Title: PATTERNS IN CURRICULUM CHOICES:
PRE-CALCULUS CURRICULA IN THE ARCHDIOCESE OF WASHINGTON

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This study aims to learn more about the choices made by mathematics teachers in the Archdiocese of Washington, given their unique independence from state or district curricular control. To study these choices, pre-calculus teachers completed a survey and submitted their course's summative assessments. These responses were then compared to themselves, to each other, and to the Common Core to study the choices teachers made, both in the scope of their curricula and in the expectations they had for student performance. This study concludes that teachers choose pre-calculus curricula within two major archetypes, either advancing students' algebraic skill or exploring new topics. Further, the study found that teachers' assessments are well aligned to their stated curriculum, but that contrary to recent education trends, teachers have largely chosen to ignore statistics. Consequences of these choices are discussed, as well as implications for policy and future research.

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Chapter 1: Rationale and Significance

For many cultural, political and constitutional reasons, the United States does not have a national education curriculum. The debate over whether or not this is a positive restriction of government is a long-standing one, with the two sides often balancing a desire for efficiency and the spread of best practices against the benefits of autonomy and the self-direction of states and districts. The Common Core State Standards have been a recent attempt to standardize curricula across state boundaries, but even this effort has restricted itself to content and practice standards, staying clear of designations of pedagogy and sequencing. Despite this, the backlash against the Common Core has been significant.

Even in this confederated system, many or most of the curricular decisions about the content, expectations and scope of the mathematics taught in classrooms are made by states and school districts, rather than schools and teachers (Stevenson & Baker, 1991, p. 2). That is, across grades and courses in many locales, states and districts determine what mathematics content is taught and when it is taught, and these administrative structures often directly or indirectly convey how it should be taught. Then, states and districts define and administer standardized assessments to measure the success teachers have had in teaching the curriculum given to them. In these settings, curricular decisions are made not by teachers, but by states and districts. In contrast, this research was designed to explore what mathematics curriculum would look like when curricular decisions were made by the teachers themselves.

If a teacher was given full control over what to teach, when to teach it, how to teach it, and how to measure student learning and achievement, what would that classroom look like? This study was designed to measure the impact of teachers' decisions about mathematical scope, expectations and assessment, quantifying each teacher's choices in comparison to their peers, to themselves, and to a common standard. This research has determined the level of variability between teachers' content choices, as well as the alignment of those content choices against the teachers' own summative assessments and against the Common Core State Standards for Mathematics (CCSS-M) (Common Core State Standards Initiative, 2010).

It is rare to find teachers with such curricular freedom to study. Nearly all public school teachers are subject at least to a curriculum guide and to standardized testing. Private schools typically do not operate in this way. Many private schools operate with complete independence and require no curricula or testing beyond those created by the school itself, and all private schools, including those of the Archdiocese of Washington, are exempt from the Common Core State Standards, regardless of the decisions of their state and local governments. Religious and parochial schools, especially Catholic schools, often exist in a hybrid approach, with schools operating within the context of the diocese, which functions in much the same way as a school system or district. In this sense, the Archdiocese of Washington is unique. Due in part to its history as a part of the Archdiocese of Baltimore, the original Catholic political entity in the United States (Russell, 1907), the Archdiocese of Washington does not maintain broad control over the non-theological curricula of its high schools (Archdiocese of Washington, 2014b).

As a result, the high school mathematics curricula in the Archdiocese of Washington are authored exclusively by the schools, mathematics departments, and teachers themselves, as are the assessments designed to evaluate student attainment of that content. This unique situation provides a rich opportunity for study in several domains. This research has quantified both teachers' perceptions of their enacted curricula and the assessed pre-calculus curricula of these several teachers across the Archdiocese of Washington and determined whether there are identifiable patterns of similarity or contrast between them. This research has been limited to pre-calculus not only for reasons of focus but also because pre-calculus as a subject provides its own unique content flexibility. Pre-calculus can mean different things to different people (Adelman, 2006, p. 97). Some teachers consider pre-calculus to be a culminating high-school course designed to solidify concepts of algebra, functions, and trigonometry, while some see pre-calculus as preparing the student for further study, introducing the student to advanced topics such as series, matrices, polar coordinates and limits.

Furthermore, it is a stated Archdiocesan goal that the high schools within the Archdiocese of Washington will prepare students for collegiate study (Archdiocese of Washington, 2014a). Therefore, pre-calculus is not considered an elite course within these schools and has broad curricular implications, reaching a far higher proportion of students than in many public schools, whose highest requirement of mathematics is often Algebra II. For the purposes of this study, pre-calculus will be defined as the course taken by the average student during their senior year, or taken by more mathematically advanced students in preparation for AP Calculus. This course could be labeled as "Pre-Calculus", "Functions and Analysis", "Algebra III", or some other similar title.

Rationale

In discussing teachers' reactions to the freedom and curricular flexibility provided by the Archdiocese of Washington, two major questions arise. First, how effective is each teacher's curriculum at accomplishing the broad goal of preparing students for collegiate mathematics? A consistent argument in favor of standardized curricula is that they ensure that all relevant topics within the subject are taught (Porter, 2011). If teachers are consistently able to construct effective and comprehensive curricula on their own, then this may provide some evidence that teachers are able to tailor curricula to the needs of individual students at the classroom level and still meet expectations for rigor and scope of content, without requiring the guidance of standardization.

Secondly, how effective are each teacher's summative assessments at evaluating student success in learning the topics most relevant to pre-calculus? Certainly, students benefit from taking examinations that are both valid and reliable measures of their learning because good assessment allows good teachers to construct good learning plans. While standardized testing is often criticized for providing no flexibility in assessing how students and teachers approach problems in creative ways (Kohn, 2000), it is often taken as given that these tests are effective and consistent within the skills they promote. If, however, teachers' examinations are equally effective and consistent, then the benefits of the standardized tests may not be strong enough to overcome the value of the focus on the student that comes with teacher-built assessments.

The field of curriculum research will benefit from this research because of the unique focus and population of this study. Studying teachers in the Archdiocese of Washington provides a set of subjects who experience true curricular freedom, both in content and in assessment. Studying pre-calculus teaching provides focus on a subject matter whose definition is not intuitive or broadly agreed upon. If there is consensus among these teachers on the content and focus of pre-calculus, then this provides evidence that teachers value similar priorities within this course. If there is not a consensus among these teachers, then that would provide evidence that teachers value a proprietary approach to pre-calculus.

Furthermore, the data collected for this research regarding teachers' assessments of their own curricula may inform districts and schools making decisions about the necessity of standardized testing. If this study suggests that teachers' assessments are reliable measures of the content in their own classes, then this provides evidence that standardized tests may not be the only way to provide reliable measures of student and teacher performance. And in comparing teachers' curricular choices to a common benchmark such as the CCSS-M, this study can inform the field about how closely teachers' chosen curricula align with new standards being adopted across 43 states, 4 territories, and the District of Columbia.

Research Questions

By quantifying the choices that each teacher makes when constructing his or her curriculum, one can achieve a general sense of the priorities of the teacher. There are several broad, philosophical choices teachers can make that affect the construction of their curriculum in various ways. Determining how teachers define pre-calculus as a subject, and specifically understanding the level of agreement among various different teachers of the same course, will be a valuable objective of this research. Assessments both reflect and drive curriculum choices. By quantifying the alignment between the enacted and assessed curricula of these teachers, this research will discuss that particular symbiotic relationship and its implications for other curriculum/assessment relationships in other pre-calculus settings. And finally, the CCSS-M, while not a curriculum, provides a list of topics expected of high school students before graduation. This research discusses how similar teachers' priorities are to the Common Core's, with one objective being to inform a larger discussion on how necessary a common pre-calculus curriculum may be.

Therefore, this study has addressed three research questions:

1. How similar or different are the various teacher-built pre-calculus curricula of the Archdiocese of Washington?
    a. What is the topic-specific variance among the various teacher-built pre-calculus curricula of the Archdiocese of Washington?
    b. Do the similarities and differences of the various teacher-built pre-calculus curricula of the Archdiocese of Washington reflect several prototypical curricular approaches, or is each curriculum independent and unique?
2. What is the level of agreement between the stated curricula and the tested curricula in the pre-calculus classes in the Archdiocese of Washington?
3. How closely does each teacher's perception of their enacted curriculum align to the objectives and standards of the Common Core State Standards for Mathematics?

Approach

This research was designed to address these questions through the statistical analysis of two sources of data, each quantifying either the enacted or assessed curriculum of teachers in the Archdiocese of Washington. Teachers provided insight into their perceptions of their enacted curricula through the completion of a survey addressing the scope of their curriculum and the expectations they have for their students' performance, and they provided evidence of their assessment approach by contributing their summative midterm and final examinations for their last year teaching pre-calculus, either 2012-13 or 2013-14.

The survey asked teachers a series of questions requiring them to recall the curricular choices that they made over the school year, in an effort to quantify each teacher's enacted curriculum. Two survey questions were asked addressing the scope and performance expectations of each teacher for each of 37 pre-calculus topics. This approach was an efficient data collection method, relying on the memory, integrity and self-reflection of each teacher for valid data. Survey data has been found to be an acceptable technique for measuring instructional intent (Mayer, 1999). Nevertheless, subsequent research may benefit from supplementing this survey approach with in-classroom observation-based assessment of teaching, not only to validate the enacted curriculum reported by teachers, but to assess enacted curriculum as well. These survey data were accessed in the analysis addressing Research Questions 1 and 3 above.

In addition to the survey, summative examinations were collected as data regarding teachers' assessed curricula choices and priorities. In contrast to the survey, which required teachers to self-report an analysis of their own choices, which can be subjective, the examinations were stable and objective. These were the actual examinations given to students in the middle and at the conclusion of year-long pre-calculus courses, and as such were accurate and valid measures of these teachers' assessed curricula. By collecting these data, this research explored potential relationships between individual teachers' enacted and assessed curricula. A strong correlation would be evidence that teachers are designing their examinations to measure student understanding of the curriculum that they are claiming to teach. These summative examinations were accessed along with the curriculum survey data in the analysis addressing Research Question 2 above.

Scope

For each of the 37 pre-calculus topics identified in the curriculum survey, the first survey question assessed curriculum scope by asking teachers for an estimation of time on topic, measured in class days. By answering this question, teachers communicated those topics on which they dedicated the highest proportion of class time, which topics required only minimal instructional emphasis, and which topics they decided were not valuable enough to be included in the course. By doing this, teachers effectively communicated their content priorities.

The collected examinations were also used to assess the scope of each teacher's curriculum. By analyzing which topics were addressed by the questions asked on the examination, and by indicating the relative point values of those questions, this study assessed precisely what proportion of the assessed curriculum's scope was dedicated to each topic. This gave a more complete picture of the scope of each teacher's curriculum than with the survey alone.

Performance Expectations

The second question within each topic on the curriculum survey measured teachers' curricular choices in terms of expectations for students. For each of the 37 topics, the survey provided teachers with a mathematical question, task, or problem related to the topic under consideration and ask the responding teachers to evaluate the level of student understanding or facility required to correctly answer this question. These survey questions were intended to provide a measure of the performance expectations the teacher has for their students on each specific topic. Certainly, the quality of the students entering the class would have a significant effect on their potential for answering mathematical questions on each topic, but these data, combined with the scope data, will construct a more complete portrait of the teachers' curricular assumptions and expectations, as well as their effects on students.

This survey expectations data (enacted curriculum) was compared to expectations data collected via the administered examinations (assessed curriculum). The researcher coded each question of each semester and final examination using the same scale the teachers used to identify the performance expectations of their curriculum for each topic during the survey. Once each examination question or item was coded, the assessed expectations for each topic were identified across the level of the expectations for all of a teacher's examination questions/items on that topic. As with the scope data, this approach helped provide a more complete picture of the performance expectations of each teacher by quantifying both the expectations of the enacted curriculum and the expectations of the assessed curriculum.

Intent

Collecting these data led to meaningful comparisons along three dimensions. The first comparison was across teachers' enacted curricula. The study assessed the variability of the curricula chosen by each individual teacher and interpreted that variability as evidence of dispersion or consensus among teachers' curricular choices. A low level of variability would imply that each teacher independently chooses similar topics to teach in pre-calculus, while a high level of variability would provide evidence that teachers are tailoring their curricula to the individual needs of, or expectations for, their students.

Within the 37 pre-calculus topics, there were several intuitive groupings of topics. For example, a teacher who believes the purpose of pre-calculus is to prepare the students for the rigors of collegiate mathematics with a mastery of algebra skill would spend a similar amount of time on topics such as "Irrational Numbers, Radicals and Rational Exponents" and "Solving Rational and Radical Equations," In contrast, a teacher who believes the purpose of pre-calculus is to prepare the students for collegiate mathematics by exposing them to many new advanced topics might spend a similar amount of time on both "Polar Coordinates and Polar Functions" and "Parametric Equations." A visual analysis of the scope and expectations data collected in this study allowed for the possibility of a typographical schematic of curriculum approaches. If these approaches existed, they would provide a valuable interpretation tool for this research.

Second, teachers' enacted curricula were compared in a pairwise fashion to their own assessed curricula, as demonstrated through their midterm and final examinations. If the data showed evidence of a high correlation between the enacted and the assessed curricula for each teacher, then that would imply that teachers are generally successful at constructing summative assessments that accurately represent what they thought they were teaching. Low correlation would imply the opposite, that teachers are ineffective at assessing their own enacted curriculum.

Finally, the teachers' perceptions of their enacted curricula were compared to the CCSS-M to allow for meaningful comparison against a standard benchmark. Teachers' chosen curricula included some topics not on the Common Core and ignored some topics that are included within the Common Core. The primary inference gained from these data was the general scope of pre-calculus curricula. That is, does the average pre-calculus teacher in the Archdiocese teach as much as the CCSS-M indicates as necessary to prepare students to be college and career ready? The data collected in service of this question are interesting as commentary both on the teachers' curriculum choices and on the scope of the Common Core.

Significance

Not only does the field of curriculum study benefit from the results of this research, but the teaching community is informed by these results as well. These data have provided evidence regarding teachers' curricular choices if provided the freedom to modify their teaching to match their own priorities, and this is relevant for any administrator or educator who constructs systems within which teachers make these decisions. This includes leaders of education corporations creating curriculum materials such as standardized assessments and textbooks. Further, high schools themselves will find value in this evidence regarding the relationship of teachers' priorities and choices as compared to those of the CCSS-M.

The field of curriculum study benefits from the results of this study, because it provides insight into the relationships between teacher autonomy and curriculum, offers a reliable tool for assessing the alignment of curricula, and explicates the curricular decisions of a population of teachers with curricular freedom and of the value of studying these teachers. Teacher autonomy is an aspect of school culture that has been studied with respect to turnover and employment issues. The evidence from this study informs the field of curricular research as to the influence and variability of teacher autonomy.

Important to note is the ubiquity of pre-calculus among many levels of education. As explored in this study, many students take pre-calculus in high school. However, pre-calculus is also a fixture at the community college and university levels, and there is no consistent curriculum governing the choices of these professors. Often, colleges and universities require pre-calculus for incoming students as a prerequisite for further mathematics, and this role as a gatekeeper course drives the choices professors make regarding the curricula of their courses. However, the public high school structures designed to provide guidance and consistency to teachers across the district, state, and nation do not exist for higher education. Therefore, the results of this study inform these professors and their schools as well, providing information on others' choices in similarly free curricular situations.

This study is valuable to commercial education companies as well, as they design and market the curriculum materials teachers use to build and execute curricula in their classrooms. The results of this study inform textbook publishers and authors as to what topics teachers themselves choose to be part of a pre-calculus curriculum, which is valuable information when developing pre-calculus textbooks, as well as for authors of Algebra II and calculus textbooks for purposes of vertical planning.

These data also include a comparison of teachers' enacted and assessed curricula against the CCSS-M, which is relevant as a consistent standard for inter-school comparisons. High schools, especially private high schools such as those in the Archdiocese of Washington, have a need to evaluate and market themselves as successful educational institutions. A school that has been demonstrated to have a senior mathematics curriculum that is near, at, or above the demands of the CCSS-M may have an advantage over a similar school with less rigorous standards. If teachers given curricular freedom do indeed adhere to these high standards on their own, then their high schools should be looked upon as equally rigorous as schools with more standardized curriculum documents.

The vast majority of students in the United States are not taught in schools where the teacher creates his or her own curriculum and builds his or her own semester or final examinations to assess that curriculum. Despite this, learning more about the actions of teachers who are given this level of creative freedom and curricular flexibility is valuable for all teachers and school systems. By better understanding the choices made by the teachers in the Archdiocese of Washington, better informed decisions can be made regarding assumptions defining the mandated curricula and standardized testing prevalent in public school systems today. This research is intended to contribute to this pressing nationwide policy and curriculum discussion.

Chapter 2: Literature Review

This research was designed to explore a topic that informs some questions that many researchers have pursued about curriculum: Is the ideal curriculum of a course in mathematics an objective reality or does it vary based on teacher, school, and situation? There are compelling, research-supported arguments for each of several positions. Some researchers advocate for a strongly centralized curriculum, which can more easily react to inefficiencies and can provide support to inexperienced teachers, more closely resembling the successful educational systems of Europe and East Asia (Stevenson & Baker, 1991, 1996; Schmidt, Houang & Cogan, 2002). Other researchers point to the benefits of teacher autonomy, stressing that matching curricula to the needs of individual students and to the instructional style and capabilities of the teacher is essential to success (Westbury & Hsu, 1996a, 1996b; McCaffrey et al, 2001, Porter, 1989). Other researchers see merit in both perspectives, arguing that the efficacy of standards depends on myriad factors and context (Hiebert, 1999). Both perspectives will be discussed here.

In addition, this research was also designed to discuss the relationship between each teacher's enacted and assessed curriculum, which is relevant in an archdiocese that does not require nor provide standardized testing for high school students. Several studies have been conducted to clarify and define the borders between various curricular distinctions (Hirsch & Reys, 2009; Taras, 2005), and Porter (2006) has established a valid and reasonable method for measuring curricular distinctions and their relationship to each other. Other research has been conducted to analyze the practices and beliefs of teachers regarding assessment and its relationship to instruction (Abrams, Pedulla & Madaus, 2003; Slavin, Lake & Groff, 2009; Senk, Beckmann & Thompson, 1997). All of this will be useful in providing context to the work proposed for this study.

