

ABSTRACT

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This dissertation explores the properties of neutron matter for baryon chemical potential of the order of a hundred MeV in the context of neutron star interior. Neutron stars are known to be cold ($< 10^9$ K) dense objects composed mostly of neutrons with densities of the outer core reaching 10^{13} g/cc to 10^{14} g/cc. For such densities neutron matter is speculated to form a superfluid condensate in the triplet channel of angular momentum. The implication of this superfluidity for the transport properties of the core of a neutron star is dramatic as the neutrons get gapped leading to a Boltzmann suppression in most observables at low temperature and the low energy properties of the system is dictated by ungapped low energy modes. The first part of the thesis discusses the effective theory of the low energy modes of triplet neutron matter. We also derive the masses of the slightly high lying massive modes as a function of temperature. This is followed by a calculation of neutrino emissivity of the Goldstone modes in the presence of realistic magnetic fields in magnetars.

Topics in Neutron Star Physics

by

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Dedication

To my parents.

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Chapter 1: Introduction

1.1 What is neutron matter ?

A significant portion of nuclear physics research today is aimed at understanding matter at densities ranging from subnuclear densities to densities much higher than nuclear densities. This is because matter at these densities is speculated to exist in the universe under various circumstances. One such laboratory of state of matter is the core of neutron stars in which the density of matter can well exceed nuclear densities. But before we go into that let us look at what happens to matter at nuclear densities. If we take ordinary matter and crush it to nuclear densities what we are left with is a liquid of neutrons and protons, the dynamics of which is described by effective theories derived from the underlying theory of strong interactions known as quantum chromodynamics (QCD). QCD is a theory which encapsulates all the dynamics of the constituents of nucleons, i. e. quarks and gluons. The theory is a quantum field theory with Fermionic degrees of freedom known as quarks which are in the fundamental representation of $SU(3)$ gauge group, and Bosonic degrees of freedom in the adjoint representation of the $SU(3)$ gauge group. The Lagrangian

is given by

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu)\psi - \frac{1}{4}(F_{\mu\nu}^a)^2 \quad (1.1)$$

where ψ stands for the quark fields and A_μ^a is the gauge field. Also $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf_{abc}A_\mu^b A_\nu^c$ and $D_\mu = \partial_\mu - igA_\mu^a t^a$. t^a are the generators of the $SU(3)$ gauge group and g is the coupling constant. The theory is asymptotically free, which is to say that the coupling constant grows weak as a function of increasing energy scale and rigorous calculations can be used to make predictions using perturbation theory in this system. But at low energies the theory is strongly coupled making perturbation theory calculations from first principles useless. At these energies the degrees of freedom are no longer quarks and gluons but hadrons. Hadrons are composite particles made up of quarks held together by the strong force. The hadrons can be fermionic, known as baryons, examples being neutrons and protons. They can also be bosonic, known as mesons, examples being pions, rho etc. At low energies and zero density, the dynamics is well described by the effective theory of the hadrons. The low energy constants of this theory needs to be extracted from the microscopic theory of QCD. But it is not possible to achieve this analytically as the theory is strongly coupled. To circumvent this problem numerical simulations on discretized space-time (lattice) is used. Lattice simulations have been remarkably successful in some scenarios. One such success story was in cosmology where lattice QCD provided answers to questions relating to the deconfinement phase transition as the universe cooled from a quark-gluon plasma phase to hadronic phase at zero density. It was found that there was no phase transition but a rapid cross-over along the tem-

perature axis. But lattice QCD has not been very successful in regimes where there is a finite baryon density and temperatures are low. This is because in this case lattice algorithms involving the method of importance sampling break down. But this region of the phase diagram is of utmost importance when it comes to exploring the core of neutron stars as the core has a finite baryon density and low temperatures. The transport properties of a neutron star and the mass-radius relation, which is dictated by the equation of state, both depend on the state of matter in the core of the star. There is only one scenario where analytic calculations are possible and it is the regime of very high density. Asymptotically high baryon density is believed to give rise to deconfined quark matter which can be treated using perturbation theory. The reason to believe this is that at these densities and low temperatures, the quarks being fermions form Fermi spheres and only the quarks with momentum close to the Fermi surface contribute to the low energy transport properties. These quarks have energy close to the Fermi energy and are weakly coupled. Also the equation of state of high density quark matter is mostly determined by weakly coupled quarks as at very high chemical potential, most of the Fermi sphere consists of quarks with very high energy. However, densities as high as this might not be present at the core of neutron stars and at moderate densities perturbation theory loses its validity again. As mentioned before numerical methods using lattice QCD also break down in this regime.

There have been several attempts to capture the physics at moderate densities using model calculations, for example the Nambu-Jona-Lasinio model [1–5], Quark-Meson model [6], Polyakov loop extended Nambu-Jona-Lasinio models [7–11], Polyakov

loop extended quark-meson model [12, 13], but these models do not follow from a controlled approximation. In the absence of a first principle calculation, other phenomenological ways to deal with the problem have been explored at nuclear densities. One such idea is to treat nucleons as the fundamental degrees of freedom which are strongly interacting and then use the Fermi liquid theory to describe the system. The statement of Fermi liquid theory is as follows. As we are interested to look at small fluctuations about the Fermi surface, the effects of the strong coupling between bulk nucleons can be encapsulated in parameters of the theory such as the effective mass, Fermi velocity of the nucleon quasiparticles and other Landau parameters. The low energy effective action for the relevant degrees of freedom, for example the Goldstone modes if a spontaneous breakdown of symmetry takes place, can then be calculated using Fermi liquid theory description. We will see an example of the above mentioned approach in this thesis.

1.1.1 Neutron matter in the context of neutron stars

Neutron stars are extremely dense and massive stellar remnants that result from the gravitational collapse of a massive star after a supernova explosion. The average density of neutron stars is of the order of 10^{12} g/cm³. The mass of a typical neutron star is between 1.4 solar mass to 2 solar mass and the radius is about 10 kms. The surface temperature of a visible neutron star can range from about $\sim 10^8$ K to 10^5 K. Neutron stars, as evident from the name, are mostly made of neutrons.

They are born of a collapse of the core of a massive star. Before the collapse the density of the core is of the order of 10^6 g/cc and the proton fraction is of the order of 0.4. During the collapse the protons capture electrons giving rise to neutrons and electron neutrinos.

$$p + e^- = n + \nu_e \quad (1.2)$$

This process reduces the proton fraction in the star. As the density rises, the neutrino mean free path becomes smaller than the size of the core trapping them inside. This helps the decay of Eq. 1.2 to reach equilibrium. About 10 seconds after the collapse the neutrinos escape the core leading to a new equilibrium condition.

This condition is given by

$$\mu_n = \mu_p + \mu_e \quad (1.3)$$

where μ_n, μ_p, μ_e are the chemical potentials of neutrons, protons and electrons respectively. This is true because the decay of Eq. 1.2 is possible only when the combined energy of the proton and the electron is equal to that of the neutron and neutrino energy. At finite density the particles participating in the decay are at the Fermi surface. This means that the typical energy of the particles participating in the decay is given by their Fermi energy. However, the Fermi energy of neutrinos is zero as the neutrinos escape the core quickly. This leads to the condition of Eq. 1.3. Note that, due to overall charge neutrality the density of protons and electrons are the same. As the density of fermions is proportional to the cube of the Fermi momentum, we have $k_F^e = k_F^p \equiv k_F$ where k_F^e, k_F^p, k_F^n are the Fermi momenta of electrons, protons and neutrons. This implies that $\mu_p = \frac{(k_F)^2}{2m_p}$ is significantly smaller

than $\mu_e = \frac{(k_F)^2}{2m_e}$ and we have $\mu_n \sim \mu_e$ and $\mu_p \sim \frac{m_e}{m_p} \mu_n$. The masses of neutrons and protons can be taken to be the same for our purpose here and we can conclude that the Fermi momentum of neutrons is much much larger than that of the protons which in turn means that the density of protons is much smaller than that of neutrons. Hence, matter at the core of neutron stars is mostly made of neutrons with proton fraction around 1% to 7%.

1.1.2 Fermi Liquid Theory

The properties of degenerate neutrons in neutron star cores are well captured by the neutron matter Fermi liquid theory. The basic idea behind this description goes as follows. In almost all of the arguments above what is emphasized over and over again that the neutrons that participate in any of the processes mentioned have momenta close to the Fermi surface. This statement can be understood easily. The degenerate fermions that participate in any process have to jump to a state of different momentum which differs from its earlier momentum by a small amount. For fermions that are in the bulk of the Fermi sphere this is not possible due to the Pauli exclusion principle, that is there are no unoccupied states in the vicinity of its momentum to jump to. However fermions close to the Fermi surface can jump out of the Fermi sphere as there are unoccupied states with momentum greater than the Fermi momentum. This picture is reasonable for free fermions gas. But what about interacting fermions? This is where Landau Fermi liquid theory comes in. The physical picture is as follows. In the absence of any interaction, the fermions have a

dispersion relation $\epsilon_0(p)$. The ground state has a distribution function $n_0(\epsilon)$ which is a unit step function of the form $\theta(-\epsilon + E_F)$ at zero temperature. Landau argued that in the presence of interactions, virtual particle and hole pairs are created. This changes the distribution function from $n_0(p)$ to $n(p)$. This change was assumed to be a smooth or analytic function of the interaction. This means that as the interaction is turned on adiabatically, the noninteracting states transform into the interacting ones smoothly. The transformation would proceed without encountering any singularity in general. If a singularity was encountered, it should be viewed as a phase transition. Hence in the absence of a phase transition the evolution of non-interacting states to interacting ones should be smooth. This would also imply that the quantum numbers used to represent the noninteracting states should be good quantum numbers for the interacting states as well. The one-fermion particle state would become a quasi particle state that would carry the same charge and spin as the noninteracting fermion but would have a different effective mass. The distribution function $n(p)$ characterises the interacting system. Let us define

$$\delta n(p) \equiv n(p) - n_0(p). \quad (1.4)$$

For a stable interacting system, $\delta n(p)$ is non-zero only in the vicinity of $|p| \sim p_F$. The ground state energy then can be characterized in terms of $\delta n(p)$. If we have $n(p) = n_0(p) + \delta n(p)$ with $\frac{\delta n}{n} \ll 1$, the total energy of the interacting system E can be expanded in powers of δn as follows:

$$E = E_0 + \delta E \quad (1.5)$$

$$= E_0 + \sum_p \epsilon_p \delta n(p). \quad (1.6)$$

δE is a measure of the energy cost to add an excitation of momentum near the Fermi surface. Hence the quasi particle energy is given by

$$\epsilon(p) = \frac{\delta E}{\delta n(p)}. \quad (1.7)$$

Now, we know that the cost of adding one particle to the ground state of N fermions is by definition

$$E(N + 1) - E(N) = \mu, \quad (1.8)$$

where μ is the chemical potential. Hence we can see that $\mu = \epsilon(p)$, where $|p| = p_F$. The higher order corrections of δE in δn can tell us about the interactions of the quasi-particles. This is known as the Landau expansion. Thus we expand the energy in terms of δn as follows

$$E(\delta n) = \sum_p \epsilon_0(p) \delta n(p) + \frac{1}{2} \sum_{p,p'} f(p, p') \delta n(p) \delta n(p') + \mathcal{O}(\delta n(p))^3. \quad (1.9)$$

Here, $f(p, p')$ satisfies $f(p, p') = f(p', p)$ and f is known as the Landau parameter.

Now we can define an effective mass by noticing that close to $|p| = p_F$, the Fermi velocity is $v_F(p) = \partial_p \epsilon(p)$. The effective mass is given by

$$m^* = \frac{p_F}{|v_F(p_F)|} \quad (1.10)$$

Hence we can see from 1.6 and 1.9 that $\epsilon(p) \equiv \frac{dE}{dn}$, can be expressed as

$$\epsilon(p) = \epsilon_0(p) + \sum_{p'} f(p, p') \delta n(p'). \quad (1.11)$$

The above equation can be understood as follows: The quasi-particle energy is equal to the noninteracting Fermi energy up to lowest order in fluctuation δn . The first

order corrections to this energy corresponds to the change in the energy of the quasiparticle due to the presence of other quasi-particles. $f(p, p')$ measures the strength of the interaction between quasiparticles of momentum p and p' where both p and p' are close to the Fermi surface.

Note that, the Fermi liquid description of fermions does not require the fermion quasiparticles near the Fermi surface to be weakly coupled because power counting arguments guarantee that their contribution to the total energy is smaller compared to the leading order terms. Having described the main idea behind the Fermi liquid theory, we can now review what we already know about the degenerate fermionic matter in the core of neutron stars. A detailed account of how the Fermi liquid theory helps our calculations is illustrated in chapter 1 of this thesis.

1.2 Superfluidity

As neutron stars are extremely dense and cool and made of neutrons which are Fermionic, any attraction can lead to the formation of Cooper pair in the core of the star. This can affect the properties of the core significantly as we will see below. But before we go into that let us look at the possibility of Cooper pair formation. Cooper pairs are basically bound state of fermions at the Fermi surface with energies lower than that of the Fermi energy. BCS theory states that an arbitrarily small attractive interaction between fermions at the Fermi surface can lead to the formation of bound states of paired fermions with opposite momenta which lowers their energy compared to the Fermi energy. The theorem was stated first in the context

of superconductivity in terrestrial metals at small temperatures. The bound pairs in the metals are that of electrons. The Cooper pairs being bosons form a Bose-Einstein condensate giving rise to the phenomenon of superconductivity at low temperatures. Something very similar can happen in neutron matter provided there are attractive angular momentum channels available at the Fermi surface between nucleons.

When we consider the possibility of Cooper pairing between nucleons, there are two possible scenarios that come to mind. The first is a pairing between neutrons and protons. The second is the pairing between two neutrons or two protons. The former is ruled out as the Fermi energies of the neutrons and the protons are very different in neutron matter. In order to decide whether two neutrons or protons can form Cooper pairs we have to look for the availability of attractive channels at the Fermi surface between the constituents of the pair. It is difficult to figure this out using analytic techniques and we have to look towards nucleon scattering experiments for clues. Note that Cooper pairs are bosonic and the lowest energy state corresponds to having the net momentum of each pair equal to zero. This means that the fermions that constitute the Cooper pair have momenta equal to the Fermi momentum and oppositely directed. Hence we look at the two nucleon scattering phase-shift data in laboratory experiments at lab energies twice the Fermi energy of a single Fermion for different partial waves. If the phase shift is positive in some partial wave, it indicates that there is an attractive interaction in the corresponding angular momentum channel. Fig. 1.1 and 1.2 show the scattering phase-shift in the 1S_0 and 3P_2 angular momentum channels. The 1S_0 channel corresponds to two nucleons pairing in orbital angular momentum zero state and spin zero state

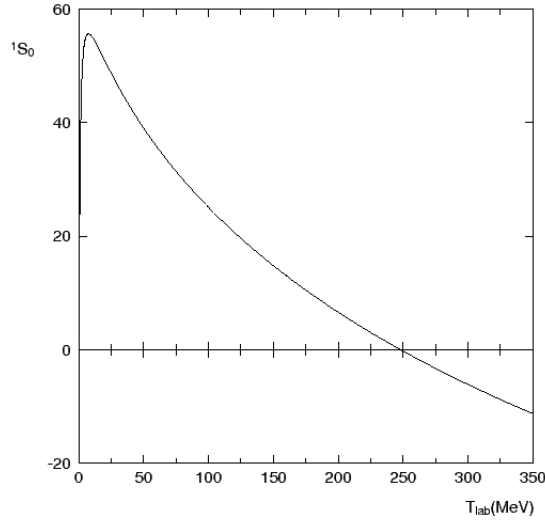


Figure 1.1: This figure shows the singlet phase shift in degrees as a function of the lab energy. As is seen, the phase shift is positive for lower lab energy which corresponds to particles with lower Fermi momentum. The phase shift decreases with increasing lab energy and eventually becomes negative for 250 MeV. This means that the singlet channel is attractive at lower energies and is repulsive at high energies.

resulting in total angular momentum zero state. 3P_2 or the triplet channel involves nucleons pairing in spin one state, orbital angular momentum one and total angular momentum two state.

As can be seen from the plots 1.1 and 1.2, at lower energies the 1S_0 channel has a positive phase shift and with increasing lab energy it decreases and becomes negative around ~ 250 MeV. The triplet channel on the other hand has an increasing phase shift with lab energy and becomes greater than the 1S_0 phase shift for lab energies ~ 150 MeV. This means that at lower energies which correspond to lower Fermi energies and densities, the more attractive channel is the singlet channel, but

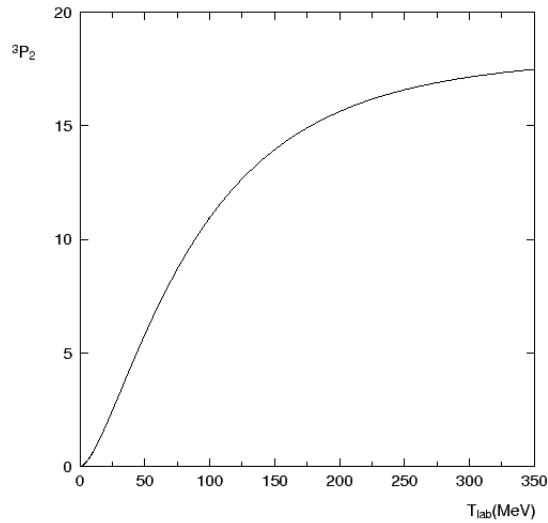


Figure 1.2: This figure shows the 3P_2 phase shift as a function of the lab energy. The phase shift is smaller at lower energies and monotonically increases with the lab energy. At one point the phase shift becomes greater than the singlet phase shift. The 3P_2 channel is always attractive, however is more attractive than the 1S_0 channel only at high energies.

at higher Fermi energies the triplet channel becomes more attractive compared to the singlet channel. As the density of neutrons is much higher than that of the protons in the neutron star cores. This means that protons in the core can form Cooper pairs in the 1S_0 angular momentum channel and the neutrons pair in the 3P_2 channel. In the outer core the neutrons can also form 1S_0 Cooper pairs as the density of neutrons is smaller there.

The Cooper pair of protons carries an electric charge twice the charge of a single proton. Hence, the core of the star where the proton Cooper pairs form and condense is superconducting. This means that there is a Meissner effect for the magnetic fields and they get confined in vortices in the 1S_0 proton superconductor. The Cooper pairs of neutrons on the other hand are neutral and hence the resulting condensate is a superfluid but not a superconductor. The superfluidity of neutrons can affect the transport properties of the core in a drastic manner. We concentrate on the effects of pairing on the cooling of the star. It turns out that there can be three major effects of the pairing on cooling and these are given by : 1. suppression of neutrino emissivity of paired Fermions, 2. change in specific heat of paired fermions and 3. triggering of a new neutrino emission process known as the pair breaking formation process near the critical temperature.

The suppression of neutrino emissivity can be explained in the following manner. Neutrons can decay to neutrinos through a Z boson exchange and the energy emitted in such a process is proportional to the available phase space to the momentum of the neutron particle and hole states. However, when the neutrons are paired, their

spectrum acquires a gap, that is

$$E = \sqrt{\left(\frac{p^2}{2m_n} - \mu\right)^2 + \Delta^2} \quad (1.12)$$

where E is the neutron quasi-particle energy and Δ is the gap. This means that the available phase space for the neutrons gets restricted and the emissivity is suppressed exponentially by a factor of $e^{-\frac{\Delta}{T}}$ at a temperature T lower than the gap.

The specific heat on the other hand shows two major effects of pairing. The first happens to be near the critical temperature and is related to the fact that as the temperature is lowered from above the critical temperature to a value lower than the critical temperature, the state of neutron matter goes through a second order phase transition. The typical energy carried by quasi-particles changes continuously during this phase transition, however the specific heat which corresponds to a derivative of the typical energy carried by the nucleons changes discontinuously with the temperature. Also, at lower temperatures the specific heat gets suppressed exponentially just like the emissivity due to the restricted phase space.

The pair breaking formation process is a neutrino emission process that involves two free neutrons lowering their energy by releasing neutrinos with energies of the order of the gap or the binding energy through a Z boson exchange and forming a Cooper pair. This process can contribute significantly to neutrino emissivity only at temperatures close to the critical temperature. This is because the PBF process requires available free neutrons which can form Cooper pairs and such free neutrons are abundant at temperatures close to the critical temperature as the pairs of neutrons can break frequently due to the thermal excitations.

1.3 Proton superconductivity and magnetic fields in neutron stars

Other than the extreme density of the matter present at the core, the neutron stars also harbour extreme magnetic fields. A typical neutron star has a magnetic field of the order of 10^{12} Gauss or higher. Such high magnetic fields are created during the collapse of a massive star as follows. When a massive star collapses through a supernova explosion into a neutron star, its magnetic fields increase in strength. Halving the size of a star increases the magnetic field four fold which is why such high magnetic fields are found in neutron stars. These huge fields are sustained by persistent superconducting currents of the proton superconducting core of the star.

A class of stars known as magnetars can have magnetic fields as high as 10^{15} Gauss on their surface. An upper bound on the magnetic field of any star can be found by comparing the magnetic energy of the field lines and that of the gravitational binding energy of a star.

