeling chapter, the model developed in this research applies *American Rule* for allocation of litigation costs. The alternative method is the *British Rule*, where the loser at trial pays for all litigation costs. There are many researchers who based their model on the *British Rule* including Shavell (1982). In case of using *British Rule*, a player's outcome losing the case will be deducted by both players' litigation costs, k_A and k_B, and player's outcome given wining the case becomes the expected award at the trial. The following equation illustrates the player's analysis:

$$Player's\ expected\ outocme = \begin{cases} x \times y & If\ the\ player\ wins \\ x \times y - k_A - k_A & If\ the\ player\ Loses \end{cases}$$

7.3. Risk Preference

This study assumes parties to be risk neutral. An alternative is to assume a risk preference for players over the outcomes. There are number of studies analyzed games with risk aversion in different ways. Binmore et al. (1986) analyzed this factor as a relative risk aversion of the players to define the portion of outcome they may receive. Another approach is to consider player's risk preference as a private information (Farmer & Pecorino, 1994). Bayesian Network is a helpful tool to assess the impact of private information on the outcome. Considering utilities for players' outcomes, rather than monetary values, could build on this model.

7.4. Optimistic Behavior

Optimistic behavior toward an expected outcome causes divergent expectations, or gaps between parties' prior beliefs. Yildiz (2004) analyzed this concept using a sequential bargaining model, where players were optimistic about their bargaining power. Watanabe

(2006) successfully employed this idea in his pretrial negotiations model on a medical claim case. In a later study Yildiz (2004) analyzed the impact of optimistic behavior on delays in settlement negotiations x. Deeporter and Mot (2011) provided a comprehensive literature review on this idea. There are significant opportunities to compare the optimism behavior models with advanced models developed in cognitive sciences literature. This comparison analysis may provide inside on concepts in behavioral game theory.

7.5. Considering Business Decisions

This research analyzes claims and settlement negotiations from the pure economic view point, and assumes parties have full intentions and capabilities to litigate the case. Business decisions and any other considerations that can be expanded in this research is considering parties' current economic status into the decision model. A plaintiff may believe that he will be entitled to damages if he pursue the claim to the trial, but he may settle with a lesser amount that expected from the trial due to his present financial issues. There are multiple other scenarios that a claim party may decide if to pursue the claim because of factors other than the issues directly related to the claim itself.

There may be other considerations such as work relationship and future opportunities between the claim parties. Measuring the relationship in monetary values is another way to advance the game theoretic models. For example, a contractor may consider maintaining its relationship with a project owner in hope of making profit in the owner's future project. Predictions of the future profit and calculating the Net Present Value is a logical way to model the relationship aspect.

7.6. Nuisance Suits

Nuisance Suit is the term used for a claim where the plaintiff pursues a case, understanding there is little or negative expected value in pursuing the case. This research assumes a party will not bring a Nuisance Suit, meaning if settlement negotiations fail, the party will only proceed to trial if the expected outcome from trial exceeds litigation costs. However, the literature suggests players may pursue a Nuisance Suit for a variety of reasons.

Several papers, starting with Nalebuff (1987), suggested players may pursue a case even if the litigation costs exceeds their expected outcome from the trial. The analysis of Nuisance Suits is evolved from P'ng's research (1983) to Rosenberg and Shavell's research (2006). Incorporation of the possibility of a Nuisance Suit could improve this model.

7.7. Multiple Litigants and Claims

This study considered one plaintiff and one defendant as the only two players of the game. A model could be developed considering multiple individuals or entities (i.e. joint ventures) for either party. The individuals that form either the plaintiff or the defendant may have different prior beliefs on the claim elements. Speirs and Prescott et al. (2010) proposed a model where two plaintiffs non-cooperatively respond to defendant's actions. The challenge is splitting the award amount between the players of one side of the game. One solution may be applying bargaining negotiations with complete information such as a Nash Bargaining Solution. For example, the bargaining model will need to include a concealed complete information game between the individuals or entities who form the plaintiff. These individuals as the plaintiff still participate in an asymmetric information game.