The specific context of this study is the curriculum of pre-calculus teachers in the Archdiocese of Washington. This context provides its own unique relationship to the curriculum, content, and assessment discussed here. Specifically, it will be instructive to consider questions of what pre-calculus is and its role in the mathematical pathway of college-bound students. This review will include the work of Woodward (2004), who discusses the history of mathematics education reform and the changes made necessary by the focus on college-bound students, and it will include the work of Thompson (1994), who specifically unpacks the role of functions as a topic and the instruction required to communicate this topic effectively.

Furthermore, the administrative decisions of the Archdiocesan schools are affected by not only a desire to educate their students in mathematics, but also by an adherence to Catholic thought and teaching on education and social justice. In specific, the Catholic doctrine of Subsidiarity, defined in the Catechism of the Catholic Church (1997), designates that “a community of a higher order should not interfere in the internal life of a community of a lower order, depriving the latter of its functions” (§1883). This teaching informs Catholics of the role of individual autonomy and decision making and provides further context for the relationship of central and proprietary curriculum that is discussed in this research.

What is Pre-Calculus?

Pre-calculus is often defined as all of "the mathematics that are necessary for success in calculus", but it is rarely so straightforward as this. Calculus has never been a requirement for the typical high school student, but beginning with the New Math reforms of the late 1950s and early 1960s, mathematics curricula began to be informed by how to best prepare students for collegiate mathematics, such as calculus (Woodward, 2004, p. 17). Each of these advances and attempts at reform for mathematics curricula was via a broad, nationally targeted approach, advocated by funding or recommendations from the federal government or from professional organizations such as the National Council of Teachers of Mathematics (NCTM). These recommendations were conveyed through policy documents such as *A Nation at Risk* (Gardner, 1983, p. 20), the 1989 *Curriculum and Evaluation Standards* (National Council of Teachers of Mathematics, p. 20), and *Goals 2000* (1994, p. 22), but each was implemented by states and districts on an ad-hoc basis.

This somewhat loose confederation changed with the introduction of the No Child Left Behind Act of 2001 (p. 25), which, for the first time, provided explicit expectations for the development of centralized state-level accountability standards for education. Subsequently, the Common Core State Standards (National Governors Association, 2010) provided an explicit, national goal: Prepare all students to be college and career ready by the end of high school. The CCSS-M specifically recognizes a three-year mathematics sequence in high school - either Algebra I, Geometry, Algebra II or an integrated course sequence Mathematics 1, Mathematics 2, Mathematics 3 (2010, p. 84) - in order to accomplish this goal. And while it may be possible to teach and learn all of the high school mathematics content in the CCSS-M effectively in three years, many schools will find that several students have not mastered all of the content by the end of Algebra II. Filling these gaps then becomes the purpose of a pre-calculus course.

One of the primary purposes of this research was to discuss just which topics the teachers in the Archdiocese of Washington believe to be contained under the title of "pre-calculus." However, one topic that is unequivocally within the scope of this course is functions. Thompson (1994) reflects on the role that a deep understanding of function may have to success in calculus, emphasizing that a knowledge of function is foundational to many calculus concepts, such as the Fundamental Theorem of Calculus (p. 4), rate and ratio (p. 16), differentiation and integration as operators (p. 19), and exponential growth (p. 21). This firmly cements function as a core topic in pre-calculus.

However, function is not the only topic necessary for success in calculus. Students must be capable of advanced algebraic manipulation, as well as quasi-algebraic skills regarding logarithms and trigonometry. Second-semester calculus often involves sequences, series, and non-rectangular function displays, such as polar coordinates and parametrics. And beyond that, multivariate calculus requires an understanding of linear algebra, vectors, and matrices. Therefore, each of these topics could be defended as part of a pre-calculus curriculum, as could elementary calculus topics such as limit.

Further complicating the definition of pre-calculus is a discussion of whether the goal of the course should even be in preparation for calculus. Following tradition, many high schools use pre-calculus as a capstone course of sorts, with the intention being to provide a comprehensive structure to the skills and concepts learned in Algebra II.

However, this tradition has recently come under scrutiny for inappropriately prioritizing calculus above other advanced mathematics, such as statistics, and for requiring advanced mathematics of students who may not be required to enroll in calculus as part of their collegiate coursework. According to the CBMS Report (2010, as cited in Blair, Kirkman & Maxwell, 2013), only 25% of undergraduate students at four-year colleges and less than 30% of undergraduate students at two-year colleges were enrolled in a mathematics or statistics course in the Fall 2010 semester. Further, many of these students were taking courses below the level of calculus, such as college algebra or pre-calculus, or were pursuing statistics instead of calculus. All of this is to say that preparing all high school students for calculus, even college-intending students, may not adequately serve the needs of these students in their undergraduate coursework.

In contrast to the finality of many high school pre-calculus courses, pre-calculus taught at the community college or university levels is often as a prerequisite for higher mathematics, making the purpose of pre-calculus an exclusively preparatory one. However, even in these situations, universities may have various pathways students can follow, each with different goals based on the major choices and skills of the students. A recent joint position statement released by the Mathematical Association of America and the National Council of Teachers of Mathematics (2012) calls for mathematics courses previous to calculus to have as their goal the development of "the mathematical foundation that will enable students to pursue whatever course of study interests them," rather than solely being focused on preparation for calculus. This would, again, lead to a diverse set of goals for the course labeled as "pre-calculus," further calling into question the efficacy of one stable definition for the course.

International Comparisons

In 1991, Stevenson and Baker presented research on the Second International Mathematics Study discussing the relationship between the level of government that exhibits control over mathematics curriculum and the consistency of instruction for the various classrooms in each country studied. They discussed the United States (as a decentralized system) and France (as a centralized system) as specific exemplars and came to the conclusion that France, along with other centralized education systems, was more successful in having the same material taught to students across the country. The implication was that this made local factors, such as the demographic characteristics of the students, less relevant to students' learning (p. 8), and that this was beneficial for students.

In contrast, a subsequent analysis of the same data (Westbury & Hsu, 1996a) , using "course" as the unit of analysis rather than the student's grade level, (i.e. separating students in 8th-grade algebra from those taking general 8th-grade mathematics) found significantly reduced variability in the data. Westbury and Hsu provided a convincing rebuttal to Stevenson and Baker (1991) arguing that even locally-controlled school systems were reasonably consistent in the delivery of content to students. After this article was released, both Stevenson and Baker (1996) and Westbury and Hsu (1996b) participated in a debate through research extensions, cross-examining each argument and criticizing methodologies. However, neither pair of authors focused much energy on determining whether or not consistent curricula across classrooms was even a worthy goal to pursue.

This question was at the heart of a study addressing the relationship between curriculum and instruction (McCaffrey, et al., 2001), specifically whether a change in curriculum affected the effectiveness of reform instruction practices advocated by the NCTM. This study found that despite controlling for demographics and prior achievement, students who were taught using reform instructional practices (community-based mathematics learning, student-centered mathematical authority, and a focus on connections, applications and problem-solving) only saw statistically significant gains in achievement if the curriculum of the class they were in was reform-centric (integrated mathematics course in contrast to the traditional Algebra I-Geometry-Algebra II sequence) as well. Indeed, most curriculum reforms across the world are in the direction of application and practicality of mathematics, especially as it applies to science and technology (Atkin & Black, 1997). In addition, a subsequent study (Slavin, Lake & Groff, 2009) examined the effect of curriculum, technology and various instructional techniques on student achievement and found that the only one that had a significant effect on achievement was cooperative learning. This brings new clarity to the debate about consistent curriculum, as this research implies that this curricular consistency implies nothing but consistency itself, and there is no guarantee for learning success.

While these studies sought to quantify the success (or lack thereof) of various curricula, in 1997, an Organisation for Economic Cooperation and Development (OECD) study attempted to understand educational reforms pursued in each of 13 different countries by asking the representatives of countries themselves. A primary finding of this work (Atkin and Black, 1997) was that even countries who scored higher than the United

States on the TIMSS were unsatisfied with the current state of their mathematics education program, and many of these nations were pursuing moderate to profound changes. When asked why, specifically, improving mathematics education was a priority for them, these countries' education ministries or federal agencies had two general perspectives. Some countries, such as the United States, identified economic competitiveness as a primary impetus for improving mathematics education, while others, such as Japan and Germany, cited the social benefits of education.

Perhaps the most compelling argument for a strong central curriculum is made by Schmidt, Houang & Cogan (2002) in their article written for *American Educator*. In it, these authors contend that the curriculum used by the United States is not just fractured into various states and districts, but that the loosely organized nature of the U.S. curriculum makes it more likely that the resulting state and district curricula will be inefficient, unfocused, and incoherent. They echoed the argument that helped policy makers consider the need for a common curriculum: The U.S. curriculum as evidenced across the states is "a mile wide and an inch deep" (p. 3). In their view, this lack of focus manifests in a spiraling curriculum that teaches the same topic for far longer than in countries with a focused curriculum, spending an average of six years on a topic in contrast to the other countries' three years. This produces broad curricula that cover far too much mathematics in far too little time, using textbooks that are far too big to be covered in depth in one year. Textbook companies react to the diverse curricula across the states by building books that cover everything in each state's curriculum standards, producing gigantic, unfocused texts that are somewhat useful for everyone but ideal for no one.

One important point regarding the difference between the research of Stevenson and Baker (1996) and the research of Schmidt, Houang and Cogan (2002) is the starting point of each of their paths of inquiry. Stevenson and Baker (1996) began by specifically identifying two countries with different approaches to standardized curriculum and sought to explain the consequences of these decisions. In contrast, Schmidt, Houang and Cogan (2002) began by specifically identifying high-performing countries and then sought to find the similarities between them. One of several findings identified was the standardization of their curriculum.

It is important to remember that each of these approaches is correlational and does not prove causation. It is eminently possible that the countries with standardized curricula are successful for other reasons (such as a strong teacher education system or a population with a culture of respect for teachers and education), and that this success, or these underlying reasons themselves, are why standardized curricula have flourished in these countries. Atkin and Black (1997) summarize:

> In mathematics… eight of the top 10 and nine of the bottom 10 [countries in the TIMSS study] have national curricula. Of course, whether or not a curriculum is centrally directed is part and parcel of the historical and cultural tradition of a country, and the question of whether or not a change in a particular country would produce a benefit cannot be answered by comparing the status quo across different countries (p. 26).

Standard vs. Proprietary Curriculum

The study of international comparisons is important and provides necessary evidence informing the debate about the efficacy of standardized curriculum. However, Hiebert (1999) cautions against drawing conclusions that are too definitive without simultaneously considering surrounding factors. It is difficult to state definitively that research supports the implementation of standards, not due to the nature of the standards themselves but due to the difficulty in drawing such conclusions in a scientifically rigorous way. While it seems that evidence is generally in support of reform-teaching standards, the success of the standards depends heavily on context such as who is teaching, who is learning, and the history and society of the school setting and community.

One such contextual element is curriculum flexibility. Opponents of standardized curriculum, such as Porter (1989), argue that standard curricula can channel efforts in inappropriate directions, "not allowing for individual differences among students in their interests, needs, and aptitudes" (p. 348). Porter's primary criticism derives from his contention that while standards of education are a broad, general policy, excellence is a deeply personal level of attainment: "Excellence is not the opposite of minimum competence… Given the individualistic nature of excellence, legislating excellence may not be possible" (p. 353). Porter goes further to claim that "higher order thinking and problem solving are antithetical to central control and standard setting" (p. 350).

Ultimately, the issue of curriculum reform is being resolved at the classroom level (Atkin & Black, 1997). Especially in content of curriculum, new reforms are being driven by teachers and educators, rather than by academia.

> It has always been assumed that teachers are the experts in devising pedagogical strategies but that they have no particular expertise or authority when it comes to selecting content. This belief is being challenged. In several of the cases, teachers assert that, because they know the students and the local conditions best, they have an important role in choosing the topics that meet their needs (p. 24).

Schmidt, Houang and Cogan (2002) would likely agree that excellence is a desirable goal, but would disagree that leaving the construction of the curriculum to the teachers is the most effective way to achieve that goal. Their argument that the United States has a curriculum, largely built at the local level, that is “highly repetitive, unfocused, unchallenging, and incoherent” (p. 3) can be interpreted as a condemnation of individual teacher- and district-built curricula writ large. Without a central authority to make the difficult choices necessary to maintain focus, rigor, and coherence, the diverse and varied nature of individual teachers’ decisions contribute to these negative attributes of the U.S. curriculum through natural entropy.

Another of Porter’s (1989) criticism of standards regards motivation, both of students and of teachers. When top-level education institutions, such as states and districts, create standards, the locus of control over curriculum shifts from teachers to administrators, perhaps by necessity creating an extrinsic motivation structure. Porter discusses the effect of standards on students’ motivation by contending that “minimum standards that work for some may stifle others, inappropriately shifting motivation from primarily intrinsic to primarily extrinsic" (p. 349). Regarding teachers, Porter decries external motivation as well: "Telling teachers what to do through state and district standard setting policies is seen as antithetical to empowering teachers and strengthening the teaching profession" (p. 345).

One issue driving much of the tone and topic of modern educational reform is the concept of accountability. With limited budgets and under high pressure to demonstrate success, states and districts desire clear, tangible markers that prove reforms are effective. This can create a challenge for reformers and reform-minded teachers, who may find the current educational climate adverse to the risk necessary for growth and reform:

> There is, however, a strong tension between an uneasy public's demand for evidence that the education reforms are working and the need to support teachers who are willing to be innovators. Teachers are frequently the ones who carry the burden of striving for new and often unfamiliar aims and then justifying them to parents and the public (Atkin & Black, 1997, p. 24).

It is important to note that while Porter (1989) does not argue the solution is to remove all curriculum controls, and does contend that standards serve well the purpose of raising student achievement. Nevertheless he questions whether these policies are as effective at encouraging excellence as is the goal:

> Standards may assure student achievement, but that which is achieved may not be most important (i.e. facts and skills, not

> higher-order thinking and problem solving). Standards may ensure that instruction covers important content but in so doing may sacrifice depth of coverage for breadth of coverage. Standards may assure worthwhile content for poorly motivated and low aptitude students but stifle the learning experiences for more gifted students. Standards may motivate students to work harder by holding them accountable, but in holding students accountable, teachers may come to accept less responsibility themselves for what students learn (p. 353).

Enacted vs. Assessed Curriculum

While curriculum for a course may be adequately discussed as a singular object, more completely, one may understand curriculum as comprised of several distinct concepts based on who created the curriculum and for whom the curriculum is relevant. In this way, a single pre-calculus curriculum can be dissected into "intended curriculum," the pattern of concepts the teacher, school or district wishes to teach for the class, "textbook curriculum," the pattern of concepts advocated by the textbook used in the class, "enacted curriculum," the actual instruction that is affected by the teacher in the classroom, and "assessed curriculum," the pattern of concepts required to be learned in order to be successful on the summative assessments of the class (Hirsch & Reys, 2009).

This research study analyzed the differences between teachers' perceptions of their own enacted curriculum and of their assessed curricula by comparing teachers' claims as to how much time they spend teaching each topic to the actual topics assessed by their midterm and final examinations. Future research may extend this by including an analysis of the teachers' textbook curriculum (by securing a copy of and analyzing each teacher's textbook or curriculum materials) and implemented curriculum (by coupling the survey measure with in-class observations of the teachers at work). It is important to note that many teachers, particularly in school systems with more district-level control over curriculum, do not select their own textbooks, and that this is a major factor in translating the intended curriculum into the enacted curriculum actually experienced by students in the classroom (Tarr, et al., 2006, p. 191).

Another distinction that is worthy of mention is the separation between summative and formative assessment. Formative assessment is vital to a teacher's daily work, as it provides relevant feedback on the effectiveness of her teaching and the state of knowledge of her students. Furthermore, it provides meaningful feedback to the students so they can monitor their own performance. Generally, formative assessments are smaller, such as quizzes, and focus on providing feedback and status updates. In contrast, summative assessments are larger and focus less on providing feedback to teacher and student, but rather assign a value judgement on the quality of a student's learning. These summative assessments are typically tests or examinations and are major vehicles for giving students grades for the class (Taras, 2005).

One interesting informative aspect of this research is the relative value of different types of assessment. Senk, Beckmann and Thompson (1997) found that in a majority of mathematics classrooms, grades were determined by tests and quizzes, rather than by the more open-ended written projects that these authors advocated as superior assessments. Atkin and Black (1997) agree:

> External assessments in particular, whether statewide or national, have traditionally exerted powerful control over what teachers feel obliged to do. Yet the impact of such assessments on efforts to engage students in original and complex work can be devastating if the tests measure only memory. In such cases teachers' efforts to do more than teach to the test are discouraged, and public support for changes becomes more difficult to generate (p. 24).

Perhaps future research can find a connection between curriculum and the type of assessment that is best suited for each topic and teaching style. Often, teachers default to these types of assessments due to familiarity. If teachers are to expand their assessments beyond tests and quizzes, teachers must be exposed to them in a productive and practical way:

> Opportunities to learn about assessment and time to actually use alternative assessment are necessary if teachers are to implement the forms of assessment recommended in the [NCTM] standards-based curricula (Bay, Reys & Reys, 1999, p. 505).

Coupled with the discussion of the relevance of the individual teacher to the effectiveness of these broad statements of success and failure in curriculum and assessment are the opinions and beliefs of the teachers themselves. Teachers are independent, creative workers who will be implementing the results of this research for the sake of their students. A study examining the views of teachers regarding state-mandated testing programs found a profoundly negative reaction to the pressures of test-based accountability (Abrams, Pedulla & Madaus, 2003). These standardized tests are having a serious impact on teacher and student morale, and are accumulating a severe human cost that may outweigh the benefits of increased teacher and student accountability (p. 27).

The Catholic social justice principle of subsidiarity is clearly prevalent here, in both the discussion of central or local curriculum and the discussion of standardized testing. The Catholic Church believes that the most effective decision-maker is the one closest to the implementation itself. The implication is that since teachers and schools know their students, know their subject, and know themselves, they will be more effective arbiters of the what and how of mathematics curriculum. While the principle of subsidiarity typically applies to matters of governance and law (Vischer, 2001), it would be appropriate to view it as an aspect of Catholic theology that informs the way the teachers and schools in the Archdiocese of Washington build their curricula within their individual classrooms. The question then simply becomes if this is, at its core, in the best interests of their students. This research is intended to inform that value judgement.