1.4 Triplet paired neutron matter

As discussed earlier, some regions of the core of a neutron star are expected to have densities of the order of a few times nuclear density. By looking at the vacuum two to two nucleon scattering phase shift at lab energies equal to twice the Fermi energy of neutrons we are interested in, we can conclude that the 3P_2 angular momentum channel is the most attractive interaction at the Fermi surface.

For the purpose of our discussion here we concentrate on this triplet paired phase of neutron matter. Although we have substantial evidence in favour of the conjecture that in the inner core of the star neutrons pair up in the 3P_2 channel, there is still some ambiguity as far as the exact form of the order parameter is concerned. This is to say that, a 3P_2 Cooper pair state can be any linear combination of total angular momentum projections $+2, +1, 0, -1, -2$ states. Although all possible linear combinations of these 5 projections belong to the same subspace $J = +2$, as $J = +2$ corresponds to an irreducible representation of the rotation group, any two randomly picked linear combinations of these 5 projections cannot necessarily be transformed into each other by rotations. This is because the only necessary and sufficient condition for a spin vector to belong to the subspace of $J = +2$ is that, after a rotation it can still be expressed as a linear combination of the 5 projection basis vectors. This does not mean that a particular vector that belongs to the $J = +2$ subspace can be transformed into any other vector that also belongs to the subspace merely by applying rotations. Hence, it is not necessary for all these possible linear combinations to be degenerate. This means that we are yet to figure out which linear combination of these 5 basis vectors (angular momentum projection vectors) minimizes the energy. In order to do this we need to write down the most general form of the order parameter explicitly in the triplet(3P_2) phase. The simplest way to express the 3P_2 order parameter, which also makes the spin and the orbital angular momentum components of the total angular momentum 2 explicit, is given by

$$\psi^T \sigma^2 \sigma^i \overleftrightarrow{\nabla}^j \psi = \Delta(k) = \sum_{m=+2,+1,0,-1,-2} a_m \Delta_m(k) \quad (1.13)$$

where

$$\begin{aligned} \Delta_{+2}(k) &= \begin{pmatrix} Y_1^1(k) & 0 \\ 0 & 0 \end{pmatrix}, \Delta_{+1}(k) = \begin{pmatrix} \sqrt{2}Y_0^1(k) & Y_1^1(k) \\ Y_1^1(k) & 0 \end{pmatrix}, \\ \Delta_0(k) &= \begin{pmatrix} Y_{-1}^1(k) & \sqrt{2}Y_0^1(k) \\ \sqrt{2}Y_0^1(k) & Y_1^1(k) \end{pmatrix}, \\ \Delta_{-1}(k) &= \begin{pmatrix} 0 & Y_{-1}^1(k) \\ Y_{-1}^1(k) & \sqrt{2}Y_0^1(k) \end{pmatrix}, \Delta_{-2}(k) = \begin{pmatrix} 0 & 0 \\ 0 & Y_{-1}^1(k) \end{pmatrix} \end{aligned} \quad (1.14)$$

. These two by two matrices are in spin space and Y_m^l s are spherical harmonics, $Y_{\pm 1}^1(k) = \mp \frac{1}{\sqrt{2}}(k_1 \pm ik_2)$ and $Y_0^1 = k_3$ where k_1, k_2, k_3 are the components of the unit vector of momentum \hat{k}_i . For our purpose however, a different basis to express the order parameter is useful. Any orbital angular momentum one state can be written as $\delta(k) = \sum_{\mu, \nu=1,2,3} i\sigma^\mu \sigma^2 A_{\mu\nu} k^\nu$ where σ are Pauli matrices and A is a complex three by three matrix. Note that since the order parameter is in total angular momentum 2 state, it transforms as a spherical tensor of rank 2. As a spherical tensor of rank 2 can be expressed as a complex symmetric traceless tensor so can be our order parameter. In order to determine what the form of the order parameter is we need to evaluate the free energy as a function of $A_{\mu\nu}$. The form of $A_{\mu\nu}$ that minimizes the free energy should correspond to the ground state. This was done using a Ginsburg Landau analysis in [32]. Basically, they considered temperatures close to the critical temperature where the magnitude of the order parameter $A_{\mu\nu}$ is small. In this regime the free energy can be expanded in the magnitude of the

order parameter over the temperature. The form of this expansion is known as the ‘Ginsburg-Landau free energy’. The coefficients of the terms in this expansion were derived using a BCS calculation as a function of $A_{\mu\nu}$ at finite temperature. If the free energy upto fourth order in the order parameter is minimized, it is found that the order parameter is unitary, which is to say that the matrix $A_{\mu\nu}$ that minimizes the free energy is real upto an overall phase. A symmetric traceless order parameter that is real upto an overall phase can be expressed in a basis where it is diagonal. Using such a basis and also ensuring that the trace of the order parameter is zero restricts the form of the order parameter to

$$\psi^T \sigma^2 \sigma^i \overleftrightarrow{\nabla}^j \psi = \bar{\Delta} = \bar{\Delta} e^{i\alpha} \begin{pmatrix} r & 0 & 0 \\ 0 & -1 - r & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (1.15)$$

Here σ are Pauli matrices. Let us now look at the symmetries of the order parameter in spin space, orbital angular momentum and overall phase. When $\bar{\Delta} = 0$, the order parameter has the symmetry $SU_s(2) \otimes SO_L(3) \otimes U(1)$ where the subscript s stands for spin and L stands for orbital angular momentum. The tensor and spin orbit nuclear forces break $SU_s(2) \otimes SO_L(3) \otimes U(1)$ explicitly to $SO_J(3) \otimes U(1)$ where J stands for total angular momentum. When $\bar{\Delta} \neq 0$ the $U(1)$ symmetry is spontaneously broken and we have a Goldstone mode that is known as the superfluid phonon and depending on the value of r , $SO_J(3)$ is broken giving us Goldstone modes that correspond to the spontaneous breaking of the generators of rotation. However, there is still a degeneracy in the form of the order parameter as r can take any value between 0 and $-\frac{1}{2}$. In order to lift this degeneracy the sixth order term in the gap

expansion in the free energy needs to be taken into account which fixes $r = -\frac{1}{2}$. Although this was derived for temperatures close to the critical temperature, this form of the order parameter with $r = -\frac{1}{2}$ has been argued to persist even at lower temperatures [14,18]. In the presence of a magnetic field in the problem the form of the order parameter changes depending on how strong the magnetic field. However in order to describe these effects we need to introduce the notion of the orientation of a unitary 3P_2 order parameter. The meaning of orientation of the order parameter is as follows. The order parameter is a traceless symmetric matrix in coordinate space. If we take the matrix in Eq. 1.15 and substitute $r = -\frac{1}{2}$ we can see that two of the eigenvalues in Eq. 1.15 are identical and equal to $-\frac{1}{2}$ and the third one is different and is equal to 1. If we denote the rows and columns of the matrix by x, y, z instead of 1, 2, 3, it is the zz eigenvalue which is different from the xx and yy eigenvalues. This means that the z direction in space is different from x and y in the presence of the condensate and this is what we mean by the orientation of the condensate. In the absence of a magnetic field the orientation of the condensate is arbitrary, which is to say that the direction in space that corresponds to the eigenvalue of 1 is arbitrary. However, when a magnetic field is introduced, it orients the condensate parallel to itself, that is, the z direction aligns with the magnetic field. Also, the value of r gets modified to $r = -\frac{1}{2} + (0.017B_{15})^2$. B_{15} is magnetic fields in units of 10^{15} Gausa. As a consequence, even for the largest magnetic fields corresponding to the magnetars, the order parameter can be well approximated by $r = -\frac{1}{2}$.

We previously mentioned that depending on what the entries of 3.1 are, the number of broken generators may vary. For $r = -\frac{1}{2}$ only 2 of the generators are broken. To

be more specific let us write down how the order parameter transforms under an $SO_J(3)$ rotation,

$$\Delta \rightarrow R\Delta R^T \tag{1.16}$$

where R is a rotation matrix given by $R = \exp(iJ^i\alpha_i)$ where J^i are the generators of $SO_J(3)$. Plugging $r = -\frac{1}{2}$ in the condensate we notice that the condensate remains invariant under any rotation about the z axis but changes under rotations about x and y axes. This means that in the absence of a magnetic field the condensate spontaneously breaks $SO(3)$ rotational symmetry down to $SO(2)$ giving rise to two goldstone modes which we call angulons. As mentioned before, the temperature of a neutron star is usually much smaller than the critical temperature for condensation. This implies that the properties of the core of a neutron star will be dictated by the lowest energy excitations of the triplet phase. The angulons being massless are the lowest energy modes of the triplet phase and hence are expected to be dominant contributors to the low energy observables. But in order to predict these observables correctly we need a low energy effective theory of angulons. The first part of this thesis concerns itself with deriving the EFT of angulons and the regime of validity of this EFT. Also a finite temperature calculation is presented which addresses a controversy regarding the mass of angulons at finite temperature. The point of contention is as follows. It was claimed in [27] that there are no massless modes at finite temperature despite the spontaneous breaking of space-time symmetries by the triplet condensate. This claim is made for temperatures not just greater than the critical temperature where the claim is clearly valid and self-evident, but

for temperatures smaller than the critical temperature. Although such a statement seems unlikely to be correct, it cannot be ruled out as there is no generic proof of the Goldstone's theorem that holds for the spontaneous breaking of space-time symmetries. In order to settle this puzzle we do an explicit calculation deriving the propagator for the goldstone modes at nonzero temperatures. Also we find the masses of the higher lying massive modes at finite temperature to confirm that the effective theory of the angulons is valid for temperatures smaller than the gap as we find that the massive modes have masses of the order of the triplet gap.

The latter part of the thesis elaborates a calculation of one of the observables, namely the neutrino emissivity which is relevant to neutron star cooling. Before we start exploring the details of the effective theory or the calculation of emissivity, a brief discussion on what role neutrino emissivity plays in the cooling of a neutron star is in order.

1.5 Neutrino emissivity and cooling

The thermal evolution of neutron stars consists of two phases. The first one lasts for upto a 10^5 years after the birth of the star and is characterised by cooling due to neutrino emission from the core of the star. The second phase follows the first and is characterised by cooling due to photon emission from the surface of the star. As we are interested in the core of neutron stars, we concentrate on the neutrino emission processes. There can be various neutrino emission processes with their respective temperature dependence. Some of these basic neutrino emission

processes are discussed below. The emissivity scales roughly as $\sim T^n$, where the power n depends on the number of degenerate fermions involved in the emission. As we will see below, the processes that involves five degenerate fermions go as T^8 which are labeled as ‘slow’ and the processes that involve three degenerate fermions go as T^6 which are labeled as ‘fast’.

1.5.1 Direct Urca

The simplest neutrino emission process is the direct Urca process which involves the following two parts,

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{1.17}$$

and

$$p + e^- \rightarrow n + \nu_e. \tag{1.18}$$

Energy conservation is ensured by the beta equilibrium condition. However, in order to satisfy momentum conservation the proton fraction is required to be greater than 11% [16]. However the proton fraction merely reaches 4% [16] for matter densities close to the nuclear density. As the proton fraction is found to increase with density, it may be possible for this process to be realized deep within the core of the star where the density is expected to be greater than the nuclear density. As an aside, the direct Urca process does not necessarily have to involve electrons and electron neutrino and can involve other leptons such that,

$$n \rightarrow p + \mu^- + \bar{\nu}_\mu \tag{1.19}$$

and

$$p + \mu^- \rightarrow n + \nu_\mu. \quad (1.20)$$

The temperature dependence of direct Urca process can be estimated as follows.

The expression for the emissivity for the process can be written as

$$\epsilon^{DU} = \int \frac{d^3 p_{\bar{\nu}}}{(2\pi)^3} \frac{d^3 p_e}{(2\pi)^3} \frac{d^3 p_n}{(2\pi)^3} \frac{d^3 p_p}{(2\pi)^3} (1 - f_p)(1 - f_e) f_n \delta(P_f - P_i) |M_{fi}|^2 E_{\bar{\nu}} \quad (1.21)$$

where p_i stands for the momentum of the i^{th} species of particles. P_f and P_i are the momentums of the final and initial states. M_{fi} stands for the amplitude of the Direct Urca process. f_i is the distribution function of the i^{th} particle species. In the expression for the emissivity, no distribution function for neutrino has been included as the neutrinos leave the star as soon as they are released as their mean free path is much larger than the radius of the star. This means that the distribution function for neutrinos is equal to '0' and the neutrinos are not Pauli blocked. In the expression for emissivity all the phase space integrals except that of the antineutrino, contribute one power of T (temperature) each. This is because the fermions participating in the process have momentum distributed in a thin shell of thickness T about the Fermi momentum. The antineutrino phase space integral contributes a factor of T^3 . $E_{\bar{\nu}}$ contributes a factor of T and the energy conserving delta function contributes T^{-1} . Counting all these factors it is found that the temperature dependence of the direct Urca process is given by T^6 .

1.5.2 Modified Urca

At densities below the required density for the Urca process to occur, a variant of the Urca process known as the modified Urca can contribute to neutrino emission. The process is the following,

$$n + n \rightarrow n + p + e + \bar{\nu}_e. \quad (1.22)$$

It involves five degenerate fermions which allows momentum conservation to be much less restrictive than direct Urca process. The process can take place at any density when both neutrons and protons are present and does not require proton fraction to be high as in direct Urca. Also, the involvement of five degenerate fermions ensures that the temperature dependence of the process goes as T^8 making the process slower than direct Urca. The two extra powers of temperature compared to that of the Urca process comes from the phase space integral of the two extra degenerate fermions.

1.5.3 Bremsstrahlung

The bremsstrahlung process is very similar to the modified Urca and is given by

$$n + n \rightarrow n + n + \nu_e + \bar{\nu}_e. \quad (1.23)$$

The difference from the modified Urca process is that it involves the neutral current and a pair of neutrino-antineutrino is released. The neutrinos can be of any flavor. Bremsstrahlung involves four degenerate fermions contributing four powers of T

to the emissivity. The two neutrinos contribute factors of T^6 through their phase space integrals. The amplitude for this process however involves the propagator of a neutron which is off shell by $\sim T$. This contributes a factor of T^{-2} through the square of the amplitude. The energy carried by neutrino-antineutrino pair contributes a factor of T and the energy conserving delta function contributes a T^{-1} . Again counting all powers of temperature, the emissivity of the Bremsstrahlung process is found to scale as T^8 with temperature. This process is slower compared to the modified Urca by two orders of magnitude. However, when the nucleons in the core are paired in Cooper pairs, neutrino emission through Bremsstrahlung from the crust of neutron stars may become dominant.

1.5.4 Pair breaking formation process

When two neutrons form a 3P_2 Cooper pair the binding energy can be released through a neutrino-antineutrino pair emission. This process involves the neutral current and is known as the pair breaking formation (PBF) process as it involves frequent breaking and formation of the Cooper pairs. The PBF process can be the dominant cooling process under certain conditions. The efficiency of the PBF process can be attributed to the second order nature of the pairing phase transition. Because of this second order nature, near the critical temperature of transition there are enough unpaired neutrons for this process to take place frequently. The thermal fluctuations near the critical temperature ensures that the pairs keep on breaking giving rise to unpaired neutrons. The emissivity for the PBF process can

be expressed as

$$\epsilon^{DU} = \int \frac{d^3 p_{\bar{\nu}}}{(2\pi)^3} \frac{d^3 p_{\nu}}{(2\pi)^3} \frac{d^3 p_n}{(2\pi)^3} \frac{d^3 p_p}{(2\pi)^3} (1 - f_n)(1 - f'_n) \delta(P_f - P_i) |M_{fi}|^2 E_{\bar{\nu}} \quad (1.24)$$

where f_n and f'_n stand for the distribution function for the two neutrons involved in pair formation. The temperature dependence of the PBF emissivity can be ascertained as follows. The antineutrino and the neutrino phase space contribute T^6 together. One of the neutrons contribute T through the phase space integral. The other neutron does not contribute any power of T as its momentum is fixed by momentum conservation unlike the direct Urca or modified Urca which involved 3 to 5 degenerate fermions. The energy carried by neutrinos is again $\sim T$ and the energy conserving delta function gives a factor of T^{-1} . This implies that the PBF emissivity goes as $T^7 R(\frac{\Delta}{T})$ where Δ is the superconducting gap and R is a dimensionless function. R captures the fact that PBF process gets suppressed for temperatures below the gap as the neutrons are paired and the small temperature fluctuations are not able to break them.

1.5.5 Cooling

During the birth of a neutron star gravitational energy of the order of 10^{51} ergs is converted to thermal energy. During the first one minute 98% of this energy is carried away by neutrinos. 1% gets transferred to the supernova ejecta and rest of the energy is stored in the new-born neutron star. The star cools down via various energy emission processes through out its evolution. The rate of decrease in

temperature can be expressed as

$$\frac{dE}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu + H. \quad (1.25)$$

Here E is the total thermal energy of the star, t stands for time. C_v is the specific heat, L_ν is the neutrino emissivity and L_γ is the photon luminosity. H stands for heating processes, including dissipation through differential rotation of the star or decay of magnetic field. The above equation assumes the core and the crust of the star to be isothermal, which is a reasonable approximation for stars that are a couple of decades old. The major contribution to the specific heat comes from the core of the star which makes up 90% of the volume of the star and contains 98% of the mass. The contribution of all leptons, baryons, mesons, to the specific heat are added up to give the total specific heat per unit volume. Baryons when paired at temperatures lower than the critical temperature become gapped and their contribution to the specific heat gets suppressed by the Boltzmann factor $\exp(\frac{-\Delta}{T})$. However as the leptons cannot pair, the specific heat does not get reduced by more than an order of magnitude when pairing between baryons occur.

The thermal luminosity of photons is given approximately by that of a blackbody

$$L_\gamma = 4\pi R^2 \sigma_{SB} T_e^4 \quad (1.26)$$

where T_e is an effective temperature, σ_{SB} is the Stefan-Boltzmann constant. Thermal photons mostly get emitted from the photosphere of the star which is also called the atmosphere. Sometimes the photosphere can be located on a solid surface, especially in the presence of a strong magnetic field. T_e gives an estimate for the average effective temperature for the photosphere. The outermost layers of a neutron

star are together known as the envelope. The thermal time scale of the layers in the envelope is shorter than that of the core. The interior of the star usually consists of degenerate matter, which causes the interior to have high thermal conductivity. This in turn makes the interior of the star to have a uniform temperature. However, the envelope acts as a layer of insulator separating the colder surface from a relatively hot interior. If the temperature at the bottom of these layers be T_b and the temperature of the photosphere be T_e , the two temperatures can be related as follows,

$$T_e = 10^6 K \left(\frac{T_b}{10^8} \right)^{0.5+\alpha} \quad (1.27)$$

where $\alpha \ll 1$ [16].

With all the the required ingredients C_v , L_γ , L_ν in hand, a basic cooling calculation proceeds as follows. C_v , L_γ , L_ν have all power law dependence on temperature. Hence they can be expressed as

$$C_v = C_9 T_9, L_\nu = N_9 T_9^8, L_\gamma = S_9 T_9^{2+4\alpha} \quad (1.28)$$

where $T_9 = \frac{T}{10^9 \text{K}}$. The temperature dependence of the specific heat corresponds to that of unpaired degenerate fermions. The expression for the neutrino emissivity used here corresponds to the modified Urca process. The typical values for C_9 , N_9 and S_9 are given by 10^{39} ergs/K, 10^{40} erg/s and 10^{33} erg/s respectively. As mentioned before, the first 10^5 years after the birth of a neutron star, cooling is dominated by neutrino emission. After 10^5 years photon cooling takes over.

Cooling by neutrino emission: Ignoring L_γ and H we can write,

$$\frac{dE_{th}}{dt} = -L_\nu. \quad (1.29)$$

Eq. 1.29 can be solved using Eq. 1.28 to obtain,

$$t = \frac{10^9 C_9}{6N_9} \left(\frac{1}{T_9^6} - \frac{1}{(T_9^0)^6} \right). \quad (1.30)$$

Here, T_9^0 is the temperature at the birth of the star, or at $t = 0$. If $T_9 \ll T_9^0$, we have $T_9 = \left(\frac{\tau_{M.U}}{t} \right)^{1/6}$, where $\tau_{M.U}$ is known as the modified URCA time scale and defined as $\tau_{M.U} \equiv \frac{10^9 C_9}{6N_9}$. For the typical values mentioned above for C_9, N_9 etc, $\tau_{M.U}$ is of the order of one year. This means that in about one year after the birth the temperature of the star reaches an asymptotic value where $T \ll T_9^0$.

Cooling by photon emission: Photon luminosity dominates over the neutrino emissivity in the second phase of cooling of a neutron star. In this phase we can write down

$$\frac{dE_{th}}{dt} = -L_\gamma. \quad (1.31)$$

which can again be solved using 1.28. As $\alpha \ll 1$, $T_9 = T_1^9 e^{\frac{t_1-t}{\tau_\gamma}}$ where T^1 is the temperature at $t = t_1$ and $T_9^1 = \frac{T^1}{10^9 \text{K}}$. τ_γ is known as the photon cooling time scale and is defined as

$$\tau_\gamma = \frac{10^9 C_9}{S_9}. \quad (1.32)$$

For typical values of S_9 and C_9 , $\tau_\gamma \sim 10^7$ years.

