ABSTRACT

Title of dissertation:	A DECISION MODEL FOR STUDENT-ATHLETE ENTRY INTO THE NBA DRAFT
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We develop a Markov Decision Process model using the framework of an optimal stopping problem to describe whether or not a student-athlete should enter the National Basketball Association (NBA) draft early. Our model uses a simulation algorithm for estimating the draft value of a student-athlete to inform his decision as he evaluates whether or not he should enter the NBA draft and forgo his remaining college eligibility. The model incorporates the shift in player evaluation for the draft that is now heavily focused on a student athlete's potential rather than the talent that a student-athlete displays in the collegiate game. The algorithm generates two estimates, one biased high and one biased low, both asymptotically unbiased as the computational effort increases and converging to the student-athlete's draft value.

A DECISION MODEL FOR STUDENT-ATHLETE ENTRY INTO THE NBA DRAFT

by

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1 Introduction

The National Basketball Association (NBA) is the premier professional basketball league in North America and home to some of the best basketball talent in the world. Since its inception in August of 1949, the landscape of the NBA has changed drastically, as the once struggling league is now a global force in the world of professional sports generating steady profits from an assortment of revenue streams (NBA.com, 2014). Today the NBA (also referred to as the League) consists of 30 organizations scattered throughout the United States and Canada with a record-breaking 92 international players from 39 countries and territories featured on opening-night rosters for the 2013-14 NBA season (NBA.com, 2014). The League has enjoyed increasing popularity with fans around the world the last few decades, as innovative global marketing campaigns, technological advances, and electrifying talent have helped the NBA become a multi-billion dollar business (Zola, 2011, p. 164). NBA teams generated $4.6 billion in basketball related revenue last season (2012-13), compared to just $118 million during the 1982-83 season. Former NBA commissioner David Stern, who stepped down from his role on February 1, 2014 after 30 years, left the League in a different state compared with the one he inherited in February 1984. The current structure of professional basketball in the United States is heavily attributed to Stern's advocacy on the NBA age minimum, labor relations, team expansion and other issues. NBA playoff games were once shown on tape-delay in the early 1980s, but are now broadcast live in 215 countries around the globe (Badenhausen, 2014). Last season (2012-13), the Miami Heat defeated the San Antonio Spurs in Game 7 (best out of seven series) of the NBA Finals to repeat as NBA Champions in the second highest-rated NBA Game in ABC history (Kondolojy, 2003).

Much of the NBA's success is determined by its competitive balance – the concept that the outcome of a particular game is unknown at the onset, thereby captivating fan interest to watch NBA games. Last year's NBA Final's runner-up, the San Antonio Spurs, fielded a team that included future Hall of Famer Tim Duncan from the U.S. Virgin Islands, All-Star players in Tony Parker from France, Manu Ginobili from Argentina, and a rising talent in Kahwi Leonard. But the Miami Heat, led by future Hall of Famers Lebron James and Dwyane Wade and All-Star Chris Bosh, prevailed in a best-of-seven series. While a variety of factors determine a team's ability to compete, the most important factor is the set of skills of the individual players who compete. Accordingly, the allocation of this talent is the lifeblood that helps ensure and maintain the NBA's competitive balance – and, thus, its popularity (Zola, 2011, p. 164). The amateur draft allows NBA franchises a mechanism to select new players with whom they may replenish their teams' talent pools. As the talent and skills of older players diminish, younger players are brought in to replace them, allowing the League to maintain its competitive balance and continue to flourish (Zola, 2011, p. 165). For much of the amateur draft's existence, the National Collegiate Athletic Association (NCAA) and its membership of colleges and universities have been the primary source of this basketball talent.

1.1 NBA Draft Entry and the Collective Bargaining Agreement (CBA)

In 1971, the United States Supreme Court ruled in the case of *Haywood v. the National Basketball Association, 401 U.S. 1204,* against the requirement that entrants into the NBA draft had to wait until their college class graduated to declare for the draft. For a brief period, the NBA allowed only early entrants (i.e. high school players) who requested and were approved entry based on “financial hardship.” As a result, two high school players, Darryl Dawkins and Bill Willoughby, entered the NBA draft in 1975 (NBA Player Profiles, 2014). The American Basketball Association (ABA), a rival professional league, drafted a high school player in Moses Malone in 1974. The two leagues merged in 1976 and Malone went on to become a perennial All-Star, three-time League Most Valuable Player (MVP), World Champion, and Hall of Famer (NBA.com, 2014). In 1976, the hardship requirement was eliminated and the “early entry” procedure was adopted by the NBA (Zola, 2011). This provision allowed any athlete with college eligibility to enter the draft by sending the Commissioner a letter formally forfeiting his remaining NCAA eligibility at least forty-five days before the draft (Zola, 2011). While all three previous early entrants achieved success in the NBA, it would be another 20 years until the next high school student-athlete would bypass college for the NBA draft (NBA.com, 2014). From 1976 through the 1994 draft, less than 18.1% of the first-round draft picks were early entrants; of these early entrants, 79.5% of the first-round draft picks into the NBA were college juniors. During this time period, a rookie player individually negotiated a contract with the team that drafted him. Salaries and contract length varied greatly based on player ability and franchise need (Groothius, Hill, & Perri, 2007, p. 224).

The NBA Collective Bargaining Agreement (CBA), first started in 1970, is the legal contract between the League and the National Basketball Player's Association (NBPA) that sets up the rules by which the League operates (Coon, NBA Salary Cap FAQ, 2014). The CBA defines the team salary cap, the procedures for determining how it is set, the minimum and maximum salaries, the rules for trades, the procedures for the NBA draft, and hundreds of other things that need to be defined in order for a league like the NBA to function. The CBA also prevents the NBA from being in violation of federal antitrust laws. Many of the League's practices (such as the salary cap, draft rules, and age minimums) would violate antitrust laws were they not agreed to via collective bargaining (Coon, NBA Salary Cap FAQ, 2014). The introduction of team salary caps in the 1983 CBA was a compromise between owners and the players' union, in return for 53% of NBA gross revenue being allocated to player salaries. The earlier cap level was below the payroll of five teams whose cap levels were frozen at their existing payroll (Groothius, Hill, & Perri, 2007). The introduction of team salary caps led to some inequities in rookie salaries. Teams could only pay a rookie either the League minimum, if they were at the cap or the available amount if they were under the cap. In 1987, for example, the third pick in the draft, Dennis Hopson, was paid a reported $400,850 for his rookie season, whereas the fifth pick in the draft, Scottie Pippen, earned $725,000 his first season. The first pick in the draft in 1987, David Robinson, earned a reported salary of $1,046,000 from San Antonio, the highest salary on his team by more than $250,000. Discontent among veterans at the prospect of unproven rookies earning more than they did and dissatisfaction with the inequities the salary cap imposed on the distribution of rookie salaries led to the introduction of a rookie pay scale in the 1995 NBA CBA (Zola, 2011, p. 226).

The draft mechanism limits amateur players from negotiating with multiple teams, thereby intentionally keeping salaries below free market rates. Under the rules of the NBA draft, a single team is granted the exclusive rights to negotiate with the players it selects. If a drafted player does not want to play for that team or live in that particular city under the proposed financial package offered, that player may refrain from playing until a contract is reached, but under no circumstances is he allowed to negotiate with other teams (Zola, 2011, p. 165). The 1983 CBA also reduced the draft from ten to seven rounds, beginning in 1985. The 1985 draft was also the first year the League instituted a "draft lottery" in which the worst seven teams (as determined by NBA records the previous year) were entered to determine the first seven picks in the draft. This modification was devised to address the perception that teams were intentionally losing games at the end of the season to enhance their draft prospects. Finally, the number of rounds in the draft was further reduced to three rounds in 1988 and to the current two round system beginning in 1989. While "modern" amateur draft rules have been in place since 1989, the way NBA franchises use the draft has changed significantly since then. The largest shift is that teams have moved away from selecting college seniors; instead teams now focus on "upside" and "potential" when evaluating draft prospects (Zola, 2011, p. 169).

1.1.1 Impacts of the NBA Draft Age Limit

The NBA abandoned the "hardship rule" five years after the Haywood ruling. In its place, the NBA adopted the "Early Entry Process." Essentially, this opened the amateur draft to seventeen-year-old players; but from 1949 to 1994, only two players entered the NBA directly from high school (McAleavey, 2010, p. 283). In 1995, over twenty years after the *Haywood v. the National Basketball Association* ruling, Kevin Garnett was selected out of high school with the fifth pick by the Minnesota Timberwolves in the first round of the NBA Draft. Garnett's jump to the professional ranks inspired other high school standouts to make that transition. Kobe Bryant followed his path and was drafted in the first round of the 1996 Draft by the Charlotte Hornets. Tracy McGrady, Amar'e Stoudemire, and LeBron James were among the most visible early entrants who went on to become NBA all-stars. The success of young players who skipped college to go straight to the NBA created a wave in which a total of thirty-nine players were drafted by the NBA immediately out of high school between 1995 and 2005. Despite some successes, several high-profile early entrants like Korleone Young (drafted in 1998), Leon Smith (drafted in 1999), Kwame Brown (drafted number one overall in 2001), and Eddy Curry (drafted in 2001) never achieved the success franchises had envisioned. Such failures increased the volatility and uncertainty surrounding the NBA Draft and player selection (NBA.com, 2014). Concerned with the influx of high school player bypassing college for the NBA, Commissioner Stern publicly argued for an age limit of twenty before an individual could be draft eligible, offering the need for more mature and "seasoned" League (Zola, 2011, p. 170).

As result, in 2005 the NBA and National Basketball Player's Association (NBPA) ultimately agreed upon a provision, which was then incorporated into the League's CBA, for the 2006 draft requiring that a player be "at least nineteen years of age during the calendar year in which the Draft is held" (Zola, 2011, p. 171). Consequently, amateur players could no longer make the jump from high school into the NBA, allowing the rookie salary scale to consume more of a player's prime athletic years, and give NBA franchises more time to assess the talent of young players before investing in them. The rule forced stars like Greg Oden (drafted number one overall in 2007) and Kevin Durant (drafted second overall in 2007), both of whom could have competed in the NBA earlier than they did, to attend Ohio University and the University of Texas, respectively, for one year before entering the Draft (NBA.com, 2014). Brandon Jennings (drafted tenth overall in 2009), one of the top high school point guards in the class of 2008, chose to leverage his economic value and played professionally in Italy as an alternative to attending college for one year (NBA.com, 2014).

The new provision was well-received by the National Collegiate Athletic Association (NCAA), which was in favor of high profile basketball stars being diverted back to college campuses across America to enhance the NCAA's visibility and increase the value of its basketball media rights (Zola, 2011, p. 170). College basketball has long been the "minor league" for the NBA. As a minor league, it serves a dual purpose. First, it is a training ground where players can hone their skills and become more productive. It is where players move from playing in front of small crowds to playing in front of large crowds and on national television. Second, college basketball serves as a signaling device to provide information on a player's NBA potential. When players leave early, they have less experience and provide a noisier signal or higher degree of draft uncertainty than a player who stays in college (Groothius, Hill, & Perri, 2007, p. 228). Despite this fact, NBA franchises are still reluctant to bypass a young student-athlete in the draft who may turn into an all-star within a few years in favor of a more seasoned college player (Zola, 2011, p. 170).

The byproduct of this paradigm shift in draft philosophy is that each year a student-athlete plays in college, his value actually decreases as he loses the ability to claim "upside" or "potential" (Zola, 2011, p. 170). In addition, we assume that on average, an NBA player plays to the age of 27. This means that every year of NBA competition becomes crucial to a student-athlete's career earning potential (Coon, ESPN.com, 2011).

Early entry provides an opportunity for elite college players to get acclimated to NBA competition earlier and sign lucrative free agent contracts in the prime of their playing careers. This earning potential increases the opportunity cost of remaining in school and makes entering the NBA draft early the better alternative and the rational choice for these student athletes (McAleavey, 2010, p. 11). In fact, over the past eight years (2006 through 2013), a total of 45 collegiate seniors have been taken in the first round of a draft out of a total of 240 selections, meaning that underclassmen and international players comprise 81% of first round draft picks. Although a successful college career may increase a student-athlete's discipline and provide useful life experience, it does not compare to the opportunity to take care of themselves and their families for the rest of their lives.

