I describe two studies in firm dynamics and macroeconomics. Chapter 1 reports on the large decline in entrepreneurial activity that preceded and accompanied the Great Recession and proposes a model relating this decline to the housing collapse. The collapse in entrepreneurial activity coincided with a historic decline in home values that preceded the onset of the broad recession by at least nine months. I describe a heterogeneous agent general equilibrium model with both housing and entrepreneurship. The model is characterized by financial frictions that affect both credit supply and credit demand. I consider the consequences of a “housing crisis” as compared to a “financial crisis.” The model produces a negative response of entrepreneurship to a housing crisis via a housing collateral channel; this mechanism can account for at least a quarter of the empirical decline in entrepreneurs’ share of activity. A financial crisis (which works through credit supply) has more nuanced effects, causing economic disruption that entices new low-productivity entrepreneurs into production.
Chapter 2 describes a theory of endogenous firm-level risk over the business cycle based on endogenous firm market exposure. Firms that reach a larger number of markets diversify market-specific demand shocks at a cost. The model is driven only by total factor productivity shocks and captures the observed countercyclicality of firm-level risk. Consistent with the model, data from Compustat and the Longitudinal Business Database show that market reach is procyclical and that the countercyclicality of firm-level risk is driven mostly by those firms that adjust their market reach. This finding is explained by a negative elasticity between firm-level volatility and various measures of market exposure.
ESSAYS ON FIRM DYNAMICS AND MACROECONOMICS

By

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This thesis consists of two essays that emphasize the role of firm heterogeneity in accounting for aggregate fluctuations. Chapter 1 explores the relationship between housing and entrepreneurship, with a focus on the role of housing as collateral for entrepreneurial credit access. Various measures of entrepreneurial activity declined precipitously leading into and during the Great Recession in the U.S., and I describe a model that can explain a portion of the decline in entrepreneurship as being the result of the decline in the value of housing. Chapter 2 explores the relationship between firm-level volatility and the business cycle. In response to a large research literature that models the business cycle as being endogenous to volatility, I describe a model (constructed with coauthors) in which the business cycle actually drives volatility through a firm-level market diversification mechanism.

I summarize the chapters in more detail below, but first it is useful to take a step back and briefly describe the development of the firm dynamics research context in which the chapters fit. Much of modern macroeconomics involves the study of economic growth and aggregate fluctuations through the lens of formal models. In such models, the production of goods is often conducted by a single representative firm. This modeling approach can yield significant insights—both qualitative and quantitative—into the workings of market economies. However, a large body of research suggests that the ways in which firms differ, both in the cross section and over the lifecycle, have consequences for aggregate economic outcomes.
Firm dynamics is the study of the relationship between individual firms and the broader economy. This firm-based approach allows for models motivated by microeconomic data and accommodates rich interactions of firms with labor, financial, and product markets. The interactions can depend on individual firm characteristics including age, size, industry, and productivity; in turn, the nature of macroeconomic fluctuations may depend in part on the economy’s composition of firms along these dimensions (e.g., Pugsley and Şahin (2014)). Moreover, extensive reallocation of resources routinely occurs even within narrow classes of firms. Firm dynamics is the study of creative destruction—the costly but productivity-enhancing process by which firms, industries, and economies are constantly reinvented, originally defined and described by Joseph Schumpeter (Schumpeter (1935, 1942)).

The modern approach to modeling firm dynamics goes back at least to Jovanovic (1982). Motivated by evidence about the high and volatile growth of surviving young firms, Jovanovic constructed a model in which young firms must gradually learn about their own productivity and expand or contract according to what they learn. This framework can explain the high growth rates among young firms observed in the data. The selection mechanism through which productive firms grow and unproductive firms shrink or exit is a key concept in firm dynamics, and the idiosyncratic nature of firm-level productivity can account for heterogeneity in observed firm size distributions. Related work by Ericson and Pakes (1992) generates similar results with rich entry and exit dynamics. Lambson (1991) showed that firm-level sunk costs and uncertain future factor prices can give rise to wide within-industry dispersion in productivity, an ongoing topic of interest in firm dynamics.
research. While these studies occurred in the context of industrial organization research, they would have large implications in macroeconomics. Indeed, firm dynamics sits at the intersection of macroeconomics with industrial organization, labor, corporate finance, and other subfields.

Hopenhayn (1992) innovated on the firm dynamics framework by providing a tractable model that can account for generally high rates of gross job flows and the stochastically increasing size distribution of firms by age class. This class of models would prove extremely useful for understanding the relationship between firm dynamics and macroeconomics. For example, Hopenhayn and Rogerson (1993) showed that frictions that impede the job destruction process reduce total employment in the long run. Cooley and Quadrini (2001) showed that persistent firm-level productivity shocks in an environment characterized by financial frictions can generate the empirical regularities of declining rates of dynamism by firm size (conditional on age) and by firm age (conditional on size).

The development of models of firm dynamics provided a lens through which to interpret increasingly available microeconomic data on firm behavior. Davis, Haltiwanger, and Shuh (1996) and later work document a U.S. economy characterized by a high pace of job creation and job destruction, which theory suggests can be the result of constantly changing idiosyncratic conditions faced by individual firms—whether through demand shocks, factor price shocks, productivity
shocks, or the acquisition of new knowledge—and the ways in which firms respond to those conditions.¹

Davis, Haltiwanger, and Shuh (1996) show that the bulk of variation in plant-level outcomes cannot be explained by observable firm characteristics. Moreover, the data on firm dynamics present several serious challenges to the representative agent approach to the study of business cycles. Gross job destruction exhibits more volatility and cyclicality than gross job creation.² Total reallocation (the sum of job creation and job destruction) is typically countercyclical. The business cycle involves not only changes in average firm growth rates but also changes in the dispersion and skewness of the growth rate distribution. Furthermore, some of these facts are subject to change over time; for example, while most recent recessions were characterized by countercyclical job destruction and (relatively) acyclical job creation, the Great Recession involved a dramatic decline in job creation and a decline in total reallocation, resembling some pre-1970s recessions. Both the similarities and the differences between recessions suggest the importance of heterogeneity for growth and business cycle dynamics. Additionally, recent research indicates that various measures of gross flows and reallocation have seen declines in recent decades, particularly since 2000.³

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¹ Here, “job creation” refers to jobs added by entering or expanding business establishments; “job destruction” refers to jobs eliminated by shrinking or exiting business establishments. Reallocation is the sum of job creation and job destruction.

² Much progress has been made on this and related questions. For example, Caballero (2007) shows how factor specificity can explain the decoupling of job creation and job destruction.

³ See Davis and Haltiwanger (2014); Decker et al. (2014, 2015); Foster, Grim, and Haltiwanger (2014); and Hyatt and Spletzer (2013).
Among the macroeconomic implications of reallocation is its role in aggregate productivity growth. Productivity dispersion is significant, even within industries (Syverson (2011)); exploring the sources of this dispersion is a key element of the firm dynamics literature. Growth in aggregate productivity requires not only advances in technology and business methods but also movement of productive resources from low-productivity to high-productivity businesses. Evidence from the manufacturing industry suggests that about one-third of aggregate productivity growth is driven by reallocation (Foster, Haltiwanger, and Syverson (2008)). Firm entry is a key aspect of reallocation, and even the portion of productivity growth that occurs within establishments may be closely linked to questions of firm dynamics.

Changes in the distribution of productivity across firms can have large aggregate implications. Khan and Thomas (2013) study a model in which financial frictions and capital specificity facilitate large, persistent responses of aggregate productivity and output to financial shocks due to misallocation of capital among firms. Unproductive firms hold too much productive capital, and productive firms—particularly young firms—hold too little. Midrigan and Xu (2013) show that finance-driven misallocation problems are most pronounced along the margins of firm entry and technology adoption. More broadly, policy or other frictions that distort selection and reallocation can empirically account for large differences in aggregate performance across countries (Bartelsman, Haltiwanger, and Scarpetta (2013)).

Another significant contribution of firm dynamics research is a better understanding of the sources of aggregate employment growth. For a time, evidence suggested that small businesses were key contributors to job growth in the U.S.
Further research showed—consistent with theory described above—that this only reflected the facts that many young firms are small and that young firms account for a large share of both net and gross job creation (Haltiwanger, Jarmin, and Miranda (2013)). This evidence implies an important labor market role for entrepreneurship; in particular, since most entrepreneurs have little or no growth potential (Hurst and Pugsley (2011)), a large share of job growth is accounted for by a small number of high-growth, young firms.

A focus on entrepreneurship has become a significant part of firm dynamics research. Quadrini (2000) innovated on firm dynamics models by linking firms to households; his model shows that a link between firm credit access and household finances in the presence of financial frictions can explain large wealth differences between workers and entrepreneurs as well as the high wealth concentration observed in the U.S. Buera (2009) builds a similar model for exploring the relationship between personal wealth and entrepreneurship.\(^4\)

A key characteristic of entrepreneurship is the relationship between firm finances and the personal characteristics of firm owners. This relationship implies multiple feedback mechanisms between firm dynamics and the economy: broad economic conditions can affect entrepreneurial activity through household channels, while the evidence on young firms described above posits an important role for entrepreneurship in aggregate job market conditions.

The Great Recession was preceded and accompanied by a historically large decline in various measures of entrepreneurial activity. In Chapter 1, I describe

\(^4\) Empirically, the relationship between personal wealth and entrepreneurship is complicated; see Hurst and Lusardi (2004) and Fairlie and Krashinsky (2012).
original research suggesting that the collapse of house prices played a significant role in the decline in entrepreneurship. Housing is an important source of collateral for entrepreneurs, so the decline in the value of housing tightened financial conditions faced by young firms. A growing body of empirical evidence supports this proposed mechanism (Adelino, Schoar, and Severino forthcoming; Fort et al. (2013); Mehrotra and Sergeyev (2014)).

I conduct a quantitative theory exercise to understand the importance of this channel. I build a general equilibrium heterogeneous agent model in which households can choose to be workers or entrepreneurs. I use the model to run comparative static experiments contrasting the effects of a house price decline with the effects of an increase in borrowing costs across the economy. The model suggests that the decline in house prices may explain a quarter or more of the decline in entrepreneurs’ share of aggregate employment, where entrepreneurs are defined as young firms and/or sole proprietorships. More generally, my results indicate that accounting for the link between household characteristics and entrepreneurship is important for understanding the Great Recession—and it may be important in future downturns as well.

The research I describe in Chapter 1 suggests the need for further research into the relationship between housing and entrepreneurship. Quantifying the ultimate effect of the entrepreneurial collateral channel on aggregate unemployment is an important priority and can be done with extensions to the modeling framework I provide. It is also important to relate the entrepreneurial collateral channel to other channels through which housing affects the economy, such as the household
consumption channel (Mian, Rao, and Sufi (2013)) and the residential construction channel (Leamer (2007)).

Chapter 2 focuses on a different aspect of firm dynamics. There is increasing interest in the relationship between firm-level volatility and the business cycle in macroeconomics. One manifestation of this interest is a growing literature on the effects of uncertainty—the volatility of potential future outcomes—on the business cycle; this literature originated with Bloom (2009). The “Bloom Fact” is the empirical regularity that various measures of volatility, including outcome volatility at the firm level, are countercyclical—volatility rises during recessions and falls during expansions. Much of the existing research on this topic interprets causality as running from volatility to booms and busts. In Chapter 2, I describe original research (conducted jointly with Pablo D’Erasmo and Hernan Moscoso Boedo) suggesting that causality runs in the opposite direction through a mechanism that illustrates the value of the firm dynamics research agenda.

We constructed a model in which individual firms make decisions about how many product or geographic markets to service. Firms with exposure to more markets face less risk due to being more diversified against market-specific demand shocks. As a result, firm-level volatility is negatively correlated with the number of markets to which firms have exposure. Firm decisions about market reach depend in part on aggregate conditions: during booms, many firms can increase profits by expanding to more markets, thereby becoming less volatile (despite lacking an actual diversification motive). During recessions, many firms reduce their market exposure, making them more volatile. The result is countercyclical volatility—the Bloom
Fact—driven by aggregate first-moment shocks and the heterogeneous responses of firms to them.

We provide evidence in favor of our model by outlining several of its empirical predictions and comparing them with data. In the data, we find that firm-level market exposure is indeed procyclical; moreover, as predicted, firms that adjust their market exposure over the cycle experience countercyclical volatility (while firms that do not adjust experience acyclical volatility). The results of the study suggest that economists cannot treat volatility as exogenous to the business cycle—an insight that will be important in future research into the role of volatility and uncertainty in macroeconomics. More broadly, the study provides an example of how an understanding of the relationship between firm-level conditions and the broader economy can shed light on important questions in macroeconomics.

Chapter 1 contributes to the firm dynamics literature by highlighting the role of personal characteristics in entrepreneurial activity. Firm behavior—specifically the behavior of young and small firms—is closely linked to household behavior. In particular, the chapter adds to the growing list of reasons for economists to carefully study housing markets. Moreover, recessions that involve negative shocks to households are likely to be characterized by distressed entrepreneurship, and the literature that I describe above suggests that this has significant implications for aggregate employment growth. Chapter 2 innovates on the firm dynamics literature by, first, showing that the causal link between volatility and the business cycle is more nuanced than is typically assumed, and second, highlighting the role that product or market heterogeneity plays in driving firm-level outcomes. When the Law
of Large Numbers does not hold, idiosyncratic economic shocks do not “average out” as is assumed by the representative agent framework.

The common theme across the two chapters is that heterogeneity is important at both the theoretical and empirical levels in macroeconomics. Models that treat heterogeneity as an essential characteristic of the economy have the potential to explain empirical facts that are not consistent with workhorse representative agent approaches. These facts have large implications for productivity, labor markets, growth, and business cycles. Recent events have led to questions about the future path of the macroeconomics discipline (Caballero (2010)); a focus on heterogeneity has the potential to supplement the insights of the workhorse approach without discarding the important contributions of past research.
Dedication

For Tiffany and Richard.
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College Park, Maryland
May 8, 2015
Disclaimer

Any opinions and conclusions expressed herein are those of the author and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.
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Chapter 1: Housing, Entrepreneurship, and Job Creation

Section 1: Introduction

The Great Recession was preceded by a simultaneous decline in house prices and entrepreneurial activity, both of which were largely unprecedented in scale. Given the crucial job market role played by young firms, understanding the cause of the collapse in entrepreneurship is important. A growing empirical literature finds a close relationship between house prices and entrepreneurial activity and suggests the importance of housing collateral for entrepreneurial finance (Adelino, Schoar, and Severino (2013), Fort et al. (2013), Mehrotra and Sergeyev (2014)). Other literature points to credit supply frictions as key drivers of the Great Recession (Adrian, Colla, and Shin (2013)). What can account for the collapse in entrepreneurship? In particular, is the housing crisis to blame, or is the broader financial crisis that followed more likely to have caused the decline?

In this chapter, I explore the collapse in entrepreneurship by constructing a model to contrast the effects of a housing crisis (which affects entrepreneurship through credit demand) with a financial crisis (which affects all firms through credit supply). The contribution of the model is to show that entrepreneurial activity declines in response to a housing crisis but not a financial crisis. Housing is an important source of collateral for entrepreneurs, while corporate firms are financed independently of the personal assets of their owners. A broad financial crisis has
roughly the same direct effect on all types of firms, so it is incapable of causing a disproportionate decline in entrepreneurship. More generally, the model’s results suggest that recessions associated with a housing crisis are likely to be accompanied by depressed employment among entrepreneurs, with potentially broad implications for aggregate job growth.

Understanding the cause of the collapse in entrepreneurship is important because young firms play a disproportionate role in aggregate job creation (Haltiwanger, Jarmin, and Miranda (2013); Haltiwanger (2012)). Fort et al. (2013) suggest that young firms account for about 22 percent of the decline in net job growth associated with the Great Recession. Consider the simple, rough counterfactual reported in Figure 1. Define an “entrepreneur” as any firm that is less than six years old or is a sole proprietorship. In 2006, the share of non-construction employment accounted for by entrepreneurs was 17 percent.1 Suppose this share had stayed constant through 2011, while non-entrepreneurial employment remained as its actual annual levels observed in the data.2 As Figure 1.1 shows, counterfactual employment has a higher peak in 2008, and the counterfactual trough in 2010 exceeds actual employment by about two million jobs. The observation for 2011 further shows a stronger recovery in the counterfactual. This is not a rigorous counterfactual, but it illustrates the large role played by entrepreneurs in the Great Recession.3

1 Data are from the Census Bureau’s Longitudinal Business Database. Employment data are reported as of March 12 of a given year. The construction industry is defined as two-digit NAICS code 23.

2 Specifically, the counterfactual is constructed as follows: In 2006, non-construction entrepreneurs’ share of total non-construction employment was 17 percent. Then the non-entrepreneurial share of employment was 83 percent. Let e_t be non-entrepreneurial employment at time t, and let \( \hat{E}_t \) be counterfactual aggregate employment in time t. Then \( \hat{E}_t = e_t / 0.83 \) for every t.

3 A more detailed counterfactual is given by Sedlacek and Sterk (2014).
In principle, two key channels could link a housing crisis with the share of economic activity accounted for by entrepreneurs. The first is a housing demand mechanism, where entrepreneurs in the construction sector and related sectors become distressed when demand for new housing falls.\(^4\) In the present study I abstract from this channel. The other possible channel operates through the balance sheets of entrepreneurs. Houses form a significant share of household assets, and entrepreneurial credit access depends heavily on the household assets of firm owners. In the decade prior to the housing crisis, households were increasingly able to “use

\(^4\) In 2006 entrepreneurial firms accounted for 30 percent of construction employment, compared with 17 percent of non-construction employment and 18 percent of employment generally (LBD).
their house as an ATM,” including for business finance. In the model I describe below, entrepreneurial borrowing is constrained by the value of entrepreneurs’ personal asset holdings, including housing. The corporate sector faces no such constraints. This modeling approach is motivated by the notion that large, established firms can access credit through bond and commercial paper markets or large-scale bank borrowing, sources that do not depend on the asset holdings of firm owners. Conversely, entrepreneurs face constraints on borrowing based on their personal ability to supply collateral.

The effects of house prices on bank balance sheets and consumer spending have been studied extensively, and there is a large literature on the effect of financial constraints on job flows, the firm age (and size) distribution, and entrepreneurship. However, the relationship between housing and entrepreneurship has received less attention. I construct a heterogeneous agent DSGE model, based on the macroeconomic literature studying entrepreneurship, in which housing plays a collateral role for potential entrepreneurs. In the model, I define a “housing crisis” as (1) a decline in the house price caused by a taste shock, (2) a decline in the house price caused by a housing supply shock, or (3) a decline in the maximum loan-to-value ratio available to households. I define a “financial crisis” as a shock to intermediation costs that manifests itself through a higher credit spread. Thus, the housing crisis operates through a credit demand channel by affecting household balance sheets, while the financial crisis operates through a credit supply channel and affects all firms (rather than only entrepreneurs). I conduct a comparative static exercise by comparing stationary distributions and find that a housing crisis can
account for at least a quarter of the decline in entrepreneurs’ share of employment. In contrast, the financial crisis has large effects on total output but does not reduce entrepreneurial activity disproportionately.

The model includes a corporate sector that is not subject to collateral constraints. As a result, shocks to the entrepreneurs’ collateral constraint need not affect aggregate production, since corporate firms can increase output in response to reduced entrepreneurial activity. In this respect, the quantitative results I describe may be thought of as a lower bound on the effects of housing crises (and collateral shocks broadly) on entrepreneurial job creation. In the absence of a healthy non-entrepreneurial sector, aggregate demand effects could result in further reductions in entrepreneurial activity following a housing crisis.

The present study does not attempt to explain the path of house prices during the last decade, taking the large post-2006 decline as exogenous to aggregate entrepreneurial activity. I generate a housing price decline either with a shock to housing preferences assuming constant housing supply or with shock to housing supply assuming constant preferences.

The chapter proceeds as follows: Section 2 describes evidence and literature relevant to housing and entrepreneurship. Section 3 describes the model in detail. Section 4 describes the model calibration. Section 5 describes results from stationary distribution experiments. Section 6 concludes.
Section 2: Evidence and Previous Literature

2.1: Data on Housing and Young Firms

The Great Recession was preceded by a sharp decline in the number of new firms and the number of jobs created by startups and entrepreneurs (see Haltiwanger, Jarmin, and Miranda (2011)); this fact holds even when the construction sector is ignored. Figure 1.2 shows data on startups as a share of firms and employment, detrended linearly to abstract from long-term trends. While startup activity may be a leading indicator generally, the most recent startup collapse was historically large. The timing of the decline in startup activity coincides broadly with the peak and deterioration of house prices and home equity. Figure 1.3 plots measures of startup activity (omitting construction) against the S&P/Case-Shiller national house price index and the value of owners’ equity in real estate (from the Federal Reserve’s Flow of Funds), with all series normalized by their year-2000 levels. The top panel of Figure 1.3 shows the numbers of startup firms and startup jobs, and the bottom panel

---

5 I define a startup as a firm with age zero. Data on startups are from Business Dynamics Statistics, a publicly available dataset starting in the late 1970s that aggregates administrative data on the universe of private nonfarm establishments; establishments are observed in March of each year. Recession bars in Figure 1.2 are also based on annual data, where the following years count as recessions: 1981-1982, 1990, 2001, and 2008-2009. According to the NBER, the most recent recession began in December 2007 and ended in July 2009.

6 Many startups are small, and the decline in small firm job creation in the mid-2000s expansion has been documented before (see Moscarini and Postel-Vinay (2008) and Moscarini and Postel-Vinay (2012)). These explanations fail to distinguish between young and small firms, however, and given the specific role of young firms in job creation, additional focus on young firms is warranted. For data on the secular decline in entrepreneurship see Haltiwanger, Jarmin, and Miranda (2012) and Decker et al. (2014).

7 Federal Housing Finance Agency (FHFA) home prices show similar timing.
Figure 1.2: Startups’ Share of Activity

Startups' share of firms
Difference from linear trend


Startups' share of employment
Difference from linear trend

Figure 1.3: Startup Activity and House Prices

**Startup quantities (left) and housing (right)**

*Excluding construction*

- Startup firms
- Startup jobs
- Case Shiller Index
- Real home equity


**Startup shares (left) and housing (right)**

*Excluding construction*

- Startup job creation component
- Startup firm share
- Case Shiller Index
- Real home equity

shows startup firms and startup job creation as shares of overall firm totals. Both the house price series and the real estate equity series are reported quarterly; Figure 1.3 annualizes the quarterly data by simply using the first quarter of each year. Startup firm and jobs data are reported annually in March.

Readers should interpret the timing as follows: in the top panel of Figure 1.3, the number of startup firms and startup job creation peak in March 2006, and these peaks coincide with the peaks in first-quarter house prices and home equity. The bottom panel of Figure 1.3 shows that the share of aggregate job creation due to startups also peaks in March 2006. In short, both panels of the figure show that house prices and startup activity fell between March 2006 and March 2007, while the NBER defines the Great Recession as beginning in December 2007. Hence, the declines in house prices and startup activity led the recession by between nine and twenty-one months.

The timing indicates that demand-side factors or broad financial sector explanations for the link between startups and house prices may be inapplicable. For example, Figure 1.4 plots fixed nonresidential investment, investment in equipment and software, and fixed residential investment. The slowdown in startup activity that began between March 2006 and March 2007 was coincident with the decline in residential investment but preceded the decline in other measures of investment.

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8 The job creation measure is actually the startup component of the overall job creation rate, where the job creation rate is constructed as in Davis, Haltiwanger, and Shuh (1996).

9 Note that the quarterly S&P/Case-Shiller house price index is actually a three-month trailing moving average (S&P (2014)).

10 This measure is constructed using data reported in both March 2006 and March 2005; it is the number of jobs accounted for by startups as of March 2006 divided by the average of the number of economywide jobs reported in March 2006 and March 2005.
As another example, Figure 1.5 plots two measures of corporate spreads over time. Spreads did not begin rising until late 2007, and extreme highs were not reached until late 2008. The recession and broad financial sector stress started too late to explain the decline in entrepreneurial activity.

The empirical relationship between house prices and young firm activity is documented and explored more formally by Fort et al. (2013). The authors employ panel vector autoregression to isolate the effect of temporally exogenous variation in house prices on young firm activity at the state level, concluding that “the collapse in housing prices play[ed] a critical role” in the dramatic decline in new firm formation and young firm job flows prior to and during the Great Recession. More evidence is provided by Adelino, Schoar, and Severino (2013). Using estimates of housing supply
elasticities from Saiz (2010) as an instrument for house price changes at the Metropolitan Statistical Area (MSA) level, the authors find that exogenous increases in house prices between 2002 and 2007 (where “exogenous” is defined in terms of the elasticities instrument) were associated with increased employment in small businesses relative to large businesses. This effect is particularly strong for businesses in industries that typically rely heavily on external financing, and the effect is strong even in tradeable industries, implying that the effect of house prices is not solely due to the impact of house prices on local consumption demand. Mehrotra and Sergeyev
(2014) find similar MSA-level results for young firms.\textsuperscript{11} The present study provides a structural explanation for the empirical relationship between house prices and entrepreneurship.