Chapter 2: Effective theory of angulons and dispersion relation of massive modes

2.1 Introduction

This chapter deals with the effective theory of the angulons and the low lying massive modes. Effective theory is one of the most useful tools in theoretical physics. It utilises the idea of separation of scales to isolate the interesting physics at the energy scale relevant to the problem at hand while averaging over higher energy scales. Effective field theory treatment of a problem fails when there is no separation of scales and consequently such regimes are difficult to handle analytically. The problem we have at hand involves energy scales that are widely separated making the use of a set of effective theories possible. What we are interested in here is a state of matter known as the triplet paired neutron matter at finite density. Some of the properties of matter at finite density in any state whatsoever is dictated by the lowest energy excitations in that state. This means that we need an effective theory for excitations at these low energy scales. This chapter elaborates the basic ideas and various steps involved in identifying the degrees of freedom relevant to the low energy triplet matter and derive an effective theory for these degrees of freedom

from an underlying microscopic theory. As mentioned before the low energy degrees of freedom for the problem at hand, that is the triplet paired neutron matter are the Goldstone modes, known as the angulons. The latter part of this chapter deals with the effects of having a nonzero temperature on the properties of the angulons and also calculates the masses of higher lying modes at finite temperature. The motivation for this is twofold. Firstly, there is no proof of the Goldstone's theorem when space-time symmetries are broken and it is unclear whether there exist gapless modes and their number. A way to settle this contradiction is to do an explicit calculation of the propagator of the modes that are expected to be the Goldstone modes if the Goldstone's theorem were to hold in this scenario. Also, there is a possibility that some of the modes that seem irrelevant at temperatures much below the critical temperature on account of being massive may actually be light enough to contribute to transport properties at finite temperature. In order to be able to decide whether they do so or not, a finite temperature calculation of their masses is required. If their masses turn out to be larger than the temperature in consideration, then they are irrelevant for the physics at that temperature. But before we can delve into the finite temperature calculation, it is important to look at some of the details of the effective theory of angulons. The effective theory of the angulons described here was derived in [14] and some of the consequences on transport properties were discussed in [17, 18].

2.2 Effective theory of angulons

In principle the underlying microscopic theory from which the low energy coefficients of the effective theory of angulons are to be calculated is QCD. But, due to the nonperturbative nature of QCD at low energies, this is not feasible. However, it should be noted that neutrons being fermionic form a Fermi sphere and for calculating observables related to the transport properties at small temperatures, only excitations close to the Fermi surface matter. So, all we need for our purpose is a theory describing the lowest energy excitations around the neutron Fermi surface. In order to obtain this theory, we begin with a model Lagrangian that captures all the basic properties of neutrons close to the Fermi surface, such as the Fermi speed and the mass of the neutron and also reproduces the 3P_2 gap. The model Lagrangian has two flavors (spin up and down) of non-relativistic neutrons (ψ) with a chemical potential, interacting via a short range potential in the 3P_2 angular momentum channel

$$\mathcal{L} = \psi^\dagger (i\partial_0 - \epsilon(-i\nabla)) \psi - \frac{g^2}{4} \left(\psi^\dagger \sigma_i \sigma_2 \overleftrightarrow{\nabla}_j \psi^* \right) \chi_{ij}^{kl} \left(\psi^T \sigma_2 \sigma_k \overleftrightarrow{\nabla}_l \psi \right) , \quad (2.1)$$

where $\chi_{ij}^{kl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl})$ is the projector onto the 3P_2 channel satisfying $\chi_{ij}^{kl}\chi_{mn}^{ij} = \chi_{mn}^{kl}$. σ_i are the Pauli matrices. The above model Lagrangian can be interpreted in the following way. If we were to take, $\epsilon(p) = p^2/2M - \mu$ and tune the coupling g^2 in such a way that it reproduced the vacuum two to two nucleon phase shift in the 3P_2 channel at lab energies equal to twice the Fermi energy of the neutrons we would obtain our model Lagrangian. The model Lagrangian above

can also be put in the frame work of the Fermi-liquid theory. To elaborate on this further, note that, the Lagrangian of Eq. 2.1 is describing neutron excitations around the Fermi surface only. For a free fermionic theory we know that a finite density of fermions can be described as a free Fermi gas forming a Fermi sphere and the properties of such a system at low temperatures is dictated by the free fermions at the Fermi surface. Fermi-liquid theory suggests that, the properties of a gas of interacting fermions is also governed by fermionic excitations near the Fermi surface only. However, there is a difference between a free fermion theory and an interacting Fermi liquid. The difference is that an interacting fermion near the Fermi surface can be described by a free fermion with a renormalized mass and a renormalized speed. The renormalized parameters describing the interactions among the quasiparticles of the theory are called Landau parameters. The low energy constants of the theory encapsulate all the effects of the strongly interacting bulk fermions which had to be integrated out to obtain this description in terms of fermionic quasiparticles. Although this effective theory is an interacting theory of quasi-particles near the Fermi surface and it may appear that this description is not particularly useful because of the presence of interactions, there is a power counting argument which shows that interaction terms involving more fields are suppressed compared to the ones involving fewer fields. The suppression parameter is given by $\frac{p}{k_F}$, where p is the momentum of quasiparticle excitation about the Fermi surface, and k_F is the Fermi momentum. The expansion parameter can also be expressed as a ratio of the quasi particle density, $k_F^2 p$ to density of fermions of the entire Fermi sphere k_F^3 . Now, imagine that we are trying to write down the Fermi liquid

theory description of interacting fermions about the Fermi surface. The lowest order term that we can write down is the quadratic term in the quasi-particle fields, which gives the kinetic part of the quasi-particle Lagrangian. If we wish to include the next order in the quasi particle density expansion, we have interaction terms involving only two to two scattering between the quasiparticles. Note that the terms involving derivatives on the quasiparticle fields are not suppressed compared to the ones without any derivatives if both contain equal number of quasiparticle fields in them. This is because every derivative brings in a power of k_F in momentum and hence the terms without derivatives are of the same order as the ones without derivatives. However as can be seen in Eq. 2.1, only two to two interactions in the 3P_2 angular momentum channel has been included. Other angular momentum channels for two to two interactions, such as the 1S_0 , 3P_0 or 3P_1 etc have not been included. The reason behind this is that all these other channels are either repulsive or less attractive than the 3P_2 channel. Provided we assume that neutrons are paired in the 3P_2 channel and condensates in all these other channels are zero, which is to say that the phase of the system does not get altered by the presence of these other interactions, the low energy constants for the effective theory of angulons only get perturbative corrections from these other interactions(1S_0 , 3P_0 or 3P_1 etc.). In the language of renormalization group, the 3P_2 interaction is the marginally relevant operator where as all the other interactions are irrelevant. Eq. 2.1 should reproduce the results of an effective theory obtained by integrating out neutron modes far from the Fermi surface. The explicit degrees of freedom, that is the quasiparticles have kinetic energy given by $\epsilon(p) = \frac{p^2}{2M} - \mu \sim v_F(p - k_F)$ where v_F is the fermi velocity,

QCD \rightarrow effective theory of neutron interactions \rightarrow neutron matter Fermi liquid
theory \rightarrow angulon effective theory \rightarrow phenomenology.

Figure 2.1: Flowchart of effective theories

$= \frac{k_F}{M}$ where M is the mass of neutron quasiparticle.

Although the weak coupling assumption here is motivated by a Fermi liquid theory like power counting argument, there is also phenomenological evidence that supports a weakly coupled Lagrangian [28]. For example, model calculations using realistic nuclear potentials predict pairing gaps that are much smaller than the Fermi energy in all attractive channels. This means that a Lagrangian as in Eq. 2.1 would have to be weakly coupled to produce such a small gap. Also if we consider the unrenormalized bare two to two phase shifts, we see that they are small, which indicates again that the effective theory description may be weakly coupled in the density range we are looking at. We will find that the low energy constants of the theory we derive will only depend on the density of states at the Fermi surface and the Fermi velocity and some geometrical factors coming from the fact that the condensate is a spherical tensor of rank two. The derivation of the effective theory of angulons here is the penultimate step in a chain of effective theories starting from the fundamental theory of QCD and ending in phenomenology. The chain goes as shown in Fig. 2.1.

Our Lagrangian of Eq. 2.1 is in principle equivalent to the third link in this chain. The fact that the Lagrangian of Eq. 2.1 gives rise to a nonzero gap in the

3P_2 channel is not self-evident and a BCS (Bardeen-Cooper-Schrieffer) calculation is required to confirm the existence of a gap. The triplet channel BCS calculation is a little bit involved and hence is presented in the appendix .1. A simpler BCS calculation of gap in the singlet channel using a similar Lagrangian with contact interaction is presented here. We start with a Lagrangian with two species of fermions ψ_1, ψ_2 and a contact interaction between them in the singlet channel,

$$\mathcal{L} = \sum_{a=1,2} \psi_a^\dagger \left(\partial_0 + \frac{\nabla^2}{2M} + \mu \right) + g^2 \psi_2^\dagger \psi_1 \psi_1^\dagger \psi_2. \quad (2.2)$$

As this interaction is attractive the coupling $g^2 > 0$. In order to show that there is a nonzero condensate of Cooper pairs, we need to prove that $\langle \psi_1 \psi_2 \rangle$ is nonzero. For this, we define $\langle \psi_1 \psi_2 \rangle = \sigma$. σ is known as the auxiliary field. We rewrite Eq. 2.2 in terms of the auxiliary field σ as follows

$$\mathcal{L} = \sum_{a=1,2} \psi_a^\dagger \left(\partial_0 + \frac{\nabla^2}{2M} + \mu \right) + \sigma \psi_1^\dagger \psi_2 + \sigma^\dagger \psi_2 \psi_1 - \frac{|\sigma|^2}{g^2}. \quad (2.3)$$

The above step also implies introduction of an integration over the auxiliary field σ in the corresponding path integral. In other words, if we perform the gaussian integral in the path integral with the action of Eq. 2.3 over σ , we obtain a path integral without any remaining integration over σ with the action of Eq. 2.2. This trick is called the auxiliary field trick. Now, we perform a one loop expansion, for small g^2 . This is implemented by shifting σ to $\sigma_0 + \delta\sigma$ where σ_0 is a constant. $\delta\sigma$ contains small spatial fluctuations but is discarded as it only affects higher loops. Now the Lagrangian can be written in momentum space as

$$\mathcal{L} = -\frac{|\sigma_0|^2}{g^2} + \int \frac{d^4p}{(2\pi)^4} \begin{pmatrix} \psi_1^\dagger(p) & \psi_2(-p) \end{pmatrix} \begin{pmatrix} p_0 - \frac{p^2}{2M} + \mu & \sigma_0 \\ \sigma_0^\dagger & p_0 + \frac{p^2}{2M} - \mu \end{pmatrix} \begin{pmatrix} \psi_1(p) \\ \psi_2^\dagger(-p) \end{pmatrix} \quad (2.4)$$

The one-loop effective potential is given by,

$$-iV(\sigma_0) = -i\frac{|\sigma_0|^2}{g^2} + \int \frac{d^4p}{(2\pi)^4} \text{tr} \log \begin{pmatrix} p_0 - \frac{p^2}{2M} + \mu & \sigma_0 \\ \sigma_0^\dagger & p_0 + \frac{p^2}{2M} - \mu \end{pmatrix}. \quad (2.5)$$

Taking a derivative of Eq. 2.5 with respect to σ_0 and setting $\frac{dV}{d\sigma_0}$ to zero, we arrive at the ‘‘gap equation’’

$$\frac{\sigma_0}{g^2} = \int \frac{d^3p}{(2\pi)^3} \frac{\sigma_0}{2\sqrt{(\omega_p - \mu)^2 + |\sigma_0|^2}}. \quad (2.6)$$

Other than the trivial solution of $\sigma_0 = 0$, Eq. 2.6 has another solution no matter how small g^2 is when the corresponding integral on the right hand side of Eq. 2.6 is dominated by $\omega_p \sim \mu$. This integral can be computed as follows

$$\begin{aligned} \int \frac{d^3p}{(2\pi)^3} \frac{\sigma_0}{2\sqrt{(\omega_p - \mu)^2 + |\sigma_0|^2}} &\approx \frac{Mk_F^2}{\pi^2} \int_0^\infty dp \frac{1}{\sqrt{(p^2 - k_F^2)^2 + 4M^2\sigma_0^2}} \\ &\approx \frac{Mk_F^2}{\pi^2} \int_0^\infty dp \frac{1}{\sqrt{4k_F^2(p - k_F)^2 + 4M^2\sigma_0^2}} \\ &\approx \frac{Mk_F^2}{2\pi^2} \int_0^\lambda dq \frac{1}{\sqrt{q^2 + \frac{M^2\sigma_0^2}{k_F^2}}} \\ &= \frac{Mk_F}{2\pi^2} \text{arcsinh} \left(\frac{\lambda k_F}{M\sigma_0} \right). \end{aligned} \quad (2.7)$$

Here, λ is an ultraviolet cutoff. From Eq. 2.7 we find that

$$\sigma_0 = \frac{2\lambda k_F}{M} e^{-\frac{4\pi^2}{Mk_F g^2}}. \quad (2.8)$$

Hence we see that there is a nonzero gap for any nonzero g^2 . In the singlet pairing case, the ground state breaks $U(1)$ invariance and the fermion spectrum picks up a mass of the order of the gap. There is also a $U(1)$ Goldstone mode associated with this spontaneous symmetry breaking and it is known as the superfluid phonon. If

the $U(1)$ was gauged, the corresponding gauge field would pick up a mass due to the Higgs mechanism and the Goldstone mode would disappear. Now, we are ready to look at the triplet channel pairing of neutron matter. Just like in the case of the singlet pairing, we are interested in excitations about the condensate or the 3P_2 order parameter, we introduce an auxiliary field Δ_{ij} to rewrite 2.1 as

$$\begin{aligned}
S[\Delta, \psi] &= \int d^4x \left[\psi^\dagger (i\partial_0 - \epsilon(-i\nabla)) \psi + \frac{1}{4g^2} \Delta_{ij}^\dagger \Delta_{ji} + \left(\frac{\Delta_{ij}^\dagger}{4} (\psi^T \sigma_2 \sigma_i \overleftrightarrow{\nabla}_j \psi) + c.c. \right) \right] \\
&= \int d^4x \left[\frac{1}{4g^2} \Delta_{ij}^\dagger \Delta_{ji} + \frac{1}{2} \begin{pmatrix} \psi^\dagger & \psi \end{pmatrix} \begin{pmatrix} i\partial_0 - \epsilon(-i\nabla) & -\Delta_{ji} \sigma_i \sigma_2 \nabla_j \\ \Delta_{ij}^\dagger \sigma_2 \sigma_i \nabla_j & i\partial_0 + \epsilon(-i\nabla) \end{pmatrix} \begin{pmatrix} \psi \\ \psi^* \end{pmatrix} \right].
\end{aligned}$$

Eq. 2.1 can be recovered from the above equation if the Gaussian integral over the auxiliary field Δ is performed. As discussed before, Eq. 2.9 is nothing but a way to rewrite Eq. 2.1 that can help us obtain the effective theory of the fluctuations about the ground state with paired fermions. The 3P_2 projectors have been excluded from the above equation, which is justified if we intend to keep the functional integral over the order parameter Δ restricted to only the space of real symmetric matrices. Now, if we integrate out the fermions, we are left with an action involving the order parameter alone and it is given by

$$S[\Delta] = \int d^4x \left[\frac{1}{4g^2} \Delta_{ij}^\dagger \Delta_{ji} - i \text{Tr} \ln \begin{pmatrix} i\partial_0 - \epsilon(-i\nabla) & -\Delta_{ji} \sigma_i \sigma_2 \nabla_j \\ \Delta_{ij}^\dagger \sigma_2 \sigma_i \nabla_j & i\partial_0 + \epsilon(-i\nabla) \end{pmatrix} \right]. \quad (2.9)$$

The action in 2.9 is exact but is complicated and nonlocal for a space-time dependent order parameter. As our intention is to derive a low energy effective theory for fluctuations about a spatially uniform condensate, we can systematically expand the action of 2.9 about the ground state: $\Delta = \Delta_0 + \delta\Delta = \bar{\Delta} (\hat{\Delta}_0 + \delta\hat{\Delta})$, where the

order parameter for the ground state is given by

$$\Delta = \bar{\Delta} \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (2.10)$$

Keeping only the leading order terms with two derivatives in this expansion we obtain a local action for the fluctuations. Parametrizing the fluctuations in terms of the effective degrees of freedom, yield the effective action for the angulons.

The derivative expansion yields the following action up to terms of the order $\frac{\bar{\Delta}^2}{v_F^2}$ which is given by [14]

$$\begin{aligned} S_2[\Delta] = & \frac{Mk_F}{12\pi^2\bar{\Delta}^2} \int d^4x \left[\mathcal{I}_{ij}^{(1)}(\hat{\Delta}^\dagger \hat{\Delta}) [\partial_0 \Delta \cdot \partial_0 \Delta^\dagger]_{ij} - v_F^2 \mathcal{I}_{ijkl}^{(1)}(\hat{\Delta}^\dagger \hat{\Delta}) [\partial_k \Delta \cdot \partial_l \Delta^\dagger]_{ij} \right. \\ & + \frac{1}{2} \mathcal{I}_{ijkl}^{(2)}(\hat{\Delta}^\dagger \hat{\Delta}) \left(-2 [\hat{\Delta} \cdot \partial_0 \Delta^\dagger]_{ij} [\hat{\Delta} \cdot \partial_0 \Delta^\dagger]_{kl} \right) + \\ & \frac{1}{2} \mathcal{I}_{ijkl}^{(2)}(\hat{\Delta}^\dagger \hat{\Delta}) \left([\partial_0 \Delta^\dagger \cdot \partial_0 \Delta^*]_{ij} [\hat{\Delta} \cdot \hat{\Delta}^T]_{kl} \right) \\ & + \frac{v_F^2}{2} \mathcal{I}_{ijklmn}^{(2)}(\hat{\Delta}^\dagger \hat{\Delta}) \left(2 [\hat{\Delta} \cdot \partial_k \Delta^\dagger]_{ij} [\hat{\Delta} \cdot \partial_l \Delta^\dagger]_{mn} \right) \\ & \left. + \frac{v_F^2}{2} \mathcal{I}_{ijklmn}^{(2)}(\hat{\Delta}^\dagger \hat{\Delta}) \left(- [\partial_k \Delta^\dagger \cdot \partial_l \Delta^*]_{ij} [\hat{\Delta} \cdot \hat{\Delta}^T]_{mn} \right) \right. \\ & \left. + \text{h.c.} \right], \quad (2.11) \end{aligned}$$

where $\hat{\Delta} \equiv \Delta/\bar{\Delta}$.