1.1.2 National Collegiate Athletic Association (NCAA) Basketball

The NCAA champions itself as a membership-driven organization dedicated to safeguarding the well-being of student-athletes and equipping them with the skills to succeed on the playing field, in the classroom and throughout life (NCAA.org, 2014). The NCAA governs college eligibility and the concept of amateurism for all student-athletes for every college and university across the country. Because the NCAA is a non-profit with voluntary membership, its ability to impose rules and restrictions on student-athletes is virtually absolute. The NCAA has operating and administrative bylaws that define the manner in which all college players must gain initial eligibility; maintain their academic and athletic eligibility; follow the constraints of the NCAA's self-defined "amateur" status; and guidelines to represent their university and the NCAA on the court. Under the auspices of protecting amateurism, the NCAA membership has very clearly established lines that define the rules by which all student-athletes who transition from college to professional athletics must abide if they are to maintain their NCAA eligibility. Any player who hopes to play in the NBA must comply with these rules, as NBA rules now practically force players to attend a minimum of one year of college. (Zola, 2011, p. 174 & 175).

The NCAA holds a post-season basketball tournament every year in March where 68 Division I teams square off at a shot at winning the national championship. The tournament, referred to as “March Madness” for its excitement and the slew of games played throughout the country during the month, provides a platform for student-athletes to play on the largest stage in college basketball. This single elimination event generates a substantial buzz, because there is always a certain amount of uncertainty as new teams and new players get an opportunity to shine in the national spotlight. This three-week stretch of college basketball is a major source of revenue for the NCAA. In 2010, the NCAA Time Warner’s Turner Broadcasting and Columbia Broadcast System (CBS) signed a $10.8 billion, 14-year deal to televise the March Madness beginning in 2011 (FoxBusiness.com, 2010). In addition, the NCAA earns money from ticket sales, concessions, and merchandise. Colleges and Universities often pay contract bonuses to their head coaches when their teams make the NCAA tournament field because the tournament exposure often generates revenues for these campuses and serves as a platform to attract elite high school talent.

Although championed primarily as a means of protecting the young players, the NBA’s minimum age requirement provides some benefits to the NCAA and NBA. NBA franchises no longer have to commit resources to scouting high school players, and now benefit from at least one additional year to assess “upside” and “potential” when evaluating draft prospects. Thus, the risk inherent in drafting and developing high school players is removed and enables franchises to make better draft decisions. In addition, the NBA franchises inherit players with greater marketing potential as many of these players are also more recognizable based on exposure in the NCAA.

The combination of money and the change in the way that NBA teams evaluate talent virtually dictates that players turn professional as early as possible, greatly affecting the landscape of college basketball as a result. The elite NBA players who have completed four years of college have all but disappeared. Looking at the players who have played in the NBA All-Star game shows how the League has evolved over the past two decades. Every player in the 1991 NBA All-Star game spent at least two years in college. Ten years later, in the 2001 NBA All-Star game, four of the twenty-four participants had skipped college altogether – Tracy McGrady, Kobe Bryant, Kevin Garnett and European star Vlade Divac. By the 2011 NBA All-Star Game, fourteen of the twenty-four players came into the League from high school, from overseas, or with only one year of college on his resume. In fact, only one player, Tim Duncan, had spent four years in college (Zola, 2011, p. 178). This “one and done” reality is a common complaint amongst college basketball fans. Because of this one-year requirement, colleges and universities face an increasing number of student-athletes whose sole reason for attending school is to build their brand and pass the time before declaring for the NBA draft (Zola, 2011, p. 174). The premature departure of an underclassman from school is viewed with disdain by the institution’s alma mater, academia, and even their own coaches (Zola, 2011, p. 178). These players are criticized for either a lack of allegiance or chasing an early payday and tarnishing the value of a four-year education. However, the reality is that players are actually being forced to make this decision by the NBA and NCAA governance (Zola, 2011, p. 178). Prior to the one-year requirement, NBA franchises were forced to commit resources to scouting high school players, where they are now afforded the opportunity to observe a player's development at least one additional year for free at the collegiate level to enhance draft decisions. Unlike the typical student, athletes do not have much free time to experience college and ascertain the general life experience that the NBA promotes. Nor do athletes necessarily benefit from the academic learning experience the NCAA advocates. Many elite players recognize that they are in college for only one year and thus only need to meet minimum academic requirements for one semester to sustain a full year of academic eligibility (McAleavey, 2010). These players also seek to capitalize on earlier entry into the draft, knowing that they will be forced to compete under the 3-year rookie scale prior to attaining the lucrative free-agent salaries, as they look to profit on their ability to compete in the prime of their career (Groothius, Hill, & Perri, 2007, p. 226)

1.1.3 "One and Done" Phenomenon

The NCAA attempted to address the "one and done" situation by developing an "evaluation period" whereby student-athletes could declare for the NBA draft, evaluate their "draft value", and return to college with their eligibility intact, so long as they followed certain rules (Zola, 2011, p. 179). The NCAA determined college players needed to decide whether or not they would remain in the draft before the draft was held—thus forcing them to guess their draft status rather than waiting for the results and then deciding. Many student-athletes spend the better part of March traveling for high profile conference tournament games, and then to NCAA sites around the country for March Madness. These tournaments generate tremendous amounts of money for the NCAA and its member institutions. The student-athletes then return to campus as their academic year is winding down and final examinations are beginning. It is during this time that players have approximately a three-week window in which to decide whether to submit their name for the NBA draft. Not surprisingly, many opt to submit and make their true decision later (Zola, 2011, p. 183).

Once a player decides to declare, he must navigate the legal regulations established between the NBA and NCAA. It requires expertise to properly advise student-athletes as to what they are allowed to do during this time period if they wish to obtain an evaluation of their prospects, yet retain their college eligibility. The NCAA has rules specifically targeted towards men's basketball players flirting with decision to pursue a career in professional basketball. First, as already noted, college basketball players may enter their names into the NBA draft one time during their college career without jeopardizing their eligibility. However, under NCAA amateurism rules, these students may not sign with an agent; if they do they forfeit their remaining college eligibility. Additionally, these players must withdraw their name from the draft prior to the draft should they want to return to college. (Zola, 2011, p. 183 & 184) The rule also benefits college basketball coaches who can determine what their rosters will look like for the upcoming year, which helps them when recruiting new talent.

During this evaluation period, NBA teams may meet with potential draftees. In fact, players may try out with NBA teams, at the NBA team's expense, so long as the student-athletes are enrolled full-time and do not miss any classes. Prospects may also submit their names to the NBA's "Undergraduate Advisory Committee," which is composed of NBA team executives, and receive a confidential projection of their likely draft position. The challenge is that the evaluation received from this Committee is neither 100% accurate nor particularly helpful because of the variance in the feedback of NBA scouts and executives. An assessment indicating that a player may be taken "between the middle of the first round and the middle of the second round" provides no true insight as to whether an individual should make this transition. The difference between a first-round selection and a second-round selection is enormous. Under the recent CBA, the first-round pick is provided a guaranteed three-year contract, while the second round pick is guaranteed nothing. Consequently, without considering a player's individual circumstances, the standard interpretation is that a player who leaves early and is selected in the first round made a "good" choice based on a positive signal, while the one who is either drafted in the second round or undrafted made a "mistake" by sacrificing his eligibility based on a weaker signal (Zola, 2011, p. 184).

1.2 Research Question

In 2011, the NCAA adopted Proposal No. 2010-24, which required student-athletes interested in assessing their NBA draft prospects to remove their name from consideration before the first day of the spring National Letter of Intent signing period (usually in mid-April). Current rules at the time allowed student-athletes to keep their name in the draft until May 8. During that period, collegiate teams were often in limbo regarding the status of their rosters for the upcoming season. By moving the withdrawal deadline to mid-April before the signing period, NCAA coaches now had the flexibility to address roster issues at the beginning of the spring signing period while viable prospects are still available. This change, which became effective in 2012, was intended to help keep student-athletes focused on academics during the spring semester and allow coaches an opportunity to fine tune their roster for the upcoming season before the recruiting period closed (Hosick, 2011) requiring student-athletes to declare their intentions to remain in the draft or return to school almost a month earlier than they would have had to decide in previous years hinders student-athletes from having the best information available to make informed decisions. It was during this “evaluation period” that student-athletes were given an opportunity to impress NBA teams through individual and pre-draft combine workouts to assess their NBA potential. Under the new rules, the NCAA concludes that evaluations by professional scouts and others during preseason practices, regular season games and postseason games provide student-athletes with adequate information to credibly determine their NBA draft status.

The first day of the 2014 spring National Letter of Intent signing period for NCAA basketball is April 16, meaning that any student-athlete that makes himself eligible for the NBA draft before then must remove it by April 15th in order to retain his collegiate eligibility (Givony, 2014). Since many student-athletes spend the better part of March traveling for high profile conference tournament games, and then to NCAA sites around the country for March, this essentially means is that there is no longer an opportunity for student-athletes to assess their draft prospects. In addition, this date falls less than two weeks before the NBA's own early-entry deadline of April 27th. Until the NBA officially disperses their list of underclassmen who have made themselves eligible for the NBA draft, which is usually a few days after the deadline, teams cannot have any type of contact with potential players, which obviously includes pre-draft workouts in preparation for the June 26th draft. Early draft entrants are now left to depend on the evaluations of the NBA's "Undergraduate Advisory Committee", consisting of executives from 20 NBA franchises, which has traditionally been very conservative with the evaluations they've provided players. These estimates are understandable, given the uncertainty surrounding the order in which a player might be drafted. For example, the "Committee" could inform a student-athlete that he is a definite first rounder in early April, only to see him slip into the second round or go undrafted in late June (Givony, 2014). Under the new rules, the NCAA concludes that evaluations by professional scouts and others during preseason practices, regular season games and postseason games provide student-athletes with adequate information to credibly determine their NBA draft status. However, the financial implications of a "bad choice" can be catastrophic for the student-athlete, as only the first 30 draft positions receive guaranteed salaries (players selected in the second round are not guaranteed contracts or roster spots with the NBA team; while the business interests of the NCAA and NBA remain protected.

The purpose of this study is to present an optimal decision framework to determine whether or not a student-athlete should enter the NBA draft early or continue to utilize his remaining collegiate eligibility. A college underclassman has the ability to exercise his option to enter the draft early and relinquish his collegiate eligibility or return to school for another year to improve his draft stock and earning potential. This dilemma to "go pro" or "stay in school" is very similar to problems found in option pricing theory as financial practitioners have been faced with the challenge of early exercise decisions in a world of uncertainty for a long time. Academics and practitioners have long recognized the usefulness of option pricing theory as a tool to explain behavioral decisions (Arel &

Thomas, 2012). It is our belief that the theory and applications of option pricing can be used by student-athletes to make rational decisions about early draft entry.

The option decision framework has been explored in economic and labor literature to explain the decision of the employer to hire the most apt employee, and likewise the applicant has the decision to determine to sell his labor or wait for a more lucrative opportunity. We expand this decision framework for the applicant by using a number of observable factors that influence a student-athlete's early entry decision, to calculate the optimal time to enter the draft based on the information available to them.

1.3 Literature Review

In many entry-level labor markets, the timing of transactions plays a large role in their ability to function. Because transactions involve two or more parties, much of the market's benefit has to do with bringing the buyers and sellers together so that they can pursue the best transaction. New entrants in many professional sports make the choice to expose themselves to the labor market through the draft. The "draft" concept employed in many entry-level labor markets for professional athletes usually consists of teams who take turns choosing student-athletes (usually high school or college) from a pool of eligible candidates, one player at a time, until every team has had a choice, after which the process begins again. Typically the rule for determining the order in which teams make choices is designed to promote balance and create competition among the teams in the same league (Roth & Xing, 1994). Much attention has been focused on the trend of drafting younger and less experienced players in recent NBA drafts. In the 1997 NBA draft, the draft set a record in which 17 of the 29 first-round were not collegiate seniors (Li & Rosen, 1998). By the 2013 draft, 27 of the 30 first-round selections were underclassmen or foreign entrants. Despite the posturing by Commissioner Stern and League executives urging student-athletes to stay in school and finish their education, student-athletes continue the trend of early entrance into the NBA, in large part due to knowing that the rookie salary scale will delay their ability to attain large free-agent salaries.