2.2: Young and Small Business Credit

Firm size has a large impact on credit access (Cole and Wolken (1995)). Small businesses borrow primarily from commercial banks rather than by issuing debt securities. Small firms also frequently use credit cards and other “nontraditional” finance sources (Mishkin (2008); Small Business Administration (2011)). Many young firms are small and may therefore be subject to such financial constraints. Adrian, Colla, and Shin (2013) report that large firms were largely able to replace bank financing with bond financing in the wake of the financial crisis, but this is not an option available to most young and small firms.

Entrepreneurial credit often lies at the intersection of business and household lending (Cole and Wolken (1995)). The Federal Reserve Board conducts a quarterly survey of senior loan officers at approximately sixty domestic banks and some foreign banks operating domestically (all past surveys are available at Federal Reserve Board (2014)). The 2007 surveys indicate a gradual but steady move toward tighter mortgage lending standards, although they have no direct questions about collateral. The January 2008 survey found that “between about 70 percent and 80

\textsuperscript{11} In the present study I focus on the relationship between house values and the ability of entrepreneurs to finance business activities; a related literature examines the relationship between house values and consumer spending. That housing matters for consumer decisions and aggregate fluctuations has been well documented (see, e.g., Cooper (2013); Mian and Sufi (2011); Mian, Rao, and Sufi (2013); and Mian, Sufi, and Trebbi (2014)). However, the consumption channel cannot explain the disproportionate decline in entrepreneurial activity. Moreover, Fort et al. (2013) note that, given the tendency of young firm finances to be linked to household finances, the Mian et al. results are not inconsistent with a firm credit channel of house prices, with the link depending on firm age.
percent of domestic respondents expect the quality of their prime, nontraditional, and subprime residential mortgage loans, as well as of their revolving home equity loans, to deteriorate in 2008.” The April 2008 survey found that “all respondents pointed to declines in the value of the collateral significantly below the appraised value for the purposes of the HELOCs [home equity lines of credit] as reasons for tightening terms on those lines.” Subsequent surveys indicated ongoing tightening of HELOC standards. Overall, while the surveys do not provide direct evidence about housing collateral and business lending, they do indicate steadily stricter loan-to-value ratios and other standards starting in late 2006 as lenders began to notice that lending against houses was becoming riskier.12

The relationship between house prices and entrepreneurship has changed over the last two decades. Few small businesses reported reliance on mortgage credit in 1993 (Cole and Wolken (1995)), but this number increased during the 1990s (Bitler, Robb, and Wolken (2001)). Growth in entrepreneurs’ reliance on credit lines and bank loans during this period is also evident (Mach and Wolken (2006)), but the nature of the collateral backing these lines is difficult to determine in the data. By 2007, about one quarter of small business owners reported that they used home equity for business financing (Schweitzer and Shane (2010)); the evidence I report in

---

12 In June 2012, I collected some limited anecdotal evidence by interviewing several community bankers. Each of them reported that collateral was the key criterion used to evaluate loan applications from prospective business creators. The bankers gave some weight to earnings history for older firms, but new firms lack such history. The dominant (and almost sole) sources of collateral for new firms were structures, with personal homes being very common. One banker reported having watched local housing conditions carefully during the house price decline, even in the context of non-construction business lending decisions. The bankers also reported sharp declines in loan-to-value ratios. One banker reported that, prior to 2006, small businesses often received loans in excess of 100 percent of collateral value, while all bankers reported that loan-to-value ratios were typically below 80 percent during and after the collapse in house prices.
footnote 12 suggests that the share may be even higher for young businesses specifically.

I take these changes in entrepreneurial finance as exogenous; that is, I do not investigate why home equity-related business borrowing grew in recent decades. My model is not intended to explain entrepreneurial activity prior to the wide availability of home equity-based borrowing, so I would not expect the relationship between house prices and startups to be as strong in previous years. Regardless, my findings are likely to have broad implications not only for the Great Recession but also for future periods of housing crisis or collateral crisis generally.

While it is difficult to investigate the role of housing collateral for startup credit directly, it is clear that (a) new (and small) firms largely lack access to the types of credit facilities enjoyed by large firms, and (b) young and small firms rely heavily on external credit for operation (Robb and Robinson (2012)). Lacking any earnings history, new firms’ access to credit therefore depends in large part on collateral. These observations are consistent with an important role for the decline in housing collateral value in the recent decline of entrepreneurial activity.

2.3: Financial Constraints and Entrepreneurship

Several empirical studies have found evidence of financially constrained behavior among young firms, or at least dependence of growth paths on initial assets. These include Angelini and Generale (2008) (using Italian data); Huynh and Petrunia (2010) (using Canadian data); Chaney, Sraer, and Thesman (2012); and Kleiner (2014). The conditional size distribution of financially constrained firms is different
from the distribution of unconstrained firms, and the number of firms facing financial constraints is nontrivial. Firms with more initial assets grow faster, and real estate holdings in particular often serve as collateral and therefore can constrain investment and growth.

Theoretical research using heterogeneous agent models following Hopenhayn (1992) has found that including financial constraints can improve the models’ ability to match U.S. data. Firm dynamics models find that financial frictions can account for the conditional size and age distributions of firms (Cooley and Quadrini (2001); D’Erasmo (2011)) and can cause persistent drops in productivity and output (Khan and Thomas (2013)). Entrepreneurship in particular has been studied with a variety of structural models (see Quadrini (2009)). Quadrini (2000) divides production between corporate and non-corporate sectors, allowing for separation between firm and household decisions for large, established firms while allowing household characteristics to have a strong impact on startup activity. Models with entrepreneurship typically find significant consequences of borrowing constraints for households. Several recent studies focus on collateral and entrepreneurship in the Great Recession specifically (see Siemer (2014) and Mehrotra and Sergeyev (2014)).

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13 Buera (2009) estimates the welfare costs of borrowing constraints to be about 6 percent of lifetime consumption. Cagetti and De Nardi (2006) argue that “the tightness of borrowing constraints and voluntary bequests are the main forces in determining the number of entrepreneurs, the size of their firms, the overall wealth concentration in the population, and the aggregate capital accumulation” (866). See also Bassetto, Cagetti, and De Nardi (2015).

14 Hurst and Lusardi (2004) raised some controversy by finding no evidence of a strong relationship between household wealth and the entrepreneurship decision among PSID households. However, Fairlie and Krashinsky (2012) argue that the Hurst and Lusardi (2004) results are mitigated when one distinguishes between entrepreneurial entry decisions driven by job loss and other reasons for entry. Fairlie (2013) finds that the impact of declining home equity on entrepreneurship was partially offset
A growing literature studies housing in general equilibrium models (e.g., Iacoviello (2010), Iacoviello and Neri (2010), Iacoviello (2014), Guerrireri and Lorenzoni (2011), and Iacoviello and Pavan (2013)). Corradin and Popov (2013) construct a simple partial equilibrium model in which housing collateral matters for entrepreneurial decisions, but the model is highly stylized and largely abstracts from housing choice and differences between entrepreneurs and other firms.

To summarize, quantitative theory exercises and empirical evidence suggest that borrowing constraints are highly relevant for new businesses, and there is evidence that such constraints were among the key drivers of the real economy before and during the Great Recession. Moreover, as the primary element of household balance sheets, home equity plays a potentially significant role in the financing constraints facing new firms. The unique contribution of the present study is to investigate the pre-Great Recession collapse in entrepreneurial activity by comparing the consequences of a housing crisis (which acts as a credit demand friction) with a financial crisis (which acts as a credit supply friction) in a general equilibrium model that recognizes differences between entrepreneurs and larger, established firms.

Section 3: Model

Consider a model of entrepreneurship based on Buera, Kaboski, and Shin (2011) but augmented with housing (as in Iacoviello and Pavan (2013)) and a corporate production sector (as in Quadrini (2000)). Households choose whether to be

by increased job loss in the Great Recession, as weak labor markets drove some people into self-employment.
workers or entrepreneurs. Housing can be owned or rented; owned housing can be used as collateral for consumer loans or capital rental. Housing supply is exogenous and constant. Households are distributed over a four-dimensional state space based on financial assets, owned housing, previous occupational status (so that new entrepreneurs may differ from incumbents) and entrepreneurial productivity (which is responsible for generating a wealth distribution). Output is produced by both entrepreneurs and a representative corporate firm.

A zero-profit financial sector borrows from households with positive savings to supply loanable funds, rental housing, and productive capital to entrepreneurs and the corporate firm. Loanable funds and rental capital are produced at an exogenously determined cost; in equilibrium this cost is manifested as a credit spread paid by borrowers and firms.

3.1: Environment

Households

There is a unit measure of infinitely lived households, each of which chooses either to be an entrepreneur or to supply one unit of labor. Preferences are given by:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t, m_t)\right],$$

where $c_t$ denotes nondurable consumption, $m_t$ denotes the flow of housing consumption, and $\beta \in (0,1)$ is the discount factor. Hereafter I use recursive notation such that $x$ indicates current-period flows and $x'$ indicates intertemporal choices made in the current period. The period utility function has the following form:
\[ u(c, m) = \frac{c^{1-\sigma_c}}{1 - \sigma_c} + \varepsilon \frac{(km)^{1-\sigma_h}}{1 - \sigma_h}, \]  

(1)

where \( \kappa \) governs a utility penalty for renting, with \( \kappa < 1 \) for renters and \( \kappa = 1 \) for homeowners (as in Iacoviello and Pavan (2013)). In the computational exercises described below, housing is discrete and can be owned in increments of 500 square feet, and rented houses cannot exceed 2,500 square feet.\(^{15}\) The household’s owned housing stock is given by state variable \( h \), where

\[ h = \begin{cases} 
  m & \text{if owner} \\
  0 & \text{if renter} 
\end{cases} \]  

(2)

Whether a household owns or rents and, if owning, the amount of owned housing consumed are chosen in the previous period. This reflects time costs associated with home purchases and is consistent with the timing of the borrowing constraint described below. Note that this implies that the binary tenure decision is also made one period in advance, but the rental housing quantity decision is made in the same period in which it is consumed.

Housing is durable and does not depreciate. Nondurable consumption is the numeraire. Housing can be purchased at price \( q \) or rented from the financial intermediary at rate \( r^h \) (such that rented housing in quantity \( m \) costs \( r^h qm \)). Owned housing adjustment is costly as in Iacoviello and Pavan (2013) with cost equal to \( \psi qh \) (this cost applies whenever \( h' \neq h \)). This cost may be thought of as including realtor fees or renovation inconveniences, costs that are proportional to the market value of housing and housing materials. Note that a household changing from owning to renting pays this fee, as does an owning household that changes the size of its house.

\(^{15}\) The 2007 American Housing Survey reports that less than 9 percent of renting households live in houses larger than 2,500 square feet. In the model, the cap on rental unit size captures the notion that homeowners have greater latitude to choose large houses.
A household changing from renting to owning does not pay this cost as \( h = 0 \) in this case. Rental housing contracts last only one period and are opened and closed without friction, so renting households can change their housing consumption freely. This setup is an attempt to capture the notion that housing consumption is much more flexible for renters than for owners.

Households can be entrepreneurs \((e = 1)\) or workers \((e = 0)\); entrepreneurs have profits from decreasing returns to scale production while workers receive wage \( w \). The household has access to asset \( a \) for saving or borrowing at rate \( r^a \).

All household borrowing—both loanable funds and capital rental—is subject to a borrowing constraint:

\[
k < a + \phi q h, \tag{3}
\]

where \( k \) is capital demand for entrepreneurial production (with \( k = 0 \) for workers). Positive \( a \) indicates positive savings while negative \( a \) reflects borrowing; households may borrow up to the collateral value of their owned housing, given by \( \phi q h \) (so renters may not borrow at all and must have positive savings to engage in entrepreneurial production). When \( k = 0 \), the borrowing constraint may be thought of as a simple mortgage with loan-to-value ratio \( \phi \). The borrowing constraint is the model’s mechanism for making entrepreneurial decisions and profits dependent on household balance sheets.

Capital is not owned by firms, so in effect entrepreneurial households face a requirement that working capital must be financed through borrowing. However, consider a special case in which \( h = 0, a > 0, \) and the rental rate on capital is not subject to a credit spread. This is equivalent to a model in which part (or all) of a
household’s financial wealth is held as physical capital, so entrepreneurial capital is not rented but owned by the entrepreneurial household. Therefore, the working capital constraint setup differs from entrepreneurial ownership of capital when there is an intermediation cost (credit spread) or when the household owns housing.

Housing is a versatile asset, since the ability to borrow against it to rent capital makes housing similar to owning productive capital.

The borrowing constraint given by (3) may be thought of as a reduced-form simplification of borrowing conditions motivated by agency problems. For example, Buera, Kaboski, and Shin (2011) derive a capital rental constraint from an exogenous default recovery parameter faced by banks along with a no-default condition. In the event of default, banks can recover a fixed portion of entrepreneurial profits along with wealth deposited with the financial intermediary; the bank derives the borrowing constraint by choosing the maximum amount of loaned capital at which the household prefers repayment to default. The resulting level of permissible capital rental is increasing in household wealth and entrepreneurial productivity. My simplified borrowing constraint has similar properties except that it discards the role of productivity in the constraint for tractability purposes. Further, I apply the maximum loan-to-value parameter $\phi \leq 1$ to housing (but not liquid wealth) based on the notion that liquid wealth is relatively easy for the bank to seize and use while foreclosed housing is characterized by costs and risks that render it less valuable as loan security (from the bank’s perspective). In my specification, the borrowing constraint works like a technology allowing households to extract cash from their home (“use it as an ATM”).
Production

Entrepreneurs and the corporate sector produce the same final good which can be consumed, traded for housing, or (via the financial intermediary) used as productive capital. All firms rent capital from the financial intermediary at rate $r^k$ and hire labor from a common labor market at wage $w$. Capital is rented, utilized, and returned to the financial intermediary within the period, so it is not an intertemporal decision for firms. Capital depreciates at rate $\delta_k$.

The corporate sector consists of a representative firm with the following constant returns to scale technology:

$$Y_c = Z_c K_c^\xi N_c^{1-\xi},$$

where $\xi \in (0,1)$, $Z_c$ is (unchanging) corporate productivity, which will be normalized to 1, and $K_c$ and $N_c$ are corporate capital and labor demand, respectively. Corporate production does not involve fixed costs.

Entrepreneurs are households that choose to operate their decreasing returns to scale productive technology:

$$y_e = zk^\alpha n^\theta,$$

where $\alpha \in (0,1)$, $\theta \in (0,1)$, and $\alpha + \theta < 1$. Capital demand is given by $k$ and labor demand is given by $n$. All households receive an idiosyncratic entrepreneurial productivity draw $z$ from a $Pareto(x_m, \eta)$ distribution. Household productivity is persistent: each period, there is probability $\gamma$ that the household keeps its productivity draw from the previous period. Households observe their entrepreneurial productivity before making their occupational decision. For the sake of generality, entrepreneurs may pay a fixed setup cost $v$ upon entry; however, in the calibration described below
I set $\psi = 0$. The state variable $s$ tracks entrepreneurial entry and is defined as follows:

$$s = \begin{cases} 
0 & \text{if the household was an entrepreneur in the previous period} \\
1 & \text{if the household was a worker in the previous period} 
\end{cases}$$

Financial Sector

A representative financial intermediary borrows from savers at interest rate $r$ and uses the savings to finance loans, capital, and rental housing. The financial intermediary owns all of the economy’s capital and rental housing and therefore suffers depreciation losses. Additionally, conversion of savings into capital and loanable funds is subject to an exogenous marginal intermediation cost $\tau$. Technology for the production of capital, the allocation of loans, and the acquisition of rental housing is linear.

3.2: Recursive Competitive Equilibrium

Define $\mu(s, a, h, z)$ as the distribution of households over the state space. Then

$$\sum_{s=0}^{1} \int_{a} \int_{h} \int_{z} \mu(s, a, h, z) dzdhd\alpha = 1.$$  

The distribution of households over the state space follows the law of motion

$$\mu'(s, a, h, z) = \Psi(\mu(s, a, h, z))$$

where $\Psi$ depends on optimal policy rules $a'$ and $h'$; the law of motion for $z$, given by

$$z' = \begin{cases} 
z & \text{with probability } \gamma \\
z' \sim P(z) & \text{with probability } 1 - \gamma 
\end{cases}.$$  

---

16 Setup costs are likely significant for new entrepreneurs. However, as Buera, Kaboski, and Shin (2011) note, setup costs increase the influence of borrowing constraints. Since setup costs are difficult to measure empirically, I take a conservative route by omitting them.
and the law of motion for \( s \), given by

\[
s' = \begin{cases} 
1 & \text{if } e(s, a, h, z) = 0 \\
0 & \text{if } e(s, a, h, z) = 1 
\end{cases}
\]

Consider the following descriptions of model aggregates. \( A_s' \) is the total savings of households with positive financial assets (savers), while \( A_b' \) is the total borrowing of households with negative financial assets (so \( A_b' < 0 \)). \( H' \) and \( M_r \) are total housing demand among buyers and renters, respectively, and \( H_{adj} \) is the total owned housing stock among households that adjusted their housing holdings. \( C \) is total nondurable consumption. \( N_s \) is total labor supply (i.e., the number of households that are workers), and \( N_e \) is the total labor demand among entrepreneurs. \( K_c \) and \( Y_e \) are entrepreneurial capital demand and output, respectively. \( S_e \) is the total number of startups. These aggregates are constructed using state variables, policy functions, and the household distribution \( \mu(s, a, h, z) \), and I define them in detail in Appendix A1.1.

Assume that \( Y_c > 0 \). The representative corporate firm maximizes profits with the following first-order conditions:

\[
r^k = \xi Z_c \left( \frac{K_c}{N_c} \right)^{\xi-1}
\]

\[
w = (1 - \xi) Z_c \left( \frac{K_c}{N_c} \right)^\xi.
\]

This implies the following equilibrium relationship between the wage and the capital rental rate:

\[
w = (1 - \xi) Z_c \left( \frac{r^k}{\xi Z_c} \right)^{\frac{\xi}{\xi-1}}.
\]

Since \( \xi < 1 \), this implies an inverse relationship between \( r^k \) and \( w \) for a given \( Z_c \).
Households maximize the value of lifetime utility, which is given by

$$v(s, a, h, z) = \max_{e \in \{0, 1\}} \{v^{e=0}(s, a, h, z), v^{e=1}(s, a, h, z)\}$$

where $v^{e=0}(s, a, h, z)$ is the value of being a worker and $v^{e=1}(s, a, h, z)$ is the value of being an entrepreneur.

Households that choose to be workers ($e = 0$) solve the following problem:

$$v^{e=0}(s, a, h, z) = \max_{c \geq 0, a'} \{u(c, m) + \beta[\gamma v(1, a', h', z) + (1 - \gamma)e^{z'}]v(1, a', h', z')\}$$

subject to

$$c + a' + q(h' - h) + \mathbb{I}_{\text{rent}} r^h q \psi h + \mathbb{I}_{h' \neq h} \psi q h \leq w + (1 + r^a) a$$

$$0 \leq a + \phi q h,$$

where $\mathbb{I}_{\text{rent}}$ indicates housing tenure with

$$\mathbb{I}_{\text{rent}} = \begin{cases} 1 & \text{if } h = 0 \text{ (renter)} \\ 0 & \text{if } h = m \text{ (owner)} \end{cases}$$

and $\mathbb{I}_{h' \neq h}$ indicates housing adjustment with

$$\mathbb{I}_{h' \neq h} = \begin{cases} 1 & \text{if } h' \neq h \\ 0 & \text{if } h' = h \end{cases}.$$

The $s$ variable in the worker’s value function $v^{e=0}(s, a, h, z)$ is trivial, since $s$ does not affect the worker’s choices. The definition of $\mathbb{I}_{\text{rent}}$ given by (10) provides mutually exclusive possible values if the housing utility term satisfies the Inada condition, as in my specification (therefore $m > 0$).

For households that own housing ($h > 0$), the term $m$ in the utility function is predetermined. Households that hold the same amount of owned housing from period to period do not pay adjustment cost $\psi q h$. 
Households that choose to be entrepreneurs \((e = 1)\) solve the following problem:

\[
v^{e=1}(s, a, h, z) = \max_{c \geq 0, a', h'} \{ u(c, m) + \beta [\gamma v(0, a', h', z) + (1 - \gamma) E_z' v(0, a', h', z)] \}
\]

subject to

\[
c + a' + q(h' - h) + \Pi_{rent} r^h q m + \Pi_{h'xh} \psi q h
\]

\[
\leq zk^\alpha n^\theta - r^k k - wn - sv + (1 + r^a) a
\]

\[
k \leq a + \phi q h,
\]

where \(\Pi_{rent} \) and \(\Pi_{h'xh} \) are defined in (10) and (11), respectively, and \(v\) is the fixed entry cost paid by new entrepreneurs \((s = 1)\). Again, \(m\) is predetermined for homeowners \((h > 0)\). In general the household policy functions must be obtained numerically. However, the entrepreneurial capital and labor demand functions can be solved analytically using Kuhn-Tucker conditions. First, assume that the borrowing constraint does not bind. Then the entrepreneurial profit maximization problem is independent of both the household’s state and the household’s other decisions. The first-order conditions are:

\[
k^u(z; r^k, w) = \left[ \frac{r^k}{az} \left( \frac{\theta z}{w} \right)^{\theta - 1} \right]^{\frac{1}{\theta - 1}}
\]

\[
n^u(z; r^k, w) = \left[ \frac{w}{z^\theta k^u(z; r^k, w)^\alpha} \right]^{\frac{1}{\theta - 1}},
\]

where \(k^u\) and \(n^u\) indicate unconstrained capital and labor demand, respectively. The unconstrained factor demand functions depend only on the wage \(w\), the capital rental rate \(r^k\), and productivity \(z\). I have intentionally expressed labor demand as a function
of capital demand, which simplifies the intuition in the constrained case. Now suppose that the borrowing constraint binds. Then labor demand still directly depends only on factor prices, productivity, and capital demand, but capital demand is given by the binding constraint. Hence, actual capital and labor demand functions are given by

\[
k(z, a, h; r^k, w, q) = \begin{cases} 
\left[ \frac{r^k}{\theta z \left( \frac{w}{a} \right)} \right]^\frac{\theta - 1}{1 - \alpha - \theta} & \text{if } k^u(z, r^k, q) \leq a + \theta q h \\
\alpha + \theta q h & \text{otherwise}
\end{cases} \tag{15}
\]

\[
n(z, a, h; r^k w, q) = \frac{w}{z \theta k(z, a, h; r^k, w, q)^\alpha} \tag{16}
\]

That is, capital demand equals its unconstrained level if that level is consistent with the borrowing constraint. Labor demand depends indirectly on the borrowing constraint because it is a function of capital demand. When the borrowing constraint binds, both factor demand functions depend on household assets, the house price, and the loan-to-value ratio. The constraint on capital and labor demand implies a constraint on both entrepreneurial output and profits any time the entrepreneurs’ optimal scale is larger than that allowed by the borrowing limit. This is the key mechanism for this model: a household with little financial and housing wealth can receive a large productivity draw but be restricted to operating at a scale that is well below the unconstrained optimal level, or may even choose to be a worker instead.

Taking the cost of funds \( r \) as given, the financial intermediary solves the following profit maximization problem:

\[
\max_{K_c, K_e, A_h, M_r} r^k (K_c + K_e) - r^a A_h + r^h q M_r - \delta_k (K_c + K_e) - \tau (K_c + K_e - A_h) - r A_s
\]
subject to

\[ K_c + K_e - A_b + qM_r = A_s, \]

that is, the financial intermediary lends out the total amount of household savings, dividing it between physical capital, loanable funds, and rental housing. Here I have defined \( r \) as the rate at which the financial intermediary borrows from saving households. The first-order conditions corresponding to the intermediary’s choice variables are:

\[ K_c, K_e : \quad r^K = r + \delta_K + \tau \quad (17) \]
\[ A_b : \quad r^a = r + \tau \text{ (for borrowers)} \quad (18) \]
\[ M_r : \quad r^h = r \quad (19) \]

Thus, the intermediation cost manifests itself as a credit spread with respect to the interest rate paid to savers. Agents that borrow from the bank must pay the basic interest rate along with extra payment to cover relevant depreciation and intermediation costs. Note that from the households’ perspective, \( r^a \) is defined as follows:

\[ r^a = \begin{cases} r & \text{if } a \geq 0 \\ r + \tau & \text{if } a < 0 \end{cases} \]

It is simple to show that the financial intermediary, while owned by the households, has zero profits (see Appendix A1.1).

In addition to the set of policy functions that solve the household’s problem, equilibrium requires the following. The financial market clears:

\[ K_c + K_e + qM_r = A_s + A_b \quad (20) \]

The labor market clears:
\[ N_c + N_e = N_s . \]  
\[ (21) \]

Define \( H_s \) as the exogenously set supply of housing. The housing market clears:

\[ H + M_r = H_s . \]  
\[ (22) \]

The aggregate resource constraint holds:

\[
Y_e + Y_c - \psi q H_{adj} - \tau(K_c + K_e - A_b) - S_e \psi \\
= \mathcal{C} + (K'_c - (1 - \delta_k)K_c) + (K'_e - (1 - \delta_k))
\]  
\[ (23) \]

where \( S_e \) is the number of startups.\(^{17}\) The resource constraint simply requires that consumption and investment be equal to total output after the costs of frictions—credit intermediation costs, housing adjustment costs, and firm entry costs. Housing investment does not appear in the resource constraint because the aggregate housing stock does not change over time. While it is uncommon for prices to appear in aggregate resource constraints, observe that the house price \( q \) enters here because housing adjustment involves the sale or purchase of housing materials. The house price is the rate at which output goods can be exchanged for housing materials. Hence, a reduction in \( q \) makes agents wealthier in aggregate by reducing the costs of adjusting house holdings, leaving more units of output available for consumption and investment. A model with an actual construction sector may not have this property.