The coefficients $\mathcal{I}^{(\alpha)}$ can be expressed as follows,

$$\begin{aligned} \mathcal{I}_{ij}^{(\alpha)}(\hat{\Delta}^\dagger \hat{\Delta}) &= A^{(\alpha)} \delta_{ij} + B^{(\alpha)} (\hat{\Delta}^\dagger \hat{\Delta})_{ij} \\ \mathcal{I}_{ijkl}^{(\alpha)}(\hat{\Delta}^\dagger \hat{\Delta}) &= C^{(\alpha)} \delta_{ij} \delta_{kl} + D^{(\alpha)} \delta_{ij} (\hat{\Delta}^\dagger \hat{\Delta})_{kl} + E^{(\alpha)} (\hat{\Delta}^\dagger \hat{\Delta})_{ij} (\Delta^\dagger \Delta)_{kl} + \text{perm.} \\ \mathcal{I}_{ijklmn}^{(\alpha)}(\hat{\Delta}^\dagger \hat{\Delta}) &= F^{(\alpha)} \delta_{ij} \delta_{kl} \delta_{mn} + G^{(\alpha)} \delta_{ij} \delta_{kl} (\hat{\Delta}^\dagger \hat{\Delta})_{mn} + H^{(\alpha)} \delta_{ij} (\hat{\Delta}^\dagger \hat{\Delta})_{kl} (\hat{\Delta}^\dagger \hat{\Delta})_{mn} + \text{perm.} \\ &+ J^{(\alpha)} (\hat{\Delta}^\dagger \hat{\Delta})_{ij} (\hat{\Delta}^\dagger \hat{\Delta})_{kl} (\hat{\Delta}^\dagger \hat{\Delta})_{mn} + \text{perm.} , \quad (2.12) \end{aligned}$$

‘+perm’ indicates that all possible permutations of the indices need to be included in the above expressions for $\mathcal{I}^{(\alpha)}$. The numerical values of A^α, B^α etc are given by

$$A^{(1)} = \frac{4}{3} \left(\frac{\pi}{\sqrt{3}} - 1 \right), \quad B^{(1)} = \frac{8}{3} - \frac{16\pi}{9\sqrt{3}} \quad (2.13)$$

$$\begin{aligned} C^{(1)} &= -\frac{4}{27} + \frac{145\pi}{1458\sqrt{3}}, & D^{(1)} &= \frac{14}{27} - \frac{220\pi}{729\sqrt{3}}, & E^{(1)} &= -\frac{10}{27} + \frac{152\pi}{729\sqrt{3}} \\ C^{(2)} &= \frac{11}{36} - \frac{25\pi}{243\sqrt{3}}, & D^{(2)} &= -\frac{10}{9} + \frac{128\pi}{243\sqrt{3}}, & E^{(2)} &= \frac{8}{9} - \frac{112\pi}{243\sqrt{3}} \\ F^{(2)} &= \frac{43}{1080} - \frac{263\pi}{13122\sqrt{3}}, & G^{(2)} &= -\frac{59}{270} + \frac{256\pi}{2187\sqrt{3}}, & & \\ H^{(2)} &= \frac{16}{45} - \frac{424\pi}{2187\sqrt{3}}, & J^{(2)} &= -\frac{8}{45} + \frac{640\pi}{6561\sqrt{3}}. & & \end{aligned} \quad (2.14)$$

(2.15)

In our problem, the ground state breaks $SO(3)$ rotational symmetry to $SO(2)$ giving rise to two Goldstone modes or angulons. We would like to parametrize the condensate in terms of the angulons fields. Note that we are ignoring the phonon that arises due to the breaking of $U(1)$ or baryon number. This is justified as we are only interested in the effective theory of angulons and the effective theory for the phonon resulting from the breaking of $U(1)$ decouples from the theory of angulons. Going back to the breaking of $SO(3)$, we can parametrize the condensate as follows,

$$\Delta = e^{-i(\alpha_1(x)J_1 + \alpha_2(x)J_2)/f} \Delta^0 e^{i(\alpha_1(x)J_1 + \alpha_2(x)J_2)/f}, \quad (2.16)$$

Here, J_1 and J_2 stand for the generators of rotation about the ‘x’ and ‘y’ axes. f is an arbitrary constant with a mass dimension same as that of α . It will be fixed by requiring a certain normalization for the angulons later. Substituting 2.16 in 2.11 and expanding in α_i gives us the following Lagrangian

$$S_2[\Delta] = \frac{1}{f^2} \frac{Mk_F}{6\pi^2} \int d^4x \left[\frac{9}{16} (8A^{(1)} + 5B^{(1)} + 80C^{(2)} + 62D^{(2)} + 53E^{(2)}) [(\partial_0\alpha_1)^2 + (\partial_0\alpha_2)^2] \right]$$

$$\begin{aligned}
& + v_F^2 \left[-\frac{9}{64} (8(32C^{(1)} + 14D^{(1)} + 5E^{(1)} + 288F^{(2)} + 162G^{(2)} + 90H^{(2)}) \right. \\
& + 333J^{(2)}) [(\partial_y \alpha_2)^2 + (\partial_x \alpha_1)^2] \\
& - \frac{9}{32} (8(16C^{(1)} + 4D^{(1)} + E^{(1)} + 96F^{(2)} + 30G^{(2)} + 9H^{(2)}) \\
& + 21J^{(2)}) [\partial_x \alpha_1 \partial_y \alpha_2 + \partial_y \alpha_1 \partial_x \alpha_2] \\
& - \frac{9}{8} (64C^{(1)} + 58D^{(1)} + 52E^{(1)} + 912F^{(2)} + 852G^{(2)} \\
& - 801H^{(2)} + 759J^{(2)}) [(\partial_z \alpha_1)^2 + (\partial_z \alpha_2)^2] \\
& - \frac{9}{64} (8(64C^{(1)} + 22D^{(1)} + 7E^{(1)} + 480F^{(2)} + 222G^{(2)} \\
& + 108H^{(2)} + 375J^{(2)}) [(\partial_y \alpha_1)^2 + (\partial_x \alpha_2)^2] \\
& = \int d^4x \left[\left(3 + \frac{\pi}{\sqrt{3}} \right) [(\partial_0 \alpha_1)^2 + (\partial_0 \alpha_2)^2] + v_F^2 \left[\left(\frac{\pi}{9\sqrt{3}} - \frac{3}{2} \right) [(\partial_z \alpha_1)^2 + (\partial_z \alpha_2)^2] \right. \right. \\
& - \frac{4\pi}{3\sqrt{3}} [(\partial_y \alpha_1)^2 + (\partial_x \alpha_2)^2] + \left. \left(\frac{2\pi}{9\sqrt{3}} - \frac{3}{2} \right) [(\partial_y \alpha_2)^2 + (\partial_x \alpha_1)^2] \right. \\
& \left. \left. + \left(\frac{3}{2} - \frac{14\pi}{9\sqrt{3}} \right) [\partial_x \alpha_1 \partial_y \alpha_2 + \partial_y \alpha_1 \partial_x \alpha_2] \right] \right], \tag{2.17}
\end{aligned}$$

provided we keep only terms upto quadratic in α_i . In the second line in the above equation we have chosen $f^2 = \frac{Mk_F}{6\pi^2}$. The action of 2.17 is not diagonal in α_1 and α_2 as the condensate mixes the two angulons through spatial derivatives. We have to diagonalize 2.17 to obtain the dispersion relation for the two fields. The matrix to be diagonalized is the following,

$$G(p) = \begin{pmatrix} ap_0^2 + v_F^2(bp_z^2 + cp_y^2 + dp_x^2) & ev^2 p_x p_y \\ ev^2 p_x p_y & ap_0^2 + v_F^2(bp_z^2 + cp_x^2 + dp_y^2) \end{pmatrix}, \tag{2.18}$$

with

$$\begin{aligned}
a &= 3 + \frac{\pi}{\sqrt{3}}, & b &= -\frac{3}{2} + \frac{\pi}{9\sqrt{3}}, & c &= -\frac{4\pi}{3\sqrt{3}}, \\
d &= -\frac{3}{2} + \frac{2\pi}{9\sqrt{3}}, & e &= \frac{3}{2} - \frac{14\pi}{9\sqrt{3}}.
\end{aligned} \tag{2.19}$$

The process of diagonalization will give us two fields which are linear combinations of the two original angulons for which dispersion relations will be given by setting the corresponding eigen values to zero. These can also be obtained by solving for p_0 for which the determinant of 2.17 goes to zero. The dispersion relations obtained are found to be such that the energy of the modes is proportional to the spatial momentum. The proportionality constant gives us the speed of the angulons. There is no reason to expect the speeds to be independent of direction and they indeed are not. Let us denote the speed of angulon i in the j th direction to be v_j^i . Then we have

$$\begin{aligned}
v_{x,y}^{(1)} &= \frac{v_F}{3} \sqrt{\frac{117}{18 + 2\sqrt{3}\pi}} - 2 \approx 0.477v_F, \\
v_{x,y}^{(2)} &= 2v_F \sqrt{\frac{\pi}{9\sqrt{3} + 3\pi}} \approx 0.709v_F, \\
v_z^{(1,2)} &= \frac{v_F}{3} \sqrt{\frac{99}{18 + 2\sqrt{3}\pi}} - 1 \approx 0.519v_F.
\end{aligned} \tag{2.20}$$

The dispersion relations are given by

$$\begin{aligned}
p_0^{(1)} &= \frac{\sqrt{27\sqrt{3}|p|^2 - 2\pi[2(p_x^2 + p_y^2) + p_z^2]}}{3\sqrt{2(3\sqrt{3} + \pi)}} v_F \\
p_0^{(2)} &= \frac{\sqrt{24\pi(p_x^2 + p_y^2) + 27\sqrt{3}p_z^2 - 2\pi p_z^2}}{3\sqrt{2(3\sqrt{3} + \pi)}} v_F.
\end{aligned} \tag{2.21}$$

The information we have is already sufficient to calculate some observables like the low temperature specific heat of angulons and we present this calculation here. At low temperature the dispersion relation can be linearized to simplify the calculation. The specific heat is found to be

$$c_v = \sum_{a=1,2} \frac{d}{dT} \int \frac{d^3p}{(2\pi)^3} \frac{\epsilon_a(p)}{e^{\epsilon_a(p)/T} - 1}$$

$$\approx 16.16 \frac{T^3}{v_F^3} = 1.44 \times 10^{-13} \left(\frac{T/K}{v_F/c} \right)^3 \frac{\text{erg}}{K \text{cm}^3}, \quad (2.22)$$

where $\epsilon_{1,2}$ is the energy of the two modes that diagonalized the 2.17 action. From this point onwards, we call these two uncoupled modes as angulons. The form T^3/v_F^3 can be obtained from dimensional analysis alone. It turns out that the specific heat of angulons is much smaller than the specific heat of electrons at temperatures relevant to neutron stars and is irrelevant for cooling.

2.2.1 Interaction of angulons

The interaction of angulons can be obtained from 2.11 by expanding it beyond second order in angulons fields. This is useful as the low temperature transport properties may get significant contribution from the interaction of the angulons and to know if they do or not we need to estimate their contribution to these transport properties. Expanding the action of 2.11 to quartic order gives us the following interaction terms

$$\begin{aligned} S_4[\Delta] &= \frac{1}{f^2} \int d^4x \left[\left(3 + \frac{\pi}{\sqrt{3}} \right) (\alpha_2^2 (\partial_0 \alpha_1)^2 + \alpha_1^2 (\partial_0 \alpha_2)^2) \right. \\ &+ \left(12 + \frac{4\pi}{\sqrt{3}} \right) (\alpha_1^2 (\partial_0 \alpha_1)^2 + \alpha_2^2 (\partial_0 \alpha_2)^2) \\ &+ \left(18 + 2\sqrt{3}\pi \right) \alpha_1 \alpha_2 \partial_0 \alpha_1 \partial_0 \alpha_2 + v_F^2 \left[\left(\frac{\pi}{9\sqrt{3}} - \frac{3}{2} \right) (\alpha_2^2 (\partial_x \alpha_1)^2 + \alpha_1^2 (\partial_y \alpha_2)^2) \right. \\ &+ \left(\frac{8\pi}{9\sqrt{3}} - 6 \right) (\alpha_1^2 (\partial_x \alpha_1)^2 + \alpha_2^2 (\partial_y \alpha_2)^2) - \frac{4\pi}{3\sqrt{3}} (\alpha_1^2 (\partial_x \alpha_2)^2 + \alpha_2^2 (\partial_y \alpha_1)^2) \\ &+ \left(3 - \frac{28\pi}{9\sqrt{3}} \right) (\alpha_1^2 \partial_x \alpha_1 \partial_y \alpha_2 + \alpha_2^2 \partial_x \alpha_1 \partial_y \alpha_2 + \alpha_1^2 \partial_x \alpha_2 \partial_y \alpha_1 + \alpha_2^2 \partial_x \alpha_2 \partial_y \alpha_1) \\ &+ \left(3 - \frac{10\pi}{3\sqrt{3}} \right) (\alpha_1 \alpha_2 \partial_x \alpha_1 \partial_y \alpha_1 + \alpha_1 \alpha_2 \partial_x \alpha_2 \partial_y \alpha_2) \\ &- \left(\frac{16\pi}{9\sqrt{3}} + 6 \right) (\alpha_1 \alpha_2 \partial_x \alpha_1 \partial_x \alpha_2 + \alpha_1 \alpha_2 \partial_y \alpha_1 \partial_y \alpha_2) \\ &\left. - \left(\frac{35\pi}{9\sqrt{3}} + \frac{3}{2} \right) (\alpha_2^2 (\partial_x \alpha_2)^2 + \alpha_1^2 \partial_y \alpha_1^2) + \left(\frac{2\pi}{9\sqrt{3}} - \frac{3}{2} \right) (\alpha_2^2 (\partial_z \alpha_1)^2 + \alpha_1^2 (\partial_z \alpha_2)^2) \right] \end{aligned}$$

$$- \left[\left(\frac{\pi}{\sqrt{3}} + \frac{9}{2} \right) (\alpha_1^2 (\partial_z \alpha_1)^2 + \alpha_2^2 (\partial_z \alpha_2)^2) - \left(\frac{22\pi}{9\sqrt{3}} + 6 \right) \alpha_1 \alpha_2 \partial_z \alpha_1 \partial_z \alpha_2 \right] . \quad (2.23)$$

As mentioned before neutrino emission is the major source of energy emission from a neutron star and hence it will be useful to derive the weak interaction of the angulons to determine whether they couple to neutrinos. Again there is a chain of effective theories involved in order to extract the weak interaction of the angulons. Since the angulons are neutral, the only possible coupling can be with the neutral current via Z boson. Here we derive this coupling. In order to do this we have to remember that the angulons couple to weak current and their interactions with the weak current can be derived from looking at the neutron coupling with the weak current. The neutrons couple to the weak current due to the presence of up and down quarks. Let us first start with the coupling of SM Lagrangian between leptons and Z bosons,

$$\mathcal{L}_{Z\text{-lep}} = \frac{gZ_\mu}{\cos \theta_W} [4\bar{\nu}\gamma^\mu(1 - \gamma_5)\nu - 4\bar{e}\gamma^\mu(1 - \gamma_5)e + \sin^2 \theta_W \bar{e}\gamma^\mu e] . \quad (2.24)$$

Here g stands for the weak coupling constant, θ_W is the Weinberg angle, the fields ν and e stand for neutrinos and electrons respectively. The standard model coupling between the Z boson and the quarks is given by

$$\mathcal{L}_{Z\text{-q}} = \frac{gZ_\mu}{\cos \theta_W} \left[4\bar{u}\gamma^\mu(1 - \gamma_5)u - \frac{2}{3} \sin^2 \theta_W \bar{u}\gamma^\mu u - 4\bar{d}\gamma^\mu(1 - \gamma_5)d + 3 \sin^2 \theta_W \bar{d}\gamma^\mu d \right] . \quad (2.25)$$

The next step in the series of effective interactions is the interaction between the nucleons and the Z bosons. The interaction between the nucleons and the Z bosons

is given by

$$\begin{aligned} \mathcal{L}_{Z\text{-had.}} = & \frac{gZ_\mu}{\cos\theta_W} \left[\frac{1}{2}\bar{N}\gamma^\mu T^3 N - (g_A + \Delta_s)\bar{N}\gamma^\mu\gamma_5 T^3 N \right. \\ & \left. - \sin^2\theta_W\bar{N}\gamma^\mu Q N + \dots \right]. \end{aligned} \quad (2.26)$$

where $g_A \sim 1.26$ is the nucleon axial charge. $\Delta_s = -0.16 \pm 0.15$ is the strange axial charge. $T^3 = \tau^3/2$ stands for the third component of *weak* isospin, Q is the electric charge in units of the fundamental charge. g_A, Δ_s are extracted from experimental data. The vector coupling equals 1 as being the coefficient of a conserved current is not renormalized. Now we need to take the non-relativistic limit as we are dealing with non-relativistic neutrons. Doing so we obtain the following interaction terms,

$$\mathcal{L}_W = C_V Z_0^0 N^\dagger N + C_A Z_i^0 N^\dagger \sigma_i N, \quad (2.27)$$

where Z_0^0 is the temporal component of the Z boson and Z_i^0 are the spatial components of the Z boson. C_V and C_A are given by

$$C_{V,A}^2 = \tilde{C}_{V,A}^2 \frac{G_F M_Z^2}{2\sqrt{2}}, \quad (2.28)$$

where $\tilde{C}_V = -1$ and $\tilde{C}_A \sim 1.1 \pm 0.15$. As not much is known about the interaction when modes far from the Fermi surface are integrated out, we use the vacuum form of the interaction. Again expanding the angulon fields gives us the angulon-Z boson vertices

$$\begin{aligned} S_W[\Delta] &= C_A \int d^4x \left[\frac{27}{8}(4A^{(1)} + 3B^{(1)})f(Z_2^0\partial_0\alpha_2 - Z_1^0\partial_0\alpha_1) \right. \\ &+ \frac{27}{8}(4A^{(1)} + 3B^{(1)})Z_3^0(\alpha_2\partial_0\alpha_1 - \alpha_1\partial_0\alpha_2) + \dots \\ &= C_A \int d^4x [9f(Z_2^0\partial_0\alpha_2 - Z_1^0\partial_0\alpha_1) + 9Z_3^0(\alpha_2\partial_0\alpha_1 - \alpha_1\partial_0\alpha_2) + ..] \end{aligned} \quad (2.29)$$

2.2.2 Validity of effective theory

The effective action in Eq. 2.11 is equivalent to Eq. 2.1 provided the higher order derivative terms in 2.11 can be neglected. It is justified to use Eq. 2.11 at tree level to calculate transport properties, as we know that a generic loop will have to have vertices coming from Eq. 2.23 and each vertex brings a negative power f or $\sqrt{Mk_F}$ with it. These negative powers of f have to be accompanied by a positive powers of external momentum Q . This means that loops are suppressed compared to the tree level as long as $Q \ll f$. When Q reaches f , the effective theory starts breaking down. However, this is not the only way this theory can break down. In action 2.11 we have also neglected higher power of $\bar{\Delta}/v_F$ and these terms may have a larger contribution to the action of Eq. 2.11 than that of the terms with powers of $\frac{Q^2}{f}$. However, for astrophysical estimates, these corrections are irrelevant as there are other sources of uncertainties which are bigger than these corrections.

The form of the effective action could have been guessed by noticing that the form in Eq. 2.11 indeed is the most generic form of the effective action upto terms of the order of two derivatives if we impose rotational invariance and use real symmetric order parameter Δ . There are terms, such as Δ^2 that never appear in Eq.2.11. The reason behind their exclusion is described below. Note that besides being invariant under equal rotations of spin and orbital angular momenta, given by

$$\Delta \rightarrow R\Delta R^T, \tag{2.30}$$

the microscopic action of 2.1 is invariant under opposite rotations of the spin and the orbital part as well,

$$\Delta \rightarrow R_S \Delta R_L^T, \quad (2.31)$$

. However Δ^2 is a term which is not invariant under unequal rotations on spin and orbital angular momentum. As the Lagrangian of Eq. 2.11 is an effective theory derived from the microscopic action of Eq. 2.9, it should respect the symmetries of 2.9 and hence does not contain Δ^2 terms. However, we know that nuclear forces don't exhibit this enhanced symmetry (separate rotations for spin and orbital angular momentum) due to the presence of spin orbit forces and tensor forces. This enhanced symmetry in the Lagrangian of Eq. 2.1 is an artifact of not including other interactions of the same mass dimension as the 3P_2 interaction, for example the ${}^3P_0, {}^3P_1$ interactions which are not invariant under the transformation 2.31. They have not been included in 2.9 as they are expected to change the low energy constants of the effective theory by a perturbatively small amount. In the next section we will evaluate the masses of modes corresponding the transformation of Eq. 2.30 (angulons) and also the masses of other real symmetric traceless fluctuations which do not correspond to any rotations of the order parameter. We expect the angulons to remain massless both at finite and zero temperature and expect the remaining modes to acquire a mass proportional to the gap that goes to zero at $T \rightarrow T_c$.

2.3 Finite temperature

Most of the material covered in this section appeared in the paper [15]. Before we get into the details of the finite temperature calculation, let us look at the normal modes first. As mentioned before we are interested in fluctuations of the order parameter that keeps it real, symmetric and traceless. This means that there are five possible modes of fluctuation of the order parameter. Some of the above modes correspond to physical rotations and the rest don't. We list all five modes

$$\begin{aligned} \mathcal{M}^{(1)} &\equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & \mathcal{M}^{(2)} &\equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \mathcal{M}^{(3)} &\equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \mathcal{M}^{(4)} &\equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \mathcal{M}^{(5)} &\equiv \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \quad (2.32)$$

\mathcal{M}^1 and \mathcal{M}^2 are angulons and correspond to physical rotations about x and y axes. \mathcal{M}^3 is the non diagonal fluctuation in the $x - y$ block of the order parameter. \mathcal{M}^4 changes the relative magnitude of the diagonal elements as is evident from its form and \mathcal{M}^5 is an overall scaling of the order parameter. There is an interesting relation between \mathcal{M}^3 and \mathcal{M}^4 and it is that under a $\frac{\pi}{4}$ rotation about the z axis, \mathcal{M}^3 turns into \mathcal{M}^4 . A consequence of this will appear when we find the masses of all these modes at nonzero temperature. In figures Fig. 2.2, 2.3, 2.4, 2.5, 2.6 there are pictorial representations of all these modes. In order to understand the pictures, we have to remember that an order parameter of the form $(\Delta_0)_{ij}$ can be expressed as

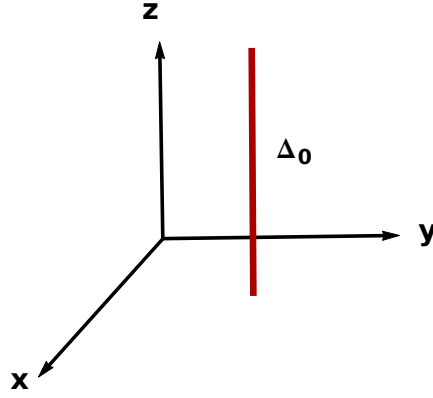


Figure 2.2: condensate

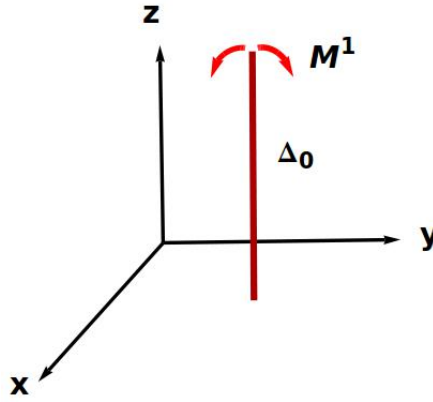


Figure 2.3: mode 1

$\hat{z}_i \hat{z}_j - \frac{1}{3} \delta_{ij}$. The order parameter hence, can be characterized as a line or a headless arrow parallel to z direction but not pointing in the $\pm z$ direction. Also, it is to be noted that only the first four modes can be pictorially represented here, the fifth one is just a scaling of the order parameter.

