ABSTRACT

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Directed By: Professor, Elise Miller-Hooks,

Department of Civil and Environmental Engineering

There are a variety of circumstances in which large numbers of people gather and must disperse. These include, for example, carnivals, parades, and other situations involving entrance to or exit from complex buildings, sport stadiums, commercial malls, and other type of facilities. Under these situations, people move on foot, commonly, in groups. Other circumstances related to large crowds involve high volumes of people waiting at transportation stations, airports, and other types of high traffic generation points. In these cases, a myriad of people need to be transported by bus, train, or other vehicles. The phenomenon of moving in groups also arises in these vehicular traffic scenarios. For example, groups may travel together by carpooling or ridesharing as a cost-saving measure. The movement of significant numbers of people by automobile also occurs in emergency situations, such as transporting large numbers of carless and mobility-impaired persons from the impacted area to shelters during evacuation of an urban area.

This dissertation addresses four optimization problems on the design of facilities and/or operations to support efficient movement of large numbers of people who travel in groups. A variety of modeling approaches, including bi-level and nonlinear programming are applied to formulate the identified problems. These formulations capture the complexity and diverse characteristics that arise from, for example, grouping behavior, interactions in decisions by the system and its users, inconvenience constraints for passengers, and interdependence of strategic and operational decisions. These models aim to provide: (1) estimates of how individuals and groups distribute themselves over the network in crowd situations; (2) an optimal configuration of the physical layout to support large crowd movement; (3) an efficient fleet resource management tool for ridesharing services; and (4) tools for effective regional disaster planning. A variety of solution algorithms, including a meta-heuristic scheme seeking a pure-strategy Nash equilibrium, a multi-start tabu search with sequential quadratic programming procedure, and constraint programming based column generation are developed to solve the formulated problems. All developed models and solution methodologies were employed on real-world or carefully created fictitious examples to demonstrate their effectiveness.

EFFICIENT PEOPLE MOVEMENT THROUGH OPTIMAL FACILITY CONFIGURATION AND OPERATION

By

Lei Feng

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Advisory Committee:

Professor Elise Miller-Hooks, Chair/Advisor
Professor Paul Schonfeld
Professor James Milke
Associate Professor Cinzia Cirillo
Dr. Erica Kuligowski

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Chapter 1 Introduction

1.1 Introduction and Motivation

There are a variety of circumstances in which large numbers of people gather and must disperse. These include, for example, carnivals, parades, shopping centers or markets, inaugurations, rock concerts, football games, and other situations involving entrance to or exit from complex buildings, sport stadiums, commercial malls, and other type of facilities. Under these situations, people move on foot and *en masse*. Within the crowds, there are groups of people who wish to travel together. For example, family members walk beside each other. Friends or colleagues tend to stay together and maintain communication with each other while walking.

Other circumstances related to large crowds involve high volumes of people waiting at transportation stations, airports, docks and other types of high traffic generation points. In these cases, a myriad of people need to be transported by bus, train, van, ship or other vehicles. The phenomenon of moving in groups also arises in these vehicular traffic scenarios. For example, a family will travel within the same vehicle or larger groups will travel in a bus. In other cases, groups may travel together by carpooling or ridesharing as a cost-saving measure.

The movement of significant numbers of people by automobile also occurs in evacuating an urban region due to natural or human-made disaster events, like flooding, hurricanes, and industrial or nuclear accidents. To reduce the adverse consequences of these disasters on humans, evacuating a large region by automobile, which is the most commonly available evacuation mode, is often the most viable response action for protecting the affected people. However, urban areas often involve large volumes of carless evacuees and a significant portion of them are mobility-impaired. Many of these carless people require transport from the impacted area to safe places, including shelters. The evacuation planning consists of two components. First, decide the locations of shelters and assign people the shelters (facility design), and second, dispatch available public transit vehicles transport them to the shelters (operation design).

Optimal design of facilities or facility locations and operations that support the movement of large numbers of people are critical to public safety and efficiency. In addition to the numerous disasters associated with crowding due to poor crowd and evacuation management, efficient control and guidance of the movement of large numbers of people can provide crucial support toward meeting ingress, egress and safety goals. Furthermore, optimal design of efficient and low-cost ridesharing or other mechanisms for moving individuals within a single vehicle can alleviate congestion on the roadways. In emergency situations, optimal design of shelter locations and operations for evacuating large numbers of carless and mobility-impaired persons are critical components of evacuation planning for a large urban area.

Modeling and decision support for these crowd-related circumstances, however, can be difficult, and related optimization problems are likely intractable. This intractability is, in part, due to (1) existence of a complex physical environment with interdependent passageways, (2) assembly of large numbers of people with complicated, collective and heterogeneous behaviors, (3) interdependence and interaction between decisions from different people who play different roles in the system (crowd manager vs. system users, operators vs. passengers), and (4) the large-scale nature of the problem instances with significant demand for service within large geographic regions, particularly as it relates to traffic and emergency events.

This dissertation will provide tools to support the efficient movement of large numbers of people under a variety of situations. Specifically, mathematical models of pedestrian movements in crowds are developed and optimization tools are proposed to control crowd movement and prevent disorder from breaking out. The movement of large numbers of people to and from transportation stations (specifically airports) through ridesharing services is addressed, supporting the movement of unrelated persons in single vehicles. Finally, optimal design of facilities (shelter location and allocation) and operations (routes and schedules of paratransit vehicles) in a large-scale transit-based mass evacuation of an urban area is addressed.

1.2 Specific Problems Addressed

The problems addressed in this dissertation arise from diverse, yet increasing concerns in facility and/or operations design for efficient movement of large numbers of people. This section provides concise statements about each addressed problems. The detailed problem descriptions, mathematical formulations and solution approaches are given in Chapters 2 through 5.

1.2.1 Pedestrian Route Choice in Crowds

In large public gatherings, crowds are directed through passageways within the facilities. The physical layout of these passageways provides a set of route options from which pedestrians can choose for ingress or egress. A pedestrian’s preference for an alternative route depends on the route's utility and its utility depends both on its attributes and the pedestrian's sensitivity to each such attribute type. In addition, some attributes, like travel speed, depend on the choices made by others who simultaneously seek passage along the same routes. Moreover, in the context of crowd movement, groups must make a concerted effort to move together and not be split apart.

Network optimization-based modeling and solution frameworks are proposed for assessing pedestrian response to the physical layout of a venue's ingress and egress routes during large public gatherings. The frameworks involve the modeling and solution of a pedestrian assignment problem. These approaches support the movement of both individuals and groups. A distinction is made between two broadly categorized group types: separable and clustered. The former can be, for example, a group of friends/colleagues who have a predilection for staying together, wherein each person within the group is free to make his or her own decision in response to the physical environment. The latter describes groups that will not be separated, such as parent and child. Such group decisions and movements are crucial to developing realistic models of pedestrian movement (Hamacher et al., 2011; Qiu and Hu, 2010). The effects of separable and clustered group movements on flow distributions through the physical layout are studied.

Two methodologies are proposed to model these effects: a stochastic user equilibrium pedestrian assignment (SUE) approach to model separable groups and an n-player non-cooperative game seeking a pure-strategy Nash equilibrium to model clustered groups. A solution scheme that combines the method of successive averages with group movements is proposed for solving the SUE assignment and a Best Response Dynamics-based Tabu Search procedure is proposed for obtaining a pure strategy Nash equilibrium for clustered groups.

Details of model formulations, solution approaches, as well as results of numerical experiments conducted to demonstrate the effectiveness of the proposed methodologies and investigate the impact of groups on flow efficiency, are provided in Chapter 2.

1.2.2 Crowd Management in Large Public Gatherings

Effective crowd management during large public gatherings is necessary to enable pedestrians to have access to and from the venue and to ensure their safety. A number of previous studies focus on determining optimal routes for the movement of pedestrians through a given physical layout. An alternative management strategy might be to reconfigure the physical layout to facilitate pedestrian movement in pursuit of a particular goal. Such redesign can both limit pedestrian choice and enhance or restrict capacity along routes to facilitate efficient movement and prevent crowd crush or other unsafe situations. Changes to the physical layout might be achieved through opening or closing gates/doorways, placing or removing barriers or changing illumination intensity to coerce pedestrians along certain paths. No prior work has suggested such an approach in the context of crowd movement.

In this dissertation, the problem of reconfiguring the physical layout of the facility to support efficient crowd movement, conceptualized as Redesign for Efficient Crowd Movement (RECM), is formulated as a bi-level integer program. The upper-level seeks a reconfiguration of the physical layout that will minimize total travel time incurred by system users (e.g. evacuees) given utility maximizing route decisions that are taken by individuals in response to physical offerings in terms of infrastructure at the lower-level. The lower-level formulation seeks a pure-strategy Nash equilibrium that fills in grouping behavior in crowds. A Multi-start Tabu Search with Sequential Quadratic Programming procedure is proposed for solutions of the bi-level Mixed Integer Program. This procedure guarantees a locally optimal solution to this nonlinear program.

The details of formulation, numerical experiments on a hypothetical network conducted to illustrate the proposed solution methodology and the insights it provides are given in Chapter 3.

1.2.3 Optimizing Ridesharing Services for Airport Access

Airports often have large numbers of departure and arrival passengers that can cause congestion on roadways, environmental pollution, and greater difficulty accessing the facility. Like traditional public transit, ridesharing can serve more than one passenger with one vehicle. Thus, it can aid in limiting the volume of traffic, thereby reducing congestion and mitigating environmental impact. Moreover, ridesharing can provide higher quality of service than traditional public transit through flexible routes and schedules as well as door to door pick-ups and drop-offs. Furthermore, reduced total passenger-miles traveled resulting from ridesharing and efficiently designed routes can increase profitability of the service provider and aid in diminishing traffic congestion along with its negative externalities, including environmental pollution.

The Airport Access Ridesharing Problem (AARP) is conceptualized in this dissertation. TheAARP seeks to determine a set of routes and schedules that meet service quality, resource, labor and vehicle capacity constraints while minimizing total cost in terms of vehicular use and total wages in the context of airport ridesharing services. The AARP is formulated as a nonlinear, mixed integer program. An exact solution approach applying constraint programming within a column generation framework, as well as adaptations of two existing heuristics, are proposed for its solution. Implementations of the mathematical program and proposed solution approaches for three different operational policies are also presented.

The details of formulation, proposed solution approaches and numerical experiments on a real-world case study involving service records for one service day of Supreme Airport Shuttle, Inc. out of Washington Dulles International Airport are given in Chapter 4.

1.2.4 Facility and Operations Design for Mass Evacuation Planning

This dissertation addresses the problem of providing safe locations for mobility-impaired persons in an evacuation and the transportation for these persons from their homes to such facilities. To state and local governments, important issues for facilities and operations in an mass evacuation planning associated with mobility-impaired population include: (1) how many and where shelters should be opened to this population, (2) to which shelter each mobility-impaired evacuee should be assigned, and (3) how to optimally dispatch and route paratransit vehicles to serve this population. No prior analytical models have been proposed in the literature to help the government with decision-making on these critical issues.

To fill this gap, the Sheltering and Paratransit Evacuation Problem (SPEP) is studied. The SPEP is formulated as a mixed integer program. The problem consists of two interdependent and integrated subproblems: 1) Capacitated Shelter Location-Allocation Problem (CSLAP) and 2) Multi-depot Pickup and Delivery Problem (MPDP). To solve a large-scale instance of the SPEP, a tabu search metaheuristic is proposed.

Details of the problem conceptualization and formulation, as well as the proposed tabu search algorithm and numerical experiments on a real-world case study involving hurricane evacuation planning for New York City, are given in Chapter 5.

1.3 Contributions

Address vital aspects in the design of facilities and operations to support the movement of large numbers of people. This dissertation seeks to provide tools that can be used for: (1) Estimating the distribution of groups and individuals over the physical layout network, considering that people move in groups. (2) Redesigning the physical layout to facilitate crowd movement in pursuit of a particular goal, considering both goals of the system and the users. (3) Optimally and efficiently matching passengers to vehicles, and routing and scheduling their trips for an airport ridesharing service operation system. (4) Optimally locating and assigning shelters and optimally routing and scheduling available paratransit vehicles to support mobility-impaired populations in a large-scale regional evacuation.

Develop optimization models for these identified problems. Mathematical models are proposed and optimization problems are formulated. These models capture the complexity and diverse characteristics that arise from, for example, grouping behavior, interactions in decisions by the system and its users, inconvenience constraints for passengers, and interdependence of strategic and operational decisions. A variety of modeling approaches, including bi-level and nonlinear programming are applied to formulate the identified problems. These models aim to provide: (1) estimates of how individuals and groups distribute themselves over the network in crowd situations; (2) an optimal configuration of the physical layout to support large crowd movement; (3) an efficient fleet resource management tool for ridesharing services; and (4) tools for effective regional disaster planning.

Provide conceptual framework and specific methodological procedures for solution of identified optimization problems. A variety of solution algorithms, including a meta-heuristic scheme seeking a pure-strategy Nash equilibrium, a multi-start tabu search with sequential quadratic programming procedure, and constraint programming based column generation are developed to solve the formulated problems. All developed models and solution methodologies were employed on real-world or carefully created fictitious examples to demonstrate their effectiveness.

1.4 Dissertation organization

The remainder of this dissertation is organized into five chapters. Chapter 2 presents the modeling and solution frameworks of pedestrian route choice in crowds, while Chapters 3 through 5 address the RECM, AARP and SPEP, respectively. Finally, conclusions and extensions for future research are given in Chapter 6.

Chapter 2 Pedestrian Route Choice in Crowds

2.1 Introduction

Large gatherings of people arise for a variety of purposes and may be held in a myriad of venues, including for example, complex buildings, transportation stations, sports stadiums, commercial malls, and other type of facilities. In such gatherings, crowds are directed through passageways within the facilities. The physical layout of these passageways provides a set of route options from which pedestrians can choose for ingress or egress. The speed with which a pedestrian will move through the passageway depends on its physical capacity, the person's physical well-being, and the number of other pedestrians utilizing it at the same time. The time for ingress or egress to or from the event depends on the series of choices the pedestrian makes in navigating the physical layout and competition with other pedestrians for passageway capacities. A pedestrian's preference for an alternative route depends on the route's utility and its utility depends both on its attributes and the pedestrian's sensitivity to each such attribute type. Moreover, some attributes, like travel speed, are affected by the choices made by competing system users. The selection of a route is assumed to be rational, meaning that the pedestrian will choose the route with the maximum utility based on his/her preference function. The overall problem of estimating which routes all travelers will take is known as a traffic assignment problem, and is referred to as a Pedestrian Route Choice in Crowds (PRCC) problem in this context.

The concept of route choice in vehicular traffic flow is well developed. Pedestrians, however, have more degrees of freedom in movement and often move en masse, or in groups. Such groups arise in vehicular traffic scenarios, but these groups are typically housed within a single vehicle. For example, a family will travel within the same car or larger groups will travel in a bus. These groups, thus, will never be faced with the possibility of being split apart. Others who seek to access the venue together but in different vehicles will often need to be willing to meet at the destination. In the context of pedestrian movement, however, groups must make a concerted effort to move together and not be split apart. For example, parents will not wish to be separated from their children. Thus, while each person within the family is an individual (i.e. a unit of flow) and is free to make his or her own decisions in response to directives from crowd managers or the physical layout, any effective crowd management plan must facilitate the movement of all members of the family as a group. That is, the group must be permitted to stay together and accommodations must be made to support this group movement. In this chapter, a distinction is made between two broadly categorized group types: separable and clustered (Aveni, 1977). The former can be, for example, a group of friends/colleagues who have a predilection for staying together, but each person within the group is free to make his or her own decision in response to the physical environment. It is likely but not guaranteed that individuals in this group type will travel together. The latter describes groups that will not be separated, such as parent and child. Such group decisions and movements are crucial to developing realistic models of pedestrian movement (Hamacher et al., 2011; Qiu and Hu, 2010).

This chapter describes a network optimization-based modeling and solution framework for estimating pedestrian flows within a network representation of a physical environment. Movements by individuals and groups must be captured in the flows produced by this method. That is, the framework involves the modeling and solution of a pedestrian assignment problem.

Before proceeding to descriptions of these two modeling approaches, traffic assignment problem is briefly reviewed, followed by general introduction to utility maximization concepts in the context of route choice.

2.2 Traffic Assignment Problem

Assignment problems for vehicular traffic have received enormous attention in the literature. The majority of traffic assignment models seek user equilibrium (UE) solutions, where no traveler can select an alternative path with higher utility by unilaterally switching routes (Sheffi, 1985). Deterministic user equilibrium (DUE) and stochastic user equilibrium (SUE) models are two common UE approaches. DUE assumes that travelers have perfect information on the performance of all alternative routes when choosing a route. SUE, on the other hand, presumes that each user makes his/her selection of a route based on perceived features of the alternatives. It is generally accepted that SUE approaches provide more realistic predictions of traveler route choice behavior (Chen and Alfa, 1991). Both modeling approaches assume that travelers are homogenous in terms of their preference functions. And both assign travelers to paths probabilistically, with higher likelihood of choosing a path with higher utility. That is, the frequency of path use can be set by the probability of its selection.

An alternative approach might be to employ a Nash equilibrium based methodology. Both pure- and mixed-strategy Nash equilibriums have been considered in the context of vehicular traffic assignment (Rosenthal, 1973a, b). In (Rosenthal, 1973a, b), players have their own preference functions. Formulations seeking such equilibriums involve concepts of non-cooperative games. In these prior works, group behavior is not considered and, therefore, the developed models and algorithms for traffic assignment cannot be applied directly in the movement of pedestrians where group behavior must be considered. One reason for this is that the marginal impact of the decision of one flow unit in pedestrian assignment where group behavior is modeled must account for the impact of group size.

Several works in the context of vehicular traffic take the heterogeneity of users into consideration. For example, the assignment problem for multiclass user traffic networks is considered in (Huang and Li, 2007; Nagurney, 2000). In this multiclass user equilibrium problem, each class of travelers (e.g. trucks, buses, passenger cars) has an individual preference function and each class makes decisions based on path utilities derived from this function. Travelers are assigned to paths probabilistically, as in DUE and SUE methods, again with higher likelihood of choosing a route with higher utility. Users in the same class will have the same probability of selecting route alternatives. Thus, the multiclass user equilibrium assignment method does not guarantee that members in the same class will make the same decisions.

While there is a significant body of work existing in the vehicular traffic assignment area, these works cannot be directly extended for use in modeling clustering (or group) behavior as is required for many pedestrian traffic assignment contexts. On the contrary, within the literature on pedestrian modeling, numerous works consider group behavior. The majority use simulation and often involve a leader and set of followers (e.g. (Qiu and Hu, 2010)). In an alternative network flow-based approach, Hamacher et al. (2011) incorporate group movements in solving a dynamic quickest cluster flow problem. However, travel times are not flow-dependent and thus competition among travelers for limited capacity is not considered.

In this chapter, the effects of separable and clustered group movements on flow distributions through the physical layout are studied. Two methodologies are proposed to model these effects: an SUE pedestrian assignment approach to model separable groups and an n-player non-cooperative game seeking a pure-strategy Nash equilibrium to model clustered groups. In terms of separable groups, all group members are assumed to have identical (homogenous) preference functions, but as mentioned previously, they behave independently. In terms of clustered groups, all members of the same group make the same route decision. Note that the SUE pedestrian assignment problem used to model separable groups can be reformulated as a game in which a mixed-strategy Nash equilibrium is sought (Devarajan, 1981). In this game, each player represents a single pedestrian. The solution produces the probability that each player chooses each strategy (i.e. route), producing the fraction of total flow distributed over the network. Numerical experiments were conducted to demonstrate the impact of pedestrian route choice under both separable and clustered group situations on movement efficiency within the venue's physical layout.

2.3 Utility Maximization in Route Choice

Route choice, sometimes referred to as wayfinding, involves choosing an option from a finite set of alternative routes for given origin-destination (O-D) pairs. The concept of route choice in vehicular traffic is well developed (Bovy and Stern, 1990). Utility maximization-based discrete choice models are widely used to model route decisions by drivers. The basic assumption underlying this model is that a traveler's preference for each alternative route can be described by a utility (or disutility) that is a function of the attributes of the alternative routes and sensitivity parameters of the traveler to these attributes (Sheffi, 1985). The traveler is assumed to choose the route with maximum utility (or minimum disutility).

In the context of pedestrians, a number of works consider pedestrian route choice behavior (Al-Gadhi, 1996; Antonini et al., 2006; Bierlaire and Robin, 2009; Hoogendoorn and Bovy, 2004a; Løvås, 1998). A couple of these works employ utility maximization-based choice models (Antonini et al., 2006; Bierlaire and Robin, 2009; Hoogendoorn and Bovy, 2004a). Pedestrians are very sensitive to route characteristics that are related to physical effort, such as walking distance, walking time and the exertion involved in climbing stairs or ramps. As discussed in (Daamen et al., 2005; Seneviratne and Morrall, 1985), walking distance and time are the most important route attributes in pedestrian route choice. Furthermore, in discrete choice models, the independence of irrelevant alternatives property is assumed to hold. The concept of path size factor proposed in (Ben-Akiva and Bierlaire, 1999) is adopted herein to deal with overlap in alternative routes due to the sharing of arcs.

In the next section, two types of utility functions that incorporate these elements (group size, travel distance, travel time and overlap) are proposed for separable and clustered groups. The pedestrian route choice problems involving separable and clustered groups are formulated as an SUE assignment problem and n-player non-cooperative game seeking a pure-strategy Nash equilibrium, respectively. Solution methodologies for obtaining flows for each problem class are also provided.

2.4 Two Proposed Approaches to Determine Pedestrian Routes

In this section, the pedestrian assignment problems involving separable and clustered groups are formulated as an SUE assignment problem and n-player non-cooperative game seeking a pure-strategy Nash equilibrium, respectively. Solution methodologies for obtaining flows for each problem class are also provided.

2.4.1 Preliminaries

The physical layout is represented by a network $G=(N,A)$, where N is a set of nodes representing locations at which decisions can be taken, and A is a set of directed arcs connecting the nodes. The arcs represent passageways along which movement is possible. Let $O,D \in N$ be the set of origins and destinations, respectively. Each arc $a \in A$ has an associated length l_a, capacity c_a, and a nonnegative travel time $t_a(x_a,c_a)$, which is a continuously differentiable and strictly increasing function of arc flow x_a and capacity c_a. The BPR-based form (Branston, 1976) is adopted:

$$t_a(x_a,c_a)=t_a^0\cdot[1+k_a(\frac{x_a}{c_a})^2] \qquad \forall a \in A, \tag{2-1}$$

where k_a is a coefficient that scales the rate at which congestion increases with time, and t_a^0 denotes free-flow travel time. For free-flow speed v_a, t_a^0 can be calculated as in equation (2-2).

$$t_a^0 = l_a / v_a \qquad \forall a \in A. \tag{2-2}$$

For specific O-D pair $w \in W$, where W is the set of O-D pairs, there is a set of groups of pedestrians $G_w (g = 1,...,|G_w|)$ and set of routes connecting O-D pair *w*, $R_w (r = 1,...,|R_w|)$. Let S_w^g denote the size of group $g \in G_w$ between pair *w*.

Further, let f_w^r, T_w^r and L_w^r denote the flow, travel time and distance on route $r \in R_w$ for pair *w*, respectively. According to the route-arc incidence relationships, route travel time and distance on route *r* connecting pair *w* can be written as in equations (2-3) and (2-4), respectively.

$$L_w^r = \sum_{a \in A} l_a \cdot \delta_w^{r,a} \qquad \forall r \in R_w, w \in W \tag{2-3}$$

$$T_w^r(f_w^r) = \sum_{a \in A} t_a(x_a) \cdot \delta_w^{r,a} \qquad \forall r \in R_w, w \in W \text{ ,} \tag{2-4}$$

where $\delta_w^{r,a}$ equals 1 if route *r* passes through arc *a*, and 0 otherwise.

2.4.2 Pedestrian Assignment with Separable Groups

An SUE-based assignment formulation in which separable groups can be modeled is given in program (P1). The skeleton of the formulation is from (Fisk, 1980). This formulation is expanded to address group movements. Thus, group assignment and group flow conservation are added as in (P1).

(P1)

$$\min_{\mathbf{x}} \quad Z_1(\mathbf{x}) = \sum_{a \in A} \int_0^{x_a} u_a(\omega) d\omega + \frac{1}{\theta} \sum_{w \in W} \sum_{r \in R_w} f_w^r \cdot \ln f_w^r \tag{2-5}$$

$$\text{s.t.} \qquad \sum_{r \in R_w} f_w^{g,r} = S_w^g \qquad \forall g \in G_w, w \in W \tag{2-6}$$

$$x_a^g = \sum_{r \in R_w} f_w^{g,r} \delta_w^{r,a} \qquad \forall a \in A, g \in G_w, w \in W \tag{2-7}$$

$$x_a = \sum_{w \in W} \sum_{g \in G_w} x_a^g \qquad \forall a \in A \tag{2-8}$$

$$f_w^{g,r} \geq 0 \qquad \forall g \in G_w, r \in R_w, w \in W, \tag{2-9}$$

where $u_a(\cdot)$ is the disutility on arc a, $f_w^{g,r}$ is the flow of group *g* on route *r* between O-D pair *w*, and x_a^g is the flow of group *g* on arc *a*. Objective function (2-5) seeks to minimize the perceived disutility subject to flow conservation constraints (2-6)-(2-8). Constraints (2-9) restrict path flows to be non-negative. Note that the objective function does not have any intuitive economic or behavioral interpretation. It is only a mathematical structure that is used to solve the SUE problem.

Due to its closed form, a logit-based route choice model has been widely employed in computing SUE flows. In a logit-bsed route choice model, flows along the routes are proportionally assigned to routes according to their corresponding utility. The perceived disutility of route r to each individual in group *g* between O-D pair *w* is given in equation (2-10),

$$U_w^{g,r} = -\theta^g \cdot u_w^{g,r} + \varepsilon_w^{g,r} \qquad \forall g \in G_w, r \in R_{od}, w \in W, \tag{2-10}$$

where $u_w^{g,r}$ denotes the measured disutility of route *r* to each individual in group *g* between O-D pair *w*, θ^g is positive scaling parameter indicating disutility perception variations between perceived disutility and real disutility (a higher θ^g means a smaller variation), and $\varepsilon_w^{g,r}$ is a random term presenting the perception errors which are assumed to be independent Gumbel distributed with mean zero.

At SUE equilibrium, the probability of group *g* choosing route *r* between pair *w* can be calculated as in equation (2-11):

$$p_w^{g,r} = \frac{\exp(-\theta^g \cdot u_w^{g,r})}{\sum_{k \in R_w} \exp(-\theta^g \cdot u_w^{g,k})} \qquad \forall g \in G_w, r \in R_w, w \in W. \tag{2-11}$$

From conservation of flow constraints (2-6), flow associated with group g and assigned to route r between O-D pair w, $f_w^{g,r}$, can be deduced through equation (2-12).

$$f_w^{g,r} = S_w^g \cdot p_w^{g,r} \qquad \forall g \in G_w, r \in R_w, w \in W. \tag{2-12}$$

Arc flows can be deduced from route flows through equations (2-7) and (2-8).

From subsection 2.3, the measured disutility, $u_w^{g,r}$, is further expressed as in equation (2-13).

$$u_w^{g,r} = \alpha^g L_r + \beta^g T_r + \gamma^g \ln(PS_r) \quad \forall g \in G_w, r \in R_w, w \in W, \tag{2-13}$$

where $\alpha^g, \beta^g, \gamma^g$ are parameters of walking distance, walking time and path size factor of group g, respectively, and represents the preference (sensitivity) of group g to these attributes. PS_r is the path size factor of route r proposed by (Ben-Akiva and Bierlaire, 1999):

$$PS_r = \sum_{a \in r} \frac{l_a}{L_r} \frac{1}{N_a}, \tag{2-14}$$

where a is index of an element arc of the route, and N_a is the number of alternative routes that pass through arc a.

Substituting equations (2-3) and (2-4) into equation (2-13), $\forall g \in G_w, r \in R_w, w \in W$, leads to equation (2-15):

$$
\begin{aligned}
u_w^{g,r} &= \sum_{a \in A} [\alpha^g l_a + \beta^g t_a(x_a)] \cdot \delta_w^{r,a} + \gamma^g \ln(PS_r) \\
&= \sum_{a \in A} u_a^g(x_a) \cdot \delta_w^{r,a} + \gamma^g \ln(PS_r),
\end{aligned} \tag{2-15}
$$

where $u_a^g(x_a)$ denotes the disutility of group g on arc a, which is a function of arc flow, x_a. Then the arc disutility $u_a(x_a)$ in Equation (2-5) can be expressed by:

$$
u_a(x_a) = \sum_{w \in W} \sum_{g \in G_w} u_a^g(x_a^g) \qquad \forall a \in A. \tag{2-16}
$$

2.4.3 Pedestrian Assignment with Clustered Groups

For clustered groups, the disutility of each route r connecting O-D pair w to group g can be expressed as in equation (2-17).

$$
\upsilon_w^{g,r}(S_w^g, f_w^r) = S_w^g \cdot [\lambda^g L_w^r + \chi^g T_w^r(f_w^r)] \quad \forall g \in G_w, r \in R_w, w \in W, \tag{2-17}
$$

where $\upsilon_w^{g,r}(\cdot)$ represents the disutility of route r for group g with O-D pair w. It is a function of group size, S_w^g, route distance, L_w^r, and walking time, T_w^r. λ^g and χ^g are parameters indicating group g's sensitivity to walking distance and time, respectively.

Let decision variable $\xi_w^{g,r}$ equal 1 if group g chooses route r for O-D pair w, and 0 otherwise. Flow along route r for O-D pair w, f_w^r, is the sum of the sizes of groups that choose the route:

$$
f_w^r = \sum_{g \in G_w} S_w^g \cdot \xi_w^{g,r} \quad \forall r \in R_w, w \in W\ . \tag{2-18}
$$

From Equations (2-3) and (2-4), for each $g \in G_w, r \in R_w, w \in W$, equation (2-17) can be rewritten as equation (2-19).

$$
\begin{aligned}
\upsilon_w^{g,r}(S_w^g, f_w^r) &= S_w^g \cdot [\lambda^g \sum_{a\in A} l_a \delta_w^{r,a} + \chi^g \sum_{a\in A} t_a(x_a)\cdot \delta_w^{r,a}] \\
&= \sum_{a\in A} S_w^g \cdot [\lambda^g l_a + \chi^g t_a(x_a)] \cdot \delta_w^{r,a} \\
&= \sum_{a\in r} \upsilon_w^{g,a}(x_a) \cdot \delta_w^{r,a},
\end{aligned}
\tag{2-19}
$$

where $\upsilon_w^{g,a}(x_a)$ measures the disutility incurred by group g using arc a.