The difficulties of controlling the timing of recruiting new entrants of highly trained professionals have been explored in many labor-related studies. Problems originate in entry-level markets with the incentives that some market participants gain by trying to "jump the gun" and arrange transactions just a little earlier than their competitors (Roth & Xing, 1994). The timing associated with these transactions is predicated on participants competing for a limited supply of the best-qualified candidates or best positions in strategically timing when to execute their offers. For instance, employers sometimes make lucrative offers to potential candidates that quickly expire, while job candidates often try to delay making a decision on existing offers in the hope of receiving a better one. Another aspect of timing is the unraveling of the appointment date, where employment begins only after the attainment of certification of professional qualification (Li & Rosen, 1998). An example of unraveling are the summer internship programs for law students that often serve as an assessment period for firms to identify potential candidates for longer term positions upon graduation (Li & Rosen, 1998). The two timing aspects are related, as jumping the gun can sometimes be the principal reason for the unraveling of the appointment date. This was evident as the NBA saw a mass exodus of high school players flood the League over a 10-year period that was followed by an unraveling of the market where the League opted to delay the entry of these student-athletes and allow NBA franchises to reduce the uncertainty associated with their investment in labor by establishing an age minimum for market entry.

Groothuis, Hill, and Perri (2007) assess early entrance into the NBA by examining how the landscape of the League has changed as franchises have drafted players earlier and earlier in their college careers or from high school in the pursuit of talent. They explore two models: (a) the human capital model and (b) the option value model. The human capital model suggests that student-athletes enter the NBA once a certain skill level is obtained. This model implies that elite student-athletes reach the NBA earlier because they do not need as much time developing their basketball skillset in college as other student-athletes may require. The option value model suggests that college basketball provides signals such that as a player stays in college, their signal becomes less noisy and NBA franchises incur less risk in drafting them. Franchises will then exercise the option to choose student-athletes who-have a more varied signal (less college experience) if they can minimize the downside risk and capitalize on the upside potential. The 1995 and 1999 CBAs introduced 3-year rookie pay scales that allowed teams to pay new entrants less than their "fair market value" (The 1999 CBA lowered the rookie scale contract salaries and added a 4th-year option for teams, at a predetermined percentage pay increase) and salary minimums for veteran players that grew based on League experience. As a result, franchises began to shift their business models, as very talented new entrants became the preference for teams, because these student-athletes cost much less than veteran players and were now becoming more productive in the long-term and more profitable in the short-term. Groothuis et al. (2007) go on to cite how Roth and

Xing (1994) predicted that unraveling accelerates when senior candidates are not close substitutes for new entrants.

They test the human capital model using a regression of salary on performance statistics, years of experience, and whether a not a player is playing under the rookie salary scale. The results of the salary regressions lend support to the human capital model as first-round draft picks were apparently paid more under the rookie scale than their performance would indicate in their 1st year in the League. During their second season, their performance and pay tend to approximate that of others in the League not under a rookie scale contract. However, in the 3rd year in the League under the 1995 CBA and in the 3rd and 4th years in the League under the 1999 CBA rookie scale, the players are paid less than the value of their performance. Groothuis et al. (2007) goes on to highlight that through CBA, the NBA set up an institutional arrangement that allows teams to capture the cost of general training that takes place during the first season in the League.

Groothuis et al. (2007) test the option value model by estimating the likelihood of an early entrant being on the ‘‘All-Star’’ team as a way to measure whether the player has high "upside" or "potential". The "All-Star" status was used because it represented a measure of a player being in the upper echelon of NBA talent. They considered two groups: early entrants and four-year college entrants; where the signal for the early entrant was expected to be noisier and riskier than for the four-year college performer. Their estimation lends some support to the option value model of unraveling in the labor market.

Much of the existing research explaining the unraveling of the labor market in professional sports do so from the perspective of the franchise and ignore the early entry decisions facing student-athletes. Arel and Thomas (2011) examine early entry into the NBA draft for college student-athletes and highlight its similarity to the decision to exercise an American style put option early (i.e. the draftee option to sell his remaining time in college early). Each year at the conclusion of the NCAA tournament, exceptional student-athletes are faced with the decision to determine if their draft stock has elevated enough for them to be a potential first round pick in the draft. However, since their draft position is not guaranteed, players need to make an optimal choice based on the information available to them prior to the draft. That is, they must either exercise their option to enter the draft early and forgo their collegiate eligibility or return to school for another year to improve their draft stock. Arel and Thomas (2011) believe that the predictions from option pricing theory could be used to help student-athletes make rational decisions to stay in school or go pro. They formalize the option decision framework for the applicant using data from the NBA draft to examine the perspective of the draftee as an exerciser of this option.

Arel and Thomas (2011) explain that athletes have a limited number of years in which they can physically compete and that player turnover in the NBA is very high. They cite that at the start of the 2009-2010 NBA season, 257 of the 434 or 59% of the players on opening day rosters had 5 or less years of NBA experience. Thus, players who are typically underpaid in their rookie contracts enter the draft early to maximize the time in their post rookie free agent contracts that are not scaled (Groothius, Hill, & Perri, 2007). The choice to enter the NBA draft early is similar to the decision to exercise a put option before maturity. In the case of a put option on a stock, the owner of the option has the right to sell the stock (the asset) at a predetermined price at or before the maturity date. As the stock falls in price the put gains value. With respect to the athlete, the asset he owns is his collegiate eligibility (and the associated knowledge, skills, and experiences that come with it). If this remaining collegiate time has little value (will not help improve his draft position in the NBA), it may be worth forgoing it and entering the draft early. Since the salary schedule for the first 30 draft positions is fixed and known, the indications of a likely draft position have meaningful value for the player. The best (highest) draft position, number 1, is equivalent to maximizing the payoff of a put option (i.e. when the student-athlete's remaining collegiate eligibility has no financial value).

Arel and Thomas (2011) hypothesize that if participating in the draft is similar to exercising an American style put option then they should see a higher proportion of underclassmen in the early draft positions (1 to n) relative to those in the later draft positions (n to 30). If not then there should be roughly equal percentages of freshmen, sophomores, juniors, and seniors at all draft positions. Using NBA draft data from 2006-2010, they employed the binomial model of Cox, Ross, and Rubinstein (1979) to estimate the option value by draft position. The inputs to the Cox, Ross, and Rubinstein model are (a) the stock price, (b) the strike price, (c) the risk-free rate, (d) the time to maturity, and (e) the volatility. For the draftee, the corresponding inputs were (a) the draft position equivalent stock price (the value of the remaining collegiate eligibility) (b) the salary for the first draft position (c) the risk-free rate, (d) the time remaining to graduation, and (e) the volatility of the draft equivalent stock price. Arel and Thomas (2011) use their model to find early exercise to be rational for all class ranks; though the early exercise boundaries were considerably different for freshmen and juniors. Their results also suggest that while most players are making reasonable choices, there are still some players that seem to be entering the draft in the face of information that would suggest they do otherwise.

1.4 Contributions of the Research

For student-athletes, making a rational decision regarding the timing for early entry into the draft requires identifying and appropriately weighing the alternatives based on the information available. In the past, most players have taken the advantage of the opportunity to "test the waters" for their draft position in order to gather relevant information pertinent to their draft status so that they can accurately determine whether they should withdraw their name prior to the draft deadline. This option has become virtually obsolete for student-athletes as the NCAA has instituted strict guidelines that make it extremely difficult for student-athletes to determine their draft status.

Since the NCAA bars third parties (even family members) of college players from contacting NBA teams to discuss their draft status, and the NBA itself has strict no-contact rules regarding the way teams can communicate with players who are not officially draft-eligible (before the early-entry list is released in early May), the only way an underclassman can "legally" gather information about his draft stock is through his college head coach. Furthermore, the head coach, according to NBA rules, is only allowed to talk with the principal basketball operations executive from each team (i.e., the general manager), , and the underclassman may not participate in or be present during any such conversation (Givony, 2014). As a result, even an honest assessment provides no true insight as to whether an underclassman should declare for the draft.

In this paper we propose a general algorithm, based on Monte Carlo simulation, for estimating the draft value of a student-athlete to inform his decision as he evaluates whether or not he should enter the NBA draft and forgo his remaining collegiate eligibility. The algorithm can be applied to models with multiple path dependent state variables and is specifically designed to handle American-style options, i.e., financial securities with opportunities for early exercise. We develop a Markov Decision Process (MDP) model using the framework of an optimal stopping problem to describe the sequential decisions a student-athlete must consider as he contemplates forgoing his amateur status to enter the draft. We apply the algorithm to a MDP model with multiple path dependent state variables that inform a student-athlete's decision as they evaluate the decision for early entry. The challenge for student-athletes however, is how to calculate the optimal choice from the information available to them. While the factors that influence the decision will be different for different players, observable variables (e.g., age/class, size, position, offensive/defensive strengths/weaknesses, and level of competition) may exist that can help student-athletes make rational draft entry decisions and reduce the volatility of a bad decision. The algorithm we use generates two estimators, one biased high and one biased low, both asymptotically unbiased as the computational effort increases.

2 Methodology

Rational decision making is an essential concern for prospective draft entrants who must determine what is in their best interests – an immediate career in the NBA or a return to college to hone their skills and become more productive. The goal of our research is to help student-athletes, faced with making life altering decisions under extreme time constraints and limited resources, make better informed decisions. "For the purposes of this problem we construct a modeling lens to try and guide the decision process, so that relevant assumptions are considered and the process is properly evaluated. In doing so, we do not aim to automate the decision-making process or diminish the role of the decision maker. Instead we aim to present a model for sequential decision making under uncertainty which takes into account both outcomes of current decisions and future decision making opportunities." (Puterman, 2005)

2.1 Markov Decision Processes (MDP)

Decisions must not be made in isolation; today's decision impacts tomorrow's decision, and tomorrow's decision impacts the next day's decision and so on. By not accounting for the relationship between present and future decisions, as well as present and future outcomes, we make it very difficult to make optimal decisions. MDPs provide a mathematical framework for modeling sequential decision making in situations where outcomes are uncertain (Puterman, 2005). MDPs, also referred to as stochastic dynamic programs, popularized in the late 1950's and early 1960's, have gained recognition because of the analytic flexibility they provide in modeling the most realistic sequential decision-making problems. MDPs have been applied to model problems in such diverse fields as inventory management, financial securities, insurance claims, health epidemics, and professional sports (White, 1993).

2.1.1 Problem Definition

The qualifier “Markov” is used to describe MDPs because the transition probability and reward functions depend on the past only through the current state of the system and the action selected by the decision maker in that state (Puterman, 2005). For example, a decision maker is faced with the problem of influencing the behavior of a probabilistic system as it evolves through time. He does this by making decisions at various points in time. His goal is to choose a sequence of actions which causes the system to perform optimally with respect to some predetermined criterion. Take for instance airplane maintenance. Airline companies periodically inspect the condition of their airplanes, and based on the age and condition of the airplane, decide on the extent of maintenance, if any, to carry out. Since the system is ongoing, the state prior to tomorrow’s decision is based on today’s decision. As a result, decisions must anticipate the opportunities and rewards/costs associated with future system states; and cannot depend on shortsighted rationale (Puterman, 2005).

MDP models consist of specific fundamental elements:

- **Decision Epochs:** Specified points of time where the decision maker makes decisions.
- **States and Actions:** At each decision epoch, the system occupies a state, where the decision maker is allowed to observe the system prior to choosing an action.

- **Rewards and Transition Probabilities:** As a result of choosing an action in a given state, the decision maker receives a reward and the system state at the next decision epoch is determined by a probability distribution based on the current state and action.

We describe the sequential decision-making process in the model as follows. At each decision epoch (i.e. April at the completion of the freshman NCAA season), a decision maker (i.e. student-athlete) observes the state of the system (i.e. draft status) and based on this state (i.e. a high draft signal or low draft signal), he chooses an action (i.e. enter the NBA draft or remain in college for an additional year). The action choice produces two results: the decision maker receives an immediate reward (i.e. is drafted and receives an NBA contract or remains in college for an additional year), and the system evolves to a new state (i.e. draft outcome or draft status for the following year's draft) at a subsequent point in time (April at the completion of the sophomore season) according to a probability distribution defined by the action choice.