Chapter 3: Methodology

This thesis study addressed pre-calculus teachers' decisions about mathematical scope, performance expectations, and assessment, quantifying each teacher's choices in comparison to their peers, to themselves, and to a common standard. This research determined the level of variability between teachers' content choices, as well as the alignment of those content choices against the teachers' own summative assessments and against the CCSS-M. This chapter describes the methodology for the study, characterizing participants and their recruitment, data sources, and analyses.

Subjects

The pool for subjects of this study was those high school mathematics teachers in the Archdiocese of Washington who taught pre-calculus during the 2012-13 or 2013-14 academic years. This study solicited volunteers from this pool via direct solicitation and email. The opportunities to directly solicit teachers to participate in this study were limited by time and distance, so the researcher supplemented direct solicitation by contacting teachers directly through their professional email.

Direct Solicitation of Participants

Each year, the Archdiocese of Washington sponsors a professional development day hosted by the High School Principal's Association (HSPA). One of the features of the 2014 HSPA conference was a series of structured roundtables called Affinity Groups. Affinity Groups are led by teachers or administrators, either from within or beyond the Archdiocese. Several weeks before HSPA, teachers across the Archdiocese received the list of roundtables available to them. For the 2014 HSPA, an Affinity Group was granted to be used for the recruitment of pre-calculus teachers for this research project.

This Affinity Group session presented attending teachers with the rationale for this research. These teachers had been informed by the Archdiocese that this Affinity Group would be taking place, and teachers were free to choose whether or not to attend. The Affinity Group was allotted 60 minutes to introduce these teachers to this research. A power-point presentation informed the teachers of the goals of this research and of the methodology that would be used in an attempt to accomplish those goals. At the conclusion of the power-point presentation, the teachers were provided with an opportunity to ask questions. Then, two copies of the informed consent form, which is provided as Appendix A, were distributed to each teacher.

Once the consent forms were distributed, each item on the form was carefully and deliberately reviewed. The teachers were informed that a $25 gift card was being offered to each participant who completes the study. The protections used to ensure their data would be secure and confidential were discussed. They were reminded that they were under no obligation to participate in this study and that they were free to leave the study at any time, for any reason. At this point, they were told that if they were interested in participating in this study, they should sign the consent form and provide their email address on the form. Every teacher was then instructed to turn in one of the copies of the consent form they were given into a cardboard box provided on their way out of the meeting, either a signed copy if they choose to participate, or a blank copy if they choose to decline. In this way, any teacher who chose not to participate would be able to remain anonymous.

Solicitation of Participants via Email

It was anticipated that not every pre-calculus teacher in the Archdiocese would attend the Affinity Group, so additional recruitment efforts were planned beyond this initial meeting. A flyer, located in Appendix B, had been prepared and was sent via email to the mathematics department chairs at each high school in the Archdiocese. In the email used to distribute the flyer, each department chair was asked to forward the flyer to each of that school's pre-calculus teachers. The flyer simply asked for an email in return from interested pre-calculus teachers. Unlike during the Affinity Groups, where teachers who provided their email address were signifying their consent, teachers who provided their email address during this secondary recruitment process were simply signifying interest.

Once the flyer was distributed, several individual responses from pre-calculus teachers were received. A standard email was crafted and sent to each teacher who expressed interest, explaining the general purposes of the research as well as the expectations of the teacher if they were interested in participating. A copy of the Summary and Rationale sheet, located in Appendix C, was attached to the email, as was a copy of the informed consent form. As detailed in the discussion below, this email also contained a link to the online survey, within which the subjects signalled their consent to participate. Once again, the email explained that these teachers were in no way obligated to participate and detailed the three-step process of providing consent, uploading examinations, and completing the survey.

Communication with Subjects

The success of the study depended upon constant clear communication between the researcher and the subjects. This communication largely depended on email, since the subjects were provided the researcher's email address both through the direct email solicitation of their participation and as stated on the informed consent form. Email was a primary method of data collection, and the researcher used email to communicate expectations of the study to the subjects. Furthermore, any questions the subjects had about the study at any time were addressed via email. All communication using the researcher's email was kept secure under password protection.

Data Sources

After recruiting each teacher to participate in the study, the researcher sent an email containing both an attached copy of the Informed Consent form and a link to the survey, hosted on the third-party online survey site Qualtrics. Qualtrics is a partner of the University of Maryland, which has determined that Qualtrics is a safe and secure data collection vehicle. The link connected the teacher to the survey that was designed by the researcher, and which is provided in full within Appendix D.

In the email sent to each teacher, the researcher reminded the subjects to carefully read through the project's informed consent form and to have available the midterm and final examinations from the most recent complete year of pre-calculus they had taught, either 2012-2013 or 2013-2014. The subjects then were directed to follow the link to the survey.

Informed Consent

Once at the study's survey site, the first question referenced the informed consent form and asked the subjects to consent electronically, typing their name to indicate that they had read and understood the informed consent form. This way, recruited subjects could not take the survey without having previously signed and returned the informed consent form to the researcher.

Collecting Examination Data

After providing their consent, subjects were directed to upload an electronic copy of their midterm and final examinations from the most recent complete year they taught pre-calculus, either 2012-2013 or 2013-2014. Qualtrics provided a survey option wherein the subject could upload the document directly into the survey itself, which was then attached to the subject's name and data, allowing the researcher to analyze the examinations specific to each teacher. By collecting the examinations within the survey itself, the process became streamlined and less likely to cause attrition of participants between stages of a two-step collection model.

Collecting Survey Data

Once informed consent had been secured and the examinations had been provided, the survey was presented. The survey was be designed to take approximately one hour, although since the construction of the survey was an important part of the research itself, an item within the survey asked the participants to report how long it took them to complete it. Once submitted, the survey data were available to the researcher alone, and the data were stored in a password-protected file separate from the examinations.

Professional and Demographic Information. After the questions regarding consent and the uploading of the examinations, the first four questions on the survey collected professional and demographic data. These data provided context, allowing for a richer and more informed data analysis. These questions asked the teacher to indicate his race, gender, years of experience teaching high school mathematics, and years of experience teaching pre-calculus. These questions, like all of the questions on this survey, were optional and could have been skipped.

Scope and Time-on-Topic Information. The second type of data characterized time-on-topic data, defining the scope of each teacher's enacted curriculum. These survey items asked teachers to approximate how much time they spent teaching each topic. Five options were provided for the teachers to choose from, including "I do not teach this topic," "Very little: 1-2 days," "Some time: 2-4 days," "Quite a bit: 4-7 days," and "A great deal: 7+ days." Both a subjective and an objective descriptor were included in each answer choice because it improved reliability. Teachers were asked to make estimations based on their records and on their personal curricular priorities, as they were more likely to be confident in their answer if it matched both their planning records and their subjective estimation of the importance of the topic. There was an attempt to control for a conflict between these two types of estimation by providing for overlap between the number of days. For example, 4 days could be considered either "Some time" or "Quite a bit," depending on the teacher's impression about the topic.

Student Performance Expectation Information. Finally, the survey asked the respondent to provide student performance expectation data, which was a measure of the expectations each teacher has for students on each topic taught. On the same page as the time-on-topic question for each topic, teachers were asked to determine the level of student that they believed would be capable of answering a question on that topic. One question was presented for each topic, constituting an average item difficulty for the types of problems teachers might ask during the lessons on these topics. Teachers were not asked to answer the problems, but instead were to identify the expected performance level for a student to be likely to answer the question correctly. Teachers were again given five choices: “All of my students can answer this”, “Most students, perhaps not my lowest, can answer this”, “Solid students can answer this”, “Only my strongest students can answer this”, and “I do not teach this topic.” If only the “strongest” students were expected to answer correctly, then it followed that the subject was not an integral part of the curriculum; if “all” students were expected to answer correctly, the expectation on this topic was high. Note that this rating was not to communicate a value judgement on the quality of teaching but was instead a representation of the priority each teacher gave to teaching each topic.

Once the survey was complete and submitted, a teacher had completed their responsibilities for the project. As teachers submitted their surveys, emails were sent thanking them for their service, and their gift card was mailed to them at their school address. Any questions teachers submitted at or after that point were answered, otherwise teachers will not be contacted again until they are sent the final report, after the completion of the study.

Data Coding

Pre-calculus was chosen as the subject of this research for several reasons, including the researcher's personal experience with the subject. More importantly, the subject of pre-calculus is broad and varied in its definition. Different teachers have different impressions of what is essential for preparation for higher mathematics, and in a setting with as much curricular flexibility as the Archdiocese of Washington, this makes pre-calculus an interesting curriculum choice. Specifically, this research was designed to identify which topics each teacher used in their own definition of pre-calculus and how that definition was reflected in their teaching and on their assessments.

Pre-Calculus Topics. In order to identify these topics, several pre-calculus curriculum materials, textbooks, and the CCSS-M were referenced to build a comprehensive list of pre-calculus topics. This list, which is provided in Appendix E, contains 37 topics in 5 distinct categories: advanced algebra, functions and modeling, trigonometry, statistics and probability, and advanced topics (such as limits, matrices, and polar coordinates). The topics are defined broadly enough to contain multiple lessons and related ideas, but are defined narrowly enough to avoid being vague or too general. For instance, one topic in the functions and modeling category is "Composition, Transformations, and Inverse Functions." These are three individual topics, and each can be the subject of several lessons, but the three are clearly connected and interdependent, as inverse functions are defined via composition and demonstrated graphically via reflection, a type of function transformation.

The Common Core State Standards. In addition to defining these 37 topics, each standard within the CCSS-M high school standards has been assigned to one and only one pre-calculus topic. Many topics have several CCSS-M objectives associated with them, while some, such as limits, are not connected to any CCSS-M objectives. This ensures that these 37 topics are comprehensive enough not only to include every topic the CCSS-M would require for a high school mathematics class, but also to include additional topics that teachers may elect to include.

Coding Scope

Coding Scope from Survey Data. When answering questions on the survey regarding scope, teachers answered using one of five answer choices: "I do not teach this topic," "Very little: 1-2 days," "Some time: 2-4 days," "Quite a bit: 4-7 days," and "A great deal: 7+ days." Each of these choices was assigned a representative value score, as denoted in Table 1 below:

Table 1: Representative Value Score Coding for Teacher Scope Responses

Teacher Response	Representative Value Score
I do not teach this topic	0 days
Very little: 1-2 days	1.5 days
Some time: 2-4 days	3 days
Quite a bit: 4-7 days	5.5 days
A great deal: 7+ days	8.5 days

After each score was translated to its representative value, each teacher's responses were totalled for a combined number of "days" spent teaching pre-calculus. Each topic's response was then scored as a percentage of this total curriculum value. This percentage was the teacher's curriculum's score for the scope of each topic.

Coding Scope from Examination Data. Scope was determined from the teachers' midterm and final examinations through an analysis of the point values associated with each problem. Each teacher's examinations were tallied up in order to identify the total points awarded by the two examinations combined. Then, each problem, itself assigned a point value, was identified as a percentage of the total point value of the combined examinations. Finally, each problem was also identified as assessing one of the 37 topics of pre-calculus. The total of these scores provided data as to what percentage of assessed curriculum each teacher assigned to each of the 37 pre-calculus topics.

Coding Performance Expectations

Coding Expectations from Survey Data. When answering questions on the survey regarding performance expectations, teachers answered using one of five answer choices: "All of my students can answer this", "Most students, perhaps not my lowest, can answer this", "Solid students can answer this", and "Only my strongest students can answer this" and "I do not teach this topic." Each of these choices was assigned a representative value score, as denoted in Table 2 below:

Table 2: Representative Value Score Coding for Teacher Performance Expectation Responses

Teacher Response	Representative Value Score
All students	4 points
Most students	3 points
Solid students	2 points
Strongest students	1 point
I do not teach this topic	0 points

After each score was translated to its representative value, each teacher's responses were totalled for a combined number of "points" regarding the expectations within their enacted curriculum. The points were again tallied to identify a total number of points for the entire curriculum, and each topic was scored as a percentage of these total points.

Coding Expectations from Examination Data. In addition, the difficulty of every question on each exam was coded using the same coding mechanism asked of the teachers during the survey, i.e. the performance level of student who should be capable of answering each question correctly was identified. As opposed to the expectations survey, which was designed to determine information about the teacher using a single constant question and asking for the differences in teachers' opinions, this measure of expectations within examinations kept the coding mechanism (i.e. the evaluation of the difficulty of the question) constant, while varying the questions being asked by the teachers.

To achieve a total performance expectations score for each topic, a weighted average score was found from the questions previously coded as assessing each of the 37 topics, weighted by each question's point value as used to determine time on topic. In this way, the difficulty of one rich, complex problem was valued appropriately against a potential abundance of easy, low point value skill problems. Once all data had been collected, each teacher was associated with 148 scores, four for each of the 37 topics: survey scope, survey expectations, examination scope, and examination expectations. Each of these 148 scores was represented as a percentage of the total curriculum, with the 37 scores within each category (survey scope, examination scope, survey expectations, and examination expectations) individually totalling 100%. The intention of this research to analyze these 148 scores in relation to themselves and in relation to other teachers' scores in order to answer the three research questions. However, for statistical power, for much of the analysis, the 37 topics were condensed into the five topic categories, as exemplified in Appendix E. For this reason, each teacher's score matrix for much of the analysis contained 20 scores.

Analysis

The three research questions asked by this research are as follows:

1. How similar or different are the various teacher-built pre-calculus curricula of the Archdiocese of Washington?
    a. What is the topic-specific variance among the various teacher-built pre-calculus curricula of the Archdiocese of Washington?

b. Do the similarities and differences of the various teacher-built pre-calculus curricula of the Archdiocese of Washington reflect several prototypical curricular approaches, or is each curriculum independent and unique?

2. What is the level of agreement between the stated curricula and the tested curricula in the pre-calculus classes in the Archdiocese of Washington?

3. How closely does each teacher's perception of their enacted curriculum align to the objectives and standards of the Common Core State Standards for Mathematics?

The approaches used by this research to answer these questions are summarized in Table 3 below:

Table 3: Analysis Approaches by Research Question

Research Question	Analysis
1a	Convert data to z-scores, analyze variance by topic
1b	Perform a visual quantitative analysis of variance data from 1a
2	Compute the Porter Alignment Index for each curriculum's enacted scope vs. assessed scope, enacted expectations vs. assessed expectations, and overall enacted vs. overall assessed
3	Designate each topic as "taught" for each teacher's curriculum. Compare this list of taught topics to the 33 topics identified within the CCSS-M

The first research question addressed the degree of curriculum similarity or difference across the participating teachers. The second question had two components, the general alignment of the enacted curriculum to the assessed curriculum, and the specific points at which these curricula are the same or different. Each of these components was addressed in different ways. The third question was a direct comparison of each teacher's perceptions of their enacted curriculum to the content standards of the CCSS-M. This required a proprietary definition to allow this alignment to be measured despite the differences in the types of data.

Research Question #1: Intercurricular Variance Among Teachers

To address the first question, each teacher's 20 data points, as defined by the five category scores from each of the four categories of data (enacted scope, enacted expectations, assessed scope, and assessed expectations) were represented as *z*-scores (using the μ and σ for each of the four data sources individually). The four scores for each teacher by category were then summed. At this point, each teacher had five topic category scores. To identify which topics were taught most and least consistently, the variance of each topic's *z*-scores across teachers was computed. High values of σ^2 indicate a wide disagreement among teachers regarding the priority given to a topic, while low values indicate a broad consensus about the relevance of a topic to pre-calculus.

These data were displayed visually, allowing conclusions determining topics with high and low agreement. A visual analysis of these data permitted consideration of broad typographical patterns describing curricula. Specifically, each teacher's curriculum was represented as a bar graph with five values, each representing the percentage of curriculum dedicated to each topic category, as determined by the combined *z*-score described above. Teachers who designed similar curricula had curriculum display graphs with similar contours, emphasizing some common topics while ignoring others.

Research Question #2: Stated vs. Assessed Curriculum

Andrew Porter's Alignment Index (2002, 2011) permits computation of the degree of alignment of two different data matrices populated by percentage data:

$$Alignment = 1 - \tfrac{1}{n}\sum_{i=1}^{n}\left|X_i - Y_i\right|$$

where each alignment score would range from 0.0 to 1.0 and would identify 1.0 as perfect alignment. Porter's Alignment Index is a well-regarded statistical construct that has been used to define and interpret alignment between curricula, standards, and assessments (Fulmer, 2011). Since, before combining topics by topic category, each teacher was associated with 148 different scores, the grain size of the analysis was reduced by collapsing the 37 topics into five broad categories, as demonstrated within the survey provided in Appendix D and as explained with the Topic Alignment Key in Appendix E. This reduced the number of total matrix values to 20, which increased the feasibility of using Porter's Alignment Index to detect differences in approach. This Index was used in three comparisons: survey time on topic vs. examination time on topic, survey performance expectations vs. examination performance expectations, and overall stated curriculum vs. overall assessed curriculum.

Note that while the first and second alignment indices, since the data are similar, could have been accomplished with the raw percentage data, the third alignment required a scaling to accommodate adding the percentage values. This was done by dividing each teacher's combined scores by a constant value to maintain the total percentage value of 100%. The second research question was answered by these alignment index values, primarily in a descriptive fashion. Porter (2011) has described 0.30 as a "moderate" (p. 105) alignment, so this was the primary context for the alignment index data.

In addition to this low-level, threshold analysis of Porter's Alignment Index, the researcher also developed a Generalized Linear Model (GLM) analysis of the Alignment Index similar to the work of Fulmer & Polikoff (2014). The GLM approach allowed for an in-depth analysis of the interaction effects present in the alignment of the two matrices, while also accounting for the assumptions present in the data, such as that the percentage data cannot be assumed to be normal.

In order to accomplish this analysis, the data was organized into a matrix such that each teacher was assigned 20 percentages, each representing the percent of his or her curriculum associated with either stated or assessed curriculum, either scope or performance expectations, and each of the five curricular categories: Advanced Algebra, Functions and Modeling, Trigonometry, Statistics and Probability, and Advanced Topics. These percentages were the dependent variable in the GLM analysis, while "source" (survey or examinations), "type" (scope or performance expectations), and "category" were categorical predictor variables.