Fig. 2.2 portrays the ground state being parallel to the z axis. Fig. 2.3 shows how the mode \mathcal{M}^1 which is one of the angulons corresponding to the breaking of rotational symmetry about the x axis creates nondiagonal elements in the $y - z$

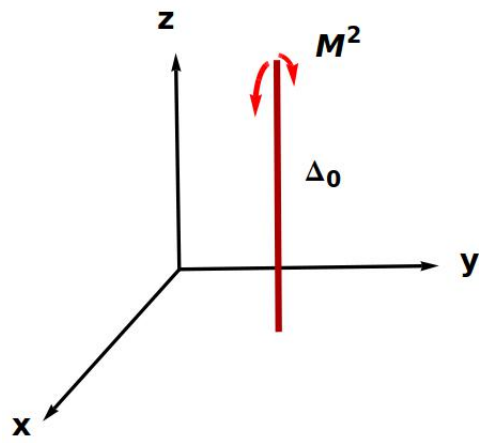


Figure 2.4: mode 2

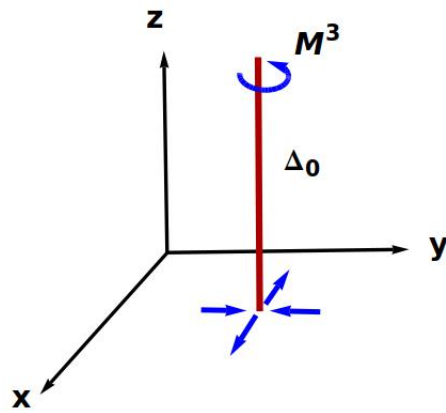


Figure 2.5: mode 3

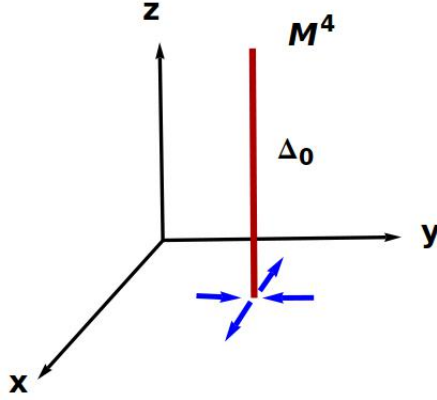


Figure 2.6: mode 4

block. Similarly Fig. 2.4 shows how the other angulon creates nondiagonal elements in the $x - z$ block. Fig. 2.5 is the mode that creates nondiagonal elements in the $x - y$ block and Fig 2.6 shows how the diagonal elements of the order parameter vary due to \mathcal{M}^4 . \mathcal{M}^3 is represented in the picture through its relation with \mathcal{M}^4 . This is because creating nondiagonal elements in the $x - y$ block does not correspond to a rotation about the z axis and hence this mode could not be portrayed like the first two modes were. It has been portrayed as a rotated \mathcal{M}^4 where the blue arrows show the change in the diagonal elements xx and yy , with arrows pointing outward parallel to the y axis conveying increase in the magnitude of the yy element and inward pointing arrows parallel to the x axis conveying the decrease in the magnitude of the xx element.

2.4 Action for the fluctuations around the ground state

We begin with the same microscopic model in 2.1 describing non-relativistic fermions with an attractive short-range interaction,

$$\mathcal{L} = \psi^\dagger (i\partial_0 - \epsilon_{(-i\nabla)}) \psi - \frac{g^2}{4} (\psi^\dagger \sigma_i \sigma_2 \nabla_j \psi^*) \chi_{ij}^{kl} (\psi^T \sigma_2 \sigma_k \nabla_l \psi) , \quad (2.33)$$

where the projector $\chi_{ij}^{kl} = \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl})$ projects the interaction onto the 3P_2 state. M and μ are the mass and chemical potential of the neutrons. $\epsilon_p = \sqrt{\frac{p^2}{2M} - \mu}$ is the kinetic energy of excitations with neutron quantum number known as the neutron quasiparticles near the Fermi surface. As we are interested in finite temperature properties of the neutron quasi-particle excitations, we have to do the calculation in imaginary time or euclidean space. At the end of the calculation our results have to be analytically continued to Minkowski space or real time. As we are interested in lower temperatures, for temperatures below the threshold of Cooper pair breaking, the only excitations that matter are bosonic. We introduce an auxiliary field Δ in order to characterize these bosonic excitations such that, $\Delta_{ij} = \Delta_0 + \delta\Delta(x)$, where $\Delta_0 = \langle \psi^T \sigma_2 \sigma_i \overleftrightarrow{\nabla}_j \psi \rangle$ is the order parameter at ground state, ψ is the neutron field and $\delta\Delta(x)$ contains the bosonic fluctuations about Δ_0 .

Rewriting the Lagrangian 2.33 in terms of the order parameter we obtain

$$S = \int d^4x \left[\psi^\dagger (i\partial_0 - \epsilon(-i\nabla)) \psi + \frac{1}{4g^2} \Delta_{ij}^\dagger \Delta_{ji} + \frac{\Delta_{ij}^\dagger}{4} (\psi^T \sigma_2 \sigma_i \nabla_j \psi) - \frac{\Delta_{ji}}{4} (\psi^\dagger \sigma_i \sigma_2 \nabla_j \psi^*) \right] (2.34)$$

. As we already know the ground state we are expanding about [19], we rewrite Δ_0 as

$$\Delta_0 = \bar{\Delta} \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2.35)$$

where $\bar{\Delta}$ is the magnitude of the gap [20]. In this phase as has been stated earlier, cylindrical invariance about the z axis is maintained, and we find two massless Goldstone modes, that are generated due to the spontaneous breaking of rotational symmetry by the condensate about x axis and y axis. We will do a generic calculation for as long as possible before specializing to the phase of 2.35. There is one small assumption regarding the order parameter. We only look at real symmetric matrices.

¹. Before presenting our results we will specialize to the phase of condensate given by 2.35. We can now integrate the fermions out and obtain

$$S[\Delta] = -T \int \frac{d^3p}{(2\pi)^3} \sum_{p_0} \left(\frac{1}{4g^2} \Delta_{ij}^\dagger(p) \Delta_{ji}(p) \right) + \text{Tr} \log(D^{-1}), \quad (2.36)$$

in momentum space, where the kernel of D^{-1} is

$$D^{-1}(p, k) = \begin{pmatrix} (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{k}) \delta_{p_0, k_0} (ip_0 + \epsilon_p) & i\Delta_{ji}(\mathbf{k} - \mathbf{p}) \sigma_i \sigma_2 p_j \\ -i\Delta_{ij}(-\mathbf{k} + \mathbf{p}) \sigma_2 \sigma_i p_j & (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{k}) \delta_{p_0, k_0} (ip_0 - \epsilon_p) \end{pmatrix} \quad (2.37)$$

Now as we begin to study small fluctuations of the order parameter about the ground state Δ_0 , we expand the action,

$$S[\Delta] = S[\Delta_0] + \left. \frac{\partial S}{\partial \Delta} \right|_{\Delta=\Delta_0} \delta\Delta + \left. \frac{1}{2} \frac{\partial^2 S}{\partial \Delta^2} \right|_{\Delta=\Delta_0} (\delta\Delta)^2 + \dots \quad (2.38)$$

¹Note that if Δ is assumed to be a real matrix it will not be able to describe the $U(1)$ superfluid phonon associated with broken baryon number. If we were to allow $\Delta = e^{i\gamma} \tilde{\Delta}$ where $\tilde{\Delta}$ is a real matrix, γ would describe the superfluid phonon field.

. Δ_0 satisfies the gap equation, $\frac{\partial S}{\partial \Delta}|_{\Delta=\Delta_0} = 0$, so that the first nonzero term in the action after $S[\Delta_0]$ is quadratic in $\delta\Delta$.

Using

$$\frac{\delta^2 \text{Tr} \log(D^{-1})}{\delta \Delta_{ij}(s) \delta \Delta_{kl}(r)} \Big|_{\Delta=\Delta_0} = \text{Tr} \left(D(p, q') \frac{\delta D^{-1}(q', q)}{\delta \Delta_{ij}(s)} D(q, p') \frac{\delta D^{-1}(p', k)}{\delta \Delta_{kl}(r)} \right) \quad (2.39)$$

where ‘Tr’ denotes the trace in Gorkov, spin, and momentum space, we find,

$$\begin{aligned} & \frac{\delta^2 \text{Tr} \log(D^{-1})}{\delta \Delta_{ij}(s) \delta \Delta_{kl}(r)} \Big|_{\Delta=\Delta_0} \\ = & T \sum_{p_0} \int \frac{d^3 p}{(2\pi)^3} \text{tr} \left[\frac{\sigma^m \sigma^i \sigma^{m'} \sigma^k (\mathbf{p} - \mathbf{r})^j (\mathbf{p} - \mathbf{r})^{n'} p^l p^n (\Delta_0)_{mn} (\Delta_0)_{m'n'}}{(p_0^2 + E_{\mathbf{p}}^2)((p_0 - \omega)^2 + E_{\mathbf{p}-\mathbf{r}}^2)} \right. \\ & + \frac{\sigma^i \sigma^l (\mathbf{p} - \mathbf{r})^j \mathbf{p}^k (ip_0 + \epsilon_{\mathbf{p}})(i(p_0 - \omega) - \epsilon_{\mathbf{p}-\mathbf{r}})}{(p_0^2 + E_{\mathbf{p}}^2)((p_0 - \omega)^2 + E_{\mathbf{p}-\mathbf{r}}^2)} + \frac{\sigma^j \sigma^k (\mathbf{p} + \mathbf{r})^i \mathbf{p}^l (ip_0 - \epsilon_{\mathbf{p}})(i(p_0 + \omega) + \epsilon_{\mathbf{p}+\mathbf{r}})}{(p_0^2 + E_{\mathbf{p}}^2)((p_0 + \omega)^2 + E_{\mathbf{p}+\mathbf{r}}^2)} \\ & \left. + \frac{\sigma^m \sigma^j \sigma^{m'} \sigma^l (\mathbf{p} + \mathbf{r})^i (\mathbf{p} + \mathbf{r})^{n'} p^k p^n (\Delta_0)_{mn} (\Delta_0)_{m'n'}}{(p_0^2 + E_{\mathbf{p}}^2)((p_0 + \omega)^2 + E_{\mathbf{p}+\mathbf{r}}^2)} \right] \delta^3(\mathbf{s} + \mathbf{r}) \frac{\delta_{r_0+s_0,0}}{T} \quad (2.40) \end{aligned}$$

where $\omega \equiv r_0$, $\sum_{p_0} f(p_0) \equiv \sum_{n=-\infty}^{\infty} f((2n+1)\pi T)$, $E_{\mathbf{p}}^2 \equiv \epsilon_{\mathbf{p}}^2 + \mathbf{p} \cdot \Delta_0 \cdot \Delta_0 \cdot \mathbf{p}$, and the remaining trace is over spin indices only. In this work we are merely interested in finding the masses of the angulons and the remaining symmetric massive modes which is why we work in the limit of zero spatial momentum. We compute the traces and find the following simplified form,

$$\begin{aligned} & \frac{\delta^2 \text{Tr} \log(D^{-1})}{\delta \Delta_{ij}(s) \delta \Delta_{kl}(r)} = \delta^3(\mathbf{s} + \mathbf{r}) \frac{\delta_{r_0+s_0,0}}{T} \\ & T \sum_{p_0} \int \frac{d^3 p}{(2\pi)^3} \left[\frac{8\mathbf{p}_i [\Delta_0 \cdot \mathbf{p}]_j \mathbf{p}_k [\Delta_0 \cdot \mathbf{p}]_l - 4\mathbf{p}_i \mathbf{p}_l (p_0(p_0 + \omega) + E_{\mathbf{p}}^2)}{(p_0^2 + E_{\mathbf{p}}^2)((p_0 + \omega)^2 + E_{\mathbf{p}^2})} \right] \quad (2.41) \end{aligned}$$

Combining the second order terms in $\delta\Delta$ expansion coming from $-T \int \frac{d^3 p}{(2\pi)^3} \sum_{p_0} \left(\frac{1}{4g^2} \Delta_{ij}^\dagger(p) \Delta_{ji}(p) \right)$ and $\text{Tr} \log(D^{-1})$ and also using the gap equation (see appendix .1) to replace the

coupling with the magnitude of the gap we find

$$\begin{aligned}
S_2 &= T \sum_{p_0} \int \frac{d^3 p}{(2\pi)^3} \left[\frac{4(\mathbf{p} \cdot \delta\Delta \cdot \Delta_0 \cdot \mathbf{p})^2 + (\omega^2 - 2(p_0^2 + E_{\mathbf{p}}^2)) (\mathbf{p} \cdot \delta\Delta \cdot \delta\Delta \cdot \mathbf{p})}{(p_0^2 + E_{\mathbf{p}}^2) ((p_0 + \omega)^2 + E_{\mathbf{p}}^2)} \right. \\
&\quad \left. + \frac{\frac{4}{3} \text{Tr} [\delta\Delta \cdot \delta\Delta] (\mathbf{p} \cdot \hat{\Delta}_0 \cdot \hat{\Delta}_0 \cdot \mathbf{p})}{p_0^2 + E_{\mathbf{p}}^2} \right], \tag{2.42}
\end{aligned}$$

where $\hat{\Delta}_0 \equiv \Delta_0/\bar{\Delta}$. We now execute the sum over p_0 and end up with an expression involving trigonometric functions of ω . The expression can be simplified by setting $\omega = 2\pi mT$ ($m \in \mathbb{Z}$, corresponding to bosonic modes) within all trigonometric functions. This replacement is necessary if we wish to obtain the correct analytic continuation to Minkowski space (see appendix .2). The result for $\omega \neq 0$ is given by

$$\begin{aligned}
S_2 &= \int \frac{d^3 p}{(2\pi)^3} \tanh\left(\frac{E_{\mathbf{p}}}{2T}\right) \left[\frac{4(\mathbf{p} \cdot \delta\Delta \cdot \Delta_0 \cdot \mathbf{p})^2 - 4E_{\mathbf{p}}^2 (\mathbf{p} \cdot \delta\Delta \cdot \delta\Delta \cdot \mathbf{p})}{E_{\mathbf{p}} (\omega^2 + 4E_{\mathbf{p}}^2)} \right. \\
&\quad \left. + \frac{\frac{2}{3} \text{Tr} [\delta\Delta \cdot \delta\Delta] (\mathbf{p} \cdot \hat{\Delta}_0 \cdot \hat{\Delta}_0 \cdot \mathbf{p})}{E_{\mathbf{p}}} \right]. \tag{2.43}
\end{aligned}$$

2.4.1 Normal modes

Let us write down the five symmetric traceless fluctuations that we elaborated in the beginning of this chapter:

$$\begin{aligned}
\mathcal{M}^{(1)} &\equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & \mathcal{M}^{(2)} &\equiv \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \mathcal{M}^{(3)} &\equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
\mathcal{M}^{(4)} &\equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \mathcal{M}^{(5)} &\equiv \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.44}
\end{aligned}$$

It should be noted that, given our chosen ground state (2.35), the kinetic and potential terms in Eq. 2.42 do not mix the states corresponding to these five modes, thus, they correspond to eigenmodes of the Hamiltonian operator. To see that the states mentioned above are actually the eigen modes of the Hamiltonian, we define an operator which is not to be confused with the Hamiltonian operator and is given by,

$$\begin{aligned} \mathcal{H}_{ij}[\delta\Delta] &\equiv \frac{\delta S_2[\delta\Delta]}{\delta\delta\Delta_{ij}} = \int \frac{d^3p}{(2\pi)^3} \left[\frac{\tanh\left(\frac{E_{\mathbf{p}}}{2T}\right)}{E_{\mathbf{p}}(\omega^2 + 4E_{\mathbf{p}}^2)} \left(8\mathbf{p} \cdot \delta\Delta \cdot \Delta_0 \cdot \mathbf{p} \left(\mathbf{p}_i [\Delta_0]_{jk} \mathbf{p}_k + \mathbf{p}_j [\Delta_0]_{ik} \mathbf{p}_k \right) \right. \right. \\ &\quad \left. \left. - 8E_{\mathbf{p}}^2 (\mathbf{p}_i \delta\Delta_{jk} \mathbf{p}_k + \mathbf{p}_j \delta\Delta_{ik} \mathbf{p}_k) + \frac{4}{3} (\omega^2 + 4E_{\mathbf{p}}^2) (\delta\Delta_{ij} + \delta\Delta_{ji}) (\mathbf{p} \cdot \hat{\Delta}_0 \cdot \hat{\Delta}_0 \cdot \mathbf{p}) \right) \right] \end{aligned} \quad (2.45)$$

Here we have used the symmetric nature of $\delta\Delta$ while taking the derivative. Then, by orthogonality of the set of modes, if $\mathcal{H}[\delta\Delta] \propto \delta\Delta$ for all the modes in Eq. 2.44, there can be no mixing between modes. For example, let us we choose $\delta\Delta = \alpha_1 \bar{\Delta} \mathcal{M}^{(1)}$, where α_i is the field corresponding to fluctuations $\mathcal{M}^{(i)}$ for $i = 1, 2, \dots, 5$ then we have

$$\mathcal{H}[\mathcal{M}^{(1)}] = \int \frac{d^3p}{(2\pi)^3} \frac{2\alpha_1 \bar{\Delta} \tanh\left(\frac{E_{\mathbf{p}}}{2T}\right)}{E_{\mathbf{p}}(\omega^2 + 4E_{\mathbf{p}}^2)} T \quad (2.46)$$

where T is given by the following matrix

$$T = \begin{pmatrix} -2p_x^2 p_y p_z \alpha_1^2 \bar{\Delta}^2 & -2\alpha_1^2 \bar{\Delta}^2 p_x p_y^2 p_z - 4E_{\mathbf{p}}^2 p_x p_z & \alpha_1^2 \bar{\Delta}^2 p_x p_y p_z^2 - 4E_{\mathbf{p}}^2 p_x p_y \\ -2\alpha_1^2 \bar{\Delta}^2 p_x p_y^2 p_z - 4E_{\mathbf{p}}^2 p_x p_z & -2\alpha_1^2 \bar{\Delta}^2 p_y^3 p_z - 8E_{\mathbf{p}}^2 p_y p_z & \alpha_1^2 \bar{\Delta}^2 p_y^2 p_z^2 + \frac{4}{3} E_{\mathbf{p}}^2 (p_x^2 - 2p_y^2 + p_z^2) \\ & & + \frac{\omega^2}{3} (p_x^2 + p_y^2 + 4p_z^2) \\ \alpha_1^2 \bar{\Delta}^2 p_x p_y p_z^2 - 4E_{\mathbf{p}}^2 p_x p_y & \alpha_1^2 \bar{\Delta}^2 p_y^2 p_z^2 + \frac{4}{3} E_{\mathbf{p}}^2 (p_x^2 - 2p_y^2 + p_z^2) & 4\alpha_1^2 \bar{\Delta}^2 p_y p_z^3 - 8E_{\mathbf{p}}^2 p_y p_z \\ & & + \frac{\omega^2}{3} (p_x^2 + p_y^2 + 4p_z^2) \end{pmatrix}$$

Now let us specialize to the ground state given in 2.35. Any term containing odd powers of spatial momentum integrates to zero where the integration is performed

over all spatial momentum. The only non-zero contribution hence is given by,

$$\begin{aligned}
& \mathcal{H}[\mathcal{M}^{(1)}] \\
&= \int \frac{d^3p}{(2\pi)^3} \frac{2\alpha_1 \bar{\Delta} \tanh\left(\frac{E_{\mathbf{p}}}{2T}\right) [p_y^2 p_z^2 \alpha_1^2 \bar{\Delta}^2 + \frac{1}{3} (4E_{\mathbf{p}}^2(p_x^2 - 2p_y^2 + p_z^2) + \omega^2(p_x^2 + p_y^2 + 4p_z^2))]}{E_{\mathbf{p}} (\omega^2 + 4E_{\mathbf{p}}^2)} \mathcal{M}^{(1)} \\
&\propto \mathcal{M}^{(1)}. \tag{2.47}
\end{aligned}$$

It is simple to perform the same operation for the remaining matrices in 2.44, therefore, these matrices do represent a set of normal modes.