The stationary distribution is characterized by equilibrium prices and allocations with the distribution law of motion at a fixed point:

\[ \mu^* = \Psi(\mu^*(s, a, h, z)). \]

\(^{17}\) I provide the derivation of the resource constraint from household budget constraints in Appendix A1.1.
Section 4: Calibration

I take several parameter values from existing literature; these are reported in Table 1.1. Many are standard values. I calibrate other parameters to match key moments from U.S. data. These are reported on Table 1.2.

Table 1.1: Literature-based Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative risk aversion, nondurables</td>
<td>$\sigma_e$</td>
</tr>
<tr>
<td>Housing adjustment cost</td>
<td>$\psi$</td>
</tr>
<tr>
<td>Capital depreciation</td>
<td>$\delta_k$</td>
</tr>
<tr>
<td>Entrep. output-to-capital elasticity</td>
<td>$a$</td>
</tr>
<tr>
<td>Entrep. output-to-labor elasticity</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Corporate output capital share</td>
<td>$\xi$</td>
</tr>
</tbody>
</table>

Table 1.2: Moment-based Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.95</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Housing supply</td>
<td>$H_s$</td>
<td>2.5</td>
<td>2.5</td>
<td>AHS 2007</td>
</tr>
<tr>
<td>Housing taste</td>
<td>$\varepsilon$</td>
<td>0.20</td>
<td>Housing/GDP</td>
<td>1.8</td>
</tr>
<tr>
<td>RRA, housing</td>
<td>$\sigma_h$</td>
<td>1.93</td>
<td>Pop. &gt;4000 sq ft</td>
<td>0.10</td>
</tr>
<tr>
<td>Rent penalty</td>
<td>$\kappa$</td>
<td>0.99</td>
<td>Share renters</td>
<td>0.32</td>
</tr>
<tr>
<td>Ent. TFP scale</td>
<td>$x_m$</td>
<td>1.18</td>
<td>0.176</td>
<td>LBD 2006</td>
</tr>
<tr>
<td>Ent. TFP shape</td>
<td>$\eta$</td>
<td>7.59</td>
<td>$n_{\min}/n_{\max}$</td>
<td>0.05</td>
</tr>
<tr>
<td>TFP persistence</td>
<td>$\gamma$</td>
<td>0.78</td>
<td>$N_{\text{startups}}/N$</td>
<td>0.03</td>
</tr>
<tr>
<td>LTV ratio</td>
<td>$\phi$</td>
<td>0.92</td>
<td>Max LTV ratio</td>
<td>0.92</td>
</tr>
<tr>
<td>Lending cost</td>
<td>$\tau$</td>
<td>0.009</td>
<td>Credit spread</td>
<td>0.009</td>
</tr>
</tbody>
</table>

The discount factor $\beta$ is chosen to obtain an interest rate of 4 percent, a standard value in the literature. Other calibrated parameters are chosen such that the model resembles the U.S. economy around the year 2006, the peak of both house prices and startup activity in the U.S. The 2007 American Housing Survey (AHS) reports an average home size of 2,500 square feet (2007 is the closest year to 2006 for
which the AHS exists). As mentioned above, I assume that housing exists in units of 500 square feet, so 2.5 housing units is 2,500 square feet. Since the economy has a population normalized to one, an average house size of 2,500 square feet is equivalent to an aggregate housing supply of the same size. Hence, I set $H_s = 2.5$.

Parameters governing housing demand are calibrated as follows. I calibrate $\epsilon$ so that the model produces a realistic value for the economy’s owned housing stock (which directly relates to collateral value); I target the ratio of the value of housing to GDP, which was about 1.8 in the first quarter of 2006. The numerator is the total (nominal) market value of all real estate held by households and nonprofit organizations, reported by the Federal Reserve’s Flow of Funds data; the denominator is nominal GDP. Since the Flow of Funds measure does not include rental properties owned by businesses, in the model I define this ratio using owned housing only. Setting $\epsilon = 0.20$ yields a perfect match for this ratio. I choose $\sigma_h$ to match an aspect of the housing distribution, in particular the proportion of households living in houses larger than 4,000 square feet. An unrealistically large share of households living in large houses would result in too much collateral in the economy. The quantity is given by the 2007 AHS, which reports house size data in 500 square foot increments (as in the model). This share in the data is about 5 percent; setting $\sigma_h = 1.93$ yields a share of 10 percent in the model. I calibrate the utility penalty for renting, given by $\kappa$, so that the proportion of households that rent approximately matches the U.S. rentership rate, which the AHS reports as about 32 percent in 2007. Setting $\kappa = 0.99$ yields a perfect match of the rentership rate. In general, the model matches the targeted housing distribution moments well.
The parameters of the $Pareto(x_m, \eta)$ distribution for $z$ are chosen to match the firm size distribution. While there is no objective definition of “entrepreneur” in the data, the model is motivated by credit constraints that are faced by young firms and other firms that have close ties to their owners’ personal finances. As such, in the data I describe a firm as an entrepreneur if it is less than six years old or is legally organized as a sole proprietorship. The scale parameter $x_m$ is chosen to match the share of employment accounted for by entrepreneurs, which was 17.6 percent in 2006 according to the Longitudinal Business Database (LBD). Setting $x_m = 1.18$ achieves a perfect match. Since the driver of heterogeneity in the model is $z$, the shape parameter $\eta$ is used to ensure a wide distribution of firms. The relevant model moment is the ratio of the smallest to the largest entrepreneur in terms of employment. To avoid extreme outliers, the corresponding data moment is the ratio of the employment of the 5th percentile entrepreneur to the 95th percentile entrepreneur from the 2006 LBD. This ratio in the data is 0.04; setting $\eta = 7.59$ results in the model producing a ratio of 0.05. The model matches the firm distribution quite well.

To target the share of employment accounted for by startups I use the productivity persistence parameter $\gamma$. This parameter governs firm churn in the model because it partly determines how many existing worker households receive new productivity draws that could induce them into entrepreneurship. In the 2006 Business Dynamics Statistics (BDS), startups account for 3 percent of employment. Setting $\gamma = 0.78$ achieves this share in the model.

The remaining parameters are the loan-to-value ratio $\phi$ and the intermediation cost $\tau$. The baseline value for the maximum loan-to-value ratio $\phi$ is based on data
from Kalita (2011), which reports median loan-to-value ratios on new mortgages. The baseline value for the intermediation cost $\tau$, which determines the exogenous credit spread, is based on the average of the Bank of America Merrill Lynch US Corporate Master Option-Adjusted Spread for the year 2006. In what follows, my policy experiments consist of varying $\tau$, $\phi$, the housing taste parameter $\varepsilon$, and the housing supply $H_s$.

The solution method is described in Appendix A1.2.

Section 5: Results

5.1: Main Experiments

I use the model described above to conduct four main experiments which are described concisely on Table 1.3 (the “Baseline” scenario is the model as calibrated to 2006-2007 data above). The purpose of these exercises is to compare the effects of a housing crisis and a financial crisis on entrepreneurial activity. I consider three different “housing crisis” scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\varepsilon$</th>
<th>$H_s$</th>
<th>$\phi$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Baseline”</td>
<td>0.20</td>
<td>2.5</td>
<td>0.92</td>
<td>0.009</td>
</tr>
<tr>
<td>“Taste”</td>
<td>0.14</td>
<td>2.5</td>
<td>0.92</td>
<td>0.009</td>
</tr>
<tr>
<td>“Supply”</td>
<td>0.20</td>
<td>3.2</td>
<td>0.92</td>
<td>0.009</td>
</tr>
<tr>
<td>“LTV”</td>
<td>0.20</td>
<td>2.5</td>
<td>0.78</td>
<td>0.009</td>
</tr>
<tr>
<td>Financial crisis</td>
<td>0.20</td>
<td>2.5</td>
<td>0.92</td>
<td>0.028</td>
</tr>
<tr>
<td>Combination</td>
<td>0.14</td>
<td>2.5</td>
<td>0.78</td>
<td>0.028</td>
</tr>
</tbody>
</table>
In the first housing crisis scenario, I reduce housing tastes to induce a lower equilibrium house price. In particular, I change the housing taste parameter \( \varepsilon \) from 0.20 (the baseline value calibrated above) to 0.14. I choose this value to produce a housing-to-GDP ratio of 1.2, corresponding to Flow of Funds data for the first quarter of 2011 (all other parameters are left as in Tables 1.1 and 1.2). This scenario is called “Taste” in the tables that follow.

In the second housing crisis scenario, I induce a lower equilibrium house price by expanding the exogenously set housing supply \( H_s \). I increase \( H_s \) by the amount necessary to obtain the same lower house price as that obtained in the Taste experiment, leaving all other model parameters as in Tables 1.1 and 1.2. This requires changing \( H_s \) from 2.5 to 3.2; in other words, it requires that I expand the average house size by 700 square feet or 28 percent. Note that this is equivalent to a partial equilibrium experiment in which the house price is chosen manually. This scenario is called “Supply” in the tables that follow.

In the third housing crisis scenario, I change the permissible loan-to-value ratio parameter \( \phi \) from 0.92 to 0.78; the latter value corresponds to 2011 data from Kalita (2011). Again, all other parameters are left as in Tables 1.1 and 1.2. The loan-to-value ratio affects the household borrowing constraint in the same way as the house price, determining the collateral value of owned housing. This scenario is called “LTV” in the tables that follow. The concept that unifies the three housing crisis scenarios is the notion of a reduction in the collateral value of housing.

Table 1.4 describes key model outcomes for the housing crisis scenarios described above.
Observe that the reduction in the taste for housing causes a decline of the stationary state house price of about 32 percent. Likewise, the S&P/Case-Shiller U.S. National Home Price Index shows a 32 percent decline across the calibration period (from the first quarter of 2006 to the first quarter of 2011). The expansion of the housing supply obtains the same house price drop because it was calibrated to do so. In both scenarios, entrepreneurial activity is significantly below the baseline scenario, as hypothesized. Table 1.5 compares entrepreneurs’ and startups’ share of employment in the model—in particular, the switch from the baseline scenario to the high-house-supply scenario—with U.S. data from the LBD. The “Model” and “Data”

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Taste</th>
<th>Supply</th>
<th>LTV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>1.68</td>
<td>1.15</td>
<td>1.15</td>
<td>1.65</td>
</tr>
<tr>
<td>$r$</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$w$</td>
<td>1.16</td>
<td>1.16</td>
<td>1.16</td>
<td>1.16</td>
</tr>
<tr>
<td>Output</td>
<td>1.74</td>
<td>1.73</td>
<td>1.73</td>
<td>1.74</td>
</tr>
<tr>
<td>Entrep. output</td>
<td>0.39</td>
<td>0.35</td>
<td>0.37</td>
<td>0.38</td>
</tr>
<tr>
<td>Entrep. employment</td>
<td>0.166</td>
<td>0.147</td>
<td>0.158</td>
<td>0.161</td>
</tr>
<tr>
<td>Entrep. share</td>
<td>0.176</td>
<td>0.155</td>
<td>0.167</td>
<td>0.170</td>
</tr>
<tr>
<td>Startup employment</td>
<td>0.028</td>
<td>0.025</td>
<td>0.026</td>
<td>0.027</td>
</tr>
<tr>
<td>Startup share</td>
<td>0.030</td>
<td>0.026</td>
<td>0.028</td>
<td>0.029</td>
</tr>
<tr>
<td>Entrep. ownership rate</td>
<td>0.95</td>
<td>0.96</td>
<td>1.00</td>
<td>0.74</td>
</tr>
<tr>
<td>Worker ownership rate</td>
<td>0.66</td>
<td>0.64</td>
<td>0.98</td>
<td>0.52</td>
</tr>
<tr>
<td>Aggregate savings</td>
<td>5.94</td>
<td>5.70</td>
<td>4.85</td>
<td>6.36</td>
</tr>
<tr>
<td>Savings/GDP ratio</td>
<td>4.49</td>
<td>3.84</td>
<td>2.80</td>
<td>5.32</td>
</tr>
<tr>
<td>Housing/GDP ratio</td>
<td>1.80</td>
<td>1.20</td>
<td>2.08</td>
<td>1.58</td>
</tr>
<tr>
<td>Constrained entrepreneurs</td>
<td>0.89</td>
<td>0.90</td>
<td>0.92</td>
<td>0.87</td>
</tr>
<tr>
<td>Renters</td>
<td>0.32</td>
<td>0.34</td>
<td>0.02</td>
<td>0.46</td>
</tr>
</tbody>
</table>
columns corresponding with the initial 2006 state are calibrated to match each other, but other columns report model and data shares that are endogenous model results.

Table 1.5: Model Results Compared to Data (“Supply” scenario)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Entrep. share</td>
<td>0.176</td>
<td>0.176</td>
<td>0.167</td>
<td>0.141</td>
</tr>
<tr>
<td>Startup share</td>
<td>0.030</td>
<td>0.030</td>
<td>0.028</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Entrepreneur data from LBD; startup data from BDS.

As Table 1.5 shows, the housing crisis driven by supply expansion (or, equivalently, through partial equilibrium) can account for about one quarter of the decline in the share of activity accounted for by entrepreneurs and the share of activity accounted for by startups. The taste shock scenario can account for 60 percent and 50 percent of the declines in entrepreneurs’ and startups’ shares, respectively. The LTV scenario can account for 17 percent and 13 percent.

Changes in employment levels (as opposed to shares) are also empirically plausible. From 2006 to 2011 the level of entrepreneurial employment in the LBD fell from about 21 million to about 16 million, a 24 percent drop. The housing crisis scenarios consist of entrepreneurial employment drops ranging from 3 to 11 percent, accounting for between one sixth and (almost) one half of the decline of entrepreneurial employment in the data. From 2006 to 2011 the level of startup employment in the BDS fell from about 560,000 to about 400,000, a 29 percent drop. The housing crisis scenarios generate startup employment drops again ranging from 3 percent to 11 percent. Broadly speaking, the housing crisis has a significant amount of explanatory power. Generally, these results are not surprising; the number of
entrepreneurs who are at the borrowing constraint (“constrained entrepreneurs”) rises in the taste shock and housing supply scenarios. It actually falls in the loan-to-value scenario, suggesting that entry deterrence is a key mechanism for reducing entrepreneurial activity.

In general, homeownership rates are higher among entrepreneurs than among workers, an outcome that is qualitatively (but not quantitatively) consistent with the data.\(^{18}\) The rise in rentership rates produced by the taste shock and loan-to-value scenarios is also quantitatively consistent with the Great Recession experience, though the housing supply scenario has the opposite effect of eliminating renting almost entirely.\(^{19}\) This scenario illustrates the value of considering multiple sources of housing collateral tightening.

The model generally produces too much aggregate savings: the ratio of savings to output in the baseline scenario is 5.94, while the corresponding number in the data is 3.47.\(^{20}\) If anything, this biases the results against my key mechanism, since savings loosen collateral constraints.

The housing crisis experiments have little effect on the interest rate, the wage, and output. While entrepreneurial output falls, corporate output rises to compensate. There is no aggregate demand shock in the model, and since the corporate sector produces the same good as entrepreneurs, total production need not fall. In that sense

\(^{18}\) In the 2007 Survey of Consumer Finances, the homeownership rates for entrepreneurs and workers are 82 percent and 67 percent, respectively.

\(^{19}\) The American Housing Survey reports an increase in the rentership rate from 32 percent in 2006 to 39 percent in 2011.

\(^{20}\) This is based on Flow of Funds data for the first quarter of 2006. The numerator for this ratio is total financial assets held by households.
the results for entrepreneurial employment can be thought of as lower bounds, since they occur within a generally healthy economy. In the absence of a healthy corporate sector to provide compensatory labor demand, it is likely that the aggregate labor market would suffer from the drop in entrepreneurship. Quantifying this next step in the causal chain is an important task for future research.

Recall that the collateral constraint is given by $k \leq a + \phi q h$, so the loan-to-value ratio enters the collateral constraint in the same way as the house price; but the decline in the multiplier on housing is different in the two experiments. Table 1.6 reports the value of $q\phi$ for the housing crisis scenarios. Observe that the collateral multiplier falls more in the taste shock and supply shock scenarios than in the loan-to-value scenario, which partly explains why entrepreneurial activity falls less in the latter than in the two former experiments.

| Table 1.6: Collateral Value of Housing |
|-------------------------------|----|----|----|----|
| $q\phi$ | Baseline | Taste | Supply | LTV |
| 1.55 | 1.06 | 1.06 | 1.29 |

I now turn to the “financial crisis” scenario, which consists of an exogenous increase in the credit spread, reflecting increased frictions to credit supply. In reality credit spreads are endogenous, but in the model experiments I abstract from the endogeneity to focus on the specific channel leading from financial market turmoil to business activity. In the financial crisis scenario, I change the credit spread $\tau$ from 0.009 (or 0.9 percentage points, the baseline value corresponding to 2006) to 0.028 (or 2.8 percentage points). The latter value is the average level of the Bank of America US Corporate Master Option-Adjusted Spread for the years 2008-2011.
In a fourth experiment, I combine several parameter changes to create a simultaneous housing and financial crisis. In particular, I combine the “Taste” experiment, the “LTV” experiment, and the high-spread experiment.

Table 1.7 describes key model outcomes for the financial crisis scenario and the combination Taste/LTV/Spread scenario.

Table 1.7: Results: Financial Crisis and Combination

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Fin. Crisis</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>1.68</td>
<td>1.40</td>
<td>0.95</td>
</tr>
<tr>
<td>r</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>w</td>
<td>1.16</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>Output</td>
<td>1.74</td>
<td>1.61</td>
<td>1.61</td>
</tr>
<tr>
<td>Entrep. output</td>
<td>0.39</td>
<td>0.38</td>
<td>0.35</td>
</tr>
<tr>
<td>Entrep. employment</td>
<td>0.166</td>
<td>0.173</td>
<td>0.159</td>
</tr>
<tr>
<td>Entrep. share</td>
<td>0.176</td>
<td>0.183</td>
<td>0.168</td>
</tr>
<tr>
<td>Startup employment</td>
<td>0.028</td>
<td>0.028</td>
<td>0.026</td>
</tr>
<tr>
<td>Startup share</td>
<td>0.030</td>
<td>0.030</td>
<td>0.027</td>
</tr>
<tr>
<td>Entrep. ownership rate</td>
<td>0.95</td>
<td>0.94</td>
<td>0.80</td>
</tr>
<tr>
<td>Worker ownership rate</td>
<td>0.66</td>
<td>0.63</td>
<td>0.61</td>
</tr>
<tr>
<td>Aggregate savings</td>
<td>5.94</td>
<td>4.27</td>
<td>4.63</td>
</tr>
<tr>
<td>Savings/GDP ratio</td>
<td>4.49</td>
<td>4.77</td>
<td>3.42</td>
</tr>
<tr>
<td>Housing/GDP ratio</td>
<td>1.80</td>
<td>1.54</td>
<td>1.02</td>
</tr>
<tr>
<td>Constrained entrepreneurs</td>
<td>0.89</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Renters</td>
<td>0.32</td>
<td>0.35</td>
<td>0.38</td>
</tr>
<tr>
<td>Entrepreneurs</td>
<td>0.055</td>
<td>0.057</td>
<td>0.053</td>
</tr>
</tbody>
</table>

The financial crisis experiment produces results which differ starkly from the housing crisis scenarios. Recall that both entrepreneurs and the corporate sector are subject to the exogenous credit spread. The increase in the spread results in a 7 percent decline in output. The interest rate and wages both fall (the former by an amount obscured by rounding), along with household savings. Thus, the financial crisis scenario is characterized by broad economic stress.
Measures of entrepreneurship show significant increases under the high-spread scenario. To understand this, recall from the model that the entrepreneurial decision is heavily dependent on factor prices. The cost of capital rental includes the interest rate and the spread, while the wage acts both as the cost of hired labor and the opportunity cost of being an entrepreneur. So both corporate and entrepreneurial profits depend on factor prices, but only entrepreneurship is associated with the wage-as-opportunity-cost channel. The high-spread scenario is associated with a labor market that is less friendly to workers, which results in lower wages in this model, making entrepreneurship more attractive. Figure 1.6 illustrates this concept by showing the minimum productivity draw that is necessary to nudge a household into choosing entrepreneurship, plotted against household wealth, for several of the scenarios described above. Relative to the baseline, the taste shock and loan-to-value scenarios involve high thresholds for low levels of wealth, while the financial crisis scenario reduces the threshold.

The high-spread scenario illustrates a central claim of this study: supply-side financial frictions that affect all firms cannot explain the relative decline of entrepreneurial activity that occurred in the U.S. economy prior to and during the Great Recession. A financial crisis can have devastating effects on the economy generally, reducing output and wages, but it does not uniquely harm entrepreneurs and can even boost entrepreneurship by creating opportunistic entrepreneurs who want to escape an unhealthy labor market (these would include “entrepreneurs of necessity,” though the model does not include unemployment). The key to explaining a decline in entrepreneurial activity lies in the close relationship between household
and business finances for entrepreneurs, which makes entrepreneurial activity heavily dependent on the value of household assets. Hence a housing crisis can reduce entrepreneurship, but a financial crisis cannot. In the next section I relax the assumption that corporate and entrepreneurial firms face the same credit spread.

Figure 1.6: Productivity Thresholds for Entrepreneurship

Since the Great Recession involved a decline in both housing prices and the loan-to-value ratio as well as a heightened credit spread, the final experimental scenario involves a combination of the low house price, the low loan-to-value ratio, and the high credit spread. Unsurprisingly, the results of this experiment resemble a compromise between housing crisis and financial crisis. Output falls dramatically, depressing wages as in the high-spread scenario, but the wage-as-opportunity cost
mechanism for increasing entrepreneurship is offset by tighter borrowing constraints on entrepreneurs which affect the extensive margin as well as firm scale. Hence, entrepreneurial activity falls relative to the baseline scenario, but it does not fall as much as in the taste shock or housing supply scenarios. Compared with the baseline scenario, the combination scenario still explains about a quarter of the decline in entrepreneurs’ share and about 40 percent of the decline in startups’ share seen in the data. In general, the experiment further confirms the importance of the housing collateral channel.

5.2: Robustness Check: Credit Spread

A key conclusion of the main analysis in this paper is that a housing crisis can significantly reduce entrepreneurship, while a general financial crisis fails to do so (and can even increase entrepreneurial activity). The main driver of this result is the fact that entrepreneurial activity is irrevocably tied to household balance sheets, while corporate firms are independent of their owners’ personal assets. Since the credit supply friction—modeled here as an intermediation cost that gives rise to a credit spread—applies equally to all firms, it does not disproportionately affect entrepreneurs directly. In reality, it is likely that entrepreneurs and corporate firms face differential frictions even on the credit supply side. In terms of my model, this notion can be operationalized by allowing entrepreneurs and corporate firms to face a different credit spread.

In this general equilibrium experiment, I revise the model to include an additional intermediation cost or risk premium, again manifest in the form of a credit
spread, that applies only to entrepreneurs. This results in entrepreneurs facing a higher capital rental rate than the corporate sector. In particular, the capital rental rates are now as follows:

\[ r^k_c = \delta k + \tau + r \]
\[ r^k_e = \delta k + \tau + \tau_e + r \]

where \( r^k_c \) is the rental rate faced by the corporate sector, \( r^k_e \) is the rental rate faced by entrepreneurs, and \( \tau_e \) is the extra intermediation cost or risk premium that applies only to entrepreneurs. I can determine the potential of this difference in credit supply terms to explain the empirical reduction in entrepreneurship by choosing a value for \( \tau_e \) such that, in general equilibrium, entrepreneurship falls to the level seen in the housing taste/house price decline experiment described above (“Taste”). Table 1.8 reports results from this experiment under the column labeled “Spread \( \tau_e \); results from several previously discussed experiments are also reported for comparison.