Another approach is to consider two different claims that are intertwined. It means that the actions and outcomes of one game impacts the other game. For instance, Che and Shearman (1993) have developed a model that a plaintiff claims against two separate defendants and the outcome of each opponent may affect the other. These situations can also be modeled as private information for each party. The private information can be assessed applying Bayesian Network structures as discussed in chapter 5.

7.8. Including the Judge as a Player in the Game

This research assumed the Judge learns actual damages in the discovery process. It means that the player's private information is assumed to be fully revealed in the pleadings. This assumption can be changed by assuming partial information transfer from the parties to the Judge. The partial information transfer may result from the Judge's error in receiving and processing the information. This case can be modeled by considering random errors for the Judge's process in finding the truth. For example, Baker et al. (2007) studied the relationship between court errors and settlement timing.

If the players hide the information from the Judge, the model requires input to include the Judge as a third player to the game. Adding the Judge as a player will require defining all the game theory elements described in Chapter 3 for the Judge including: actions, outcomes, timing, information and predictions. In this case the Judge's point of interest in the game would be finding the true level of Damages and Liability to grant an accurate award to the damaged party. However, the Judge's view is significantly different than parties' interest, which is maximizing their monetary outcome at trial. Therefore, the challenge is to find a method that can measure these two different goals in a same scale. Another solution may be designing a multi-objective game theory model including Attorneys and Experts in the Game

7.9. Including Attorneys and Experts as Players in the Game

Attorneys or experts can also be considered as players in the settlement negotiation games but will require defining all the game theory elements described in Chapter 3 for the added player(s) including: actions, outcomes, timing, information and predictions. This model assumes the attorneys want to maximize the outcome for their clients (the plaintiff or the defendant), not maximize their own profit by extending the duration of the claim. This assumption is consistent with the attorney code of ethics, and the belief that attorneys maximize their long-term profit by maintaining a positive reputation. However, there are number of researchers that assume attorney's fees is a tool for maximizing their own outcome (Watts, 1994) (Miller, 1987). The attorney fees can be arranged as contingent fees or hourly fees. Contingent fees are usually set to be a certain percentage of the Judge's award. Hourly fees are flat rates that an attorney charges to its client in exchange of each hour of work on the case. Hourly fee arrangement is a more common approach in the industry practice in construction claims.

Contingent fees are common in claims that involves individuals versus corporations. There are numbers of studies analyzing contingent fee contract attorneys. For example, Polinsky and Rubnifield (2001) studied the impact of contingent fee on settlement time and amount.

Timing is not the only aspect of the attorneys and expert involvement that can impact the settlement negotiations. Friehe and Baumann (2016) analyze the impact of the relationship of a party and its attorney in a discovery process. Analyses like this research provide insight to a new area of expanding on settlement negotiation games. However, further research requires legal background and understanding the common industry practice.

7.10. Historic Data to Form Prior Beliefs

Gathering historical data from different sources aids in assessing parties' prior beliefs over the claim elements, leading to an accurate prediction of parties' decisions. Using this method requires data gathering from diverse resources including sources from a project level, corporate level, general counsels, law firms, experts, and eventually the public court documents. However, not all the disputes are discussed with the attorneys or experts or go through discovery, or which limits the method of calculating the parties' decisions. Furthermore, considering the root causes in construction claims can be a helpful tool in classifying the data.

7.11. Estimating Judge's Award

This model assumes the award at trial is calculated as product of Liability and Damages. This is not the only acceptable approach the Judge may employ. For example, the Judge may choose awards for the purpose of maximizing the overall social efficiency, or minimizing the probability of trial. Daugherty and Reinganum (2008) provided a comprehensive literature review on this matter.

7.12. Mechanism Design for Contract Adjustments

Mechanism Design is a game theoretic approach applying engineering methodology to designing economic incentives toward desired objectives in strategic settings. The Mechanism Design starts at the end of the game, from the outcomes, and goes backwards to find the routes that leads (or does not lead) to the desired outcome. This approach is widely used to draft a contract agreements. Chatterjee (2014) provides a selective literature review of the mechanism design with a focus on Bayesian games.