2.4.2 Angulons

We know that the angulons correspond to rotations about the x - and y -axes. Hence, they can be identified with the corresponding generators $J_{1,2} = \mathcal{M}^{(1,2)}$. We intend to show that the angulons remain massless for all temperatures below the critical temperature. For this we need to show that the potential term in Eq. 2.42 is zero when $\delta\Delta$ is replaced by corresponding fluctuations. We perform the angular integrations, which can be done analytically before performing the sum over p_0 . Therefore, we begin with 2.42 and set $\omega = 0$ and $\delta\Delta = \alpha_{(1,2)} \bar{\Delta} \mathcal{M}^{(1,2)}$ to find the potential. This gives,

$$\begin{aligned}
S_2 &= T\alpha_{(1,2)}^2 \bar{\Delta}^2 \sum_{p_0} \int \frac{d^3p}{(2\pi)^3} \left[\frac{4(\mathbf{p} \cdot \mathcal{M}^{(1,2)} \cdot \Delta_0 \cdot \mathbf{p})^2 - 2(p_0^2 + E_{\mathbf{p}}^2)(\mathbf{p} \cdot \mathcal{M}^{(1,2)} \cdot \mathcal{M}^{(1,2)} \cdot \mathbf{p})}{(p_0^2 + E_{\mathbf{p}}^2)^2} \right. \\
&\quad \left. + \frac{\frac{8}{3}(\mathbf{p} \cdot \hat{\Delta}_0 \cdot \hat{\Delta}_0 \cdot \mathbf{p})}{p_0^2 + E_{\mathbf{p}}^2} \right] \\
&= T\alpha_{(1,2)}^2 \bar{\Delta}^2 M k_F^3 \sum_{p_0} \int \frac{d\epsilon dxd\phi}{(2\pi)^3} [16(\epsilon^2 + p_0^2)(3x^2 - 1) + 4(k_F \bar{\Delta})^2(3x^4 + 6x^2 - 1) \\
&\quad \pm 12(x^2 - 1)(4\epsilon^2 + 4p_0^2 + (k_F \bar{\Delta})^2(x^2 + 1)) \cos(2\phi)] / \left(3(4\epsilon^2 + 4p_0^2 + (k_F \bar{\Delta})^2(3x^2 + 1))^2 \right) \\
&= \pm T\alpha_{(1,2)}^2 \bar{\Delta}^2 M k_F^3 \sum_{p_0} \int \frac{d\epsilon d\phi}{(2\pi)^3} \frac{16 \tan^{-1} \sqrt{\frac{3(k_F \bar{\Delta})^2}{4\epsilon^2 + (k_F \bar{\Delta})^2 + 4p_0^2}} \cos(2\phi)}{3k_F \bar{\Delta} \sqrt{3(4\epsilon^2 + (k_F \bar{\Delta})^2 + 4p_0^2)}}, \tag{2.48}
\end{aligned}$$

where $x = \cos \theta$. We have used the standard BCS formalism where the integral is dominated by the singularity at $p = k_F$ for small $\bar{\Delta}/v_F$ to approximate $p \approx k_F$, $\epsilon \approx v_F(p - k_F)$, and the \pm correspond to $\delta\Delta = \mathcal{M}^{(1,2)}$, respectively.

Now we do the remaining integration over the angle ϕ . We can see that it is clearly 0 for both modes. This tells us that the potential for angulons is flat. This is achieved due to the geometry of the ground state, depending only on the angular integration. This implies that, the angulons must remain massless not only at zero temperature, but for all temperatures T , $T < T_c$. Our result here disagrees with that in [27] where it is claimed that the angulons pick up a mass as at nonzero temperatures below T_c . We do not understand the origin of this discrepancy.

2.4.3 Massive modes

Now we intend to calculate the masses of the remaining modes. In order to do so, we take $\omega \neq 0$ and then we look for ω for which the inverse propagator has a zero or the propagator has a pole. The inverse propagator for the i th mode, corresponding to ω_i , is given by substituting $\delta\Delta = \mathcal{M}^{(i)}$ into 2.43.

We find that there are two modes that are degenerate. These are given by $\mathcal{M}^{(3,4)}$. This is due to the remaining symmetry in the (x, y) -plane after the spontaneous breaking of rotations about the x, y axes by our choice for the ground state (2.35). This is also a consequence of \mathcal{M}^3 and \mathcal{M}^4 being related by a rotation about the z axis as pointed out in the beginning of this chapter. The inverse propagator for

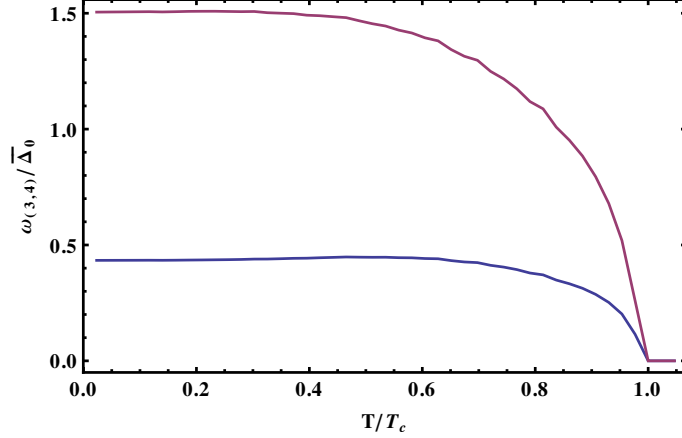


Figure 2.7: Here we plot the masses of the modes associated with $\mathcal{M}_{(3,4)}$ in units of the magnitude of the gap at zero temperature as a function of the temperature in units of the critical temperature. The two curves show the two poles found for a given state. Also there are two degenerate corresponding to each curve.

these modes is given by,

$$\begin{aligned}
& \Pi_{(3,4)}^{-1}(\omega_{(3,4)}) \\
&= \bar{\Delta}^2 \frac{Mk_F^3}{4\pi^2} \int d\epsilon dx \tanh\left(\frac{E}{2T}\right) \left[\frac{\left(\frac{(k_F\bar{\Delta})^2}{2}(1-x^2)^2 + \omega_{(3,4)}^2(1-x^2)\right)}{E(4E^2 + \omega_{(3,4)}^2)} \right. \\
&\quad \left. + \frac{2(x^2 - \frac{1}{3})}{E} \right]. \tag{2.49}
\end{aligned}$$

Here $E = \sqrt{\epsilon^2 + (k_F\bar{\Delta})^2(1+3x^2)/4}$. In order to find the poles of the propagator we must go to Minkowski space ($\omega \rightarrow i\omega$) and integrate numerically. We solve for $\omega_{3,4}$ from the equation $\Pi_{(3,4)}^{-1}(i\omega_{(3,4)}) = 0$. Numerical solutions for $\omega_{(3,4)}$ are plotted in Fig. 2.7.

We see that the masses of both modes are smaller than the energy required for pair breakup. The masses turn out to be as expected of the order of the gap for low temperatures $T \ll T_c$. Therefore, the effective theory of angulons and properties

calculated using this effective theory being valid for energies much smaller than the gap will not be affected by the presence of these modes. This is a result that validates the approach of the previous section. This result seems agree roughly with ref. [27].

The inverse propagator corresponding to the mode \mathcal{M}_5 is,

$$\begin{aligned} & \Pi_5^{-1}(\omega_5) \\ &= \bar{\Delta}^2 \frac{Mk_F^3}{4\pi^2} \int d\epsilon dx \tanh\left(\frac{E}{2T}\right) \left[\frac{((k_F \bar{\Delta})^2(1+3x^2) + \omega_5^2) \left(\frac{1+3x^2}{4}\right)}{E(4E^2 + \omega_5^2)} \right]. \end{aligned} \quad (2.50)$$

We find that there are no real solutions to the equation $\Pi_5^{-1}(i\omega_5) = 0$, therefore, there is no real pole associated with \mathcal{M}_5 . This result also agrees with ref. [27].

2.5 Summary

We have found the spectrum of bosonic modes for 3P_2 condensed neutron matter using a simple model calculation. We find the existence of two massless Goldstone modes associated with spontaneously broken rotational symmetry in two planes for all temperatures below the critical temperature. This is in contrast to the result found in [27], where these modes acquire masses for all nonzero temperatures. In addition, we find two massive modes whose masses are of the order of the (zero-temperature) gap as long as $T \ll T_c$. The fact that the massive modes have a minimum energy of the order of the zero temperature gap justifies the use of the effective theory developed in ref. [14]. Due to their large masses, their contributions to processes at temperatures $T \ll T_c$ are exponentially suppressed. For this reason we did not compute their properties at non-zero spatial momentum. The contributions

from the massless modes to neutron star physics should be much more relevant and were discussed in ref. [17]. We have not considered modes corresponding to complex (as was done in [27]), non-symmetric, or non-zero trace deformations of the condensate, the latter two corresponding to an admixture of 3P_0 and 3P_1 pairing to the 3P_2 background. There is no reason to expect them to be particularly light, but a calculation of their masses would require the relative strengths of the pairing force in these different channels as an input.

.1 Gap equation and critical temperature

The gap equation, $\frac{\delta S}{\delta \Delta}|_{\Delta=\Delta_0} = 0$, gives us,

$$\frac{\Delta_{ij}}{2g^2} = 2T \sum_{p_0} \int \frac{d^3p}{(2\pi)^3} \frac{(\mathbf{p} \cdot \Delta)_i \mathbf{p}_j + (\mathbf{p} \cdot \Delta)_j \mathbf{p}_i - \frac{2}{3} \mathbf{p} \cdot \Delta \cdot \mathbf{p} \delta_{ij}}{p_0^2 + \epsilon^2 + \mathbf{p} \cdot \Delta^2 \cdot \mathbf{p}}. \quad (51)$$

This leads to the following equation for the magnitude of the gap, using our chosen ground state (groundState),

$$\frac{\bar{\Delta}}{4g^2} = \frac{2Tk_F^2 M}{3\pi} \sum_{p_0} \int_{-1}^1 dx \frac{(1+3x^2)/4}{\left((p_0/(k_F \bar{\Delta}))^2 + (1+3x^2)/4 \right)^{1/2}}, \quad (52)$$

where $x = \cos \theta$ and we have used the fact that the integral is dominated by the singularity at $p = k_F$ for small $\bar{\Delta}/v_F$ to approximate $p \approx k_F, \epsilon \approx v_F(p - k_F)$. In 2.8 we plot the value of the gap as a function of temperature. We find the critical temperature $T_c \approx 0.43 \bar{\Delta}_0$, where $\bar{\Delta}_0$ is the magnitude of the gap at zero temperature.

.2 Real time formalism

The imaginary time formalism used in this work is only one out of many methods used in studying field theories at finite temperature. It allows for the computation of field correlators using imaginary time and (anti-)periodic boundary conditions in the Euclidean “time” direction. When information about real time correlators is required, an analytic continuation must be made from imaginary and discretized energies $k_0 = 2\pi in, n \in \mathbb{Z}$ to real, continuous ones. In general this continuation is not unique, but for two-point functions the correct behavior of the correlator at asymptotically large $|k_0|$ does specify a unique continuation [21]. In practice, this continuation may be difficult to find. In order to verify that we have the correct analytic continuation, we have repeated the calculation described in the main text using the real time formalism (RTF) for finite temperature field theories [22–25] (for a review see, for example, ref. [26]). In the RTF the number of fields is doubled and each copy is denoted by an index “+” or “-”. Propagators acquire a 2×2 matrix structure corresponding to the doubling of the number of fields. The construction of the perturbation series follows the usual diagrammatic rules familiar to the zero temperature case with the addition of vertices involving the “-” fields (which come with an opposite sign). Fortunately, in our calculation only the “+” fields appear. The “++” component of propagator of the fermions is given by

$$D^{++}(p) = \left[\frac{1}{p_0^2 - E_p^2 + i\epsilon} + i2\pi n(p)\delta(p_0^2 - E_p^2) \right] \begin{pmatrix} p_0 + \epsilon_p & (\mathbf{p} \cdot \Delta_0)_j \sigma_i \sigma_2 \\ (\Delta_0 \cdot \mathbf{p})_j \sigma_2 \sigma_i & p_0 - \epsilon_p \end{pmatrix}, \quad (53)$$

with

$$n(p) = \frac{1}{e^{\beta|p_0|} + 1}. \quad (54)$$

The 2×2 structure of the propagator above refers to ‘‘Gorkov space’’, not the doubling of fields due to the RTF. Repeating the steps in the main text, now in the RTF, we have

$$\begin{aligned} & \frac{\delta^2 \text{Tr} \log(D^{-1})}{\delta \Delta_{ij}(s) \delta \Delta_{kl}(r)} \Big|_{\Delta=\Delta_0} \\ &= \int \frac{d^4 p}{(2\pi)^4} [D_{11}^{++}(p+k) D_{22}^{++}(k) + D_{22}^{++}(p+k) D_{11}^{++}(k) + D_{12}^{++}(p+k) D_{21}^{++}(k) \\ &+ D_{21}^{++}(p+k) D_{12}^{++}(k)] \\ &= \int \frac{d^4 p}{(2\pi)^4} \times (8\mathbf{p}_i [\Delta_0 \cdot \mathbf{p}]_j \mathbf{p}_k [\Delta_0 \cdot \mathbf{p}]_l - 4\mathbf{p}_i \mathbf{p}_l (p_0(p_0 + \omega) + E_{\mathbf{p}}^2)) \delta(s+r) \\ & \quad \left[\frac{1}{(p_0^2 - E_{\mathbf{p}}^2 + i\epsilon)} + 2\pi i n(p) \delta(p_0^2 - E_{\mathbf{p}}^2) \right] \\ & \quad \left[\frac{1}{(p_0 + \omega)^2 - E_{\mathbf{p}}^2 + i\epsilon} + 2\pi i n(p+k) \delta((p_0 + k_0)^2 - E_{\mathbf{p}}^2) \right] \end{aligned} \quad (55)$$

Separating the real from the imaginary part using

$$\frac{1}{x + i\epsilon} = \frac{x}{x^2 + \epsilon^2} - i \frac{\epsilon}{x^2 + \epsilon^2} = \mathcal{P} \left(\frac{1}{x} \right) - i\pi \delta(x), \quad (56)$$

performing the p_0 integral and adding the contribution from the $\Delta_{ij}^\dagger \Delta_{ji} / (4g^2)$ term we find for the real part of the action (for $\omega \neq 0$)

$$\begin{aligned} S_2^R &= \int \frac{d^3 p}{(2\pi)^3} \tanh \left(\frac{E_{\mathbf{p}}}{2T} \right) \left[\frac{4(\mathbf{p} \cdot \delta \Delta \cdot \Delta_0 \cdot \mathbf{p})^2 - 4E_{\mathbf{p}}^2 (\mathbf{p} \cdot \delta \Delta \cdot \delta \Delta \cdot \mathbf{p})}{E_{\mathbf{p}} (\omega^2 - 4E_{\mathbf{p}}^2)} \right. \\ & \quad \left. + \frac{\frac{2}{3} \text{Tr} [\delta \Delta \cdot \delta \Delta] (\mathbf{p} \cdot \hat{\Delta}_0 \cdot \hat{\Delta}_0 \cdot \mathbf{p})}{E_{\mathbf{p}}} \right]. \end{aligned} \quad (57)$$

In agreement with eq. 2.43. The imaginary part, which we do not compute here, describes the thermal width of the quasi-particles.

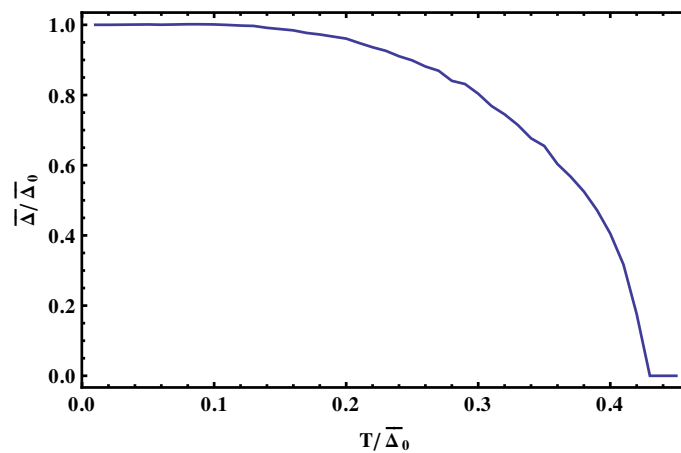


Figure 2.8: We plot the magnitude of the gap as a function of temperature, in units of the magnitude of the gap at zero temperature.

Chapter 3: Neutrino Emissivity

3.1 Introduction

The purpose of this chapter is to compute observables using the effective theory derived in the previous chapter. As discussed earlier, the knowledge of neutrino emissivity is necessary in order to determine the cooling history of a star. As different phases of dense matter can have different neutrino emission processes, all of which have different neutrino emissivities, the processes can affect the cooling in different ways to give us clues about which phase of matter is actually realized in the core of the star.

Here we intend to calculate the neutrino emissivity from angulon decays in situations where such a decay is possible. To put things in context, the pair-breaking-formation process, already described in the introduction, is only effective at temperatures close to the critical temperature where unpaired neutrons exist in substantial numbers. At lower temperatures, processes that involve angulons are expected to dominate. In [29] the emissivity due to bremsstrahlung of neutrino pairs following angulon-angulon collisions was estimated. The emissivity was found to scale with temperature as $\sim T^9$ and was small for most of the relevant temperatures and densities. On the other hand the decay of angulons into neutrino pairs is kinematically forbidden as

the angulons are massless. However, the presence of a magnetic field can alter this scenario considerably. As we will see below the dispersion relation of the angulons changes in the presence of strong magnetic fields. One of the angulons develops a mass of the order of eB/M (B is the magnetic field, e the electron charge and M the neutron mass, corrected by Fermi liquid effects). The gapped angulon can then kinematically decay into a neutrino pair. The other angulon remains massless, however its energy is proportional to the square of the spatial momentum for low momentum.

Before we proceed with the calculation of emissivity, let us revisit some of the properties of the condensate in the presence of a magnetic field. It was mentioned earlier that near the critical temperature, where Guinzburg-Landau arguments are valid, the ground state is [30]

$$\bar{\Delta} = \Delta_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & -1 - r \end{pmatrix}, \quad (3.1)$$

with $r = -1/2$ (assuming that certain parameters exhibit values close to the ones obtained from BCS) and it has been argued that this form of the order parameter is stable as the temperature is lowered [31]. We will assume in our calculation that the condensate has the form in Eq. 3.1 with $r = -\frac{1}{2}$ and comment on how our results get affected if we chose a different form of the order parameter. The presence of a magnetic field affects the condensate in two ways. First, it becomes energetically favorable for the direction corresponding to the diagonal element 1 in eq. 3.1, which otherwise would be arbitrary, to align with the magnetic field [32].

Second, the magnitude of r gets altered to $r = -1/2 + CB^2 \approx -1/2 + (0.017B_{15})^2$, where C is a combination of parameters of the Guinzburg-Landau free energy and $B_{15} = B/(10^{15}G)$ [32]. We will neglect the corrections to r due to the magnetic field, which is a reasonable approximation for $B < 10^{17}G$. But other than these two effects on the condensate, the magnetic field also affects the dispersion relations of the angulons. In order to see how this takes place let us write down the leading (two derivatives) term quadratic in α_i in the effective Lagrangian for angulons in the presence of a magnetic field in momentum space

$$S = \int \frac{d^4p}{(2\pi)^4} \begin{pmatrix} \alpha_1(p) & \alpha_2(p) \end{pmatrix} \begin{pmatrix} ap_0^2 + v_F^2(dp_x^2 + cp_y^2 + bp_z^2) & ev_F^2p_xp_y - i\frac{eg_NBp_0}{2M} \\ ev_F^2p_xp_y - i\frac{eg_NBp_0}{2M} & ap_0^2 + v_F^2(dp_x^2 + cp_y^2 + bp_z^2) \end{pmatrix} \begin{pmatrix} \alpha_1(-p) \\ \alpha_2(-p) \end{pmatrix} \quad (3.2)$$

where a, b, c, d, e are given by [14]

$$\begin{aligned} a &= 3 + \frac{\pi}{\sqrt{3}} \approx 4.81, \\ b &= -\frac{3}{2} + \frac{\pi}{9\sqrt{3}} \approx -1.30, c = -\frac{4\pi}{3\sqrt{3}} \approx -2.42, \\ d &= -\frac{3}{2} + \frac{2\pi}{9\sqrt{3}} \approx -1.10, e = \frac{3}{2} - \frac{14\pi}{9\sqrt{3}} \approx -1.32, \end{aligned} \quad (3.3)$$

v_F is the Fermi velocity of the neutrons. B is the magnetic field pointing along the z direction, e stands for the charge of an electron and g_N and M stand for the magnetic moment and mass of the neutrons which include the Fermi liquid corrections. The symbol e has been used to denote both a low energy constant and the electric charge.

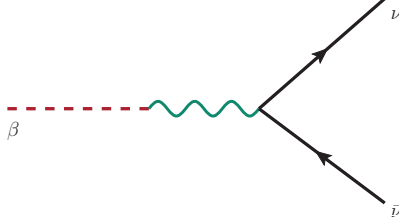


Figure 3.1: Feynman diagram demonstrating the decay of massive angulon (dashed line) into a neutrino pair (solid line). The wavy line represents a Z_0

However, it will always be clear from the context which of the two quantities the symbol represents. In order to extract the coupling of the angulons to the magnetic field we can look at the results in [14] and we find that magnetic fields couples to neutron through their magnetic moment:

$$\mathcal{L}_{B-\alpha} = \frac{eg_N}{2M} n^\dagger \mathbf{S} \cdot \mathbf{B} n, \quad (3.4)$$

where $g_N = -1.913$ is the neutron anomalous magnetic moment (in unit of the nuclear magneton) and \mathbf{S} stands for the spin operator. The interaction has the same form as the angulon interaction to the spatial part of the Z_0 boson worked out in [14], from which the terms proportional to B in eq. 3.2 can be read off.