The assignment of clustered groups to routes can be formulated as in program (P2). Program (P2) seeks (objective (2-20)) the set of path flows over all O-D pairs with the minimum total disutility (weighted by group size). Derived from equations (2-8) and (2-18), constraints (2-21) relate arc flows to path flows, thus, ensuring flow conversation. Constraints (2-22) force each group to choose one route. Binary restrictions are guaranteed through constraints (2-23).

(P2)

$$\min \sum_{w\in W} \sum_{r\in R_w} \sum_{g\in G_w} \sum_{a\in A} S_w^g \cdot [\alpha_g l_a + \beta_g t_a(x_a)] \cdot \delta_w^{r,a} \tag{2-20}$$

$$\text{s.t.} \quad x_a = \sum_{w\in W} \sum_{r\in R_w} \sum_{g\in G_w} S_w^g \cdot \xi_w^{g,r} \cdot \delta_w^{r,a} \qquad \forall a \in A \tag{2-21}$$

$$\sum_{r\in R_w} \xi_w^{g,r} = 1 \qquad \forall g \in G_w, w \in W \tag{2-22}$$

$$\xi_w^{g,r} = 0 \text{ or } 1 \qquad \forall g \in G_w, r \in R_w, w \in W \tag{2-23}$$

Program (P2) can be viewed as an n-player, pure-strategy, non-cooperative game, where each group is a player, and the possible routes between each O-D pair form the strategy space. It must be shown that at least one pure-strategy Nash equilibrium for the game modeled as (P2) exists and that the optimal solution to (P2) is a pure-strategy Nash equilibrium, i.e. the solution to (P2) is the equilibrium with smallest total disutility. Proof of this is as follows. The proof builds directly on a related proof given in (Rosenthal, 1973a, b).

Theorem: There exists at least one solution to (P2) that achieves a pure-strategy Nash equilibrium. Additionally, such an equilibrium is achieved by the optimal solution to (P2).

Proof: Since **x**=**0** is a feasible solution to (P2), a feasible solution to (P2) always exist. Let $\{\xi_w^{g,r}\}^*$ be the optimal solution to (P2), and $\{x_a\}^*$ be the associated link flows. Assume that $\{\xi_w^{g,r}\}^*$ does not result in an equilibrium. Then, it must be possible for some group $p \in G_w$ traveling between w along a route $r_1 \in R_w$ to reduce its disutility by switching routes to some other route $r_2 \in R_w$. By Equation (2-19),

$$\sum_{(a \in r_2) \cap (a \notin r_1)} \upsilon_w^{p,a}(l_a, x_a^* + S_w^p, c_a + y_a) < \sum_{(a \in r_1) \cap (a \notin r_2)} \upsilon_w^{p,a}(l_a, x_a^*, c_a + y_a) \,. \qquad (2\text{-}24)$$

Let $\{\xi_w^{g,r}\}'$ denote the resulting solution to (P2) given that group p switches from route r_1 to route r_2.

$$\xi_w^{g,r\prime} = \begin{cases} \xi_w^{g,r*} + 1, & \text{if } g = p, r = r_2 \\ \xi_w^{g,r*} - 1, & \text{if } g = p, r = r_1 \\ \xi_w^{g,r*} & \text{otherwise} \end{cases} \qquad (2\text{-}25)$$

Link flow $\{x_a\}'$ is updated accordingly:

$$x_a' = \begin{cases} x_a^* + S_w^p, & \text{if } a \in r_2 \text{ and } a \notin r_1 \\ x_a^* - S_w^p, & \text{if } a \in r_1 \text{ and } a \notin r_2 \\ x_a^*, & \text{otherwise} \end{cases} \qquad (2\text{-}26)$$

Let $r_1^c = R_w - r_1$ and $r_2^c = R_w - r_2$. Given updated link flows, it can be shown that

$$
\begin{aligned}
&\sum_{w\in W}\sum_{r\in R_w}\sum_{g\in G_w}\sum_{a\in r}\upsilon_w^{g,a}(l_a, x'_a, c_a + y_a)\\
&=\sum_{w\in W}\sum_{g\in G_w}\sum_{a\in(r_1\cap r_2)\cup(r_1^C\cap r_2^C)}\upsilon_w^{g,a}(l_a, x_a^*, c_a + y_a)\\
&\quad+\sum_{w\in W}\sum_{g\in G_w}\sum_{(a\in r_2)\cap(a\notin r_1)}\upsilon_w^{g,a}(l_a, x_a^* + S_{od}^p, c_a + y_a)\\
&\quad+\sum_{w\in W}\sum_{g\in G_w}\sum_{(a\in r_1)\cap(a\notin r_2)}\upsilon_w^{g,a}(l_a, x_a^* - S_w^p, c_a + y_a)\\
&=\sum_{w\in W}\sum_{r\in R_w}\sum_{g\in G_w}\sum_{a\in A}\upsilon_w^{g,a}(l_a, x_a^*, c_a + y_a) + \sum_{(a\in r_2)\cap(a\notin r_1)}\upsilon_w^{p,a}(l_a, x_a^* + S_w^p, c_a + y_a)\\
&\quad-\sum_{(a\in r_1)\cap(a\notin r_2)}\upsilon_w^{p,a}(l_a, x_a^*, c_a + y_a)\\
&<\sum_{w\in W}\sum_{r\in R_w}\sum_{g\in G_w}\sum_{a\in A}\upsilon_w^{g,a}(l_a, x_a^*, c_a + y_a) = \sum_{w\in W}\sum_{r\in R_w}\sum_{g\in G_w}\upsilon_w^{g,r}(S_w^g, f_w^r)
\end{aligned}
$$

This contradicts the assumption that $\{\xi_w^{g,r}\}^*$ is the optimal solution to (L). ||

2.5 Solution Schemes to Determine Chosen Routes

The solution approaches to programs (P1) and (P2) both begin with the generation of an efficient route set (Sheffi, 1985) for each O-D pair. It is presumed, as in (Bovy and Stern, 1990), that when faced with a route decision, a traveler selects his/her route from a limited choice set. Since complete enumeration of all possible routes is impractical and given that most people do not consider all alternatives in making their decisions, only the efficient route set is considered.

2.5.1 Efficient Route Set Definition

Based on [4], an efficient route is defined as a route passing only through efficient arcs, and an efficient arc is defined as follows. For each arc a connecting i to j, if r(i)<r(j), for r(k) the shortest distance from the origin to node k, and s(i)>s(j), for s(k) the shortest distance from k to the destination, then arc a is efficient (eff(i,j)=1);

otherwise, it is inefficient (eff(i,j)=0). The efficient routes, , between each O-D pair w are obtained with a depth-first-search (DFS) on the network of efficient arcs (i.e. the subgraph , where is the set of efficient arcs). Routes with cycles are not generated, because by definition any efficient arc transports travelers to locations that are further from the origin and closer to the destination.

2.5.2 Solution Approach for Program (P1)

The Method of Successive Averages (MSA) (Sheffi, 1985) has been successfully used in solving various stochastic user equilibrium problems. In this chapter, a solution scheme that combines the MSA with group movements is proposed for solving the SUE assignment. The main procedure of MSA is given below.

Step 0: ***Initialization.*** For each $g \in G_w, w \in W,$ use equations (2-11) and (2-12) to perform a logit assignment based on the initial disutility, $u_a^{g\ [0]}(0), \forall a \in A$. The result of this assignment is a set of route flows $f_w^{g,r\ [0]}, \forall r \in R_w$. Generate initial arc flows $x_a^{[1]}, \forall a \in A,$ through equations (2-7) and (2-8) and set iteration count *n*=1.

Step 1: ***Update.*** According to current arc flows $x_a^{[n]}, \forall a \in A$, update the arc disutility, $u_a^{g\ [n]}(x_a^{[n]}), \forall a \in A, g \in G_w, w \in W$.

Step 2: ***Find direction.*** For each $g \in G_w, w \in W,$ perform a logit assignment based on current disutility $u_a^{g\ [n]}(x_a^{[n]}), \forall a \in A$, and find auxiliary arc flow $d_a^{[n]}, \forall a \in A$.

Step 3: ***Move.*** Compute new arc flow as $x_a^{[n+1]} = x_a^{[n]} + (1/n)(d_a^{[n]} - x_a^{[n]}), \forall a \in A$.

Step 4: ***Convergence check.*** Compute $gap^{[n]} = \sum_{a \in A} \left| d_a^{[n]} - x_a^{[n]} \right| \Big/ \sum_{a \in A} x_a^{[n]}$. If $gap^{[n]} \leq \kappa$, stop; otherwise, n=n+1, κ = 0.001, and go to step 1.

2.5.3 Solution Approach for Program (P2)

To solve program (P2), the Best Response Dynamics-Based Tabu Search procedure proposed by (Sureka and Wurman, 2005) for obtaining a pure strategy Nash equilibrium in normal form games in the context Combinatorial Auctions is adapted.

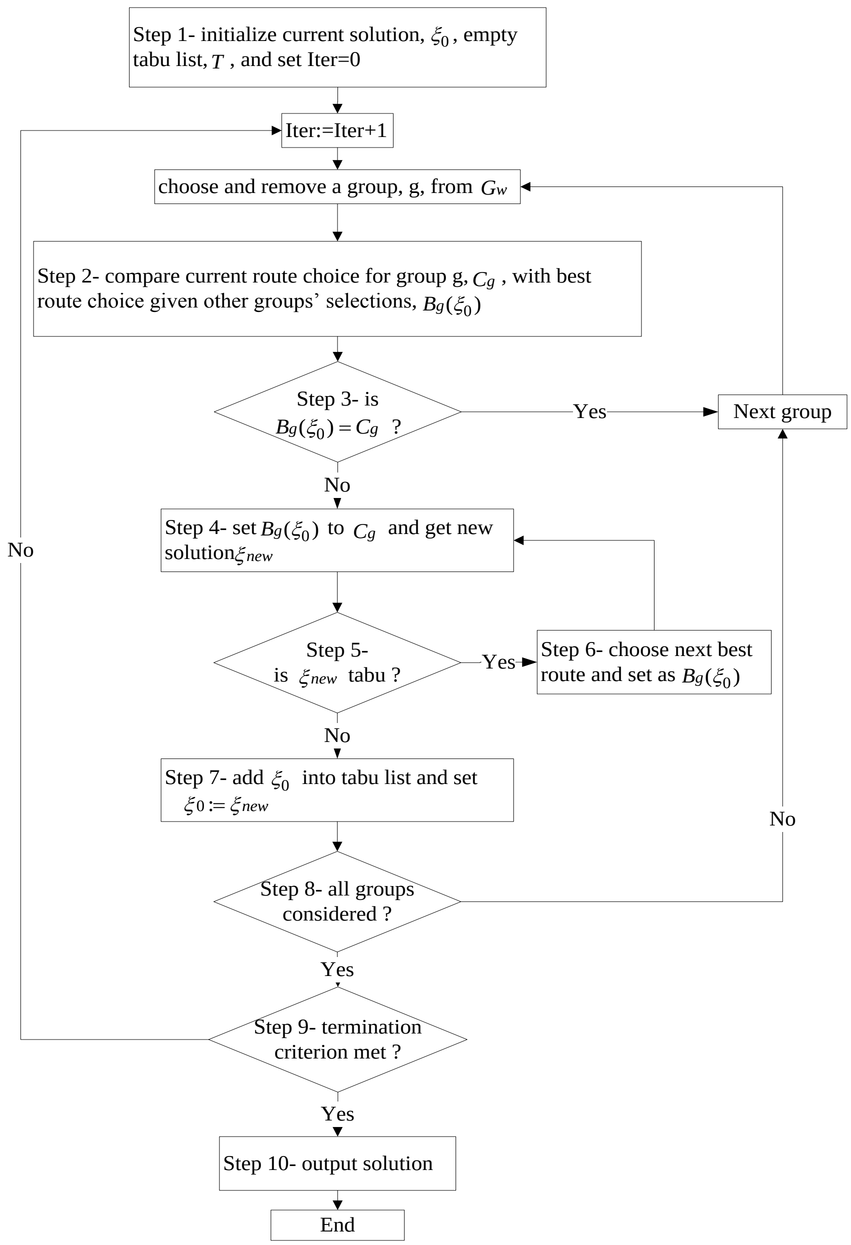


Figure 2-1 Flowchart of TS Algorithm for Program (P2)

In solving problem (P2), the best response (choice) is defined as the route chosen by a group that minimizes total disutility. This differs from the definition of the best response in Sureka and Wurman's approach, where each player exhaustively explores the solution space to find the best response that maximizes each player's payoff. This difference is important, because the use of the total disutility reduces the search space, eliminating the need for an exhaustive search. A flowchart of the proposed method is provided in Figure 2-1, followed by details of important procedural steps.

1) Initialization

Randomly generate starting (current) solution ξ_0, a $G_w \times R_w \times W$ matrix with elements of 0 and 1. According to constraint (2-22) in problem (P2), each row in ξ_0 includes only one 1. All other entries are 0. For example, one possible solution to a specific O-D pair where 4 groups choose 3 routes can be expressed as

$$\xi_0 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} ,$$

representing the selection of route 1 by groups 1 and 2 and route 3 by groups 3 and 4. No group chooses route 2.

Initialize tabu list, *T*, as empty. The tabu list is a list of matrices representing visited solutions. The length of the tabu list, *T*, is a predefined fixed number (nT=10). For each iteration (indicated by *Iter*), all groups explore route options, choosing the best route given the route choices of other groups.

2) Finding the best route

Selection of a best route is made once for each group as follows. Randomly choose group, *g*. Let the route chosen by group *g* in the current solution be C_g. The best choice of group *g*, $B_g(\xi_0)$, under the current solution, ξ_0, can be obtained through exploration of the route choice space of group *g*. For group 1 in the above example, to find $B_1(\xi_0)$, the objective function is evaluated for the following 3 solutions.

$$\xi_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \ \xi_0^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{and } \xi_0^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

If ξ_0^3 is the solution with the minimum total disutility, $B_1(\xi_0) = 3$. After exploration of the route choice space, if group g cannot find a better solution (i.e. $C_g = B_g(\xi_0)$), move to the next group. If it is able to find a better solution (i.e. $C_g \neq B_g(\xi_0)$), replace C_g by $B_g(\xi_0)$ forming new solution ξ_{new}.

3) **Checking tabu**

Check if ξ_{new} is tabu. If yes, choose the route with the next lowest total disutility. If not, add the current solution to the tabu list and set the current solution to the new solution.

4) Termination criteria

If all groups are able to obtain their first choice routes, i.e. $C_g = B_g(\xi_0), \forall g \in G_w, w \in W,$ then terminate; otherwise, begin the next iteration.

While even a locally optimal solution to problem (P2) is not guaranteed, the resulting solution will be an equilibrium solution. That is, no group can unilaterally switch routes to reduce the total disutility of travel. Numerical experience indicates that just a few iterations are required to achieve convergence.

2.6 Computational Experiments

2.6.1 Experimental Design

The efficiencies and differences between flows generated by modeling and solution methodologies designed for separable and clustered group behaviors are investigated on an illustrative example network representation of a facility layout.

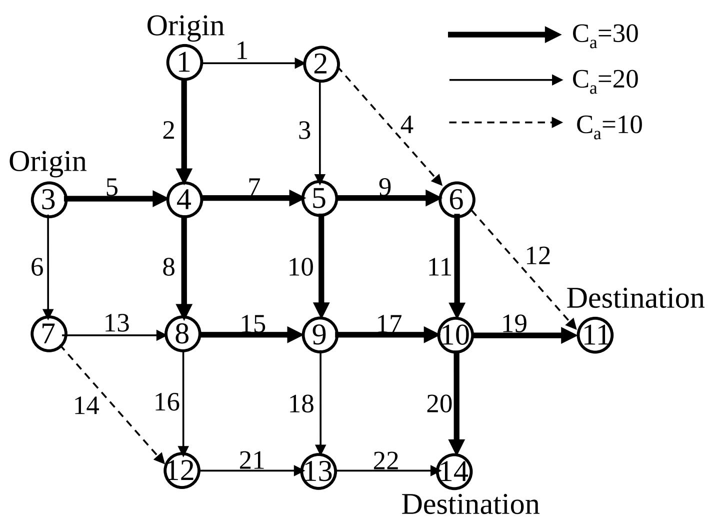


Figure 2-2 Network Configuration

The network consists of 14 nodes, 22 arcs and 4 O-D pairs as portrayed in Figure 2-2. The capacity of each arc is indicated in the network. With the exception of arcs 4, 12 and 14, all arcs are assumed to have a length of 100 meters. Arcs 4, 12 and 14 are 200 meters in length. The free-flow speed is set be 1.42m/s (Thalmann and Musse, 2007) and coefficient k_a = 0.008 for travel time calculations.

To explore the effects of separable and clustered groups on flow distributions over the network, four grouping scenarios listed in Table 2-1 are considered. Scenario 1 is an extreme case of scenario 2, where all pedestrians belong to the same group.

Scenario 3 can be viewed as an extreme case of scenario 4, where each group consists of only one individual.

Table 2-1.Experiment Scenarios

Separable groups	scenario 1	All pedestrians are treated as members of one single large group and all individuals have the same preference parameters.
	scenario 2	All pedestrian can be divided into a finite number of groups composed of one or more individuals; preferences between groups are heterogeneous, but homogeneous within each group.
Clustered groups	scenario 3	Each pedestrian is viewed as a group and each has his/her own preference function.
	scenario 4	All pedestrian can be divided into a finite number of groups composed of one or more individuals; preferences between groups are heterogeneous, but homogeneous within each group; individuals within a group will stay together.

Table 2-2 gives the demand information for each O-D pair. For scenarios 2 and 4, it is assumed that for each O-D pair there are 20 groups each with group size randomly chosen on the interval [1, 40]. The preference parameters for each group in scenario 2 were generated incrementally, while λ^g in scenarios 3 and 4 is randomly generated between 0 and 1 and $\chi^g = 1 - \lambda^g$. Scenarios 1 and 3 have the same total demand (indicated in the O-D column in Table 2-2). In scenario 1, $\alpha^g = 0.053$, $\beta^g = 0.535$, $\gamma^g = 3.475$, and $\theta^g = 1.050$, computed from the average values of similar parameters in scenario 2.

Table 2-2 Demand for Each O-D Pair

O-D	g	S_g	scenario 2				scenario 4		O-D	g	S_g	scenario 2				scenario 4	
			α^g	β^g	γ^g	θ^g	λ^g	χ^g				α^g	β^g	γ^g	θ^g	λ^g	χ^g
1 to 11 (400)	1	20	0.005	0.05	3.00	0.01	0.5	0.5	1 to 14 (350)	1	7	0.005	0.05	3.00	0.01	0.4	0.6
	2	23	0.010	0.10	3.05	0.02	0.7	0.3		2	27	0.010	0.10	3.05	0.02	0.3	0.7
	3	29	0.015	0.15	3.10	0.03	0.3	0.7		3	16	0.015	0.15	3.10	0.03	0.6	0.4
	4	23	0.020	0.20	3.15	0.04	0.2	0.8		4	19	0.020	0.20	3.15	0.04	0.6	0.4
	5	19	0.025	0.25	3.20	0.05	0.4	0.6		5	22	0.025	0.25	3.20	0.05	0.3	0.7
	6	25	0.030	0.30	3.25	0.06	0.6	0.4		6	27	0.030	0.30	3.25	0.06	0.1	0.9
	7	21	0.035	0.35	3.30	0.07	0.5	0.5		7	28	0.035	0.35	3.30	0.07	0.6	0.4
	8	40	0.040	0.40	3.35	0.08	0.4	0.6		8	30	0.040	0.40	3.35	0.08	0.1	0.9

	9	15	0.045	0.45	3.40	0.09	0.7	0.3		9	21	0.045	0.45	3.40	0.09	0.7	0.3
	10	1	0.050	0.50	3.45	0.10	0.7	0.3		10	16	0.050	0.50	3.45	0.10	0.2	0.8
	11	14	0.055	0.55	3.50	0.11	0.3	0.7		11	16	0.055	0.55	3.50	0.11	0.8	0.2
	12	30	0.060	0.60	3.55	0.12	0.3	0.7		12	1	0.060	0.60	3.55	0.12	0.1	0.9
	13	12	0.065	0.65	3.60	0.13	0.6	0.4		13	14	0.065	0.65	3.60	0.13	0.5	0.5
	14	30	0.070	0.70	3.65	0.14	0.7	0.3		14	18	0.070	0.70	3.65	0.14	0.3	0.7
	15	4	0.075	0.75	3.70	0.15	0.5	0.5		15	10	0.075	0.75	3.70	0.15	0.4	0.6
	16	18	0.080	0.80	3.75	0.16	0.8	0.2		16	11	0.080	0.80	3.75	0.16	0.2	0.8
	17	22	0.085	0.85	3.80	0.17	0.2	0.8		17	18	0.085	0.85	3.80	0.17	0.8	0.2
	18	14	0.090	0.90	3.85	0.18	0.6	0.4		18	1	0.090	0.90	3.85	0.18	0.2	0.8
	19	24	0.095	0.95	3.90	0.19	0.2	0.8		19	18	0.095	0.95	3.90	0.19	0.1	0.9
	20	16	0.100	1.00	3.95	0.20	0.3	0.7		20	30	0.100	1.00	3.95	0.20	0.5	0.5
3 to 11 (350)	1	18	0.005	0.05	3.00	0.01	0.8	0.2	3 to 14 (300)	1	10	0.005	0.05	3.00	0.01	0.2	0.8
	2	29	0.010	0.10	3.05	0.02	0.8	0.2		2	13	0.010	0.10	3.05	0.02	0.7	0.3
	3	29	0.015	0.15	3.10	0.03	0.8	0.2		3	16	0.015	0.15	3.10	0.03	0.1	0.9
	4	15	0.020	0.20	3.15	0.04	0.2	0.8		4	13	0.020	0.20	3.15	0.04	0.7	0.3
	5	27	0.025	0.25	3.20	0.05	0.4	0.6		5	19	0.025	0.25	3.20	0.05	0.8	0.2
	6	20	0.030	0.30	3.25	0.06	0.4	0.6		6	18	0.030	0.30	3.25	0.06	0.5	0.5
	7	17	0.035	0.35	3.30	0.07	0.3	0.7		7	29	0.035	0.35	3.30	0.07	0.5	0.5
	8	20	0.040	0.40	3.35	0.08	0.1	0.9		8	24	0.040	0.40	3.35	0.08	0.0	1.0
	9	19	0.045	0.45	3.40	0.09	0.6	0.4		9	28	0.045	0.45	3.40	0.09	0.4	0.6
	10	6	0.050	0.50	3.45	0.10	0.2	0.8		10	6	0.050	0.50	3.45	0.10	0.3	0.7
	11	19	0.055	0.55	3.50	0.11	0.6	0.4		11	12	0.055	0.55	3.50	0.11	0.7	0.3
	12	27	0.060	0.60	3.55	0.12	0.5	0.5		12	17	0.060	0.60	3.55	0.12	0.4	0.6
	13	8	0.065	0.65	3.60	0.13	0.2	0.8		13	15	0.065	0.65	3.60	0.13	0.8	0.2
	14	1	0.070	0.70	3.65	0.14	0.3	0.7		14	9	0.070	0.70	3.65	0.14	0.3	0.7
	15	20	0.075	0.75	3.70	0.15	0.6	0.4		15	20	0.075	0.75	3.70	0.15	0.8	0.2
	16	22	0.080	0.80	3.75	0.16	0.2	0.8		16	28	0.080	0.80	3.75	0.16	0.3	0.7
	17	21	0.085	0.85	3.80	0.17	0.8	0.2		17	8	0.085	0.85	3.80	0.17	0.8	0.2
	18	7	0.090	0.90	3.85	0.18	0.2	0.8		18	1	0.090	0.90	3.85	0.18	0.7	0.3
	19	23	0.095	0.95	3.90	0.19	0.3	0.7		19	8	0.095	0.95	3.90	0.19	0.3	0.7
	20	2	0.100	1.00	3.95	0.20	0.1	0.9		20	6	0.100	1.00	3.95	0.20	0.2	0.8

Logit-based SUE assignment is employed for obtaining solutions under scenarios 1 and 2, while the Best Response Dynamics-Based Tabu Search procedure is used to address the n-player non-cooperative games of scenarios 3 and 4. Results are discussed in the next section.

2.6.2 Results and Analysis

Table 2-3 gives the efficient routes for each O-D pair.

Table 2-3 Efficient Routes Set for Each O-D Pair

O-D	Index	Route	O-D	Index	Route
1-11	1	1→2→5→6→10→11	1-14	1	1→2→5→6→10→14
	2	1→2→5→6→11		2	1→2→5→9→10→14
	3	1→2→5→9→10→11		3	1→2→5→9→13→14
	4	1→2→6→10→11		4	1→2→6→10→14
	5	1→2→6→11		5	1→4→5→6→10→14

	6	1→4→5→6→10→11		6	1→4→5→9→10→14
	7	1→4→5→6→11		7	1→4→5→9→13→14
	8	1→4→5→9→10→11		8	1→4→8→9→10→14
	9	1→4→8→9→10→11		9	1→4→8→9→13→14
				10	1→4→8→12→13→14
3-11	1	3→4→5→6→10→11	3-14	1	3→4→5→6→10→14
	2	3→4→5→6→11		2	3→4→5→9→10→14
	3	3→4→5→9→10→11		3	3→4→5→9→13→14
	4	3→4→8→9→10→11		4	3→4→8→9→10→14
	5	3→7→8→9→10→11		5	3→4→8→9→13→14
				6	3→4→8→12→13→14
				7	3→7→8→9→10→14
				8	3→7→8→9→13→14
				9	3→7→8→12→13→14
				10	3→7→12→13→14

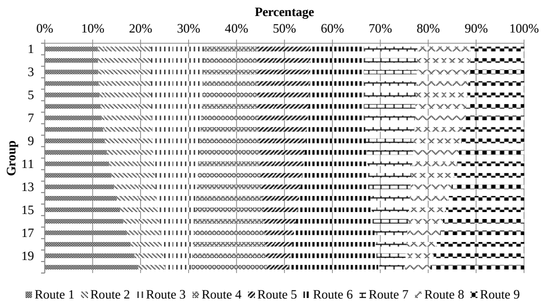
Table 2-4 shows the arc flows by scenario. Similar arc flow results for separable single- (scenario 1) and separable variable-size groups (scenario 2). This is because scenario 1 relies on parameters taken from the average of parameter values assigned in scenario 2 - each pedestrian in scenario 1 will have identical parameter values. Note that the total travel time under the latter scenario (2) is slightly lower than that under the former scenario (1). This is because pedestrians in scenario 1 assign the same utility to every path. Thus, the lowest utility paths will be highly sought after and, therefore, highly congested. The greater variability in parameter settings of scenario 2 cause the pedestrians to disperse over a larger number of routes, reducing total travel time. A greater difference between arc flows exists between single-pedestrian groups (scenario 3) and clustered variable-size groups (scenario 4). The total travel time under scenario 3 is much lower than that under scenario 4. The reason is that individuals in scenario 3 have greater flexibility compared with those in scenario 4.

Table 2-4 Flows for Scenarios

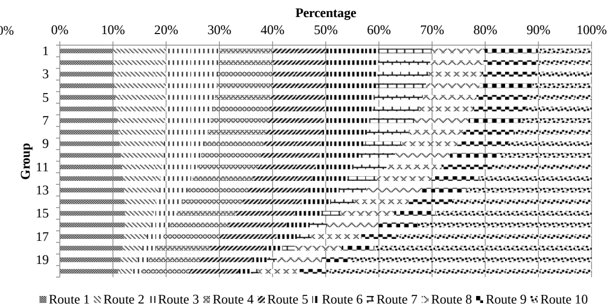
	scenario 1				scenario 2				scenario 3				scenario 4			
	Arc	x_a	Arc	x_a	Arc	x_a	Arc	x_a	Arc	x_a	Arc	x_a	Arc	x_a	Arc	x_a
	1	342	12	165	1	341	12	165	1	340	12	164	1	357	12	187
	2	408	13	192	2	409	13	191	2	410	13	197	2	393	13	179
	3	217	14	81	3	217	14	81	3	219	14	84	3	243	14	84
	4	124	15	381	4	124	15	380	4	121	15	387	4	114	15	348
	5	378	16	148	5	378	16	148	5	369	16	154	5	387	16	152
	6	272	17	502	6	272	17	501	6	281	17	498	6	263	17	494
	7	449	18	134	7	449	18	134	7	435	18	132	7	459	18	120
	8	337	19	585	8	337	19	585	8	344	19	586	8	321	19	563
	9	411	20	287	9	411	20	287	9	411	20	280	9	436	20	294
	10	255	21	228	10	255	21	229	10	243	21	238	10	266	21	236
	11	370	22	363	11	370	22	363	11	368	22	370	11	363	22	356
TT	1,268,507				1,268,050				1,271,253				1,275,991			

$$*TT = \sum_{a \in A} x_a \cdot t_a(x_a)$$

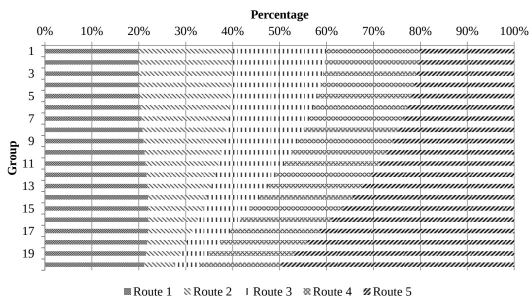
Figure 2-3 (a)-(d) shows the distribution of flows by group over route alternatives between each O-D pair for the scenario involving separable variable-size groups (scenario 2). Consider for example Fig. 2 (d). 10 efficient routes exist for O-D pair 3-14. Of individuals in group 20, approximately 70% chose Route 10, while only 10% of group 1 chose a common route. For group 1, chosen routes are evenly distributed over all efficient options. This differs from group 20 in which the majority of individuals chose the same route and other routes are chosen by very few individuals. This can be attributed to differences in group preference function parameters, i.e. individual sensitivity to route attributes. Group 1's parameters are all very small. Thus, route choice is almost random, since individuals are not very sensitive to route attributes. Parameter settings for group 20 are more significant, which is also reflected in the route decisions. Also contributing to these differences in route choice between groups 1 and 20 is that the value of θ for group 1, indicating the level of discrepancy between actual and perceived utility, is smaller than for group 20. The smaller the value of θ, the larger the difference between perceived and measured disutilities. Similar patterns in flow distribution over routes can be observed for other O-D pairs.