A student-athlete's decision to enter the NBA labor market at an early age allows the player to the opportunity to adjust to the pace of the League and experience the limitations of the rookie salary cap earlier in his career. Assuming that on average, an NBA player plays to the age of 27, this means that every year of NBA competition becomes crucial to a student-athlete's career earning potential. As a result, a student-athlete is faced with determining when his earning potential exceeds the opportunity cost of maintaining his college eligibility, and thus makes entering the draft the better alternative and rational choice. At each draft decision, a given student athlete will occupy a state that will be an indicator of his draft value. We determine this state by constructing a continuous function that assigns a mathematical relationship to a set of random variables $X_1, X_2, \ldots, X_k$. These predictive variables serve as observable attributes and provide a mechanism for the student-athlete to assess his draft value prior to making the decision to enter the draft or remain in school. The X_i's (for $i \leq k$) used in our model are based on real student-athlete data and include attributes like seasonal performance statistics, height, position, and level of competition. For simplicity, we limit the number of observable attributes used in our model and assume they are independent with incremental changes in X_i (for $i \leq k$) that are independent and identically normally distributed. In theory, each X_i (for $i \leq k$) occupies its own state and provides some measure of uncertainty in its prediction of draft value. However, we utilize an aggregate variable Z in our model to take advantage of the algorithm's framework for pricing financial securities with opportunities for early exercise.

MDP models function with a set of decision rules that prescribe a procedure for the decision maker to select actions in each state at a specified decision epoch. These decision rules range in generality from deterministic Markovian to randomized history dependent, depending on their degree of dependence on past information and on their method of selecting actions. Our focus in this study will be on an MDP model with deterministic Markovian decision rules. The decision is said to be *Markovian* ("memoryless") because it depends on the past history of the previous states and actions only through the current state of the system, and *deterministic* because it chooses an action with certainty (Puterman, 2005). A policy specifies the decision rule to be used at all decision epochs, and provides the decision maker with a prescription for action selection under any possible future system state or history. We will use our MDP model to develop a policy for student-athletes to answer the fundamental question many face with respect to their collegiate eligibility: "Under what conditions is it optimal to enter the NBA draft each year?"

2.1.2 Optimal Stopping Times

Optimal stopping problems are characterized by a decision maker who observes the current state of the system and decides whether to stop or continue. The underclassman's decision to enter the draft early is similar to an optimal stopping problem. Since the current NCAA early entry draft guidelines prevent potential NBA players from leveraging the opportunity to "test the waters" and gather accurate information about their draft status, the decision to enter the draft has converged to an absolute decision to forego your collegiate eligibility and ability improve as a student-athlete. A key aspect of most optimal stopping problems is the Markovian nature of the decision to stop based upon the value of the current state. The system has either a reward or cost for each additional period, and once the decision has been made to continue with the system, the reward for the next period is received, and the system transitions according to a probability distribution to the next state. At this next state, the decision must be revisited (Hall, 2009).

In the optimal stopping problem for the student-athlete, we explore how the choice to enter draft early is similar to the decision to exercise a labor option before the completion of college. The problem will be modeled as an MDP with finite state space and fixed time horizon. The fixed time horizon is a reasonable modeling assumption; since on average, a student-athlete has four years of collegiate eligibility to actively participate in NCAA basketball. The exception is when a student-athlete receives a redshirt year, where he may attend classes, practice with his team, and dress for play but may not compete in games. Under this mechanism, a student-athlete has up to five academic years to use his four years of eligibility, thus becoming a fifth-year senior. Our model will assess eligibility over a four-year period and analyze optimal stopping policies for each of the five basketball positions (i.e. Point Guard (PG), Shooting Guard (SG), Small Forward (SF), Power Forward (PF), and Center (C)), looking at sensitivities to both draft value and salary. The MDP used to model the student-athlete's early entry decision can be effectively solved by current simulation techniques used in American option pricing theory. Simulation provides a framework for valuing and optimally exercising American options. For example, simulation is readily applied when the value of the option depends on multiple factors, and can also be used to value problems with both path-dependent and American-exercise features (Longstaff & Schwartz, 2001).

2.2 Simulation-Based Early Exercise Decisions for Financial Options

One of the most important problems in option pricing theory is the valuation and optimal exercise of derivatives with American-style exercise features. These types of derivatives are found in all major financial markets including the equity, commodity, foreign exchange, insurance, energy, mortgage, credit, and real estate markets. Despite recent advances, however, the valuation and optimal exercise of American options remains one of the most challenging problems in derivatives finance, particularly when more than one factor affects the value of the option (Longstaff & Schwartz, 2001).

2.2.1 Background

Options are traded on financial exchanges throughout the world. The underlying assets include stocks, stock indices, foreign currencies, and commodities. There are two basic types of options. A *call option* gives the holder the right to buy the underlying asset (i.e. stock, stock index, commodity, foreign currency, etc.) by a certain date for a certain price. A *put option* gives the holder the right to sell the underlying asset by a certain date for certain price. The price in the contract is known as the *exercise price* or *strike price* and the expiration date in the contract is known as the *exercise date* or *maturity*. For a call option, if the asset price at the exercise date ($40) is less than the exercise price ($50), the *call option* is worthless. This is because the owner of the call can buy the asset for less in the open market than he can by exercising the option. For a *put option*, if the asset's price at ($50) the exercise date is greater than the exercise price ($40) the option is worthless since the owner would have to buy the asset at higher price to then sell it at cheaper price. As a way to remove market risk, market participants who anticipate an appreciation of the underlying asset will buy a call or sell a put option; and conversely they will short the call or buy the put if they foresee a decline of the underlying asset.

In general call and put options are defined one of two ways: American or European. American options can be exercised at any time up to the expiration date, while European options can only be exercised on the expiration date itself. The terms American and European have no connection to geographic locations, but instead serve as adjectives that differentiate when the exercise of the option can occur. Most of the options traded on exchanges are American options (Hull, 1997). However, these options are harder to analyze than their European counterpart due to modeling complexity of their early exercise features.

2.2.2 American Options

In this study we view the student-athlete's decision to enter the NBA draft early as being similar to the decision to exercise an American option. The student-athlete's decision should reflect a high signal that his labor or skill set is highly valued in the NBA market and the option to enter draft early reflects the optimal financial decision for the player. If remaining in college and playing NCAA basketball has little value (i.e., it will not help improve his draft position in the NBA), it may be worthwhile for the student-athlete to forgo his remaining eligibility and enter the draft early. We examine the player's perspective using this option model framework and establish the factors that affect the early exercise decision. At maturity, the owner of an option would rationally choose to exercise the option if it is ''in-the-money" (i.e. when the underlying asset is exceeds the exercise price). However, when evaluating American options, it may be rational to exercise the option early. The decision to exercise early is influenced by several factors, including:

- How deep in-the-money the option is,
- How much time to maturity remains,
- How volatile the underlying asset is, and
- The interest rate.

In the case of call options, whether the asset pays a cash flow during the life of the option is a necessary condition for early exercise, but this is not so for put options. However, we will not address this condition in our option model. If an American style option is sufficiently deep in-the-money (the asset has gained sufficient value (call) or loss sufficient (put)) it should be exercised before maturity. We will refer to the point at which the option is sufficiently deep in-the-money as the early exercise boundary, which is depicted in the Figure 1 below:

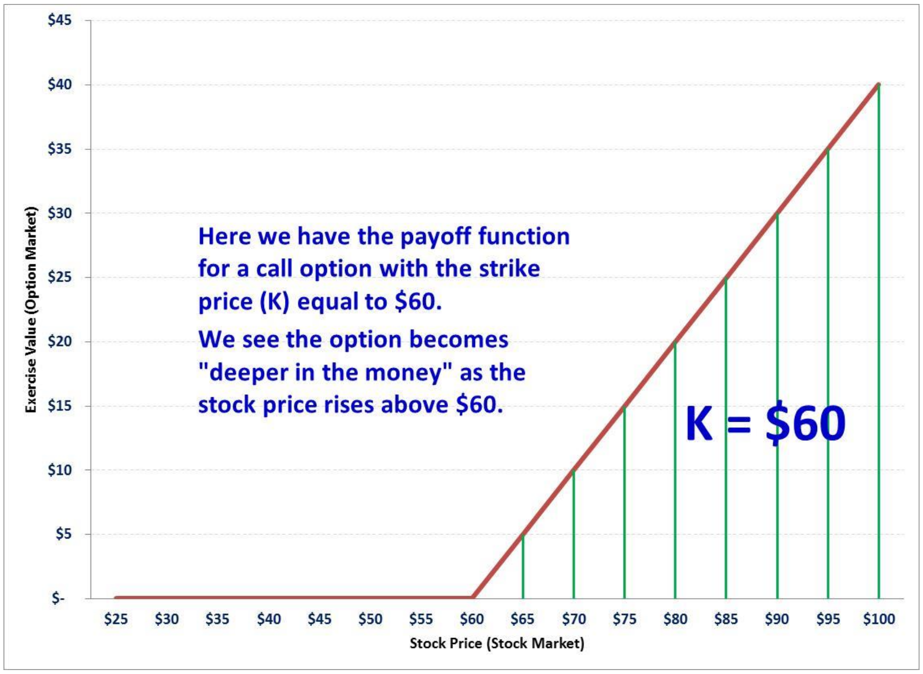

Figure 1 – Payoff Function for a Call Option

The student-athlete's draft value operates in a similar fashion. As his draft value rises his option for early entry becomes "deeper in the money". If the student-athlete is projected to be a lottery pick (top seven picks) then his option is considered to be deep in-the-money and he has the ability to maximize his career earnings and accelerate his independence from rookie salary cap restrictions. The student-athlete's payoff function is depicted below:

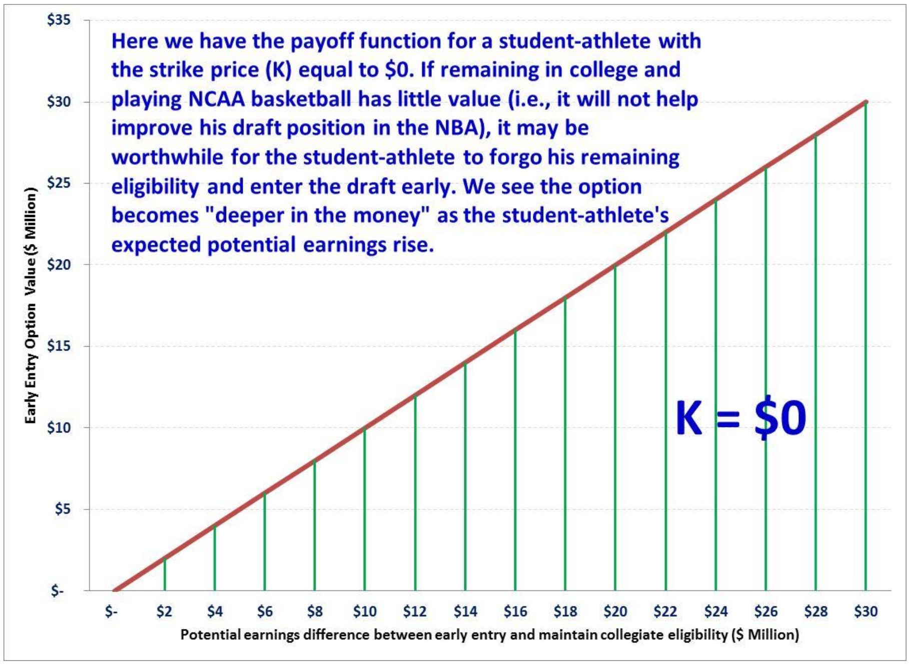

Figure 2 – Student-Athlete's Payoff Function

2.2.3 Simulation

Simulation is a widely used method for studying complex models. Most complex, real-world systems with stochastic elements cannot be accurately described by a mathematical model that can be evaluated analytically. In many cases, simulation is often the only type of reasonable model available. Simulation allows one to estimate the performance of an existing system under some projected set of operating conditions (Law & Kelton, 2000). By its nature, simulation is a promising alternative to traditional finite difference and binomial techniques and has many advantages as a framework for valuing and optimally exercising American options (Longstaff & Schwartz, 2001). Simulation can also be used to value derivatives with both path-dependent and American-exercise features. To understand the intuition behind this approach, recall that at any exercise time, the holder of an American option optimally compares the payoff from immediate exercise with the expected payoff from continuation, and then exercises if the immediate payoff is higher. Thus the optimal exercise strategy is fundamentally determined by the conditional expectation of the payoff from continuing to keep the option alive, or in the student-athlete's case, remaining in college and retaining his eligibility.