<table>
<thead>
<tr>
<th>( \tau_e )</th>
<th>Baseline</th>
<th>Taste</th>
<th>Fin. crisis</th>
<th>Spread ( \tau_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau )</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.020</td>
</tr>
<tr>
<td>( q )</td>
<td>1.68</td>
<td>1.15</td>
<td>1.40</td>
<td>1.40</td>
</tr>
<tr>
<td>( r )</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>( w )</td>
<td>1.16</td>
<td>1.16</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td>Output</td>
<td>1.74</td>
<td>1.73</td>
<td>1.61</td>
<td>1.61</td>
</tr>
<tr>
<td>Entrep. output</td>
<td>0.39</td>
<td>0.35</td>
<td>0.38</td>
<td>0.32</td>
</tr>
<tr>
<td>Entrep. employment</td>
<td>0.166</td>
<td>0.147</td>
<td>0.173</td>
<td>0.147</td>
</tr>
<tr>
<td>Entrep. share</td>
<td>0.176</td>
<td>0.155</td>
<td>0.183</td>
<td>0.155</td>
</tr>
<tr>
<td>Startup employment</td>
<td>0.028</td>
<td>0.025</td>
<td>0.028</td>
<td>0.025</td>
</tr>
<tr>
<td>Startup share</td>
<td>0.030</td>
<td>0.026</td>
<td>0.030</td>
<td>0.026</td>
</tr>
</tbody>
</table>

As Table 1.8 reports, the scenario in which entrepreneurs face an additional spread generates changes in house prices, wages, and output that are the same as the
common-spread scenario. As targeted, entrepreneurial activity matches the low level of the housing taste/low-house price scenario. The additional spread required to achieve this target is \( \tau_e = 0.02 \). In other words, a credit supply friction can generate the same reduction in entrepreneurship as the low-house price experiment provided that the increase in the credit spread faced by entrepreneurs is 2 percentage points or 105 percent larger than the increase in the credit spread faced by the corporate sector. Running the experiment with an entrepreneurial employment share target based on the housing supply shock scenario yields \( \tau_e = 0.012 \), or 1.2 percentage points. Data on lending by commercial banks by loan size class suggests that such a large difference in the spread increase between small and large firms is implausible.\(^{21}\)

**Section 6: Conclusion**

I review evidence consistent with a relationship between home values and entrepreneurship, with a key mechanism being the dependence of entrepreneurial activity on housing collateral. I construct a model characterized by rich financial frictions, a housing sector, and entrepreneurship. The model suggests that the disproportionate decline in entrepreneurial activity prior to and during the Great Recession was more likely to have been caused by the housing crisis than the broad financial crisis. For a financial crisis to reduce entrepreneurship as much as a housing crisis, the increase in the credit spread faced by entrepreneurs is 2 percentage points or 105 percent larger than the increase in the credit spread faced by the corporate sector. Running the experiment with an entrepreneurial employment share target based on the housing supply shock scenario yields \( \tau_e = 0.012 \), or 1.2 percentage points. Data on lending by commercial banks by loan size class suggests that such a large difference in the spread increase between small and large firms is implausible.\(^{21}\)

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\(^{21}\) The Federal Reserve Board reports size-weighted interest rate averages by size class in its Survey of Terms of Business Lending. Consider the interest rates, averaged annually for 2005-2011, for small loans (less than $100,000) and large loans (greater than $10 million). The difference between these ranges from 1.99 to 2.47 percentage points. In other words, the spread for small loans increases by less than 0.5 percentage points more than the spread for large loans, compared with 1.2 to 2 percentage points in the model. The results are similar if “large loans” are defined as being between $1 million and $10 million.
crisis, entrepreneurs must face an increase in their credit spread that is 1.2 to 2.0 percentage points higher than the change in the spread faced by corporate firms. Recessions associated with large house price declines are likely to be characterized by disproportionate reductions in entrepreneurial activity, with the potential for quantitatively significant implications for job creation (though I do not model this). This is a plausible mechanism for which there is growing evidence in the literature.

In the present study I have defined “housing crisis” and “financial crisis” in specific, narrow ways: A housing crisis is a contraction in the collateral value of housing (through a drop in either house prices or loan-to-value ratios), while a financial crisis is a positive, exogenous shock to credit spreads. In reality, these concepts are broad and interrelated. In the Great Recession, the housing crisis had large effects on banks and was a key driver of the financial crisis. Both credit spreads and loan-to-value ratios are equilibrium objects in reality. For my purposes, though, it is important to model the financial sector parsimoniously to allow for clean comparisons of the effects of various types of credit stress on entrepreneurs.

With the caveat that this model necessarily simplifies key aspects of the financial sector, the model results may be thought of as a lower bound on the effects of housing collateral shocks on entrepreneurial job creation and of the effects of distressed entrepreneurs on the broader economy. One reason for this is that, in the model, the house price acts as a technology for converting output to housing, and a decline in the house price leaves more output available for other uses. This can boost consumer demand, offsetting the pressures on entrepreneurial firms. Additionally, the model has a corporate sector that is not distressed by household collateral shocks.
Reallocating factors from entrepreneurs to the corporate sector is frictionless, so the corporate sector can compensate for low entrepreneurial activity and labor demand to allow households to continue to consume at high levels. The Great Recession, on the other hand, was characterized by stress on large firms associated with, for example, credit supply restrictions and low aggregate demand. In reality and my model, these effects can indirectly boost entrepreneurship through a labor market channel, but in reality the resulting increase in entrepreneurship is not likely to be characterized by the kind of high-growth entrants that drive job creation. Overall the model suggests that periods characterized by significant declines in home values are likely to be accompanied by lower entrepreneurial activity. More broadly, the close link between entrepreneurial credit access and the personal characteristics and assets of entrepreneurs themselves is important for understanding the relationship between entrepreneurs and the broader economy.
Section 1: Introduction

A key insight of the firm dynamics research agenda is that the composition and behavior of individual firms exist in a feedback relationship with the aggregate economy. This chapter describes an economic mechanism through which aggregate first-moment conditions affect the choices of firms, and these choices have consequences for aggregate second-moment conditions. In particular, aggregate productivity drives firms’ decisions about market participation, and the number of markets in which firms participate influences the volatility of firm-level and aggregate outcomes. The model described here has implications for an ongoing debate in macroeconomics about uncertainty and the business cycle.

Firm-level risk is countercyclical. Bloom (2009) and an extensive literature that followed have analyzed how exogenous changes in uncertainty (or second-moment shocks) can be key drivers of the business cycle. In this chapter, we evaluate the possibility that causality operates in the opposite direction. Our hypothesis is that part of the observed change in measured firm-level risk over the business cycle is an endogenous response to first-moment shocks.

We propose a theoretical model in which firms endogenously determine the number of markets in which to participate and thereby expose themselves to market-
specific demand shocks.\textsuperscript{1} As long as the market-specific shocks are not perfectly correlated, the endogenous dynamics of market exposure imply changes in firm-level volatility. A continuum of competitive firms face stochastic market demands, differ in their level of idiosyncratic productivity, and face an aggregate productivity shock. They can participate in many markets by incurring selling expenses before demand shocks are realized. Thus, these endogenous per-period sunk costs determine the pool of suppliers in each market. Incentives to expand are higher in good times (\textit{i.e.}, when aggregate productivity is high) than in bad times. Since broader market exposure results in lower firm-level volatility, the model captures the observed countercyclicality of firm-level risk (with a business cycle correlation ranging from -0.20 to -0.42 in the model versus -0.46 in the data).\textsuperscript{2}

We test the model in several dimensions and find that it is broadly consistent with the evidence. In order to do so, we work with novel data on market presence that link Compustat with the U.S. Census Bureau’s Longitudinal Business Database (LBD). Though the firms in our model are risk neutral and lack any risk diversification objective, we find that they increase their revenues and diversify market-specific shocks by reaching more markets. The model is consistent with the procyclicality of measures of market exposure as well as the observed negative

\begin{itemize}
  \item \textsuperscript{1} We adopt a broad definition of “market” that applies to the market-product space, the market-location space, and a combination of the two. The empirical measures we use are intended to capture this broad definition of “market.”
  \item \textsuperscript{2} In the data, we follow Castro, Clementi, and MacDonald (2009) and measure firm-level idiosyncratic risk as the portion of growth in sales that cannot be explained by firm-level characteristics (such as age or size), industry, or year effects. The core data set for our empirical analysis is Compustat Fundamental (a sizeable panel of large, public U.S. firms). We construct two samples based on the core Compustat file: in one sample, we match Compustat Fundamental with Compustat Segment or “line of business” data; in another sample, we add matched firm-year data from the Longitudinal Business Database (LBD). See the following section as well as Appendix A2 for a detailed description of each data source and the matching procedure.
\end{itemize}
elasticity of firm-level risk to measures of market expansion (that ranges from -7.5 percent to -30.1 percent in the data and from -5.4 percent to -57.9 percent in the model).

A key prediction of the model is that the negative correlation between firm-level risk and the business cycle is mostly driven by those firms that adjust the number of markets they operate over time, and that these firms happen to be, on average, larger than those firms that do not adjust. We test this prediction in the data and find that, consistent with the model, among firms that adjust market exposure in a given year, the correlation between firm-level risk and detrended GDP is between -0.311 and -0.422, depending on our market definition, while risk is acyclical for firms that do not adjust. Also consistent with our theory, when we split the sample of firms by size, we find that firm-level risk for large firms is countercyclical while it is not for small firms. In addition, the model presented here is consistent with the evidence at levels of aggregation other than the firm. First, it captures the fact that the dispersion of prices is countercyclical, as described by Berger and Vavra (2011). Second, it generates a countercyclical cross-sectional variance of plant-level productivity, as reported by Kehrig (2011). Finally, the model captures the fact that firm-level market exposure is procyclical. This has been documented by Broda and Weinstein (2010) based on the number of products per firm (derived from bar code data) over the cycle. We also find that the number of establishments per firm, another correlate of market exposure (especially for large firms), is procyclical.

This work brings together two relatively recent streams in the literature. The first is the literature regarding business cycles and uncertainty that began with the
work by Bloom (2009) but also includes Arrellano, Bai, and Kehoe (2012); Bloom et al. (2012); Bachmann and Bayer (2013, 2014); Christiano, Motto, and Rostagno (2014); Chugh (2014); and Schaal (2012). In this literature, in contrast to our research, exogenous changes in volatility are key to generating business cycles. The second stream of literature is composed of studies that empirically analyze firm-level risk.\(^3\) Leahy and Whited (1996) analyze the relationship between firm-level risk and total market exposure and associated expenditures (after controlling for industry effects). Comin and Philippon (2006) document the increasing trend in firm-level volatility using Compustat, whereas Davis et al. (2007) show that the increase in firm-level risk is related to selection issues: the trend among privately held firms is actually downward, and the trend among publicly traded firms is driven in large part by cohort effects.

Our research is related to previous work analyzing the possibility of reverse causation between measured uncertainty and business cycles, which has already been described by Bachmann, Elstner, and Sims (2013) and Bachman and Bayer (2013, 2014).\(^4\) We offer an alternative explanation to Van Nieuwerburgh and Veldkamp (2006), Bachmann and Moscarini (2012), and Tian (2012). In Van Nieuwerburgh and Veldkamp (2006), procyclical learning about productivity generates the observed countercyclicality in firm-level volatility. In Bachmann and Moscarini (2012), downturns offer firms the opportunity to experiment and learn about their firm-

\(^3\) Bachmann and Bayer (2013, 2014) show that if countercyclical firm-level risk is imposed as a second driving force and propagated through a wait-and-see mechanism in capital adjustment costs, it does not generate large aggregate fluctuations.

\(^4\) Baker and Bloom (2013) address the issue of causality between first- and second-moment shocks using disasters as natural experiments, finding that both are significant in explaining GDP growth.
specific demand function; that experimentation is the driver of additional volatility. In Tian (2012), periods of recession are accompanied by more risk-taking behavior at the firm level. In our model, positive first-moment shocks (aggregate TFP) enable firms to expand into more markets and expose firms to an increased number of market-specific shocks, reducing volatility through an intuitive diversification mechanism. More recently, Alessandria et al. (2014) study the role of exogenous first- and second-moment shocks to productivity as drivers of export dynamics and business cycles.

The notion that agents are exposed to a limited number of shocks and that, therefore, the law of large numbers does not apply is not unique to our work. Among the papers that use this assumption are Gabaix (2011), Acemoglu et al. (2012), and Koren and Tenreyro (2013). These papers argue that a small group of firms (as in Gabaix (2011)), a small number of sectors (as in Acemoglu et al. (2012)), or a small number of inputs (as in Koren and Tenreyro (2013)) are the drivers of aggregate volatility. We also build on the literature of multiproduct firms. For example, Bernard, Redding, and Schott (2010) allow for the endogenous expansion of the firm but do not consider the risk dimension of activity. Other related papers include Arkolakis (2010), Bloom et al. (2013), and Gourio and Rudanko (2014). Arkolakis (2010) develops a model of customer capital through advertisement, which is one of the elements of our intangible expenditures measure. Bloom et al. (2013) measure the effects of management expenditures (also within our definition of market exposure costs) on Indian firms, and Gourio and Rudanko (2014) develop a search model to analyze how intangible expenses affect firm dynamics.
The chapter is organized as follows: Section 2 presents the empirical facts regarding the risk distribution across firms and over the business cycle, using both Compustat and Kauffman Firm Survey (KFS) data. Sections 3 and 4 describe a firm dynamics model with endogenous expansion and contraction of firms to capture the evidence presented in Section 2. Section 5 describes our calibration of the model to the distribution of firms in the United States and discusses the workings of the model. Section 6 presents the main results and compares the empirical evidence with model outcomes. Section 7 concludes the chapter.

Section 2: Idiosyncratic Risk and Business Cycles

In this section, we present evidence on the level of idiosyncratic risk and its cyclical components using our sample. These are well-known facts in the literature; specifically, firm-level risk is countercyclical and is related to firm size (larger firms tend to be less volatile).

Our main empirical facts come from Compustat and consist of annual accounting data for publicly listed U.S. firms. We use data from 1960 to 2012, consisting of an unbalanced panel of more than 8,400 firms for a total of 241,308 firm-year observations. Compustat data are subject to selection bias as described by Davis et al. (2007). Because these firms are relatively larger and older than those that are not in Compustat, they are likely to be less volatile (see Castro, Clementi, and Lee (2014)). We try to address these differences by controlling for age and size and by

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5 Appendix A2 provides a detailed description of our sample and the construction of key variables and the matching procedure.
using data from the KFS, which is based on a sample of small firms, to derive some
of our results. The KFS provides a large panel of data on “young” businesses. Firms
in the sample were founded in 2004 and have been tracked annually. This panel was
created using a random sample from Dun and Bradstreet’s database of new
businesses. The target population consisted of new businesses that were started in
2004 in the United States and excluded any branch or subsidiary that was owned by
an existing business or was inherited from someone else. The sample for the first
survey consisted of 4,928 businesses.

As in Castro, Clementi, and MacDonald (2009), our proxy for firm-level
idiosyncratic risk is the portion of sales growth that is not explained by industry
effects, time effects, or firm characteristics associated with growth such as age or size
(measured by employment). The first step toward obtaining our measure of
idiosyncratic risk is to estimate the following equation:

\[ \Delta \ln(sales_{ijt}) = \mu_i + \delta_{jt} + \beta_1 \ln(size_{ijt}) + \beta_2 \ln(age_{ijt}) + \epsilon_{ijt} \]  (1)

---

6 As we discuss in Section 6.4, we combine the Compustat Fundamental file with Compustat Segment
data and the U.S. Census Bureau’s LBD to provide direct evidence on the mechanism in the model.
While the LBD contains nearly every firm in the U.S. economy, it does not provide information on
total revenues at the firm level (and, regardless, we only report results from LBD firms that have been
matched to Compustat data). Other Census Bureau datasets, such as the Longitudinal Research
Database (LRD), include revenue data but consist of limited samples for specific sectors only. These
limitations prevented us from conducting the full experiment (all sectors, all firms) using only the LBD
and/or the other data sources.

7 Data are currently available for the years through 2008. See http://www.kauffman.org/kfs/ for a
detailed description of the data and for the actual public-use microdata.

8 Results are robust to a measure of idiosyncratic risk derived from total factor productivity (TFP) at
the firm level. However, due to measurement issues associated with physical capital and factor shares
in Compustat Fundamental and KFS data, our preferred firm-level volatility measure is based on sales
growth.

9 We are able to explicitly control for age in our Compustat sample; however, because all firms in the
KFS are of the same age (all firms began operating in 2004), this effect has already been factored in.
where $\Delta \ln (sales_{ijt})$ is the growth of real sales for firm $i$, in industry $j$, between period $t$ and period $t+1$. The variable $\mu_i$ is a firm fixed effect that accounts for unobserved persistent heterogeneity at the firm level (such as higher productivity or higher human capital of the entrepreneur). The variable $\delta_{jt}$ denotes a full set of time- and industry-specific fixed effects.\(^{10}\) We allow for industry-specific size effects. The estimation of equation (1) is done using the fixed effects panel estimator with robust standard errors. In the KFS sample, we use revenues from sales of goods, services, or intellectual property as our measure of sales. In the Compustat Fundamental sample, our measure of sales is item #12, net sales.\(^{11}\) As is standard in the literature, size is defined in both samples as the number of employees. Age corresponds to the amount of time since a firm first appeared in the sample.

Once equation (1) is estimated, we can compute $\epsilon_{ijt}$, the pure idiosyncratic and unpredictable component of firms’ sales growth. Following Castro, Clementi, and MacDonald (2009), we proxy firm-level risk by $\epsilon_{ijt}^2$.\(^{12}\)

Figure 2.1 shows the relationship between detrended GDP and two different aggregate measures of idiosyncratic risk: the detrended log-median $\epsilon_{ijt}^2$ and the detrended cross-sectional standard deviation of $\epsilon_{ijt}$.

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\(^{10}\) We use two-digit NAICS codes for firms in our KFS and Compustat samples.

\(^{11}\) The sample selection and the definition of all variables used in the analysis are described in detail in Appendix A2. Nominal variables are deflated using two-digit sector-specific price deflators for value added from the U.S. Bureau of Economic Analysis (BEA).

\(^{12}\) The estimated dispersion for the Compustat sample is consistent with the estimates in Castro, Clementi, and MacDonald (2009) and Castro, Clementi, and Lee (2014). Consistent with the estimates in Comin and Philippon (2006) and Davis et al. (2007), we find that idiosyncratic risk for publicly traded firms increased for several decades until the early 2000s.
The correlation between log-real GDP and the median ln($\epsilon_{ijt}^2$) and between log-real GDP and the cross sectional standard deviation of $\epsilon_{ijt}$ (our estimated measures of idiosyncratic risk) equals -0.46 (p=0.00) and -0.23 (p-value=0.09), respectively. The 10 percent confidence intervals for these correlations are [-0.62, -0.26] and [-0.44, -0.01], respectively. The finding of countercyclical risk at the firm level is common in the literature; a survey can be found in Bloom (2014).

Figure 2.1: Idiosyncratic Dispersion and Business Cycles

![Figure 2.1: Idiosyncratic Dispersion and Business Cycles](image.png)

Note: This figure shows correlations between detrended log-real GDP and the detrended cross-sectional standard deviation of $\epsilon_{ijt}$, and between detrended log-real GDP and median ln($\epsilon_{ijt}^2$), where $\epsilon_{ijt}$ is the unexplained portion of sales growth from equation (1). All variables are detrended using the Hodrick-Prescott (HP) filter with a parameter of 6.25. Firm data are from Compustat Fundamental.

We present our results based on the median ln($\epsilon_{ijt}^2$) and cross sectional standard deviation of $\epsilon_{ijt}$. The results are robust to different definitions of volatility. In particular, the correlation between the average ln($\epsilon_{ijt}^2$) and the log-real detrended GDP is -0.22 (significant at the 10 percent level), and the correlation between the sales-weighted standard deviation of $\epsilon_{ijt}$ and the log-real detrended GDP is -0.09 (significant only at the 25 percent level).
In what follows, we explore the relationship between firm-level risk and the business cycle through the lens of our model.

Section 3: Environment

Consider an economy with \( N \) markets (where \( N \) is large but finite), a representative consumer, and a continuum of competitive firms. Time is discrete, and a period is set to one year. Firms can service each of the different markets by incurring sales and marketing expenses. We adopt a broad definition of “market” that applies to the market-product space, the market-location space, or a combination of the two.

3.1: Household Preferences and Endowments

The representative household derives utility from the consumption of the composite good \( C_t \). More specifically, its preferences are given by \( U(C_t) \), where \( C_t \) is a composite of the consumption goods associated with each market \( n \):

\[
C_t = \left[ \sum_{n=1}^{N} (\xi_{n,t} c_{n,t})^\rho \right]^{\frac{1}{\rho}}, \quad 1 > \rho > 0, \tag{2}
\]

where \( c_{n,t} \) refers to consumption in market \( n \), \( \xi_{n,t} \) is a taste shock associated with market \( n \) in period \( t \), and \( 1/(1 - \rho) > 1 \) is the elasticity of substitution across different markets. The taste shock is stochastic according to
\[
\log \left( \xi_{n,t}^{1 - \alpha \rho} \right) \sim N(0, \sigma^2_\xi)
\]
where \( \alpha \) is the degree of decreasing returns to scale in production.\(^{14}\)

The household is endowed with one unit of labor that it supplies inelastically every period at wage \( w_t \) and receives dividends \( D_t \) through ownership of firms in the economy.\(^{15}\)

The ideal Dixit-Stiglitz price index is then

\[
P_t = \left[ \sum_{n=1}^{N} \left( \frac{p_{n,t}}{\xi_{n,t}} \right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} \cdot (3)
\]

The budget constraint faced by consumers is therefore

\[
P_t C_t \leq w_t + D_t.
\]

\(^{(4)}\)

3.2: Firms and Technology

Firms are described by their productivity parameter \( s \), which is constant over time. Production requires only one factor, labor. Given aggregate productivity \( z_t \), a firm with productivity \( s \) and serving to market \( n \) produces with technology given by

\[
q_{n,t}(s) = z_t s l_{n,t}^\alpha,
\]

where \( l_{n,t} \) is labor employed in the production of goods in period \( t \). We assume that firm-level productivity takes values on a finite set \( S \), is drawn from a distribution with probability density function \( \mu(s) \), and is constant over the lifespan of the firm.

\(^{14}\) This normalization of the exponent of \( \xi_{n,t} \) only makes the analysis cleaner below.

\(^{15}\) Note that firms generate profits due to the assumption of decreasing returns to scale.
Firms can reach and sell to consumers in market $n$ by incurring sales, marketing, and other intangible expenses. We assume that these expenses are measured in units of labor and are convex in the number of markets that the firms serve.\footnote{One interpretation of the demand differences corresponds to geographical distance or differences in products. Another interpretation is an increasing cost that arises from the complexity of serving many markets.} The total costs paid by a firm that serves $m$ markets, measured in relation to labor costs, is

\[ w_t \Phi_t(m) = w_t \frac{\psi}{z_t} (m - 1)^{1+\nu}. \]  

Firms have incentives to participate in more markets to access more customers; this results in diversification of market-specific risk even though diversification is not the (risk neutral) firm’s objective. We are assuming that the firm runs an establishment (or has a physical presence) in each market it serves (a reasonable assumption for most industries with the possible exceptions of manufacturing, online trade, and certain sectors of the finance, insurance, and real estate industries). The assumption that marketing and sales expenses are convex in the number of markets that a firm serves reflects the notion that complexity in management is tied to some resource that is in fixed supply. This is consistent with the evidence that shows that a considerable set of firms operates in a small number of markets, but the distribution does not place all mass on only one point.\footnote{In the quantitative section of the chapter, we use SIC codes, MSAs, and the number of establishments as different measures of market exposure. All three measures present a distribution of firms in a wide range with the median to mean ratios significantly below one.} A different assumption (such as a linear or strictly concave functional form) would result in all firms expanding to all markets.

Moreover, while we do not have access to a direct measure of the total cost of...
expansion to a new market, we observe selling, general, and administrative expenses (SGA) that are a proxy for market expansion costs since they refer to expenses on advertising, marketing, brand development, and research and development, among other items. Using this information and our measures of market presence, we estimate a cost function below that links changes in SGA with firm size and changes in market presence and find evidence consistent with a convex functional form.

3.3: Timing

The timing within the period is as follows:

1. $z_t$ is realized.
2. Firms choose the number of markets in which to operate.
3. Consumer taste shocks $\xi_{n,t}$ are realized.
4. Taking prices as given, firms choose labor and produce.
5. Households consume.

This assumed timing simplifies the model solution because it abstracts from the specific market in which the firm chooses to participate and reduces the problem to choosing the number of markets the firm wants to reach. This choice is a function of aggregate productivity $z_t$ and the firm’s idiosyncratic productivity $s$. These assumptions imply that the solution to the dynamic problem of the firm boils down to solving a sequence of one-period problems.
Section 4: Equilibrium

In this section, we present the definition and characterization of the competitive equilibrium of the model.

4.1: Consumer’s Problem

The household’s optimal conditions imply that its demand for consumption good \( n \) in period \( t \) is:

\[
c_{n,t} = \xi_n \left( \frac{p_{n,t}}{p_t} \right)^{\frac{1}{\rho - 1}} (w_t + D_t).
\] (7)

4.2: Firm’s Problem

Firms are perfect competitors in each market in which they participate. It is most intuitive to start by solving the firm’s problem at the production stage and then deriving the optimal condition for the number of markets. After the shocks \( z_t \) and \( \xi_{n,t} \) are revealed, the firm optimizes over the amount of labor to demand in each market they have previously chosen to serve.