7.13. Multi Criteria Decision Making Models

Decision trees are one of the most relevant and sound approaches to perform a decision analysis in claims. Traditional decision tree models cannot analyze the interaction between claim parties, since they only consider the outcomes for one party. Using the multi-criteria decision making approach can include additional parameters such as opponent's beliefs and Judge's decision.

The Judge's goal is to properly apply the law to a case, where player's goal are often maximizing the monetary outcomes of a claim. These two goals completely different in nature. Hypothetically, the multi-criteria decisions making analysis can capture these goals simultaneously. The challenge in this method would be quantifying Judge's goal or his considerations in ruling the verdict. This topic needs to be addressed by working hand-in hand with law scholars who are familiar with the fundamentals of Judge's decision process.

7.14. Game Theory and Influence Diagrams

Influence diagrams are a different type of graphical and mathematical representations of decision making analysis. Since the decision making processes are combined with probabilistic inferences in influence diagrams, these diagrams are a generalization of Bayesian Networks. There are multiple examples where influence diagrams are used in game theoretic approaches including Koller and Milch (2001), who introduce Multi-Agent Influence Diagram (MAID), where they claim MAIDs provide a complete graphical representation of any extensive form games.

The influence diagrams are also used in analyzing non-cooperative games to model opponent's decision behavior in context of adversarial risk analysis (Banks, & Rios, 2016). Some of the concepts provided in this research can be applied to settlement negotiation games in future research

7.15. Artificial Neural Network (AAN) Model for Updating Beliefs

The model in this research relies on Bayesian thinking approach for updating parties' beliefs over the claim elements once they have anew observation. An alternative to Bayesian Networks is Artificial Neural Network (ANN), a computational model based on the structure and functions of biological neural networks. The ANN model contains a dynamic structure which changes based on the information that flows through the network.

ANNs are capable of applying nonlinear statistical modeling to define complex relationships between inputs and outputs and provide system updates using a learning process. The ANN model can replaces the BN model discussed in chapter 6. If ANN is applied, the parametric models introduced in chapter 5 will remain unchanged.

8. Conclusion and Remarks

This dissertation provided a model to predict outcomes in construction claims using economic theory to analyze construction disputes. This researched defined construction disputes in quantifiable elements, categorized the elements into measurable variables given complexities and interrelationships, and input the variables into the model to ascertain each party's decision making process based on their beliefs over the specifics of the dispute.

This research provided a broad literature review on construction claims to ascertain the root causes for claims. It identified construction claims often occur due to divergent expectations, or parties' belief gap over Liability and Damages. It was shown that the Bayesian Network model for legal reasoning accurately measures parties' beliefs over these two variables. In addition, the Bayesian Network updates parties' beliefs based on new information they may receive during the claim processes. The belief update is a key tool for parties to bring their opponent to settlement before litigation.

The analysis in the case studies show the parties continuously exchange information during the claim process. As parties advance in the claim processes, they update their beliefs and expected outcome from trial. In both cases the updated beliefs resulted in less gap between parties' expected outcome from trial. Therefore, a party that is motivated to reach settlement, attempts to update their opponent's beliefs through information exchange processes.

The following are main remarks of this dissertation:

- Party's best action depends on his belief over the specifics of the case and beliefs over its opponent's beliefs.
- Claim parties update their opponent's beliefs over Liability and Damages to get closer to their desirable outcome
- A successful settlement negotiation requires the following actions:
  - Finding all relevant contract articles
  - Having Experts to find pieces of evidence relevant to the contract articles
  - Inquire attorney's opinion on credibility and strength of arguments
  - Use robust methods to calculate the damages
  - Estimating Litigation Costs and use them as a leverage toward settlement

Disclaimer

This document is only intended to be for academic used and does not represent legal interpretation, guidance or advice. While efforts have been made to ensure accuracy of the models, these methods are not applicable to any legal dispute and shall not be addressed in any dispute resolution or legal proceeding.

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