In this study we use a general method based on Monte Carlo simulation presented by Broadie and Glasserman (2007) for valuing assets with early exercise features. Monte Carlo simulation is a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results; typically one runs simulations many times over in order to obtain the distribution of an unknown probabilistic entity. Monte Carlo simulation can be easily applied to models with multiple state variables and possible path dependencies. The major difficulty in valuing early exercise features is the need to estimate exercise policies as well. Standard simulation procedures are "forward" algorithms, i.e., paths of state variables are simulated forward in time. By contrast, pricing procedures for assets with early exercise features are generally "backward" algorithms. That is, the optimal exercise strategy at the maturity of the contract is easily determined (Broadie & Glasserman, 1997). Once the student-athlete has exhausted his college eligibility he will make himself eligible for the NBA draft and framework for his future earnings can be established using the rookie salary scale. Using their method, we generate two estimators of the student-athlete's option value based on random samples of future state trajectories and increasingly refined approximations to optimal exercise decisions. One estimator is biased high and one is biased low; both estimators are asymptotically unbiased as the computational effort increases. These estimators are based on simulated trees parameterized by b, the number of branches per node, and converge to the student-athlete's true draft value as b approaches infinity.

3 Optimal Early NBA Draft Entry for Student Athletes

The current "evaluation period" that the NCAA affords its student-athletes to determine between an immediate career in the NBA or returning to college has been severely hampered by the dates and restrictions that the NCAA, not the NBA, has imposed. Under NBA rules, League teams cannot hold workouts for early entry players until the list of early entrants is published by the League in late-April. The date by which these potential draftees must withdraw their names from the draft in order to maintain their NCAA eligibility is usually in mid-April. These deadlines force student-athletes to make career defining decisions without receiving a fair appraisal from the NBA scouts and executives.

Additionally, while the NBA draft is an important talent-recruitment mechanism for the League, sixteen NBA teams are beginning the playoffs in April, making it virtually impossible for these teams to concentrate on providing accurate player evaluations. Many NBA teams have limited personnel, and front offices are usually preoccupied by the team's playoff run. The result is that NBA teams truly do not have a sense of where potential players may be selected two months before the draft. It is not until they have had adequate time to concentrate on and evaluate these college players– something that realistically happens much closer to the day of the draft - that NBA teams have any sense of a player's likely draft prospects (Zola, 2011, p. 186).

Each collegiate basketball season, a student-athlete's performance serves a signaling device to NBA teams to provide information and sort players into the category of being "NBA ready." When players leave early, they have less experience and a noisier signal than a player who stays in college. When a franchise chooses an early an entrant, they choose a player who is both riskier and with less experience (Groothius, Hill, & Perri, 2007, p. 228). In 2010 and 2011 prior to new NCAA early entry guidelines, 149 NCAA student-athletes declared for the draft as early entry participants. A total of fifty-seven of them (or 38%) decided to return to college - either because of a poor reception from NBA franchises or not having enough time to properly evaluate this decision. However, only 42% of the original 149 student-athletes were drafted so, in retrospect, another thirty players should have pulled their names from the NBA draft. Furthermore, if you add the twenty underclassmen drafted in the second round, a total of fifty of the 149, or 34%, probably would have been better served by returning to school. Given the high numbers of individuals improperly evaluating their NBA prospects, it seems right for student-athletes to consider a better process when making this life-changing decision (Zola, 2011, p. 187).

3.1 Modeling Student-Athletes' Decisions to Enter the Draft

We develop an MDP model using the framework of an optimal stopping problem to describe whether or not a student-athlete should enter the NBA draft after each year of collegiate eligibility. The early entry decisions of student-athletes may be influenced by many factors, but clearly financial compensation will be a major consideration. It is assumed that student-athletes will choose optimally between entering the NBA draft and maintaining their eligibility to participate in collegiate athletics. In our model we assume draft decisions for student-athletes are made at discrete time points observed annually in April between the end of the collegiate season and the NCAA deadline for early entrants to withdraw their names from the draft. Each year is modeled as one period, and at the conclusion of each period, a student-athlete knows his current state which includes information on his draft value, years of remaining of eligibility. At each draft decision, a given student athlete will occupy a state that will be an indicator of his NBA draft potential and future career earnings as a NBA player.

3.2 Overview of Method

The combination of money and the change in the way that NBA teams evaluate talent virtually dictates that players turn professional as early as possible, greatly affecting the landscape of college basketball as a result. The elite NBA players who have completed four years of college have all but disappeared (Zola, 2011). Our simulation approach is to measure:

$$\textbf{Draft Value (DV)} = \max_{\tau} (V_{\tau}^{EE}, V_{\tau}^{SS}) \textbf{ over all stopping times } \tau \leq 4.$$

The above equation has the properties of the American option pricing problem where V_t^{EE} denotes the expected discounted career earnings from declaring for the NBA draft at time t, and V_t^{SS} is the expected discounted career earnings from staying in school at time t and declaring for the draft in the future. In this paper we focus on a discrete time approximation to this problem where we restrict the draft declaration decisions to lie in the finite set of times $0 < t_1 =$ *End of 1st Year of Collegiate Eligibility* $< t_2 =$ *End of 2nd Year of Collegiate Eligibility* $< t_3 =$ *End of 3rd Year of Collegiate Eligibility* $< t_4 =$ *End of 4th Year of Collegiate Eligibility* $= T$.

Let Z_t denote the set of possible states for a given student-athlete impacting his decision to enter the NBA draft decision at time t. Let $X_1, X_2, \ldots, X_k$ be a set of observable attributes at time t impacting a given student-athlete's decision to enter the NBA draft and let f be a continuous function such that $f_t(X_1, X_2, \ldots, X_k) = Z_t \in [0,1]$. The observable attribute variables used to estimate the Z values in our model are discussed in detail in Section 3.3.

We simulate a path of draft values Z_1, Z_2, Z_3, and Z_4 at corresponding times $0 = t_1 < t_2 < t_3 < t_4 = T$; then compute the *discounted* draft value corresponding to this path, and finally average the results over multiple simulated paths. The main question is how to determine when to enter the NBA draft based on the corresponding draft value path. If the optimal stopping policy were known, the path estimate would be when the discounted expected earnings from declaring for the NBA draft was greater than the discounted expected earnings from staying in school and declaring in the future. But the optimal stopping policy is not known and must be determined via the simulation. We compute the optimal stopping time for the simulated path which gives the path estimate:

$$\mathbf{max}_{t_i}\,(\mathbf{V}_{t_i}^{\mathbf{EE}}, \mathbf{V}_{t_i}^{\mathbf{SS}}) \textbf{ for i = 1, 2, 3, 4.}$$

However, this path estimate corresponds to the perfect foresight solution and tends to overestimate the student-athlete’s draft value. Naturally, the overestimate follows from the inequality:

$$\mathbf{max}_{t_i}\,(\mathbf{V}_{t_i}^{\mathbf{EE}}, \mathbf{V}_{t_i}^{\mathbf{SS}}) \geq \mathbf{max}\,(\mathbf{V}_{\tau}^{\mathbf{EE}}, \mathbf{V}_{\tau}^{\mathbf{SS}})$$

This phenomenon can be further explained by considering the optimal but unknown early entry strategy where the student-athlete does not declare for the draft until the completion of his senior year where it was never optimal to enter the draft early. However, the student-athlete’s draft value may have been higher in his freshman, sophomore or junior year and under the perfect foresight strategy the student-athlete could receive a higher path value by exercising earlier. The expected draft value of the student-athlete is not equal to his true draft value in this scenario. Increasing the simulation effort by simulating many paths does not remove the bias in the problem either. This example illustrates the difficulties involved in applying standard simulation methodology to our problem which is similar to pricing an American option. In order to remove bias from our estimate of a student-athlete's draft value we develop valid simulation error bounds on the estimated draft value. Thus we introduce two estimators, one biased high and one biased low, but both asymptotically unbiased as the computational effort increases. These estimators are based on simulated trees parameterized by b, the number of branches per node. (Broadie & Glasserman, 1997) The observable attribute variables are simulated at each of the four decision points (i.e. declaration times) and the connection between nodes indicate the dependence structure of the draft values.

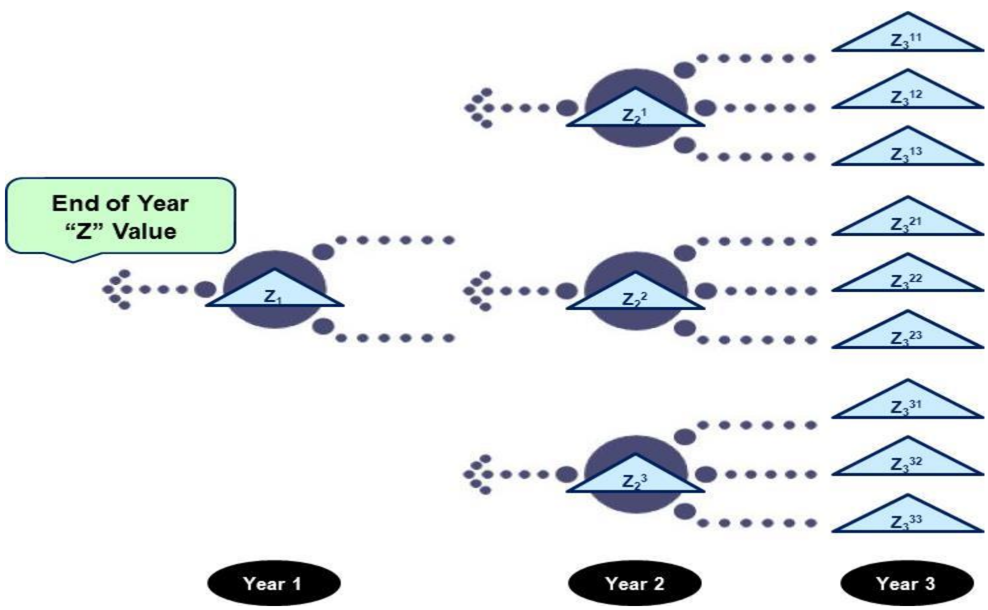

Figure 3 – Simulated Z Value Tree for b = 3

State variables are simulated at the finite number of possible draft declaration points. An illustration of a tree for b = 3 is given in Figure 3 above. The connections between the nodes indicate the dependent structure of a student-athlete's draft value. For example, both Z_3^{11}, Z_3^{12} depend on Z_2^1 but neither depends on Z_2^2. It is important to understand that the nodes in Figure 3 represent fixed times and appear according to the order in which they are generated. For example, the node labeled Z_2^1 need not correspond to a higher draft value than the node Z_2^2.

3.3 Early Entry Draft Model

Our objective in this section is to give a precise specification of estimators and to state their theoretical properties. We choose five collegiate seasons of statistical data taken from the 2007-2008 through 2011-2012 to design a model to simulate a student-athlete's decision to enter the NBA draft or remain in college. We analyze these data for each season to estimate the draft value of a student-athlete to inform their decision as they evaluate whether or not they should enter the draft and forgo their remaining collegiate eligibility.

3.3.1 States and Actions

Let Z_t denote the NBA draft potential of a given student-athlete at time period t where Z_t is a process that is observed at, discrete time periods in a student-athlete's collegiate career, to be in any of one of N possible states, which we represent by 1,2,…,N. We develop a MDP model using the framework of an optimal stopping problem to describe whether or not a student-athlete should enter the NBA draft based on their draft value after each collegiate season. Optimal stopping problems are characterized by a decision maker who observes the current state of the system and decides whether to stop or continue. A key aspect of most optimal stopping problems is the Markovian nature of the decision to stop based upon the value of the current state. We utilize an aggregate variable Z_t in our model, based on predictive variables that serve as observable attributes and indicators of draft value, to take advantage of the algorithm's framework for pricing financial securities with opportunities for early exercise. The system has a draft value at each decision point, and once the decision has been made to continue with his collegiate eligibility, the system transitions according to a probability distribution to the next state. At this next state, the decision must be revisited. Z_t is defined as a Markov Process if and only if $P[Z_{t+1} \mid Z_t] = P[Z_{t+1} \mid Z_1,\ldots,Z_t]$ (i.e. “The future is independent of the past given the present“). (Ross, 1997).

