Defining parameters and functions;

Clear All;

s =.; h =.; c =.;
a_i =.; a_j =.; ai =.; aj =.;
x =.; x_i =.; x_j =.; xi =.; xj =.;
w =.; w_i =.; w_j =.; wi =.; wj =.;

Defining the profit and marginal profit functions;

profit function (as revenue minus cost);

$$\text{NP[ai\_, wi\_, xi\_, aj\_, wj\_, xj\_]} = \left(\text{ai wi} - \frac{\text{wi}^2}{2}\right) - \text{c wi}\left(\text{h} - \frac{\text{xi} + (1 - \text{s})\,(\text{xi} - \text{wi}) + \text{s}\,(\text{xj} - \text{wj})}{2}\right);$$

Motion function : not necessary for this one round game;
SM[xi_, wi_, xj_, wj_] = (1 – s) (xi – wi) + s (xj – wj) + R;

The marginal profit (function) from an extra unit of water;

$$\text{DNP[ai\_, wi\_, xi\_, aj\_, wj\_, xj\_]} = \text{ai} - \text{wi} - \text{c}\left(\text{h} - \frac{\text{xi} + (1 - \text{s})\,(\text{xi} - 2\,\text{wi}) + \text{s}\,(\text{xj} - \text{wj})}{2}\right);$$

Double checking the marginal profit;

$$\text{FullSimplify}\left[\left\{\partial_{w_i}\text{NP}[a_i, w_i, x_i, a_j, w_j, x_j] - \text{DNP}[a_i, w_i, x_i, a_j, w_j, x_j], \partial_{w_j}\text{NP}[a_j, w_j, x_j, a_i, w_i, x_i] - \text{DNP}[a_j, w_j, x_j, a_i, w_i, x_i]\right\}\right]$$

{0, 0}

Players' Decision and the Stock Value Functions;

One round game under competition;

Solving for the extraction decisions then Rewriting it as linear functions of stock levels;

$$\text{sols} = \text{Solve}\left[\left\{\partial_{w_i}\text{NP}[a_i, w_i, x_i, a_j, w_j, x_j] == 0, \partial_{w_j}\text{NP}[a_j, w_j, x_j, a_i, w_i, x_i] == 0\right\}, \{w_i, w_j\}\right];$$

$$\{w_{i,\text{comp}}, w_{j,\text{comp}}\} = \text{Simplify}[\{w_i, w_j\} /. \text{sols}[[1]]];$$

$$\alpha_{1,\text{comp}} = -\frac{1}{3} - \frac{c}{2+2c-cs} + \frac{2-c}{6+6c-9cs};\ \alpha_{2,\text{comp}} = \frac{1}{3} - \frac{c}{2+2c-cs} - \frac{2-c}{6+6c-9cs};$$

$$\alpha_{3\,i,\text{comp}} = \frac{a_i+a_j}{2+2c-cs} + \frac{a_i-a_j}{2+2c-3cs};\ \alpha_{3\,j,\text{comp}} = \frac{a_i+a_j}{2+2c-cs} + \frac{a_j-a_i}{2+2c-3cs};$$

$$\text{FullSimplify}\left[\left\{w_{i,\text{comp}} - \left(\alpha_{1,\text{comp}}(h-x_i) + \alpha_{2,\text{comp}}\left(h-x_j\right) + \alpha_{3\,i,\text{comp}}\right), w_{j,\text{comp}} - \left(\alpha_{1,\text{comp}}\left(h-x_j\right) + \alpha_{2,\text{comp}}(h-x_i) + \alpha_{3\,j,\text{comp}}\right)\right\}\right]$$

{0, 0}

$$\alpha_{3\,\text{comp}}[\text{ai\_}, \text{aj\_}] = \frac{ai+aj}{2+2c-cs} + \frac{ai-aj}{2+2c-3cs};$$

$$w_{\text{comp}}[\text{ai\_}, \text{xi\_}, \text{aj\_}, \text{xj\_}] = \alpha_{1,\text{comp}}(h-xi) + \alpha_{2,\text{comp}}(h-xj) + \alpha_{3\,\text{comp}}[ai, aj];$$

$$\text{FullSimplify}\left[\left\{w_{\text{comp}}[a_i, x_i, a_j, x_j] - w_{i,\text{comp}}, w_{\text{comp}}[a_j, x_j, a_i, x_i] - w_{j,\text{comp}}\right\}\right]$$