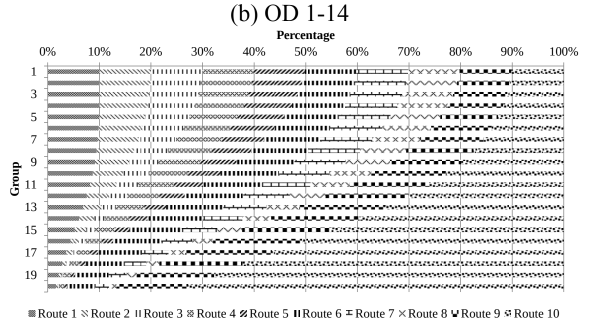
(a) OD 1-11



(b) OD 1-14



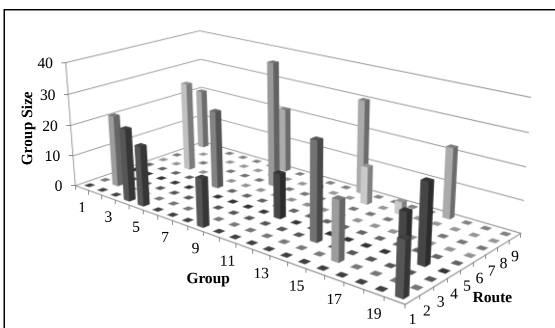
(c) OD 3-11



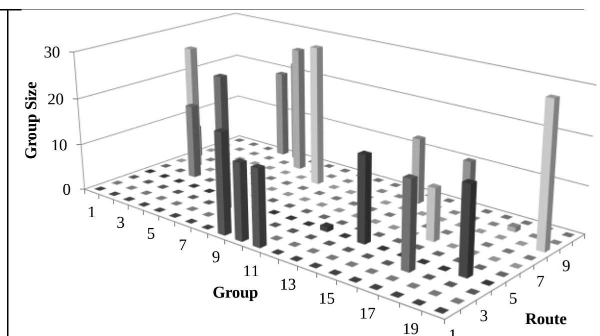
(d) OD 3-14

Figure 2-3 Distribution of Groups over Routes by O-D Pair for Scenario 2

The distribution of flows for clustered variable-size groups (scenario 4) is depicted in Figure 2-4. Although the same group size is used in scenario 4 as in scenario 2, each group in Figure 2-4 selects only one route and there is no group that can decrease its total incurred disutility by unilaterally switching routes.



(a) OD 1-11



(b) OD 1-14

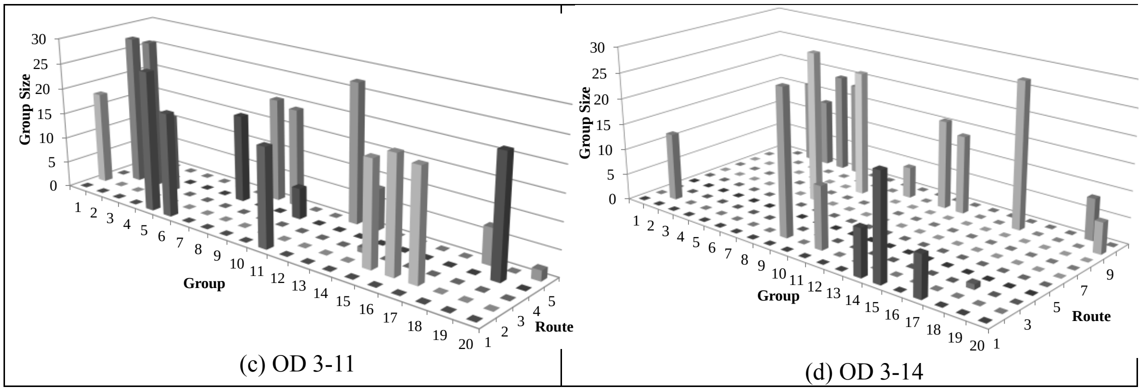


Figure 2-4 Distribution of Groups over Routes by O-D Pair for Scenario 4

2.7 Conclusions and Extensions

In this chapter, pedestrian route choice is modeled using a traffic assignment type of framework. Methods for estimating the distribution of groups and individuals over "efficient" routes for two types of groups, separable and clustered, are proposed. These methods employ formulations using logit-based SUE assignment and a pure-strategy Nash equilibrium game for separable and clustered groups, respectively. Solution methodologies for solving problems so formulated involves an MSA with groups procedure (for solution to the SUE assignment of separable groups) and a meta-heuristic scheme based on best response dynamic and tabu search (to find the pure-strategy Nash equilibrium of the game formulated for clustered groups). The conceptual framework, and specific models and corresponding solution schemes were tested on an illustrative example. The results from the experiments show the effectiveness and efficiency of the proposed approaches.

There are a number of directions in which the proposed models and solution approaches might be extended. For example, in this chapter, the parameters are assumed to be homogeneous within a group. In reality, however, the parameters associated with each group might follow a distribution over individuals. This heterogeneity within each group can be further explored with the proposed models and solution schemes. Furthermore, in this chapter, pedestrians make decisions based on route-based performance and once a route is selected, it is assumed that each pedestrian will follow the route in its entirety. The developed model and solution methodology might be extended to address a dynamic pedestrian assignment problem, where physical changes in the network and user goals affect the optimality and choice of routes.

Chapter 3 Crowd Management in Large Public Gatherings

3.1 Introduction

Effective management of pedestrian movement during large public gatherings can provide crucial support toward meeting pedestrian access and safety goals. As stated in Chapter 2, large public gatherings are held in a variety of venues. Poor execution of crowd management within these venues can frustrate the people in a crowd by thwarting their goals. At the extreme, poor crowd management has caused many instances of crowd crush, injuries and fatalities involving high volumes of people in a wide array of circumstances, ranging from rock concerts and sales events at stores to the offering of free food and clothing. A few specific examples where better crowd management may have saved lives include: the 1979 Who concert in Ohio in which 11 people perished, the 1989 U.K. Hillsborough Stadium sporting event where 96 deaths may have been prevented, 362 deaths resulting in the 2006 Hajj in Saudi Arabia, and the 2010 incident in Northern India where 63 people perished while seeking free food and clothing at a temple. In addition, in some circumstances, such as in the event of fire, explosion, occurrence of natural or human-induced disaster event, or crowd violence, well-designed systems for moving large crowds quickly are needed to support quick egress from dangerous situations.

The majority of works related to crowd management propose methods for modeling crowd movements during emergency evacuation. Such models can be used to quantify the performance, in terms of measures like evacuation time, of a given facility's architectural layout during such an event. These models can be broadly categorized as: fluid dynamics-based approaches (Colombo and Rosini, 2005; Hughes, 2002), optimization and network flow-based methods (Choi et al., 1988; Fahy, 1994), and simulation-based techniques, which include rule-based methods (Blue and Adler, 2001; Helbing, 1995), agent-based modeling (Shi et al., 2009) and virtual reality (Shih et al., 2000). Additional information can be found in (Gwynne et al., 1999; Kuligowski and Peacock, 2005; Zheng et al., 2009) Other works, including for example (Hoogendoorn and Bovy, 2004b), focus on simulation of pedestrian movement under non-emergency situations. Whether created to support analysis in emergency or non-emergency situations, techniques described in these works are designed for use in evaluation of, for example, architectural designs and other elements of the physical layout. They do not provide strategies for managing the crowd.

Techniques have been proposed to support crowd management. In the context of pedestrian movement, these techniques determine optimal routes to which pedestrians should be guided within an existing physical environment. Route guidance is created through network optimization-based methods. Simplistic, static methodologies based on minimum cost network flows have been developed, e.g. (Yamada, 1996). More sophisticated techniques that capture problem dynamics, time-dependencies and other problem characteristics have been proposed specifically for building evacuation (Cai et al., 2001; Mamada et al., 2003). A variety of objectives have been considered, including for example maximizing throughput by a given end time (Miller-Hooks and Sorrel, 2008) and maximizing the minimum probability of arrival at an exit for any evacuee (Opasanon and Miller-Hooks, 2008). Other works have considered the role of real-time information in updating routing instructions

(Miller-Hooks and Krauthammer, 2007). Chen and Miller-Hooks (2008) developed a dynamic network flow-based model that forces instructions to reflect how shared information will be used. A review of optimization techniques proposed for use in building and regional evacuation is provided in (Hamacher and Tjandra, 2002). Relevant network optimization-based techniques developed for regional evacuation are described in (Kimms and Bretschneider, 2011). Unlike the simulation and fluid dynamics-based methods that are used in modeling pedestrian movement, optimization-based techniques provide strategies for pushing flow through the network to achieve system optimal performance.

Related techniques have been proposed for use in guiding vehicular traffic in both emergency and non-emergency circumstances. See, for example, (Kesting et al., 2008; Liu et al., 2007). Dynamic traffic management approaches, such as ramp metering, adaptive speed limits, and provision of real-time information, are widely used to support efficient vehicular traffic movement during peak traffic flow. These strategies are also used in emergency evacuation scenarios. Although tools developed for vehicular evacuation have relevance, there are significant distinctions in behavior and degrees of freedom between vehicular and pedestrian modes that make direct application of traffic tools insufficient for use in the pedestrian environment.

Approaches discussed thus far focus on influencing the movement of pedestrians through a given physical layout. An alternative might be to reconfigure the physical layout to facilitate pedestrian movement in pursuit of a particular goal. Such reconfiguration can both limit pedestrian choice and enhance or restrict capacity along routes to facilitate efficient movement and prevent crowd crush or other unsafe situations. Changes to the physical layout might be achieved through opening or closing gates/doorways, placing or removing barriers or changing illumination intensity to coerce pedestrians along certain paths. No prior work has suggested such an approach in the context of pedestrian movement; however, reconfiguring methodologies, such as the use of contraflow, have been proposed for evacuation by automobile. See (Abdelgawad and Abdulhai, 2009) for a review.

In this chapter, a network optimization-based methodology that seeks the optimal reconfiguration of a physical (architectural) layout to support efficient crowd movement during large events is proposed. This methodology takes into consideration pedestrian response to route offerings as controlled through the architectural design. Further, it incorporates findings from the social sciences and psychological studies on grouping behavior in crowds (Aveni, 1977; Qiu and Hu, 2010). That is, the methodology recognizes that families, friends and emergent groups will act together, and control strategies that separate such groups will be ineffective. This approach seeks a system optimal solution based the crowd manager's goals; however, it explicitly recognizes the utility maximizing behavior of individuals in the crowd as is consistent with user equilibrium. In contrast to prior fluid dynamics-based techniques that model aggregate pedestrian flows, often requiring extraordinary computational effort to solve embedded differential equations, the proposed approach captures individual movements and goals with significantly reduced computational time. Alternative simulation-based methodologies offer an ability to replicate complex behaviors, but do not provide guidance; rather, they support performance assessment given chosen guidance mechanisms. The proposed technique builds on concepts of network optimization, but accounts for behavioral norms often only included in computationally expensive simulation-based approaches.

A bi-level integer program is presented that, at the upper-level, seeks a reconfiguration of the physical design that will minimize total travel time incurred by system users (e.g. evacuees) given route decisions that are taken by individuals in response to physical offerings in terms of the infrastructure at the lower-level. The lower-level formulation seeks a pure-strategy Nash equilibrium that respects grouping behavior. The general overview and mathematical program is presented in detail in section 2. In Section 3, the bi-level program is reformulated as a nonlinear integer single-level program for which determination of a globally optimal solution is formidable. Thus, a Multi-start Tabu Search with Sequential Quadratic Programming (MTS-SQP) procedure is proposed for its solution. This procedure is described in this section. Numerical experiments were conducted on a hypothetical example to assess this technique. Results of these experiments are given in Section 4. Conclusions and directions for future work are discussed in Section 5.

3.2 Problem Overview

The general structure of the proposed bi-level program (Stackelberg Leader-Follower program) for the problem of reconfiguring physical layout to support efficient crowd movement, referred to herein as the Reconfigure for Efficient Crowd Movement (RECM) Problem, is depicted in Figure 3-1.

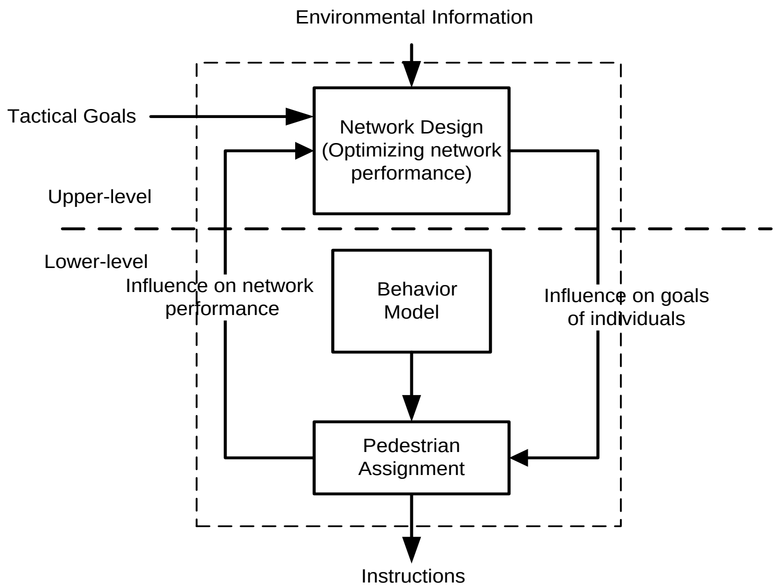


Figure 3-1 Overview of the RECM Problem

The upper-level describes a network design problem whose decision variables represent actions in terms of system reconfiguration that the leader (i.e. crowd manager) might take to optimize network performance (e.g. minimizing total travel time or maximizing throughput). The lower-level is a pure-strategy Nash equilibrium pedestrian assignment problem in which the followers (i.e. pedestrians in the crowd) are presumed to follow paths that minimize disutility in terms of related path characteristics. Solution at the upper-level provides optimal measures for changing configuration of the network through, for example, opening or closing doorways/gates, changing the capacity of passageways through use of barriers, closing or opening new passageways, changing illumination to accentuate a route, and removing interactions between persons in the crowd through implementation of lanes from the upper-level. Given the network configuration determined in the upper-level, solution at the lower-level predicts the flow along the passageways assuming that pedestrians will choose their paths to minimize disutility. Predictions of network flows from the lower-level provide input to the upper-level problem, creating interaction between levels.

This bi-level approach permits the modeling of objectives of both the crowd manager and pedestrians in the crowd. However, the bi-level structure gives priority to the upper-level objective, thus, providing suitable designs from the crowd manager's perspective while simultaneously recognizing that the individuals in the crowd will exploit the configuration so as to achieve their own selfish objectives (goals). Prioritization is given to the objective of the crowd manager to encourage system efficient designs. The route choice behaviors that follow the goals are described mathematically in the behavior model component.

Details of the bi-level formulation of the RECM Problem are provided next.

3.3 The Upper-Level Problem

Consider a network representation of the physical environment, $\Gamma = (N, A)$, where N is the set of nodes, representing locations at which decisions must be taken in regard to movement and A is the set of directed arcs connecting the nodes representing passageways along which movement is possible. Let $O, D \in N$ be the set of origins and destinations, respectively. Each arc $a \in A$ has an associated length, l_a, initial capacity, c_a, arc flow, x_a, potential change in capacity, y_a, and nonnegative travel time, $t_a(x_a, c_a + y_a)$. As discussed in (Schomborg et al., 2011) in the context of macroscopic modeling of pedestrian and vehicular traffic, a similar structure for the velocity-density fundamental diagram for each can be utilized; only the parameter values will differ. For a fixed value of $c_a + y_a$, a BPR-based travel time function (Branston, 1976) with assumed parameters is adopted:

$$t_a(x_a, c_a + y_a) = t_a^0 \cdot [1 + k_a (\frac{x_a}{c_a + y_a})^e] \qquad \forall a \in A \tag{3-1}$$

where k_a is a coefficient scaling the rate with which congestion increases travel time, e is a parameter. k_a and e would require calibration using data from actual observations, and t_a^0 denotes free-flow travel time along link *a*. For free-flow walking speed, v_a, t_a^0 can be calculated as:

$$t_a^0 = l_a / v_a \qquad \forall a \in A. \tag{3-2}$$

This approach supports the use of alternative equations that capture the relationship between travel time and density.

Let $\mathbf{x} = (x_1, x_2 \ldots x_a \ldots x_{|A|})$ be the vector of link flows and $\mathbf{y} = (y_1, y_2 \ldots y_a \ldots y_{|A|})$ be the change in capacity vector. Capacity expansion for a link is limited by physical barriers. For each link, $a \in A$, c_a^{up} denotes the upper-limit of capacity on link *a*. A non-negative per unit cost, b_a, is imposed for any change made to capacity of link *a*. This unit cost may reflect, for example, resources required to open or close the link, or may be the monetary cost of providing additional capacity. A budget, B, is imposed to limit such effort or monetary spending. The upper-level problem is formulated with this notation as follows.

(U)

$$\min \; Z(\mathbf{x}, \mathbf{y}) = \sum_{a \in A} x_a \cdot \; t_a(x_a, c_a + y_a) \tag{3-3}$$

s.t.

$$\sum_{a \in A} b_a \cdot |y_a| \le B \tag{3-4}$$

$$\sum_{a \in A} y_a = 0 \tag{3-5}$$

$$0 \le c_a + y_a \le c_a^{up},\ \forall a \in A \tag{3-6}$$

Objective function (3-3) seeks an optimal vector **y** that minimizes the total travel time required to ship a given flow **x** over the network. **y** obtained from the upper-level problem is employed in setting **x** at the lower-level. Constraint (3-4) ensures that incurred costs required for the chosen changes in arc capacities do not exceed the budget. The absolute value of y_a is used, because y_a can take positive or negative values. The budget, *B*, is set sufficiently large to accommodate total changes. The total available space is forced to remain fixed through Constraint (3-5). When a capacity increase is warranted in one section of the layout, a decrease in capacity elsewhere is required, since space is fixed. This constraint can be omitted in circumstances in which space is essentially unlimited. Constraints (3-6) guarantee that link capacities remain within their lower and upper limits.

3.4 The Lower-Level Problem

For a given upper-level design, expressed in terms of design vector **y**, the lower-level is a traffic (pedestrian) assignment problem seeking the vector of link flows **x** that minimizes disutility for all pedestrians. The disutility of each route to each user depends on user preference characteristics and the performance attributes on each route. The performance on each route further depends on the number of pedestrians who choose each passageway. That is, when many pedestrians use a particular passageway, travel time along the passageway will increase, rendering it less desirable. Additionally, many pedestrians travel in groups and, thus, will seek the same route for their groups. The pedestrian assignment problem is modeled as a pure-strategy Nash equilibrium assignment problem. The use of the pure-strategy approach permits the modeling of this critical grouping behavior.

3.4.1 Route Choice and Group Behavior

The process of selecting a route involves choosing an option from a finite set of alternative routes with the desired origin and destination. The selection of a route by a pedestrian is sometimes referred to as wayfinding (e.g. (Bovy and Stern, 1990)). A number of works consider route choice behavior (or wayfinding) in the context of crowds (Bierlaire and Robin, 2009; Løvås, 1998). A small portion of these works (Antonini et al., 2006; Hoogendoorn and Bovy, 2004b) apply utility maximization theory for the purpose of forecasting route decisions. This approach is widely used to model route choice for vehicular traffic. A review is provided in (Bovy and Stern, 1990). The basic assumption underlying these choice models is that a traveler's preference for each potential alternative can be described by a mathematical function of the route's utility (or disutility). The utility of a path in a pedestrian network is derived from attributes of distance, time, required physical effort, safety, and physical appeal, among others. The preference function on those attributes is indivualized. The preference function is formulated to capture the relative importance of each attribute for the user. Pedestrian sensitivities to such attributes are discussed in (Daamen et al., 2005; Seneviratne and Morrall, 1985). These works suggest that walking distance and time are the most important route attributes in route choice.

Some attributes, such as travel time, depend on the number of users. In general, the greater the number of users choosing a route, the greater its travel time. Thus, route choice models are often embedded within a traffic assignment model that seeks an assignment of vehicles to the network based on congestion-dependent route utilities so as to achieve a user equilibrium. An equilibrium is reached when no user can improve his/her performance in terms of route utilities by unilaterally switching routes. The majority of traffic assignment models in the literature seek such user equilibrium (UE) solutions. A deterministic user equilibrium (DUE) model presumes perfect knowledge of the performance of all alternative routes and all users perceive route performance in an identical manner. To provide greater realism, stochastic user equilibrium (SUE) models have been suggested in which each user is presumed to have only probabilistic information about the route choices and each has his/her own utility function regarding route performance (Sheffi, 1985).

Users in UE approaches (DUE or SUE) are treated either continuously or as individuals. No mechanism exists to support group behavior (e.g. desire by a family, group of friends/colleagues or emergent groups to travel en masse). Such group behavior, however, is common and can have significant impact on crowd movement. Even if each member of a group has the same utility function within the employed route choice methodology, there is no guarantee that members of the group will be assigned to the same path.

The problem of predicting route choice given the impact of user interactions on link performance can be treated as an n-player non-cooperative game in which players selfishly choose strategies from their own strategy sets (Haurie and Marcotte, 1985). The payoff for each player depends on his/her chosen strategy, as well as on the strategies chosen by others. The solution of such a game in which there is a finite number of players will result in a mixed-strategy Nash equilibrium. In the context of traffic assignment, travelers correspond to the players in the game. The strategy set is composed of the available potential routes from origin to destination. Payoff is gained through quality route performance.

A mixed-strategy Nash equilibrium presumes that decisions taken by each player in the n-player game have identical impact on strategy performance. Such an approach, therefore, cannot account for the impact of group movements. Thus, an n-player, pure-strategy Nash equilibrium game (Rosenthal, 1973a, b) is proposed herein that can capture the impact of group behavior. When a pure-strategy Nash equilibrium is achieved, each player, representing a group composed of one or more pedestrians, cannot benefit from unilaterally switching strategies (or routes).

In applying the concept of pure-strategy Nash equilibrium in this context of crowd management, a number of assumptions are required: (1) the crowd consists of a finite number of groups, the members of which will travel together; (2) preference functions may be heterogeneous across groups, but are homogeneous among members of the same group; (3) groups behave rationally, choosing a route that minimizes disutility for the group; (4) all groups make their route choice decisions simultaneously (Bierlaire and Robin, 2009) and the ultimate choice depends on the choice of competing groups; and (5) link disutility is additive.

3.4.2 Formulation

For an O-D pair, $w \in W$, W the set of O-D pairs, there are $G_w(g = 1, ..., |G_w|)$ groups of pedestrians and $R_w(r = 1, ..., |R_w|)$ routes. Let S_w^g denote the size of group, $g \in G_w$, which can be as small as one. For each w, the disutility of each route r for group g can be expressed as:

$$u_w^{g,r}(S_w^g, f_w^r) = S_w^g \cdot [\alpha_g L_w^r + \beta_g T_w^r(f_w^r)] \quad \forall g \in G_w, r \in R_w, w \in W, \tag{3-7}$$

where $u_w^{g,r}(\cdot)$ represents the disutility of route r for group g with O-D pair w. The disutility of route r between O-D pair w is a function of group size, S_w^g, the corresponding route distance, L_w^r, and walking time, T_w^r. Walking time, T_w^r, is a function of the flow on route r, f_w^r. α_g and β_g are parameters indicating group g's sensitivity to walking distance and time, respectively.

Let lower-level decision variable $\xi_w^{g,r}$ equal 1 if group g chooses route r for O-D pair w, and 0 otherwise. Flow along route r for O-D pair w, f_w^r, is computed from the sum of group sizes of groups that choose the route:

$$f_w^r = \sum_{g \in G_w} S_w^g \cdot \xi_w^{g,r} \quad \forall r \in R_w, w \in W. \tag{3-8}$$

From the incidence relationship of links and routes, walking distance and walking time on route r between pair w can be further written as in Equations (3-9) and (3-10), respectively.

$$L_w^r = \sum_{a \in A} l_a \cdot \delta_w^{r,a} \qquad \forall r \in R_w, w \in W \tag{3-9}$$

$$T_w^r(f_w^r) = \sum_{a \in A} t_a(x_a, c_a + y_a) \cdot \delta_w^{r,a} \qquad \forall r \in R_w, w \in W, \tag{3-10}$$

where $\delta_w^{r,a}$ equals 1 if route r passes through link a, and 0 otherwise. Flow on link a, x_a, is given as:

$$x_a = \sum_{w \in W} \sum_{r \in R_w} f_w^r \cdot \delta_w^{r,a} \qquad \forall a \in A. \tag{3-11}$$

From Equations (3-9) and (3-10), for each $g \in G_w, r \in R_w, w \in W$, Equation (3-7) can be written as:

$$\begin{aligned} u_w^{g,r}(S_w^g, f_w^r) &= S_w^g \cdot [\alpha_g \sum_{a \in A} l_a \delta_w^{r,a} + \beta_g \sum_{a \in A} t_a(x_a, c_a + y_a) \cdot \delta_w^{r,a}] \\ &= \sum_{a \in A} S_w^g \cdot [\alpha_g l_a + \beta_g t_a(x_a, c_a + y_a)] \cdot \delta_w^{r,a} \\ &= \sum_{a \in A} u_w^{g,a}(l_a, x_a, c_a + y_a) \end{aligned} \quad (3\text{-}12)$$

where $u_w^{g,a}(l_a, x_a, c_a + y_a)$ measures the disutility incurred by group g using link a.

The lower-level problem can, thus, be formulated as binary, nonlinear, integer program (L):

$$\textbf{(L)} \quad \min \sum_{w \in W} \sum_{r \in R_w} \sum_{g \in G_w} \sum_{a \in A} S_w^g \cdot [\alpha_g l_a + \beta_g t_a(x_a, c_a + y_a)] \cdot \delta_w^{r,a} \quad (3\text{-}13)$$

$$s.t. \quad x_a = \sum_{w \in W} \sum_{r \in R_w} \sum_{g \in G_w} S_w^g \cdot \xi_w^{g,r} \cdot \delta_w^{r,a} \qquad \forall a \in A \quad (3\text{-}14)$$

$$\sum_{r \in R_w} \xi_w^{g,r} = 1 \qquad \forall g \in G_w, w \in W \quad (3\text{-}15)$$

$$\xi_w^{g,r} = 0 \text{ or } 1 \qquad \forall g \in G_w, r \in R_w, w \in W \quad (3\text{-}16)$$

Objective function (3-13) seeks the set of path flows over all O-D pairs with the minimum total disutility (weighted by group size). Derived from Equations (3-8) and (3-11), constraints (3-14) relate link flows to path flows, thus, ensuring flow conservation. Constraints (3-15) force each group to choose one route. Binary restrictions are guaranteed through constraints (3-16).

The optimal solution to (L) is a pure-strategy Nash equilibrium attaining the smallest total disutility, proof of which is provided in subsection 2.4.3. Note that there might be several pure-strategy Nash equilibria for the game. Problem (L) seeks the one with the smallest total disutility.

3.5 Single-Level Reformulation

Similar bi-level modeling approaches have been employed in vehicular transport network design applications. Chiou (2005) developed a gradient-based methodology to obtain the Karush-Kuhn-Tucker (KKT) points required for converting the bi-level program to a single mixed integer programming (MIP). Gao et al.(2005) employed a generalized Benders decomposition method for a similar problem formulation. A similar bi-level mathematical model is used to make decisions related to increasing or decreasing link capacities in (Karoonsoontawong and Waller, 2006). Capacity change decisions are fed to a simulation model designed to capture traffic dynamics. Comparison between solutions obtained by MIP reformulation with heuristic approaches is made. While there are similarities between these models and the RECM model, these existing solution methodologies cannot be directly applied, in part because determination of the KKT conditions associated with (L) are difficult to derive due in part to the inclusion of binary decision variables, which are needed for the determination of link flows. Thus, an alternative solution methodology is proposed herein.

In the RECM problem, a Stackelberg game is played between the leader (crowd manager in (U)) and follower (pedestrians in the crowd in (L)). In essence, the game is played out in such a way that the leader chooses a solution for (U) that minimizes his/her objective function given that the followers, after observing the leader's actions, will respond rationally and selfishly. Direct solution of this bilevel optimization problem is difficult. However, the RECM problem can be reduced to a single-level program in which the lower-level program (L) is incorporated within the constraints of (U). This approach of converting a bilevel program to a single-level program in this way is described in (Bard, 1998). This single-level form of the RECM problem is given by program (SL):

$$(\mathbf{SL}) \qquad \min \quad Z(\mathbf{x},\mathbf{y}) = \sum_{a \in A} x_a \cdot t_a(x_a, c_a + y_a) \tag{3-17}$$

$$s.t. \qquad \text{Constraints (3-4), (3-5), and (3-6)}$$

$$x_a - Lower(c_a + y_a) = 0 \qquad \forall a \in A \tag{3-18}$$

Objective function (3-17) seeks vectors **x** and **y** that minimize total travel time, subject to budget (3-4) and capacity ((3-5) and (3-6)) limitations. Link flows **x** associated with vector **y** are implicitly derived from the solution of problem (L), which is expressed within $Lower(\cdot)$ in Equation (3-18). $Lower(\cdot)$ returns solution matrix $\{\xi_w^{g,r}\}$.

3.6 Solution Methodology

Program (SL) is a nonlinear mixed integer program with nonlinear objective function and nonlinear constraints. Solution approaches exist that can guarantee a global optimum for nonlinear programs possessing specific characteristics, like convexity, or that can be shown to possess certain properties. No solution methodology with applicability to program (SL) exists that can guarantee a global optimum. Instead, a solution methodology is presented herein that guarantees a locally optimal solution and takes advantage of global search strategies to increase the likelihood of finding the globally optimal solution. Specifically, the proposed methodology embeds an exact Sequential Quadratic Programming (SQP) procedure within a tabu search environment.