Draft decisions for student-athletes will occur at discrete time points t observed as follows:

t_1 = The NBA “Draft Early Entry Eligibility Deadline” date corresponding to a student-athlete’s first year of collegiate eligibility (usually between April 15^{th} – 30^{th})

t_2 = The NBA “Draft Early Entry Eligibility Deadline” date corresponding to a student-athlete’s second year…

t_3 = The NBA “Draft Early Entry Eligibility Deadline” date corresponding to a student-athlete’s third year…

t_4 = The NBA “Draft Early Entry Eligibility Deadline” date corresponding to a student-athlete’s fourth year…

At each decision point, the undergraduate student-athlete must choose between one of two actions:

Action 1 = Declare for the NBA draft

Action 2 = Stay in school and maintain their remaining college eligibility

It is assumed that student-athletes will choose optimally between entering the NBA draft and maintaining their eligibility to participate in intercollegiate athletics. At each draft decision, a given student athlete will occupy a state that will be an indicator of his NBA potential.

Let Z_t denote the set of possible states for a given student athlete impacting his decision to enter the NBA draft decision at time t. Let $X_1, X_2, \ldots, X_k$ be a set of random variables that serve as observable attributes impacting a given student athlete's decision to enter the NBA draft and let f be a continuous function such that $f_t(X_1, X_2, \ldots, X_k) = Z_t = n \in [0,1]$. A list of observable attribute variables is provided in Table 1. The variables used in our model differed for each of the five basketball positions (PG, SG, SF, PF, and C) to reflect the specific attributes unique to each position. There are a number of other variables that we could have included in our model. However, we focused our efforts on a number of tangible game statistics to streamline our data collection efforts and simplify our aggregate state variable. This is a limitation of our model since game statistics can sometimes be misleading because intangibles like attitude, work ethic, teamwork, desire, and mental toughness, while hard to measure, are just as important in determining NBA potential as points, rebounds and assists.

Table 1 – Observable Attribute Variables

Variable	Description		Variable	Description
X_1	= Points per Game		X_7	= Assists per Game
X_2	= 2 point Field Goal %		X_8	= Steals per Game
X_3	= 3 Point Field Goal %		X_9	= Blocks per Game
X_4	= Offensive Rebounds per Game		X_{10}	= Turnovers per Game
X_5	= Total Rebounds per Game		X_{11}	= Height
X_6	= Remaining Eligibility		X_{12}	= Level of Competition

3.3.2 Transitions

We model each year of the student-athlete's collegiate eligibility as one period. At the conclusion of each period, he knows his current state; which includes years of remaining of eligibility and NBA performance indicators like size, position, offensive/defensive strengths/weaknesses, and level of competition. We model the student-athlete's progression through his collegiate career through the Markov decision tree described in Figure 3. Let $P_{n,m,t} = P\,[Z_{t+1} = m \mid Z_t = n] = P\,[Z_{t+1} \mid Z_1,\ldots,Z_t]$ denote the probability of transitioning from the current state n in period t to state m in the next period t + 1, where $n,m \leq N$. The function $f_t(X_1,X_2,\ldots,X_k) = Z_t \in [0,1]$ used to describe our process is dependent on the transitional probabilities for the observable attribute variables, X_i (for $i \leq k$); which are normally distributed with estimated mean and variance parameters. We estimate these parameters using historical data drafted players to capture the variation in these attributes from year to year. These attributes either increase or decrease each time period to determine the student-athlete's new state and to provide an indicator of his current NBA potential. We restrict a student-athlete's height from decreasing, allowing this attribute to remain constant or increase from year to year. The student-athlete's draft value is not dependent on the path taken to any state and is only contingent on the observable attributes used to generate his current state.

3.3.3 Reward

At the conclusion of each season, a given student-athlete will occupy a state that will be an indicator of his NBA draft potential and future career earnings as a NBA player. He will make a decision to either declare for the NBA draft or remain in college for an additional year to increase his draft value. The value of declaring for the draft for a student-athlete in state n is defined by

$$\mathbf{V_t^{EE}(n) = (p_n^1 FE_t^1 + p_n^2 FE_t^2 + p_n^{ND} FE_t^{ND});}$$

where p_n^1 is the probability of being drafted in the first round in state n, p_n^2 is the probability of being drafted in the second round in state n, and p_n^{ND} is the probability of not being drafted in state n. FE_t^1 and FE_t^2 represent the expected future discounted earnings over an eight year period for student-athletes who were drafted in the first and second rounds at time t. For the purposes of this analysis, FE_t^{ND} will equal zero for all values of t.

The value for staying in college for an additional year for a student-athlete in state n is recursively defined by

$$\mathbf{V_t^{SS}(n) = e^{-r} \sum_{m=1}^{N} P_{n,m,t} \max(V_{t+1}^{EE}(m), V_{t+1}^{SS}(m));}$$

where V_t^{SS} is the expected discounted career earnings from staying in school at t and declaring for the draft in the future; and V_{t+1}^{EE} is the expected discounted career earnings from declaring for the NBA draft at time t+1. Discounting the next period value function at the riskless interest rate r, and summing over all possible transitions from the current state n to possible future states m (where N is the total number of aggregate states for Z) at time t captures the period t expected value of compensation for remaining in school an additional year. Our model assumes an eight-year period to represent the earning potential available to a student-athlete at the conclusion of his freshman year, based on a typical player who is 19 years old at the end of his freshman year and the average NBA player playing until the age of 27. This earning potential increases the opportunity cost of remaining in school and makes entering the NBA draft early the better alternative in some instances.

3.3.4 Draft Value Function

After each season a student-athlete will make the decision to declare for the draft or remain in school to complete his collegiate eligibility. We assume that each student-athlete will exhaust his eligibility after four seasons of NCAA basketball. It is also assumed that financial compensation will be the major consideration as student-athletes choose optimally between entering the NBA draft and maintaining their eligibility to participate in collegiate athletics. The optimal stopping policy will be determined by evaluating the annual draft value of student-athletes at each state and choosing optimally between early draft entry and continuing with school. In order to utilize the principle of optimality from dynamic programming, the draft value of a student-athlete at the completion of his senior season, T = 4, must be calculated first. This decision epoch indicates the end of his collegiate playing career and the NBA draft represents his opportunity to determine his professional value at an elite level. Hence his draft value is now determined by his final state:

$$\mathbf{DV_4(n) = V_4^{SS}(n) = V_4^{EE}(n) = (p_n^{1}FE_4^{1} + p_n^{2}FE_4^{2} + p_n^{ND}FE_4^{ND}).}$$

Creating the T= 3 value function is the choice of the student-athlete playing one additional season of collegiate basketball, or entering the draft at the completion of his junior season:

$$\mathbf{DV_3(n) = max\{\, V_3^{EE}(n), V_3^{SS}(n) \,\}}$$

$$= \max\{ (p_n^{1}FE_3^{1} + p_n^{2}FE_3^{2} + p_n^{ND}FE_3^{ND}); e^{-r} \sum_{m=1}^{N} P_{n,m,3} * \max(V_4^{EE}(m), V_4^{SS}(m))\}.$$

Solving for the value of each period by backwards induction, the draft value function for each collegiate freshman in state n is represented as:

$$DV_1(n) = \max\{ V_1^{SS}(n), V_1^{EE}(n)\}$$

$$= \max\{ (p_n^{1}FE_1^{1} + p_n^{2}FE_1^{2} + p_n^{ND}FE_1^{ND}); e^{-r} \sum_{m=1}^{N} P_{n,m,2} * \max(V_2^{EE}(m), V_2^{SS}(m))\}.$$

4 Analytical Results

We solved the early draft entry model using a dynamic programming formulation in addition to obtaining the expected value of entering the NBA draft early or remaining in college at each time period. The draft value function is the optimal solution for a student-athlete for a given state at the conclusion of each collegiate season. We generate two estimates of a student-athlete's draft value based on random samples of future state trajectories and increasingly refined approximations to optimal exercise decisions. One estimate is biased high and one is biased low; both estimates are asymptotically unbiased as the computational effort increases. These estimators are based on simulated trees parameterized by b, the number of branches per node, and converge to the student-athlete's true draft value as b approaches infinity.

4.1 High Estimator θ

Let DV denote the draft value of a student-athlete with four exercise opportunities to declare for the draft at times t_1, t_2, t_3, and t_4. We denote our first estimator by θ. It is defined as the draft value estimate for a student-athlete obtained by using a dynamic programming algorithm to evaluate the simulated Z value tree in Figure 3. At the conclusion of a student-athlete's senior season, his draft value is known; similar in theory to the value of an American call option at the terminal date.

At the conclusion of each prior season, the draft value of the student athlete is designed to be the maximum value of declaring for the NBA draft and the expected value of remaining in college and declaring for the draft in the future. Finally, θ is the estimated value calculated at the initial node.

At the conclusion of each season, the estimator chooses the maximum of the immediate draft value of the student-athlete entering the draft early and the expected future draft value of the student athlete retaining his collegiate eligibility. A numerical illustration is given in Figures 4 and 5. In Figure 5, the expected earnings for a student-athlete are listed next to each generated Z value. The boxes colored "green" represent the decision to "declare for the draft", while the boxes colored "red" represent the decision to "stay in school." Simulated Z_t values are generated to estimate expected earnings of an NBA player who plays on average until he is 27 years old. The θ estimator gives an estimate of the true draft value which is biased upward, i.e. $E(\theta) \geq DV_1$.

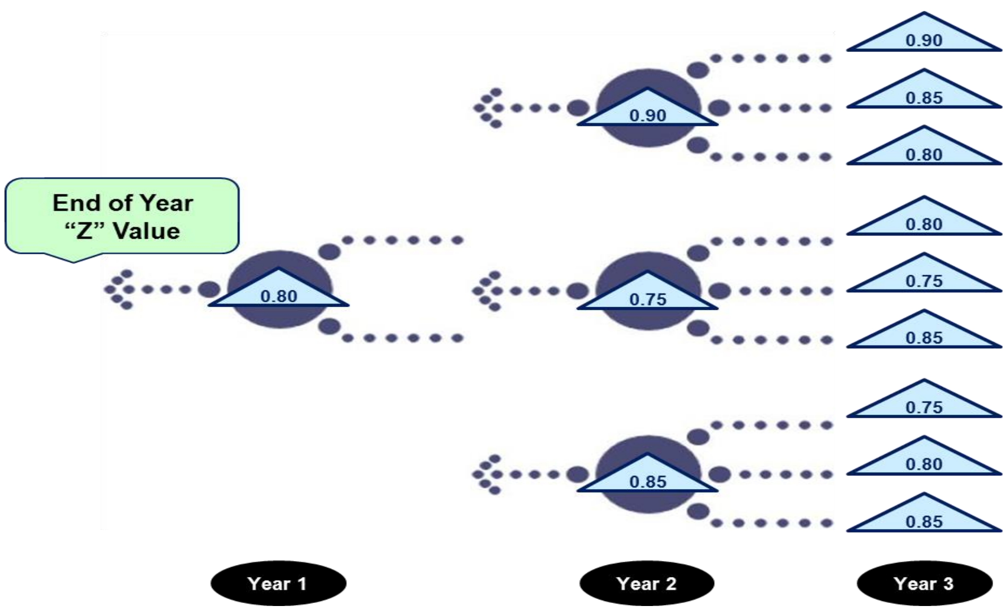

Figure 4 – Simulated Z Value Tree

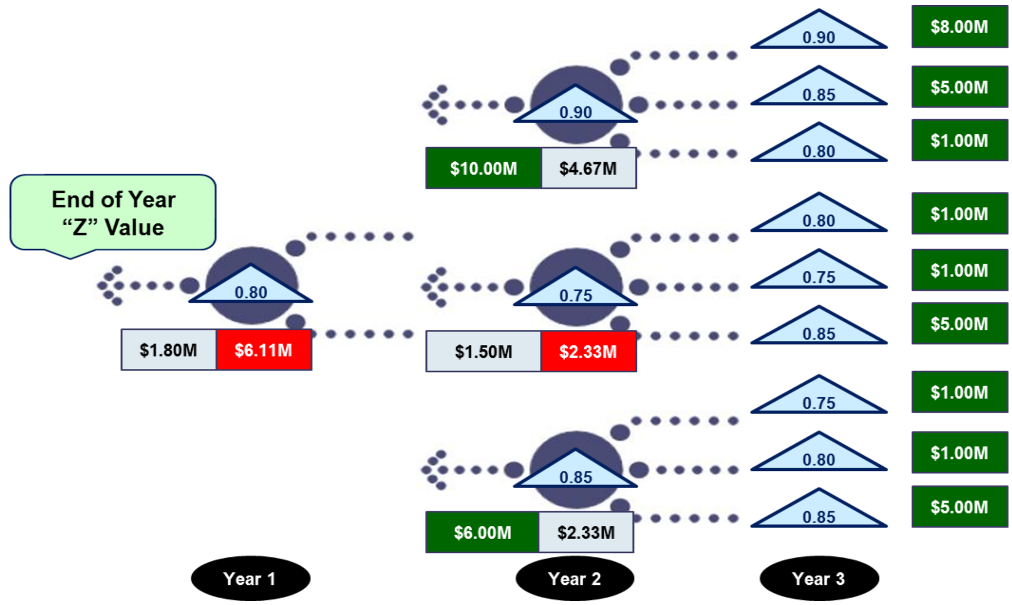

Figure 5 – θ ("High") Estimate Construction in Dollars Million (M)