The profit function for a firm in market \( n \) is given by:

\[
\pi_{n,t}(s) = \max_{l_{n,t}} \{ p_{n,t} q_{n,t}(s) - w_t l_{n,t} \}
\] (8)

subject to

\[
q_{n,t}(s) = z_t s l_{n,t}^\alpha.
\] (9)

This delivers a standard labor demand in market \( n \) for a firm with productivity \( s \):
\[ l_{n,t}(s) = \left( \frac{w_t}{p_{n,t}sz_t \alpha} \right)^{1 \over \alpha-1}. \]  

(10)

This implies that profits for a firm with productivity \( s \) in market \( n \) are the following:

\[ \pi_{n,t}(s) = (p_{n,t}sz_t)^{1 \over 1-\alpha}w_t^\alpha \left( \frac{\alpha}{\alpha^{1-\alpha}} - \frac{1}{\alpha^{1-\alpha}} \right). \]  

(11)

At the beginning of the period (i.e., before firms observe \( \xi_{n,t} \) but after they observe \( z_t \)), firms can derive the optimal number of markets in which to operate by taking the expected value of (11). More specifically, firms enter the \( m^{th} \) market as long as

\[ \mathbb{E}\left( \pi_{m,t}(s) \right) \geq w_t \left( \Phi(m) - \Phi(m - 1) \right). \]  

(12)

In other words, (12) stipulates that the firm will expand into \( m \) markets as long as the expected profit in the last market is larger than the additional cost required to manage the last market. We denote by \( m_t(s) \) the number of markets in which a firm with productivity \( s \) chooses to participate in period \( t \).  

4.3: Definition of Equilibrium

In any given period \( t \), the Competitive Equilibrium is a set of labor \( l_{n,t}(s) \) and market exposure \( m_t(s) \) decision rules, a wage rate \( w_t \), a vector of goods prices \( \{p_{n,t}\}_{n=1}^N \), and a vector distribution of firms with productivity \( s \) participating in each market \( n \), \( \{\lambda_{n,t}(s)\}_{n=1}^N \), such that:

1. Given the wage rate and prices, the labor decision rules and market exposure decision rules of all firms are the solution to problems (8) and (12).

---

18 Our convexity assumption on the cost function \( \Phi(m) \) ensures that the solution to the firm’s problem is unique.
2. The distribution of firms in market \( n \) equals

\[
\lambda_{n,t}(s) = \frac{\mu(s)m_t(s)}{N},
\]

(13)

3. The labor market clears, that is,

\[
\sum_{s=\xi}^{\bar{s}} \sum_{n=1}^{N} \lambda_{n,t}(s)l_{n,t}(s) + \sum_{s=\xi}^{\bar{s}} \mu(s)\Phi(m_t(s)) = 1.
\]

(14)

4. Each price \( p_{n,t} \) is such that it clears the \( n^{th} \) market, that is,

\[
\sum_{s=\xi}^{\bar{s}} \lambda_{n,t}(s)q_{n,t}(s) = c_{n,t},
\]

(15)

where \( c_{n,t} \) is given by equation (7).

5. Aggregate dividends are

\[
D_t = \Pi_t - w_t \sum_{s=\xi}^{\bar{s}} \mu(s)\Phi(m_t(s)),
\]

(16)

where \( \Pi_t \) denotes the sum of profits across markets and is given by

\[
\Pi_t = \sum_{s=\xi}^{\bar{s}} \sum_{n=1}^{N} \lambda_{n,t}(s)\pi_{n,t}(s).
\]

(17)

With this definition established, we can characterize firms’ behavior and the aggregate equilibrium objects.

### 4.4: Characterization of the Equilibrium

From the market clearing condition (15) and the optimal demand of goods (7),

the equilibrium price in market \( n \) is

\[
p_{n,t} = \xi_{n,t}^\rho(1-\alpha)^{\frac{1}{1-\alpha}} A_t,
\]

(18)
where

$$A_t = \left[ P_t^{\rho} \left( \frac{(w_t + D_t)w_t^{1-\alpha}}{\bar{s}_t z_t^{1-\alpha} \alpha^{1-\alpha}} \right)^{1-\rho} \right]^{1-\alpha},$$

and

$$\bar{s}_t = \frac{1}{N} \sum_{s=1}^{\bar{s}} \mu(s) m_t(s) s^{1-\alpha}.$$ 

Note that $A_t$ is a function of aggregate productivity as well as the endogenous wage. Under the calibrated parameters that follow, $A_t$ is countercyclical and is one of the driving forces of the countercyclicality of cross sectional price dispersion mentioned above.

Combining (12) (i.e., the equation that determines the number of markets for a firm of productivity $s$) with the market clearing price in market $n$ given by (18), we find that firms will expand into at least $m$ markets only if

$$\frac{1}{\bar{s}^{1-\alpha} B_t} \geq w_t \left( \Phi(m) - \Phi(m - 1) \right),$$

(19)

where

$$B_t = e^{2 \sum \sigma_z^2} \frac{1}{\bar{s}^{1-\alpha} w_t^{\alpha-1}} \left( \alpha \frac{\alpha}{1-\alpha} - \alpha \frac{1}{1-\alpha} \right) A_t^{1-\alpha}.$$ 

The expected marginal profit has two components: One is firm-specific and is a function of firm productivity $s$, and the other is tied to the economy generally and depends on parameters at time $t$ (such as the wage rate $w_t$ and aggregate productivity $z_t$). Larger firm-specific productivity implies larger expected profits (given the assumption of decreasing returns to scale). The effects from the economywide
parameters go in the following direction: higher levels of aggregate productivity
generate higher expected profits, while higher wages reduce expected profits. Both
aggregate effects are multiplied by the firm-specific productivity, generating an
asymmetric response of productivity to the aggregate environmental parameters.

The labor market clearing condition (14) implies that

\[ w_t + D_t = \frac{w_t}{\alpha} \left[ 1 - \sum_{s=\bar{s}}^{\bar{s}} \mu(s) \Phi(m_t(s)) \right] \]

\[ \Rightarrow \frac{\Pi_t}{w_t} = \left[ 1 - \sum_{s=\bar{s}}^{\bar{s}} \mu(s) \Phi(m_t(s)) \right] \left( \frac{1}{\alpha} - 1 \right) . \]

so, in equilibrium, the price index \( P_t \) becomes

\[ P_t = \left[ 1 - \sum_{s=\bar{s}}^{\bar{s}} \mu(s) \Phi(m_t(s)) \right]^{1-\alpha} \frac{w_t}{az_t} \left( N e^{\frac{\sigma^2}{2}} \right)^{\frac{\alpha \rho - 1}{\rho}} . \quad (20) \]

Finding an equilibrium requires solving a system of three aggregate equations

((14), (16), (17)) and three unknowns:

\[ \sum_{s=\bar{s}}^{\bar{s}} \mu(s) \Phi(m_t(s)) \]

\[ w_t \]

\[ \bar{s}_t \]

such that they are consistent with firm-level decisions.

Prior to describing our calibration, we now discuss an empirical implication of
the equilibrium described above. The coefficient of variation of firm-level total factor
productivity (TFPR) can be derived in closed form.\textsuperscript{19} Conditional on the aggregate shock $z_t$, the model predicts a relationship between the firm’s idiosyncratic productivity $s$ and its volatility. The coefficient of variation of the weighted sum of TFPR to which the firm is exposed, conditional on serving $m$ markets, is

$$CV_t(s) = \sqrt{\frac{\text{Var}\left(\sum_{n=1}^{m}(sz_{t}p_{n,t})^{1/1-\alpha}\right)}{\mathbb{E}\left(\sum_{n=1}^{m}(sz_{t}p_{n,t})^{1/1-\alpha}\right)}}.$$ \textsuperscript{(21)}

Note that (21) is presented as the coefficient of variation for a given firm with productivity $s$. However, an identical expression can be derived if we focus on the coefficient of variation across establishments, conditional on firm-level idiosyncratic physical productivity. The analysis that follows is consistent with either interpretation.\textsuperscript{20} From (19), it is evident that the firm will participate in an increasing number of markets as a function of its productivity $s$. Therefore, the coefficient of variation of the firm’s TFPR is a function of the firm’s productivity through its effect on the optimal number of markets that the firm will serve. Then, using the optimal market exposure decision, the coefficient of variation for a firm with productivity $s$ can be written as

$$CV_t(s) = \frac{e^{\sigma_t^2/2} - 1}{\sqrt{m_t(s)}}.$$ \textsuperscript{(22)}

This result is based on the assumption that the shocks $\xi_{n,t}$ are \textit{i.i.d.} However, as long as the shocks are not perfectly correlated (which would make them, in fact, one aggregate shock), the coefficient of variation falls as the firm is exposed to an

\textsuperscript{19} Following calibration we can develop more testable implications to connect the model to data with moments that are more empirically reliable but lack closed form expressions.

\textsuperscript{20} As we discuss in the next section, consistent with the model, the data show that volatility is countercyclical at both levels of aggregation.
increasing number of shocks. This can be seen by analyzing the variance-covariance matrix of the shocks $\xi_{n,t}$. Given that they have the same variance, the variance covariance matrix can be rewritten in terms of the correlation coefficient between two shocks multiplied by the common variance term. In this case, the coefficient of variation is given by the following expression:

$$CV_t(s) = \frac{\sqrt{\left(e^{\sigma_{\xi}^2} - 1\right) \sum_{u=1}^{m} \sum_{v=1}^{m} \rho_{uv}}}{m_t(s)},$$

(23)

where $\rho_{uv}$ is the correlation coefficient between the shocks $u$ and $v$. In the case of $i.i.d.$ shocks the double sum equals the number of shocks, and in the case of perfectly correlated shocks, it equals the square of the number of shocks. Anything in between means that the coefficient of variation falls as the number of varieties increases.

A key model prediction implied by (22) is that for firms that expand in booms and contract in recessions, the coefficient of variation at the firm level is countercyclical. Thus, the model asks us to split the data sample between those firms that adjust the number of markets to which they are exposed and those that do not (as opposed to, for example, splitting the sample by firm size). We perform this critical empirical test in Section 5.1 and show that the model is consistent with the empirical evidence. Moreover, we also observe that the variance of the weighted sum of TFPR to which the firm is exposed is countercyclical, given that the variance of prices follows the term $A_{t}$ in (18) (which, at the calibrated parameters described below, is countercyclical). The fact that $A_{t}$ is countercyclical implies a countercyclical dispersion in prices across markets, which is consistent with the evidence described by Berger and Vavra (2011). Further, under our assumptions of one establishment per
market, the cross-sectional variance of TFPR at the plant level is also countercyclical, as reported by Kehrig (2011).

Section 5: Calibration

This section presents the calibration of the model. Using this calibration, we then explore the workings of the model to study the business cycle properties of firm-level risk.

We assume that firm-level productivity is distributed following a log-normal distribution with mean $\bar{s}$ and standard deviation $\sigma_s$, so $\ln(s) \sim N(\bar{s}, \sigma_s^2)$. The number of markets, $N$, only determines the scale of the problem. We set its value to 100, but this number is irrelevant to our results. We assume that $z_t \in \{z_B, z_G\}$ with transition probability $\Gamma(z', z)$ and denote by $\Gamma_{kj}$ the $(j, k)$th element of $\Gamma(z', z)$. We normalize $z_G = 1$. This leaves us with 10 parameters to calibrate:

$$\{\rho, \sigma_\xi, \alpha, \nu, \psi, \bar{s}, \sigma_s, z_B, \Gamma_{GG}, \Gamma_{BB}\} \quad (24)$$

We calibrate the preference $\rho$ to 0.83, a standard parameter in the trade literature.\(^{21}\) We set $\alpha = 0.64$, also a standard value in the literature and determines the labor share of output. To choose a value for $\sigma_\xi$, we obtain $\hat{\varepsilon}$ by estimating (1) on our KFS sample and evaluate the standard deviation of $\ln \hat{\varepsilon}^{\frac{\rho}{1-\alpha}}$; this results in $\sigma_\xi = 3.04$. This is a good approximation under the assumption that these very small firms are exposed to

\(^{21}\) In a model with monopolistic competition, this would imply a 20 percent markup. Recall, though, that there is no markup in our model since we look for a competitive equilibrium.
only one market, and it allows us to pin down the dispersion of market-specific risk.\footnote{\textsuperscript{22}} To calibrate $\Gamma_{GG}$ and $\Gamma_{BB}$, we estimate the fraction of booms and recessions with data from the NBER. More specifically, for a given year, we set a recession indicator to one if two or more quarters in that year were dated as part of a recession by the NBER. Then, we identify years in which the indicator equals one with our periods of \( z = z_B \) and construct a transition matrix. The estimate of $\Gamma_{kJ}$ is the ratio of the number of times the economy switched from state $j$ to state $k$ to the number of times the economy was observed to be in state $j$. We find that $\Gamma_{GG}$ is 0.86 and that $\Gamma_{BB}$ is 0.43. This implies that the unconditional probabilities of $z_G$ and $z_B$ are 0.80 and 0.20, respectively. Finally, we set $z_B=0.90$. This amplitude of the support for $z$ is consistent with data presented by Gordon (2005), where the average peak-to-trough distance in terms of the output gap is 7.9 percent for the period 1945-2005.\footnote{\textsuperscript{23}}

The four remaining parameters are \{\( \bar{\zeta}, \sigma_s, \nu, \psi \)\}: the mean and standard deviation of the idiosyncratic productivity distribution as well as the two parameters that control the cost of firm expansion. These we jointly calibrate so that the model produces a firm size distribution matching the firm size distribution in the 2008

\footnote{\textsuperscript{22} The KFS sample does not contain information on market exposure. The KFS focuses strictly on new businesses that most likely operate in only one market. Note also that from our Compustat-LBD link, we observe firms with only one establishment (\textit{i.e.}, exposed to only one market). They represent approximately 1 percent of total sales and total workers in 2011. However, since Compustat covers only publicly traded firms, we believe that these are not representative of firms exposed to a single market. Moreover, due to data limitations, the group of firms that only have one establishment likely includes firms in Compustat that are not perfectly matched with establishments in the LBD sample, introducing an additional source of measurement error into the calibration.}

\footnote{\textsuperscript{23} This is computed by averaging the peaks and the troughs during the sample time period and taking their difference. Note that this time period excludes the 2007-2009 recession and the Great Depression, as well as all the previous recessions that had a much stronger impact on GDP.}
The identification of these parameters derives from the fat tail in the firm size distribution. The BDS covers all employer firms (about 5 million) that, in total, employ approximately 120 million workers. For 2008, we observe 5,185 firms in our Compustat Fundamental sample, which is less than 0.1 percent of the total number of firms. In particular, we choose the parameters by minimizing the sum a sum of squares of the distance between model and data moments, where each moment is the fraction of firms in a given size bin (as specified by the BDS). Effectively, since we have four parameters and six moments, this is an overidentified model.

To compute the average distribution in the model, we draw 100,000 firms from the idiosyncratic productivity distribution and simulate the model 20 times for 50 periods in each simulation in which aggregate shocks are drawn from \( \Gamma(E, E) \).

Given that in the model the total measure of firms is equal to one, the model distribution reported was adjusted for the difference in overall means.

### Table 2.1: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference parameter ( \rho )</td>
<td>0.83</td>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>Dispersion taste ( \sigma_\xi )</td>
<td>3.04</td>
<td>( SD(\ln(\hat{\theta}^{\rho/1-\alpha}) ) from KFS</td>
</tr>
<tr>
<td>Labor share ( \alpha )</td>
<td>0.64</td>
<td>Standard value</td>
</tr>
<tr>
<td>Aggregate prod. ( z_G )</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>Aggregate prod. ( z_B )</td>
<td>0.90</td>
<td>Peak to trough amplitude</td>
</tr>
<tr>
<td>Transition prob. ( \Gamma_{GG} )</td>
<td>0.86</td>
<td>NBER boom/recession</td>
</tr>
<tr>
<td>Transition prob. ( \Gamma_{BB} )</td>
<td>0.43</td>
<td>NBER boom/recession</td>
</tr>
<tr>
<td>Cost function ( \nu )</td>
<td>0.56</td>
<td>Firm size dist. (see Table 2.2)</td>
</tr>
<tr>
<td>Cost function ( \psi )</td>
<td>0.46</td>
<td>Firm size dist. (see Table 2.2)</td>
</tr>
<tr>
<td>Mean productivity ( \bar{s} )</td>
<td>( \ln(1.7) )</td>
<td>Firm size dist. (see Table 2.2)</td>
</tr>
<tr>
<td>Std. dev. productivity ( \sigma_s )</td>
<td>0.4</td>
<td>Firm size dist. (see Table 2.2)</td>
</tr>
</tbody>
</table>

The accuracy of the firm size distribution match is shown in Table 2.2. We observe that the model adequately replicates the firm size distribution with a close match for all size classes.

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24 The BDS covers all employer firms (about 5 million) that, in total, employ approximately 120 million workers. For 2008, we observe 5,185 firms in our Compustat Fundamental sample, which is less than 0.1 percent of the total number of firms.

25 In particular, we choose the parameters by minimizing the sum a sum of squares of the distance between model and data moments, where each moment is the fraction of firms in a given size bin (as specified by the BDS). Effectively, since we have four parameters and six moments, this is an overidentified model.

26 To compute the average distribution in the model, we draw 100,000 firms from the idiosyncratic productivity distribution and simulate the model 20 times for 50 periods in each simulation in which aggregate shocks are drawn from \( \Gamma(z', z) \).

27 Given that in the model the total measure of firms is equal to one, the model distribution reported was adjusted for the difference in overall means.
Table 2.2: Firm Size Distribution (Employment)

<table>
<thead>
<tr>
<th>Employment size</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms with 1-4 employees</td>
<td>0.610</td>
<td>0.601</td>
</tr>
<tr>
<td>Firms with 5-9 employees</td>
<td>0.176</td>
<td>0.209</td>
</tr>
<tr>
<td>Firms with 10-19 employees</td>
<td>0.107</td>
<td>0.100</td>
</tr>
<tr>
<td>Firms with 20-99 employees</td>
<td>0.089</td>
<td>0.064</td>
</tr>
<tr>
<td>Firms with 100-499 employees</td>
<td>0.015</td>
<td>0.021</td>
</tr>
<tr>
<td>Firms with 500 or more employees</td>
<td>0.003</td>
<td>0.005</td>
</tr>
</tbody>
</table>

The model also performs well when compared with other moments related to the core story of the chapter—in particular, to moments related to the joint distribution of markets and employment. In the data, 90 percent of firms have fewer than 20 employees and are almost all single-units (i.e., firms with only one establishment; in fact, the average number of establishments per firm conditional on firms employing fewer than 20 employees is 1.01 establishments). In the BDS the average number of establishments per firm is equal to 1.267. In the model, 91 percent of firms are single-units, and firms have an average of 1.186 establishments. These numbers reassure us that the functional forms and calibrated parameters in the model are generating reasonable quantities the dimensions relevant for the main results. Other dimensions worth examining are related to business cycle dynamics. The relevant items are real wages and dispersion of productivity and prices. Real wages in the model are procyclical and, at the calibrated parameters, the term $A$ in (18) is countercyclical, resulting in countercyclical cross-sectional price variance and firm-level TFPR. Moreover, consumption, investment, and productivity are by
construction procyclical. Finally, the price level is countercyclical. These items are consistent with the business cycle data for the post-war US.  

5.1: Workings of the Model

In this section, we further explore the workings of the model and present intuition for the main result along with a set of testable implications that we will compare with data in the following section.

We first describe how movement in \( E \) affects market exposure and the cross-sectional distribution of firms in the model. Changes in \( z \) have a nonmonotone effect on firm-level decisions. The most productive firms expand in response to an increase in the aggregate shock, while less productive firms change their market exposure only slightly. This cyclical expansion and contraction is consistent with the procyclical net entry rate found for the manufacturing sector by Lee and Mukoyama (2012). Moreover, the uneven response of the change in the number of markets by firm size is also consistent with the data (see Figure 2.3 and Table 2.4 that follow).

Consistent with the characteristics of equilibrium, changes in \( z \) generate an endogenous change in firm-level risk. The impact of changes in \( z \) (from boom to bust) on the coefficient of variation from (22), by firm productivity level \( s \), is depicted in Figure 2.2.

It is clear that the impact of TFP shocks on the coefficient of variation is not monotone by productivity level. When the economy moves from boom to recession, the average firm-level volatility rises by 0.023 percent, and the average firm-level risk

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28 These moments are close to those generated by the standard real business cycle model.
for the top 10 percent and 1 percent of firms rises by 1.4 percent and 3.9 percent, respectively. This uneven change in volatility is a direct consequence of firms’ asymmetric response to variations in \( r \). The fixed cost of expanding to an additional market creates regions of inaction. The business cycle has no effect on the volatility of firms with productivity \( q \) less than about 3.3 since these firms never find it profitable to adjust their market exposure (indeed, those firms are operating only one market). Above that productivity level, regions of inaction still exist due to the cost of market reach adjustment.

Observe that the regions of inaction shrink as productivity increases, disappearing entirely around \( s = 6.5 \) (after which nonlinearities still exist as some firms adjust by more than one market at a time). Moreover, the impact of recessions on the coefficient of variation decreases with productivity at the thresholds at which firms adjust the number of markets in which they operate, since conditional on \( r \), high-productivity firms are better diversified than those with low productivity. Changing from operating two markets to operating just one market, as firms with productivity around \( s = 3.3 \) do, effects a large reduction in firm-level market diversification and, therefore, results in much higher firm-level volatility. However, adjusting from ten to nine markets does not reduce diversification as much.

The above discussion implies that (a) the idiosyncratic volatility of firms engaged in market expansion and contraction should be countercyclical, and (b) the idiosyncratic volatility of firms not adjusting should be acyclical. As we show in Section 6.2, this is consistent with the empirical evidence.
In this model with firm heterogeneity, the endogenous variation in the number of markets also has an effect on measured aggregate TFP. In this model, aggregate measured TFP is computed as aggregate production divided by aggregate labor (the only input of production) raised to the power $\alpha$. Since the household supplies a unit measure of labor, measured aggregate TFP in period $t$ equals total output

$$\sum_n \sum_s q_{t,n}(s) \lambda_{t,n}(s).$$

As $z$ increases and more productive firms expand proportionally to a larger set of markets, there is an additional positive effect on measured TFP beyond that due to the change in $z$. This endogenous amplification effect on measured TFP is nonnegligible and amounts to a further 13 percent increase in measured TFP beyond the effect of the aggregate shock $z$. 

Figure 2.2: Change in Firm Volatility over the Business Cycle

Note: Percentage change in the coefficient of variation of firm-level risk when the economy moves from $z_{e}$ to $z_{B}$ (see equation (22)).
In summary, the predictions of the model include:

1. Firm-level risk is countercyclical.
2. Market exposure is procyclical.
3. Firms that adjust their market exposure are larger than those not adjusting.
4. Small and less productive firms display larger firm-level risk than large firms.
5. Firm-level risk is countercyclical only for those adjusting the number of markets and not for those exposed to the same number of markets over the cycle.
6. Large firms’ risk is more countercyclical than small firms’ risk.
7. The elasticity of firm-level risk with respect to market exposure is negative.

We confront these model predictions with the data in the following section.

Section 6: Main Results and Testable Implications

In this section, we present the main results of our research—cyclical properties of firm-level risk and market participation measures—together with a description of how the model’s testable implications compare with the data.

Our analysis combines several data sources to cover many angles. As discussed in Section 2, we derive our measures of firm-level risk from the Compustat Fundamental and KFS data sets after estimating (1). Since the core Compustat sample covers five decades, this allows us to analyze how firm-level risk moves over the business cycle. Furthermore, we match our Compustat Fundamental panel with two

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29 Appendix A2 includes a detailed description of how our sample is constructed and how the match across data sets is performed.
other data sets. First, we link Compustat Fundamental with the Compustat Segment data. The Segment data provide information on sales for each firm by four-digit SIC code, at an annual frequency for most years in our sample. Like Bloom, Schankerman, and Van Reenen (2013), we use line of business (i.e., SIC codes) information as one of our direct measures of product and market exposure. Second, we match Compustat Fundamental with the U.S. Census Bureau’s Longitudinal Business Database. This allows us to obtain information on the number of establishments as well Metropolitan Statistical Area (MSA) locations for the firms in our Compustat sample. The number of establishments and number of MSAs in which a firm is operating provide two additional measures of market exposure. Finally, we also use the Census Bureau’s Business Dynamics Statistics (BDS) data which enable us to derive annual data starting in 1977 on the average number of establishments by firm size (employment) and to analyze its cyclical properties.

6.1: Firm-Level Risk and Business Cycles

We start by presenting our main result, the negative correlation between firm-level risk and the business cycle. Using the pseudo panel of firms from the model, we estimate firm-level volatility as we did in the data. That is, we compute the sales

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30 The Segment sample starts in 1977 and provides 183,991 firm-year observations.

31 On average, each firm reports 2.68 industry codes every year (approximately five industry codes per year when weighted by sales).

32 These measures of market reach (SIC codes, establishments, and MSA) do have weaknesses. For example, business lines, even at the four-digit level, can be associated with R&D; the construction of a plant has an investment component that we do not consider in the model, and firms in some industries might ship their goods to various locations from only one establishment (this is more problematic in manufacturing than in services or retail). However, the fact that most of our results are robust across market exposure measures is evidence that the mechanism in the model is present in the data.
growth (in logs) from period $t$ to period $t+1$ and regress it against a firm fixed effect, size (number of employees), and a time dummy capturing aggregate conditions (booms or recessions). That is, we estimate
\[
\Delta \ln(sales_{it}) = \delta_0 s_t + \delta_1 z_t + \delta_2 \ln(size_{it}) + \epsilon_{it}
\]
and obtain the residuals $\epsilon_{it}$ from (25) to derive our measure of firm-level risk $\ln(\epsilon_{it}^2)$, as in Castro, Clementi, and MacDonald (2009). We study the cyclical properties of this measure of firm-level risk. The findings are summarized in Table 2.3.