This approach builds on the solution frameworks of two works: (Chelouah and Siarry, 2000) and (Chen et al., 2008). Chelouah and Siarry (2000) proposed a tabu search-based (Glover and Taillard, 1993) metaheuristic, called the Enhanced Continuous Tabu Search (ECTS) algorithm, with the goal of obtaining a global optimum for unconstrained optimization problems. Chen et al. (2008) extended Chelouah and Siarry's continuous tabu search (CTS) approach for constrained math programs. They employ a methodology based on Lagrangian relaxation in which a term involving the square of each constraint is included and penalized in the objective function. The procedure aims to minimize this term to produce a feasible solution. SQP is used to produce such feasible solutions. A multi-start strategy involving exploration around a current best solution within concentric hyper-rectangles is employed within the diversification stage of the CTS. This procedure produces a set of starting points for the SQP, leading to a set of likely feasible solutions. The best solution among this set is chosen and the multi-start procedure is repeated.

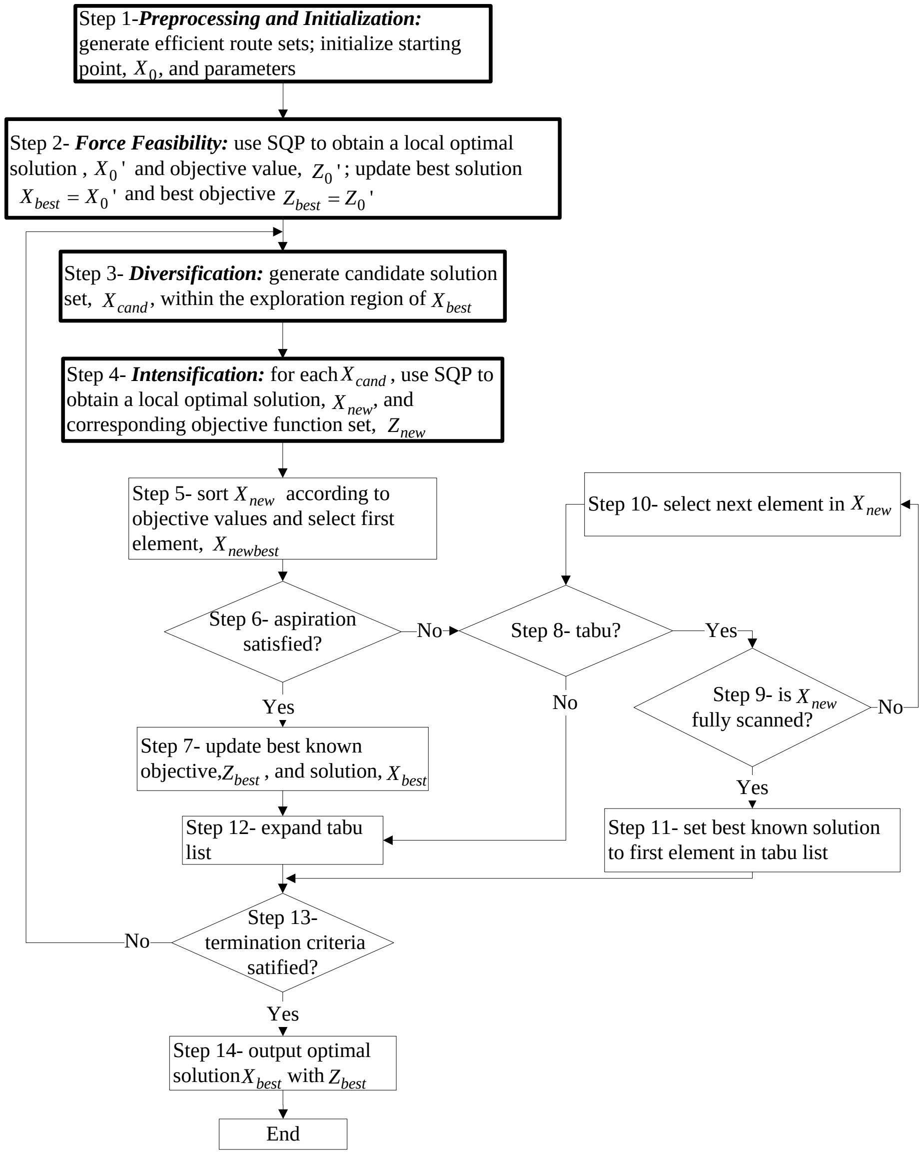


Figure 3-2 Flowchart of the MST-SQP Procedure

The proposed methodology for solving the RECM problem employs a similar framework as in (Chen et al., 2008), involving a multi-start SQP procedure within a CTS framework. Moreover, an adaptation of concentric hyper-rectangles structure developed in (Teh and Rangaiah, 2003) is embedded within this framework. However, instead of relaxing the constraints and seeking a set of feasible solutions from which the optimal solution can be obtained, the original constrained math program is solved directly by SQP. Additionally, a secondary tabu search methodology is employed within the proposed methodology (during identification and intensification stages) to evaluate $Lower(\cdot)$. This proposed approach is referred to herein as the Multi-start Tabu method with SQP (MST-SQP). Figure 3-2 provides a flowchart of the steps of the main procedure. Details of its key steps follow.

3.6.1 Preprocessing and Initialization (Step 1)

The procedure begins with the generation of an efficient route set (Sheffi, 1985) for each O-D pair. It is presumed, as in (Bovy and Stern, 1990), that when faced with a route decision, a traveler selects his/her route from a limited choice set. The more comprehensive the choice set, the more likely he/she will choose the optimal route given his/her goals. Since complete enumeration of all possible routes is impractical and given that most people do not consider all alternatives in making their decisions, only the efficient route set is considered. Based on Sheffi's work, an efficient route is defined as a route passing only through efficient arcs, and an efficient arc is defined as follows. For each arc a connecting i to j, if $r(i)<r(j)$, for $r(k)$ the shortest distance from the origin to node k, and $s(i)>s(j)$, for $s(k)$ the shortest distance from k to the destination, then arc a is efficient ($\mathrm{eff}(i,j)=1$); otherwise, it is inefficient ($\mathrm{eff}(i,j)=0$). The efficient routes, R_w, between each O-D pair w are obtained with a depth-first-search (DFS) on the network of efficient arcs (i.e. the subgraph $\Gamma' = (N, A')$, where A' is the set of efficient arcs). Routes with cycles are not generated, because by definition any efficient arc transports travelers to locations that are further from the origin and closer to the destination.

Once the efficient route set is generated, an initial starting point, X_0, must be chosen. X_0 consists of two vectors: link flow **x** and capacity change **y**. To produce X_0, the elements of **x** and **y** are chosen randomly given restrictions on their bounds.

The aspiration, tabu and termination criteria employed herein are adopted directly from (Chen et al., 2008). These criteria are summarized for completeness.

Aspiration criterion

Any candidate solution that has the best objective value of all discovered solutions will become the best solution regardless of its tabu status.

Tabu list

A list of solutions, each of which is given by a pair of vectors (**x**,**y**), considered in the last n iterations (the tabu tenure) of the tabu search procedure is maintained. Thus, an explicit memory approach is used. The best found solution obtained thus far will not enter the tabu list, unless it is identified twice, until a better solution is found. This construction of the tabu list prevents revisiting of solutions within the iterations associated with its tabu tenure. A solution may be removed from the tabu list prematurely if no neighboring solution of the best solution outperforms the best solution. A solution is tabu if

$$\left\| X - X_j^{tabu} \right\| \le h_0 \quad \forall j = 1, 2, ..., n \ , \tag{3-19}$$

where X_j^{tabu} is the j^{th} solution in the tabu list and h_0 is defined in equation (3-21) of subsection 3.6.3.

Termination criteria

When either a predefined maximum number of iterations or a predefined maximum number of iterations without improvement is reached, the procedure terminates.

Parameter settings

The tabu parameters were tuned through initial experiments. The best found settings, and the settings that will be used in the remainder of the Chapter, are: maximum iteration number = 50; maximum number of iterations without improvement = 10; number of candidate solution points to be explored = 10; tabu tenure = 20.

3.6.2 Force Feasibility (Step 2)

Using X_0 obtained from step 1 in Figure 3-2 as the starting point, SQP is employed to find the corresponding locally optimal solution X'_0 with objective value Z'_0 for program SL. The best known solution, X_{best}, and objective value, Z_{best}, are set to X'_0 and Z'_0, respectively. The SQP algorithm requires evaluation of $Lower(\cdot)$ within Equations (3-18). Details of the process to solve the lower-level problem are discussed in subsection 2.5.3.

3.6.3 Diversification (Step 3)

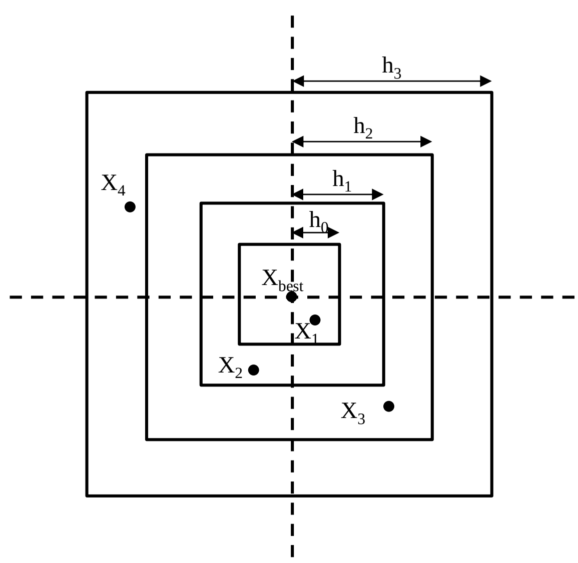


Figure 3-3 Hyper-rectangles adapted from (Chelouah and Siarry, 2000)

A diversification strategy generates a set of candidate solutions within the exploration space of the current best solution, X_{best}. That is, the diversification process involves a multi-start strategy, where a set of candidate solution points, given by array X_{cand}, are randomly generated around the current best solution. The solution space around the current solution, as defined in (Chelouah and Siarry, 2000) and (Teh and Rangaiah, 2003), is partitioned by a set of concentric hyper-rectangles. The structure of hyper-rectangles around X_{best} in two dimensions is illustrated in Figure 3-3. The relationship between the radii of concentric hyper-rectangles is expressed as

$$h_k = 2 \cdot h_{k-1}, k = 1, 2, \ldots N_{cand} - 1 \tag{3-20}$$

$$h_0 = 0.01 \cdot (UB - LB) / 2 \tag{3-21}$$

where N_{cand} is number of candidate solutions, h_0 is the half-width of the inner-most rectangle, and UB and LB are the upper- and lower-bound vectors of X, respectively.

In exploration of solution points within a vicinity of X_{best}, one candidate solution is randomly generated within each region enclosed by two adjacent hyper-rectangles (the innermost region is enclosed only by the inner-most hyper-rectangle).

3.6.4 Intensification (Step 4)

The candidate solution points generated in the diversification stage are not guaranteed to be feasible for (SL). Thus, they are used as starting points for the SQP algorithm through which neighboring feasible solutions are obtained. The intensification process seeks a set of such feasible solutions (see Figure 3-4), employing SQP for each such starting point. An updated candidate solution array X_{cand} is generated.

Intensification starts with selecting the 1st element, X, of X_{cand} generated in the diversification process. If X is tabu, then the process is applied to the next element in X_{cand}. If X is not tabu and it is feasible, X and its objective function value Z, are directly added into the new feasible solution set, X_{new}, and objective set, Z_{new}, respectively; otherwise, (SL) is solved through SQP using X as the starting point and resulting locally optimal solution X' with corresponding objective value Z'. X' and Z' will be added into X_{new} and Z_{new}, respectively. This process is repeated until all elements of X_{cand} have been investigated.

After obtaining a new feasible set, X_{new}, it is sorted in nondecreasing order according to objective values. The best (first) new feasible solution $X_{newbest}$ is selected. The aspiration criterion is used to update the best known solution. If the aspiration criterion is satisfied (i.e. $Z_{newbest} < Z_{best}$), then the best known solution X_{best} will switch to $X_{newbest}$ and the best known objective Z_{best} will change to $Z_{newbest}$. The previous best solution will be placed in the tabu list. Termination criteria will be assessed. If one of the termination criteria is met, the procedure stops; otherwise, continue to the next iteration. If the aspiration criterion is not satisfied, the subsequent elements in X_{new} cannot be better than $X_{newbest}$, and the tabu criterion will be checked for all elements in X_{new}. If any is not tabu, it will be placed in the tabu list. If all of elements in X_{new} are tabu, the first element in the tabu list will be selected as the best known solution. The tabu list aids in preventing the search from being trapped at a local solution. The SQP algorithm requires evaluation of $Lower(\cdot)$ within Equations (3-18). Details of the process to solve the lower-level problem are discussed in subsection 2.5.3.

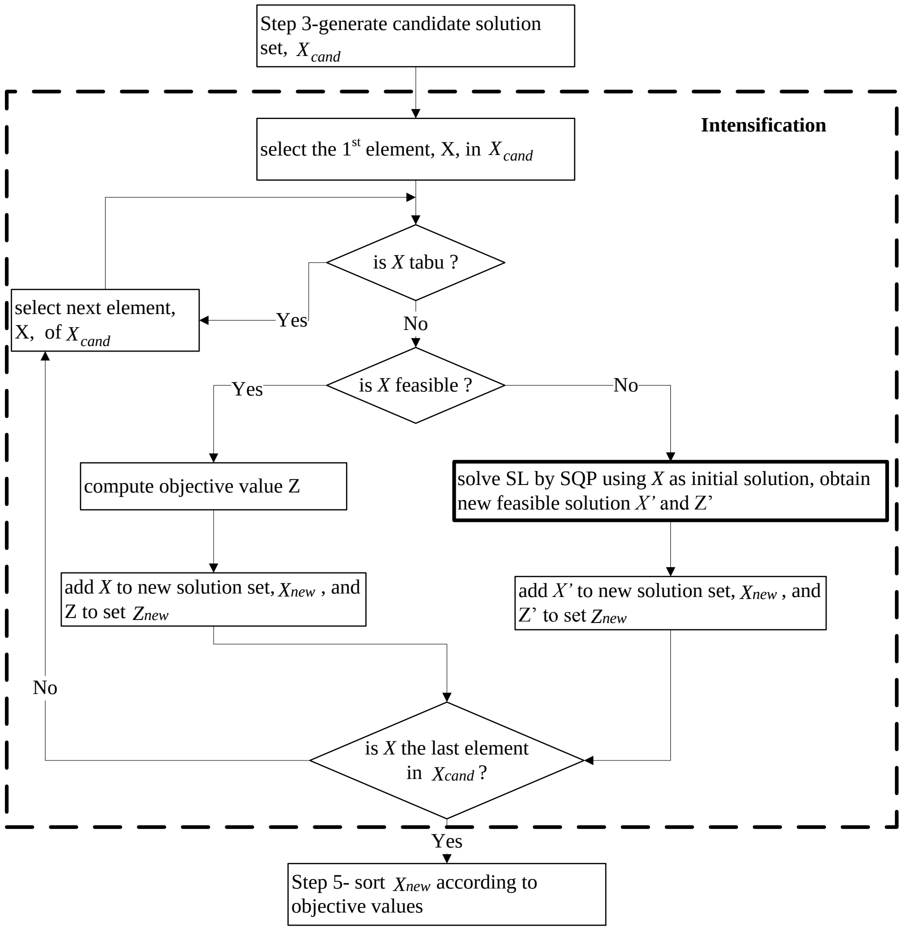


Figure 3-4 Flowchart of Intensification Process (Step 4)

3.7 Numerical Experiments

3.7.1 Experiment Design

To investigate the efficiency of the proposed model and solution methodology, the MTS-SQP procedure with embedded TS algorithm for solution of $Lower(\cdot)$ is applied on a numerical example consisting of 14 nodes, 22 links and 4 O-D pairs, as shown in Figure 3-5. The example network is acyclic; however, the methodology supports solution in networks with cycles.

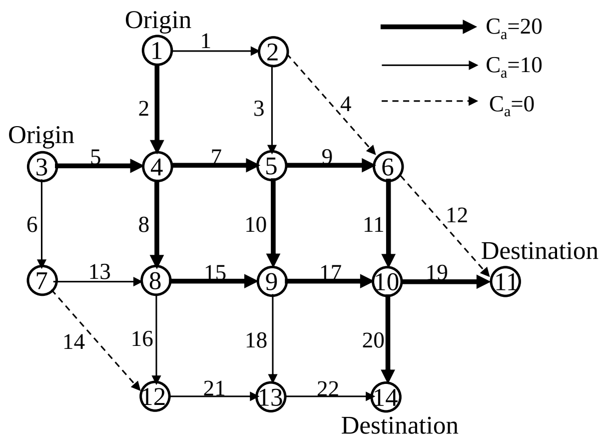


Figure 3-5 Test Network Configuration

As indicated in Figure 3-5, some links begin with zero capacity. An increase in capacity from zero is akin to opening or constructing the link. Detailed information of the network is listed in Table 3-1. The free-flow speed is set to be 1.42m/s (Thalmann and Musse, 2007) and coefficient k_a = 0.0008 for travel time calculations. The total budget B is 1500 cost units.

Table 3-1 Network Information

Link	l_a (m)	t_a^0 (s)	c_a	b_a	c_a^{up}	Link	l_a (m)	t_a^0 (s)	c_a	b_a	c_a^{up}
1	100	70.42	10	3	50	13	100	70.42	10	3	50
2	100	70.42	20	3	50	14	200	140.85	0	5	50
3	100	70.42	10	3	50	15	100	70.42	20	3	50
4	200	140.85	0	5	50	16	100	70.42	10	3	50
5	100	70.42	20	3	50	17	100	70.42	20	3	50
6	100	70.42	10	3	50	18	100	70.42	10	3	50
7	100	70.42	20	3	50	19	100	70.42	20	3	50
8	100	70.42	20	3	50	20	100	70.42	20	3	50
9	100	70.42	20	3	50	21	100	70.42	10	3	50
10	100	70.42	20	3	50	22	100	70.42	10	3	50
11	100	70.42	20	3	50	23	100	70.42	10	3	50
12	200	140.85	0	5	50	24	200	140.85	0	5	50

Table 3-2 gives the demand information for each O-D pair. There are 20 groups of pedestrians for each O-D pair. The group size is uniformly chosen on the interval [1, 30]. Traveling distance sensitivity parameter α^g is uniformly distributed between 0 and 1 and travel time sensitivity parameter $\beta^g = 1 - \alpha^g$.

Table 3-2 Demand for Each O-D Pair

O-D pair	Group	Size	α^g (/m)	β^g (/s)	O-D pair	Group	Size	α^g (/m)	β^g (/s)
1-11	1	10	0.5	0.5	1-14	1	7	0.4	0.6
	2	12	0.7	0.3		2	27	0.3	0.7
	3	29	0.3	0.7		3	16	0.6	0.4
	4	23	0.2	0.8		4	19	0.6	0.4
	5	19	0.4	0.6		5	22	0.3	0.7
	6	4	0.6	0.4		6	27	0.1	0.9
	7	10	0.5	0.5		7	28	0.6	0.4
	8	25	0.4	0.6		8	30	0.1	0.9
	9	15	0.7	0.3		9	21	0.7	0.3
	10	1	0.7	0.3		10	16	0.2	0.8
	11	14	0.3	0.7		11	16	0.8	0.2
	12	30	0.3	0.7		12	1	0.1	0.9
	13	12	0.6	0.4		13	14	0.5	0.5
	14	30	0.7	0.3		14	18	0.3	0.7
	15	4	0.5	0.5		15	10	0.4	0.6
	16	18	0.8	0.2		16	11	0.2	0.8
	17	22	0.2	0.8		17	18	0.8	0.2
	18	14	0.6	0.4		18	1	0.2	0.8
	19	13	0.2	0.8		19	4	0.1	0.9
	20	16	0.3	0.7		20	30	0.5	0.5
2-11	1	18	0.8	0.2	2-14	1	10	0.2	0.8
	2	29	0.8	0.2		2	13	0.7	0.3
	3	29	0.8	0.2		3	16	0.1	0.9
	4	15	0.2	0.8		4	13	0.7	0.3
	5	27	0.4	0.6		5	19	0.8	0.2
	6	7	0.4	0.6		6	18	0.5	0.5
	7	17	0.3	0.7		7	9	0.5	0.5
	8	20	0.1	0.9		8	4	0.0	1.0
	9	19	0.6	0.4		9	28	0.4	0.6
	10	6	0.2	0.8		10	6	0.3	0.7
	11	19	0.6	0.4		11	2	0.7	0.3
	12	27	0.5	0.5		12	17	0.4	0.6
	13	8	0.2	0.8		13	2	0.8	0.2
	14	1	0.3	0.7		14	9	0.3	0.7
	15	20	0.6	0.4		15	20	0.8	0.2
	16	22	0.2	0.8		16	28	0.3	0.7
	17	21	0.8	0.2		17	1	0.8	0.2
	18	7	0.2	0.8		18	3	0.7	0.3
	19	23	0.3	0.7		19	8	0.3	0.7
	20	2	0.1	0.9		20	4	0.2	0.8

The proposed MST-SQP procedure with embedded TS algorithm was coded in the MATLAB 2010a environment and run on a personal computer with Intel(R) CPU 3.10GHz and 4.0GB RAM. The procedure takes advantage of an existing SQP tool available within the Optimization Toolbox of MATLAB (Coleman et al., 1999).

3.7.2 Results and Analysis

Table 3-3 gives the set of 34 efficient routes among the four O-D pairs. Three links with no prior capacity are included. The distances required to traverse the routes are identical with 500 m.

Table 3-3 Routes Set for Each O-D Pair

O-D	Index	Route	O-D	Index	Route
1-11	1	1→2→5→6→10→11	1-14	1	1→2→5→6→10→14
	2*	1→2→5→6→11		2	1→2→5→9→10→14
	3	1→2→5→9→10→11		3	1→2→5→9→13→14
	4*	1→2→6→10→11		4*	1→2→6→10→14
	5*	1→2→6→11		5	1→4→5→6→10→14
	6	1→4→5→6→10→11		6	1→4→5→9→10→14
	7*	1→4→5→6→11		7	1→4→5→9→13→14
	8	1→4→5→9→10→11		8	1→4→8→9→10→14
	9	1→4→8→9→10→11		9	1→4→8→9→13→14
				10	1→4→8→12→13→14
3-11	1	3→4→5→6→10→11	3-14	1	3→4→5→6→10→14
	2*	3→4→5→6→11		2	3→4→5→9→10→14
	3	3→4→5→9→10→11		3	3→4→5→9→13→14
	4	3→4→8→9→10→11		4	3→4→8→9→10→14
	5	3→7→8→9→10→11		5	3→4→8→9→13→14
				6	3→4→8→12→13→14
				7	3→7→8→9→10→14
				8	3→7→8→9→13→14
				9	3→7→8→12→13→14
				10*	3→7→12→13→14

* indicates that a link that originally had zero capacity is included within the route

Assignment Results before Reconfiguration

Convergence to an equilibrium solution with total disutility of 600,000 is obtained after 7 iterations of evaluation of *Lower* $(\cdot)$ for the original network design, requiring 3.84 CPU seconds in total, as shown in Figure 3-6.

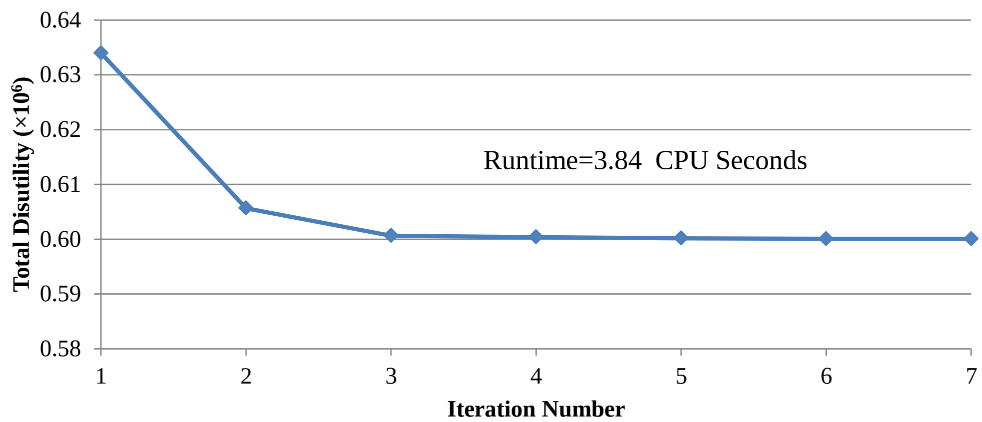
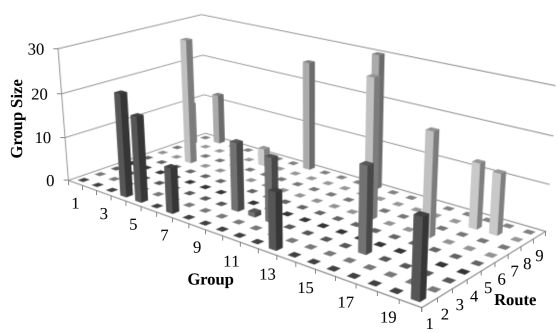
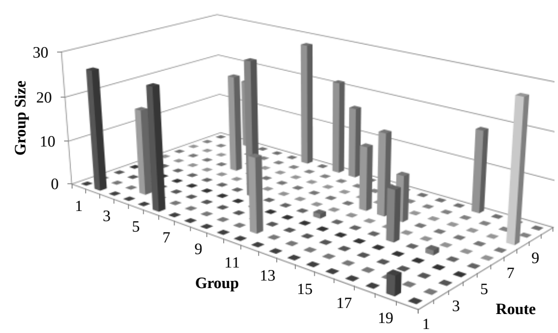


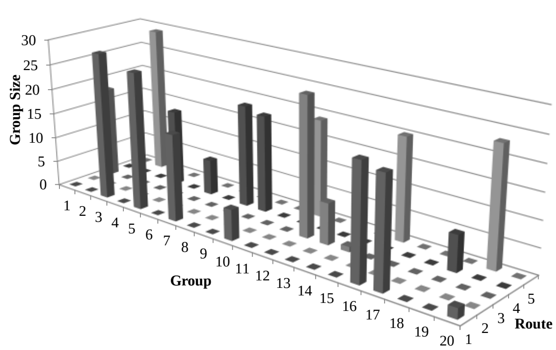
Figure 3-6 Convergence Process of Lower-Level Solution Algorithm



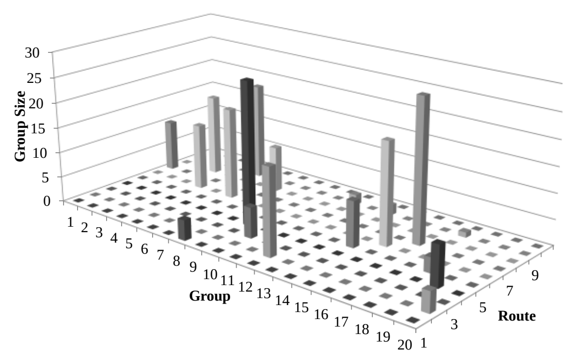
(a) OD 1-11



(b) OD 1-14



(c) OD 2-11



(d) OD 2-14

Figure 3-7 Distribution of Groups over Routes by O-D Pair before Reconfiguration

Solution of the lower-level problem is obtained for the existing system configuration. Figure 3-7 shows the distribution of groups over the route options between each of the O-D pairs. Note that no group is assigned to a route with any link with zero capacity. For example, in Figure 3-7(a), no groups are assigned to routes 2, 4, 5, or 7 related to O-D pair 1-11. Furthermore, there is no group that can decrease its total incurred disutility by unilaterally switching routes.

Results from Solving the RECM allowing for Reconfiguration

The MST-SQP solution methodology is applied, where reconfiguration is permitted. As shown in Figure 3-8, the procedure terminates after 20 iterations, because no improvement in solution value is obtained for more than 10 iterations. The resulting solution has a total travel time of 495,240. The disutility at the lower-level is 565,260 (total disutility before reconfiguration is 600,000 as shown in Figure 3-6). The procedure required 1,955 CPU seconds.

Table 3-4 shows capacity changes needed to minimize total travel time as suggested by the solution methodology. As shown in the table, the entire budget (1500) need not be used to obtain an improvement in total travel time by 18 percent (from 603,730 to 495,240 time units). The sum of the capacity changes equals zero, indicating that no more space than exists will be used. Those links with larger capacity increases also supported larger increases in flows. If capacity limitations are relaxed, one would expect the entire budget to be used, and total travel time to decrease further.