4.2 Low Estimator β

Next we utilize an estimator that is biased low. The idea is to separate the branches at each node into two sets. The first set of branches is used by the student-athlete to decide whether or not to enter the draft, and the second set is used to determine whther to stay in school. The separation removes the upward bias of the estimator, but instead leads to an estimator with downward bias. This idea is illustrated in Figure 6. The numerical Z values are based on Figure 4. At each node the first branch is used to determine the continuation value, if necessary. For example, at the middle node at t = 2 the early entry decision is made by comparing the expected value of entering the NBA draft ($1.5 million (M)) with discounted expected value of not excercising ($3.00M). Hence, the decision is made based on expected earnings assigned to the first branch ($1.00M). The exercise decision is based on unbiased infromation based on a student athlete's draft value at the end of their eleigibility. If the correct decision is inferred from this information, the estimator would be unbiased. But with a finite sample there is a positive probability of inferring a suboptimal decision.

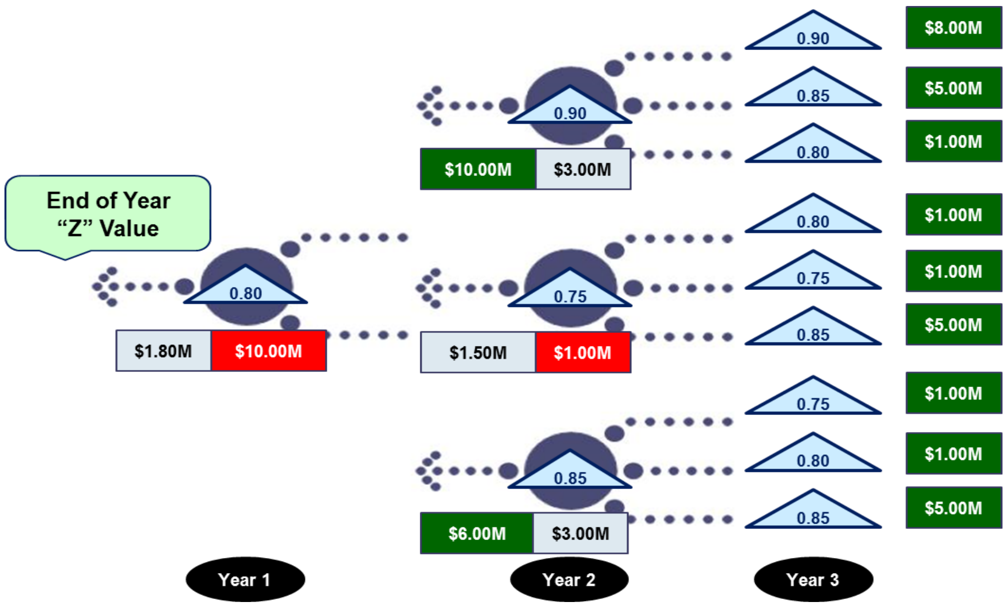

Figure 6 – Simple "Low" Estimate Construction in Dollars Million (M)

We use a slight modification to estimate β. At each node, we use branch 1 to estimate the value of remaining in school, and the other b-1 branches to estimate the decision to declare. This process is repeated b-1 times, using branch 2 to estimate the value of remaining in school, then branch 3, etc. The b values obtained are averaged to determine a student-athlete's draft value at each decsion epoch. Consider the bottom node at t = 2. As before,using the middle node at t = 2, when branch 1 is used to determine the value to remain in school (and branches 2 and 3 are used to determine the decision "stay in school"), the estimate is $1.00M. When branch 2 is used to determine the value of remaining in school (and branches 1 and 3 are used to detrmine the decision to "stay"), the estimate is $1.00M. When branch 3 is used to determine the value of remaining in school, branches 1 and 2 are used to make the decision to "declare for the draft", so the estimate is \$1.50M. These values are averaged to give an estimate for the node of \$1.16M (= (\$1.00M + \$1.00M + \$1.50M)/3) We refer to the resulting prameter β as the "low" estimator, where $E(\beta) \leq DV_1$.

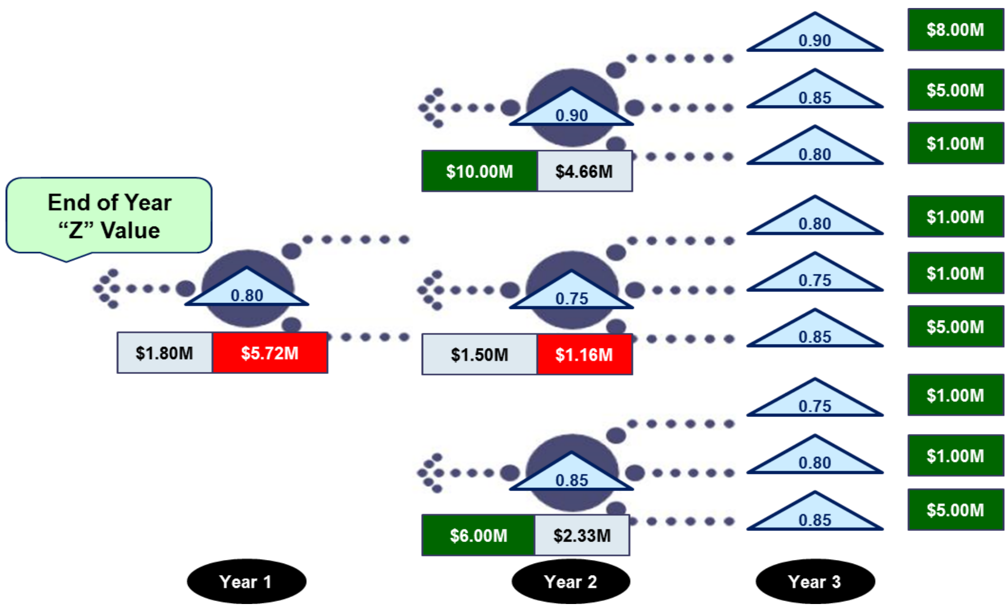

Figure 7 – β ("High") Estimate Construction in Dollars Million (M)

4.3 Analysis of Estimators

Our objective in this section is to give the precise specification of the estimators and assumptions used in our model and to state their analytical properties.

We use the following elements to create our model.

- Time is indexed by times $0 = t_1 < t_2 < t_3 < t_4 = T$. We restrict the draft declaration decisions to lie in the finite set of times $0 < t_1$ = *End of 1st Year of Collegiate Eligibility* $< t_2$ = *End of 2nd Year of Collegiate Eligibility* $< t_3$ = *End of 3rd Year of Collegiate Eligibility* $< t_4$ = *End of 4th Year of Collegiate Eligibility* = T.

- Z_t is an aggregate Markov chain recording all state variables impacting a student-athlete's decision to enter the NBA draft.
- $f(X_1,X_2,\ldots,X_k) = Z_t \in [0,1]$ is a continuous function used to compute sample Z values based on the linear equation $f(X_1,X_2,\ldots,X_k) = w_1X_1 + w_2X_2 + \ldots + w_KX_k$; where the X_i (for $i \leq k$) are observable attribute variables used to determine a student-athlete's draft value. These attribute variables differ by position and are used as NBA performance indicators to assess to assess draft value. For simplicity, we limit the number of attributes variables used in our model and assume they are independent with incremental changes in X_i that are independent and identically normally distributed.[1] We utilize an aggregate variable Z in our model to take advantage of the algorithm's framework for pricing financial securities with opportunities for early exercise. Z values were calculated for both early entrants student-athletes drafted between 2008 and 2013. These attribute variables were normalized using attribute data for student-athletes drafted in the first round (i.e. drafted among the first 30 draft picks) between 2008 and 2013. The weights, w_i, were generated using the Analytic Hierarchy Process (AHP). AHP is a theory of measurement through pairwise comparisons and relies on informed judgments to derive priorities and weights (Saaty, 2008). These priorities were used to measure the intangibles of our attribute variables in relative terms and served as a guide to measure its contribution to a given

[1] This assumption simplifies the formulation of our problem and limits the scope and complexity of our model. As result, we realize that this may have a strong effect on our conclusions and recommend further generalization in any practical application of this method.

student-athlete's draft value. Weights for our attribute variables varied by position to prioritize the attributes that best reflect NBA potential.

- The probabilities p_n^1, p_n^2, and p_n^{ND} are generated based using observed Z values for early draft entrants who declared between 2008 and 2012. We use a logistic regression model to help generate probabilities for student-athletes drafted and not drafted. NBA franchises are still reluctant to bypass a student-athlete in the draft who may turn into an all-star within a few years in favor of a more seasoned college player.
- FE_t^1, FE_t^2 represent the expected future discounted career earnings (ranging from five to eight years) for student-athletes drafted in the first and second rounds at time t. Expected future career earnings were estimated using historical salary data for players drafted between 2008 and 2012. The one-year discount rate r for our model is 3.5%. We estimated r by calculating the annual salary increase for first round draft picks using the rookie salary scales from 2012 – 2013 and 2013 – 2014 NBA seasons.

The framework for our model is sufficiently broad enough to encompass most pricing models which allow for early exercise opportunities. A random tree with b branches per node is represented by the array:

$$\{ Z_t^{l_1,\ldots,l_t} : t = 1,2,3,4;\ l_j = 1,\ldots,b;\ j = 1,2,3,4 \}.$$

See Figure 3 for an illustration. The joint distribution of the elements of this array is specified as follow: Z_1 is the fixed initial state; $Z_t^{l_1,\ldots,l_t,j}$, j = 1,…b, are conditionally independent of each other and of all $Z_t^{l_1,\ldots,l_u}$ with u < t, each $Z_t^{l_1,\ldots,l_t,k}$ has the distribution of [$Z_t \mid Z_{t-1} = Z_{t-1}^{l_1,..,l_{t-1}}$]. Thus, the sequence { $Z_1^{l_1}$, $Z_2^{l_1,l_2}$, $Z_3^{l_1,l_2,l_3}$, $Z_t^{l_1,l_2,l_3,l_4}$ } is a realization of the Markov chain { Z_t: t = 1,2,3,4 }, and two such sequences evolve in dependently of each other once they differ in some l_t.

The high estimator θ is defined recursively by:

$$\boldsymbol{\theta_4^{l_1,..,l_4} = DV_4 = V_4^{SS} = V_4^{EE}}$$

and

$$\boldsymbol{\theta_t^{l_1,..,l_t} = \max \{ V_t^{EE}, \frac{1}{b} \sum_{j=1}^{b} e^{-r}\theta_{t+1}^{l_1,..,l_{t+1}} \},}$$

for t = 1,2,3. At each node, this estimator chooses the maximum of the early entry draft and the continuation value estimated from all future draft entry nodes. The estimator depends on the branching parameter b, and consequence of this result is that:

$$\boldsymbol{E[\ \theta_1(b)\ ] \to DV_1 \text{ as } b \to \infty,}$$

so the estimator is asymptotically unbiased and $\boldsymbol{E[\ \theta_1(b)\ ] \geq DV_1}$ for all b.