Two issues present themselves that prevent a traditional OLS regression analysis of these data. First, the variables cannot be assumed to be normal, and a low sample size ($n<30$) does not overcome this requirement for robustness to the normality assumption. Second, since the output variable is percentage data, a significant floor effect is present due to the relative proximity of much of the data to the minimum value of 0%. As a result, the most appropriate distribution for this data was the Poisson log-linear distribution, which can only be analyzed within the context of a Generalized Linear Model.

The objective of this analysis is to determine the parameter coefficients associated with both the "source" variable and the "type" variable. If these coefficients are significant, it would imply that there is a difference between teachers' stated and assessed curriculum, or a difference between the way teachers' curriculum approaches scope and performance. If neither of these coefficients are found to be statistically significant, it would then be evidence that teachers' curricula are adequately aligned.

Research Question #3: Comparison to the Common Core

In order to compare each teacher's enacted curriculum to the CCSS-M, a coding mechanism was created that provided a binary approach to each topic, labeling it as "taught" or "not taught" alone. In this system, a topic was "taught" if the following three conditions were met: First, the teacher must have indicated on the survey that he or she taught the topic for at least one day. Second, the teacher must have indicated that the problem presented in the survey to assess the performance expectations of the enacted curriculum could be answered correctly by at least his or her strongest students. And finally, the teacher must have included at least one problem from the topic on his or her midterm or final examination. If all three of these conditions were met, then the teacher was judged to have taught the topic in question.

Note that of the 37 topics, only 33 topics were associated with content standards from the CCSS-M. Therefore, an enacted curriculum that is in perfect alignment with the Common Core would teach these 33 topics, no more and no less. This perfectly aligned curriculum would be assigned a score of zero, and every topic that does not match, either not teaching one of the 33 topics that the Common Core recommends or including one of the four topics not described by the Common Core, increases the score by one. Therefore, the curriculum that is least aligned with the Common Core, teaching none of the topics recommended and all four of the topics ignored, would score 37 on this scale.

In addition, 7 of the 33 aforementioned topics are populated exclusively with honors-cited Common Core standards. These seven topics, dealing with ideas such as logarithms, vectors, and advanced trigonometry, could be considered as "optional," even in the Common Core. Therefore, the researcher also analyzed each teacher's curriculum against the "standard" Common Core curriculum, which consisted of 26 of the 37 topics tested, using the same process as described above.

Once each teacher's enacted curriculum was compared to the CCSS-M, the various alignment scores were compared to each other, and conclusions were drawn based on the average alignment across teachers.

The data, collected in two meaningful and distinct ways, were strong and stable. This research had been structured to take advantage of the data collected and to transform it into a meaningful statistical analysis, from which conclusions were drawn about the nature of pre-calculus curricula in the Archdiocese of Washington.

Chapter 4: Analysis

This chapter presents the results of an analysis of the data that were collected in an effort to address the three research questions of this thesis study. After reporting descriptive statistics, this chapter provides information characterizing the variability of teachers' curricular choices. Further, this chapter summarizes the nature of a series of statistical analyses that were performed along with the quantitative findings.

Data Sources

There were two major sources of data: surveys of teachers' impressions of their own curricula and summative assessments provided by these same teachers. The survey was hosted on the online survey website Qualtrics through its agreement with the University of Maryland. This ensured not only that the survey was secure and that the data collected remained confidential, but also that the survey was broadly accessible to any teacher who was invited to participate. The examinations were collected via direct upload to the survey and were each coded individually by the researcher. These data were then unified and analyzed along several dimensions.

Curricula surveys were completed by 13 pre-calculus teachers in the Archdiocese of Washington. There are 20 high schools in the Archdiocese of Washington, with no high school having more than 3 pre-calculus teachers. Many of the smaller schools in the Archdiocese employ only one or two pre-calculus teachers. As a result, the population being studied is approximately 50 teachers. This sample of 13 teachers is approximately 26% of the population.

Survey Data Collection

The survey consisted of 80 questions, 74 of which were from the main body of the survey, two questions each on 37 topics defined as pre-calculus topics by the researcher. Before reaching these main questions, subjects were asked four questions regarding their personal information. Then, after completing the main body of the survey, subjects were asked two questions about the time and comprehensiveness of the survey itself. On these two final questions, all 13 respondents answered that the survey had taken them less than an hour to complete, and each respondent answered that the topics presented in the survey were comprehensive.

Demographic Descriptive Statistics. As reflected in Table 4, the 13 subjects were a group that demonstrated the gender and age diversity of the teaching corps within the Archdiocese of Washington. However, the sample was not especially diverse as 10 of the respondents are White/Caucasian, while two respondents are Black/African-American and one is Hispanic/Latino:

Table 4: Racial Distribution of Subjects

	Black/ African-American	Asian/ Asian-American	Hispanic/Latino	White/ Caucasian	Other
Subjects	2	0	1	10	0

Scope and Performance Expectation Descriptive Statistics. The survey questions addressed the participating teachers' choices in curricular scope and in performance expectations. For each topic, the survey asked one question regarding scope (i.e. How many class days are spent teaching this topic?) and one question regarding performance expectations (i.e. What level of student would be able to answer this question?). Each question's answer was recorded on a 1-5 Likert-format scale, with high values for scope questions signifying a large amount of time spent, and low values for performance expectations signifying that most students were expected to be able to answer a given problem on that topic.

The 37 topics were broken into five broad categories, as described in Appendix D, representing the five major topics found in most pre-calculus course curricula. The mean score among all subjects for each category is in Table 5.

Table 5: Mean (Standard Deviation) Survey Scope and Performance Expectation Scores by Category

Topic Category	Scope	Performance Expectation
Advanced Algebra	2.45 (1.027)	2.60 (1.439)
Functions and Modeling	2.90 (1.111)	2.69 (1.215)
Trigonometry	3.22 (1.169)	2.47 (1.336)
Statistics and Probability	1.43 (0.728)	4.17 (1.364)
Advanced Topics	2.11 (1.253)	3.55 (1.521)

These scope data indicate that, for example, trigonometry is a shared emphasis across these teachers' pre-calculus curricula, but that statistics and probability are of generally lower priority. The performance expectation data indicate a different set of priorities, as teachers asked, in general, the easiest questions on the topic of trigonometry, but the hardest questions on the topic of statistics. However, these data are misleading because this characterization does not take into account the length of each teacher's total curriculum. The data presented a much clearer picture after they had been adjusted to account for this difference in teachers' responses.

Teachers' scope and performance expectation data by topic requires comparison to other topics and to other teachers. Teaching a topic for 5 days reflects a different proportional emphasis if the course is expected to be 90, 160, or 180 total class days. Therefore, these data were converted from raw data scores to values representing each score as a percentage of the teacher's overall curriculum. In this way, each teacher's curricula could be compared to other teachers' curricula, as well as to each teacher's own assessed curriculum as expressed in the submitted examinations.

Survey Data Coding

In order to accomplish this, each teacher's responses were totalled to determine the overall expectations for the year. Then, each subject's topic scores were divided by this total, resulting in a percentage value for each score. This value represented the proportion of the curriculum dedicated to that topic. These values across each mathematical category are listed in Table 6.

Table 6: Mean (Standard Deviation) Percent of Survey Scope and Performance Expectation Scores by Category

Topic Category	Scope	Performance Expectation
Advanced Algebra	26.1% (0.092)	34.1% (0.114)
Functions and Modeling	29.4% (0.108)	25.4% (0.054)
Trigonometry	25.2% (0.102)	19.9% (0.066)
Statistics and Probability	3.7% (0.032)	5.6% (0.042)
Advanced Topics	15.7% (0.104)	14.9% (0.062)

When proportional duration of topic focus was evaluated, trigonometry is no longer the highest-scoring category, falling to third in both measures. Instead, these data express that pre-calculus curricula in the Archdiocese of Washington focus primarily on functions, advanced algebra, and trigonometry at approximately equal levels. But these data, though clearer, may also be misleading, since they only take into account the teachers' stated curricula. Comparing these data to the data collected from teachers' examinations provides meaningful and relevant context.

Examination Data Collection

Teachers were asked to submit their pre-calculus summative assessments. These were primarily midterm and final examinations, but if one or both was unavailable, chapter tests were substituted. Since each exam was written by the teachers themselves, the data from these summative assessments presented a clearer and more complete picture of each teacher's chosen curriculum. The topics tested on these examinations were often slightly different than the topics discussed on the survey, either in scope or in performance expectation.

In the process of collecting these examinations, it became evident that there were two major assessment types: long, multiple-choice tests and shorter, free-response tests. 23% of the teachers surveyed presented multiple-choice examinations in excess of 40 questions each. These tests largely had less rigorous questions by design which would allow students to complete the exam in a reasonable time. In contrast, most examinations were free-response in nature, with much more complex, but fewer, questions. No grading rubrics were provided, but teachers did provide point scales to assign relative value and priority to each question.

Another dichotomy in the assessments collected was in whether they were summative examinations. One teacher administered a final examination, but no midterm, and another teacher gave a midterm examination, but no final exam. One other teacher administered neither a midterm nor a final examination. In an effort to extract information regarding these teachers' assessment of skills and concepts, they were asked to provide chapter tests for the period of time in question. To maintain fairness, the point values for the chapter tests were scaled so that they were, combined, equal in value to a final examination.

Examination Data Coding

Once collected, the examination data needed to be transformed into data comparable to the collected data from the survey. This required the researcher to categorize each question from each examination as adhering to one of the 37 pre-calculus topics, and then to assess the problem's difficulty along the same scale used by the subjects in the survey.

The process of categorizing each question within the 37 pre-calculus topics was more difficult than anticipated. Several questions measured multiple skills and could have been accurately placed in two or more topics. Other questions assessed a skill that was not explicitly described in the 37 topics, most notably a topic that could be named "inverse trigonometric functions and solving trigonometric equations." Since most of the data would eventually be compressed into the 5 category-level bands, it was the goal of the researcher to ensure that every question was accurately placed into the correct category, while placing questions into topics as accurately as possible with the considerations above.

Once questions had been assigned topics, each question also needed to be assigned a performance expectation level, similar to the levels of performance expectation asked of subjects in the survey. In practice, this amounted to an assessment by the researcher of the difficulty of each problem. To do so, the researcher considered the nature of the topic being assessed, the cognitive demand of the problem (e.g. analysis was considered to be a higher expectation than memorization), and the skill difficulty built into the problem by the teacher. Each problem was then assigned a value between one and four, with one representing the lowest performance expectation and four representing the highest performance expectation.

Each question was assigned a point value by the teacher, essentially giving each question a relative priority among the assessment topics. Note that for multiple-choice examinations, each question was assigned a point value of 1, since each question was equal in value. To transform the coded examination data into a format comparable to that of the survey data, each teacher's examination points were totalled, and each question's point value was found as a percentage of that total. For example, if a teacher assigned 30 midterm exam questions worth a total of 120 points and 35 final exam questions worth a total of 130 points, then her overall assessed curriculum would be worth 250 points. In that example, a 5-point question would be scored as 2% of her overall assessed curriculum.

In this way, a teacher's assessed curriculum can be converted to percentage data, which were then comparable to that same teacher's survey data measuring stated curriculum. Scope data was computed simply as a total percentage dedicated to each category. Performance expectation scores were assigned percentage data in the same way using a weighted mean based on each question's point value. These percentage values are noted in Table 7.

Table 7: Mean (Standard Deviation) Percent of Examination Scope and Performance Expectation Scores by Category

Topic Category	Scope	Performance Expectation
Advanced Algebra	25.0% (0.124)	26.4% (0.259)
Functions and Modeling	31.2% (0.168)	28.7% (0.251)
Trigonometry	25.6% (0.162)	25.5% (0.279)
Statistics and Probability	2.1% (0.020)	2.1% (0.033)
Advanced Topics	16.1% (0.098)	17.3% (0.179)

Notice here that the scope and performance expectation values for each category are remarkably similar, with differences between the percentages of each no more than 2.5%, as well as two categories exhibiting differences of no more than 0.1%. Contrast this to the same data reported on teachers' stated curricula, where three categories exhibited differences of 4.0% or higher. This could imply that teachers in the Archdiocese of Washington are extremely effective at designing assessments that match both scope and difficulty well, or at least that these teachers are more effective at matching scope to difficulty in assessments than in their stated curricula. This relationship is of particular interest in the analysis of Research Question #2.

These data describe the emphasis and priorities of these pre-calculus teachers' curricula. This information is interesting as the Archdiocesan schools search for and define the nature of pre-calculus as a course. However, the purpose of this study was not to define pre-calculus as the crowd-sourced consensus of various teachers' curricula but to study the variability in these teachers decisions, informed by their individual definitions of pre-calculus. The analysis of these data necessary to answer those questions follows.

Inter-Teacher Variance

The first research question addressed by this study was: "How similar or different are the various teacher-built pre-calculus curricula of the Archdiocese of Washington?" In the Archdiocese of Washington, no school is required to follow any standards or curricula set by either the state, the district, or the Archdiocese itself. As a result, schools, and often teachers, construct their own conceptions of the nature of classes such as pre-calculus. This question sought to discover how different those conceptions were from each other.

Topic Variance

The four types of data collected - survey scope, survey performance expectations, examination scope, and examination performance expectations - were each transformed into percentage data. This allowed for accurate comparison of the data from each source and type, but it did not provide an intuitive justification for combining the data as needed to provide individual scores for teachers on various dimensions. In order to provide that justification, each score needed to be represented as a *z*-score. In this way, teacher scores, either combined by data source or by type (i.e. scope or performance expectation) were comparable to each other.

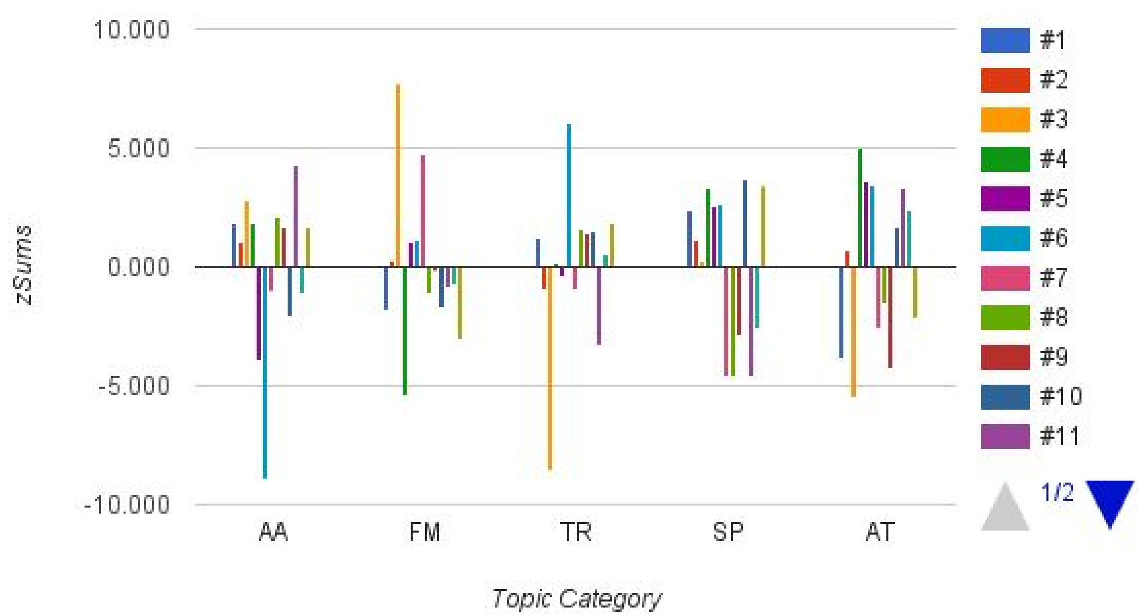
Standardized scope and performance expectation *z*-scores were computed for the survey and examination data separately by category. The resulting 20 means and standard deviations are shown in Table 8.

Table 8: Mean (Standard Deviation) of Topic Categories by Data Collection Method and Type

Topic Category	Survey		Examinations	
	Scope	Performance Expecations	Scope	Performance Expectations
Advanced Algebra	0.261 (0.092)	0.341 (0.114)	0.250 (0.124)	0.617 (0.357)
Functions and Modeling	0.294 (0.108)	0.254 (0.054)	0.312 (0.168)	0.672 (0.346)
Trigonometry	0.252 (0.102)	0.199 (0.066)	0.256 (0.162)	0.596 (0.384)
Statistics and Probability	0.037 (0.032)	0.056 (0.042)	0.021 (0.042)	0.048 (0.045)
Advanced Topics	0.157 (0.104)	0.149 (0.062)	0.149 (0.062)	0.405 (0.246)

Each teacher's scores were then converted to *z*-scores using these values. Each teacher therefore had 20 individual *z*-scores, four for each mathematics topic category based on data source (survey or submitted examinations) and type (scope or performance expectations). A *z*-sum was then found for each topic category by adding together the four *z*-scores. Displayed in Figure 1, these *z*-sums can be interpreted as the total curricular attention paid to each category by each teacher.

Figure 1: Teacher z-Sums by Topic Category



Notice the extreme variability and distinct lack of pattern in this data. Priorities of various teachers are shown to be distinct and unique regarding which categories they consider to be important. To quantify this variability, the variance across these topics was used to measure the degree of similarity of teachers' priorities about the content of pre-calculus. Table 9 lists the variance values for the *z*-sums within each category among the 13 teachers.

Table 9: Topic Variance by Topic Category

	Advanced Algebra	Functions and Modeling	Trigonometry	Statistics and Probability	Advanced Topics
Variance	12.16	12.34	11.04	11.33	11.30

These variance levels exhibit only a marginal increase in variability for the algebra-heavy topics from the advanced topics. However, this top-level analysis of the data is obfuscated by the collection of various teachers with various priorities. A further compression of the data reveals an intuitive split of the subjects into two categories: Those who emphasize algebra and function, and those who emphasize trigonometry, statistics, and advanced topics. The first group has been labeled as the "Algebra III" group and the second as the "New Topics" group. One teacher has been excluded from both groups because the data from that teacher fit neither typography.