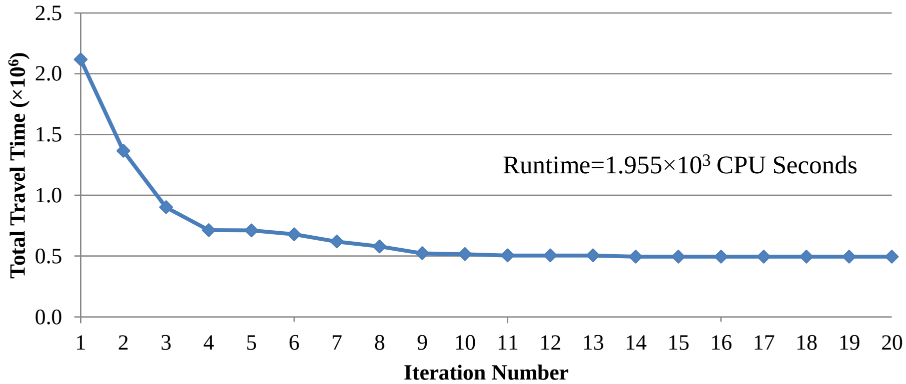


Figure 3-8 Termination of the MST-SQP Solution Algorithm

Table 3-4. Results before and after Capacity Increase

Link	x_a before redesign	y_a	x_a after redesign	b_a	Cost
1	221	15.86	451	3	48
2	436	-8.00	206	3	24
3	221	-8.94	19	3	27
4	0	31.25	432	5	156
5	376	-6.22	235	3	19
6	191	9.36	332	3	28
7	445	-3.72	282	3	11
8	367	-10.86	159	3	33
9	397	-6.71	233	3	20
10	269	-16.02	68	3	48
11	397	-12.57	165	3	38
12	0	36.05	500	5	180
13	191	-1.25	148	3	4
14	0	13.31	184	5	67
15	371	-4.54	270	3	14
16	187	-7.86	37	3	24
17	533	-6.19	244	3	19
18	107	-4.63	94	3	14
19	658	-13.25	158	3	40
20	272	-5.61	251	3	17
21	187	2.45	221	3	7
22	294	8.10	315	3	24
Total travel time	before	after		Cost spent =860	
$\sum_{a\in A} x_a \cdot t_a(x_a)$	6.0373×10^5	4.9524×10^5			

Figure 3-9 pictorializes changes in network configuration, specifically capacity allocation and flow patterns, resulting from the application of the solution methodology under two initial capacity settings, where the second setting involves 10 more units of capacity along each link as compared with the original network. With increased capacity, travel times will generally decrease. One can observe dramatic changes in capacity and flow after reconfiguration, especially for links with original capacities of zero. For the different starting conditions, flow distributions and allocation of budget differ as expected. For both starting conditions, increases in capacities occur only on links with relatively low capacities. This supports a larger dispersion of flow over the network. Flows, thus, follow capacity changes, illustrating the interactions between upper- and lower-levels. Flow conservation is respected both before and after reconfiguration.

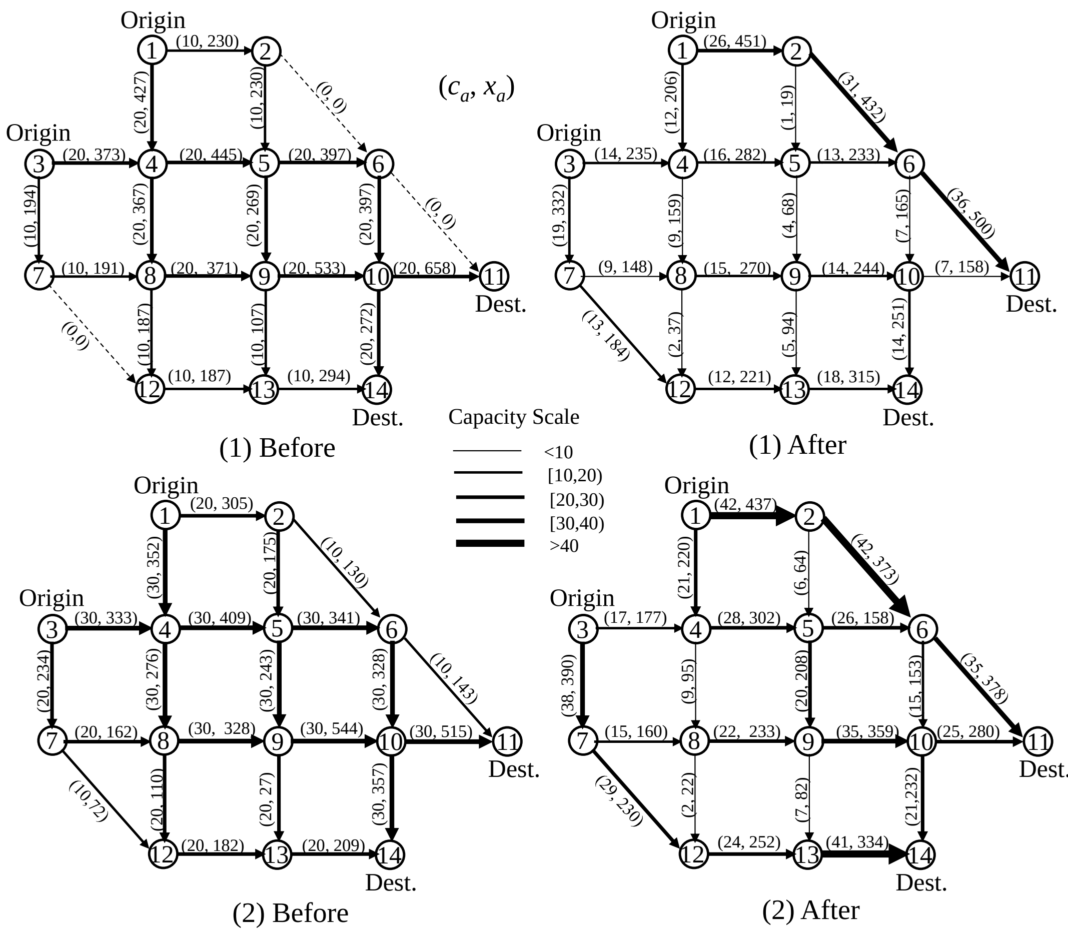


Figure 3-9 Comparison of Capacity Allocation and Flow Distribution

3.8 Conclusions and Extensions

In this chapter, the crowd control problem is formulated as a bi-level program. A network design problem and an assignment problem based on pure-strategy Nash equilibrium are considered in the upper- and lower-level, respectively. The lower-level problem incorporates characteristics of crowd grouping behavior. A MST-SQP Procedure is proposed for solution of the bi-level program. In the proposed procedure, a metaheuristic based on best response dynamic and tabu search methods is proposed to identify the pure-strategy Nash equilibrium solution of the lower-level game. The model and solution algorithm are tested on a numerical example, results from which show the effectiveness and efficiency of the proposed methodology.

The main contributions of this paper include: a modeling framework that simultaneously takes the crowd manager and pedestrian goals into consideration; crowd control strategies created from the solution of a network design problem, a type of mathematical decision problem; incorporation within a mathematical framework of key behavioral rules, including group dynamics, and the desire by system users to choose utility maximizing routes; and more generally, a solution framework that obviates the need for simulation.

The proposed modeling approach has practical utility in crowd control. The outcome of implementing this methodology is a set of strategies for reconfiguring the physical layout to better support likely pedestrian response to the physical offerings. It does not attempt to control pedestrian decisions, but instead recognizes that the pedestrians will make selfish decisions that support their personal (individual or group) goals. The outcome of the model can be implemented through, for example, the placement of portable barriers and barricades, opening and closing of gates, and use of other devices such as ropes with posts and signage.

Proposed model and solution framework might be improved or extended in several directions. First, additional experiments are required to assess the utility of the proposed methodology on larger problem instances. The procedure guarantees a local optimal solution and employs heuristic steps in seeking a global optimum. This modeling framework permits alternative solution approaches, such as linear approximation, that may be useful in addressing large problem instances. Alternatively, large problem instances can be addressed by replacing the SQP approach of the MST-SQP procedure with a heuristic. However, such an approach will not guarantee even local optimality.

In addition, it is assumed that all pedestrians within a group have the same preference function including parameter settings and that pedestrians within a group always stay together as they move on the network. These assumptions were used to investigate the maximum marginal impact of group size. Such assumptions can be relaxed to model other type of grouping behaviors. For example, within the proposed framework, one might model separable groups, where pedestrians within a group are allowed to split. In this case, instead of seeking a pure strategy Nash equilibrium, the objective of the lower-level problem will seek a mixed strategy Nash equilibrium and a UE or SUE based assignment can be sought in the solution of the lower-level problem. Additionally, the heterogeneity of preference parameters of pedestrians within a group can be further explored within the proposed framework. The heterogeneous preference parameters and attributes of routes that affect route choice might be estimated using a survery-based approach (Daamen et al., 2005; Seneviratne and Morrall, 1985).

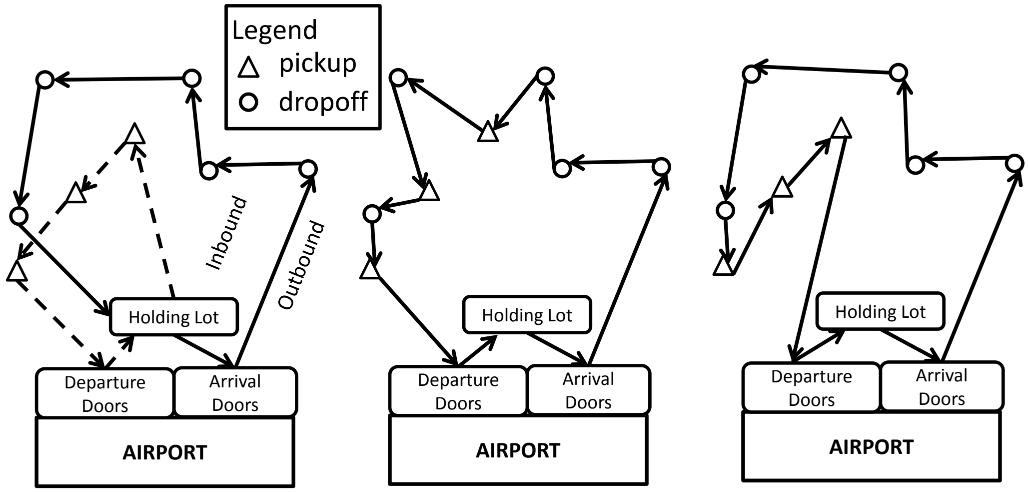
Moreover, one might extend the developed model and solution methodology to address a dynamic crowd control problem, where the physical environment changes dynamically and pedestrians make decisions on splitting or grouping at each node according to dynamically updated utilities. In dynamic settings, a more sophisticated travel time function would be necessary to capture pedestrian dynamics at intersections as well as the impacts of bi-directional flows.

Finally, pedestrians are assumed to move on a network representation of a facility. One might explore the interdependencies in space restrictions between abutting or adjoining links of the network, this might be modeled within the proposed framework through the addition of constraints in (U). One might also extend the proposed framework to model movements of pedestrians over a continuous space by including heading direction and neighborhood density in the utility function. This would, however, require longer computational time to solve the lower-level.

Chapter 4 Optimizing Ridesharing Services for Airport Access

4.1 Introduction

This chapter addresses the problem of optimally routing and scheduling airport shuttle vehicles that offer pickup and dropoff services to customers through ridesharing. This work was motivated by a need for tools to support efficient resource management at Supreme Airport Shuttle, Inc. This company provides ridesharing services to customers travelling to/from two major airports in the Washington, D.C. area. In its outbound operations, they have a fleet of vehicles used to pick up customers from the airport's arrival doors and drop them at customer-chosen destinations. The vehicles also provide inbound services in which they pick up customers at multiple origins outside the airport and drop them at the airport's departure doors. Customers request services by phone, online or at a kiosk in the airport or hotel. Each request includes information on the number of passengers, pickup location and time, and (or) dropoff location and time. A single request can be for a one-way trip (outbound or inbound) or a round trip (outbound and inbound). Each request results in a trip from the arrival door of the airport to the trip's destination or from the trip's origin to the departure door of the airport. Thus, requests can be made in advance or may arise dynamically on the same day of service. One or more trips are served by one vehicle through a route, which is defined by a circuit that is travelled by a vehicle starting from and ending at the holding lot in the airport. Each vehicle may have multiple routes during a shift.



(a) Policy 1 **(b) Policy 2** **(c) Policy 3**

Figure 4-1 Illustration of One Vehicle Route

Reduced total passenger-miles traveled resulting from ridesharing and efficiently designed routes can increase profitability of the service provider and aid in diminishing traffic congestion and related negative externalities, including environmental pollution. Thus, an optimization model is proposed for the problem of determining a set of routes and schedules that meet service quality, resource, labor and vehicle capacity constraints while minimizing total cost in terms of vehicular use and total wages in the context of airport ridesharing services. This problem, which is a version of the Dial-A-Ride Problem (DARP), is called the Airport Access Ridesharing Problem (AARP) here.

The AARP is considered under three different operational policies illustrated in Figure 4-1. **Policy 1** handles outbound and inbound trips separately, assigning different sets of vehicles to each. This policy is under consideration, because in current operations demand for outbound service far outweighs the demand for inbound service. **Policy 2** handles outbound and inbound trips simultaneously, permitting a single vehicle to drop off outbound and pick up inbound customers at all points along the route. Under **Policy 3**, all outbound trips must be dropped off before the same vehicle starts picking up inbound trips. This last policy gives preference to outbound customers, the majority of the company's actual customers. While one can show that Policy 2 will always produce the most efficient routes for the operator, other policies must be considered under certain contractual agreements or passenger service policies.

The AARP is a difficult combinatorial optimization problem; the number of possible solutions for it grows exponentially with increasing problem size. Even obtaining a single feasible solution by hand can be quite challenging. Yet, efficient use of limited resources is key to providing profitable, quality service that, by contract, meets service level agreement requirements. Thus, for real-world problem instances, tools to support the identification of feasible and optimal or near-optimal solutions can be crucial. An exact solution algorithm and two heuristics are proposed to solve the AARP. The exact solution applies a Constraint Programming based Column Generation (CPCG) approach (Junker et al., 1999). The first heuristic is a variant of the sequential insertion heuristic proposed by Jaw (Jaw et al., 1986) for a related dial-a-ride problem (DARP). The second is adapted from Solomon's work on the Vehicle Routing Problem with Time Windows (Solomon, 1987). Performance of the proposed heuristics is compared in a case study involving data from one day's operation of the Supreme Airport Shuttle fleet at one airport. The solution approaches were implemented and adapted to the three operational polices. Results from runs of the algorithms are analyzed and compared based on a variety of performance measures.

Related works from the literature are reviewed in Section 4.2. Notation and problem formulations for the three polices are introduced in Section 4.3, followed by the description of the proposed solution approaches in Section 4.4. In Section 4.5, results of numerical experiments conducted on the real-world case study are provided. Finally, conclusions and extensions are discussed in Section 4.6.

4.2 Related Literature

The AARP shares several characteristics of a variety of established optimization problems. First, the AARP is related to the Vehicle Routing Problem with Time Windows (VRPTW) (Kolen et al., 1987), which is an extension of the traditional Capacitated Vehicle Routing Problem (CVRP) (Dantzig and Ramser, 1959). In the VRPTW, a vehicle must arrive within given time window at each customer. Where soft time window constraints are permitted, a penalty for early or late arrival may be incurred. In this case, the total costs for routing and scheduling include not only the travel distance and time costs, but also the penalty costs. For comprehensive reviews of optimization algorithms for VRPTW, the reader is referred to (Braysy and Gendreau, 2005a, b; Desrochers et al., 1992; Prescott-Gagnon et al., 2009). The AARP similarly has time windows; however, these constraints are hard. Thus, any solution that violates these constraints is infeasible. The AARP differs from the VRPTW by also including maximum ride time constraints needed to control the time spent by each passenger traveling in the vehicle, as well as maximum shift durations.

Moreover, customer stops are paired in the AARP, since each customer has a pair of pickup and dropoff locations, and the pickup must be completed before the dropoff. This creates additional precedence constraints. Such pairing and precedence of customers are captured in a generalization of the VRPTW, the Pickup and Delivery Problem with Time Windows (PDPTW) (Dumas et al., 1991).

As in the AARP, in the PDPTW the origin of each request must precede its destination on each vehicle tour, and both locations must be visited by the same vehicle. Among the PDPTWs in the literature, those that address the DARP (Jaw et al., 1986) are most relevant. Comprehensive surveys of optimization algorithms on the PDPTW are provided in (Wallace, 1978) and (Cordeau and Laporte, 2007), and on the DARP in (Parragh et al., 2008b) and (Berbeglia et al., 2007). The DARP involves passenger transportation between paired pickup and delivery points and takes user inconvenience into account. The AARP can be viewed as a special case of the DARP with one-to-many and many-to-one operations. See (Gribkovskaia and Laporte, 2008) for a discussion of this variant for a single vehicle. The AARP with Policy 1 or 2 can be treated as a PDPTW; however, operational Policy 3 requires that inbound movements cannot begin until outbound movements are complete. This variant does not appear to have been addressed previously.

The Vehicle Routing Problem with Backhauls and Time Windows (VRPBTW), another variant of the VRPTW, specifically captures the one-to-many and many-to-one characteristics of the AARP. The VRPBTW involves linehaul and backhaul operations. In the linehaul operations, the loading of goods onto a vehicle is completed at one or more depots, and goods are transported to one or more destinations. In backhaul operations, once linehaul operations are complete (or partially complete), goods are loaded at the linehaul destinations or other convenient locations, and transported to the depot, the destination of the backhaul deliveries. A comprehensive survey of algorithms and applications for the VRPB, including the VRPBTW, is given in (Parragh et al., 2008a). Following the classification scheme in (Parragh et al., 2008a), the AARP using Policy 1 can be viewed as two separated DARPs, one addressing the outbound trips and the other the inbound trips. The AARP using Policy 2 can be defined as a Vehicle Routing Problem with Simultaneous Delivery and Pickup and Time Windows (VRPSDPTW). The AARP using Policy 3 can be classified as a Vehicle Routing Problem with Clustered Backhauls and Time Windows (VRPCBTW). The VRPCBTW does not deal with pairing and precedence of service points, a crucial characteristic of the AARP.

Limited works in the literature address the specifics of the VRPBTW and its VRPSBTW and VRPCBTW variants. The VRPBTW has been formulated as a mixed integer linear program, but only relatively small instances can be solved to optimality. An exact solution approach for the VRPSBTW is presented in (Angelelli and Mansini, 2002). Specifically, a column generation framework is proposed in which the problem is decomposed into a Master Problem (MP) and Subproblem (SP). The MP is formulated as a set covering problem and branch-and-price is proposed for solution of the SP. Solution of the SP supplies a feasible route for inclusion in the set of possible routes considered in the MP. The largest instance solved to optimality had 20 customers. Yano et al. (1987) formulated the VRPCBTW as a set partitioning problem. They proposed an exact solution method based on branch-and-bound to generate optimal tours, each with a maximum of four linehaul and four backhaul customers. For the same problem with possibly more than four customers, Gelinas et al. (1995) proposed an exact algorithm based on column generation for solving a similar set partitioning formulation of the VRPCBTW. The algorithm found optimal solutions for problems with up to 100 customers.

While promising, exact solution methods cannot be applied to typical problems of a size seen in real-world operations. Thus, numerous works have proposed heuristics for these problems. The majority of these heuristics include construction and improvement schemes based on classical greedy methods. More powerful methods have been proposed based on metaheuristics. For example, Dethloff (2001) proposed an extension of the cheapest insertion heuristic for the VRPSBTW. Thangiah et al. (1996) proposed a heuristic for solution of the VRPCBTW. In the construction phase, the insertion procedure of (Kontoravdis and Bard, 1995) proposed for the VRPTW is used to obtain initial solutions. Then, the initial solutions are improved through the application of λ-interchanges and 2-opt* exchanges in the improvement phase. Duhamel et al. (1997) uses an insertion procedure proposed in (Solomon, 1987) for initial solution construction, but proposed a tabu search heuristic for the improvement phase. An augmented objective function for the VRPCBTW is presented by Zhong and Cole (2005), where violations of time windows, capacity and linehaul-backhaul precedence constraints are penalized. The cluster-first route-second method is used for initial route construction and intra- and inter-route operators are described for use in improving the tours. Metaheuristics, such as Tabu Search (Duhamel et al., 1997), genetic algorithms (Tasan and Gen, 2012) and ant colony optimization (Paraphantakul et al., 2012), have also been proposed.

In the next section, a general formulation is presented that can be used to solve all three variants of the AARP. This formulation combines aspects of previously proposed formulations for the PDPTW and VRPBTW. It fills the need for a formulation of a PDPTW with linehaul and backhaul operations or the VRPCBTW with additional pairing and precedence constraints, i.e. AARP with Policy 3. The proposed formulation does not rely on a complete enumeration of the feasible tours, which is a requirement of the set partitioning formulation of the VRPCBTW and set covering approach for the VRPSBTW. The formulation given next includes additional constraints specific to an application involving passengers as opposed to cargo, including maximum passenger ride times and restrictions on idling with passengers onboard.

4.3 Problem Formulation

In this section, the AARP is formulated. The formulation is preceded by the introduction of notation. Additional adaptations needed for different operational policy implementations are given.

Notation

n_O number of outbound trips/requests

n_I number of inbound trips/requests

P_O set of outbound pickup nodes located at the arrival door, $P_O = \{1, ..., n_O\}$

P_I set of inbound pickup nodes, $P_I = \{n_O + 1, \ldots, n_O + n_I\}$

D_O set of outbound dropoff nodes, $D_O = \{n_O + n_I + 1, ..., 2n_O + n_I\}$

D_I set of inbound dropoff nodes located at the departure door,

$D_I = \{2n_O + n_I + 1, ..., 2n_O + 2n_I\}$

P set of all pickup nodes, $P = P_O \cup P_I$

D set of all pickup nodes, $D = D_O \cup D_I$

V set of available vehicles

q_i demand/supply at node i; for pickup nodes, $q_i > 0, \forall i \in P$; for dropoff nodes, $q_i < 0, \forall i \in D$; for the holding lot, $q_0 = q_{2n_O+2n_I+1} = 0$.

e_i earliest service time at node i, i.e. the start of the time window

l_i latest service time at node i, i.e. the end of the time window

s_i service duration or dwell at node i

c_{ij}^v cost to travel from node i to node j with vehicle v

t_{ij}^v travel time from node i to node j with vehicle v

Q^v capacity of vehicle v

T^v shift duration of vehicle/route v

R_i maximum ride time of request i

Decision Variables

$$x_{ij}^v = \begin{cases} 1, & \text{if arc}(i, j) \text{ is traversed by vehicle } v \\ 0, & \text{otherwise} \end{cases}$$

L_i^v load of vehicle v when depature node i

A_i^v arrival time of vehicle v at node i

B_i^v time of beginning service of vehicle v at node i

With this notation, the AARP problem can be modeled on a digraph $G=(N,A)$, where N is the set of all nodes, $N=P\cup D\cup\{0,2n_O+2n_I+1\}$ and A is the set of directed arcs, $A=\{(i,j):i,j\in N, i\neq 2n_O+2n_I+1, j\neq 0, i\neq j\}$.

4.3.1 The General AARP Formulation

The general formulation of the AARP builds on the existing formulations for the VRPBTW (Parragh et al., 2008a) and PDPTW (Ropke and Cordeau, 2009).

$$\min \sum_{v\in V}\sum_{(i,j)\in A} c_{ij}^v\cdot x_{ij}^v+\sum_{v\in V}\sum_{j\in P} C_w^v\cdot(B_j^v-A_j^v) \tag{4-1}$$

$$s.t. \quad \sum_{v\in V}\sum_{j:(i,j)\in A} x_{ij}^v=1 \qquad \forall i\in P, \tag{4-2}$$

$$\sum_{j:(i,j)\in A} x_{ij}^v-\sum_{j:(i,j)\in A} x_{n_O+n_I+i,j}^v=0 \qquad \forall i\in P, v\in V, \tag{4-3}$$

$$\sum_{j:(0,j)\in A} x_{0j}^v=1 \qquad \forall v\in V, \tag{4-4}$$

$$\sum_{j:(i,j)\in A} x_{ji}^v-\sum_{j:(i,j)\in A} x_{ij}^v=0 \qquad \forall i\in P\cup D, v\in V, \tag{4-5}$$

$$\sum_{i:(i,2n_o+2n_I+1)\in A} x_{i,2n_O+2n_I+1}^v=1 \qquad \forall v\in V, \tag{4-6}$$

$$-M(1-x_{ij}^v)\le B_j^v-B_i^v-s_i-t_{ij}^v\le M(1-x_{ij}^v) \qquad \forall (i,j)\in A, v\in V, \tag{4-7}$$

$$-M(1-x_{ij}^v)\le A_j^v-A_i^v-t_{ij}^v\le M(1-x_{ij}^v) \qquad \forall (i,j)\in A, v\in V, \tag{4-8}$$

$$L_0^v=L_{2n_O+2n_I+1}^v=0 \qquad \forall v\in V, \tag{4-9}$$

$$-M(1-x_{ij}^v)\le L_j^v-L_i^v-q_j\le M(1-x_{ij}^v) \qquad \forall (i,j)\in A, v\in V, \tag{4-10}$$

$$e_i-A_i^v<M(1-y) \qquad \forall i\in N, v\in V, \tag{4-11}$$

$$-(L_i^v-q_i)\le M\cdot y \qquad \forall i\in N, v\in V, \tag{4-12}$$

$$\max(0,q_i)\le L_i^v\le\min(Q^v, Q^v+q_i) \qquad \forall i\in N, v\in V, \tag{4-13}$$

$$\max(e_i,A_i^v)\le B_i^v\le l_i \qquad \forall i\in N, v\in V, \tag{4-14}$$

$$B_i^v+t_{i,n_O+n_I+i}^v\le B_{n_O+n_I+i}^v \qquad \forall i\in P, v\in V, \tag{4-15}$$

$$B^v_{2n_O+2n_I+1} - B^v_0 \le T^v \qquad \forall v \in V, \qquad (4\text{-}16)$$

$$B^v_{n_O+n_I+i} - (B^v_i + s_i) \le R_i \qquad \forall i \in P, v \in V, \qquad (4\text{-}17)$$

$$x^v_{ij} \in \{0,1\} \qquad \forall (i,j) \in A, v \in V. \qquad (4\text{-}18)$$

The objective function (4-1) minimizes total routing cost. C_w is unit cost of vehicle waiting. The cost c^v_{ij} in the function is expressed in equation (4-19), which includes costs related to vehicle travel distance and time.

$$c^v_{ij} = C_d * d_{ij} + C_t * t^v_{ij}, \qquad (4\text{-}19)$$

where C_d , and C_t are unit costs of vehicle travel distance and travel time, respectively. d_{ij} is the distance between node *i* and *j*.

Constraints (4-2) and (4-3) ensure that every node is visited exactly once and pickup and dropoff nodes associated with a particular request are visited by the same vehicle, respectively. Each route starts and ends at a holding lot as required in Constraints (4-4) and (4-6), respectively. Constraints (4-5) enforce flow conservation. Constraints (4-7)-(3-8) and (4-9)-(4-10) guarantee consistency between time and load variables. Constraints (4-11) and (4-12) ensure that a vehicle does not idle while carrying passengers. Capacity and time window constraints are imposed by inequalities (4-13) and (4-14), respectively. Constraints (4-15) force the pickup node to be visited before the dropoff node for each request. The maximum route duration is restricted in Constraints (4-16). The passenger maximum ride time constraints are specified in inequalities (4-17), followed by integrality constraints expressed by Constraints (4-18). Constraints (4-2), (4-4)-(4-6), (4-13) and (4-15) are used in both formulations of (Parragh et al., 2008a) and (Ropke and Cordeau, 2009). Constraints

(4-7), (3-9) and (4-15) are included in the formulation given in (Parragh et al., 2008a), while constraints (4-3) and (3-14) are included in the formulation of (Ropke and Cordeau, 2009). Constraints (4-8), (4-10) - (4-12) and (4-16)-(4-17) are unique to the AARP. Note that if all vehicles are identical, superscript v in $c_{ij}^{v}, t_{ij}^{v}, Q^{v}$ and T^{v} can be eliminated for the AARP formulation.

The formulation is designed to be general and directly applicable for Policy 2. Small adaptations are required for the application of Policies 1 and 3 as described next.

4.3.2 Adaptation for Policy 1

To apply the formulation where Policy 1 is implemented, the problem can be posed as two separate DARPs, one for outbound trips and the other for inbound trips. To specify the DARP for outbound trips, the AARP formulation can be applied by presetting certain variables. Specifically, $n_I = 0, P_I = D_I = \phi$ for the outbound problem and $n_O = 0, P_O = D_O = \phi$ for the inbound problem.

4.3.3 Adaptation for Policy 3

For the AARP under Policy 3, additional constraints (4-20) are required to ensure that each vehicle drops off its outbound passengers before picking up its inbound passengers. This can be implemented by restricting arcs between inbound and outbound customer location nodes. That is, no arc can exist in a route directly connecting any inbound pickup location to an outbound dropoff location. This precludes any tour from allowing a sequence in which an inbound request is served before all outbound dropoffs are completed.

$$x_{ij}^{v} = 0 \qquad \forall i \in P_I, j \in P_O \cup D_O, v \in V \tag{4-20}$$

The AARP is difficult to solve directly, since the number of decision variables increases exponentially with increasing problem size (number of nodes to be visited). The proposed formulation was implemented directly in the IBM IOG CPLEX package on a personal computer with Intel(R) CPU 3.10GHz and 4.0GB RAM. The required computational time was exceptionally long. In a reduced version of the problem instance with only 10 outbound trips and 10 available vehicles, the solution was obtained after more than 6 hours, which is unacceptable in practice. Thus, in the next section, an alternative exact solution method is proposed.

4.4 Exact Solution Method

A CPCG solution methodology is proposed for exact solution of the AARP. A column generation mechanism is employed wherein the AARP is decomposed into a master problem (MP) and a subproblem (SP). A restricted linear relaxation of the MP (LMP) is solved and optimal dual variables associated with the requests served Constraints (26) are set in solving the SP. Solution of the SP is obtained through a constraint programming (CP) methodology. An overview of the proposed CPCG is provided in Figure 4-2.