{0, 0}

Defining the individuals valuation of stock based on individual profits ;

$$V_{\text{comp}}[\text{ai\_}, \text{xi\_}, \text{aj\_}, \text{xj\_}] = -\left((-1+c(-1+s))(4(1+c-cs)ai + c(-4(1+c)h + 6chs - 2s\,aj + (4-2s+c(4+(-6+s)s))xi + s(2-cs)xj))^2\right)\Big/\left(2(-2+c(-2+s))^2(-2+c(-2+3s))^2\right);$$

$$\text{FullSimplify}\Big[\big\{V_{comp}[a_i, x_i, a_j, x_j] - NP[a_i, w_{comp}[a_i, x_i, a_j, x_j], x_i, a_j, w_{comp}[a_j, x_j, a_i, x_i], x_j],\; V_{comp}[a_j, x_j, a_i, x_i] - NP[a_j, w_{comp}[a_j, x_j, a_i, x_i], x_j, a_i, w_{comp}[a_i, x_i, a_j, x_j], x_i]\big\}\Big]$$

{0, 0}

$$\text{FullSimplify}\Big[\Big\{V_{comp}[a_i, x_i, a_j, x_j] - \frac{1}{2}(1 + c - c\,s)\,w_{i,comp}{}^2,\; V_{comp}[a_j, x_j, a_i, x_i] - \frac{1}{2}(1 + c - c\,s)\,w_{j,comp}{}^2\Big\}\Big]$$

{0, 0}

Rewriting the individuals valuation of stock as quadratic function of stock levels;

$$\overset{comp}{\Gamma}_1 = \frac{c^2 (1 + c - c\,s)\left(4 - 2\,s + 4\,c - 6\,c\,s + c\,s^2\right)^2}{2\,(2 + 2\,c - c\,s)^2\,(2 + 2\,c - 3\,c\,s)^2}; \qquad \overset{comp}{\Gamma}_2 = \frac{c^2 (1 + c - c\,s)\,s^2\,(2 - c\,s)^2}{2\,(2 + 2\,c - c\,s)^2\,(2 + 2\,c - 3\,c\,s)^2};$$

$$\overset{comp}{\Gamma}_3 = \frac{c^2 (1 + c - c\,s)\,s\,(2 - c\,s)\left(4 - 2\,s + 4\,c - 6\,c\,s + c\,s^2\right)}{(2 + 2\,c - c\,s)^2\,(2 + 2\,c - 3\,c\,s)^2};$$

$$\overset{comp}{\Gamma}_4[ai\_, aj\_] = -\frac{c\,(1 + c - c\,s)\left(4 - 2\,s + 4\,c - 6\,c\,s + c\,s^2\right)}{(2 + 2\,c - c\,s)\,(2 + 2\,c - 3\,c\,s)}\left(\frac{ai + aj}{2 + 2\,c - c\,s} + \frac{ai - aj}{2 + 2\,c - 3\,c\,s}\right);$$

$$\overset{comp}{\Gamma}_5[ai\_, aj\_] = -\frac{c\,(1 + c - c\,s)\,s\,(2 - c\,s)}{(2 + 2\,c - c\,s)\,(2 + 2\,c - 3\,c\,s)}\left(\frac{ai + aj}{2 + 2\,c - c\,s} + \frac{ai - aj}{2 + 2\,c - 3\,c\,s}\right);$$

$$\overset{comp}{\Gamma}_6[ai\_, aj\_] = \frac{1}{2}(1 + c - c\,s)\left(\frac{ai + aj}{2 + 2\,c - c\,s} + \frac{ai - aj}{2 + 2\,c - 3\,c\,s}\right)^2;$$

$$\text{FullSimplify}\Big[V_{comp}[a_i, x_i, a_j, x_j] - \Big(\overset{comp}{\Gamma}_1 (h - x_i)^2 + \overset{comp}{\Gamma}_2 (h - x_j)^2 + \overset{comp}{\Gamma}_3 (h - x_i)(h - x_j) + \overset{comp}{\Gamma}_4[a_i, a_j]\,(h - x_i) + \overset{comp}{\Gamma}_5[a_i, a_j]\,(h - x_j) + \overset{comp}{\Gamma}_6[a_i, a_j]\Big)\Big]$$

0

$$\text{FullSimplify}\Big[V_{comp}[a_j, x_j, a_i, x_i] - \Big(\overset{comp}{\Gamma}_1 (h - x_j)^2 + \overset{comp}{\Gamma}_2 (h - x_i)^2 + \overset{comp}{\Gamma}_3 (h - x_i)(h - x_j) + \overset{comp}{\Gamma}_4[a_j, a_i](h - x_j) + \overset{comp}{\Gamma}_5[a_j, a_i](h - x_i) + \overset{comp}{\Gamma}_6[a_j, a_i]\Big)\Big]$$