Figure 2 depicts the *z*-sum data for the Algebra III group, and Figure 3 depicts the z-sum data for the New Topics group:

Figure 2: z-Sums of Teachers in the Algebra III group

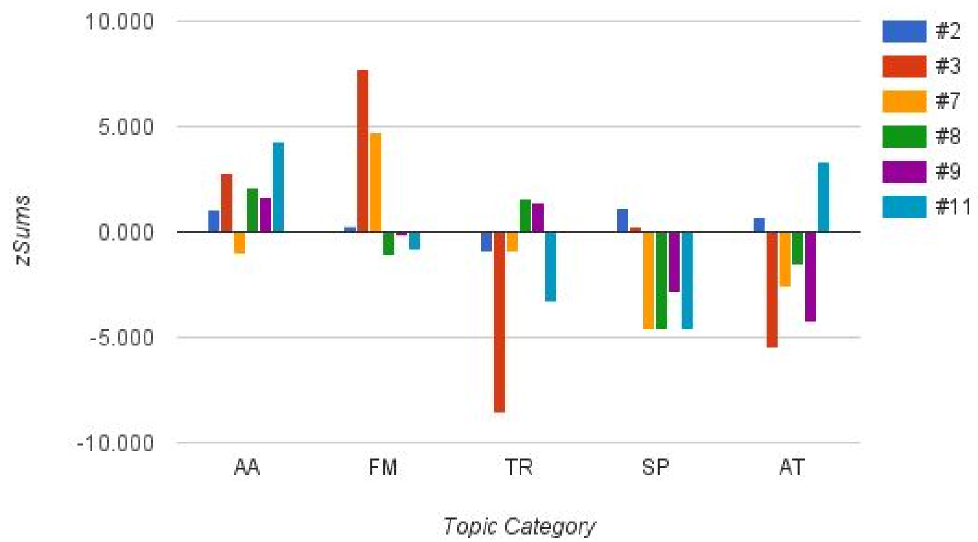
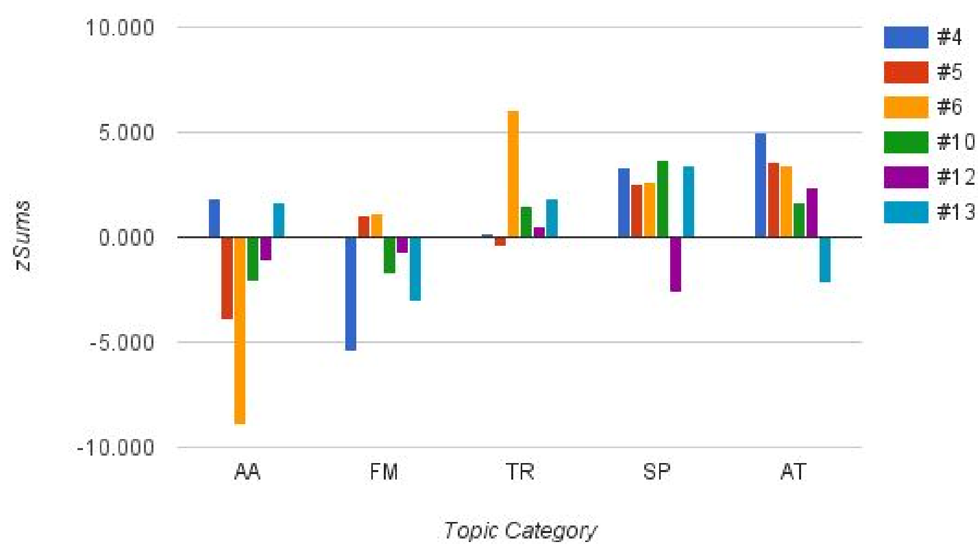


Figure 3: z-Sums of Teachers in the New Topics Group



By splitting these data into two figures, a more clear and distinct pattern emerges. Algebra III teachers emphasize Advanced Algebra and Functions and Modeling and give a lower priority to Trigonometry, Statistics and Probability, and Advanced Topics. In contrast, New Topics teachers privilege Trigonometry, Statistics and Probability, and Advanced Topics over Advanced Algebra and Functions and Modeling.

It is important to note that this study is not designed to make value judgments on the effectiveness of one curricular archetype in comparison to the other. Teachers in each archetype designed their curriculum to match their students and their own personal philosophies of pre-calculus. Since this study has not collected any evidence of student achievement, this research can not make any claims addressing the relative effectiveness of any curriculum type.

Combined Topic Variance

Figures 2 and 3 grouped teachers based on the relative priority they gave either to Advanced Algebra and Functions and Modeling or to Trigonometry, Statistics and Probability and Advanced Topics, Figures 4 and 5 demonstrate an even clearer relationship after both the Advanced Algebra and Functions and Modeling topic categories are combined and the Trigonometry, Statistics and Probability, and Advanced Topics topic categories are combined:

Figure 4: Combined z-Sums of Teachers in the Algebra III Group

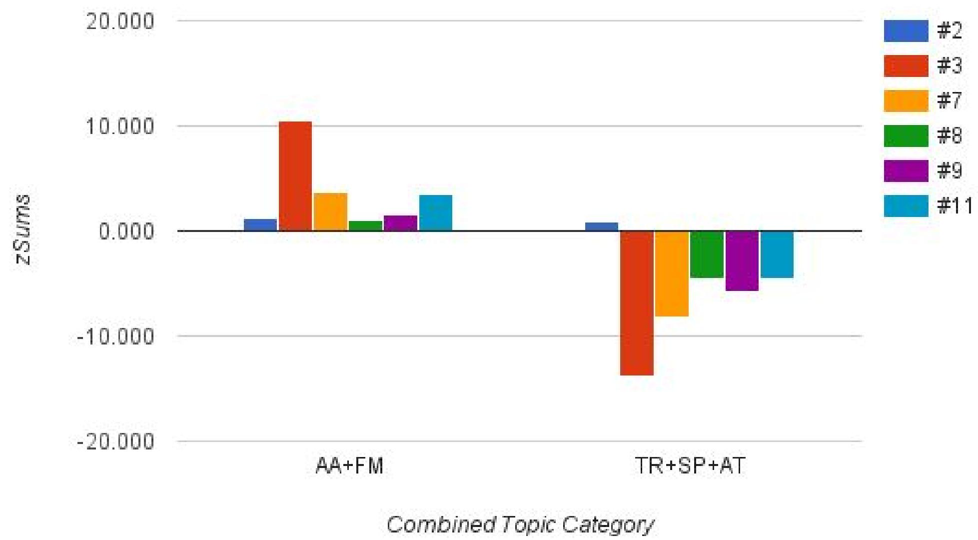


Figure 5: Combined z-Sums of Teachers in the New Topics Group

These data imply that there is fairly strong agreement between teachers on the role of pre-calculus within two distinct schools of thought. One group of teachers contends that the purpose of pre-calculus is to solidify and expand students' algebraic skills learned in Algebra II, while the other group believes that the role of pre-calculus is to expose students to new topics not previously presented. Again, the role of this research is not to present value judgements on which approach is preferable for students. Indeed, teachers are making these decisions with the varied needs of their individual students in mind. It is, however, valuable and interesting to note the typography of pre-calculus as split into two fairly robust definitions of the course based on two distinct goals.

The Relationship Between Stated and Assessed Curriculum

The second research question addressed by this research was: "What is the level of agreement between the stated curricula and the tested curricula in the pre-calculus classes in the Archdiocese of Washington?" One aspect of being independent of a state or district in terms of curriculum is that the individual teachers - those who created the curriculum - were the ones to design the tests their students completed. These tests also comprise an element of the teachers' curricula and have been included in this analysis as equivalent to the value of the survey in determining each teacher's curricular priorities. However, this analysis has so far ignored the potential differences in the priorities of a participating teacher's stated, in-classroom curriculum and the one reflected by his or her summative assessments.

To test the alignment of teacher's stated curricula to their assessed curricula, the data were analyzed using Andrew Porter's Alignment Index (2002, 2011) for the alignment of two matrices populated by percentage data:

$$Alignment = 1 - \frac{1}{n}\sum_{i=1}^{n}\left|X_i - Y_i\right|$$

In this study, this index was defined as a measure of the relationship between teachers' stated and assessed curricula on three dimensions: survey scope vs. examination scope, survey performance expectations vs. examination performance expectations, and overall stated curriculum vs. overall assessed curriculum.

Each teacher's matrix was comprised of 20 values, one for each category over both data sources and both curriculum types. This was populated with percentage data. For example, Teacher #1's curriculum matrix is listed in Table 10.

Table 10: Collected Percentage Data Example, Teacher #1

Topic Category	Survey		Examinations	
	Scope	Performance Expecations	Scope	Performance Expectations
Advanced Algebra	0.353	0.405	0.290	0.301
Functions and Modeling	0.059	0.041	0.120	0.117
Trigonometry	0.235	0.270	0.210	0.168
Statistics and Probability	0.059	0.095	0.030	0.031
Advanced Topics	0.294	0.189	0.350	0.383

These percentages represent the proportion of this teacher's curriculum dedicated to each topic. In this example, Teacher #1 reported on the survey that he or she dedicated about 35.3% of his or her curriculum time to teaching advanced algebra, but that only approximately 29% of his or her examination was on the same topic.

The means and standard deviations of the data for all three Alignment Indices are listed in Table 11. Alignment Index #1 corresponds to the relationship between survey scope and examination scope, Alignment Index #2 corresponds to the relationship between survey performance expectations and examination performance expectations, and Alignment Index #3 corresponds to the relationship between overall stated curriculum and overall assessed curriculum.

Table 11: Porter Alignment Index Means and Standard Deviations

	Alignment Index #1	Alignment Index #2	Alignment Index #3
Mean (S.D.)	0.933 (0.039)	0.914 (0.036)	0.856 (0.069)

The first reaction to these data is that these values are extraordinarily high. In his 2011 work, Porter described a "moderate" alignment as 0.30 (p. 105), but this was in a work describing the relationship between different states' mathematics standards compared against each other and against the Common Core. Here, the teachers of these curricula are writing their own summative assessments, so the high level of alignment makes intuitive sense.

The minimum value in each of the first two alignment indices is approximately 0.845, with the mean values at 0.933 and 0.915, respectively. This implies that while teachers are slightly less proficient at aligning the performance expectations of their examinations with those of their stated curricula than they are at aligning the scope of their examinations with that of their stated curricula, the difference is small and the overall success is high.

Tellingly, the lowest Porter Index mean is for Alignment Index #3, comparing the overall stated curriculum to the overall assessed curriculum for each teacher. This suggests that teachers are quite proficient at writing tests that match both the scope of topics taught with the value of those topics on the examination, and that teachers are also skilled in matching their expectations for students in class with their expectations for students on examinations, but that teachers are less proficient at matching the scope of their curriculum with their expectations. For example, many teachers may have assigned a relatively high value to success on low-level trigonometric skills, such as evaluating ratios such as $cos\,\frac{\pi}{3}$, but may have attributed fewer points to more difficult skills, such as solving the equation $2\,cos\,x - 1 = 0$. This may cause a disparity between the scope of a teacher's curriculum and the performance expectations of the same topics.

Importantly, even these alignments are exceptionally high. It seems clear after this analysis that teachers are effective and proficient at matching their own stated curricula with summative assessments. To confirm this analysis, the researcher conducted a Generalized Linear Model analysis of the data, as described in Chapter 3.

In order to perform this analysis, each teacher was assigned a matrix of 20 percentage values, each associated with the 20 possible combinations of curriculum "source" (survey or examination), "type" (scope or performance expectation), or "category" (Advanced Algebra, Functions and Modeling, Trigonometry, Statistics and Probability, and

Advanced Topics). Because these percentages are subject to a significant floor effect, and because the low sample size (n=13) failed to achieve the minimum threshold for robustness to the normality assumption, a Generalized Linear Model, using a Poisson log-linear distribution, was the appropriate statistical tool.

The first model tested, Model Run #1, was as follows:

$$Y = \beta_0 + \beta_1 \text{*Source} + \beta_2 \text{*Type} + \varepsilon$$

This model attributed all of the variation in percentage to the source of the data, the type of data, and residual error. The results of this model, shown below in Table 12, demonstrate that neither the source of the data nor the type of data was significant, which matches the conclusions of the Porter Alignment Index analysis, in that both analyses find teachers' stated and assessed curricula and teachers' scope and performance expectations to be adequately aligned.

Table 12: Parameter Estimates for Model Run #1

Parameter Estimates

			95% Wald Confidence Interval		Hypothesis Test		
Parameter	B	Std. Error	Lower	Upper	Wald Chi-Square	df	Sig.
(Intercept)	1.391	.5592	.295	2.487	6.192	1	.013
[Source=0]	-.574	.7247	-1.994	.847	.627	1	.429
[Source=1]	0[a]	.	.	.	.	.	.
[Type=0]	.910	.7090	-.479	2.300	1.648	1	.199
[Type=1]	0[a]	.	.	.	.	.	.
(Scale)	1[b]						

Dependent Variable: Percentage
Model: (Intercept), Source, Type

a. Set to zero because this parameter is redundant.

b. Fixed at the displayed value.

However, the model used in this statistical test was far from perfect. Much of the variation was unexplained and the model was not a particularly tight fit, as demonstrated in Table 13.

Table 13: Goodness of Fit for Model Run #1

Goodness of Fit[a]

	Value	df	Value/df
Deviance	608.798	30	20.293
Scaled Deviance	608.798	30	
Pearson Chi-Square	896.746	30	29.892
Scaled Pearson Chi-Square	896.746	30	
Log Likelihood[b]	-325.378		
Akaike's Information Criterion (AIC)	656.756		
Finite Sample Corrected AIC (AICC)	657.583		
Bayesian Information Criterion (BIC)	661.245		
Consistent AIC (CAIC)	664.245		

Dependent Variable: Percentage
Model: (Intercept), Source, Type

a. Information criteria are in small-is-better form.

b. The full log likelihood function is displayed and used in computing information criteria.

Instead, it made intuitive sense to include "Category" as a predictor variable, which would limit residual error and make the model more capable of detecting significance in the relevant variables of "Source" and "Type". As a result, Model Run #2 used the following model:

$$Y = \beta_0 + \beta_1\text{*Source} + \beta_2\text{*Type} + \beta_3\text{*Category} + \varepsilon$$

This model represented a significant improvement over Model Run #1, as demonstrated by Table 14.

Table 14: Goodness of Fit for Model Run #2

Goodness of Fit[a]

	Value	df	Value/df
Deviance	217.108	26	8.350
Scaled Deviance	217.108	26	
Pearson Chi-Square	218.182	26	8.392
Scaled Pearson Chi-Square	218.182	26	
Log Likelihood[b]	-129.533		
Akaike's Information Criterion (AIC)	273.065		
Finite Sample Corrected AIC (AICC)	277.545		
Bayesian Information Criterion (BIC)	283.541		
Consistent AIC (CAIC)	290.541		

Dependent Variable: Percentage
Model: (Intercept), Source, Type, Category

a. Information criteria are in small-is-better form.

b. The full log likelihood function is displayed and used in computing information criteria.

The dramatic improvement of the goodness of fit for this model implied that Model Run #2 was more likely to detect differences between teachers' stated and assessed curricula, and between teachers' approaches to scope and to performance. Indeed, this model achieved the best goodness of fit values for any model tested, except for models that excluded the essential variables of "Source" or "Type". In addition, all interaction effects between variables were tested, and none were found to be significant. As a result, Model Run #2 was the best model available and was the most likely to detect significance. As shown in Table 15, even this model found that "Source" and "Type" were not significant. Furthermore, the significance of two items within the "Category" predictor, Functions and

Modeling, and Statistics and Probability, supports the previous evidence that these items distinguish differences between the curricular choices of pre-calculus teachers.

Table 15: Parameter Estimates for Model Run #2

Parameter Estimates

Parameter	B	Std. Error	95% Wald Confidence Interval		Hypothesis Test		
			Lower	Upper	Wald Chi-Square	df	Sig.
(Intercept)	1.360	.7414	-.093	2.814	3.367	1	.067
[Source=0]	-.095	.7215	-1.509	1.319	.017	1	.895
[Source=1]	0[a]	.	.	.	.	.	.
[Type=0]	.719	.6517	-.558	1.996	1.217	1	.270
[Type=1]	0[a]	.	.	.	.	.	.
[Category=0]	.377	.8598	-1.309	2.062	.192	1	.661
[Category=1]	1.719	.4866	.765	2.672	12.476	1	.000
[Category=2]	.473	.7572	-1.011	1.957	.391	1	.532
[Category=3]	-3.609	1.0560	-5.678	-1.539	11.680	1	.001
[Category=4]	0[a]	.	.	.	.	.	.
(Scale)	1[b]						

Dependent Variable: Percentage
Model: (Intercept), Source, Type, Category

a. Set to zero because this parameter is redundant.

b. Fixed at the displayed value.

These results provide further evidence that teachers' stated and assessed curricula are closely aligned, and that these curricula provide a similar emphasis on topics covered and the performance expected on those topics. Coupled with the Porter Alignment Index analysis from above, this evidence becomes quite strong that teachers' curricula are adequately self-aligned.

Teachers' Curricula vs. the Common Core State Standards

The third research question addressed by this research was: "How closely does each teacher's perception of their enacted curriculum align to the objectives and standards of the Common Core State Standards for Mathematics?" Each of the reported comparisons thus far are intracurricular comparisons; whether comparing teachers' pre-calculus curricula to each other or comparing each teacher's own pre-calculus curriculum to itself, each comparison has been between teacher-designed, pre-calculus curricula of the Archdiocese of Washington. A more complete picture would include a comparison of these teacher-designed curricula to others from outside the Archdiocese. For this reason, these curricula were also compared to the Common Core State Standards for Mathematics.

Before the analysis, a few points of discussion are important regarding the Common Core. First, this study did not seek to determine the mathematical practices advocated by teachers in service of the content of their curriculum. Further, the Common Core identifies not only content standards but also practice standards. The Common Core is not, itself, a curriculum. No official Common Core examinations were developed when the state standards were created. The Common Core State Standards for Mathematics do not address expected instructional strategies nor do they clarify pacing/ordering expectations. For these reasons, this study only surveyed the content of each teacher's curriculum (both stated and assessed) and compared these to the content standards of the Common Core.

Each Common Core content standard for high school mathematics was represented by the 37 pre-calculus topics assessed in the survey, and the alignment of the Common Core standards to the 37 topics is shown in full in Appendix E. Of the 37 topics, 26 were represented by "regular" Common Core standards, an additional 7 were represented by "honors" standards, and 4 were not represented in the standards at all. If the goal of this research question is to assess whether or not teachers have chosen and built curricula that have them teaching Common Core topics, then, for that purpose, teaching one of the 26 topics represented by "regular" Common Core would imply that the teacher's curriculum taught those Common Core topics.