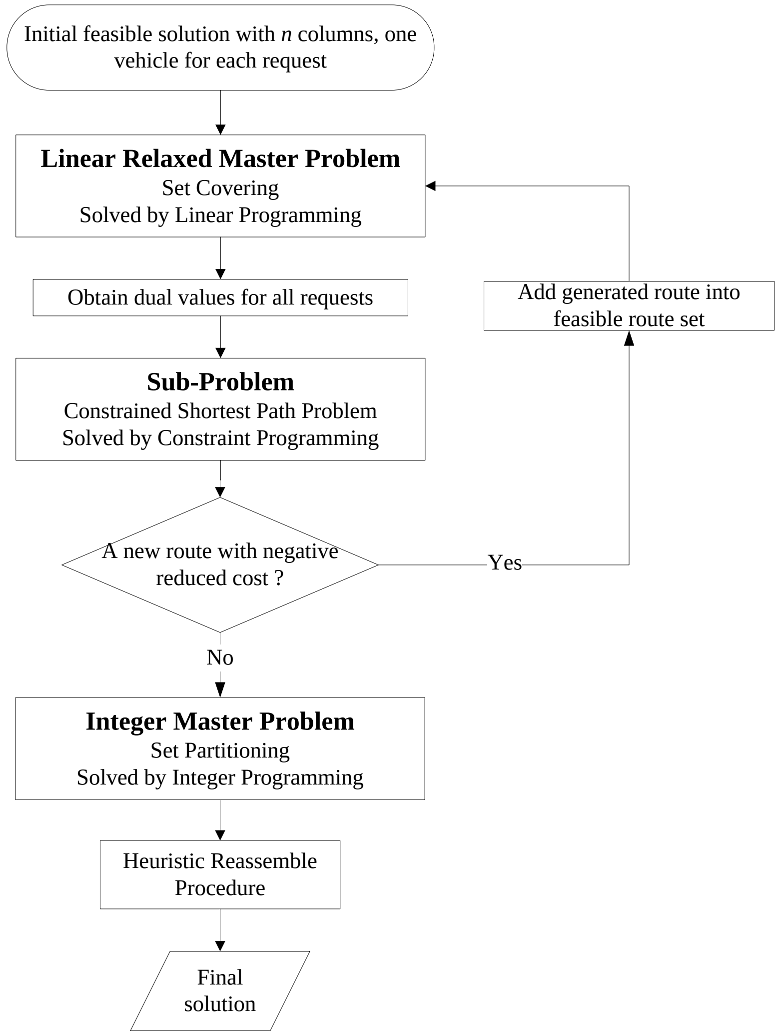


Figure 4-2 Flowchart of Exact Solution Method

The procedure starts by feeding the LMP, modeled as a set covering problem, with a feasible solution consisting of a set of vehicles, each of which serves one request. The SP is a constrained shortest path problem. Solution of the SP produces a route with negative reduced cost. This route is added to the route set (or column set) used in the next iteration in which solution of the LMP is repeated. This process terminates when solution of the SP does not produce a route that is not already included in the column pool with negative reduced cost. With the final column pool (route set), the integer MP, a set partitioning problem, is solved. The route obtained from solution of the integer MP is reassembled through a proposed heuristic procedure to generate the final solution. Details associated with the MP, SP and heuristic reassemble procedure are provided next.

4.4.1 Master Problem

Assume that all vehicles are identical and let Ω denote the set of feasible routes satisfying constraints (4-3)-(4-17). For each route, $r \in \Omega$, let c_r be the cost of the route and a_{ir} be a binary constant indicating whether or not a node $i \in P$ is visited by route *r*. Let y_r be a binary variable equal to 1 if route $r \in \Omega$ is selected, and 0 otherwise. The AARP can be reformulated as the following *set partitioning problem (MP-SPP).*

$$\text{(MP-SPP)} \qquad \min \sum_{r \in \Omega} c_r \cdot y_r \tag{4-21}$$

$$s.t \sum_{r \in \Omega} a_{ir} y_r = 1 \qquad \forall i \in P \tag{4-22}$$

$$y_r \in \{0,1\} \qquad \forall r \in \Omega \tag{4-23}$$

The objective (4-21) minimizes the cost of the chosen routes. Constraints (4-22) ensure that every request is served once.

It is impractical to explicitly enumerate all feasible routes in Ω. Instead, as is typical, only a subset $\Omega' \subset \Omega$ is considered. This subset is expanded iteratively by adding a route with negative reduced cost through solution of the SP. The reduced cost of a route is expressed by equation (4-24).

$$\hat{c}_{ij} = \begin{cases} c_{ij} - \pi_i, & \forall i \in P, (i,j) \in A \\ c_{ij}, & \forall i \in N \setminus P, (i,j) \in A \end{cases} \tag{4-24}$$

where π_i is the dual value associate with the i^{th} constraint (4-22). The MP is given next.

$$\text{(LMP)} \qquad \min \sum_{r \in \Omega'} c_r \cdot y_r \tag{4-25}$$

$$s.t. \quad \sum_{r \in \Omega'} a_{ir} y_r \geq 1 \qquad \forall i \in P \tag{4-26}$$

$$y_r \geq 0 \qquad \forall r \in \Omega' \tag{4-27}$$

This relaxation allows every request to be served more than once rather than only once. Constraints (4-27) relax integrality constraints.

4.4.2 Sub-problem

The SP formulation is given next.

$$\text{(SP)} \ \min \sum_{(i,j) \in A} rc_{ij} \cdot x_{ij} + \sum_{v \in V} \sum_{j \in P} C_w \cdot (B_j - A_j) \tag{4-28}$$

s.t. Constraints (4-3)-(4-17).

4.4.3 Constraint Programming for Sub-Problem

In the CP approach, each decision variable has a domain. For example, in the SP, the domain of each arc, x_{ij}, is $\{0,1\}$. Similarly, the domain of load L_i is $\{0, 1, \ldots, Q\}$. Initially, the search space contains all combinations of the values in the domains of all decision variables. To avoid exploring the entire search space, CP first removes inconsistent values from the domains of the variables involved in each constraint. Then a search strategy (depth first, width first or multi-start) is applied to guide the search for a solution within the reduced search space. The search process can be viewed as traversing a tree, where the root is the starting point, a leaf node is a combination of values in the reduced search space and each branch represents a move (branching) within the search. A solution is a set of value assignments to the decision variables such that each variable is assigned to exactly one value from its domain. Together these values satisfy all constraints and minimize the objective function. Each leaf node is evaluated to determine if it will produce a feasible solution.

Two measures are suggested for speeding up the process of finding a feasible solution: 1) eliminate ineligible decision variable settings from the initial search space, wherein those decisions that include starting from the end depot, ending at the starting depot, selfloops, or that would violate Constraints (4-13) - (4-15) are excluded, and 2) set branching limits for the route generation process. As mentioned in (Irnich and Desaulniers, 2005), in the context of column generation, optimality of the SP is only necessary to prove that no negative reduced cost routes exist in the last iteration, and feasible solutions to the SP are sufficient for preceding iterations. Thus, a lower branching limit (10^6) is used for these nonfinal iterations, while higher branching limits (10^8) are applied in the last iteration.

4.4.4 Heuristic Reassembly Procedure

The optimal solution to the LMP is obtained when there are no remaining routes with negative reduced cost to the SP. Unfortunately, this solution is not always integervalued. A branching scheme was proposed in (Dumas et al., 1991) to address this issue through adding additional arc flow constraints to the SP and resolving it. This process is repeated for each branching decision taken in the MP. The following observations are made:

(1) The MP starts with a feasible solution in which one vehicle serves one request;
(2) Each iteration generates a single unique route;
(3) The newly generated routes that are selected by solution of the MP-SPP are always a subset of the newly generated routes that are selected by solution of the set covering problem.
(4) The solution to the MP-SPP always includes one or more initial feasible routes.

Since solution from MP-SPP provides useful information, the following heuristic applies.

Step 1. Calculate the value of *V* = route cost/number of request served for each route selected by the MP-SPP.

Step 2. Select the route *r* with maximum *V*. Try to extract the first unvisited request on route *r* and insert it into the best feasible position on one of the other routes, *r'*. If this decreases the total cost, update *r* and *r'*, and go to Step 1. Otherwise, mark this request as visited and move to the next request in *r*, if all requests in *r* have been visited, stop.

Step 3. Repeat Step 1

4.5 Heuristic Solution Approaches

Two heuristics proposed in the literature were modified for solution of the AARP. An overview of each is given first, followed by the modifications required to address the three variants of the AARP.

4.5.1 Jaw's Heuristic

The first heuristic considered for solution to the AARP is the sequential insertion procedure originally proposed by Jaw (Jaw et al., 1986) for the DARP problem. The algorithm processes each request in an unrouted request list (URL) in sequence, and assigns each request to a vehicle until the URL is exhausted. The main steps of Jaw's sequential insertion procedure are summarized as follows.

Step 1: Sort URL by the requested pickup times in increasing order. Create a route from the depot and back to the depot. Set r =1.

Step 2: Select the first unrouted request u from URL. Find all feasible insertion positions within all existing routes, 1 to r.

(i) If a feasible insertion position is found, assign the request u to the route r* with minimum insertion cost, and update route r*.

(ii) If no feasible insertion position exists, create a new route from the depot to request u, and add a return to the depot. Set r = r+1.

Delete u from URL.

Step 3: Repeat step 2 until URL is empty.

The *additional insertion cost* to route r of inserting request u is calculated as the difference between the total cost of route r after the insertion minus its cost before the insertion. This is expressed in (4-29).

$$\sum_{i,j \in new^r} c_{ij}^r - \sum_{i,j \in old^r} c_{ij}^r , \qquad (4\text{-}29)$$

where new^r denotes route r after insertion of request u and old^r denotes route r before insertion of request u.

4.5.2 Solomon's Insertion I1 Heuristic

The second heuristic considered here, Insertion I1, was proposed by Solomon (Solomon, 1987) for the VRPTW. Insertion I1 constructs routes one at a time. For the first created route, a tour is developed from the depot to a "seed" request, which returns to the depot. Remaining requests are considered for insertion in the route. The cost of insertion of all remaining unrouted requests is computed. The request with the minimum insertion cost that can be feasibly inserted is selected. Insertion of additional requests is considered until no remaining unrouted request can be feasibly inserted. A new route is then created. The process is repeated until all requests have been included in a tour. At each iteration in which a new route is created, the remaining unrouted request with the minimum value of $-\alpha d_{0i} + (1-\alpha) l_i$, $0 \le \alpha \le 1$, $i \in D_O$ for outbound trips and $i \in P_I$ for inbound trips is selected as the seed. Trips that are far from the depot and have an earlier deadline are, thus, favored in choosing the request.

The main steps of Insertion I1 can now be summarized:

Step 1: Initialize $r = 0$.

Step 2: Set $r = r+1$. Select the 'seed' request u^* with the minimum value of $-\alpha d_{0i} + \beta l_i$ from URL for inclusion in route r. Add u^* to route r and delete it from URL. If URL is empty, stop.

Step 3: For each remaining unrouted request u in URL, find the feasible insertion position in route r, if a feasible insertion exists, that minimizes the *additional insertion cost* (equation (21)).

(i) If a feasible insertion exists, select request u^* with the minimum *additional insertion cost* (equation (21)), and insert this request at its best feasible insertion position in route r. Update route r, and delete u^* from URL.

(ii) If there is no feasible insertion of any unrouted request in route r, go to step 2.

Step 4: Repeat step 3 until URL is empty.

The two heuristics are quite similar, but differ in one fundamental aspect relating to the choice of a feasible insertion position for the unrouted requests. In Jaw's heuristic, for each selected unrouted request u, its best insertion position within all constructed routes is evaluated and the insertion is made accordingly. When a request cannot be feasibly inserted in any existing route, a new route is constructed. The request is inserted in the new route. The next unrouted request from a list that was not yet tested will be considered for inclusion in this expanded set of constructed routes. In Insertion I1 this evaluation is conducted over only the most recently constructed route. The list of unrouted requests must be considered and any possible insertions must be made in that route before considering insertion in another route.

Both heuristics as described can be used directly on the AARP with Policies 1 and 2. For Policy 3, however, feasibility is further restricted by outbound and inbound trip separation requirements. Both heuristics can be adapted to deal with this additional constraint. Specifically, modifications are made when choosing the best feasible insertion position for each unrouted request: if the selected unrouted request u is an outbound trip, its dropoff location must be inserted before the pickup of the first inbound trip, given that there are inbound trips in the current route. Likewise, if the unrouted request u is an inbound trip, its pickup location must be inserted after the dropoff location of the last outbound trip, assuming there is an outbound trip in the current route.

4.5.3 Checking Solution Feasibility

Both heuristics ensure that the problem constraints associated with time windows, precedence and pairing, maximum ride time, shift duration for drivers and vehicle capacities are satisfied during the insertion process. An insertion of a request in a route is feasible only if it does not lead to violation of any of these constraints by inclusion of this request. Moreover, its inclusion should not create other violations of these constraints for other requests already included in the route. The implementation of these constraints during this process is important and is described next.

Time Window Constraints. Time window feasibility is maintained in a route if the insertion of a new request does not push the vehicle arrival time at any node i past its latest service time l_i. While a vehicle without a passenger onboard is permitted to arrive at a pickup node earlier than its earliest service time e_i, thus incurring an additional waiting cost, no vehicle is permitted to idle while carrying passengers. The procedure proposed in (Jaw et al., 1986) is applied for the calculations of earliest service time, e_i, and latest service time, l_i.

To ensure that time window constraints are met, we must check that e_i and l_i fall within each request's time window for each i in the route and for requests considered for inclusion.

Precedence and Pairing Constraints. For any insertion of a new request, precedence and pairing constraints are ensured by simultaneously inserting both the pickup and dropoff locations associated with a single request within the route. The pickup location must precede the dropoff location.

Maximum Ride Time Constraints. For each considered insertion of a request, the insertion must not cause a violation in constraints (4-17), whether directly for the request or for other requests already inserted in the route. The maximum ride time R_i is a function of direct (shortest path) ride time DRT_i. Herein, a piecewise linear function (4-30) is applied:

$$R_i = \begin{cases} 3 \cdot DRT_i, \text{ if } DRT_i \leq 30 \\ 2 \cdot DRT_i, \text{ if } 30 \leq DRT_i \leq 60 \\ DRT_i + 30, \text{ if } DRT_i > 60 \end{cases} \tag{4-30}$$

Shift Duration Limit for Drivers. Any insertion of a new request cannot extend the route duration over the shift duration limit for a driver as expressed by constraints (4-16). Thus, shift duration must be assessed for each insertion of a request.

Vehicle Capacity Constraints. Any insertion of a new request must adhere to capacity constraints (4-13). Thus, no insertion is made if its inclusion will cause the vehicle to exceed its capacity. This must be assessed at each potential insertion location, because the number of requests handled at any point in time changes over the route duration.

4.6 Numerical Experiments

4.6.1 Experiment Design

To investigate the efficiency of the proposed solution approach, the solution methods are tested on a real-world problem instance. The test case involves service records for one service day in January of 2012 out of Washington Dulles International Airport

(IAD). It includes 164 outbound requests involving 212 passengers and 22 inbound requests involving 41 passengers. For each request, detailed information, including desired pickup time, number of passengers, latitude and longitude of pickup and dropoff locations, and assigned vehicle index are also included. All requests were served by a fleet of identical vehicles. Figure 4-3 shows the partial distributions of the requested pickup (inbound) and dropoff (outbound) locations. The service area covers Maryland, District of Columbia, Virginia and Pennsylvania. Distances and travel times between pairs of customer locations were calculated through the OD Cost Matrix Tool in the Network Analyst toolbox of ArcGIS. The travel time is computed based on the shortest distance and speed limits.

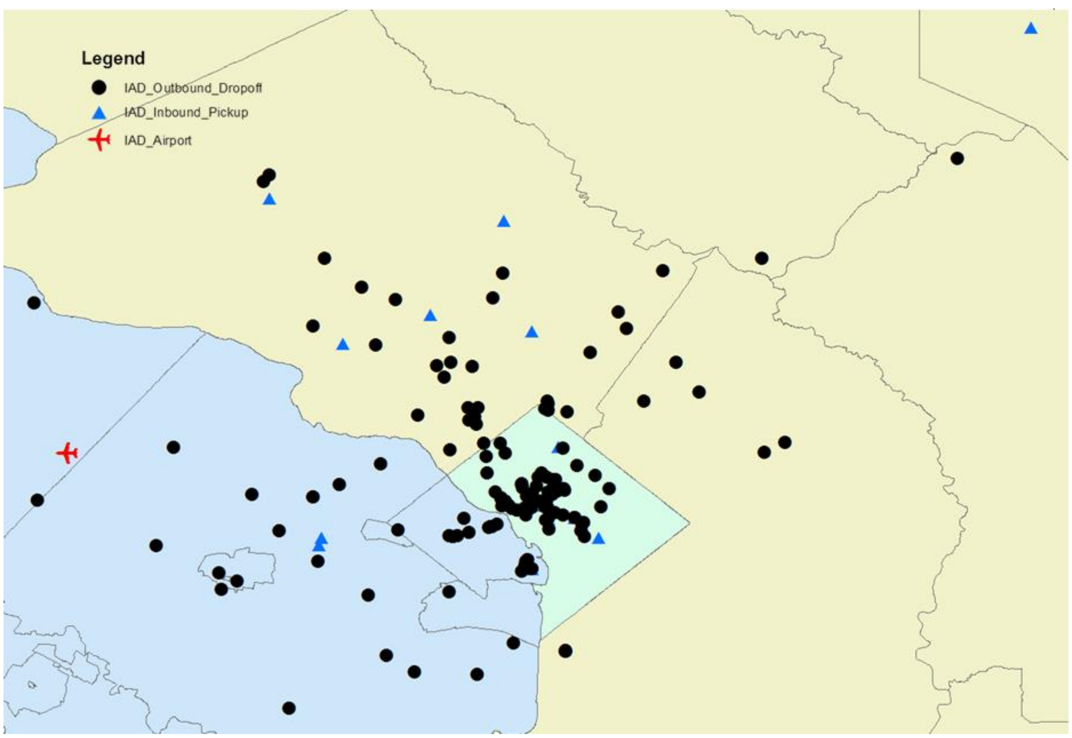


Figure 4-3 Distributions of Pickup and Dropoff Locations

Parameters of the model and the algorithms are presented in Table 4-1. The solution methods were implemented in Visual C++ 2010 and run on a personal computer with Intel(R) CPU 3.10GHz and 4.0GB RAM. The subproblem of proposed exact solution method was solved by the C++ Concert Technology of CP (Constraint Programming) solver in the IBM-ILOG CPLEX.

Table 4-1 Parameters of Proposed Model and Algorithms

Parameters	Explanation	Values
C_t	Unit cost of time	0.54 $/min
C_d	Unit cost of distance	0.72 $/mile
C_w	Unit cost of vehicle waiting time	0.23 $/min
Q	Vehicle capacity	7 passengers
V	Maximum fleet size	30
s	Identical service time	3 minutes
T	Shift duration	10 hours
α, β	Weight parameters	α=0.8, β=0.2
TW	Pre-specified maximum deviation from desired time	45 minutes

4.6.2 Algorithm Performance

The CPCG approach was tested on cases with 10, 20 and 30 requests under the most general policy, Policy 2. Computational time increases exponentially with increasing number of customers. Thus, solution of problem instances with significantly more than 30 customers is precluded. The results are compared with those obtained through the adapted Jaw's algorithm in Table 4-2. The numbers in parentheses are outbound and inbound requests, respectively. Results show that the maximum gap between the exact solution and the adapted Jaw's algorithm is approximately 7% (with 20 requests), but the computational time is around 1/1200 of that of CPCG.

Table 4-2 Comparison of Results from CPCG and Adapted Jaw's Algorithm

	CPCG			Adapted Jaw's Algorithm		
Number of Requests	10(7+3)	20(14+6)	30(22+8)	10(7+3)	20(14+6)	30(22+8)
Total Cost	314.9	581.1	839.6	314.9	625.0	853.4
Vehicle Use	2	2	4	2	3	4
Computational Time(s)	360. 5	2040.2	8940.3	0.7	3.5	7.6

4.6.3 Policy Performance

Computational results obtained by applying the two heuristics for each of the three operational policies are shown in Table 4-3. From Table 4-3, two significant conclusions can be reached. First, Jaw's heuristic outperforms Insertion I1. For all three policies, the computation time required by Jaw's heuristic is only between 14 and 25% of that required by Insertion I1. The longer computation time of Insertion I1 can be explained by the requirement of assessing the insertion of every unrouted request when each route is constructed. For each of the three operational policies, the total cost of the routes built through Jaw's heuristic is below that developed by Insertion I1. This may be due to the 'seed' selection process of Insertion I1, where the furthest unrouted request is selected for inclusion. The long distance to this request may lead to longer empty vehicle miles, and thus, longer route duration and total cost. Moreover, for each of the three operational policies, the routes built through Jaw's heuristic have higher utility factors (higher average occupancy, lower passenger miles and higher average utilization) than those from I1.

Table 4-3 Performance Comparisons of Two Heuristics

Performance Measures	Policy 1						Policy 2		Policy 3	
	Jaw's			Solomon's			Jaw's	Solomon's	Jaw's	Solomon's
	Outbd	Inbd	Total	Outd	Inbd	Total				
Number of Vehicles	17	4	21	17	4	21	17	19	20	21
Total Idle Time[1]	1051	159	1211	856	255	1110	929	1430	1201	1603
Total DH1Time[2]	0	215	215	0	350	350	52	18	151	49
Total DH1Mile[3]	0	200	200	0	327	327	49	17	141	46
Total DH2Time[4]	933	0	933	1151	0	1151	562	1029	723	1161
Total DH2Mile[5]	871	0	871	1074	0	1074	524	961	674	1083

Total EDTime[6]	1696	516	2213	2223	548	2771	1537	1802	1861	2141
Total EDMile[7]	1583	482	2065	2075	511	2586	1434	1682	1737	1998
Total LDTime[8]	3082	586	3668	2798	590	3388	3669	3424	3690	3599
Total LDMile[9]	2877	547	3424	2611	550	3162	3424	3195	3444	3359
Route Duration	6883	1414	8298	6947	1557	8505	7319	7877	7953	8615
Average Occupancy	1.5	1.1	1.3	1.2	0.8	1.0	1.6	1.3	1.4	1.1
Average Passenger Mile	25.9	25.6	25.7	26.9	30.5	28.7	24.7	25.7	25.3	25.8
Average Utilization[10]	7.6	6.1	7.3	6.9	6.1	6.7	9.0	7.5	7.7	7.1
Total Cost	6034	1372	**7406**	6282	1437	**7719**	**6523**	**6662**	**7005**	**7325**
CPU Seconds	27.1			173.1			37.4	274.8	28.6	116.0

[1]Sum of all waiting times incurred by a vehicle along its route (min); [2]Empty driving time from depot to the first pickup (min); [3]Empty driving distance from depot to the first pickup (mile); [4]Empty driving time from last dropoff to the ending depot (min); [5]Empty driving distance from last dropoff to the ending depot (mile); [6]Driving time without passengers on board (min); [7]Driving distance without passengers on board (mile); [8]Driving time with one or more passengers on board (min); [9]Driving distance with one or more passengers on board (mile); [10]Total LDTime/(24*Number of Vehicles).

Second, both heuristics reveal that Policy 2 will provide the best performance in terms of number of needed vehicles, idle time, empty/loaded driving time or miles traveled, and total cost, Policy 3 the second best performance, and Policy 1 the worst performance. That Policy 2 provides the best performance is not surprising and can be shown theoretically, because it is the least constrained of the three variants. From run results of Jaw’s heuristic, Policy 2 requires the fewest vehicles, lowest idle time, lowest empty vehicle miles, and lowest total cost of the three policies. Accordingly, Policy 2 has the highest vehicle utilization rate of the three. The vehicle utilization rate of Policy 3 is significantly above that of Policy 1, but below that of Policy 2. This difference in vehicle utilization rate is caused by requirements for ordering outbound and inbound operations with Policies 1 and 3.

To assess the value of this optimization-based approach, solutions obtained from the heuristics were compared against manually derived routes used to deploy the vehicle fleet on the date of the case study. In actual operations on the date of service, Policy 2 was employed. From records maintained for this date, 37 vehicles were employed. Stringing the vehicle routes together where feasible would permit completion by as few as 28 vehicles. Many of the routes did not comply with maximum ride time constraints and several violated constraints that prohibit waiting with a passenger onboard. Of course, violations were addressed during actual operations. By comparison, results from the proposed heuristic for the AARP under Policy 2 required only 17 vehicles to serve the same requests. This is an approximately 60% improvement in vehicle utilization

4.7 Conclusions and Future Work

In this Chapter, the AARP is formulated as a nonlinear integer program. Three implementations corresponding to three different operational policies under consideration in practice are investigated. Exact and heuristic solution procedures are proposed. The performance of the proposed solution approaches is compared in a case study involving data from one day's operation of an actual service provider. Exact solution could not be obtained for the full-version of the case study, but exact solution was obtained for a reduced version with 30 customer requests. In a comparison to the exact solution, results of the adapted Jaw's algorithm were within 7% of the exact solution, and required only 1/1200 the computation time. In the original case study, the adapted Jaw's algorithm outperformed the second proposed heuristic. Thus, the heuristic is an effective and efficient approach for addressing the AARP, yielding significantly better results than routes and schedules determined manually.

The proposed methodologies also have applicability to other routing and scheduling applications involving 'one-to-many-to-one' operations. Mail service is one example. In these services delivery of mail from a local distributor to a set of destinations and collection of mail from a set of origins for return to the local distributor is required. Other possible applications arise in reverse logistics operations, such as the delivery of full bottles and collection of empty ones between a manufacture and retailers. The reverse logistics problem is simpler than the airport shuttle and mail services applications, because the goods to be transported are identical. Thus, every unit to be picked up can equally satisfy customer demand.

While the heuristics described herein provide good results with low computational effort, more sophisticated heuristics may provide improved solutions. Both described heuristics are construction heuristics. Thus, constructed routes can be improved through the application of improvement operators, such as λ-interchange, 2-opt* exchange, trip exchange and trip reinsertion. A cluster-first route-second methodology may also address this myopic nature. Clustering can be based on both temporal and spatial characteristics of the pickup and dropoff locations. The author is currently investigating these and other improvements.

In a dynamic setting, new requests may be received on short notice while some vehicles are en route. The operator must quickly insert these new requests within previously constructed routes and schedules. In the airport operations of the case study, most inbound requests are known in advance, but almost all outbound trips arise dynamically. A fast algorithm to find a good feasible insertion for the new requests is required. The author is working to extend the developed model and solution methodologies for use in such a dynamic setting that considers not only dynamic requests, but also uncertainty in travel and service times.

Chapter 5 Sheltering and Paratransit Operations for Mobility-Impaired Populations Evacuation

5.1 Introduction

Populations in urban areas are vulnerable to disaster, whether due to natural, accidental or malicious causes. Evacuation is often the most viable response action to reduce the adverse consequences to affected populations in these circumstances. Moreover, shelters play a critical role in many evacuation situations, providing safe, temporary housing to affected individuals. They are often located at schools, municipal buildings, places of worship and other places that are easily accessed via public or private transportation by the general population. Individuals may shelter until the disaster impact has subsided or be further evacuated from the impacted area.

While in most urban areas the majority of evacuees will use an automobile to evacuate the area or seek shelter, not all people in risk-prone areas will own or have access to personal vehicles during an evacuation. According to the U.S. Census, greater than 30% of all households in several metropolitan cities, including New York, Washington, D.C., Baltimore, Philadelphia, Boston, Chicago, and San Francisco, are carless (U.S. Census Bureau, 2010). These carless people, and perhaps others seeking to shelter at an official shelter location, depend on public transit. However, there are significant numbers of people with low-mobility who cannot access a fixed route public transit system. According to the U.S. Census (U.S. Census Bureau, 2010), 2.8% of households had at least one mobility-impaired member in 2010. Furthermore, as the population ages, increasing numbers of people will have mobility restrictions.

Most of these individuals would not be able to drive even if they had access to personal vehicles. In this chapter, those who cannot neither drive nor access public transit and would require mobility assistance during evacuation are referred to as mobility-impaired persons.

The mobility-impaired population requires a greater level of assistance than does the general population when sheltering. For example, wheelchair access must be provided, medical assistance might be required and specialized personnel and equipment may be needed. The mobility-impaired often rely on specialized equipment or medical assistance. In an emergency evacuation, it may be necessary to transport such persons to shelters in which such equipment and assistance are provided. As specially trained personnel and require equipment are limited resources, it would be beneficial to concentrate sheltering efforts for this population at a subset of the potential shelters designed for the general population.

Regardless of the number and location of shelters with facilities to support the needs of the mobility-impaired, door-to-door service is required, since these individuals would find it difficult or impossible to access the general public transit stops. Vehicles attending to these individuals should also be equipped to transport wheelchairs and other medical or mobility equipment. Thus, paratransit vehicles could be an efficient, if not the only, solution. These paratransit services can be provided by local paratransit operators who have existing contracts with their local governments, have appropriate vehicle fleets within their own holding lots, and are acquainted with the needs of this population. Such contracts are maintained as part of a solution needed to ensure that all Americans have access to public transit as required by the Americans with Disability Act.

This chapter proposes optimization-based techniques for optimally deploying such a fleet of paratransit vehicles to assist the mobility-impaired population in evacuating from their homes or other chosen locations to a set of selected, specially equipped shelters. Specifically, the problem of choosing the subset of shelters at which to house the mobility-impaired population during the disaster event, assigning the mobility-impaired evacuees to the selected shelters based on their home locations and simultaneously designing a set of vehicle routes to minimize total costs is formulated as a mixed integer program. Total costs include the fixed cost of operating shelters that can support this mobility-impaired population and operational costs of transporting these individuals to their assigned shelters. This problem is referred to herein as the Sheltering and Paratransit Evacuation Problem (SPEP).

The SPEP captures many practical considerations through its constraints. Assignments of individuals to shelters are made with attention to the distance that each individual would need to travel and shelter capacity limitations for serving this population. In addition, the problem formulation accounts for the number of vehicles available to provide services, the relative location of holding lots, vehicle seating and equipment capacities, maximum driving distances, and the maximum time any passenger spends on board. Moreover, the SPEP ensures that no intermediate stops are made at shelters where only a portion of the passengers disembark. A solution that allows only a portion of the passengers in a single vehicle to reach safety, forcing other passengers to incur additional risk while continuing onward to a second shelter, would not be palatable. Finally, vehicles (and drivers) may be expected to perform multiple tours each with potentially different shelter destinations.

The SPEP can be viewed as a location-allocation problem, where the location-allocation results influence the optimal transport tours. The influence of the location-allocation decisions on optimal tour construction is illustrated in Figure 5-1, where three different tours are constructed for a specific vehicle with capacity of 7 from a given holding lot under different shelter location-allocations. In Figure 5-1(a), only one shelter S is open and all pickups are assigned to it, while in Figure 5-1(b) and (c) two shelters S1 and S2 are open instead. In Figure 5-1(b) pickups *a* through *f* are assigned to shelter S1 and *g*, *h* and *i* are assigned to shelter S2. In Figure 5-1(c), customer pickup location *c* is assigned to shelter S2. Comparison of Figure 5-1(a) and (b) indicates that the change in shelter location might affect the tours dramatically. Even with the same location decisions in Figure 5-1(b) and (c), a minor change in allocation decisions can lead to substantial changes in the optimal tours.