We have to diagonalize the angulon Lagrangian. To accomplish this we intro-

duce new fields β_1 and β_2

$$\sqrt{a} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \quad (3.5)$$

in terms of which the quadratic part of the action read

$$S = \int \frac{d^4p}{(2\pi)^4} \begin{pmatrix} \beta_1(p) & \beta_2(p) \end{pmatrix} \begin{pmatrix} (p_0 - \xi_1)(p_0 - \xi_2) & 0 \\ 0 & (p_0 + \xi_1)(p_0 + \xi_2) \end{pmatrix} \begin{pmatrix} \beta_1(-p) \\ \beta_2(-p) \end{pmatrix} \quad (3.6)$$

where, ξ_1 and ξ_2 determine the dispersion relation of the modes:

$$\xi_1 = \sqrt{A_1 + \sqrt{A_1 - A_2}}, \quad (3.7)$$

$$\xi_2 = \sqrt{A_1 - \sqrt{A_1 - A_2}} \quad (3.8)$$

where

$$A_1 = \frac{1}{2} \left(\frac{eg_N B}{2Ma} \right)^2 - \frac{c+d}{2a} v_F^2 (p_x^2 + p_y^2) - \frac{b}{a} v_F^2 p_z^2 \quad (3.9)$$

and

$$A_2 = \frac{cd}{a^2} v_F^4 (p_x^4 + p_y^4) + \frac{bc+bd}{a^2} v_F^4 (p_x^2 + p_y^2) p_z^2 + \frac{b^2}{a^2} v_F^4 p_z^4 + \frac{c^2}{a^2} v_F^4 p_x^2 p_y^2 + \frac{d^2}{a^2} v_F^2 p_x^2 p_y^2 - \frac{e^2}{a^2} v_F^2 p_x^2 p_y^2. \quad (3.10)$$

We notice that the dispersion relation of one of the Goldstone modes has turned gapped in the presence of a magnetic field while the remaining massless mode has acquired a quadratic dispersion relation at small momenta.

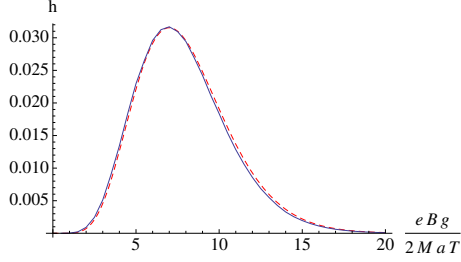


Figure 3.2: $h(x)$ as a function of $x = \frac{egNB}{2MaT}$ (solid line, blue online) and its analytic approximation $h(x) \approx 0.000042 x^7 e^{-x}$ (dashed line, red online).

Having derived the modified dispersion relations, we need the coupling between the β fields and the Z_0 gauge boson. The coupling of the angulons to the electroweak Z_0 gauge boson [14] is given by

$$\mathcal{L} = C_A 9f (Z_2^0 \partial_0 \alpha_2 - Z_1^0 \partial_0 \alpha_1) \quad (3.11)$$

where $f^2 = \frac{Mk_F}{6\pi^2}$, k_F is the neutron Fermi momentum, $C_A^2 = \tilde{C}_A^2 \frac{G_F M_Z^2}{2\sqrt{2}}$ with $\tilde{C}_A \sim 1.1 \pm 0.15$, G_F the Fermi constant and M_Z the Z_0 boson mass.

Finally, the coupling between the gauge boson and neutrinos is well know [33]:

$$\mathcal{L}_{Z-\nu} = \frac{gZ_\mu}{\cos \theta_W} \left(\frac{1}{4} \bar{\nu} \gamma^\mu (1 - \gamma^5) \nu \right) \quad (3.12)$$

where θ_W is the Weinberg mixing angle.

3.2 Emissivity

The tree level contribution to the massive angulon decay is shown in the diagram in fig. 3.1 . The amplitude for this process in a box of volume V with

appropriate box normalization¹ can be written as

$$A = \left(\bar{u}_s(p) \frac{\gamma^1(1 - \gamma^5)}{2} v_t(p') c_{11} - \bar{u}_s(p) \frac{\gamma^2(1 - \gamma^5)}{2} v_t(p') c_{21} \right) \frac{C_A 9fg}{2\sqrt{a} \cos \theta_W} \frac{k_0}{M_Z^2} \frac{(2\pi)^4 \delta^4(p + p' - k)}{V^{3/2} \sqrt{2\xi_k 2w_p 2w_{p'}}} \quad (3.13)$$

where the outgoing neutrinos have momenta p and p' and the angulon β_1 has momentum k . The definitions of u^s and v^s are as in reference [33]. The on-shell conditions for the external legs are

$$\begin{aligned} k_0 &= w_k = \xi_1(\vec{k}) \\ p_0 &= w_p = |\vec{p}| \\ p'_0 &= w_{p'} = |\vec{p}'| \end{aligned} \quad (3.14)$$

as the neutrinos here are assumed to be massless.

The decay rate is defined by

$$\Gamma = N \sum_{\substack{\text{neutrino momenta} \\ \text{and helicities}}} \frac{|A|^2}{\tau} \quad (3.15)$$

where τ is the time over which the interaction is on (to be taken to infinity at the end of the computation) and $N = 3$ stands for the number of neutrino flavors. We

¹In a three dimensional box with a volume V , $\vec{p} = \frac{2\pi}{L} \vec{n}$, where L is the length of each side of the box and n_x , n_y and n_z are integers. In continuum, the free particle states with momentum \vec{p} and energy E_p are normalized such that, $\langle p|p' \rangle = 2E_p (2\pi)^3 \delta^3(p - p')$ which in a box turns into $\langle p|p' \rangle = 2E_p V \delta_{n,n'}$

have to now evaluate the spin sum in Eq. 3.15. The spin sum is given by

$$\begin{aligned}
\sum_{s,t} |A|^2 = & \left[\sum_{s,t} \bar{u}_s(p) \frac{\gamma_1(1-\gamma_5)}{2} v_t(p') \bar{v}_t(p') \frac{\gamma_1(1-\gamma_5)}{2} u_s(p) |c_{11}|^2 \right. \\
& + \sum_{s,t} \bar{u}_s(p) \frac{\gamma_2(1-\gamma_5)}{2} v_t(p') \bar{v}_t(p') \frac{\gamma_2(1-\gamma_5)}{2} u_s(p) |c_{21}|^2 \\
& - \sum_{s,t} \bar{u}_s(p) \frac{\gamma_1(1-\gamma_5)}{2} v_t(p') \bar{v}_t(p') \frac{\gamma_2(1-\gamma_5)}{2} u_s(p) c_{11} c_{21}^* \\
& \left. - \sum_{s,t} \bar{u}_s(p) \frac{\gamma_2(1-\gamma_5)}{2} v_t(p') \bar{v}_t(p') \frac{\gamma_1(1-\gamma_5)}{2} u_s(p) c_{11}^* c_{21} \right] \\
& \left(\left(\frac{C_A 9fg}{2\sqrt{a} \cos(\theta_w)} \right)^2 \left(\frac{k_0}{M_z^2} \right)^2 \frac{(2\pi)^8 \delta^4(k-p-p')}{V^3 (2w_p)(2w'_p)(2w_k)} \right). \tag{3.16}
\end{aligned}$$

Now with some algebra we can find

$$\sum_{s,t} \bar{u}_s(p) \frac{\gamma_1(1-\gamma_5)}{2} v_t(p') \bar{v}_t(p') \frac{\gamma_1(1-\gamma_5)}{2} u_s(p) = 2(p_0 p'_0 + p_1 p'_1 - p_2 p'_2 - p_3 p'_3) \tag{3.17}$$

$$\sum_{s,t} \bar{u}_s(p) \frac{\gamma_2(1-\gamma_5)}{2} v_t(p') \bar{v}_t(p') \frac{\gamma_2(1-\gamma_5)}{2} u_s(p) = 2(p_0 p'_0 + p_2 p'_2 - p_1 p'_1 - p_3 p'_3) \tag{3.18}$$

$$\sum_{s,t} \bar{u}_s(p) \frac{\gamma_2(1-\gamma_5)}{2} v_t(p') \bar{v}_t(p') \frac{\gamma_1(1-\gamma_5)}{2} u_s(p) = 2(p_2 p'_1 + p_1 p'_2 - ip_0 p'_3 + ip_3 p'_0) \tag{3.19}$$

and

$$\sum_{s,t} \bar{u}_s(p) \frac{\gamma_1(1-\gamma_5)}{2} v_t(p') \bar{v}_t(p') \frac{\gamma_2(1-\gamma_5)}{2} u_s(p) = 2(p_1 p'_2 + p_2 p'_1 - ip_3 p'_0 + ip_0 p'_3) \tag{3.20}$$

Plugging (3.17), (3.18), (3.19) and (3.20) in (3.16) and replacing one of the delta functions $(2\pi)^4 \delta^4(k-p-p')$ by $V\tau$, we find the expression for Γ to be

$$\begin{aligned}
\Gamma = & N \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(p+p'-k)}{2w_k 2w_p 2w_{p'}} \\
& 2(P_1 |c_{11}|^2 + P_2 |c_{21}|^2 - P_{12} c_{11} c_{21} - P_{12}^* c_{11}^* c_{21}) \left(\frac{C_A 9fg}{2\sqrt{a} \cos \theta_W} \right)^2 \left(\frac{w_k}{M_z^2} \right)^2. \tag{3.21}
\end{aligned}$$

After performing the d^3p' integral in (3.21) we arrive at

$$\begin{aligned} \Gamma = N \int \frac{d^3p}{(2\pi)^3} \frac{2\pi\delta(p_0 + p'_0 - k_0)}{2w_k 2w_p 2w_{p'}} \\ 2(P_1|c_{11}|^2 + P_2|c_{21}|^2 - P_{12}c_{11}c_{21} - P_{12}^*c_{11}^*c_{21}) \left(\frac{C_A 9fg}{2\sqrt{a} \cos \theta_W} \right)^2 \left(\frac{k_0}{M_Z^2} \right)^2 \end{aligned} \quad (3.22)$$

where,

$$\begin{aligned} P_1 &= p_0p'_0 + p_1p'_1 - p_2p'_2 - p_3p'_3 \\ P_2 &= p_0p'_0 + p_2p'_2 - p_1p'_1 - p_3p'_3 \\ P_{12} &= p_1p'_2 + p_2p'_1 + i(p_0p'_3 - p_3p'_0) \end{aligned} \quad (3.23)$$

with $\vec{p}' = \vec{k} - \vec{p}$. The emissivity (the amount of energy emitted per unit of volume and time) can then be expressed as,

$$Q = \int \frac{d^3k}{(2\pi)^3} \Gamma \frac{k_0}{e^{\beta k_0} - 1} \quad (3.24)$$

where β is the inverse temperature. In order to simplify our calculations, we write Q as a dimensionless integral. To do this we normalize all dimensionfull quantities with respect to T^λ where λ is the mass dimension of the quantity the symbol stands for. Now we express the emissivity in terms of these new dimensionless symbols, which we distinguish from the unnormalized ones by a tilde

$$\begin{aligned} Q = N \int T^5 \frac{d^3\tilde{k} d^3\tilde{p}}{(2\pi)^6} (2\pi)\delta(\tilde{p}_0 + \tilde{p}'_0 - \tilde{k}_0) 2(\tilde{P}_1|c_{11}|^2 + \tilde{P}_2|c_{11}|^2 - \tilde{P}_{12}c_{11}c_{21} - \tilde{P}_{12}^*c_{11}^*c_{21}) \\ \left(\frac{\tilde{k}_0}{\tilde{M}_z^2} \right)^2 \frac{1}{2\tilde{w}_k 2\tilde{w}_p 2\tilde{w}'_p} \left(\frac{C_A 9\tilde{f}\tilde{g}}{2\sqrt{a} \cos(\theta_W)} \right)^2 \frac{\tilde{k}_0}{e^{\tilde{k}_0} - 1}. \end{aligned} \quad (3.25)$$

The factor of $\left(\frac{C_A 9\tilde{f}\tilde{g}}{2\sqrt{a} \cos(\theta_W)} \right)^2 \frac{1}{\tilde{M}_z^4}$ can be written as $\tilde{C}_A^2 \frac{81}{12a\pi^2} \tilde{G}_F^2 \tilde{M}\tilde{k}_F$. Now we plug

this back in (3.25) to obtain,

$$\begin{aligned}
Q &= N\tilde{C}_A^2 \frac{81}{12a\pi^2} \tilde{G}_F^2 \tilde{M} \tilde{k}_F \int T^5 \frac{d^3\tilde{k}d^3\tilde{p}}{(2\pi)^6} (2\pi)\delta(\tilde{p}_0 + \tilde{p}'_0 - \tilde{k}_0) \\
&\quad 2(\tilde{P}_1|c_{11}|^2 + \tilde{P}_2|c_{11}|^2 - \tilde{P}_{12}c_{11}c_{21} - \tilde{P}_{12}^*c_{11}^*c_{21}) \frac{1}{2\tilde{w}_k 2\tilde{w}_p 2\tilde{w}'_p} \frac{\tilde{k}_0^3}{e^{\tilde{k}_0-1}}. \\
&= G_F^2 M k_F T^7 h \left(\frac{eg_N B}{2aMT} \right)
\end{aligned} \tag{3.26}$$

where $h(x)$ is given by the dimensionless integral

$$\begin{aligned}
h &= N\tilde{C}_A^2 \frac{81}{12a\pi^2} \int \frac{d^3\tilde{k}d^3\tilde{p}}{(2\pi)^6} (2\pi)\delta(\tilde{p}_0 + \tilde{p}'_0 - \tilde{k}_0) \\
&\quad 2(\tilde{P}_1|c_{11}|^2 + \tilde{P}_2|c_{11}|^2 - \tilde{P}_{12}c_{11}c_{21} - \tilde{P}_{12}^*c_{11}^*c_{21}) \frac{1}{2\tilde{w}_k 2\tilde{w}_p 2\tilde{w}'_p} \frac{\tilde{k}_0^3}{e^{\tilde{k}_0-1}},
\end{aligned} \tag{3.27}$$

which is a function of the dimensionless quantity $x = \frac{Beg_N}{2MaT}$ alone. We numerically calculate the integral in equation 3.27 and plot it as a function of $\frac{Beg_N}{2MaT}$ in Fig. 3.2.

We find that the function $h(x)$ is very well approximated by

$$h(x) \approx 0.000042 x^7 e^{-x} \tag{3.28}$$

Fig. 3.3 is a plot of neutrino emissivity obtained from this calculation for three different magnetic fields as a function of temperature .

3.3 Discussion

The main result here is that neutrino emissivity due to the decay of angulons in the presence of a magnetic field is given by eq. 3.26 and as depicted in Fig. 3.3. We have come up with a neat and simple analytic form in eqs. 3.26 and 3.28. Although there are a few approximations involved in this result, all of them are controlled and precise for the application to the cooling of neutron stars. The first

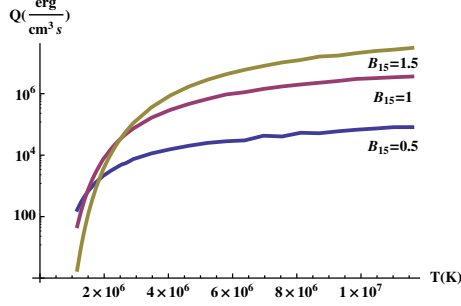


Figure 3.3: Neutrino emissivity as a function of temperature for different magnetic fields.

approximation lies in the derivation of the low energy constants of the effective theory whose regime of validity is discussed in great detail in [14]. Higher order terms in the low momentum expansion, either from the effective theory or from loops are suppressed by factors of $(T/\bar{\Delta}k_F)^2$ where T is the temperature and $\bar{\Delta}k_F$ is the value of the gap. Hence they are small at temperatures much smaller than the critical temperature. Also, we use $r = -1/2$ for our calculations. A different value of r would imply a different pattern of symmetry breaking. In general, the rotation group would be broken down to the discrete subgroup $(\mathbf{Z}_2)^3$ of inversion along the principal axes of Δ_{ij} . This would give rise to three Goldstone bosons. However, only one of them would become massive due to its interaction with the magnetic field. This would change our calculation by a small shift of the angulon's magnetic mass. For magnetic fields that are greater than $B \approx 10^{17}G$ the phase with $r = -1$ is expected to be favored [32]. This phase however, is qualitatively different from the other unitary 3P_2 phases because the neutrons are gapless along a certain direction in space. Those ungapped neutron may undergo beta decay which

would be a source of neutrinos. It would be suppressed only by the corresponding restricted phase space of that of ungapped neutrons. Our calculation for angulon emissivity would still hold. But it has to be supplemented by the ungapped neutron beta decay contribution.

Although we should like to compare the neutrino emission of the angulons with other dominating neutrino emission processes like the pair breaking formation (PBF) process in neutron stars, we should not compare the emissivity rates from pair-breaking-formation(PBF) and angulon decay processes. The reason behind this is that the critical temperature of every layer or shell of the star depends on its density. Hence, at any particular instant in time, the PBF process is effective only in a shell of the star where the temperature is close to the critical temperature. On the other hand, due to the presence of superconducting protons, magnetic fields are expected to be confined in flux tubes. This means that angulons can only decay inside the flux tubes in the core of the star. The calculation of emissivities due to both processes, PBF and the decay of angulons suffer from uncertainties as far as the value of the gap and the magnetic field in the core of the neutron star is concerned. Still, it is illuminating to look at their relative numerical values. The emissivity of neutrinos in PBF processes is given by [34], [35]

$$Q_{PBF} = \frac{4G_F^2 M k_F}{15\pi^5} T^7 N F \left(\frac{\Delta}{T} \right) \quad (3.29)$$

where F is a function of ratio of the gap Δ to temperature T . F peaks at $T \sim \Delta$ and decays exponentially at lower temperatures. The decay of massive angulons

gives rise to an emissivity equal to

$$Q_{ang} = G_F^2 M k_F T^7 h\left(\frac{B e g_N}{2 a M T}\right), \quad (3.30)$$

where the function $h(x)$ peaks at $x \sim 7$ and decays exponentially at larger values of x and as $\sim x^7$ at small x . For temperatures close to the gap $T \sim \bar{\Delta} k_F$ the emissivity due to the PBF process is much larger than the angulon neutrino emissivity (assuming $\bar{\Delta} k_F \gg e B g_N / M$). At smaller temperatures, around $T \sim e B_{15} / M \approx 3 \times 10^7 K B_{15}$, the angulon emissivity is larger than that of PBF. For temperatures even smaller than that, the emissivity due to the angulon process still dominates but the phenomenological interest of these rates becomes negligible as it is difficult to observe stars that are this cold.

Since the decay process involving angulons can occur only inside flux tubes with high magnetic fields, it is desirable to have an estimate of the volume fraction of the star that is in the vicinity of these magnetic flux tubes. Each flux tube is occupied by a flux quantum equal to $\Phi_0 = \pi/e$. We assume a dipole form for the magnetic field inside of the star. Then the total flux of magnetic field passing through the star is $\Phi = \pi R_{star}^2 B_{star}$ (where R_{star} and B_{star} are the radius and average magnetic field in the interior of the star). The number of flux tubes then can be estimated to be $N \approx \Phi/\Phi_0 \approx R_{star}^2 e B_{star}$ and the average distance by which they are separated is $L \approx \sqrt{\pi R_{star}^2 / N} \approx \sqrt{\pi / (e B_{star})}$. The magnetic field has a certain spreads around a flux tube, almost to a distance of the order of the penetration length $\lambda = \sqrt{m / (4\pi\alpha n_p)}$, where n_p is the proton density. Thus, the fraction of the volume of the star which has sizable magnetic fields is of the order

of $(\lambda/L)^2 \approx 0.04B_{15}(0.1n_0)/n_p$, where $B_{15} = B/10^{15}G$, n_p is the proton density and $n_0 = 0.16fm^{-3}$ the nuclear saturation density. We can see that only in stars with very large magnetic fields the angulon decay mechanism may be relevant for neutrino emission. Magnetars form a class of neutron stars where fields of such high magnitude are known to exist. Ordinary neutron stars on the other hand, in general exhibit much smaller long range magnetic fields, but may have magnetic fields of this order in their interior. In order to make a better assessment of the decay of angulons on the cooling curves can only be accomplished with a realistic cooling code.

Chapter 4: Summary and Outlook

The final chapter of this thesis summarises the calculations in the previous chapters while pointing out some of the interesting features of these calculations overlooked there. It also proposes possible directions for future work on angulons and ends with a short section on deconfined quark matter. The goal of this thesis was to explore the effects of neutron pairing in a particular channel on some of the transport properties of the core of a neutron star. The pairing of neutrons in neutron stars in general is expected to have considerable influence on observables. One such observable, the cooling rate of the star is related to the thermal evolution of the star. The other set of observables are related to the dynamic evolution of the star which include pulsar glitches and spin down characteristics etc. One of the major effects of neutron pairing on the thermal observables appears through the reduction in the neutrino emissivity and the specific heat of the individual constituents of the pair. However as discussed in previous chapters, the onset of pairing can also lead to a different type of neutrino emission process known as the pair-breaking-formation process. The PBF process takes place when two neutrons form a Cooper pair releasing their binding energy through weak interaction in the form of neutrino-antineutrino pair. The process is effective very close to the critical tem-

perature where plenty of unpaired neutrons are available for this process to take place. Also, it is crucial that the pairing transition is of second order for the PBF process to be effective. This is because the energy that unbound neutrons need to give up in order to form a pair at the critical temperature in the case of a second order phase transition is infinitesimal. In contrast, first order transitions have latent heat associated with them and this would require a finite energy (equal to the latent heat) to be extracted from two unbound neutrons to form a Cooper pair. This would make the PBF process inefficient.