### Table 2.3: Model Firm-Level Idiosyncratic Risk over the Business Cycle

<table>
<thead>
<tr>
<th></th>
<th>Correlation with det. GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
</tr>
<tr>
<td>Median $\ln(\epsilon_{it}^2)$</td>
<td>-0.46***</td>
</tr>
<tr>
<td>Cross-sectional Std($\epsilon_{it}$)</td>
<td>-0.23*</td>
</tr>
</tbody>
</table>

Note: *** denotes significance at the 1% level, ** at the 5% level, and * at the 10% level

We find that both for the entire sample and for the top 5 percent of firms (which are the model firms most comparable to the Compustat sample), measures of firm-level risk generating by the model are significantly countercyclical. More specifically, for the top 5 percent of model firms, the correlation between median $\ln(\epsilon_{it}^2)$ and GDP equals -0.20 (significant at the 10 percent level), and the correlation between the cross-sectional standard deviation of $\epsilon_{it}$ and GDP is -0.35 (significant at the 1 percent level). These model correlations are -0.42 (significant at the 1 percent level) and -0.18 (significant at the 7 percent level) if computed using all firms from the model instead of just the top 5 percent. Meanwhile, in our panel of U.S. firms (i.e., Compustat Fundamental), the correlation between median idiosyncratic risk and
GDP is -0.46 with a 90 percent interval of [-0.625,-0.257], and the correlation between the cross-sectional standard deviation of risk and GDP is -0.23 with a 90 percent interval of [-0.439,-0.017], as shown in Section 2.\textsuperscript{33}

6.2: Market Exposure and Business Cycles

The model predicts that high-productivity firms respond to changes in aggregate productivity by expanding and contracting the number of markets in which they operate (with corresponding changes in selling expenses). However, these observed changes in market participation do not translate into a monotone relationship in terms of the cyclicality of volatility, as shown in Figure 2.2. The reason is that high-productivity firms are already exposed to a large number of markets even in bad times. Therefore, their reaction to aggregate productivity changes does not affect their volatility dramatically, while firms that operate in relatively few markets during recessions and expand in booms experience large fluctuations in their volatility over the cycle.

We first test whether market exposure is procyclical for those firms engaged in market expansions and contractions. Figure 2.3 reports how the average number of markets that firms participate in (as measured by four-digit SIC codes, establishment counts, MSA counts, and the product of SIC codes and establishment counts) moves for firms that change their market exposure in a given year (“changers”).

As predicted by the model, the average change in market exposure among changers behaves procyclically for all of our definitions of market participation. The

\textsuperscript{33} This measure of firm-level risk derived from sales growth has been widely used in the literature. However, a firm-level risk measure derived from an AR(1) model of log(sales) is also countercyclical.
correlation coefficients between detrended GDP and average change in market
exposure for changers is 0.28 for SIC codes, 0.48 for establishments, 0.51 for MSAs,
and 0.31 for SICs×establishments.

While not as direct as the measures of market exposure presented thus far,
guided by our model we also observe how expenses associated with market reach
move with the business cycle. In the model, the cost of market exposure is given by
\( w_t \Phi_t(m) \) and is predicted to be procyclical given that it is a function of \( m \). In our
Compustat sample, we use Selling, General, and Administrative (SGA) expenditures
as the measure associated with operating the firm as a function of the firm’s
complexity. We also look at the advertising component of SGA, as it should closely
follow the market reach of the firm. Figure 2.4 shows the correlation between these
indirect measures of market exposure and detrended GDP.

Figure 2.4 shows that log real GDP and our measures of market reach
expenses are positively correlated, as predicted. The correlation is 0.283 (significant
at the 5 percent level) and 0.149 (significant at the 10 percent level) when expenses
are measured as SGA expenses and advertising expenses, respectively. In the model,
business cycle fluctuations in selling expenses result in fluctuations in the size of the
labor force employed in related activities. The labor force share engaged in these
activities reaches 17.8 percent during booms and falls to 2.65 percent during
recessions (in the model); however, the model overpredicts the correlation between
GDP and median selling expenses since this correlation (in the model) is 1 for the top
5 percent of firms.\(^{34}\)

\(^{34}\) Sampling from the top of the model’s distribution is appropriate since Compustat corresponds to the
right tail of the U.S. firm size distribution.
Figure 2.3: Average Change in Market Exposure for “Changers”

Panel (i): Market Exposure = Number of SIC Codes

Note: Detrended GDP corresponds to HP detrended log-real GDP (parameter 6.25). The market exposure measure corresponds to the number of four-digit SIC codes (i.e., line of business), number of establishments a firm operates, number of MSAs in which a firm has establishments, and the product of establishments and SIC codes. “Changers” are firms that change market exposure measure in a given year. We report the average change in market exposure conditional on being a “changer”. Source: linked Compustat Fundamental-Compustat Segment, linked Compustat-LBD.
Figure 2.4: Market Reach Expenses and Business Cycles

Note: Log-real expenses (SGA and Advertising) correspond to the median of the observed distribution in any given year. Series are detrended using an HP filter with parameter 6.25. GDP data are available since 1947. In Panel (i), log-real expenses are measured by Selling, General, and Administrative expenses. In Panel (ii), log-real expenses are measured by Advertising Expenses. Source: Compustat Fundamental.

6.3: Market Exposure and Firm Size

Another implication of the model is that firms that react to the cycle by changing the number of markets in which they participate are larger, on average, than those that do not change their market exposure. Table 2.4 shows descriptive statistics for both the linked Compustat Fundamental/Compustat Segment data and the linked Compustat/LBD data; “changers” are firms that change their market reach, and “non-changers” are firms that do not. Also, consistent with the implications of the model,
Table 2.4: Descriptive Statistics, “Changers” versus “Non-changers”

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th></th>
<th>“Changers”</th>
<th></th>
<th>“Non-changers”</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>Number of SIC codes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1607</td>
<td>8679</td>
<td>2739</td>
<td>12460</td>
<td>1388</td>
<td>7716</td>
</tr>
<tr>
<td>Emp.</td>
<td>8.4</td>
<td>33.9</td>
<td>11.7</td>
<td>41.5</td>
<td>7.8</td>
<td>32.2</td>
</tr>
<tr>
<td>SGA</td>
<td>269</td>
<td>1416</td>
<td>465</td>
<td>1979</td>
<td>230</td>
<td>1273</td>
</tr>
<tr>
<td>Adv.</td>
<td>50</td>
<td>273</td>
<td>80</td>
<td>367</td>
<td>43</td>
<td>247</td>
</tr>
<tr>
<td>SICs</td>
<td>2.6</td>
<td>1.9</td>
<td>3.3</td>
<td>2.4</td>
<td>2.5</td>
<td>1.7</td>
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<tr>
<td>Number of establishments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
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<td>8308</td>
<td>2316</td>
<td>9547</td>
<td>271</td>
<td>1828</td>
</tr>
<tr>
<td>Emp.</td>
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<td>33.4</td>
<td>12.3</td>
<td>41.2</td>
<td>1.4</td>
<td>8.2</td>
</tr>
<tr>
<td>SGA</td>
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<td>1370</td>
<td>399</td>
<td>1634</td>
<td>56</td>
<td>332</td>
</tr>
<tr>
<td>Adv.</td>
<td>49</td>
<td>265</td>
<td>77</td>
<td>308</td>
<td>9</td>
<td>71</td>
</tr>
<tr>
<td>Estabs.</td>
<td>116</td>
<td>565</td>
<td>215</td>
<td>765</td>
<td>8</td>
<td>77</td>
</tr>
<tr>
<td>Number of MSAs</td>
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<td>8308</td>
<td>2583</td>
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<td>418</td>
<td>2574</td>
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<td>13.7</td>
<td>44.1</td>
<td>2.3</td>
<td>12.4</td>
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<td>439</td>
<td>1725</td>
<td>86</td>
<td>565</td>
</tr>
<tr>
<td>Adv.</td>
<td>49</td>
<td>264</td>
<td>86</td>
<td>326</td>
<td>15</td>
<td>112</td>
</tr>
<tr>
<td>MSAs</td>
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<td>93</td>
<td>66</td>
<td>127</td>
<td>8</td>
<td>41</td>
</tr>
<tr>
<td>Number of SICs×estab</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1553</td>
<td>8308</td>
<td>2108</td>
<td>8945</td>
<td>253</td>
<td>1840</td>
</tr>
<tr>
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<td>11.4</td>
<td>39.1</td>
<td>1.4</td>
<td>7.9</td>
</tr>
<tr>
<td>SGA</td>
<td>260</td>
<td>1370</td>
<td>364</td>
<td>1544</td>
<td>53</td>
<td>331</td>
</tr>
<tr>
<td>Adv.</td>
<td>49</td>
<td>264</td>
<td>71</td>
<td>290</td>
<td>9</td>
<td>73</td>
</tr>
<tr>
<td>SIC×Estab</td>
<td>406</td>
<td>2552</td>
<td>705</td>
<td>3388</td>
<td>22</td>
<td>428</td>
</tr>
</tbody>
</table>

Note: Sales; Selling, General, and Administrative expenses (SGA); and Advertising Expenses (Adv.) are expressed in millions of dollars deflated using BEA two-digit SIC price deflators for value added. Employment (Emp.) is expressed in thousands of employees. Source: Data are from the linked Compustat Fundamental to Compustat Segment and the linked Compustat Fundamental to LBD data. We formally test for statistically significant differences to find that all the moments for “changers” are significantly different from the moments for “non-changers” at the 5% level.
we find that the fraction of firms that adjust their market participation is acyclical. This fraction is between 20 percent and 56 percent of the total number of firms, depending on the definition of market, and this fraction is uncorrelated with detrended GDP.  

We find that, conditional on the number of SIC codes, changers are about twice as big as non-changers in terms of sales, SGA expenses, and advertising expenses. They are 50 percent larger in terms of employment and 33 percent larger in terms of the number of SIC codes they operate. But changers are between five and nine times larger than non-changers when we consider the number of establishments each firm operates or the number of MSAs in which each firm is present, based on sales, employees, or expenditures. Changers are 26 times larger than non-changers in terms of establishment count and 31 times larger in terms of the product of SIC codes and establishments.

We next split the sample by firm size classes to determine whether the cyclicality of market exposure for large firms is consistent with that of changers. In this case, the measure of market exposure corresponds to the number of active establishments per firm. Using publicly available BDS data, it is possible to derive the average number of establishments by firm size (employment), annually from 1977 to 2009, and analyze its cyclical properties. The last three columns of Table 2.5

---

35 Market participation is measured by the number of products (SIC codes) for which firms report sales, the number of establishments they operate, the number of MSAs in which they have establishments, or the product of establishments and SIC codes. The fraction of changing firms is 20 percent, 52 percent, 42 percent, and 56 percent for these market definitions, respectively, and this fraction is uncorrelated with GDP.

36 The BDS is compiled from the LBD. The LBD is a longitudinal database of business establishments and firms covering the years from 1977 on. The BDS provides annual statistics on the number of establishments and gross flows, by firm size class, for the entire private nonfarm economy.
present evidence on the change in establishment count by firm size between periods when log-real GDP is above and below trend as well as the correlation between the number of establishments per firm and detrended GDP (conditional on firm size).

Table 2.5 shows that for most size categories, there is a minimal change in the number of establishments per firm between periods when GDP is above trend and those when it is below trend (the correlation with detrended GDP gives similar results). However, for large firms (those with at least 5,000 workers), the number of plants is larger when GDP is above trend than when GDP is below trend, and the elasticity with respect to detrended GDP is close to 1. This difference is economically

<table>
<thead>
<tr>
<th>Firm size (workers)</th>
<th>Avg. # Firms (1000s)</th>
<th>Fraction total emp (%)</th>
<th>Avg. emp. per plant</th>
<th>Cyclical properties, # plants per firm</th>
<th>Elast. w/ GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Avg. # plants when GDP below trend</td>
<td>Avg. # plants when GDP above trend</td>
</tr>
<tr>
<td>1-4</td>
<td>1909.7</td>
<td>6.6</td>
<td>2.3</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>5-9</td>
<td>736.4</td>
<td>7.3</td>
<td>6.6</td>
<td>1.03</td>
<td>1.03</td>
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<tr>
<td>10-19</td>
<td>407.7</td>
<td>8.4</td>
<td>12.7</td>
<td>1.1</td>
<td>1.09</td>
</tr>
<tr>
<td>20-49</td>
<td>237.0</td>
<td>11.0</td>
<td>24.4</td>
<td>1.28</td>
<td>1.27</td>
</tr>
<tr>
<td>50-99</td>
<td>71.0</td>
<td>7.5</td>
<td>40.2</td>
<td>1.77</td>
<td>1.73</td>
</tr>
<tr>
<td>100-249</td>
<td>35.0</td>
<td>8.2</td>
<td>51.4</td>
<td>3.02</td>
<td>2.95</td>
</tr>
<tr>
<td>250-499</td>
<td>9.8</td>
<td>5.2</td>
<td>59.0</td>
<td>5.85</td>
<td>5.74</td>
</tr>
<tr>
<td>500-999</td>
<td>4.7</td>
<td>4.8</td>
<td>63.2</td>
<td>10.54</td>
<td>10.63</td>
</tr>
<tr>
<td>1,000-2,499</td>
<td>3.0</td>
<td>6.5</td>
<td>66.1</td>
<td>21.56</td>
<td>21.28</td>
</tr>
<tr>
<td>2,500-4,999</td>
<td>1.1</td>
<td>4.8</td>
<td>60.5</td>
<td>46.69</td>
<td>46.77</td>
</tr>
<tr>
<td>5,000 +</td>
<td>1.5</td>
<td>29.7</td>
<td>75.8</td>
<td>167.43</td>
<td>175.19</td>
</tr>
</tbody>
</table>

Note: We extract a linear trend component from all variables. The “Avg. # firms” corresponds to the average number of firms in each size bin over our sample (in thousands). “Fraction total emp.” is computed as the average of total employees in each size bin divided by total employment. “Avg. emp. per plant” corresponds to the average number of employees per establishment in each size category. “Avg. # of plants when GDP below trend or above trend” is derived from a linear regression of the average number of establishments by firm size on a constant, a linear trend, and a dummy that identifies periods in which GDP is above trend. GDP corresponds to log-real GDP. The trend for GDP is computed using the Hodrick-Prescott (HP) filter with a parameter of 6.25. “Elast. w/ GDP” corresponds to the elasticity between the average number of plants and detrended GDP. This elasticity is derived from a linear regression of log average number of establishments by firm size on a constant, a linear trend, and log-real detrended GDP.

* denotes significance at the 10% level.

Source: U.S. Census Bureau, Business Dynamics Statistics (BDS).
and statistically significant.\textsuperscript{37} This is relevant in terms of activity and observed dispersion because, as Table 2.5 shows, these firms account for about 30 percent of total employment and comprise the top 1,450 largest firms in the economy (equal to just under half of the number of firms included in our Compustat sample).\textsuperscript{38}

Furthermore, since the change in the number of plants is approximately eight between periods when GDP is above and below trend in these 1,450 firms, and these firms employ about 75 workers per plant, the change in employment coming only from this margin amounts to 1.29 percent of total private nonfarm employment.\textsuperscript{39}

Within-firm expansions and contractions seem to be correlated with the business cycle. Broda and Weinstein (2010) report that the product portfolio of firms is procyclical based on bar code data. This observation is consistent with our evidence and our model. In recessions, firms contract their product mix; in booms, firms expand their product mix and expose themselves to more markets.

We next exploit the relationship between market exposure and size to further test the model. In the model, low-productivity firms (which are small on average) are more volatile than high-productivity firms (which are large on average) since they are

\textsuperscript{37} This is consistent with the evidence presented in Moscarini and Postel-Vinay (2012) that the net job creation of large firms or establishments commoves negatively and more strongly with aggregate unemployment than the net job creation of small employers at business cycle frequencies.

\textsuperscript{38} Note that the reported numbers in Table 2.5 correspond to detrended averages from the period 1977-2009. The total number of firms for the year 2009 was around 3,000.

\textsuperscript{39} Another margin of adjustment for firms is the number of workers per plant. As we show in Appendix A2.4, the variation in the average number of workers per establishment is positive and significant for small firms but not for large firms. Moreover, the change in the number of workers over the business cycle coming from the change in the number of plants per firm is larger than the change in the number of workers coming from adjustment in the number of workers per plant (which represents 1.15 percent of total private nonfarm employment).
exposed to a lower number of markets. To test this prediction, we examine whether a negative relationship between firm size and firm-level risk is present in the data. Figure 2.5 presents the estimated distribution of idiosyncratic risk for our two samples (i.e., the KFS and Compustat).

Figure 2.5: Distribution of Firm Risk across Sample

![Graph showing the distribution of firm risk across sample](image)

Note: Idiosyncratic dispersion is based on sales growth. Sources: Compustat Fundamental and KFS.

Consistent with the model, we find that small firms (i.e., those in KFS) are considerably more volatile than large firms (i.e., those in Compustat).\(^40\) This is

\(^{40}\)Some firms in our Compustat sample have only one establishment (these represent 4 percent of Compustat employment and 5 percent of sales). However, we choose not to study these firms alone.
consistent with the evidence presented in Haltiwanger et al. (2013). The median dispersion in the KFS is more than five times the median dispersion in Compustat.

To further explore the link between firm-level risk and firm size, Table 2.6 presents several moments of the distribution of firm-level risk conditional on size. To construct this table, we condition on a particular size class then compute moments of the distribution of firm-level risk.

Table 2.6 shows that all moments of the distribution of firm-level risk are decreasing in firm size. That is, consistent with the predictions of the model, we find that large firms tend to be less volatile than small firms. For example, the median values of $\ln(\epsilon^2)$ imply that a firm with 10 to 19 employees faces 37 percent less risk than a firm with one to four employees, and a firm with 20 to 99 employees faces 74 percent less risk than a firm with one to four employees.\(^\text{41}\)

<table>
<thead>
<tr>
<th>Size (employment)</th>
<th>Moments distribution $\ln(\epsilon^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>1-4</td>
<td>-1.97</td>
</tr>
<tr>
<td>5-9</td>
<td>-2.31</td>
</tr>
<tr>
<td>10-19</td>
<td>-2.47</td>
</tr>
<tr>
<td>20-99</td>
<td>-3.35</td>
</tr>
<tr>
<td>100-499</td>
<td>-4.18</td>
</tr>
<tr>
<td>500+</td>
<td>-5.26</td>
</tr>
</tbody>
</table>

Note: Idiosyncratic dispersion is based on sales growth.
Source: Compustat Fundamental-Compustat Segment; Compustat Fundamental-LBD.

\(^{41}\) These can be computed by dividing the values for the condition median. Specifically, $\exp(-2.21)/\exp(-1.75) - 1 = -0.3687$ and $\exp(-3.09)/\exp(-1.75) - 1 = -0.7382$. 

85
6.4: Conditional Firm-Level Risk over the Business Cycle

In this section we split the data in two different ways to study the cyclicality of firm-level risk. First, we look at the difference in cyclicality of firm-level risk between changers and non-changes (i.e., those firms that change their market exposure and those that do not). Second, we look at differences in the cyclicality of firm-level risk conditional on firm size. The model implies that changers’ and large firms’ risk should be countercyclical, while risk for non-changers and small firms should be acyclical. The evidence is consistent with the model.

We first test whether, by reacting to the business cycle, firms that change their market exposure experience a countercyclical pattern of volatility, while firms that do not change their market exposure have acyclical volatility. This is an implication of the model result described by Figure 2.2. Figure 2.6 presents the evolution of our measure of firm-level risk over the business cycle conditional on being a changer or a non-changer, as well as the correlation of each series with detrended GDP.

As Figure 2.6 illustrates, the data are consistent with the predictions of the model. Splitting the sample into changers and non-changers (using our four measures of market exposure) reveals that the correlation between firm-level risk and GDP for changers and non-changers is different. The changers’ median $\varepsilon^2$ is countercyclical, with a GDP correlation between -0.31 and -0.42 (significant at the 5 percent level). Conversely, among non-changers the correlation between median $\varepsilon^2$ and GDP is between -0.1 and -0.2 and is never statistically different from zero.\footnote{These findings are robust to a different measure of firm-level risk derived from the time series standard deviation of $\varepsilon_{it}$.}
Figure 2.6: Volatility of “Changers” versus “Non-changers”

Note: Detrended GDP corresponds to HP detrended log-real GDP (parameter 6.25). The market exposure measure corresponds to the number of four-digit SIC codes (i.e., line of business), the number of establishments the firm operates, the number of MSAs in which the firms are present, and the product of establishments and SIC codes. “Changers” refers to firms that change market exposure in a given period; “non-changers” refers to firms that do not. Data are from linked Compustat-Fundamental and linked Compustat-LBD.

The model predicts a non-monotone but broadly decreasing relationship between firm-level risk and firm size (see Figure 2.2 and associated discussion).

However, on average, large firms react to the business cycle by expanding the
number of markets while small firms do not, so firm-level risk conditional on firm size should display different cyclical properties across size categories.

Figure 2.7 presents the correlation between firm-level risk and GDP, conditional on firm size. To construct this figure we rank firms by their size (employment) and label as “small” those firms in the bottom 25 percent of the size distribution, as “medium” those firms in the interquartile range of the distribution, and as “large” those firms in the top 25 percent of the distribution.43

Figure 2.7: Firm Risk over the Business Cycle (Conditional on Size)

Note: Idiosyncratic risk based on sales growth. Sources: Linked Compustat Fundamental-LBD.

43 Results are robust to size bins based on the bottom decile, the 10-90th percentile range, and the top decile: business cycle correlations are -0.22 (p=0.21) for small firms and -0.41 (p=0.02) for large firms.
The evidence is consistent with our model and shows that the correlation is negative and stronger for large firms than for small firms. This is not surprising based on the model since, as we discussed in the previous section, not only do changers tend to be large but large firms tend to be changers. The correlation of the median \( \varepsilon^2 \) and GDP is -0.04 and -0.33 for small and large firms, respectively (within the top 5 percent of firms, which is our model analog of Compustat).

6.5: Determinants of Firm-Level Risk

To understand the properties of firm-level volatility and market exposure, we derive a testable implication that links market exposure \( m_t(s) \), selling expenses, and firm-level volatility \( \ln(\varepsilon^2_{it}) \) derived from (25). The model predicts that market exposure and selling expenses are key to understanding the evolution of firm-level risk. As we describe in detail in the following section, we observe in the data several relatively direct measures of market exposure as well as selling expenses, which in our model correspond with \( m^*_t(s) \) and \( w_t \Phi(m^*_t(s)) \), respectively. Therefore, we estimate the regression

\[
\ln(\varepsilon^2_{it}) = \gamma_0 s_i + \gamma_1 \ln(x_{it}) + u_{it2}
\]

where \( \ln(\varepsilon^2_{it}) \) is our measure of firm-level risk, \( \gamma_0 s_i \) is a firm fixed effect, and \( x_{it} \) represents either the number of markets or selling expenses.

Table 2.7 summarizes our model’s predictions. The simulated elasticities of firm-level risk with respect to the number of markets and selling expenses are -0.58

---

44 This result is robust to a measure of firm-level risk derived from an AR(1) model of \( \ln(sales) \). In particular, the correlation for “small” firms is -0.23 (p=0.20) and the correlation for large firms is -0.39 (p=0.03).
(significant at 5 percent) and -0.05 (significant at 5 percent), respectively, when looking at a sample including all model firms. Restricting attention to the top 5 percent of firms (our model counterpart to Compustat) results in firm-level risk elasticities of -0.21 and -0.15 with respect to markets and expenses, respectively. These values are very close to the ones reported in Table 2.8, which are obtained in actual data. We turn to those results next.

Table 2.7: Model Firm-Level Idiosyncratic Volatility and Market Exposure

<table>
<thead>
<tr>
<th>Dependent variable $\ln(\epsilon_{ijt}^2)$</th>
<th>All sample</th>
<th>Top 5 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln(m_{ijt})$</td>
<td>-0.579***</td>
<td>-0.210***</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0246)</td>
</tr>
<tr>
<td>$\ln(\text{expenses}_{ijt})$</td>
<td>-0.054***</td>
<td>-0.145***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0065)</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. *** denotes significance at the 1% level. $\ln(\epsilon_{ijt}^2)$ is constructed from the estimated residual of equation (25).

Given our estimate of idiosyncratic risk $\epsilon_{ijt}$ from (1) (as in Castro, Clementi, and MacDonald (2009)), we proxy firm-level by $\ln(\epsilon_{ijt}^2)$ and study how it is related to our various market exposure measures once industry-specific factors are accounted for.\(^{45}\) We estimate the following log-linear equation (as we did in the model-produced data):

\[
\ln(\epsilon_{ijt}^2) = \gamma_t + \theta_{jt} + \alpha_1 \ln(X_{ijt}) + \alpha_2 t + u_{ijt}
\]

\(^{45}\) This specification of the variance allows us to identify a value for the variance of every firm in industry $j$ and year $t$, and it is consistent with the multiplicative heteroskedasticity model analyzed by Harvey (1976). More specifically, this formulation results from assuming that $\sigma_{ijt}^2$, the variance of the disturbance in (1) (i.e., the variance of $\epsilon_{ijt}$), takes the following form:

$$
\sigma_{ijt}^2 = \exp(\gamma_t + \theta_{jt} + \alpha_1 \ln(X_{ijt}) + \alpha_2 t).
$$
where $\gamma_i$ is a firm fixed effect, $\theta_{jt}$ is an industry- and year-specific component, $\ln(X_{ijt})$ is a measure of market exposure for firm $i$ in sector $j$ at time $t$, and $t$ is a time trend.