0

Deriving the aggregate stock valuation by the two users as quadratic function of stock levels;

$$\overset{comp}{\Delta}_1 = \frac{c^2 (1 + c - c\,s)\left(\left(4 - 2s + 4c - 6cs + cs^2\right)^2 + s^2 (2 - cs)^2\right)}{2 (2 + 2c - cs)^2 (2 + 2c - 3cs)^2};$$

$$\overset{comp}{\Delta}_3 = 2 \frac{c^2 (1 + c - cs)\, s\, (2 - cs)\left(4 - 2s + 4c - 6cs + cs^2\right)}{(2 + 2c - cs)^2 (2 + 2c - 3cs)^2};$$

$$\overset{comp}{\Delta}_4[ai\_, aj\_] = -2c(1 + c - cs)\left(\frac{(ai + aj)}{(2 + 2c - cs)^2} + \frac{(1 - s)(ai - aj)}{(2 + 2c - 3cs)^2}\right);$$

$$\overset{comp}{\Delta}_5[ai\_, aj\_] = -2c(1 + c - cs)\left(\frac{(ai + aj)}{(2 + 2c - cs)^2} + \frac{(1 - s)(aj - ai)}{(2 + 2c - 3cs)^2}\right);$$

$$\overset{comp}{\Delta}_6[ai\_, aj\_] = (1 + c - cs)\left(\frac{(ai - aj)^2}{(2 + 2c - 3cs)^2} + \frac{(ai + aj)^2}{(2 + 2c - cs)^2}\right);$$

$$\overset{comp}{\mathcal{V}}[ai\_, xi\_, aj\_, xj\_, s\_] = \overset{comp}{\Delta}_1 (h - xi)^2 + \overset{comp}{\Delta}_1 (h - xj)^2 + \overset{comp}{\Delta}_3 (h - xi)(h - xj) + \overset{comp}{\Delta}_4[ai, aj](h - xi) + \overset{comp}{\Delta}_5[ai, aj](h - xj) + \overset{comp}{\Delta}_6[ai, aj];$$

$$\text{FullSimplify}\left[\overset{\text{comp}}{\mathcal{V}}[a_i, x_i, a_j, x_j, s] - (V_{\text{comp}}[a_i, x_i, a_j, x_j] + V_{\text{comp}}[a_j, x_j, a_i, x_i])\right]$$

0

One round game under cooperation;

Rewriting the objective function;

profit function (as the sum of revenues minus costs for the two players);

$$\text{NPcoop[ai\_, wi\_, xi\_, aj\_, wj\_, xj\_]} = \left(\text{ai wi} - \frac{\text{wi}^2}{2}\right) - \text{c wi}\left(\text{h} - \frac{\text{xi} + (1-\text{s})(\text{xi} - \text{wi}) + \text{s}(\text{xj} - \text{wj})}{2}\right) + \left(\text{aj wj} - \frac{\text{wj}^2}{2}\right) - \text{c wj}\left(\text{h} - \frac{\text{xj} + (1-\text{s})(\text{xj} - \text{wj}) + \text{s}(\text{xi} - \text{wi})}{2}\right);$$

$$\text{FullSimplify}[\text{NPcoop}[a_i, w_i, x_i, a_j, w_j, x_j] - (\text{NP}[a_i, w_i, x_i, a_j, w_j, x_j] + \text{NP}[a_j, w_j, x_j, a_i, w_i, x_i])]$$

0

Motion function : not necessary for this one round game;
SM[xi_, wi_, xj_, wj_] = (1 – s) (xi – wi) + s (xj – wj) + R;

The marginal profit (function) from an extra unit of water;

$$\text{DNPcoop[ai\_, wi\_, xi\_, aj\_, wj\_, xj\_]} = \text{ai} - \text{wi} - \text{c}\left(\text{h} - \frac{\text{xi} + (1-\text{s})(\text{xi} - 2\,\text{wi}) + \text{s}(\text{xj} - \text{wj})}{2}\right) - \frac{\text{c wj s}}{2};$$

Double checking the marginal profit;

$$\text{FullSimplify}\left[\left\{\partial_{w_i}\text{NPcoop}[a_i, w_i, x_i, a_j, w_j, x_j] - \text{DNPcoop}[a_i, w_i, x_i, a_j, w_j, x_j],\ \partial_{w_j}\text{NPcoop}[a_i, w_i, x_i, a_j, w_j, x_j] - \text{DNPcoop}[a_j, w_j, x_j, a_i, w_i, x_i]\right\}\right]$$

{0, 0}

Solving for the extraction decisions;

$$\text{sols} = \text{Solve}\left[\left\{\partial_{w_i}\text{NPcoop}[a_i, w_i, x_i, a_j, w_j, x_j] == 0,\ \partial_{w_j}\text{NPcoop}[a_i, w_i, x_i, a_j, w_j, x_j] == 0\right\}, \{w_i, w_j\}\right];$$

$$\{w_{i,\text{coop}}, w_{j,\text{coop}}\} = \text{Simplify}[\{w_i, w_j\} /. \text{sols[[1]]}];$$

$$\text{FullSimplify}\left[w_{i,\text{coop}} - \left(\left(-\frac{1+3\,c}{4\,(1+c)} + \frac{1-c}{4\,(1+c-2\,c\,s)}\right)(h - x_i) + \left(\frac{1-c}{4\,(1+c)} - \frac{1-c}{4\,(1+c-2\,c\,s)}\right)(h - x_j) + \frac{1}{2}\left(\frac{a_i + a_j}{1+c} + \frac{a_i - a_j}{1+c-2\,c\,s}\right)\right)\right]$$