The phenomenon of pairing in nucleons can be anticipated by looking at the pairing phenomenon in laboratory nuclei where it is observed experimentally that even-even nuclei exhibit a gap in their excitation spectrum. The even-odd or the odd-even nuclei do not exhibit a gap as they have one unpaired nucleon which can be excited easily. The typical difference in the binding energy of even-even and even-odd/odd-even nuclei is of the order of $0.5 - 3$ MeV. The gaps in the nucleon spectrum in the neutron stars are expected to be of similar magnitude. Both the singlet pairing and the triplet pairing are likely to take place in the star depending on which part of the star (core or crust) we are interested in. From the scattering phase shift data it is clear that at lower densities nucleons pair in the singlet channel and hence we expect the crust of a neutron star, where the density is smaller, to exhibit neutron pairing in the singlet channel. Similarly in denser regions like the core, pairing in the triplet channel is expected. This pairing causes a gap Δ to appear in the single particle spectrum which restricts all phase space integrals for processes involving single particle by a factor of $e^{\Delta/T}$. This leads to the strong suppression of the neu-

trino emissivity and the specific heat of the neutrons at temperatures lower than the gap. As most neutron stars are at temperatures much lower than the estimated gaps, the neutrino emissivity and the specific heat of the neutrons in them are very small. However, there is considerable uncertainty in the magnitude of the gap as it is sensitive to the potential used in the BCS calculation.

In the absence of a definite knowledge of the magnitude of the 3P_2 gap, in this thesis we chose to look at properties of the 3P_2 condensate that are not particularly sensitive to the gap. In the following paragraphs I present a summary of what was computed in the earlier chapters and clarify some minor issues that seem relevant. We considered neutrons paired in the triplet channel with a Cooper pair of total angular momentum 2 and proposed a neutrino emission process that could potentially compete with the PBF process in cold magnetars. The triplet pairing can be of various form and all these forms are almost degenerate making it very hard to establish the ground state of triplet paired neutron matter. Despite this degeneracy, it is however known that the traceless symmetric order parameter for the 3P_2 ground state is also real upto a phase. Such order parameters are called unitary. Close to the critical temperature the Ginsburg-Landau free energy can be calculated by expanding in the magnitude of the gap over the temperature and the coefficients in this expansion can be found using BCS. The Ginsburg-Landau free energy for triplet paired neutron matter suggests that the energetically favoured phase of matter close

to the critical temperature is given by

$$\Delta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \quad (4.1)$$

At lower temperatures the expansion in Ginsburg -Landau free energy breaks down. However, it was argued in the literature that this form of the order parameter persisted even at lower temperatures. We assumed this form of the order parameter to be true for our purpose of calculating the low energy properties of the system. It is known that the low temperature properties of any fermionic system are dictated by fermions close to the Fermi surface. This meant that in order to analyse the low energy properties of such a system we just had to integrate out the bulk neutrons which were strongly interacting and their only contribution to the effective theory would arise from the low energy constants of the effective theory. The first step in writing down the low energy properties of the system would be to look for the relevant degrees of freedom. To identify these degrees of freedom it is important to look at how the order parameter transforms under rotations. The order parameter of the triplet phase considered here breaks rotational symmetry $SO(3)$ of the Lagrangian down to $SO(2)$ spontaneously. This is expected to give rise to two Goldstone modes corresponding to the breaking of rotation about the x axis and the y axis if the unbroken generator corresponds to rotations about z axis. These massless modes, called angulons, are the relevant degrees of freedom when we wish to concentrate on low temperature properties of the triplet paired neutron matter as they are the only ungapped excitations of the system. Although the form of the condensate and

the excitations around it are well described in terms of the real, symmetric traceless matrices we have been using, it may be interesting to look at what these correspond to in terms of angular momentum states. In order to delve deeper into this we need to remember how we expressed the order parameter in the angular momentum basis in the second chapter. In this basis the 3P_2 order parameter was expressed as

$$\Delta(k) = \sum_{m=+2,+1,0,-1,-2} a_m \Delta_m(k) \quad (4.2)$$

where

$$\begin{aligned} \Delta_{+2}(k) &= \begin{pmatrix} Y_1^1(k) & 0 \\ 0 & 0 \end{pmatrix}, \Delta_{+1}(k) = \begin{pmatrix} \sqrt{2}Y_0^1(k) & Y_1^1(k) \\ Y_1^1(k) & 0 \end{pmatrix}, \\ \Delta_0(k) &= \begin{pmatrix} Y_{-1}^1(k) & \sqrt{2}Y_0^1(k) \\ \sqrt{2}Y_0^1(k) & Y_1^1(k) \end{pmatrix}, \\ \Delta_{-1}(k) &= \begin{pmatrix} 0 & Y_{-1}^1(k) \\ Y_{-1}^1(k) & \sqrt{2}Y_0^1(k) \end{pmatrix}, \Delta_{-2}(k) = \begin{pmatrix} 0 & 0 \\ 0 & Y_{-1}^1(k) \end{pmatrix}. \end{aligned} \quad (4.3)$$

Here Y_m^l s are spherical harmonics. The five different Δ_i correspond 5 possible total angular momentum projection of the 3P_2 state. The next step would be to compare the form of the order parameter in the basis of Eq. 4.2 with the form of the order parameter in the basis of traceless symmetric matrices, in which a 3P_2 order parameter can also be expressed as

$$\Delta(k) = \sum_{\mu,\nu=1,2,3} i\sigma^\mu \sigma^2 A_{\mu\nu} k^\nu \quad (4.4)$$

where the matrix A is symmetric and traceless. The representation of the order parameter in Eq. 4.4 is justified because we can verify that an angular momentum 2 operator is a spherical tensor of rank 2 and the operator of Eq. 4.4 is the only spherical tensor of rank 2 that corresponds to a spin of 1. Now, if we compare the forms of Eq. 4.2 and Eq. 4.4, we can relate the elements $A_{\mu\nu}$ and a_i where $i = +2, +1, 0, -1, -2$ as follows

$$A = \begin{pmatrix} \frac{a_2+a_{-2}}{2\sqrt{2}} - \frac{a_0}{\sqrt{2}} & i\frac{a_2-a_{-2}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}}(a_1 - a_{-1}) \\ i\frac{a_2-a_{-2}}{2\sqrt{2}} & \frac{a_2+a_{-2}}{2\sqrt{2}} - \frac{a_0}{\sqrt{2}} & -\frac{i(a_1+a_{-1})}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}(a_1 - a_{-1}) & -\frac{i(a_1+a_{-1})}{\sqrt{2}} & \sqrt{2}a_0 \end{pmatrix}. \quad (4.5)$$

We can solve for a_i from the order parameter that is the ground state of the triplet phase given by Eq. 4.1 and we obtain all a_i except a_0 are zero. This means that the condensate or the Cooper pairs are in $m_J = 0$ state. As expected, a rotation about the z axis does not affect the condensate. However an infinitesimal rotation about the y axes creates a nonzero value for $(a_1 - a_{-1})$, keeping $a_1 = -a_{-1}$ and $a_2 = a_{-2} = 0$ if we look at only the first order fluctuations. Similarly, an infinitesimal rotation about the x axis creates a nonzero value for $(a_1 + a_{-1})$, keeping $a_1 = a_{-1}$ and $a_2 = a_{-2} = 0$. The reason behind a_2, a_{-2} remaining zero upto fluctuations of first order is simple and is as follows: As the condensate is in a $m_J = 0$ state, a first order fluctuation due to rotation in space will correspond to application of the operators J_x or J_y only once. As J_x and J_y are composed only of single J_+ and J_- , the only possible nonzero amplitude created by this rotation will be a_1 and a_{-1} up to first order in the fluctuations.

In principle, a chain of effective theories needed to be employed starting from QCD to obtain the effective theory of angulons. The standard method is to start from QCD and then to write down the effective theory of neutron interactions, from which an effective theory of neutron quasiparticles near the Fermi surface can be obtained. This theory can then be used to obtain the effective theory of the angulons. But we skipped some of the initial steps involving finding the effective theory of neutron interactions etc. and instead wrote a model Lagrangian down which was expected to reproduce the results of the effective theory of neutron quasiparticles close to the Fermi surface. The latter theory involves an expansion parameter which is given by the density of Fermion quasiparticles near the Fermi surface over the density of all Fermions. The integrated out bulk Fermions contribute to the low energy constants of this theory and should in principle be equivalent to the Landau parameters in Landau's theory of Fermi liquid. To be a valid description of the quasi-particles close to the Fermi surface, this effective theory does not need the integrated out fermions to be weakly interacting. All that is required is that interaction energies be smaller than the kinetic energy of the quasiparticles. The quasiparticles interact weakly among themselves as the coupling between them is suppressed by $\frac{p}{k_F}$ where p is the energy scale corresponding to the fluctuations and k_F the Fermi momentum. There is another line of reasoning that can be employed to establish the weakly coupled nature of our model Lagrangian. The evidence in favour of the claim that our model Lagrangian is weakly coupled, can be found if we look at the vacuum two to two scattering phase shifts of neutrons in the relevant angular momentum channels. The unrenormalized bare phase shifts are found to be

modest and it can be expected that the in medium corrections will not change them radically. This although is an expectation is no proof that such is really the case. However, there are model calculations of the pairing gaps in the various angular momentum channels using realistic nucleon nucleon potentials which find the gaps to be much smaller than the Fermi energy. This is an indication that our model Lagrangian of neutron quasiparticle interactions is weakly coupled in reality. For our purpose here, we assumed this to be true.

The model Lagrangian describes neutron quasi-particles close to the Fermi surface with a short-range four-Fermi interaction in the 3P_2 channel. For temperatures below the critical temperature the only excitations that are relevant are bosonic excitation about the condensate. To obtain the effective theory of the bosonic gapless modes, we introduced an auxiliary field in the Lagrangian and integrated out the neutron-quasiparticles completely. Having done that, we expanded the Lagrangian in fluctuations about the ground state. The effective theory so obtained should be valid for excitations of momentum smaller than the energy scale of the gap. A series of improvements and extensions to the effective theory presented here can be useful. One of them is to consider neutrons in the presence of protons. The protons in the core of the star form singlet Cooper pairs and hence are superconducting. This means that the proton excitation spectrum is gapped just like the neutron spectrum due to pairing. However, despite being gapped the protons can mediate interactions between angulons via the strong force and the electrons through electromagnetic interactions. Also, in our derivation of the effective theory we have only included the dominant short range four Fermi interaction in the 3P_2 channel as we expect the

contributions from other channels to be perturbatively small. However, it would be interesting to quantify the effects of the interactions in these other channels on the low energy theory of the angulons.

After having obtained the effective theory of angulons we do a finite temperature calculation of the masses of the angulons and the other low lying modes that correspond to real, symmetric, traceless fluctuations of the order parameter. Our motivation behind this was two-fold. The scale of validity of the effective theory of the angulons would be restricted by the masses of the massive modes if they turned out to be smaller than the gap. Also there was a claim in the literature that the angulons did not remain massless at finite temperature which is below the critical temperature. The claim would be intuitive if the temperature was larger than the critical temperature as the condensate itself vanishes beyond T_c and there are no gapless modes. Although the claim about gapless modes acquiring mass at finite temperature seems somewhat far fetched, it cannot be disproved as there is no proof of the Goldstone's theorem for the spontaneous break-down of space-time symmetries. Hence the issue of generation of the mass of the gapless modes needed to be settled by an explicit calculation at finite temperature. We did this by again starting with our model Lagrangian describing neutron quasi-particles close to the Fermi surface with a short range four Fermi interaction in the 3P_2 channel and introduced an auxiliary field describing the condensate. Then we integrated out the fermions completely which left us with the Lagrangian in terms of the order parameter. This action was minimized with respect to the auxiliary field. The corresponding equation is known as the gap equation. The gap equation gave us a relation between the gap $\bar{\Delta}$ and

the coupling constant g . All these equations involved a sum in p_0 over discrete values given by either $\frac{(2n+1)\pi}{\beta}$ or $\frac{n\pi}{\beta}$ depending on whether fermions or bosons were involved. The action in terms of the order parameter was then expanded about the minimum upto second order in the fluctuations from which we could figure out the dispersion relations for specific modes that we were interested in. Since we were looking at real, traceless, symmetric fluctuations of the order parameter, we had five possible orthogonal modes. Two of them correspond to the angulons and the rest three don't correspond to any symmetry of the Lagrangian. We found that the angulons remained massless even at finite temperature contradicting the previous claims about the angulons acquiring a mass at finite temperature. We also found that the masses of two of the other three modes were of the order of the gap. Hence the effective theory of the angulons that we obtained earlier is valid up to $k_F \bar{\Delta}$ and the masses of the massive modes do not put additional restrictions on the validity of the effective Lagrangian of the angulons. However, we only looked at the masses of real-symmetric traceless fluctuations. In principle it would be good to have an estimate of the masses of all the other possible remaining modes. These could be the modes that correspond to anti-symmetric fluctuations of the order parameter, or could be fluctuations that are complex matrices, both symmetric and antisymmetric. We also found that two of the massive modes are degenerate. This was expected as although the condensate broke rotational symmetry partially, it kept the rotational symmetry about z axis unbroken. The modes that come out to be degenerate can actually be related by a rotation about the z axis. To be more specific, if the condensate plus the fluctuation corresponding to one of the two degenerate

modes is rotated by an angle of $\frac{\pi}{4}$ about the z axis, we obtain the condensate plus a fluctuation that corresponds to the other degenerate mode. This is why the masses of the two modes turning out to be the same is not that surprising. We do not find a real pole in the dispersion relation corresponding to the fifth mode. This may have to do with the fact that the pole is imaginary which corresponds to a damped mode.

Having obtained the finite temperature masses of the massless and massive modes and the effective theory of the angulons, we calculated a couple of the observables related to the cooling of neutron stars. We found that the specific heat of the angulons was miniscule compared to that of the electrons. However, the neutrino emissivity of these modes can be considerable under certain circumstances. In general in the triplet phase of neutrons the most dominant source of neutrino emission is the pair breaking formation process which was discussed in detail earlier. However this process is dominant only at temperatures close to the critical temperature. The critical temperature of condensation inside a star varies from layer to layer and as the star cools down, the layer with temperature close to the critical temperature starts corresponding to layers that are larger in radii. When the temperature of the entire star is below the critical temperature, the PBF process shuts down as it is suppressed by a power of $e^{-\frac{\Delta}{T}}$. This is when the neutrino emission processes involving angulons start dominating. In order to determine whether the angulons can contribute significantly to the neutrino emissivity or not, we needed the effective interaction between the angulons and the neutrinos. We figured this out by noticing how the quarks coupled to the neutral Z boson, from which we derived

how non-relativistic neutrons would couple to the Z boson. The lepton Z vertices are well known. The interaction of the neutrons with the Z boson could be used to find how the angulons couple with Z and then to the leptons. The neutrino process involving the scattering of two angulons was found to be too small to contribute to the neutrino emissivity. However, in the effective theory derived, a term was present which was a mixing between the angulon field and the Z . This term could potentially give rise to angulons decaying to neutrinos. However, there is a problem with this and it is that the angulons are massless and are not allowed to decay to neutrinos kinematically. But most neutron stars have considerably strong magnetic fields and magnetic fields can couple to neutrons through the neutron magnetic moment. When the presence of a magnetic field was taken into account, it was found that the quadratic part of the angulon effective action got modified. This changed the dispersion relation of the angulons making one of them pick up a mass of the order of the $\frac{eB}{M}$ where B is the magnetic field and M is the neutron mass and the other angulon acquired a dispersion relation where energy was proportional to the square of the apital momentum at small momentum. The massive angulon mode could now decay to neutrinos as the decay was kinematically allowed.

Having found a kinematically allowed decay process we calculated the corresponding neutrino emissivity. The neutrino emissivity of a process is defined as the energy carried by neutrinos per unit time per unit volume emitted in that process. The neutrino emissivity due to the decay of angulons can be calculated by squaring the amplitude for the process, multiplying by the energy being carried by the neutrinos summed over all momentum and spin states and then divided by time for which the

interaction is turned on. This time should be taken to infinity at the end of the calculation. As the emissivity for the process is proportional to the square of the amplitude of the process, the expression for emissivity should involve quartic power of the propagator of the Z boson. This is because the process involves a virtual Z boson exchange. For the momentums we are interested in, this contributes a factor of the mass of Z boson raised to the power of -4 , which is the same as the Fermi constant G_F^2 . The emissivity is also proportional to the density of states of the neutrons at the Fermi surface given by Mk_F where M, k_F have the usual definitions. As evident from the definition of emissivity, the mass dimension of it is 5. $G_F^2 Mk_F$ has a mass dimension of -2 and to make up the rest of the dimensions we could use one of the other two scales in the problem, temperature or $\frac{eBg_N}{M}$. We can choose temperature raised to the power of 7 to obtain our expression of emissivity. There is a dimensionless function h of the ratio $\frac{eg_NB}{2aMT}$ that multiplies the dimensionally argued expression that we already deduced above. The final expression for emissivity is given by

$$\nu = G_F^2 Mk_F T^7 h\left(\frac{eg_NB}{2aMT}\right) \quad (4.6)$$

and h is

$$h(x) \approx 0.000042 x^7 e^{-x} \quad (4.7)$$

As was seen from the plots, the neutrino emissivity of the angulons cannot compete with the PBF processes at temperatures when the PBF process peaks or the temperatures are close to the critical temperature. However, at lower temperatures the angulon neutrino emissivity can dominate over the PBF. The function h peaks for

temperatures close to $\frac{eBg_N}{M}$. For a typical neutron star, $\frac{eBg_N}{M} < \bar{\Delta}k_F$ and hence it can be inferred that the angulon emissivity peaks for smaller temperatures than the critical temperature. For higher temperatures, the angulon emissivity is suppressed. The reason behind this is straight forward. At regimes where the magnetic field is much smaller than the temperature, the angulon masses are not large enough for them to decay to neutrinos which carry a typical energy that is proportional to the temperature. Hence the emissivity due to the decay of angulons is suppressed. For temperatures smaller than $\frac{eBg_N}{2M}$, there is a Boltzmann suppression coming from the fact that the mass of the angulons restrict the phase space of angulons. Hence as the star cools down, for $T \sim \frac{eBg_N}{2M}$, the angulon emissivity peaks.

There was another important issue regarding where in the star this decay could take place. The proton superconductor in the core of the star only allows magnetic fields to be present inside superconducting proton vortices inside the star. This meant that angulons are in close proximity of high magnetic fields only close to the flux tubes or inside them. This restricts the area of the star that can participate in the decay of angulons and an estimate of what fraction of the star was occupied by magnetic fields was made for a typical magnetar. It was found that the neutrino emissivity of angulons could compete with the PBF emissivity and can even dominate in magnetars that are cold.

As mentioned before, we analysed real, symmetric, traceless modes of fluctuations about the ground state and these were found to have masses of the order of the gap. Hence, the lowest energy transport properties are not affected by the presence of these modes. However, it is reasonable to think that these may also decay to

neutrinos and since they are massive, they will not need a magnetic field to decay unlike the angulons. If they have a nonzero decay amplitude to neutrinos, we can guess what their emissivity will be. Invariably it will involve an exchange of a Z boson, contributing a factor of G_F^2 . Also it should be proportional to the density of states of the neutrons at the Fermi surface which will contribute a factor of Mk_F . Most interestingly as their mass is of the order of the gap, their phase space will be restricted by a Boltzmann suppression factor of $e^{-\frac{\Delta}{T}}$. Hence the emissivity will peak at temperatures close to the critical temperature just like the PBF process and the form of the emissivity due to the decay of these massive modes to neutrinos will exactly be like that of the PBF. But if we attempt to find the decay amplitude of these modes to neutrinos, we find that it is zero. Hence there is no question of real, symmetric and traceless massive modes decaying to neutrinos and competing with the PBF process. However, there are another set of modes the masses of which we are yet to compute. These modes are the anti-symmetric fluctuations of the order parameter. They correspond to opposite rotations of the total spin and total orbital angular momentum of the Cooper pairs. The order parameter transforms as follows

$$\Delta' = R^s \Delta R^t \tag{4.8}$$

where R^s and R^t are different rotation matrices. These transformations are symmetries of the Lagrangian considered in this thesis as the Lagrangian contains terms of the form $\Delta\Delta^\dagger$ and no terms of the form Δ^2 . Hence these modes will be found to be massless if we use the Lagrangian used here. However, this is a consequence of the fact that we have not included spin-orbit forces in our Lagrangian. We know

that in reality, $\Delta \rightarrow R^s \Delta R^t$ is not a symmetry of the nuclear forces. If we include the spin-orbit interaction terms in our Lagrangian, we will find what the masses of these modes are. Interestingly these anti-symmetric modes can decay to neutrinos through a Z boson exchange. Hence it is probably useful to do a calculation to figure out the masses of these modes. The form of the emissivity for these modes can be predicted using the previous arguments. This decay also involves the exchange of a Z boson, giving rise to a factor of G_F^2 and the emissivity should again be proportional to the density of states of the neutrons at the Fermi surface giving a factor of Mk_F . Hence, using temperature to make up the rest of the dimensions, the form of the emissivity is $G_F^2 Mk_F T^7 e^{-\frac{m}{T}}$ where m is the mass of these anti-symmetric modes. The emissivity will peak for temperatures close to m and it is crucial to make an estimate of m to be able to conclude what that temperature is.

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