As mentioned above, we use several different market exposure measures as $X_{ijt}$. From our Compustat-LBD linked sample we obtain establishment counts and MSA counts by firm. Using our Compustat Segment linked sample we obtain the number of SIC codes in which firms have sales. We also study the interaction between establishments and product markets. Finally, we consider indirect measures of market exposure, SGA expenses and advertising expenses from the Compustat Fundamental sample (and SGA expenses for small firms in the KFS sample).

Table 2.8 reports the results of these regressions.

| Table 2.8: Firm-Level Regressions with Dependent Variable $\ln(\varepsilon^2)$ |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                  | Estabs. | MSAs | SICs | Est×SIC | Adv | SGA | SGA |
| $\ln(X)$         |  -0.081 | -0.094 | -0.137 | -0.075 | -0.134 | -0.301 | -0.085 |
|                   | (0.009) | (0.010) | (0.021) | (0.008) | (0.012) | (0.009) | (0.036) |
| N                 | 129700 | 129700 | 155200 | 124400 | 67000 | 177200 | 2500 |
| R²                | 0.018 | 0.018 | 0.018 | 0.018 | 0.042 | 0.044 | 0.16 |
| Source            | LBD | LBD | Comp. | Comp.-LBD | Comp. | Comp. | KFS |

Note: All results are significant at the 1% level except for those from KFS, which are significant at the 5% level. All regressions include firm fixed effects, industry-year controls, and time trends. Sources: Number of establishments and MSAs taken from the LBD; number of SICs and expenditures taken from Compustat, except those noted as coming from KFS. $\ln(\varepsilon^2_{ijt})$ is derived from estimating equation (1) on our Compustat Fundamental sample. Regressions are run after performing relevant links between Compustat Fundamental and Compustat Segments and the LBD.

We show below that, consistent with the evidence presented in Decker et al. (2015), a time trend is necessary because the variance of idiosyncratic risk among public firms has had a trend rise and fall in recent decades.

We perform a large set of robustness checks, many of which are described in Appendix A2.
Regressions using various measures of market exposure produce reasonably similar results, in terms of both sign and magnitude. The elasticity of firm-level volatility with respect to these measures ranges from 7.5 to 13.7 percent. Even using advertising expenses as the measure of market reach delivers an estimate that is very close to those obtained using direct measures of market reach. All of the measures reported in the first five regressions use Compustat data (linked to the Segment files or the LBD), so the model counterpart for these firms is the top end of the firm size distribution. In the model, when we use the top 5 percent of firms by productivity, the elasticity of risk with respect to market exposure is -0.21, which is close to the numbers obtained from our empirical model reported above.\footnote{The estimated negative elasticity is robust to various definitions of firm-level volatility, including a definition based on time series standard deviations.}

Finally, when looking at the indirect measures of market exposure—namely SGA expenses—we find that the elasticity is -0.30 for the Compustat sample and -0.09 for the KFS sample. The model performs well along this dimension, producing an elasticity of -0.14 among the top 5 percent of firms and -0.05 among all firms.\footnote{The estimated firm-level volatility $\tilde{\sigma}_{ijt}^2 = \exp (\tilde{\gamma}_i + \tilde{\theta}_{ij} + \tilde{\alpha}_1 \ln (X_{ijt}) + \tilde{\alpha}_2 t) \tilde{E}(\tilde{a}_{ijt})$ is, like our other measures of firm-level risk, countercyclical.}

6.6: Evidence on Market Exposure Expenses

In this section, we briefly digress to provide evidence about the relationship between market exposure and expenses. The model assumes that marketing and sales expenses are increasing in the number of markets that a firm serves. The evidence is
broadly consistent with this assumption and reflects the notion that complexity in management is tied to some resource that is in fixed supply.

More specifically, we have access to selling, general, and administrative expenses (SGA), which are a proxy for market expansion and costs since they refer to expenses on, for example, advertising, marketing, brand development, and research and development. Using this information and our measures of market exposure (SICs, establishments, and MSAs), we estimate a cost function that links changes in SGA with firm size and changes in market presence. For each measure of market presence, we classify firms according to how broad their market presence is: “small”, “medium”, and “large”. We then estimate how the change in expenses is correlated with the change in market exposure plus the change in market exposure interacted with a dummy for the size category after conditioning on firm size (employment). We omit the medium size category, so a full convex functional form would result in a negative coefficient for the interaction term between changes in market exposure and the small category, and positive coefficient on the interaction term between changes in market exposure and the large category. Table 2.9 reports the results.

The results support the assumption of a convex functional form, either across all size categories or at least between two of them, for the three measures of market exposure. In particular, when using SICs as the measure of market exposure, the coefficient on ΔMarkets × Small is negative, and the coefficient on ΔMarkets × Large is positive. When using establishments or MSAs as a measure of market exposure, the cost function shows signs of being convex only across two size
categories (in the small-medium portion for establishments, and in the medium-large portion for MSAs).

The regressions provide evidence for a convex functional form, either across all size categories or at least between two of them, for the three measures of market exposure.

Table 2.9: Cost of Expansion and Market Exposure

<table>
<thead>
<tr>
<th>Market definition</th>
<th>SIC</th>
<th>Estab.</th>
<th>MSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>0.0017</td>
<td>0.0023</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>D.Markets</td>
<td>9.476</td>
<td>0.298</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(1.16)</td>
<td>(0.01)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>D.Markets*Small</td>
<td>-1.124</td>
<td>-0.312</td>
<td>2.109</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.01)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>D.Markets*Large</td>
<td>8.728</td>
<td>-0.266</td>
<td>0.368</td>
</tr>
<tr>
<td></td>
<td>(2.40)</td>
<td>(0.01)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>R²</td>
<td>0.05</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>N</td>
<td>135300</td>
<td>111300</td>
<td>111300</td>
</tr>
</tbody>
</table>

The dependent variable in all regressions is the firm-level change in Selling, General, and Administrative expenses. Independent variables are size (employment) and the firm-level change in market exposure (with interactions). Data are from the linked Compustat Fundamental-Compustat Segment and the linked Compustat Fundamental-LBD datasets.

Section 7: Conclusion

Consistent with previous literature, using a panel of U.S. firms we document the countercyclical nature of idiosyncratic firm-level risk. We propose a theory of endogenous volatility over the business cycle based on firm-level market exposure to explain this fact. In our model, firms pay a cost to expand to a larger number of
markets. The result is that high-productivity firms operate in more markets, making them less volatile than their low-productivity counterparts through an intuitive diversification mechanism. Notably, low-productivity firms do not adjust market exposure in reaction to the cycle, whereas medium-sized and large firms do, explaining the cyclical properties of firm-level volatility.

From the model, we derive a set of testable implications for measures of market exposure and firm-level volatility and show that the empirical evidence is broadly consistent with the theory. Specifically, using firm-level data from Compustat Fundamental and Compustat Segment data, as well as the LBD, we show that measures of market exposure are procyclical and that the volatility of firms that expand and contract is countercyclical, while volatility is acyclical for those not engaged in market exposure adjustments. This holds when market exposure is measured directly in terms of lines of business, establishment counts, and geographic locations, and it holds for indirect measures such as selling, general, and administrative expenses or advertising. Moreover, using data from Compustat, the LBD, and the KFS, we show that firm-level idiosyncratic risk is negatively correlated with all measures of market exposure, even when controlling for firm, year, and industry fixed effects.
Appendices

Section A1: Appendix for Chapter 1

A1.1: Derivations

Financial Intermediary Profits

The financial intermediary’s profit maximization problem is given by:

$$\max_{K_c, K_e} r^k (K_c + K_e) - r^a_{\text{borrow}} A_b + r^h q M_r - \delta_k (K_c + K_e) - \tau (K_c + K_e - A_b) - \tau A_s$$

subject to

$$K_c + K_e - A_b + q M_r = A_s.$$  

The first-order conditions follow:

- $$K_c, K_e : \quad r^k = r + \delta_k + \tau$$
- $$A_b : \quad r^a = r + \tau \quad \text{(for borrowers)}$$
- $$M_r : \quad r^h = r.$$  

Substituting the first-order conditions into the profit function yields

$$\text{Profits} = r (K_c + K_e) - \tau A_b + r q M_r - r A_s.$$  

Substituting the financial market clearing condition given by (20) in Chapter 1 yields the result of zero profits.

Aggregate Quantities

The aggregated variables used in market clearing conditions and the aggregate resources constraint are defined as follows:
\[ A'_s = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)\alpha'(s, a, h, z)I_{a'\geq 0}(s, a, h, z)dzdhda \] (A1.1)

\[ A'_b = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)\alpha'(s, a, h, z)(1 - I_{a'\geq 0}(s, a, h, z))dzdhda \] (A1.2)

\[ H' = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)h'(s, a, h, z)dzdhda \] (A1.3)

\[ M_r = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)m(s, a, h, z)I_{rent}(s, a, h, z)dzdhda \] (A1.4)

\[ C = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)c(s, a, h, z)dzdhda \] (A1.5)

\[ N_s = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)(1 - e(s, a, h, z))dzdhda \] (A1.6)

\[ N_e = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)n(s, a, h, z)dzdhda \] (A1.7)

\[ K_e = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)k(s, a, h, z)dzdhda \] (A1.8)

\[ Y_e = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)zk(s, a, h, z)\alpha n(s, a, h, z)^{\theta}dzdhda \] (A1.9)

\[ S_e = \int_a \int_h \int_s \mu(1, a, h, z)e(1, a, h, z)dzdhda \] (A1.10)

\[ H_{adj} = \sum_{s=0}^{1} \int_a \int_h \int_s \mu(s, a, h, z)I_{h'\neq h}(s, a, h, z)dzdhda \] (A1.11)

where \( I_{a'\geq 0}(s, a, h, z) \) is 1 for households with positive financial assets and 0 for borrowers, and \( I_{rent}(s, a, h, z) \) is defined by equation (10) in Chapter 1.
Aggregate Resource Constraint

The workers’ budget constraint is given by equation (9) in Chapter 1, and the entrepreneurs’ budget constraint is given by (12) in Chapter 1. These can be collapsed into a general budget constraint applying to all households:

\[ c + a' + q(h' - h) + \Pi_{rent}^{r^h} qm + \Pi_{h'xh}^{\psi qh} \leq z k^a n^\theta - r^k k - wn - es \nu + (1 - e)w + (1 + r^a)a. \]  

(A1.12)

Recall that \( k = n = e = 0 \) for workers, and \( e = 1 \) for entrepreneurs. Since utility is increasing in consumption, the budget constraint holds with equality for all households. Integrating both sides of (A1.12) across the entire state space yields the aggregate resource constraint:

\[
\sum_s \int_a \int_h \int_z (c + a' + q(h' - h) + \Pi_{rent}^{r^h} qm + \Pi_{h'xh}^{\psi qhdzdhda} \\
= \sum_s \int_a \int_h \int_z (z k^a n^\theta - r^k k - wn - es \nu \\
+ (1 - e)w + (1 + r^a)a)dzdhda. \\

\]

(A1.13)

Rearranging and using the notation for aggregate quantities given by (A1.1-A1.A11) and substituting the financial sector’s first-order conditions given by (17)-(19) in Chapter 1 simplifies the equation:

\[
C + A'_b + A'_s + q(H' - H) + rqM_r + \psi qH_{adj} \\
= Y_e - (r + \delta_k + \tau)K_e + w(N_s - N_e) - S_{e \nu} \\
+ (1 + r)A_s + (1 + r + \tau)A_b. \\

\]

(A1.14)

---

1 Optimal choice variables are functions of the state (i.e., \( a' = a'(s, a, h, z) \)); for notational simplicity, I omit the parenthetical state list in this derivation.
Transforming (A1.13) using the market clearing conditions described by (20) and (21) in Chapter 1 yields:

\[ C + K'_c + K'_e + qM'_r + q(H' - H) + \psi qH_{adj} \]

\[ = Y_e - (r + \delta_k + \tau)K_e + wN_c - S_e\psi + K_c + K_e + qM_r \]

\[ + \tau A_b + r(K_e + K_c) . \]

The CRS technology of corporate production implies that \( Y_c = (r + \tau + \delta_k)K_c + wN_c \); substituting this identity and the housing market clearing condition given by (22) in Chapter 1 into the aggregated budget constraint yields:

\[ C + K'_c + K'_e + \psi qH_{adj} + \tau(K_e + K_c) \]

\[ = Y_e + Y_c + K_c + K_e + qM_r - (\delta_k + \tau)(K_e + K_c) - S_e\psi \]

\[ + \tau A_b . \]

Rearranging yields the aggregate resource constraint given by (23) in Chapter 1.

A1.2: Computational Approach

*Tauchen Method Implementation*

Consider a Pareto\( (x_m, \eta) \) process \( z \). Discretize the \( z \) space into \( N_z \) equally spaced points, with \( z_1 = x_m \) and

\[ \frac{x_m}{0.001} \cdot \frac{1}{\eta} . \]

This is the value of \( z \) for which the cumulative density function is equal to 0.999 (this value is necessarily arbitrary, since the domain of the random variable is unbounded).

Using Tauchen’s Method, the probability associated with \( z = z_i \) is given by
Model Solution Algorithm

Solving for the model stationary distribution (for a given parameter calibration) requires the following steps:

1. Make an initial guess for $q$, the house price.

2. Make an initial guess for $r$, the interest rate paid to savers.

3. Based on the corporate firm’s optimality condition given by (8) in Chapter 1, use $w = (1 - \xi)Z_c \left(\frac{r + \delta_k + \tau}{\xi z_c}\right)^{\frac{\xi}{1-\xi}}$ to obtain the wage. The optimality condition also provides the corporate capital/labor ratio implied by the interest rate guess: $\frac{K_c}{N_c} = \left(\frac{z_c \xi}{r + \delta_k + \tau}\right)^{\frac{1}{1-\xi}}$. Define this capital/labor ratio as $X_{Kn}$.

4. Given the price vector $(r, w, q)$ and using value function iteration, solve for the entrepreneur value function $v^{e=1}(s, a, h, z)$ and associated policy functions $x_{e=1}(s, a, h, z)$ for $x \in \{a', h', m, k, n\}$. Solve for the worker value function $v^{e=0}(a, h, z)$ and associated policy functions $x_{e=0}(a, h, z)$ for $x \in \{a', h', m\}$. For all points on the (discretized) state space, define $v(s, a, h, z) = \ldots$
max(\(v^{e=0}(a, h, z), v^{e=1}(s, a, h, z)\)), and define optimal policy functions as follows:

\[ x(s, a, h, z) = \begin{cases} x_{e=1}(s, a, h, z) & \text{if } v^{e=1}(s, a, h, z) \geq v^{e=0}(a, h, z) \\ x_{e=0}(s, a, h, z) & \text{if } v^{e=1}(s, a, h, z) < v^{e=0}(a, h, z) \end{cases} \]

5. Make an initial guess \(\mu_0\) for the distribution of households, obtain \(\mu' = \Psi(\mu_0)\), then iterate until convergence (according to desired tolerance) to fixed point \(\mu^*\).

6. Clear the labor market by defining

\[ N_c = N_s - N_e, \]

then compute \(K_c = X_{KN}N_c\). Define

\[ K_c^* = A_s + A_e - K_e - qM_r. \]

7. If \(K_c = K_c^*\) (within chosen tolerance), then the financial market clears. If not, update the guess for \(r\) and return to step 3.

8. Compare aggregate housing demand \(M_r + H'\) to housing supply \(H_s\). If \(M_r + H' = H_s\) (within chosen tolerance), all markets clear and the stationary distribution equilibrium has been obtained. Otherwise, update the guess for \(q\) and return to step 2.
Section A2: Appendix for Chapter 2

A2.1: Kauffman Firm Survey Sample

The Kauffman Firm Survey (KFS) provides a large panel of data on “young” businesses. Firms in the sample were founded in 2004 and have been tracked annually. This panel was created using a random sample from Dun & Bradstreet’s database of new businesses. The target population consists of all new businesses that were started in 2004 in the United States, and it excludes any branch or subsidiary owned by an existing business or that was simply inherited from someone else. The sample for the first survey consisted of 4,928 businesses.

The KFS provides a unique opportunity to study a panel of new businesses from startup, using available data on their revenues and expenses, number of workers, products, services, innovations that they possessed and developed in their early years, and the extent to which these businesses are involved in innovative activities. One drawback of the publicly available KFS data is that some variables, such as assets (and its components) and sales, are only reported within certain ranges. We set the value of the relevant variables to the middle value of the reporting range.

Our unit of observation is the firm, as defined by the KFS. The change in sales is constructed from total revenues from sales of goods, services, and intellectual

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2 Data available at the time of writing extend through 2008. Firms will continue to be tracked through 2011. See http://www.kauffman.org/kfs/ for a detailed description of the data and the actual public-use microdata.

3 For example, ranges for revenues are $0, $1 to $1,000, $1,001 to $5,000, $5,001 to $10,000, $10,001 to $25,000, $25,001 to $100,000, and $100,001 or more.

4 The set of variables we use that present this problem are: revenue from sales of goods, services or intellectual property; expenses; wages; and assets (and their components).
property. As is standard in the literature, size is defined as the number of employees.

We use two-digit industry deflators. Our variable expenses associated with, for example, design of new products, brand development, advertising, marketing, organizational development, or management consulting. For firm-year observations with missing values of SGA expenses, we compute the average ratio of SGA expenses to total expenses and impute SGA expenses from this ratio and total expenses.

Table A2.1 reports the distribution of real sales and real SGA expenses for new firms (i.e., the distribution of firms in 2004) and for firms that survive until the end of our sample (2008).

<table>
<thead>
<tr>
<th>Table A2.1: Distribution of Sales and Expenses (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thousands of $</td>
</tr>
<tr>
<td>Sales</td>
</tr>
<tr>
<td>$0-$3</td>
</tr>
<tr>
<td>$3-$10</td>
</tr>
<tr>
<td>$10-$50</td>
</tr>
<tr>
<td>$50-$100</td>
</tr>
<tr>
<td>$100+</td>
</tr>
<tr>
<td># firms</td>
</tr>
</tbody>
</table>

Note: Sales and SGA are deflated using the GDP deflator. Source: KFS.

Observe that many firms are relatively small, with sales and selling expenses below $10,000. This is still the case even after four years of existence. However, a non-trivial number of new firms have sales and SGA above $100,000. The distributions clearly shift upward as the cohort of firms becomes older and as selection occurs.
Table A2.2 reports the distribution of newly created firms as seen in the KFS, a comparison with the size distribution of new firms from the Census Bureau’s Statistics of new Businesses data, and the distribution of firms over employment for our cohort of firms in 2008.5

Table A2.2: Distribution of Workers (%)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>74</td>
<td>77</td>
<td>65</td>
</tr>
<tr>
<td>5-9</td>
<td>15</td>
<td>13</td>
<td>18</td>
</tr>
<tr>
<td>10-19</td>
<td>7</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>20-99</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>100-499</td>
<td>0.3</td>
<td>0.4</td>
<td>5.0</td>
</tr>
<tr>
<td>500+</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>


Table A2.2 shows that a large fraction of firms start with only a few workers. More than 70 percent of new firms hire between one and four workers. As a comparison we report the distribution of new firms from the Statistics of New Businesses; note that the distributions are very similar. This reassures us that we have a representative sample of new firms, despite some differences in the distribution of new firms across industries and in the different methodologies used across the sources. Finally, and consistent with the evidence presented in Table A2.1, among active firms in the KFS in 2008 there is a sizeable reduction in the fraction of firms with less than four workers and an increase in the fraction of firms with more than 10 workers.

---

5 For comparison, we report the distribution conditional on firms having more than one worker. In the KFS data, we find that in 2004, 58 percent of active firms hired zero workers; this value was 44 percent in 2008.
Table A2.3 displays the distribution of firms across some representative industries and their one-year survival rates.

Table A2.3: Distribution of Firms Across Industries and Survival Rates

<table>
<thead>
<tr>
<th>Industry</th>
<th>Fraction of firms (%)</th>
<th>One-year survival rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>10.0</td>
<td>91.9</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>7.1</td>
<td>92.0</td>
</tr>
<tr>
<td>Wholesale</td>
<td>5.4</td>
<td>88.7</td>
</tr>
<tr>
<td>Retail</td>
<td>15.9</td>
<td>86.1</td>
</tr>
<tr>
<td>Transportation and Warehousing</td>
<td>3.4</td>
<td>84.7</td>
</tr>
<tr>
<td>Information</td>
<td>2.7</td>
<td>84.6</td>
</tr>
<tr>
<td>Finance and Insurance</td>
<td>4.7</td>
<td>95.8</td>
</tr>
<tr>
<td>Administration and Support</td>
<td>9.6</td>
<td>91.7</td>
</tr>
<tr>
<td>Accommodation and Food Services</td>
<td>4.3</td>
<td>77.7</td>
</tr>
</tbody>
</table>

Source: Kauffman Firm Survey.

A2.2: Compustat and Compustat-Segment Sample

We use Compustat’s Fundamental and Segment annual data. Our choice of firm identifier is GVKEY, and this is the variable we use for matching the Compustat segment file to the fundamentals file. The sample period for the fundamentals data ranges from 1960 to 2012, but segment data only exist from 1977 to 2012. Not all firms have segment data. Our year variable is extracted from the variable DATADATE (for both the fundamentals and the segments file). We exclude financial firms with SIC codes between 6000 and 6999, utility firms with SIC codes between 4900 and 4999, and firms with SIC codes greater than 9000 (residual categories). Observations are deleted if they do not have a positive book value of assets or if gross

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* All variable names correspond to the Wharton Research Data Services (WRDS) version of Compustat.
capital stock or sales are either zero, negative, or missing. The final sample is an unbalanced panel with more than 21,600 firms and 241,000 firm-year observations; of these, there are 18,700 firms and 184,000 firm-year observations with segment data.

Our data variables are defined as follows. The change in sales is constructed from the variable SALE. As is standard in the literature, firm size is defined as the number of employees, using the variable EMP. We use two-digit NAICS codes to control for industry effects. Firm age is proxied by the number of years since the firm’s first-year observation in Compustat (so it is really a measure of years since IPO rather than age proper). All nominal variables are deflated using BEA’s two-digit, sector-specific price deflator for value added.

Segment counts reflect the sum of primary and secondary four-digit SIC codes reported in the Compustat variables SICS1 and SICS2. Compustat reports four-digit SIC codes for segments throughout the sample. NAICS codes are also reported in later years; there are no observations in which a NAICS code is reported but a SIC code is not. BEA deflators for value added are only given for SIC codes until 1988; at that time, the BEA began reporting deflators for NAICS codes. Therefore, when possible, we deflate segment-level sales using 10 sector-level SIC code deflators; elsewhere, we deflate with 24 two-digit NAICS sector codes. Thus, SIC deflators reflect less industry detail than NAICS deflators due to the lack of a unique mapping between NAICS and SIC; for this reason, we verified that our results are robust to using SIC deflators at the next possible level of detail, for which there are more than 80 SIC codes. For our reported results, we use the sector-level SIC deflators.
Table A2.4 reports the distribution of real sales and real SGA expenses for firms in 1980 and 2008.\(^7\) Note that firms’ sales and SGE expenses are considerably larger than those in the KFS sample.

<table>
<thead>
<tr>
<th>Millions of $</th>
<th>Year 1980</th>
<th></th>
<th>Year 2008</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sales</td>
<td>SGA</td>
<td>Sales</td>
<td>SGA</td>
</tr>
<tr>
<td>&lt;$10</td>
<td>19.7</td>
<td>27.7</td>
<td>12.4</td>
<td>21.7</td>
</tr>
<tr>
<td>$10-$20</td>
<td>9.4</td>
<td>12.4</td>
<td>5.3</td>
<td>12.7</td>
</tr>
<tr>
<td>$20-$50</td>
<td>14.0</td>
<td>16.8</td>
<td>10.3</td>
<td>16.9</td>
</tr>
<tr>
<td>$50-$100</td>
<td>11.8</td>
<td>12.0</td>
<td>10.1</td>
<td>13.1</td>
</tr>
<tr>
<td>$100-$250</td>
<td>14.3</td>
<td>12.9</td>
<td>13.8</td>
<td>13.7</td>
</tr>
<tr>
<td>$250+</td>
<td>30.8</td>
<td>18.1</td>
<td>48.1</td>
<td>21.9</td>
</tr>
</tbody>
</table>

| # Firms | 4,581 | 4,150 | 5,219 | 4,741 |

Note: Sales and Expenses are deflated using BEA’s two-digit SIC price deflators for value added. Source: Compustat Fundamental.

Table A2.5 reports the distribution of employment size for 1980 and 2008. To simplify the comparison, the size bins are the same as the ones we used for the KFS sample.