For this reason, this study considered a topic to be "taught" if it met three conditions. First, it needed to be included on the scope question of the survey as being taught at least for 1 day. Second, it needed to be included on the performance expectation question of the survey as a topic that at least the strongest students can answer. And third, it needed to be included as an understanding or expectation underlying at least one question on the teacher's summative assessments. Once the data had been coded in this way, a teacher's alignment score was calculated by adding together all of the topics taught that are included on the Common Core, and subtracting all of the topics taught that are not included on the Common Core. A perfectly aligned curriculum would score a zero on this scale. The means and standard deviations of this data is presented below.

Table 16: Common Core Score Means and Standard Deviations

	Regular Common Core Score	Honors Common Core Score
Mean (S.D.)	16.7 (2.93)	18.9 (3.38)

There are several striking results from these data, which are higher than the expectations given the model. Teacher #5, for example, scored a 22, which implied that his or her curriculum differed so significantly from the Common Core as to have a combined 22 topics either missing or added to the Common Core's standards. However, this does not imply that these teachers are choosing curricula that are inferior to or less demanding than the Common Core. The mean difference between a teacher's regular Common Core score and their honors Common Core score is 2.2 points, far below the actual difference of 7 topics between regular and honors Common Core scores. This implies that teachers are building curricula that are, relatively speaking, more comparable to the honors Common Core standards than the regular standards. A deeper analysis, as reflected in Table 17, brings this to light:

Table 17: Teacher Common Core Scores and Percentages

Category	Score	Percentage
Regular Common Core Topics Not Taught	12.92 / 26	49.7%
Regular Common Core Topics Taught	13.08 / 26	50.3%
Honors Common Core Topics Taught	2.38 / 7	34.0%
Non-Common Core Topics Taught	1.38 / 4	34.5%
Regular Common Core Alignment Score	16.69	
Honors Common Core Alignment Score	18.92	

The above are the mean values for each teacher in the study. While only about half of the regular Common Core topics were taught, over a third of the honors topics were taught, and over a third of the topics that were deemed too difficult for the Common Core (graphing reciprocal trigonometric functions, polar coordinates, parametric equations, and limits) were also taught. If teachers were building easy curricula, these advanced topics would be ignored.

Another revealing analysis breaks the pre-calculus topics down by category:

Table 18: Number and Percentage of Topics Taught by Topic Category

Topic Category	Topics Taught	Maximum Topics	Topic Percentage
Advanced Algebra	63	130	48.5%
Functions and Modeling	64	104	61.5%
Trigonometry	52	78	66.7%
Statistics and Probability	6	65	9.2%
Advanced Topics	34	104	32.7%

Table 18 demonstrates that the proportion of topics taught by the participating teachers is higher in the Functions and Modeling and the Trigonometry categories than in any other category, including those of Advanced Algebra. Taking into consideration the low proportion of teachers who included two advanced algebra topics in their curricula - no teachers taught graphing points in the complex plane and only 1/13 teachers taught factoring higher-order polynomials - the Advanced Algebra proportion would be even higher. Indeed, it is clear that the pre-calculus teachers in the Archdiocese of Washington are less fond of statistics and probability than is the Common Core. But this analysis provides evidence that a low correlation to the Common Core does not necessarily imply that the curriculum is inferior in difficulty.

These data collected are multi-faceted and say many things about the variety and similarity of teacher-built curricula in the Archdiocese of Washington. These analyses have been able to transform these data into evidence, and that evidence allows this study to draw several conclusions about the nature of teacher-built curricula and of teacher-built summative assessments. Those conclusions are presented in the subsequent chapter.

Chapter 5: Conclusions

This chapter interprets the analyzed data in the context of broader topics in an attempt to draw meaningful conclusions about the nature of teacher-built pre-calculus curriculum in the Archdiocese of Washington. Research questions are addressed and additional general conclusions are presented. With appropriate limitations, these conclusions may inform larger decisions made at state and district levels regarding other teachers in different situations. In addition, this chapter addresses the potential impact this study may have on the field of curriculum research and suggestions are presented for expanding this research in the future.

Inter-Teacher Variance

This research study addressed the question, “How similar or different are the various teacher-built pre-calculus curricula of the Archdiocese of Washington?” In an effort to answer this question, the study quantified each teacher’s curriculum in four distinct ways (stated scope, stated performance expectations, assessed scope, and assessed performance expectations) and measured the variability in teachers’ curricula for each measure based on their choices to teach topics in five categories. Those variance levels are presented in Table 9 in Chapter 4.

Advanced Algebra includes topics such as solving quadratic equations, radicals, complex numbers, and logarithms. The category Functions and Modeling includes domain and range, transformations, graphing various functions, and using functions to describe phenomena. Trigonometry ranges from right triangle trigonometry to the unit circle, oblique triangles, and identities. Within a pre-calculus course, the topic of Statistics and Probability discusses elementary and conditional probability, and expected value, as well as descriptive and inferential statistics. The pre-calculus category of Advanced Topics includes vectors and matrices, sequences and series, polar coordinates, and limits. The full list of potential pre-calculus topics is included in Appendix E.

In this study, nearly all teachers placed a significant amount of emphasis on advanced algebra, functions, and trigonometry topics, so the variances within these topic categories are not a result of some teachers including functions in their curricula and others not including functions at all. Rather for these topics the variance values indicate that some teachers gave a higher relative priority to topics such as functions than others did. Furthermore, these topics are not ranked based on difficulty. It is quite reasonable that a teacher could build a very difficult curriculum based on an in-depth exploration of the complex plane, logarithms, and rational functions.

Without a comparison to other, non-Archdiocesan curricula, it is difficult to label these as more or less variable than average. However, a visual analysis of the categories prioritized by each teacher shows no intuitive pattern. This lack of an obvious pattern implies that teachers in the Archdiocese of Washington are choosing widely diverse approaches to pre-calculus with limited correlation between their curricula. This interpretation dovetails with the understanding that the Archdiocese does not provide any curricular guidance to teachers or schools at the high school level for any subject other than Theology. Given such curricular freedom, it is not surprising that there is a wide diversity of curricular priorities within the same subject.

However, a deeper analysis revealed that there was, indeed, a pattern underlying this variability. The curricula as submitted by the participating teachers could be interpreted as composing two major groups-- the "Algebra III" group and the "New Topics" group -- reflecting a prioritization of either mastering advanced algebraic skills or introducing new topics, respectively. A visual analysis presents a clear distinction between the groups, as demonstrated in Figure 2 and Figure 3 in Chapter 4 and as supported by the statistical analysis demonstrated in Table 15. It is important to note here that, since these scores are z-scores, low scores in either Advanced Algebra or New Topics do not imply that teachers ignored these topics, but rather that they were of relatively lower priority. This could be for myriad reasons, such as the professional judgement of the teacher, the culture of the school, the composition of the student body, or the other opportunities students had for learning these topics outside of the pre-calculus classroom. For example, if a student could take a separate trigonometry or statistics course in his or her high school pathway, then he or she may be best served by a teacher who chooses to ignore trigonometry or statistics, instead prioritizing other topics not included elsewhere in the school.

The purpose for categorizing this distinction in curricular emphasis is to explain the priorities of the teachers' curriculum, not to make judgements on the relative value of one curriculum or another. Indeed, some students may benefit more from advancing their algebraic skills, and some may benefit more from expanding their mathematical repertoire of topics to trigonometry, statistics, and beyond. It is also important to note that teachers who submitted survey responses and examinations were interpreted as emphasizing curricula in the "Advanced Algebra" group do not ignore advanced topics such as trigonometry, and that teachers whose responses were interpreted as emphasizing a curricula in the "New Topics" group do not ignore algebra and functions. Rather, these teachers build curricula that prioritizes one or the other.

The primary conclusion drawn from this study is that the pre-calculus teachers of the Archdiocese of Washington build diverse and unique curricula that roughly adhere to two broad typographies. Future research could expand upon this finding by either comparing these teachers' curricula to the curricula of local states or districts. For instance, the Archdiocese of Washington has ecclesiastical jurisdiction crossing six geographical school districts: Washington, DC and Prince George's, Montgomery, Charles, Calvert, and St. Mary's Counties in Maryland. It would be interesting to compare these Archdiocesan curriculum prototypes to the curriculum required by these local public school districts.

The Relationship Between Stated and Assessed Curriculum

Teachers have the continuous and challenging task of maintaining the balance between the various possible elements of curriculum. The progression from intended to enacted to assessed curriculum is a pathway through the practical realities of teaching and of students, with teachers' in-the-moment decisions and planning realities affecting the execution of the intended curriculum and how it is received. A key measure, then, is to determine the alignment between what teachers claim is a part of their pre-calculus course and what is actually assessed by their summative assessments. This was one of the objectives of this study as reflected in the second research question: "What is the level of agreement between the stated curricula and the tested curricula in the pre-calculus classes in the Archdiocese of Washington?"

To accomplish this goal, both the Porter Alignment Index (Porter, 2002, 2011) and a Generalized Linear Model were calculated along three dimensions: comparing stated curriculum scope against assessed curriculum scope, comparing stated curriculum performance expectations against assessed curriculum performance expectations, and comparing overall stated curriculum against overall assessed curriculum. Those results are presented as Table 11 and Table 15 in Chapter 4.

In a discussion of his Alignment Index, Porter (2011) describes 0.30 as a "moderate" alignment. That value was indeed moderate for the question at hand in that work, which compared Common Core topics as defined by various state curricula. Such a comparison would necessarily have significant alignment disparities that are not present in this analysis, which compares the choices of the same teacher to themselves. That being so, these Alignment Index values are still extraordinarily high, indicating a broad success that these teachers have at aligning their stated curricula to their assessed curricula.

This alignment success is strongest when comparing each teacher's stated curriculum scope to their assessed curriculum scope. This comparison reflected an average alignment analogous to a 93.3% agreement in scope. While ideally this number should be high, and since teachers in the Archdiocese of Washington write their own assessments so that they are able to define assessments that measure only and all of the material they have taught, the high level of alignment identified by the Porter Alignment Index for these teachers' curricula is higher than even those presumed expectations. This is strong evidence that teachers in the Archdiocese of Washington are effective at writing assessments that measure the same mathematical content and understandings that they are intending to teach.

The alignment between stated and assessed curricular performance expectations is also strong, a very high 91.4% agreement. This value implies that teachers in the Archdiocese of Washington are also writing assessments that effectively match the assessment expectations they have for students on each topic with the intended instructional expectations they had for those students as the topic was taught. This, combined with the high alignment in scope, implies that teachers in the Archdiocese of Washington are effective test developers, in that their assessments match their (stated) in-classroom curriculum well.

For each teacher, the overall alignment between stated and assessed curriculum was the lowest of the three alignment scores. Since the Porter Alignment Index (Porter, 2002, 2011) includes an adjustment for the number of elements in the matrix, this decreased alignment is not a product of having more data to align, but is rather an indication that these teachers' scope and performance expectations are not as closely aligned to each other as they are to themselves. Essentially, what this implies is that while the scope of teachers' assessments matches the scope of the curriculum taught in the classroom, and while the expectations each of these teachers has for students on assessments match the expectations each of these teachers has for the students in a teacher's pre-calculus classroom, the scope of the teachers' curriculum is not as strongly aligned to the expectations of their assessed curriculum.

This is intuitive and was evidenced in several ways. Often, teachers included an algebra review as a part of their curriculum, generally early in the school year. Because these topics were typically a review from Algebra II, less time was spent teaching these topics than their difficulty would imply. For example, the median teacher responded on the survey that they spent only 1-2 days teaching algebraic properties and proofs, but the median performance expectation was that most students could answer a problem about that topic. On the other side of the spectrum, difficult topics such as trigonometric identities often were taught for a relatively longer time (5 teachers taught this topic for 4-7 days or more) but expectations were relatively low (the median response was that “solid” students could answer a verifying identities question).

This evidence is confirmed and expounded upon by the Generalized Linear Model that was used to further explore the relationship between teachers’ curriculum source and type. Once the appropriate model was discovered and selected, the GLM analysis found that neither the source of the curriculum (the survey or the teachers’ examinations) nor the type of curriculum (scope or performance expectations) was a significant factor in determining the differences between the curricula. This result is further evidence that a teacher’s stated and assessed curricula are adequately aligned.

As a result, the evidence remains strong that teachers in the Archdiocese of Washington are effective at building curricula that are consistent with their own expectations and that these teachers are also effective at designing assessments to test that curricula in fair and accurate ways. Future research could expand upon this finding to compare the effectiveness and alignment of these curricula and assessments with those of local school districts to determine if the teacher-built assessments are more or less consistent with the curricula being taught in the classroom.

Teachers' Curricula vs. the Common Core State Standards

The Common Core State Standards are not a curriculum, but rather a list of intended common mathematical content and practices for which states and districts can create curricula. (Note that this study only considers the Mathematical Content Standards, rather than the Mathematical Practice Standards). However, the meaning of curriculum as examined by this research study focuses on curriculum scope, perhaps best defined as the list of topics taught within a curriculum. For this reason, comparisons and contrasts of the curricula of teachers in the Archdiocese of Washington to the Common Core State Standards are intuitive and insightful.

The primary focus of this research question is the relationship between the participating teachers' curricula and the Common Core. That is, whether when given the curricular freedom to choose which mathematical topics to teach within a precalculus course, teachers choose to teach topics that are also identified within the Common Core. Since the Common Core expects to be a comprehensive approach to high school mathematics, determining if teachers choose to teach similar topics is interesting both when characterizing the teachers and the Common Core. Furthermore, since the independent high schools of the Archdiocese of Washington must market themselves as advanced academic institutions, it is important to these schools to be able to identify themselves as academically on par with the Common Core.

As described in the Chapter 3, this study considered each of the 37 pre-calculus topics to be "taught" if teachers met three conditions: the teacher reported that he or she teaches the topic for at least 1-2 days, the teacher reported that at least his or her strongest students can answer a given problem, and the teacher included a problem from that topic on his or her summative assessments. Then, once each topic was labeled as either "taught" or "not taught", that teacher was said to have taught or not taught the associated Common Core topics, as connected on the Alignment Key presented in Appendix E.

As demonstrated in Table 13 from Chapter 4, the participating teachers in the Archdiocese of Washington do *not* teach curricula that are analogous to the Common Core State Standards. Only about 50% of the 26 pre-calculus topics associated with regular Common Core topics are reported as "taught." However, this does not necessarily imply that these teacher-built curricula are inferior in difficulty to the Common Core, because 34% of the topics associated only with Honors Common Core topics are also taught, as well as 34.5% of the topics that go beyond the Common Core itself. These are topics such as reciprocal trigonometric graphs, polar coordinates, parametric equations, and limits. The broad teaching of these topics implies that these teacher-built curricula do not in fact teach fewer topics than those identified by the Common Core, but rather that these participating teachers simply teach different topics reflecting different priorities.

That is also not to say that the teacher-built curricula are superior to the Common Core in what and how the participating Archdiocesan teachers teach pre-calculus. More information is required to make decisions about the efficacy of these teachers' curricula in promoting college and career readiness among their students, especially regarding the success teachers are having at communicating these topics at high cognitive levels. The definition of "taught" used by this study could more accurately be labeled as "covered" as truly evaluating whether a topic was taught or not, and especially evaluating whether a topic was taught well, would also require a measure of *learning* success.

Furthermore, there is evidence in this data to suggest that teachers in the Archdiocese of Washington are de-prioritizing statistics and probability, relative to the Common Core, in a dramatic way. As described in Table 14 from Chapter 4, only 6 of the possible 65 Statistics and Probability topics were actually "taught" by Archdiocesan teachers, by far the lowest percentage of any category, including Advanced Topics. One interpretation of this is as follows: top-down changes have the opportunity to react quickly to developing trends. The Common Core, as an administrative change presented to public-school teachers from the district and the state level, conveys both the intention and the opportunity to adhere to research interpretations as offered by Common Core advocates. It also advocates attending to growing mathematical trends, specifically the rapid proliferation of data and the need for analytic interpretation of data within applications across society that are likely to impact or influence most students' mathematical futures. However, it is important to note that the Common Core is a broad approach to an entire high school curricular pathway. Statistics have not traditionally been included in pre-calculus curricula, and it remains possible that students are provided an opportunity to take a separate statistics course, which could discourage teachers from including statistics as a pre-calculus topic.

Confederated, bottom-up change, however, is often evolutionary rather than revolutionary, attending to the inertia of past choices. If this interpretation is true, it would cast an interesting light on the curricular freedom given to teachers in the Archdiocese of Washington, implying that the freedom they are given is, at least in some cases, potentially less fertile for bold and dramatic change than a more hierarchical system.

General Conclusions

The three research questions addressed by this study relate to the quality and effectiveness of curriculum. What makes studying teachers in the Archdiocese of Washington interesting is that they are so independent, given the freedom they have to choose what and how to teach. Assessing the quality of these teacher-built curricula allows for contributions to the general discussion about the efficacy of district-built, state-built, and potentially nationally-built curricula.

Certainly, what was evaluated by this research was not the "quality" of these teachers' curricula. In order to evaluate how "good" these teacher-built curricula truly are, one would need to assess the impact on student learning and achievement, as well as the rigors and difficulties on the teachers themselves. What creative sacrifices must be made if planning time is taken up by building and developing curriculum? However, given the results of this study's research and informed by the debate held in the literature, the evidence supports the idea that giving teachers creative freedom over their curricula is not detrimental to the quality of the pre-calculus classroom.

First, the evidence from this research's study of inter-teacher variance suggests that teachers, given curricular freedom, will choose widely diverse curricular approaches that vary in scope and in expectations for students. This isn't, by itself, enough to permit a value judgement addressing the variability of these curricula, but it does suggest that teachers have the ability to manipulate the content of their teaching to match their students' knowledge base. No district-built curriculum will ever be all things to all classrooms, so it makes sense to consider the innate flexibility of teacher-built curriculum as a positive element in at least this particular aspect.