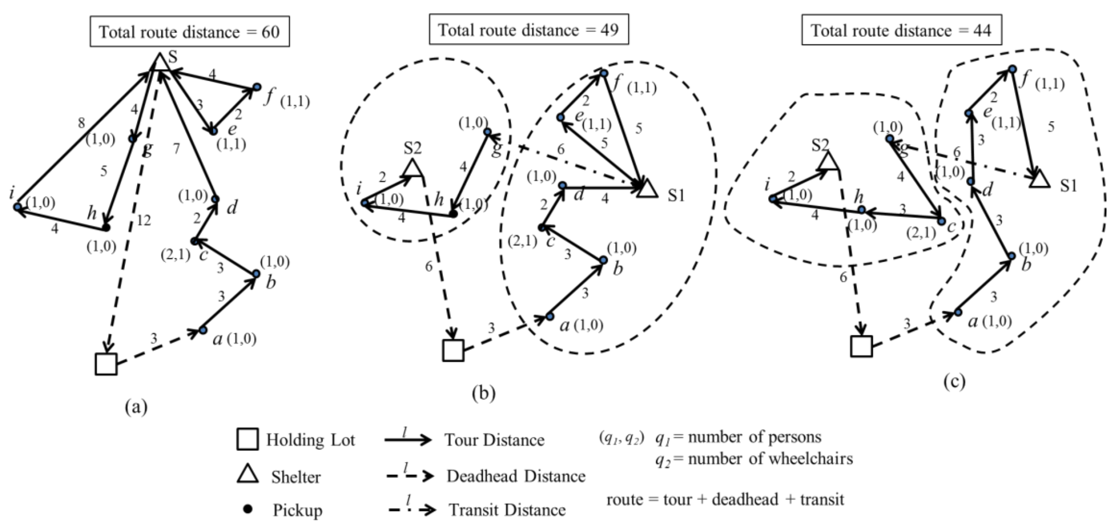


Figure 5-1 Illustration of One Vehicle Route with Different Assignments

This need to simultaneously tackle both location-allocation and routing decisions renders the SPEP a difficult combinatorial problem. Even if these problems are solved sequentially, exact solution of practical problem instances can be quite challenging. Thus, in this chapter, a sequential technique in which the location-allocation problem is solved first and the routing problem second, and a tabu search metaheuristic in which these problems are nested, are proposed. The nested structure captures the interactions between the two problem elements. The proposed approaches are applied on a case study involving large-scale evacuation of New York City (NYC). They are aimed at supporting local governments in planning for and carrying out an emergency evacuation of its residents with mobility impairments.

5.2 Related Literature

Public transit-based movement of carless people to shelters in an evacuation has received increasing attention in the literature over the last decade. This is in part due to increased awareness of the extra risks faced by carless people during emergency evacuation as became evident in Hurricanes Katrina and Rita in 2005 (Litman, 2006; Renne et al., 2008; TRB, 2008).

Several works have addressed the use of transit in evacuation. Margulis et al. (2006) developed a deterministic decision-support model for bus dispatching to maximize the number to egress in hurricane evacuation. This model assumes that evacuees are assembled at their closest pickup points and the locations of shelters are known. With a similar assumption of predetermined pickup and shelter locations, Sayyady and Eksioglu (2010) proposed a mixed integer linear program to optimize transit routing plans with the objective of minimizing total evacuation time for no-notice evacuation. In their work, transit vehicles are only allowed to perform one trip. Abdelgawad and Abdulhai (2010) and Bish (2011) formulated this transit-based evacuation problem as types of vehicle routing problems. The objectives of both models are to transport evacuees from preset pickup locations to known shelter locations in the minimal amount of time by efficiently routing and scheduling a fleet of buses from a set of bus yards. Both works propose heuristic solution concepts. More recently, Kulshretha et al. (2012) proposed a mixed integer linear program to determine the optimal pickup locations for evacuees to assemble. They simultaneously consider the allocation of available buses to transport the assembled evacuees between the pickup locations and specified shelters. While these works are related in that they assign pickup locations to shelters, they do not capture many of the elements of the SPEP (need for simultaneous routing, assignment and shelter location decisions), or they focus on aspects of an evacuation that are not relevant (selection of pickup locations at which evacuees will assemble).

The studies on public transit-based evacuation assume that the locations of shelters are known and fixed. Instead, they focus on designing a set of pickup locations where evacuees assemble to await transit services. Such assembly points cannot serve the mobility-impaired population given their mobility restrictions.

It seems that only one prior study in the literature has proposed tools to aid in decisions regarding the location of shelters for transit-based evacuation. Song et al. (2009) formulated this transit-based shelter location and evacuation problem in the context of hurricanes as a location-routing problem (LRP) with uncertain demand.

The LRP is used to determine optimal shelter locations and transit routes with the objective of minimizing total evacuation time. Shelter locations are chosen from a pool of potential locations based on their distances from the pickup locations assuming all pickup locations are assigned to their nearest open shelter. Only the transportation cost is considered in the objective and each bus is restricted to a single route per shelter. Thus, the authors did not explore the interrelations between the location-allocation problem and the routing problem.

The general LRP has received significant attention over past decades. Applications are primarily related to logistics. See (Nagy and Salhi, 2007) for a review of both models and solution methods for LRPs. Traditional LRP models determine where to locate facilities and how to distribute or collect goods to or from customers through simultaneously solving a joint location and routing problem. The SPEP differs from the LRP in several important ways. The SPEP involves two types of facilities: the holding lots for paratransit vehicles (depots) and shelters. The LRP has only depots from which vehicles start out and to which they return once the goods are distributed or collected. Additionally, the SPEP has several additional constraints, such as that evacuees should not spend an unreasonable amount of time onboard while additional pickups are made. The operating patterns of these two classes of problems also differ. Specifically, in the LRP, each vehicle is restricted to serve only one depot, while in the SPEP each vehicle is allowed to perform multiple tours for multiple shelters.

SPEP also has commonality with other ridesharing problems, including the Dial-A-Ride Problem (DARP) (see (Cordeau and Laporte, 2007) for a review of

DARP). Like SPEP, DARP is characterized by pairing and precedence constraints, such that for each request the origin must precede the destination and both locations must be visited by the same vehicle, and user inconvenience constraints, such as a maximum ride time limitation. A primary concern in DARP, however, is time windows for pickup and delivery of customers. The routing aspects of SPEP are similar, with some exceptions: (1) pickups assigned to the same shelter share an identical destination; (2) customers do not choose their time windows and instead are expected to be ready for the vehicle when it arrives; and (3) customers with different shelter destinations must be transported on different vehicles. Additionally, shelter destinations are chosen for the evacuees in the SPEP in coordination with routing decisions; whereas, customer destinations in DARP are set by the customers. Finally, the SPEP is a multi-depot type of ridesharing problem, since resources from multiple companies' fleets will be drawn upon.

Thus, it appears that no previous work has proposed optimization tools to support sheltering and routing decisions for the mobility-impaired population in an evacuation. This chapter seeks to fill this gap.

5.3 Mathematical Formulation

The SPEP is formulation next. Before proceeding to the formulation, notation is introduced.

5.3.1 Notation

w	Number of paratranist holding lots
$H = \{1, ..., w\}$	Set of paratranist holding lots
V_h	Number of vehicles at holding lot $h \in H$

$K = \{1,...,V\}, V = \sum_{h \in H} V_h$	Set of all vehicles in all holding lots
n	Number of pickup nodes
$P = \{w+1,...,w+n\}$	Set of pickup nodes
(q_i^p, q_i^w)	Number of persons and wheelchairs at pickup node $i \in P$
s_i	Service time at pickup node $i \in P$
R	Maximum onboard time for each client
m_{di}	Distance of pickup node $i \in P$ to its nearest shelter
m_t	Maximum onboard time for all passengers
η	Deviation parameter for the assigned distance to m_{d_i}
m	The number of potential shelters
$S = \{w+n+1,...,w+n+m\}$	Set of potential shelters
F_s	Fixed opening cost of shelter $s \in S$
Q_s	Capacity of shelter $s \in S$
C	Capacity of vehicle
D	Maximum driving distance for each vehicle
d_{ij}	Distance from node $i \in \{P \cup S \cup H\}$ to node $j \in \{P \cup S \cup H\}$
t_{ij}	Travel time from node $i \in \{P \cup S \cup H\}$ to node $j \in \{P \cup S \cup H\}$
C_d	Unit cost of driving distance for all vehicles
M	Arbitrary large number

5.3.2 Decision Variables

$$y_s = \begin{cases} 1, & \text{if shelter } s \in S \text{ is open} \\ 0, & \text{otherwise} \end{cases}$$

$$z_{is}^k = \begin{cases} 1, & \text{if pickup } i \in P \text{ is assinged to shelter } s \in S \text{ and is transported by vehicle } k \in K \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ij}^k = \begin{cases} 1, \text{ if arc}(i, j) \text{ is traversed by vehicle } k \in K \\ 0, \text{ otherwise} \end{cases}$$

L_i^k Load of vehicle $k \in K$ after visiting node $i \in H \cup P \cup S$

Furthermore, some auxiliary decision variables are needed for the vehicle routing.

$$u_{kh} = \begin{cases} 1, \text{ if vehicle } k \in K \text{ comes from depot } h \in H \\ 0, \text{ otherwise} \end{cases}$$

T_{ij}^k trip duration of vehicle $k \in K$, starting from node i and ending at node j

5.3.3 Formulation

Given the above notation, the SPEP can be defined on a digraph $G = (N, A)$, where N is the set of nodes, $N = H \cup P \cup S$, and A is the set of directed arcs $A = \{(i, j) : i, j \in N\}$ connecting the nodes.

$$\min \quad C_{SPEP} = \sum_{s \in S} F_s \cdot y_s + \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} C_d \cdot d_{ij} \cdot x_{ij}^k \tag{5-1}$$

$$\text{Subject to} \quad \sum_{k \in K} \sum_{s \in S} z_{is}^k = 1 \quad \forall i \in P, \tag{5-2}$$

$$\sum_{k \in K} \sum_{i \in P} (q_i^p + 2 \cdot q_i^w) \cdot z_{is}^k \le Q_s \cdot y_s \quad \forall s \in S, \tag{5-3}$$

$$\sum_{k \in K} \sum_{s \in S} d_{is} \cdot z_{is}^k \le \eta \cdot m_{di} \quad \forall i \in P, \tag{5-4}$$

$$\sum_{k \in K} \sum_{i \in N} x_{ij}^k = 1 \quad \forall j \in P, \tag{5-5}$$

$$\sum_{j \in N} x_{ji}^k - \sum_{j \in N} x_{ij}^k = 0 \quad \forall i \in N, \ k \in K, \tag{5-6}$$

$$\sum_{h \in H} u_{kh} \le 1 \quad \forall k \in K, \tag{5-7}$$

$$\sum_{j \in P} x_{hj}^k \le u_{kh} \quad \forall k \in K, h \in H, \tag{5-8}$$

$$\sum_{k\in K} u_{kh} \le V_h \quad \forall h \in H, \tag{5-9}$$

$$-M\cdot(1-x_{ij}^k) \le z_{is}^k - z_{js}^k \le M\cdot(1-x_{ij}^k) \quad \forall i\in P, j\in P, k\in K, s\in S, \tag{5-10}$$

$$z_{is}^k \ge 1 - M\cdot(1-x_{is}^k) \quad \forall i\in P, s\in S, k\in K, \tag{5-11}$$

$$L_i^k = 0 \quad \forall i\in H\cup S,\ k\in K, \tag{5-12}$$

$$-M\cdot(1-x_{ij}^k) \le L_j^k - q_j^p - 2\cdot q_j^w \le M\cdot(1-x_{ij}^k) \quad \forall j\in P,\ i\in N,\ k\in K, \tag{5-13}$$

$$L_j^k \le C\cdot \sum_{i\in N} x_{ij}^k \quad \forall j\in P,\ k\in K, \tag{5-14}$$

$$\sum_{i\in P\cup S}\ \sum_{j\in P\cup S} d_{ij}\cdot x_{ij}^k \le D \quad \forall k\in K, \tag{5-15}$$

$$T_{ij}^k = 0 \ \ \forall i\in H\cup S,\ j\in N, k\in K, \tag{5-16}$$

$$-M\cdot(1-x_{ij}^k) \le T_{ij}^k - t_{ij}\cdot x_{ij}^k - \sum_{q\in N} T_{jq}^k \le M\cdot(1-x_{ij}^k) \quad \forall i\in P,\ \ j\in P,\ k\in K, \tag{5-17}$$

$$T_{ij}^k \le M\cdot x_{ij}^k \quad \forall i\in N, j\in N, k\in K, \tag{5-18}$$

$$T_{ij}^k \le m_t \quad \forall i\in P,\ j\in N, k\in K, \tag{5-19}$$

$$\sum_{k\in K}\sum_{i\in P'}\sum_{j\in N\setminus P'} x_{ij}^k \ge 1 \quad \forall P'\subset P, 2\le |P'|, \tag{5-20}$$

$$y_s, z_{ps}^k, x_{ij}^k, u_{kh} \in \{0,1\}, L_i^k,\ T_{ij}^k, \in R^+,\ \forall h\in H, p\in P, i\in N, j\in N, s\in S. \tag{5-21}$$

Objective function (5-1) minimizes the sum of fixed and operational costs. Constraints (5-2) ensure that each pickup node is assigned to exactly one shelter. By Constraints (5-3) pickup nodes are assigned only to those shelters that are open and capacities of open shelters are not exceeded. Constraints (5-4) force that, for each pickup node, the assigned shelter is within η times the distance to its nearest shelter. Each customer is served exactly once by Constraints (5-5). Flow conservation is expressed in Constraints (5-6). Constraints (5-7) and (5-8) ensure that each vehicle is used at most once, while Constraints (5-9) force the number of vehicles that come from each holding lot do not exceed the number of available vehicles in it. Constraints (5-10) and (5-11) ensure that clients travel toward their assigned shelters without stopping intermediately at other shelters. Constraints (5-12) ensure vehicles are empty when leaving a holding lot and after each tour to a shelter. Constraints (5-13) express that vehicle load when leaving a pickup node increases by the number of passengers and wheelchairs loaded. It is assumed that one person occupies one seat and one wheelchair two seats within each vehicle. Constraints (5-14) guarantee that vehicle load does not exceed its capacity. Constraints (5-15) define that the distance that each vehicle travels from its first pickup to its last dropoff location is restricted to a maximum distance limit. Constraints (5-16) reset the incurred trip duration of each vehicle to zeros every time this vehicle leaves a holding lot or shelter. Constraints (5-17) and (5-18) express that the trip duration of each vehicle increases when it traverse the nodes. Constraints (5-19) ensure that trip duration is lower than the passenger maximum onboard time. Subtours are eliminated in Constraints (5-20). Finally, binary and integrality of the decision variables are stated in Constraints (5-21).

The formulation involves $O(|K|\cdot|N|^2)$ binary decision variables, $O(|K|\cdot|N|^2)$ integer decision variables, and $O(2^{|N|}\text{-}2^{|P|})$ constraints. Only very small-scale instances can be solved exactly. The most recent exact solution method is proposed by (Akca et al., 2008). The authors formulated a traditional location routing and scheduling problem, which is comparable in complexity to the SPEP, as a set-partitioning problem and proposed a column generation framework with two-phase pricing in the subproblem. To deal with large instances, they also proposed two heuristic pricing algorithms to solve the subproblem. The largest instances involve 5 facilities and 40 customers requiring 8 CPU hours computing time. Thus, the practical SPEP instance is solved by two heuristic strategies described in the next section.

5.4 Solving the SPEP

Although not directly applicable to solving the SPEP, the heuristics proposed for traditional LRPs with a size seen in real-world operations provide inspiration for the proposed solution strategies. Three solution strategies for LRPs were described in (Nagy and Salhi, 1996): sequential, interactive, and nested methods. Sequential methods are often 'locate first and route second' type heuristics, where the location problem is solved first and the routing problem second. Without consideration for the interrelations between the two problems, they usually obtain low-quality solutions (Laporte et al., 1988). Interactive methods treat the location and routing problems equally and iterate between the two problems until a stopping criterion is met. For example, Tuzun and Burke (1999) and Wu et al., (2002) proposed tabu search and simulated annealing solution methods, respectively, to solve traditional LRPs. Although these methods can provide better solutions than the sequential methods, these interactive methods cannot explore the neighborhood space extensively due to the equal treatment of the two problems. Instead of treating the location and routing problems as equal, (Nagy and Salhi, 1996) proposed a hierarchical structure for the LRP, where the location problem is solved in the main problem and the routing problem as a 'subproblem' to it. Based on this hierarchical structure, (Gündüz, 2011) proposed a tabu search algorithm to solve a combined location problem and muti-depot vehicle routing problem with time windows. Results show that the nested methods outperform the other two solution methods.

In this section, a sequential solution strategy and a nested tabu search strategy applying the hierarchical structure proposed in (Nagy and Salhi, 1996) are described for solution of the SPEP. The sequential strategy provides an initial solution from which the proposed nested tabu search strategy starts. Both strategies rely on decomposing the problem into subproblems. Thus, before describing the two solution strategies, descriptions of these subproblems and proposed solution approaches for each are given.

5.4.1 Subproblems

The SPEP can be decomposed into two interrelated subproblems: (1) the Capacitated Shelter Location-Allocation Problem (CSLAP) and (2) the Multi-depot Dial-A-Ride Problem (MDARP). These two subproblems will later be solved in a sequential solution strategy and iteratively in the nested tabu search strategy.

Capacitated Shelter Location-Allocation Problem (CSLAP)

By defining new binary decision variables $z_{is} = \sum_{k \in K} z_{is}^k, \forall i \in P, s \in S$, the CSLAP can now be formulated as:

$$\min \quad C_{CSLAP} = \sum_{s \in S} F_s \cdot y_s + \sum_{i \in P} \sum_{s \in S} C_d \cdot d_{is} \cdot z_{is} \tag{5-22}$$

subject to

$$\sum_{s \in S} z_{is} = 1 \quad \forall i \in P, \tag{5-23}$$

$$\sum_{i \in P} (q_i^p + 2 \cdot q_i^w) \cdot z_{is} \le Q_s \cdot y_s \quad \forall s \in S \tag{5-24}$$

$$\sum_{s \in S} d_{is} \cdot z_{is} \le \eta \cdot m_{d_i} \quad \forall i \in P, \tag{5-25}$$

$$y_s, z_{is} \in \{0,1\} \quad \forall i \in p, s \in S, k \in K \tag{5-26}$$

Objective function (5-22) minimizes the sum of fixed and assignment costs. The assignment cost is calculated through the direct distance between the pickup location and the associated evacuees' assigned shelter. This direct distance is an approximation to the actual transportation distance. Constraints (5-23), (5-24) and (5-25) play the same roles as (5-1), (5-2) and (5-3) play in the SPEP formulation, respectively. The binary decision variables are stated in Constraints (5-26).

The CSLAP can be solved exactly through a commercial solver. Two solutions can be obtained: $\mathbf{y}_0$, which indicates whether or not each shelter $s \in S$ is open, and $\mathbf{z}_0$, which indicates if a passenger $i \in P$ has been assigned to shelter $s \in S$.

Multi-Depot Dial-A-Ride Problem (MDARP)

With $\mathbf{z}_0$ from CSLAP, the MDARP can be formulated as:

$$\min \quad C_{MDARP} = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} C_d \cdot d_{ij} \cdot x_{ij}^k \tag{5-27}$$

subject to Constraints (5-5)-(5-20)

$$x_{ij}^k, u_{kh} \in \{0,1\}, L_i^k, T_{ij}^k, \in \mathrm{Z}^+, \ \forall h \in H, p \in P, i \in N, j \in N, s \in S. \tag{5-28}$$

The MDARP is a difficult problem, since its corresponding single-depot DARP is NP-hard. Thus, a cheapest insertion heuristic adapted from (Jaw et al., 1986) is proposed to solve it. This algorithm first builds optimal routes for all pickup-shelter location pairs, and then assigns the built routes to vehicles in holding lots. The objective of this assignment is to minimize total route costs subject to limitations on the number of available vehicles in each holding lot.

The main steps of the cheapest insertion heuristic can be summarized as:

Cheapest Insertion Heuristic for the MDARP

Step a: Copy each pickup and shelter pairs obtained from the CSLAP into the unrouted request list URL).

Step b: Create an empty route. Set r =1.

Step c: Select the first unrouted pair (p, s) from URL. Find all feasible insertions within all existing routes, 1 to r.

(iii) If a feasible insertion is found, insert (p, s) to the route r^* with minimum insertion cost, and update route r^*.

(iv) If no feasible insertion exists, create a new empty route and insert (p, s) in it. Set $r = r+1$.

Step d: Repeat step c until URL is empty.

Step e: Add each depot to the starting and ending point of each route, calculate updated route cost matrix.

Step f: Assign routes to holding lots according to updated route cost matrix.

Potential Feasible Insertions

Due to Constraints (5-10) and (5-11) that prevent routes stopping at intermediate shelters while en route to another destination shelter, the potential feasible insertions of pickup-shelter pair, (p, s), on route, r, in the step c can be confined to three categories:

(1) Insert (p, s) at the beginning of route r.
(2) Insert (p, s) immediately after each shelter, if there any shelter already exists on route, r.
(3) Insert p immediately before s and each of the pickup nodes before s until reach another shelter or the beginning of the route, if shelter s already exists on route, r.

Feasibility Checking

An insertion of a pickup-shelter pair needs to ensure that the constraints associated with the vehicle capacity constraints (5-14), maximum driving distance limit for drivers (5-15) and passenger maximum onboard time (5-19) are satisfied during the insertion process. An insertion of a pickup-shelter pair in a route is feasible only if it does not lead to violation of any of these constraints by inclusion of this pair. Moreover, its inclusion should not create other violations of these constraints for other nodes already included in the route.

5.4.2 Sequential Solution Strategy

The sequential solution strategy involves solving the CSLAP first and the MDARP second. Figure 5-2 depicts this sequential solution process, where the opening cost from CSLAP and transportation cost from MDARP are the fixed and operational costs in objective function (5-1), respectively.

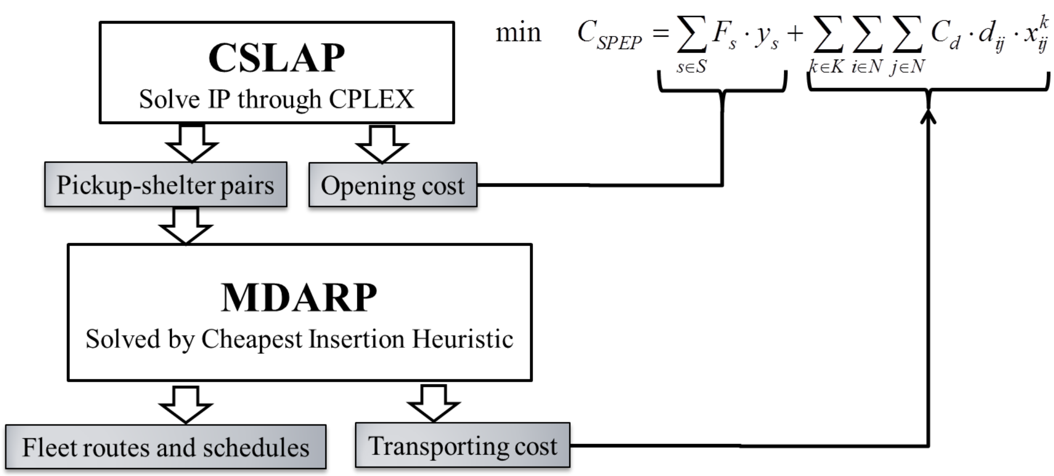


Figure 5-2 Sequential Solution Process

5.4.3 Nested Tabu Search Strategy

Improvements to the shelter location and evacuee routing solutions can be obtained by explicitly recognizing their interconnections in the solution strategy. For this purpose, a nested tabu search strategy is proposed in which the interactions between the CSLAP and the MDARP are explicitly considered. Figure 5-3 provides the overall framework of this strategy.

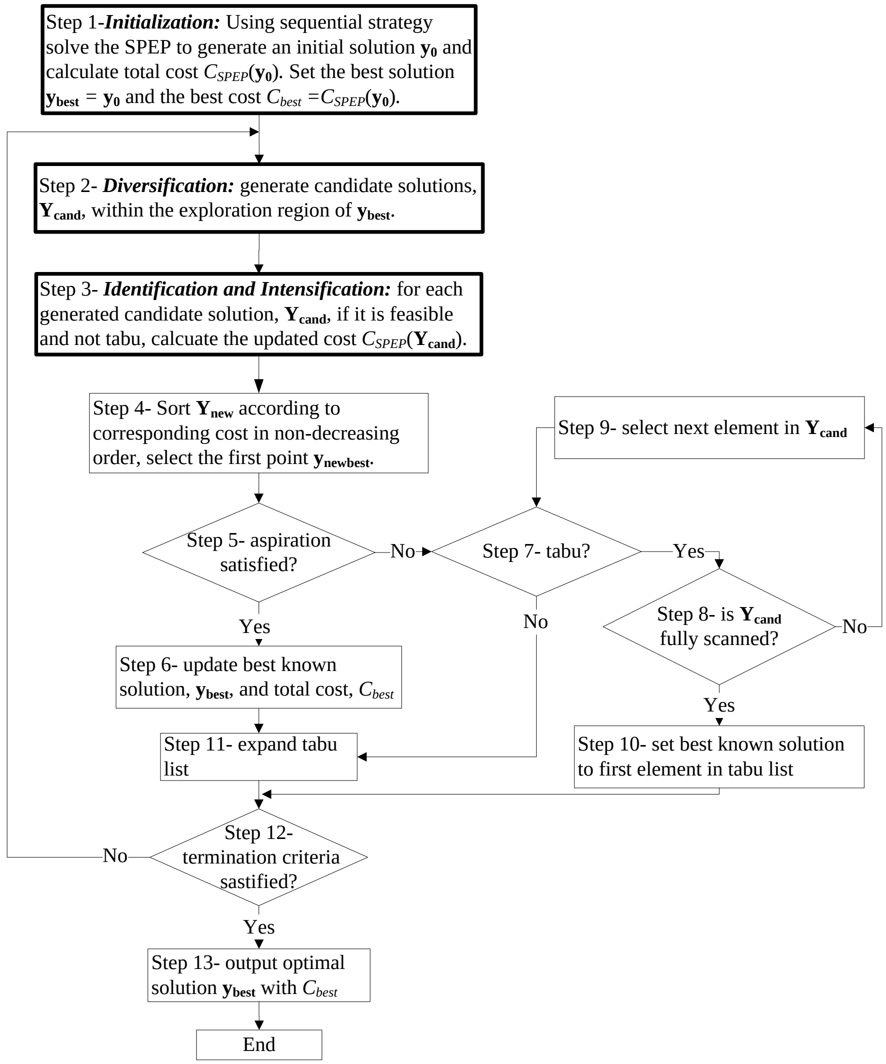


Figure 5-3 Flowchart of Proposed Tabu Search Algorithm

The procedure begins with generation of an initial solution through a sequential solution strategy, and setting the best solution to the obtained initial solution. Candidate solutions around the current best solution are generated in the diversification step. An identification and intensification procedure is employed to filter out infeasible generated candidates and calculate total costs for the feasible ones.

The best generated feasible solution will be assessed to see if it meets the aspiration criterion. If the aspiration criterion is satisfied, the best solution and tabu list will be updated and termination criteria will be assessed. If one of the termination criteria is met, the procedure stops; otherwise, it continues to the next iteration. If the aspiration criterion is not satisfied, tabu criteria will be checked for other generated feasible solutions, since they cannot be better than the selected best feasible solution. If any of them is not tabu, it will be placed in the tabu list. If all of them are tabu, the first element in the tabu list will be selected as the best solution. As in other tabu search procedures, the tabu list aids in preventing the search from being trapped at a local solution.

The aspiration, tabu list and termination criteria applied herein are summarized next.

Aspiration criterion

Any feasible candidate solution that has the best total cost of all discovered solutions will become the best solution regardless of its tabu status.

Tabu list

Two tabu lists are applied within the overall procedure. Both employ a complete memory approach. One is named *tabuList* and maintains a list of solutions, considered in the last *L* iterations (the tabu tenure) of the search procedure. This list prevents revisiting of solutions within the iterations associated with the tabu tenure. A solution may be removed from the list prematurely if no neighboring candidate solution of the best solution outperforms this best solution. The second tabu list is called *infeasibleList*, where the infeasible candidate solutions generated in the diversification step are maintained permanently during the entire search procedure. This list is used to filter out infeasible candidate solutions in steps 2 and 3.

Termination criteria

The procedure terminates when either a predefined maximum number of iterations (*ItMax*) or a predefined maximum number of iterations without improvement (*NoMax*) is reached.

Details of the steps of diversification as well as identification and intensification are given next.

Step 2-Diversification

In this step, a diversification strategy is applied to generate a set of candidate solutions $\mathbf{Y}_{\mathbf{cand}}$, a set of solutions vectors, within the neighborhood space of the best solution, $\mathbf{y}_{\text{best}}$. The candidate solutions are generated through adapted exploration moves 'drop', 'add' and 'switch', originally introduced by (Kuehn and Hamburger, 1963). Before introducing the adapted exploration moves, a neighborhood relation between two shelters is defined based on the definition from (Nagy and Salhi, 1996):

Given constraints (5-4) or (5-28), which enforce that each client cannot be sent to shelters that are beyond η times the distance to the nearest shelter, a concept of neighboring shelters is defined below and illustrated in Figure 5-4.

Neighboring Shelters: *Two shelters s_1 and s_2 are neighbors if and only if at least one pickup node p exists, such that s_1 and s_2 are the first and second nearest shelters to p, respectively, and if $d_{(p,s_2)} \le \eta \cdot d_{(p,s_1)}$, then $s_1 \in Nb(s_2)$ and $s_2 \in Nb(s_1)$, where $Nb(s)$ denotes a set of neighboring shelters of shelter s.*

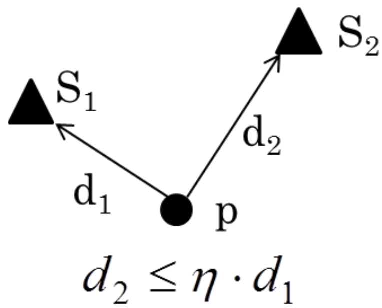


Figure 5-4 Illustration of Neighboring Shelters

Based on the definition of neighboring shelters, $\mathbf{Y}_{\mathbf{cand}}$ can be generated through the following steps.