The low estimator β is defined recursively as follows. First let:

$$\boldsymbol{\beta_4^{l_1,..,l_4} = DV_4 = V_4^{SS} = V_4^{EE}.}$$

Next define:

$$\boldsymbol{\eta_t^{l_1,..,l_t,j} = \begin{cases} = V_t^{EE} & \text{if } V_t^{EE} \geq \frac{1}{b-1} \sum_{g=1}^{b} e^{-r}\beta_{t+1}^{l_1,..,l_{t+1},g}\ ;\ g \neq j, \\ = e^{-r}\beta_{t+1}^{l_1,..,l_{t+1},j} & \text{if } V_t^{EE} < \frac{1}{b-1} \sum_{g=1}^{b} e^{-r}\beta_{t+1}^{l_1,..,l_{t+1},g}\ ;\ g \neq j \end{cases}}$$

for j = 1,…,b. Then let

$$\boldsymbol{\beta}_t^{l_1,\ldots,l_t} = \frac{1}{b}\sum_{j=1}^{b} \boldsymbol{\eta}_t^{l_1,\ldots,l_t,j},$$

for t = 1,2,3 and $\mathbf{E}[\ \boldsymbol{\beta}_1(\mathbf{b})\ ] \leq \mathbf{DV_1}$ for all b.

4.4 Numerical Results

In this section we provide some numerical results to illustrate the simulation method. We begin by generating simulated paths of Z_t values for student-athletes by position whose value is governed by the continuous function $f_t(X_1,X_2,\ldots,X_k) = Z_{t;}$ where the observable attribute variables, X_i (for $i \leq k$). Each Xi can be simulated using the process

$$\mathbf{X_i = X_{i-1} + X(p,a)(t_i - t_{i-1}),}$$

where is X(p,a) is a unique normal random variable with parameters μ and σ specific to a student-athlete's position (i.e., PG, SG, etc.) and attribute (i.e., points per game, rebounds per game, etc.) The observable attribute variables are likely to be correlated; however we assume independence for simplicity. The parameters were estimated using the annual improvement/decline for a given attribute associated with student-athletes drafted between 2008 and 2012 by position. The paths for these attribute variables were then used to generate an aggregate approximation for Z_t at time t for each position. We used the aggregate state MDP to utilize the dynamic programming pricing algorithm introduced by Broadie et al. (2007). We use the algorithm to develop future earnings estimates for estimates for θ and β for a student-athlete faced with the decision to declare for the NBA draft or remain in school at the end of his freshman year of college. The results in Tables 2 through 6 are consistent with the theoretical developments in the previous section. The β estimator is biased low and the θ estimator is biased high, where $\beta \leq \theta$ for all Z values. We generate our confidence interval by taking the upper confidence limit from the high estimator and the lower confidence limit from the low estimator. The point estimate is given by the simple average

$$\textbf{Point Estimate} = \mathbf{0.5\beta + 0.5\theta}$$

Table 2 – 1st Year Policy for Freshman Centers[2]

Z-Value	Low Est. β	Std. Err. β	High Est. α	Std. Err. β	Decision	Confidence Interval		Point Estimate
0.800	$11.57M	$0.07M	$11.68M	$0.07M	Stay	$11.46M	$11.80M	$11.62M
0.850	$14.12M	$0.07M	$14.51M	$0.07M	Stay	$14.00M	$14.63M	$14.31M
0.900	$16.42M	$0.08M	$17.32M	$0.07M	Stay	$16.29M	$17.43M	$16.87M
0.950	$19.25M	$0.03M	$20.28M	$0.01M	Declare	$19.19M	$20.31M	$19.77M

Table 3 – 1st Year Policy for Freshman Power Forwards

Z-Value	Low Est. β	Std. Err. β	High Est. α	Std. Err. β	Decision	Confidence Interval		Point Estimate
0.800	$11.57M	$0.07M	$11.68M	$0.07M	Stay	$11.46M	$11.80M	$11.62M
0.850	$14.12M	$0.07M	$14.51M	$0.07M	Stay	$14.00M	$14.63M	$14.31M
0.900	$16.42M	$0.08M	$17.32M	$0.07M	Stay	$16.29M	$17.43M	$16.87M
0.950	$19.25M	$0.03M	$20.28M	$0.01M	Declare	$19.19M	$20.31M	$19.77M

Table 4 – 1st Year Policy for Freshman Small Forwards

Z-Value	Low Est. β	Std. Err. β	High Est. α	Std. Err. β	Decision	Confidence Interval		Point Estimate
0.800	$9.74M	$0.08M	$10.00M	$0.08M	Stay	$9.61M	$10.13M	$9.87M
0.850	$12.22M	$0.09M	$12.95M	$0.08M	Stay	$12.07M	$13.08M	$12.58M
0.900	$14.57M	$0.09M	$15.95M	$0.07M	Stay	$14.42M	$16.06M	$15.26M

[2] **Draft Value Parameters:** Z-Value = Z_1 as indicated in the tables. Also, r = 3.5%, and the student-athlete has four exercise opportunities at times t_1, t_2, t_3, and t_4. **Simulation parameters:** n = 1000 and b = 3. All standard errors reflect variability in our simulations and do not take account of likely misspecification in our predictive model.

0.950	$19.22M	$0.04M	$20.07M	$0.01M	**Declare**	$19.17M	$20.09M	$19.65M

Table 5 – 1st Year Policy for Freshman Shooting Guards

Z-Value	Low Est. β	Std. Err. β	High Est. α	Std. Err. β	Decision	Confidence Interval		Point Estimate
0.800	$9.17M	$0.05M	$9.42M	$0.06M	**Stay**	$9.08M	$9.52M	$9.30M
0.850	$12.21M	$0.07M	$12.79M	$0.06M	**Stay**	$12.09M	$12.90M	$12.50M
0.900	$15.29M	$0.07M	$16.39M	$0.05M	**Stay**	$15.17M	$16.48M	$15.84M
0.950	$19.67M	$0.02M	$19.94M	$0.00M	**Declare**	$19.64M	$19.94M	$19.80M

Table 6 – 1st Year Policy for Freshman Point Guards

Z-Value	Low Est. β	Std. Err. β	High Est. α	Std. Err. β	Decision	Confidence Interval		Point Estimate
0.800	$8.56M	$0.06M	$8.80M	$0.06M	**Stay**	$8.47M	$8.90M	$8.68M
0.850	$11.48M	$0.08M	$12.21M	$0.07M	**Stay**	$11.35M	$12.32M	$11.85M
0.900	$14.44M	$0.08M	$15.77M	$0.06M	**Stay**	$14.31M	$15.87M	$15.10M
0.950	$19.26M	$0.03M	$20.09M	$0.01M	**Declare**	$19.20M	$20.10M	$19.67M

The model predicts that on average, a student-athlete with a Z value less than or equal to 0.900 should not declare for the NBA draft. That same player can expect greater career earnings by remaining in school and declaring for the draft in the future. Hence the model reflects that a student-athlete with a high signal (i.e. Z > 0.900) and a skill set highly valued in the NBA market should exercise the option to enter draft early and make the optimal financial decision. In all other cases our model shows that remaining in college and continuing to play NCAA basketball has value and will help the student-athlete improve his draft position and drat value next season. Because of the bias of the estimators, the reported confidence intervals are conservative. Results highly depend on the model and simulation parameters used which influence the bias in the estimators.

The table below depicts the Z values for student-athletes drafted in 2013 NBA Draft using our model parameters.

Table 7 – Freshman Drafted in the 2013 NBA Draft

Draft Year	Name	College	Class	Position	Round	Draft No.	Z-Value	Model Decision
2013	Anthony Bennett	UNLV	Fr	PF	1	1	0.935	**Declare**
2013	Nerlens Noel	Kentucky	Fr	C	1	6	0.967	**Declare**
2013	Ben McLemore	Kansas	Fr	SG	1	7	0.914	**Declare**
2013	Steven Adams	Pittsburgh	Fr	C	1	12	0.880	**Stay**
2013	Shabazz Muhammad	UCLA	Fr	SG	1	14	0.885	**Stay**
2013	Grant Jerrett	Arizona	Fr	PF	2	10	0.616	**Stay**

The results above are fairly consistent with my model results. As depicted above, the policy for Anthony Bennett, Nerlens Noel, and Ben McLemore calls for all three of them to declare for the draft. All three were drafted in the top-10 picks and provided instant financial security. The policy is correct for Steven Adams and Shabazz Muhammad as well. The model advised that they were expected to receive higher future career earnings had they remained in school another year. Although they both declared and were drafted in the first round, they both had considerable upside that could have made them higher draft picks in the 2014 draft. The model also exhibits the downside of the decision process as well. For instance, Grant Jarrett appears to be a student-athlete that could have remained in school and benefited from an additional year or two of collegiate basketball. Although he was drafted in the second-round of the draft, he has spent much of the 2013 season playing in the NBA's Developmental League as he remains in pursuit of the financial security afforded to his freshman counterparts.

5 Conclusions

The combination of money and the change in the way that NBA teams evaluate talent virtually dictates that players turn professional as early as possible, greatly affecting the landscape of college basketball. As a result, the elite NBA players who have completed four years of college have all but disappeared. For student-athletes, making a rational decision regarding the timing for early entry into the draft requires identifying and appropriately weighing the alternatives based on the information available. In the past, most players have taken the advantage of the opportunity to “test the waters” for their draft position in order to gather relevant information pertinent to their draft status so that they can accurately determine whether they should withdraw their name prior to the draft deadline. This option has become virtually obsolete for student-athletes as the NCAA has instituted strict guidelines that make it extremely difficult for student-athletes to determine their draft status. NBA prospects are now restricted to submitting their names to the NBA’s “Undergraduate Advisory Committee,” which is composed of NBA team executives, and receive a confidential projection of their likely draft position. An assessment indicating that a player may be taken “between the middle of the first-round and the middle of the second-round”, provides no true insight as to whether an individual should make this transition.

Our model results suggest that student-athletes should only enter the NBA draft as freshman if they possess an extremely high Z value. Because of the age requirement, colleges and universities face an increasing number of student-athletes whose sole reason for attending school is to build their brand and pass the time before declaring for the NBA draft. The volatility of the draft has shown that that is very difficult gauge your draft status with a Z value less than 0.900. The difference between a first round selection and a second round selection is enormous. Under the recent CBA, the first round pick is provided a guaranteed three-year contract, while the second round pick is guaranteed nothing. The standard interpretation is that a player who leaves early and is selected in the first round made a “good” choice, while the one who is either drafted in the second round or undrafted made a “mistake” by sacrificing his eligibility.

Our model presents a first attempt at examining the early entry decisions facing student-athletes. We will continue to refine the model assumptions and parameters in future work. The current model uses NBA performance indicators like size, position, offensive/defensive strengths/weaknesses, and level of competition to estimate draft value and future career earnings. Since our model uses an aggregate state variable, Z, it is difficult to assess the individual attribute variables that make up a student-athlete’s Z value. For instance, Steven Adams, drafted in the first round of the 2013 draft, could have performed well in rebounds per game and block shots, but failed to score enough points in games for a higher Z value. As a result, if a team is drafting to fill a need that excludes scoring, then you indeed have a high commodity in Steven Adams, who stands seven feet tall and is still relatively new to the game of basketball. Our model is not able capture his “potential” or the current climate of the draft. Further limitations to the model include:

- The ability to model and capture pertinent draft intangibles like:
  - Laziness
  - Physical Shape
  - Selfish Play
  - Lack of Focus/Concentration.

    - Attitude

- The ability to differentiate between guaranteed contracts and non-guaranteed contracts. The expected earnings for student-athlete's drafted in the second-round is skewed by the players who go on to receive lucrative free agent contracts. A number of student-athletes are forced to seek professional basketball opportunities abroad or explore life after basketball.

Unlike the typical student, athletes do not have much free time to experience college and ascertain the general life experience that the NBA promotes. Nor do athletes necessarily benefit from the academic learning experience the NCAA advocates. Many elite players recognize that they are in college for only one year. Education should play a larger role in the valuation process, but unfortunately, the synergy between academics and athletics is not as strong as it should be.

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