However, this flexibility and responsiveness to the student is only valuable if the curricula are also comprehensive. While the teachers' curricula were unique and distinct, most also fell into one of two major typographies, indicating that there is at least some consensus as to the topics necessary in a pre-calculus class. This suggests that these curricula are not lacking in scope, and that students graduating from these pre-calculus programs will be adequately exposed to enough pre-calculus to succeed in collegiate mathematics. The one exception to this conclusion is in the broad ignoring of statistics and probability by Archdiocesan teachers. The mathematical world is moving quickly into an applied construct in which data analysis will be a key skill. The ability to claim these teacher-built curricula as "comprehensive" is thus weakened because of the broad consensus to exclude these topics from pre-calculus.

Beyond the scope and expectations of the curricula themselves, this study has also evaluated the effectiveness of teachers at assessing the topics that they teach. In this aspect, the evidence is quite strong that these teachers are skilled at designing their own exams to cover the same mathematical material that they claim to address in their classroom during instruction. Any concerns that teacher-designed tests would fail to cover all of the material taught are alleviated by the high degree of alignment found between the curricula and assessments written by Archdiocesan teachers. While this statement is true, it does not represent a closing of the debate about the benefits of standardized or district-written assessments. Other considerations, such as the sacrifice of planning time to write examinations or the inability to compare student achievement across classrooms or schools also contribute to this debate. However, this study's evidence suggests that the concern or perspective that teachers may not be able to write well-aligned and comprehensive assessments measuring their intended curriculum is not supported here.

This research has been framed as an opportunity for developing insights into the effects of standardization in classrooms by studying the effects of fundamentally unstandardized classrooms. The debate over the effectiveness and desirability of standardized curriculum and standardized assessments is far from over, and this research is but one of many data points within that debate. One major point made by supporters of standardization is that teachers with standardized curricula teach in a more focused and coherent way, covering all of the necessary material and no more. However, in this research, the evidence suggests that granting teachers the ability to mold and bend their teaching to match both their students and their personal style may be equally effective at producing comprehensive results. What, then, is truly necessary is the ability for teachers in every curricular system to have consistent access to professional development on cutting-edge education research and a commitment to continuously updating their approach to provide the best value for students. The deficiency in teaching statistics in the

Archdiocese of Washington indicates that even with creative flexibility, teachers can be subject to the inertia of past success.

Far more research must be done to determine whether standardization is a net positive or a net negative for both students and teachers. This study did not contribute to discussions of teacher morale, of the validity of standardized testing, or of the potential biases of curricular freedom that could affect classroom equity (in either a positive or a negative way). However, this study did contribute evidence about the similarities and differences of teachers' choices when presented with curricular flexibility. Though this evidence does not definitively identify curricular flexibility as good for students, it does imply that curricular flexibility is not inherently bad for students.

Significance

These conclusions have the potential to inform the field of curriculum study and several parties who depend on curriculum study for educational excellence. Teachers, as well as commercial education companies and private schools, may all benefit from the research analyzed here. While none of the conclusions reached as a result of this study are definitive, each conclusion contributes to the accumulated body of scientific knowledge about the relationship of teachers to curriculum and, as a result, informs the curriculum-centered decisions made by these groups.

The field of curriculum study focuses on improving education by affecting what is taught by whom. Often, this results in a continuous addition of topics, forcing teachers to teach more and more in the same amount of time. Indeed, one major purpose of the Common Core was to thwart this very trend, instead allowing for an intuitive, coherent, and focused approach written with global goals in mind. This research provides evidence that teachers are capable of writing their own curricula to meet their goals and may be better suited to write curricula that match the needs of their students than districts and states. As a result, this study supports the idea behind the Common Core and its global approach to curriculum, while simultaneously supporting teachers' ability to mold and build their curriculum to match the standards advocated by the Common Core.

Pre-calculus is a remarkably universal mathematics course, taught at several levels of education including high school, community college, and university levels. The curriculum discussed in this research has been primarily focused on high school students, but the lessons learned about the nature of pre-calculus curriculum can be extrapolated to the higher levels of education as well. Indeed, the notion that there are two broad interpretations of pre-calculus, the "Algebra III" approach and the "New Topics" approach, could very easily convince teachers and professors at higher education levels to split traditionally one-semester pre-calculus courses into two courses, one based on reviewing, deepening, and expanding algebraic skillsets and the other introducing students to new skills in trigonometry, statistics, linear algebra, and calculus.

As these curricular decisions are made, both from high school districts and teachers reacting to the implementation of the Common Core and from community college and university professors building intuitive and focused pre-calculus curricula, educational publishing companies who provide commercial curriculum materials will be anxious to react and provide value to these teachers for these courses. Textbooks, built with this study's results in mind, would be based on the topics of the Common Core, but would be flexible enough, both in scope and approach, to allow teachers to customize and personalize their pathway through pre-calculus. In many ways, this study predicts the rise of computer and Internet-based curriculum materials, which could have advantages over paper and book-based materials in their ability to be flexible and responsive to the personal priorities of the teacher and the needs of the students.

Finally, this study has provided evidence about the relative strength of the pre-calculus curricula in the Archdiocese of Washington. Private schools have a very present need to market themselves as academically superior institutions, and the schools and teachers studied in this research make a strong argument that their pre-calculus curricula are near or matching the rigor and scope of the Common Core State Standards, with one exception: statistics and probability. Based on the findings of this report, if the schools in the Archdiocese of Washington wish to fully claim that they provide a comprehensive approach to high school mathematics in a way that matches or exceeds that of the strongest local public schools, then these schools must begin to emphasize data analysis and statistics to match these modernizing efforts of the Common Core. With the exception of this lack of emphasis on data and statistics, this research does indeed show that many of the schools in the Archdiocese of Washington are on strong academic footing when teaching pre-calculus.

Future Research

Not only did this research produce evidence from which reasonable, and reasonably restrained, conclusions could be drawn, this study can serve as a starting point from which new research can expand. The topic of teacher choice and curricular freedom is a compelling one, and is one that needs further study to be fully understood. It has been a consistent goal of this research to contribute to that effort.

The Archdiocese of Washington is a unique political entity within the United States; the rules that govern most Catholic Archdioceses do not always apply in this jurisdiction. This is part of what made this collection of teachers interesting to study. However, these teachers are not unique nationally for their ability to make curriculum decisions. Many schools that are private and secular, or which are private and otherwise independent, grant their teachers the same freedom as do high schools in the Archdiocese of Washington. Studying these teachers at these institutions could provide even more insight into the ability of teachers to select and build curricula for their own courses. One word of caution: Catholic education is unique in that it is private education, but often serves a population that is similar demographically to the public school population. Secular private schools may differ in their approach to curriculum from Catholic and public schools in some part due to their more affluent student populations.

Secondly, this study could be expanded by including classroom observations. While this research was limited to teacher-produced responses to surveys and teacher-written assessments, objective researcher interpretation of in-classroom lessons would contribute to the potential for the research to distinguish between intended, enacted, and assessed curricula. Such a study would be costly, requiring video equipment and trained data coders to digest the collected lesson tapes. However, additional funding may be worthwhile if the result is an increased understanding of the choices teachers make in their classrooms when given the freedom to choose.

Furthermore, this research was restricted to a discussion of mathematical content, when many of the differences in teachers' choices are due to personal decisions about teaching. How a teacher uses the curriculum he or she has built is crucial to studying its effectiveness and its impact on students. Future research could evaluate the cognitive demand of the lessons in the classroom, the discourse of the students, and the types of questions asked on the assessments. This information would supplement the scope and content of curriculum to provide a more complete picture of the effect of curricular freedom on teachers and students in the Archdiocese of Washington. Only then could conclusions be drawn about whether granting all students the freedom available to Archdiocesan teachers could be a beneficial change.

Finally, future research could expand upon these results by studying not only the alignment of high school pre-calculus curricula to the content standards of the Common Core, but could also compare these curricula to the objectives within the advanced college admissions assessments, such as the SAT Mathematics Level 2 Subject Test or the ACT Mathematics Test. These advanced assessments correspond to the level and objectives of pre-calculus, in that they seek to determine students' readiness for collegiate mathematics. Assessing the connections between the pre-calculus curricular choices and the relevant effects on students' preparation for these tests could inform teachers and students in their pathway to college acceptance.

The long arc of education history in the United States is bending towards national curriculum. There are strong arguments on both sides of this debate, both ultimately focused on the success of students in the long term, but both of which include other priorities as well: adherence to international norms, the intrinsic value of state and district autonomy, the morale of teachers. Nations, states, and districts charged with making these decisions would do well to collect all information available about the benefits and potential detriments of creating standardized curriculum. It is the goal of this study to contribute to that information.

Appendix A

Informed Consent Form

University of Maryland College Park

Initials_____ Date____

Project Title	Patterns in Curriculum Choices: Pre-Calculus Curricula in the Archdiocese of Washington
Purpose of the Study	This research is being conducted by Christopher Hurst, under the guidance of Dr. Patricia F. Campbell, as a requirement for his M.A. Degree at the University of Maryland, College Park (UMCP). We are inviting you to participate in this research project because you have taught pre-calculus (or an equivalent course, such as Algebra III, Functions and Analysis, or College Algebra) in 2013-2014, within the Archdiocese of Washington. The purpose of this research project is to identify similarities and differences between curriculum decisions, both in emphasis and scope, as made by pre-calculus teachers throughout the Archdiocese.
Procedures	In order to complete this study, three things will be required of you. First, you will provide your primary professional email address on this consent form. Then, you will receive an email asking you to send either an electronic or a hard copy of your previous year's (2013-2014) pre-calculus midterm and final exams. Once that is received, you will receive another email providing a link to an online survey. The survey will require approximately one hour for you to complete. This survey will consist of multiple choice questions asking you to estimate the time spent teaching a variety of pre-calculus topics, as well as your professional assessment about the relative difficulty of several pre-calculus mathematics questions. You will also be asked to provide answers to several demographic questions, including your gender, race, years of teaching experience, and years of experience teaching pre-calculus.
Potential Risks and Discomforts	There are no risks associated with this study. You will be asked to give approximately one hour of your time to taking a survey and to contribute a copy of your 2013-2014 midterm and final exams. You will not be required to complete any survey questions that make you uncomfortable.
Potential Benefits	There are no direct benefits to participants for completing this study. However, you will contribute to the general development of the field of curricular research in pre-calculus. To that end, upon completion of the study, you will be given a copy of the study report, which will contain information about the curricular decisions of your peers throughout the Archdiocese.
Confidentiality	All data provided will be kept securely on a password-protected computer. No names will be attached to any data collected, as all data will be organized by an identification (ID) number. Only the researcher will have access to the file linking names to ID Numbers. Online security is ensured by the University of Maryland survey client Qualtrics. All hard-copies of midterm and final examinations will be stored in a locked file cabinet until they are scanned by the researcher and stored electronically. After scanning, all hard copies of examinations will then be immediately destroyed. The reporting on this research project will rely upon analysis of aggregate data. Your identity will not be revealed in the thesis. If a report or article is written about this research project, your identity will be protected to the maximum extent possible. Your information may be shared with representatives of the University of Maryland, College Park or governmental authorities if you or someone else is in danger or if we are required to do so by law.
Compensation	For completing this study, you will receive a $25-40 gift card. You will be responsible for any taxes assessed on the compensation. All teachers who complete the study will receive a gift card of the same value. Gift cards will be distributed to teachers via US mail, addressed to them at their professional address. ☐ Check here if you expect to earn $600 or more as a research participant in UMCP studies in this calendar year. You must provide your name, address and SSN to receive compensation. ☐ Check here if you do not expect to earn $600 or more as a research participant in UMCP studies in this calendar year. Your name, address, and SSN will not be collected to receive compensation.

<table>
<tr><td>Right to Withdraw and Questions</td><td colspan="2">Your participation in this research is completely voluntary. You may choose not to take part at all. If you decide to participate in this research, you may stop participating at any time. If you decide not to participate in this study or if you stop participating at any time, you will not be penalized or lose any benefits to which you otherwise qualify.

If you decide to stop taking part in the study, if you have questions, concerns, or complaints, or if you need to report an injury related to the research, please contact the research investigator or his advisor:

Christopher Hurst
DeMatha Catholic High School
4313 Madison St. Hyattsville, MD
(240) 764-2200
churst@dematha.org

Patricia F. Campbell
University of Maryland, College Park
2311 Benjamin Building, College Park, MD
(301) 405-3324
patc@umd.edu</td></tr>
<tr><td>Participant Rights</td><td colspan="2">If you have questions about your rights as a research participant or wish to report a research-related injury, please contact:

University of Maryland College Park
Institutional Review Board Office
1204 Marie Mount Hall
College Park, Maryland, 20742
E-mail: irb@umd.edu
Telephone: 301-405-0678

This research has been reviewed according to the University of Maryland, College Park IRB procedures for research involving human subjects.</td></tr>
<tr><td>Statement of Consent</td><td colspan="2">Your signature indicates that you are at least 18 years of age; you have read this consent form or have had it read to you; your questions have been answered to your satisfaction and you voluntarily agree to participate in this research study. You will receive a copy of this signed consent form.

If you agree to participate, please sign your name below.</td></tr>
<tr><td rowspan="4">Signature and Date</td><td>NAME OF PARTICIPANT [Please Print]</td><td></td></tr>
<tr><td>SIGNATURE OF PARTICIPANT</td><td></td></tr>
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Appendix B

Subject Recruitment Flyers

HSPA Affinity Group for Pre-Calculus Teachers

Christopher Hurst
Mathematics Teacher, DeMatha Catholic High School
Master's Candidate, University of Maryland

What is your goal here?

I would like to recruit you to be a part of the research I am conducting for my Master's thesis.

What is this research about?

I think that mathematics teachers in the Archdiocese of Washington are in a very unique situation, and that we could learn a lot by studying how we build our classes. Very few places in the country allow schools and teachers to have as much creative freedom in determining curriculum as the Archdiocese does. I would like to see exactly what we do with that freedom.

Why Pre-Calculus?

Pre-Calculus is interesting for curriculum study because it is, for most of the country, an optional course given only to college-bound students. The fact that Pre-Calculus is so widespread in Archdiocesan schools is a statement in itself. Also, the course has such a wide definition: whether you call it "Algebra III" or "Functions and Analysis", different people think "Pre-Calculus" means different things, which is good!

What are you going to be studying?

I am pursuing these three research questions:

1) In what ways are the various Pre-Calculus curricula of the Archdiocese different, and in what ways are they the same?

2) What are the differences, if any, between the intended curricula and the assessed curricula in these Pre-Calculus classes?

3) How closely does each approach to Pre-Calculus align to the content standards of the Common Core State Standards for Mathematics?

So what do you want me to do?

If you would like to be a part of my project, I will collect data from you in two ways:

First, I would like a copy of both your 2013-2014 midterm (or winter) and final (or spring) exams. I am going to use these to determine what was your assessed curriculum for that year. I will simply send you an email asking you for these exams, and you can email them back to me as an attachment. No one else will *ever* have access to these exams besides me.

Second, I would like you to take an online survey, which should take no more than 60 minutes. In the survey, I will ask you two types of questions: "how long did you teach this topic?" and "how good would a student have to be to answer a question on this topic?" The first question is designed to look for curriculum scope, and the second is for curriculum rigor. I will ask you to take this survey sometime this winter, at whatever time is convenient for you.

It is important to note that you will be given an ID number, so that I can never associate your name or your school with either your exams or your survey. Your name will never be included in the final report, and all data that I present in that report will be in general.

Why would I want to do this?

Besides being very generous to me, there are two reasons you should want to be a part of this study. First, because you are giving an hour of your time for this research, I would like to offer you a $25 gift card to Ledo's pizza as a thank you for participating. Second, this research will produce tangible, valuable information that will be helpful for schools and teachers of Pre-Calculus. If you participate, I will give you a copy of the analysis, and my interpretation of what it means. My hope is that you will be able to use this information to inform the choices that you make about Pre-Calculus curriculum every day. But mainly pizza.

Ok, I'm in! What do you need me to do *right now*?

For legal reasons, I need to give each of you a copy of the informed consent form. I will go over all of the risks and benefits of participating, and I will ask you to sign at the bottom, as well as to provide your professional email address. I will contact you with more information at this email address. Then, please turn in a consent form in the cardboard box I have provided.

If you would not like to be a part of this study, you can simply leave a blank copy of the consent form in the box. That way, no one will know who else is and is not participating.

Thank you for your attention, and I look forward to working with you!

Attention Pre-Calculus Teachers:

If you taught Pre-Calculus last year (2013-2014),
I would like to hear from you!

I am conducting a study of Pre-Calculus curricula from all of the high schools in the Archdiocese of Washington.
I am interested in what you taught, how long you taught it, and how it was assessed.

I will ask for only about an hour of your time to complete an online survey, in addition to a copy of last year's midterm and final exams for your class.
As a thank you gift for participating, if you complete the study, you will earn a $25 gift card.

If you would like to participate, please send an email to churst@dematha.org.
Thank you for being a part of my research!

Christopher Hurst
Pre-Calculus Teacher, DeMatha Catholic High School
Master's Candidate, University of Maryland

Appendix C

Summary and Rationale

Pre-Calculus Curriculum Research Project Summary and Rationale

Christopher Hurst, DeMatha Catholic High School/University of Maryland - College Park

Topic:

Parochial and other non-public schools are often given a large degree of freedom to define their courses and curricula as the departments and teachers see fit, producing a wide variety of approaches. In an effort to compare and contrast the priorities inherent to these approaches, I would like to study the various pre-calculus curricula and summative assessments across Catholic high schools in the Archdiocese of Washington.

Research Questions:

In what ways are the various pre-calculus curricula of the Archdiocese different, and in what ways are they the same?

How closely does each approach to pre-calculus align to the objectives and standards of the Common Core State Standards for Mathematics?

What are the differences, if any, between the stated curricula and the tested curricula in these pre-calculus classes?

Method:

Two types of data will be collected. First, participating teachers will be given a survey on which they will indicate their curricular approach in two ways: the estimated time spent on each major topic and the relative quality of a student who could successfully answer questions about selected topics. For example, a teacher may indicate that he or she spends an average of 15 class days teaching trigonometric functions, and can indicate that he or she would expect that the question "Graph: $y = 2\ cos\ \pi x$" could be successfully answered at the end of their course by a B student or better.

The second type of data considers the summative assessments provided by the teachers themselves. Teachers will be asked to provide a copy of their midterm and final exams, and the assessed curriculum implied by each will be determined based on the questions asked, their relative difficulty, and the relative priority given to those questions, such as point values.