<table>
<thead>
<tr>
<th>Number of employees</th>
<th>All firms</th>
<th></th>
<th>Segment firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>1.64</td>
<td>1.38</td>
<td>1.68</td>
<td>1.38</td>
</tr>
<tr>
<td>5-9</td>
<td>1.75</td>
<td>1.80</td>
<td>1.77</td>
<td>1.75</td>
</tr>
<tr>
<td>10-19</td>
<td>2.49</td>
<td>3.14</td>
<td>2.51</td>
<td>2.79</td>
</tr>
<tr>
<td>20-99</td>
<td>11.07</td>
<td>13.26</td>
<td>11.30</td>
<td>12.49</td>
</tr>
<tr>
<td>100-499</td>
<td>23.31</td>
<td>21.63</td>
<td>23.50</td>
<td>20.62</td>
</tr>
<tr>
<td>500+</td>
<td>59.75</td>
<td>58.79</td>
<td>59.25</td>
<td>60.97</td>
</tr>
</tbody>
</table>

| # Firms | 4,581 | 5,219 | 4,469 | 4,627 |

Source: Compustat Fundamental and Compustat Segment.

\(^7\) Our data extend to 2012, but we present 2008 to allow for a comparison with the last year of our KFS sample.
Most firms in the Compustat sample have more than 500 workers, whereas in the KFS sample this value is less than 1 percent. Table A2.6 reports the distribution of firm age (computed as the number of years of presence in the sample).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>18.05</td>
<td>26.38</td>
<td>18.12</td>
<td>20.90</td>
</tr>
<tr>
<td>6-10</td>
<td>41.06</td>
<td>18.85</td>
<td>41.64</td>
<td>19.23</td>
</tr>
<tr>
<td>11-15</td>
<td>15.06</td>
<td>17.34</td>
<td>15.10</td>
<td>18.63</td>
</tr>
<tr>
<td>16-20</td>
<td>10.43</td>
<td>10.92</td>
<td>9.85</td>
<td>12.08</td>
</tr>
<tr>
<td>21-25</td>
<td>0.00</td>
<td>7.86</td>
<td>0.00</td>
<td>8.64</td>
</tr>
<tr>
<td>26+</td>
<td>0.00</td>
<td>14.60</td>
<td>0.00</td>
<td>15.97</td>
</tr>
<tr>
<td>Top censored</td>
<td>15.39</td>
<td>4.04</td>
<td>15.28</td>
<td>4.54</td>
</tr>
</tbody>
</table>

Note: "Top censored" corresponds to firms that are in our sample starting in 1960. Source: Compustat Fundamental and Compustat Segment.

We employed the following rules when constructing the Compustat Fundamental/Compustat Segment linked data. When multiple data source dates (SRCDATE) existed for one firm/data date/segment combination, we kept only the most recent source date. When multiple data dates existed for one firm-year-segment combination, we kept only the later data date unless its sales figures were missing (in which case we kept the earlier data date). When multiple segment identifiers existed for one four-digit SIC code, we combined the segments: Segment counts reflect the number of unique four-digit SIC codes, and segment-level employment reflects the sum of all reported segments within a four-digit SIC code.

Finally, Table A2.7 shows the correlations between the variables used from the Compustat Fundamental/Compustat Segment dataset.
Table A2.7: Correlations Table, Compustat Segment

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>SICs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Emp.</strong></td>
<td>0.797</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SGA</strong></td>
<td>0.893</td>
<td>0.749</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Adv.</strong></td>
<td>0.641</td>
<td>0.484</td>
<td>0.716</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>SICs</strong></td>
<td>0.227</td>
<td>0.233</td>
<td>0.228</td>
<td>0.226</td>
<td>1</td>
</tr>
</tbody>
</table>

**Entire sample, Compustat Segment**

**“Changers”**

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>SICs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Emp.</strong></td>
<td>0.789</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SGA</strong></td>
<td>0.874</td>
<td>0.728</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Adv.</strong></td>
<td>0.699</td>
<td>0.533</td>
<td>0.764</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>SICs</strong></td>
<td>0.205</td>
<td>0.240</td>
<td>0.199</td>
<td>0.206</td>
<td>1</td>
</tr>
</tbody>
</table>

**“Non-changers”**

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>SICs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Emp.</strong></td>
<td>0.799</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SGA</strong></td>
<td>0.898</td>
<td>0.753</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Adv.</strong></td>
<td>0.624</td>
<td>0.470</td>
<td>0.703</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>SICs</strong></td>
<td>0.229</td>
<td>0.227</td>
<td>0.231</td>
<td>0.228</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: All nominal variables are deflated using BEA’s two-digit, sector-specific price deflator for value added. Source: Compustat Fundamental and Compustat Segment.

A2.3: LBD-Compustat Fundamental Link

The LBD is constructed from the business register of the U.S. Bureau of the Census (see Jarmin and Miranda (2002)). It includes all nonfarm private sector employer establishments and firms in the United States from 1976 to 2011 and provides information on location, industry, and employment. Employment information reflects the payroll of establishments as of March 12 of a given year. The LBD links establishments as firms; firm identifiers reflect operational control and can span across state lines.
Both Compustat and the LBD include various firm identifiers that can be used for matching: employer identification numbers (EINs), two alternative business names (in Compustat, these are given by CONM and CONML), and addresses. We obtained further match flexibility by employing the SAS DQMATCH algorithm. We linked Compustat to the LBD by using successive “passes” that matched Compustat firms to LBD establishments using these identifiers with varying degrees of specificity. Early match passes relied on EINs and full business names and addresses (which have been standardized). Subsequent passes utilized algorithms that evaluate name similarity conditional on geographic matches. Final passes employed DQMATCH descriptors. We utilize both alternative name variables from each data source, thus allowing for potential matches along any combination of name variables. Only residual nonmatched CUSIPs are retained after each pass; by ordering such that match criteria become less specific with each successive pass, we ensure that the final data linkages are based on the highest possible match quality for each firm.

We eliminate firm-year matches that are out of scope for Compustat activity (as determined by IPODATE and DLDTE when available or by time periods of non-positive employment, sales, or share price when the former variables are missing). Instances in which a CUSIP was paired with multiple LBD firms were resolved by first dropping LBD firms with only one operating unit and then choosing the LBD firm with reported employment closes to Compustat reported employment. Since many firms have time series gaps in EIN coverage, and since business names in the

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8 For matching purposes, we first discard from Compustat all exchange traded funds (ETFs) that can be easily identified, American depositary receipts (ADRs) and American depositary shares (ADSs), CUSIPs with non-U.S. geographical identifiers, and firms that operate only outside of North America (as identified by IDBFLAG).
LBD refer to establishments rather than firms (and occasionally change over time), we make additional matches by rolling firm-year matches across years when appropriate.

We find matches for about 80 percent of relevant Compustat CUSIPs in the LBD source data; on average, we have about 4,200 observations with relevant non-missing data per year in our LBD-Compustat Fundamental linked data spanning from 1977 to 2011. The resulting comingled data include sales and industry information from Compustat with employment and geographic information from the LBD. In the LBD, an establishment is a single business location with one or more employees (sometimes called a plant in other data sources). We classify establishment locations using the Census Bureau’s 2009 definitions of Metropolitan Statistical Areas (MSAs), which are comprised of whole counties (i.e., an MSA is a collection of counties). For our estimation purposes, any county that is not included in an MSA is classified as its own MSA.\(^9\) Table A2.8 presents summary statistics from the linked dataset.

\begin{table}[h]
\centering
\begin{tabular}{lccc}
\hline
 & Mean & Std. Dev. & Corr. w/ SGA \\
Number of estabs & 125 & 617 & 0.35 \\
Number of MSAs & 34 & 99 & 0.34 \\
\hline
\end{tabular}
\caption{Summary Statistics for LBD Variables}
\end{table}

Finally, Tables A2.9, A2.10, and A2.11 show the correlations between the variables used from the Compustat-LBD data for the different market definitions.

\(^9\) An alternative to this classification is to collect all counties within a state that are not included in an MSA and defined that collection as a single MSA so that each state has a single residual “MSA” in addition to proper MSAs. We also performed our analysis on data constructed with this definition, and it did not substantially alter the results.
Table A2.9: Correlations Table, Compustat-LBD (Establishments)

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>Estabs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire sample, Compustat-LBD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.800</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.882</td>
<td>0.749</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.656</td>
<td>0.462</td>
<td>0.727</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Estabs</td>
<td>0.348</td>
<td>0.448</td>
<td>0.370</td>
<td>0.230</td>
<td>1</td>
</tr>
</tbody>
</table>

| “Changers” |       |      |       |      |        |
| Sales      | 1     |      |       |      |        |
| Emp.       | 0.837 | 1    |       |      |        |
| SGA        | 0.864 | 0.664| 1     |      |        |
| Adv.       | 0.696 | 0.565| 0.787 | 1    |        |
| Estabs     | 0.447 | 0.285| 0.550 | 0.356| 1      |

| “Non-changers” |       |      |       |      |        |
| Sales         | 1     |      |       |      |        |
| Emp.          | 0.793 | 1    |       |      |        |
| SGA           | 0.881 | 0.743| 1     |      |        |
| Adv.          | 0.655 | 0.450| 0.727 | 1    |        |
| Estabs        | 0.326 | 0.439| 0.345 | 0.208| 1      |

Note: All nominal variables are deflated using BEA’s two-digit, sector-specific price deflator for value added. Source: Compustat Fundamental and LBD.
### Table A2.10: Correlations Table, Compustat-LBD (MSAs)

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>Estabs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
<td>1</td>
<td>0.800</td>
<td>0.749</td>
<td>0.727</td>
<td>1</td>
</tr>
<tr>
<td><strong>Emp.</strong></td>
<td></td>
<td>0.883</td>
<td>0.4616</td>
<td>0.237</td>
<td></td>
</tr>
<tr>
<td><strong>SGA</strong></td>
<td></td>
<td>0.656</td>
<td>0.727</td>
<td>0.367</td>
<td>1</td>
</tr>
<tr>
<td><strong>Adv.</strong></td>
<td></td>
<td>0.367</td>
<td>0.479</td>
<td>0.385</td>
<td>0.237</td>
</tr>
<tr>
<td><strong>MSAs</strong></td>
<td></td>
<td>0.385</td>
<td>0.237</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Entire sample, Compustat-LBD**

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>Estabs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
<td>1</td>
<td>0.801</td>
<td>0.653</td>
<td>0.766</td>
<td>1</td>
</tr>
<tr>
<td><strong>Emp.</strong></td>
<td></td>
<td>0.875</td>
<td>0.487</td>
<td>0.410</td>
<td></td>
</tr>
<tr>
<td><strong>SGA</strong></td>
<td></td>
<td>0.680</td>
<td>0.766</td>
<td>0.360</td>
<td>0.230</td>
</tr>
<tr>
<td><strong>Adv.</strong></td>
<td></td>
<td>0.367</td>
<td>0.487</td>
<td>0.360</td>
<td>1</td>
</tr>
<tr>
<td><strong>MSAs</strong></td>
<td></td>
<td>0.341</td>
<td>0.462</td>
<td>0.358</td>
<td>0.203</td>
</tr>
</tbody>
</table>

**“Changers”**

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>Estabs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sales</strong></td>
<td>1</td>
<td>0.793</td>
<td>0.749</td>
<td>0.724</td>
<td>1</td>
</tr>
<tr>
<td><strong>Emp.</strong></td>
<td></td>
<td>0.882</td>
<td>0.451</td>
<td>0.385</td>
<td>1</td>
</tr>
<tr>
<td><strong>SGA</strong></td>
<td></td>
<td>0.656</td>
<td>0.724</td>
<td>0.385</td>
<td>0.237</td>
</tr>
<tr>
<td><strong>Adv.</strong></td>
<td></td>
<td>0.341</td>
<td>0.451</td>
<td>0.358</td>
<td>0.203</td>
</tr>
<tr>
<td><strong>MSAs</strong></td>
<td></td>
<td>0.341</td>
<td>0.462</td>
<td>0.385</td>
<td>1</td>
</tr>
</tbody>
</table>

**“Non-changers”**

Note: All nominal variables are deflated using BEA’s two-digit, sector-specific price deflator for value added. Source: Compustat Fundamental and LBD.
Table A2.11: Correlations Table, Compustat-LBD (Estabs*SICs)

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>Estabs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.802</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.884</td>
<td>0.748</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.658</td>
<td>0.465</td>
<td>0.727</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Est*SIC</td>
<td>0.381</td>
<td>0.444</td>
<td>0.391</td>
<td>0.258</td>
<td>1</td>
</tr>
</tbody>
</table>

Entire sample, Compustat-LBD

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>Est*SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.839</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.855</td>
<td>0.637</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.701</td>
<td>0.577</td>
<td>0.800</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Est*SIC</td>
<td>0.490</td>
<td>0.252</td>
<td>0.615</td>
<td>0.383</td>
<td>1</td>
</tr>
</tbody>
</table>

“Changers”

<table>
<thead>
<tr>
<th></th>
<th>Sales</th>
<th>Emp.</th>
<th>SGA</th>
<th>Adv.</th>
<th>Est*SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emp.</td>
<td>0.798</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA</td>
<td>0.882</td>
<td>0.740</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv.</td>
<td>0.664</td>
<td>0.461</td>
<td>0.732</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Est*SIC</td>
<td>0.362</td>
<td>0.445</td>
<td>0.363</td>
<td>0.241</td>
<td></td>
</tr>
</tbody>
</table>

“Non-changers”

Note: All nominal variables are deflated using BEA’s two-digit, sector-specific price deflator for value added. Source: Compustat Fundamental and LBD.

All reported statistics based on the LBD-Compustat link were reviewed and do not disclose confidential information.

A2.4: BDS Sample

Table A2.12 presents the cyclical properties of workers per establishment computed from BDS data. Since most variables in this sample have a trend component, we detrended them using a linear trend when reporting the averages in Tables 5 (in Chapter 2) and A2.12. Table A2.13 reports the detrended and the nondetrended average (i.e., sample average) for the variables of interest.
Table A2.12: Number of Workers per Establishment over the Business Cycle

<table>
<thead>
<tr>
<th>Firm size (workers)</th>
<th>Avg. # of firms (1000s)</th>
<th>Fraction total emp. (%)</th>
<th>Avg. emp. per plant</th>
<th>Cyclical props., Wkr. per est w/ GDP</th>
<th>Elasticity w/ GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Below</td>
<td>Above</td>
</tr>
<tr>
<td>1-4</td>
<td>1909</td>
<td>6.6</td>
<td>2.31</td>
<td>2.31</td>
<td>2.32</td>
</tr>
<tr>
<td>5-9</td>
<td>736</td>
<td>7.3</td>
<td>6.59</td>
<td>6.56</td>
<td>6.62</td>
</tr>
<tr>
<td>10-19</td>
<td>408</td>
<td>8.4</td>
<td>12.7</td>
<td>12.6</td>
<td>12.8</td>
</tr>
<tr>
<td>20-49</td>
<td>237</td>
<td>11.0</td>
<td>24.4</td>
<td>24.2</td>
<td>24.7</td>
</tr>
<tr>
<td>50-99</td>
<td>71</td>
<td>7.5</td>
<td>40.2</td>
<td>39.7</td>
<td>40.8</td>
</tr>
<tr>
<td>100-249</td>
<td>35</td>
<td>8.2</td>
<td>51.4</td>
<td>50.6</td>
<td>52.4</td>
</tr>
<tr>
<td>250-499</td>
<td>10</td>
<td>5.2</td>
<td>59.0</td>
<td>58.2</td>
<td>59.8</td>
</tr>
<tr>
<td>500-999</td>
<td>5</td>
<td>4.8</td>
<td>63.2</td>
<td>63.0</td>
<td>63.3</td>
</tr>
<tr>
<td>1000-2499</td>
<td>3</td>
<td>6.5</td>
<td>66.1</td>
<td>65.5</td>
<td>66.8</td>
</tr>
<tr>
<td>2500-4999</td>
<td>1</td>
<td>4.8</td>
<td>60.5</td>
<td>60.2</td>
<td>60.9</td>
</tr>
<tr>
<td>5000+</td>
<td>1</td>
<td>29.7</td>
<td>75.8</td>
<td>75.7</td>
<td>75.9</td>
</tr>
</tbody>
</table>

Note: We extract a linear trend component from all variables. The “Avg. # of firms” corresponds to the average number of firms in each size bin in our sample (in thousands). “Fraction total emp.” is computed as the average of total employees in each size bin divided by total employment. “Avg. emp. per plant” corresponds to the average of the number of employees per establishment in each size category. “Wkr. per est. w/ GDP below or above is derived from a linear regression of average number of workers per establishment by firm size on a constant, a linear trend, and a dummy that identifies periods when GDP is above trend. The value reported is the parameter on this dummy. GDP corresponds to log-real GDP. The trend for GDP is computed using the HP filter with parameter 6.25. “Elasticity w/ GDP” corresponds to the elasticity between the average number of workers per establishment by firm size and detrended GDP. This elasticity is derived from a linear regression of log-average number of workers per establishments by firm size on a constant, a linear trend, and log-real detrended GDP. * denotes significance at the 10% level.

Source: U.S. Census Bureau, Business Dynamics Statistics (BDS).
<table>
<thead>
<tr>
<th>Firm size (workers)</th>
<th>Avg. # firms (1000s)</th>
<th>Fraction total emp.</th>
<th>Avg. emp. per plant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Detrended</td>
<td>Non-det.</td>
<td>Detrended</td>
</tr>
<tr>
<td>1-4</td>
<td>1909</td>
<td>2437</td>
<td>6.6</td>
</tr>
<tr>
<td>5-9</td>
<td>736</td>
<td>941</td>
<td>7.3</td>
</tr>
<tr>
<td>10-19</td>
<td>408</td>
<td>542</td>
<td>8.4</td>
</tr>
<tr>
<td>20-49</td>
<td>237</td>
<td>329</td>
<td>11.0</td>
</tr>
<tr>
<td>50-99</td>
<td>71</td>
<td>102</td>
<td>7.5</td>
</tr>
<tr>
<td>100-249</td>
<td>35</td>
<td>54</td>
<td>8.2</td>
</tr>
<tr>
<td>250-499</td>
<td>10</td>
<td>15</td>
<td>5.1</td>
</tr>
<tr>
<td>500-999</td>
<td>5</td>
<td>7</td>
<td>4.8</td>
</tr>
<tr>
<td>1000-2499</td>
<td>3</td>
<td>5</td>
<td>6.5</td>
</tr>
<tr>
<td>2500-4999</td>
<td>1</td>
<td>2</td>
<td>4.8</td>
</tr>
<tr>
<td>5000+</td>
<td>1</td>
<td>2</td>
<td>29.7</td>
</tr>
</tbody>
</table>

Note: The “Avg. # firms” corresponds to the average number of firms in each size bin over our sample (in thousands). “Fraction total emp.” is computed as the average of total employees in each size bin divided by total employment. “Avg. emp. per plant” corresponds to the average of the number of employees per establishment in each size category.

Source: U.S. Census Bureau, Business Dynamics Statistics.

A2.5 Market Exposure and Volatility: Robustness Checks

Table A2.15 reports estimates from equation (27) in Chapter 2 using the number of SIC codes as well as SGA and advertising expenses from Compustat. We incorporated size and age as additional controls (in addition to firm fixed effects, year-industry fixed effects, and a time trend).

Table A2.16 reports estimates from equation (27) in Chapter 2 using the number of establishments, the number of MSAs, and the product of establishment counts and SIC codes using the Compustat-LBD linked dataset. In this table, too, we incorporated size and age as additional controls (in addition to firm fixed effects, year-industry fixed effects, and a time trend).

Observe that the relationship between our measures of market exposure (SIC codes, establishments, MSAs, SIC×establishments, SGA expenses, and advertising expenses) and firm-level volatility is robust to the incorporation of these additional controls.
controls. The coefficient on the appropriate market exposure measure is negative in all specifications other than the case of the SIC code count and the SIC code/establishment count product when size is included as a control. Moreover, the introduction of size as a control makes the estimates on market exposure nonsignificant. Note that this is not a surprising result based on the model: market exposure is highly correlated with firm size. In the data, the measures of market exposure and employment are also highly correlated, with correlation coefficients of up to 0.47.

Finally, Table A2.17 reports the elasticity of firm-level volatility to market exposure based on several different specifications involving multiple market exposure variables and interactions. These specifications deliver very close elasticities between 9 and 13 percent.
Table A2.14: Market Exposure and Firm-Level Idiosyncratic Volatility I

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Dependent variable $\ln(\epsilon_{i,t}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\ln(SICs)$</td>
</tr>
<tr>
<td></td>
<td>$\ln(SGA)$</td>
</tr>
<tr>
<td></td>
<td>$\ln(Adv)$</td>
</tr>
<tr>
<td></td>
<td>$\ln(size)$</td>
</tr>
<tr>
<td></td>
<td>$\ln(age)$</td>
</tr>
</tbody>
</table>

Note: *** denotes significance at the 1% level, ** at the 5% level, and * at the 10% level. All regressions include firm and industry-year fixed effects and a time trend. $\ln(\epsilon_{i,t}^2)$ is constructed from the estimated residual of equation (1) from Chapter 2. $\ln(SGA)$ is constructed as log-real Selling, General, and Administrative expenses, and $\ln(Adv)$ corresponds to Advertising expenses. Industry deflators are used in every case. $\ln(size)$ corresponds to log-employment as in equation (1) from Chapter 2. The age of the firm corresponds to the number of years in the Compustat sample. $\ln(SICs)$ corresponds to four-digit SIC codes as reported in the Compustat Segment data. Data are from linked Compustat Fundamental-Compustat Segment.
Table A2.15: Market Exposure and Firm-Level Idiosyncratic Volatility II

<table>
<thead>
<tr>
<th>Dependent variable $\ln(\epsilon_{ijt}^2)$</th>
<th>$\ln(Estabs)$</th>
<th>$\ln(MSAs)$</th>
<th>$\ln(SIC*Est)$</th>
<th>$\ln(size)$</th>
<th>$\ln(age)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.081^{***}$</td>
<td>$-0.046^{***}$</td>
<td>$-0.005$</td>
<td>$-0.075^{***}$</td>
<td>$-0.037^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.009)$</td>
<td>$(0.009)$</td>
<td>$(0.009)$</td>
<td>$(0.008)$</td>
<td>$(0.008)$</td>
</tr>
<tr>
<td></td>
<td>$-0.094^{***}$</td>
<td>$-0.053^{***}$</td>
<td>$-0.002$</td>
<td>$-0.002$</td>
<td>$-0.002$</td>
</tr>
<tr>
<td></td>
<td>$(0.010)$</td>
<td>$(0.010)$</td>
<td>$(0.011)$</td>
<td>$(0.011)$</td>
<td>$(0.011)$</td>
</tr>
<tr>
<td></td>
<td>$-0.282^{***}$</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
<td>$-0.558$</td>
</tr>
<tr>
<td></td>
<td>$(0.019)$</td>
<td>$(0.019)$</td>
<td>$(0.019)$</td>
<td>$(0.019)$</td>
<td>$(0.019)$</td>
</tr>
</tbody>
</table>

N: 129700, 129700, 129700, 124400, 124400, 124400, 129700, 129700, 129700

$R^2$: 0.02, 0.03, 0.02, 0.02, 0.03, 0.02, 0.02, 0.03, 0.02

Note: *** denotes significance at the 1% level, ** at the 5% level, and * at the 10% level. All regressions include firm and industry-year fixed effects and a time trend. $\ln(\epsilon_{ijt}^2)$ is constructed from the estimated residual of equation (1) from Chapter 2. $\ln(Estabs)$, $\ln(MSAs)$, and $\ln(SIC*Est)$ are the log of establishment count, MSA count, and product of SIC and establishment count. $\ln(size)$ corresponds to log-employment as in equation (1) from Chapter 2. The age of the firm corresponds to the number of years in the Compustat sample. $\ln(SICs)$ corresponds to four-digit SIC codes as reported in the Compustat Segment data. Data are from linked Compustat Fundamental-Compustat Segment and linked Compustat Fundamental-LBD.
Table A2.16: Market Exposure and Firm-Level Idiosyncratic Volatility III

<table>
<thead>
<tr>
<th>Dependent variable $\ln(\epsilon_{it}^2)$</th>
<th>$\ln(SICs)$</th>
<th>$\ln(Estabs)$</th>
<th>$\ln(MSAs)$</th>
<th>$\ln(SIC*Est)$</th>
<th>$\ln(SIC*MSA)$</th>
<th>$\ln(Est*MSA)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.137***</td>
<td>-0.081***</td>
<td>-0.094***</td>
<td>-0.075***</td>
<td>-0.085***</td>
<td>-0.045***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.008)</td>
<td>(0.009)</td>
<td>(0.005)</td>
</tr>
<tr>
<td></td>
<td>-0.007</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
</tr>
<tr>
<td></td>
<td>- (0.028)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
</tr>
<tr>
<td></td>
<td>- (0.028)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
</tr>
<tr>
<td></td>
<td>- (0.028)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
</tr>
<tr>
<td></td>
<td>- (0.028)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
<td>- (omitted)</td>
</tr>
</tbody>
</table>

Note: *** denotes significance at the 1% level, ** at the 5% level, and * at the 10% level. All regressions include firm and industry-year fixed effects and a time trend. $\ln(\epsilon_{it}^2)$ is constructed from the estimated residual of equation (1) from Chapter 2. $\ln(SICs)$, $\ln(Estabs)$, and $\ln(MSAs)$, are the log of SIC count, establishment count, and MSA count, respectively. Other included variables are corresponding products of the foregoing market exposure measures. Data are from linked Compustat Fundamental-Compustat Segment and linked Compustat Fundamental-LBD.