For each opened shelter $s \in S$, and each closed shelter $\bar{s} \in Nb(s)$:

Drop - setting $\mathbf{y}_{\text{best}}(s) = 0$, if resulting solution is not tabu, add it into $\mathbf{Y}_{\mathbf{cand}}$;

Add - setting $\mathbf{y}_{\text{best}}(\bar{s}) = 1$, if resulting solution is not tabu, add it into $\mathbf{Y}_{\mathbf{cand}}$;

Switch - setting $\mathbf{y}_{\text{best}}(s) = 0$ and $\mathbf{y}_{\text{best}}(\bar{s}) = 1$, if resulting solution is not tabu, add it into $\mathbf{Y}_{\mathbf{cand}}$.

Step 3-Identification and Intensification

Candidate solutions generated in the diversification step are not guaranteed to be adhering to Constraints (5-4). Those solutions that are not feasible need to be filtered out. However, whether or not a candidate solution is feasible cannot be known until the CSLAP is solved. In fact, for any set of location variables, $\tilde{\mathbf{y}}$, the optimal assignment $\mathbf{z}(\tilde{\mathbf{y}})$ can be obtained by solving the associated Capacitated Shelter Assignment Problem (CSAP), which is defined as:

$$\min \quad C_{CSLAP}(\tilde{\mathbf{y}}) = \sum_{s \in S} F_s \cdot \tilde{y}_s + \sum_{i \in P} \sum_{s \in S} C_d \cdot d_{is} \cdot z_{is} \tag{5-29}$$

$$\text{Subject to} \qquad \sum_{i \in P} (q_i^p + 2 \cdot q_i^w) \cdot z_{is} \le Q_s \cdot \tilde{y}_s \quad \forall s \in S \tag{5-30}$$

Constraints (5-26) and (5-28)

$$z_{is} \in \{0,1\} \quad \forall i \in p, s \in S, k \in K \tag{5-31}$$

Thus, given a generated candidate solution, instead of solving the CSLAP, its corresponding CSAP problem is solved to filter out the infeasible candidate solutions.

Figure 5-5 depicts the identification and intensification processes. The processes start with selecting the 1st element, $\boldsymbol{y}$, of $\mathbf{Y_{cand}}$ generated in the diversification process. If $\boldsymbol{y}$ is in the *tabuList* or *infeasibleList*, then the next element in $\mathbf{Y_{cand}}$ will be selected. If $\boldsymbol{y}$ is not tabu, $\boldsymbol{y}$ will be used as input to CSAP($\boldsymbol{y}$). If it is infeasible, put it into the *infeasibleList* and move to the next element in $\mathbf{Y_{cand}}$; otherwise, the MDARP will be solved with the assignment results from CSLAP($\boldsymbol{y}$) and the total cost $C_{\mathrm{SPEP}}(\boldsymbol{y})$ will be calculated. $\boldsymbol{y}$ and $C_{\mathrm{SPEP}}(\boldsymbol{y})$ will be added into the new feasible solution set $\mathbf{Y_{new}}$ and total cost set $C_{\mathrm{SPEP}}(\mathbf{Y_{new}})$, respectively. This process is repeated until all elements of $\mathbf{Y_{cand}}$ have been investigated. The new feasible set, $\mathbf{Y_{new}}$, will be sorted in nondecreasing order according to $C_{SPEP}(\mathbf{Y_{new}})$ and the best (first) one will be selected for aspiration checking in the next step.

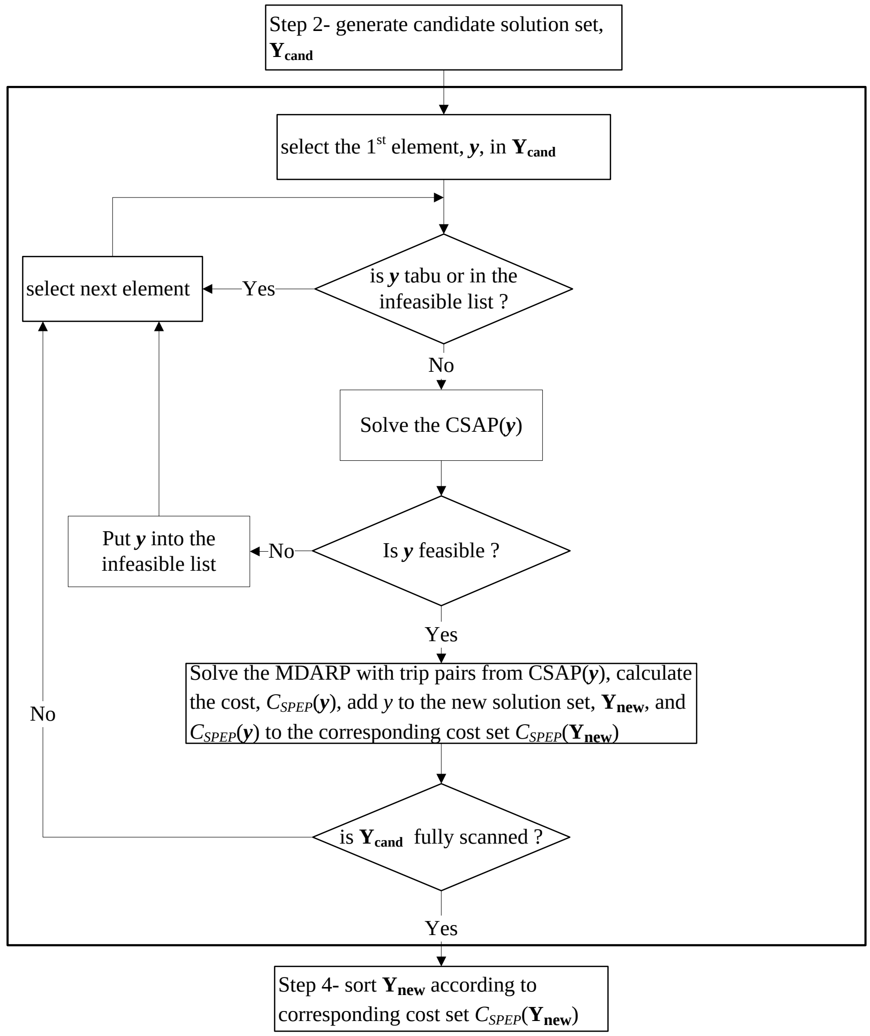


Figure 5-5 Flowchart of Identification and Intensification Process

5.5 Numerical Experiments

5.5.1 Experimental design

To investigate the efficiency of the proposed solution approach, proposed solution strategies are tested on a real-world case study. The case study involves an assumed hurricane evacuation in New York City, one of the many hurricane vulnerable areas along the coastline of the United States. The evacuation scenario is shown in Figure 5-6 and involves evacuating mobility-impaired individuals from 588 pickup locations (round nodes) within the hurricane evacuation zone (shaded area) to 238 potential shelters (triangles) through the paratransit vehicles affiliated with 39 paratransit depots (squares). The data on potential shelters, including location and capacity, as well as the hurricane evacuation zones, was obtained from a technical report from the US Army Corps of Engineers (FEMA and USACE, 2009).

The information on paratransit vehicles comes from the NYC data website (Weir, 2013), which includes information from 161 companies, including the depot locations and number of affiliated vehicles. Only 39 companies with more than 15 paratransit vehicles are considered. Each vehicle is assumed to have a capacity of seven spaces with each person occupying one space and each wheelchair two spaces. To preserve privacy issues, data on real pickup locations were not available. The 588 pickup locations were thus chosen as the centroids of census tracts in U.S. Census 2010. In reality, the needed pickup information for the mobility-impaired can be obtained information gathered through registration for paratransit services during ordinary circumstances. Random numbers were generated to determine with equal odds whether each location contained one or two evacuees awaiting assitance from each pickup location. Similarly, each passenger was assigned a wheelchair with probability 0.5. The fixed opening cost of each shelter is assumed to be proportional to the shelter's capacity.

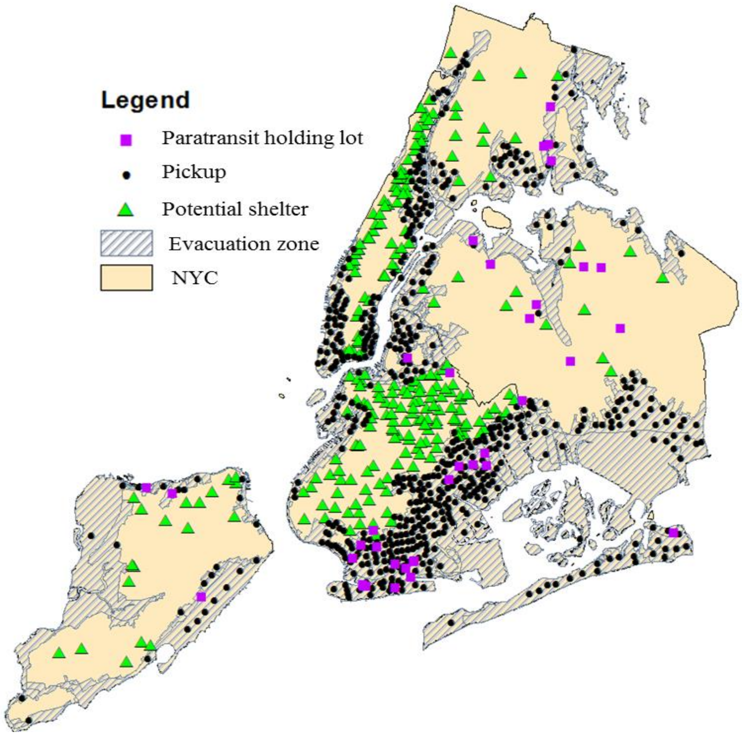


Figure 5-6 NYC Hurricane Evacuation

Both the sequential and nested tabu search solution strategies were applied. Results of the two strategies are compared. Parameters used in the proposed model and solution algorithm are presented in Table 5-1. The tabu tenure of the *tabuList* was tuned from 10 to 80 iterations in increments of 5. A setting of 15 iterations was found to have the best performance in terms of convergence and best solution found.

Table 5-1 Parameters of Proposed Model and Algorithm

Parameters	Explanation	Values
C_d	Unit cost of travel distance of vehicle	1.52 $/mile
C	Identical vehicle capacity	7 spaces
s	Identical service time	3 minutes
D	Maximum driving distance(excluding deadhead distance)	320 miles
R	Maximum passenger onboard time	2 hours
L	Tabu tenure	15
ItMax	Maximum iterations	500
NoMax	Maximum non-improvement iterations	30

The proposed solution strategies were implemented in Visual C++ 2010 and run on a personal computer with Intel(R) CPU 3.10GHz and 4.0GB RAM. The C++ Concert Technology of CPLEX in the IBM-ILOG CPLEX 12.51 was applied to solve the CSLAP and CSAP problems.

5.5.2 Results Analysis

Figure 5-7 shows the convergence process of the proposed nested tabu search strategy with $\eta = 5$. The procedure terminates after 70 iterations, because no improvement in solution value was obtained after 30 iterations. The resulting solution has a total cost of $5,688 compared with an initial total cost of $5,974 obtained from the sequential solution strategy, thus producing a relative improvement of approximately 5%.

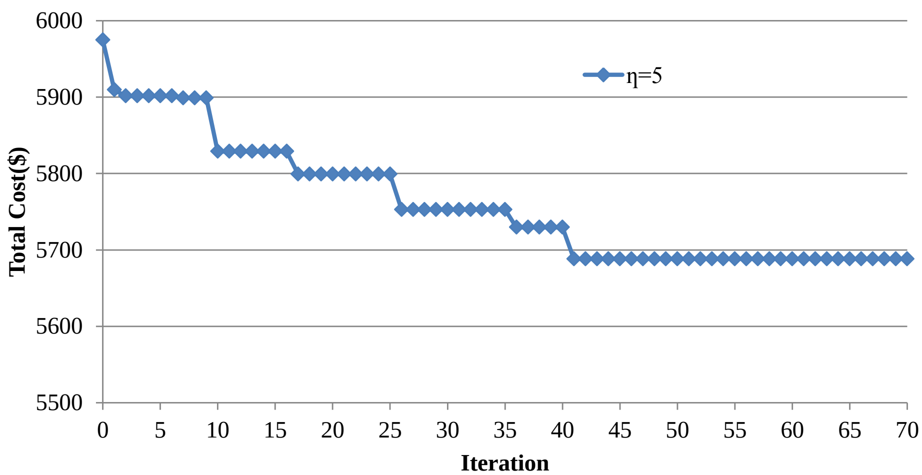


Figure 5-7 Convergence Process of Tabu Search Algorithm

To further explore the difference between the solutions obtained from the sequential and nested tabu search solution strategies, the initial and final open shelters and corresponding fixed opening costs are listed in Table 5-2. Although the total fixed opening costs are fairly close (only $2 difference), there are 6 different shelters, marked by a star symbol, between the two solutions. Table 5-2 shows that two expensive shelters (761 and 768) in the initial solution are not sufficiently equipped to support the mobility-impaired, and instead four cheaper locations (630, 659, 772 and 774) are open to them in the final solution. The difference indicates the 'myopic' of the sequential solution strategy, where an approximation of operational cost (the cost of direct distances from pickups to assigned shelters) is considered in the CSLAP. However, taking the actual operational cost into account, the final shelter location decisions are quite different. Fixed and operational costs are further explored in Table 5-3.

Table 5-2 Comparisons of Shelter Locations

	Initial Shelter Location		Final Shelter Location	
#	Notation	Fixed Open Cost($)	Notation	Fixed Open Cost($)
1	629	220	629	220
2	632	195	**630***	**65**
3	639	340	632	195
4	640	50	639	340
5	651	61	640	50
6	673	146	651	61
7	677	82	**659***	**63**
8	698	50	673	146
9	700	58	677	82
10	704	74	698	50
11	727	71	700	58
12	742	96	704	74
13	752	94	727	71
14	**761***	**90**	742	96
15	765	94	752	94
16	766	87	765	94
17	767	78	766	87
18	**768***	**138**	767	78
19	769	59	769	59
20	775	77	**772***	**52**
21	778	98	**774***	**50**
22	832	62	775	77
23	839	40	778	98
24	842	90	832	62

25	848	481	839	40
26	852	83	842	90
27	856	52	848	481
28			852	83
29			856	52
Opening Cost ($)		3066		3068

Table 5-3 shows the initial (sequential solution strategy) and final cost results (nested tabu search strategy) for different η values. The 'Δ' columns give the relative improvement (decrease) from the initial results. For both initial and final results, it is not surprising that total cost decreases with increasing value of η, because a lager value of η infers more relaxed constraints. For both initial and final results, with increasing value of η, the ratio of fixed cost to total cost decreases, while the ratio of operational cost to total cost increases. This is reasonable. A lower value of η means more shelters should be opened inferring shorter distances to travel, while a higher value of η means fewer shelters or farther but cheaper shelters can be opened, which usually means longer travel distances will be incurred, and thus higher operational (transportation) costs will exist. For all η values, compared to initial results, fixed costs increase, operational costs decrease and the total cost decreases in the final results (from the nested procedure). This indicates the benefit of considering operational costs in the CSLAP. The CPU time increases with the increasing value of η, however, even for the instance with η=10, the computational time is still acceptable for the application.

Table 5-3 Cost Results with Different η Values

	Initial Results			Final Results			Statistics				
	Fixed Cost ($)	Trans. Cost ($)	Total Cost ($)	Fixed Cost ($)	Trans. Cost ($)	Total Cost ($)	Δ Fixed Cost (%)	Δ Trans. Cost (%)	Δ Total Cost (%)	# Iter	CPU Time

η=2	5707	2572	8279	5826	2390	8216	2.09	-7.10	-0.77	38	0:58:20
η=3	4504	2641	7146	4582	2360	6942	1.73	-10.67	-2.85	52	1:06:35
η=5	3066	2908	5975	3068	2620	5688	0.07	-9.92	-4.80	70	1:26:46
η=7	1990	3198	5188	2019	2770	4789	1.46	-13.40	-7.70	87	1:57:22
η=10	1875	3163	5038	1928	2704	4632	2.83	-14.52	-8.06	108	2:33:49

Figure 5-8 shows the final route set generated by the MDARP with η=5. There are total 15 routes performed by 15 vehicles from 11 holding lots. The numbers next to the holding lots in Figure 5-8 denote the assigned holding lot. Figure 5-8 shows that no built route violates the maximum driving distance constraint. Other constraints, such as vehicle capacity and maximum onboard time constraints, were also checked for each route.

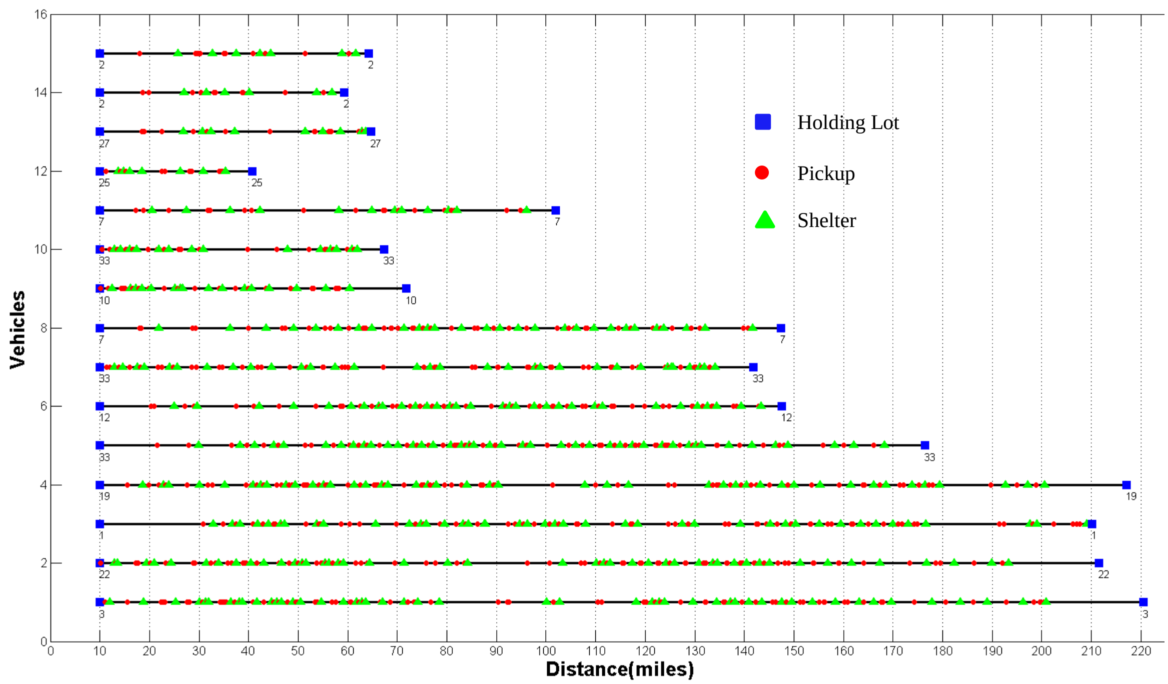


Figure 5-8 Final Route Set with η=5

5.6 Conclusions and Extensions

In this chapter, the problem of sheltering and paratransit operations for evacuation of populations with mobility impairments during disaster is addressed. An optimization problem, the SPEP, is formulated as a mixed integer program. Sequential and nested tabu search solution strategies are proposed. The former decomposes the problem into two subproblems: 1) a capacitated shelter location-allocation problem and 2) a multi-depot dial-a-ride problem (DARP). The latter approach explicitly considers the interconnection between optimal location, assignment and routing decisions. The proposed solution strategies were tested on a case study involving NYC. The results obtained indicate that the proposed nested tabu search strategy is efficient and effective for addressing the SPEP, and yields better results than the sequential solution method.

A primary outcome of the developed model and solution methodologies is the selection of shelters that can, once identified, be adequately prepared to support an evacuation. These developments have additional tactical and operational utilities. Pre-disaster, forecasts specific to a given impending hazard may be available, affecting the potential utility of shelter locations. Thus, in this tactical phase, a subset of the equipped shelters may be open, and the CSLAP model can be applied to re-allocate and route the evacuees using these services for this smaller set of destinations. Similarly, in an evacuation arising post-disaster event, this routing and allocation component of the decision problem can be resolved once knowledge of the viability of the shelter locations and/or roadways is determined. If the original locations are to be operated, the allocation of evacuees to the shelters may be maintained and only the routing decisions may be reassessed in response to information about roadway closures or other conditions affecting the predetermined routes. This latter problem can be addressed by solving an MDARP.

This study addresses emergency evacuation with notice as is the case in situations involving hurricanes; however, for no-notice emergency evacuations, an alternative objective function in which the number of pickups that can be completed in a fixed amount of time is maximized or the time required to complete the pickups is minimized given a fixed fleet might be appropriate. Additionally, emergency situations are inherently uncertain. For example, travel times, shelter capacities, demand for assistance and even available resources may be affected by the disaster and a priori knowledge of quantities may be precluded. Thus, it could be beneficial to account for such uncertainties within the proposed model. Also, as some disasters evolve over time, it may become necessary to evacuate the shelters, sending the evacuees even further from the affected area. That is, a two-stage evacuation might be required. One could explore the possibility of applying the proposed model and solution methodologies to each stage.

Chapter 6 Conclusions and Extensions

6.1 Conclusions

Motivated by increasing concerns about the safety and efficiency in the movement of large numbers of people in crowd-related circumstances, this dissertation conceptualizes and addresses four important optimization problems regarding facility and/or operational design to support efficient people movement: the Pedestrian Route Choice in Crowds (PRCC) problem, the Redesign for Efficient Crowd Movement (RECM) problem, the Airport Access Ridesharing Problem (AARP), and the Sheltering and Paratransit Evacuation Problem (SPEP). These problems are aimed at identifying opportunities to support accurate prediction of crowd movements over a facility layout, optimal reconfiguration of the facility layout for large crowd management, efficient routing and scheduling for ridesharing vehicles, and optimal shelter location-allocation and paratransit vehicle routing for effective regional evacuation planning of the mobility-impaired.

This dissertation addressed complex and diverse characteristics, not previously conceived in the literature. The PRCC problem incorporates grouping behavior described in social science and psychological studies within a network optimization-based framework with the aim of estimating the distribution of groups (separable and clustered) and individuals over efficient routes through a facility layout. The RECM problem simultaneously takes the goal of crowd manager and pedestrian route choice behavior into consideration for redesigning the physical layout to facilitate crowd movement. The AARP combines several characteristics from other related problems, such as time windows, pairing and precedence, and linehaul and backhaul operations, as well as user inconvenience constraints. The SPEP explores the interdependence and integration of the facility design (shelter location-allocation) and paratransit operations in evacuation planning for mobility-impaired populations.

The focus of this dissertation is to formulate and provide algorithmic solution approaches (exact and approximate) to tackle these complex problems with their diverse characteristics. The PRCC problem employs formulations of logit-based SUE assignment and n-player non-cooperative game for separable and clustered groups, respectively. A procedure of Method of Successive Averages (MSA) with groups and a metaheuristic scheme based on best response dynamic and tabu search were proposed for solving the formulated problems. The RECM is formulated as a bi-level mixed integer program, where the upper-level is a network design problem and the lower-level is a pure-strategy Nash equilibrium assignment problem. A Multi-start Tabu Search with Sequential Quadratic Programming (MTS-SQP) procedure is proposed for its solution. The AARP problem is formulated as a nonlinear, mixed integer program. An exact solution approach applying Constraint Programming based Column Generation (CPCG) and two insertion algorithms adapted from existing heuristics are proposed for its solution. The SPEP is formulated as a mixed integer program. To solve large-scale instances of the SPEP, a tabu search metaheuristic is proposed. This approach is based on decomposition of the entire problem into two interdependent subproblems. The mathematical formulations aim to provide precise definitions of the identified problems and permit quantitative analysis of real-world problem instances.

Numerical experiments were conducted on carefully created fictitious examples for buildings and other facilities, an actual day of ridesharing service records out of Washington Dulles International Airport and a real-world based case study of hurricane evacuation in New York City. Results of numerical experiments demonstrate the effectiveness and efficiency of the proposed methodologies. Results from these experiments show that the proposed exact solution algorithms can solve small- and moderate-size problems to optimality or near-optimality with reasonable computational time; whereas, the proposed approximate algorithm can tackle large-scale problem instances with good approximation to optimal or near optimal solutions.

6.2 Extensions

This dissertation can be extended in several directions.

The PRCC problem

In the proposed models and solution approaches, the parameters of the disutility function for a group (separable or clustered) are assumed to be homogeneous. In reality, however, the parameters associated with each group may vary by individual. Additionally, the disutility function only considers distance and travel time. In reality, other factors, such as safety, might also play a role in route choice, especially during emergency events. The heterogeneity within each group and additional factors in the disutility function can be further explored with the proposed models and solution schemes.

Furthermore, only grouping behavior is considered in the proposed models. Other collective behaviors, such as splitting, flocking and following might be incorporated in the proposed models. Moreover, the proposed models assume that pedestrians make decisions based on route-based performance. Once a route is selected, it is assumed that each pedestrian will follow the route in its entirety. The developed model and solution methodology might be extended to address a dynamic pedestrian route choice problem, where the physical environment changes dynamically and people would make decisions on splitting, flocking or following at each node en route according to current dynamic goals (utilities).

Finally, the pure- and mixed-strategy Nash equilibriums considered in the proposed models and solution approaches might be applicable in other areas. For example, the SUE assignment for separable groups might be applicable for vehicular traffic assignment that follows the mixed-strategy Nash equilibrium (Wardrop's principle). The n-player non-cooperative game that seeks pure-strategy Nash equilibrium for clustered groups might be applicable for other applications involving pure strategy interactions of several decision makers, such as auction markets.

The RECM problem

The RECM problem is formulated as a bi-level program. Due to the equilibrium constraints embedded in the formulation structure, the RECM is NP-hard. A MTS-SQP is proposed for solution of mid-size problem instances. The procedure guarantees a local optimal solution through SQP and employs multi-start strategy to increase the chances of obtaining a global optimum.

For large problem instances, one might consider replacing the SQP approach in the MST-SQP procedure with a heuristic. However, such a heuristic will not guarantee even local optimality. Another useful alternative for addressing large problem instances might be to apply linear approximations of the equilibrium constraints in the lower-level problem. This modeling framework permits both alternatives.

Additionally, a BPR-based travel time function is applied in the proposed model. A more sophisticated travel time function, however, would be necessary to capture pedestrian dynamics and intersections and impacts of bi-directional flows. Moreover, the link capacities in proposed model are assumed continuous, but the model framework allows both discrete and continuous link capacities. If discrete link capacities are introduced, the SQP should be replaced by a mixed integer program solver. Furthermore, within the framework, one might relax the constraint that total capacity is fixed and explore the interdependencies in space restrictions between adjacent links through the addition of constraints in each link upper limit.

Finally, although the proposed model and solution approach aims for practical utility in crowd control, it also has potential applicability in vehicular traffic control by omitting grouping behavior constraints in the lower-level problem. In vehicular traffic control, the outcome of the model can be implemented through, for example, opening and closing of lanes, ramp metering, adaptive speed limits, and provision of real-time information through signage or other devices.

The AARP

The AARP is conceptualized and formulated as a mixed integer nonlinear program. An exact solution approach (CPCG) and two heuristics adapted from Jaw's sequential insertion heuristic (Jaw et al., 1986) and Solomon's Insertion I1 (Solomon, 1987) are proposed for its solution. In the CPCG, a constraint programing (CP) is applied for the solution of the subproblem (SP) that is a constrained shortest path problem. To obtain an integer solution in the master problem (MP) and avoid a time-consuming branching process, a heuristic reassemble process is proposed. Numerical experiments show that this exact solution approach can guarantee near optimal solutions (within 5% to the lower bound) with reasonable computational time for a reduced version of the case study. Observations indicate that 98% computational time is spent on solving the SP. A future extension may consider a more sophisticated branching scheme in the MP that would obviate the need for resolving the SP. Additionally, instead of using CP, the proposed exact solution framework permits an alternative solution approach for the SP, such as a dynamic label setting algorithm or heuristic approach. However, the efficiency of these approaches depends on their effectiveness in identifying and discarding paths that are not useful to the MP, given applicable dominance rules.

Results of the adapted Jaw's algorithm provide good approximations to the exact solution (within 7%), but require significantly less computational time (1/1200). The gap between these results, however, might be higher for larger problem instances. The myopic nature of the proposed heuristics might be improved in several directions. For example, the constructed routes can be improved through the application of improvement operators, such as λ-interchange, 2-opt* exchange, trip exchange and trip reinsertion. A cluster-first route-second strategy may also address this myopic nature.

In addition to these extensions related to algorithm improvement, the developed model and solution methodologies might be applied in a dynamic framework that considers not only dynamic requests, but also uncertainty in travel and service times. Finally, the proposed methodologies also have potential applicability to other routing and scheduling applications involving 'one-to-many-to-one' operations, such as mail service and reverse logistics operations.

The SPEP problem

The SPEP is formulated as a mixed integer problem. Due to the included multi-depot pickup and delivery problem, the SPEP is shown to be NP-hard. A tabu search strategy with innovative diversification, identification and intensification procedures is proposed to solve large instances of the SPEP.

The objective function in the proposed model seeks to minimize total cost. While suitable for emergency evacuation situations in which there is advanced notice, such as in a hurricane, for the no-notice emergency evacuations, an alternative objective function, minimizing time to handle all pickups with a fixed fleet, might be considered.

Additionally, the proposed model and solution approach might be extended to include uncertainty in various factors as may arise in emergency evacuations. These uncertainties may be related to demand, as well as shelters and disaster characteristics. For example, the impact of the disaster event may not be known with certainty and evolution of the disaster impact over time and space may induce a second evacuation from the shelters to locations further from the disaster region. The proposed model and solution methodology might be applied for each stage in such a multi-stage evacuation.

Finally, one might relax the farthest assigned shelter and other user inconvenience constraints in the proposed model for other location-routing applications that do not include these constraints. One would need to recognize that the required computational time may increase with such omissions as was shown in the numerical experiments.

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