At this point, I will quantify the relative priorities of each class' curriculum, both intended and assessed, and will determine the similarities and differences between curricula, between each curriculum and the Common Core, and between each curriculum and the assessments used to test it.

Significance:

This information will be valuable to both teachers and administrators at these and other schools. Teachers will be able to compare their class' priorities to others', and given this information, may choose to affect their own curriculum to more closely match the consensus topics taught by their peers. Further, teachers may be able to objectively compare how effectively their summative assessments test their intended curriculum, and may be able to enact adjustments to these exams so they more closely match what is actually taught in the classroom. Furthermore, the construction of the alignment measure itself, and the test of its reliability in this study, will provide a benefit to future research in the field of pre-calculus curriculum alignment.

Appendix D

Survey Questions

Survey Questions

Notes: No question will be required, and each question will be able to be skipped by the teacher.

Selecting "I Do Not Accept" for Question CE.1 ends the survey.

Questions D.1, D.3, D.4, and CQ.2b will be open-response answers that teachers will type.

Questions CE.3 and CE.4 will be upload prompts for submitting examinations.

Questions CE.2 and T.1 are informative statements with no choices or subject response.

The multiple-choice answers for the scope questions (marked as "a") will be:

I do not teach this topic

Very little: 1-2 days

Some time: 2-4 days

Quite a bit: 4-7 days

A great deal: 7+ days

The multiple-choice answers for the rigor questions (marked as "b") will be:

All of my students can answer this

Most students, perhaps not my lowest, can answer this

Solid students can answer this

Only my strongest students can answer this

I do not teach this topic

Consent and Exam Uploads

CE.1 In the email providing you the link to this survey, you were given a copy of the Informed Consent Form.

Your signature here indicates that you are at least 18 years of age; you have read this consent form or have had it read to you; your questions have been answered to your satisfaction and you voluntarily agree to participate in this research study.

If you agree to participate, please select "I Accept" below. (Choices: I Accept, I Do Not Accept)

CE.2 You will now be asked to upload the Midterm and Final Examinations for the most recent complete year you taught pre-calculus. PDF, Microsoft Word, and Google Docs are all acceptable formats.

CE.3 Please upload your most recent pre-calculus Midterm Examination.

CE.4 Please upload your most recent pre-calculus Final Examination.

Demographics and Subject Information Questions

D.1 Please state your Identification Number as provided by the researcher.

D.2 Please state the following:

Your gender: (Choices: Male, Female)

Your race: (Choices: African-American, Asian-American, Hispanic, White/Caucasian, Mixed, Other)

D.3 How long have you been teaching mathematics at the high school level?

D.4 How long have you been teaching pre-calculus?

Technology Statement

T.1 Please note that you will be asked to rate your expectations for students given certain pre-calculus problems. For these questions, unless specifically forbidden, access to a graphing calculator is assumed. However, students are assumed to not have access to any more advanced technology, such as a computer algebra system.

Advanced Algebra

AA.1a For approximately how long do you teach:

Algebraic properties, proof, and the formal reasoning process

AA.1b What is the lowest level of student who would be likely to answer this question correctly?

Create an equation that demonstrates each of the commutative, associative, distributive, identity and inverse properties.

AA.2a For approximately how long do you teach:

Solving quadratic equations using Completing the Square and the Quadratic Formula

AA.2b What is the lowest level of student who would be likely to answer this question correctly?

Solve $x^2 - 8x - 20 = 0$ using three different methods.

AA.3a For approximately how long do you teach:

Arithmetic with polynomials, including general polynomial expansions and identities

AA.3b What is the lowest level of student who would be likely to answer this question correctly?

Without multiplying, find the polynomial expansion of $(x+3)^5$.

AA.4a For approximately how long do you teach:

Irrational numbers, radicals, and rational exponents

AA.4b What is the lowest level of student who would be likely to answer this question correctly?

Simplify: $\left(\frac{x^{3/2}y^{1/3}}{x^{-1/3}y^{1/2}}\right)^{-6}$

AA.5a For approximately how long do you teach:

Solving rational and radical equations

AA.5b What is the lowest level of student who would be likely to answer this question correctly?

Solve: $\sqrt{x+2} - 1 = 2x$

AA.6a For approximately how long do you teach:

Arithmetic and algebra with complex numbers

AA.6b What is the lowest level of student who would be likely to answer this question correctly?

Simplify: $\frac{3+4i}{5-2i}$

AA.7a For approximately how long do you teach:

Graphing points in the complex plane, and DeMoivre's Theorem

AA.7b What is the lowest level of student who would be likely to answer this question correctly?

What point on the complex plane is represented by $3\sqrt{2}\,cis\ 135^{\circ}$?

AA.8a For approximately how long do you teach:

Solving systems of equations and linear programming

AA.8b What is the lowest level of student who would be likely to answer this question correctly?

Solve: $5x+4y=7$
$3x+2y=3$

AA.9a For approximately how long do you teach:

Factoring higher-order polynomials

AA.9b What is the lowest level of student who would be likely to answer this question correctly?

Use the factor theorem to determine if $x=2$ is a root of $P(x)=x^3-3x^2+4$

AA.10a For approximately how long do you teach:

Exponentials, logarithms and Logarithmic Laws

AA.10b What is the lowest level of student who would be likely to answer this question correctly?

Solve: $log_4 x + log_4(x-6) = 2$

Functions and Modeling

FM.1a For approximately how long do you teach:

Definition of function, function notation, representations, domain and range

FM.1b What is the lowest level of student who would be likely to answer this question correctly?

Is the relationship associating each person with his or her birthday a function? Is the inverse of this relationship a function? Explain why or why not for each.

FM.2a For approximately how long do you teach:

Composition, transformations, and inverse functions

FM.2b What is the lowest level of student who would be likely to answer this question correctly?

Prove that $f(x) = x^2 - 3$ and $g(x) = \sqrt{x+3}$ are inverses.

FM.3a For approximately how long do you teach:

Understanding modeling and models as a problem-solving process

FM.3b What is the lowest level of student who would be likely to answer this question correctly?

An airplane flies round-trip from Boston to Denver, which is 2000 miles each way. Due to the jet stream, the plane flies eastward 100 mi/hr faster than westward. If the total air time was 9 hours for the round trip, how fast was the plane flying each way?

FM.4a For approximately how long do you teach:

Building models to describe situations

FM.4b What is the lowest level of student who would be likely to answer this question correctly?

If a baseball player throws a baseball, what are the relevant factors that determine the position of the ball over time? What kind of model would best describe the flight of the ball? Build a model that could incorporate all of these relevant factors.

FM.5a For approximately how long do you teach:

Graphing and modeling with linear, quadratic, absolute value and piecewise functions

FM.5b What is the lowest level of student who would be likely to answer this question correctly?

Where is the vertex of the parabola $f(x) = (x+1)^2 - 3$? Use this information to help you graph f.

FM.6a For approximately how long do you teach:

Graphing and modeling with higher-order polynomials

FM.6b What is the lowest level of student who would be likely to answer this question correctly?

Graph $P(x) = (x-1)^2(x+3) = x^3 + x^2 - 5x + 3$

FM.7a For approximately how long do you teach:

Graphing and modeling with exponential and logarithmic functions

FM.7b What is the lowest level of student who would be likely to answer this question correctly?

A cup of coffee fresh out of the pot is $205^{\circ F}$. After 20 minutes, it has cooled to a drinkable $90^{\circ F}$. Use this information to build an exponential model describing the temperature of the coffee over time. Use the graph of this model to approximate the temperature of the room.

FM.8a For approximately how long do you teach:

Graphing and modeling with other (rational, radical, logistic, etc.) functions

FM.8b What is the lowest level of student who would be likely to answer this question correctly?

Describe what happens to the graph of $R(x) = \frac{x-2}{x^2-5x+6}$ at both $x = 2$ and $x = 3$.

Trigonometry

TR.1a For approximately how long do you teach:

Right triangles, trigonometric ratios, and applications

TR.1b What is the lowest level of student who would be likely to answer this question correctly?

A surveyor notes that a tree is 157.3 m from her, and that the angle of elevation to the top of the tree is 12°. How tall is the tree?

TR.2a For approximately how long do you teach:

Radian measure and the unit circle

TR.2b What is the lowest level of student who would be likely to answer this question correctly?

Use the unit circle to explain why $sin\,\frac{\pi}{6} = sin\,\frac{5\pi}{6}$.

TR.3a For approximately how long do you teach:

Oblique triangles, the Law of Sines, and the Law of Cosines

TR.3b What is the lowest level of student who would be likely to answer this question correctly?

A ship leaves port and travels a distance of 137 km at a bearing of 235°. To avoid a storm, at this point, the ship turns to a bearing of 309° and travels for an additional 54 km. How far is the ship from the original port?

TR.4a For approximately how long do you teach:

Graphing and modeling with the sine and cosine functions, and transformations

TR.4b What is the lowest level of student who would be likely to answer this question correctly?

Graph $y = 3\,cos\,(x - \frac{3\pi}{4}) + 2$

TR.5a For approximately how long do you teach:

Graphing tangent, cotangent, secant and cosecant functions

TR.5b What is the lowest level of student who would be likely to answer this question correctly?

Explain why the graphs of tangent and secant have the same asymptotes, and why the graphs of cotangent and cosecant have the same asymptotes.

TR.6a For approximately how long do you teach:

Proving and using trigonometric identities

TR.6b What is the lowest level of student who would be likely to answer this question correctly?

Use the unit circle to prove the Pythagorean identity: $sin^2\theta + cos^2\theta = 1$.

Statistics and Probability

SP.1a For approximately how long do you teach:

Independence, conditional probability, and expected values

SP.1b What is the lowest level of student who would be likely to answer this question correctly?

You flip a fair coin 20 times. How likely is it that your flips will result in exactly 10 heads? 5 heads? 0 heads?

SP.2a For approximately how long do you teach:

Representing and interpreting univariate and bivariate data

SP.2b What is the lowest level of student who would be likely to answer this question correctly?

Sketch a scatterplot that might represent the relationship between a student's GPA and their SAT score. Discuss any choices you made in terms of correlation.

SP.3a For approximately how long do you teach:

Understanding statistics as a decisionmaking process

SP.3b What is the lowest level of student who would be likely to answer this question correctly?

For a lottery drawing, 154,803 tickets will be sold for $5 each. The winner will receive a cash prize of $300,000. Does it make sense to buy a lottery ticket? Explain why or why not.

SP.4a For approximately how long do you teach:

Building, interpreting and using linear regression models

SP.4b What is the lowest level of student who would be likely to answer this question correctly?

Given the following dataset:

Year	Price of Milk (per gallon)
1950	\$0.63
1963	\$0.95
1972	\$1.12
1987	\$1.40
1995	\$1.77
2003	\$2.09

use your calculator to build a linear regression model. Then predict the price of milk in 2014.

SP.5a For approximately how long do you teach:

The normal distribution, z-scores, and hypothesis testing

SP.5b What is the lowest level of student who would be likely to answer this question correctly?

Explain what it means to claim that "a null hypothesis was rejected at the 95% significance level".

Advanced Topics

AT.1a For approximately how long do you teach:

Vectors, vector arithmetic, and applications

AT.1b What is the lowest level of student who would be likely to answer this question correctly?

A swimmer is crossing a river by exerting 180 N of force directly eastward. The river is flowing directly southward with a force of 120 N. Find the resultant direction the swimmer is moving and the speed at which she is moving if her mass is 35 kg.

AT.2a For approximately how long do you teach:

Matrices and determinants

AT.2b What is the lowest level of student who would be likely to answer this question correctly?

Find the determinant of the following matrix:

$$\begin{bmatrix} -1 & 5 & 2 \\ 4 & 1 & 0 \\ 3 & -5 & 7 \end{bmatrix}$$

AT.3a For approximately how long do you teach:

Polar coordinates and polar functions

AT.3b What is the lowest level of student who would be likely to answer this question correctly?

Convert the following rectangular coordinates to polar coordinates:

$$(-1,\ 1) \qquad \left(\tfrac{\sqrt{3}}{2},\ \tfrac{1}{2}\right) \qquad (5,\ 0)$$

AT.4a For approximately how long do you teach:

Parametric equations

AT.4b What is the lowest level of student who would be likely to answer this question correctly?

Graph the following parametric function by plotting points:

$$x = t^2 + t \qquad y = t + 3$$

AT.5a For approximately how long do you teach:

Sequences and partial sums

AT.5b What is the lowest level of student who would be likely to answer this question correctly?

Explain what it means for a sequence to *converge*. Give an example of a convergent sequence.

AT.6a For approximately how long do you teach:

Arithmetic, geometric and harmonic series

AT.6b What is the lowest level of student who would be likely to answer this question correctly?

Find $\sum_{i=1}^{3} \frac{1}{2^i}$. Find $\sum_{i=1}^{5} \frac{1}{2^i}$. What do you think $\sum_{i=1}^{\infty} \frac{1}{2^i}$ will be?

AT.7a For approximately how long do you teach:

Conic sections: circles, ellipses, parabolas and hyperbolas

AT.7b What is the lowest level of student who would be likely to answer this question correctly?

Find the foci of the ellipse $9x^2 + 25y^2 = 225$, and sketch its graph.

AT.8a For approximately how long do you teach:

Graphical, numerical, and algebraic limits

AT.8b What is the lowest level of student who would be likely to answer this question correctly?

Use a input/output chart to find $\lim_{x \to 0} \frac{\sin x}{x}$.

Concluding Questions

CQ.1 How long did it require for you to complete this survey?
(Choices: Less than one hour, about one hour, more than one hour)

CQ.2a Did this survey contain a comprehensive representation of the content of pre-calculus?
(Choices: Yes, No)

CQ.2b If this survey was not comprehensive, what other topic do you believe should be included?

Appendix E

Topic List and Alignment Key

Topic List and Common Core Alignment Key

Advanced Algebra

- Algebraic properties, proof, and the formal reasoning process
    - A-CED.4, A-REI.1
- Solving quadratic equations using Completing the Square and the Quadratic Formula
    - N-CN.7, N-CN.8+, A-REI.4
- Arithmetic with polynomials, including general polynomial expansions and identities
    - A-APR.1, A-APR.4, A-APR.5+
- Irrational numbers, radicals, and rational exponents
    - N-RN.1, N-RN.2, N-RN.3
- Solving rational and radical equations
    - A-APR.6, A-APR.7+, A-REI.2
- Arithmetic and algebra with complex numbers
    - N-CN.1, N-CN.2, N-CN.3+
- Graphing points in the complex plane, and DeMoivre's Theorem
    - N-CN.4+, N-CN.5+, N-CN.6+
- Solving systems of equations and linear programming
    - A-REI.5, A-REI.6, A-REI.7, A-REI.12
- Factoring higher-order polynomials
    - N-CN.9+, A-APR.2
- Exponentials, logarithms and Logarithmic Laws
    - F-BF.5+

Functions and Modeling

- Definition of function, function notation, representations, domain and range
    - F-IF.1, F-IF.2, F-IF.5, F-IF.9, A-REI.10, A-REI.11
- Composition, transformations, and inverse functions
    - F-BF.1c, F-BF.3, F-BF.4
- Understanding modeling and models as a problem-solving process
    - N-Q.1, N-Q.2, N-Q.3, A-SSE.1, F-IF.4, F-IF.6
- Building models to describe situations
    - A-CED.1, A-CED.2, A-CED.3, F-BF.1a, F-BF.1b, F-LE.2, F-LE.3, F-LE.5, G-MG.1, G-MG.2, G-MG.3
- Graphing and modeling with linear, quadratic, absolute value and piecewise functions
    - A-SSE.3b, F-IF.7a, F-IF.7b, F-IF.8a, F-LE.1a, F-LE.1b
- Graphing and modeling with higher-order polynomials
    - A-SSE.3a, A-APR.3, F-IF.7c
- Graphing and modeling with exponential and logarithmic functions
    - A-SSE.3c, F-IF.7e(1), F-IF.8b, F-LE.1c, F-LE.4
- Graphing and modeling with other (rational, radical, logistic, etc.) functions
    - F-IF.7d+

Trigonometry

- Right triangles, trigonometric ratios, and applications
    - G-SRT.6, G-SRT.7, G-SRT.8
- Radian measure and the unit circle
    - F-TF.1, F-TF.2, F-TF.3+, F-TF.4+, G-C.5
- Oblique triangles, the Law of Sines, and the Law of Cosines
    - G-SRT.9+, G-SRT.10+, G-SRT.11+
- Graphing and modeling with the sine and cosine functions, and transformations
    - F-TF.5, F-TF.6+, F-TF.7+, F-IF.7e(2)
- Graphing tangent, cotangent, secant and cosecant functions
    - No Core Standards
- Proving and using trigonometric identities
    - F-TF.8, F-TF.9+

Statistics and Probability

- Independence, conditional probability, and expected values
    - S-CP.1, S-CP.2, S-CP.3, S-CP.4, S-CP.5, S-CP.6, S-CP.7, S-CP.8+, S-CP.9+, S-MD.2+
- Representing and interpreting univariate and bivariate data
    - S-ID.1, S-ID.2, S-ID.3, S-ID.4, S-ID.5, S-ID.6
- Understanding statistics as a decisionmaking process
    - S-ID.9, S-IC.1, S-IC.2, S-IC.3, S-IC.4, S-IC.5, S-IC.6, S-MD.5+, S-MD.6+, S-MD.7+
- Building, interpreting and using linear regression models
    - S-ID.7, S-ID.8
- The normal distribution, z-scores, and hypothesis testing
    - S-MD.1+, S-MD.3+, S-MD.4+

Advanced Topics

- Vectors, vector arithmetic, and applications
    - N-VM.1+, N-VM.2+, N-VM.3+, N-VM.4+, N-VM.5+
- Matrices and determinants
    - N-VM.6+, N-VM.7+, N-VM.8+, N-VM.9+, N-VM.10+, N-VM.11+, N-VM.12+, A-REI.8+, A-REI.9+
- Polar coordinates and polar functions
    - No Core Standards
- Parametric equations
    - No Core Standards
- Sequences and partial sums
    - F-IF.3, F-BF.2
- Arithmetic, geometric and harmonic series
    - A-SSE.4
- Conic sections: circles, ellipses, parabolas and hyperbolas
    - G-GPE.2, G-GPE.3+
- Graphical, numerical, and algebraic limits
    - No Core Standards

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