0

$$\text{FullSimplify}\Big[w_{j,coop} - \Big(\Big(-\frac{1+3c}{4(1+c)} + \frac{1-c}{4(1+c-2cs)}\Big)(h-x_j) + \Big(\frac{1-c}{4(1+c)} - \frac{1-c}{4(1+c-2cs)}\Big)(h-x_i) + \frac{1}{2}\Big(\frac{a_i+a_j}{1+c} + \frac{a_j-a_i}{1+c-2cs}\Big)\Big)\Big]$$

0

Rewriting the extraction decisions as linear functions of stock levels;

$$\alpha_{1,coop} = -\frac{1+3c}{4(1+c)} + \frac{1-c}{4(1+c-2cs)};\ \alpha_{2,coop} = \frac{1-c}{4(1+c)} - \frac{1-c}{4(1+c-2cs)};$$

$$\alpha_{3\,i,coop} = \frac{1}{2}\Big(\frac{a_i+a_j}{1+c} + \frac{a_i-a_j}{1+c-2cs}\Big);\ \alpha_{3\,j,coop} = \frac{1}{2}\Big(\frac{a_i+a_j}{1+c} + \frac{a_j-a_i}{1+c-2cs}\Big);$$

$$\text{FullSimplify}\big[\{w_{i,coop} - (\alpha_{1,coop}(h-x_i) + \alpha_{2,coop}(h-x_j) + \alpha_{3\,i,coop}),\ w_{j,coop} - (\alpha_{1,coop}(h-x_j) + \alpha_{2,coop}(h-x_i) + \alpha_{3\,j,coop})\}\big]$$

{0, 0}

$$\alpha_{3\,coop}[ai\_, aj\_] = \frac{1}{2}\Big(\frac{ai+aj}{1+c} + \frac{ai-aj}{1+c-2cs}\Big);$$

$$w_{coop}[ai\_, xi\_, aj\_, xj\_] = \alpha_{1,coop}(h-xi) + \alpha_{2,coop}(h-xj) + \alpha_{3\,coop}[ai, aj];$$

$$\text{FullSimplify}\big[\{\{w_{coop}[a_i, x_i, a_j, x_j] - w_{i,coop}\},\ \{w_{coop}[a_j, x_j, a_i, x_i] - w_{j,coop}\}\}\big]$$

{{0}, {0}}

Defining the individuals valuation of stock based on individual profits ;

Rewriting the aggregate valuation of stock as quadratic function of stock levels;

$$\overset{coop}{\Delta}_1 = -\frac{c^2(2+(-2+s)s+c(2+(-4+s)s))}{4(1+c)(-1+c(-1+2s))};$$

$$\overset{coop}{\Delta}_3 = \frac{c^2 s(-2+s+cs)}{2(1+c)(-1+c(-1+2s))};\ \overset{coop}{\Delta}_4[ai\_, aj\_] = -\frac{(ai+aj)c}{2(1+c)} + \frac{(ai-aj)c(-1+s)}{2+c(2-4s)};$$

$$\overset{coop}{\Delta}_5[ai\_, aj\_] = -\frac{(ai+aj)c}{2(1+c)} + \frac{(aj-ai)c(-1+s)}{2+c(2-4s)};\ \overset{coop}{\Delta}_6[ai\_, aj\_] = \frac{(ai+aj)^2}{4(1+c)} - \frac{(ai-aj)^2}{-4+c(-4+8s)};$$

$$\overset{coop}{\mathcal{V}}[ai\_, xi\_, aj\_, xj\_, s\_] = \overset{coop}{\Delta}_1(h-xi)^2 + \overset{coop}{\Delta}_1(h-xj)^2 + \overset{coop}{\Delta}_3(h-xi)(h-xj) + \overset{coop}{\Delta}_4[ai, aj](h-xi) + \overset{coop}{\Delta}_5[ai, aj](h-xj) + \overset{coop}{\Delta}_6[ai, aj];$$

$$\text{FullSimplify}\left[\overset{\text{coop}}{\mathcal{V}}[a_i, x_i, a_j, x_j, s] - (\text{NPcoop}[a_i, w_{coop}[a_i, x_i, a_j, x_j], x_i, a_j, w_{coop}[a_j, x_j, a_i, x_i], x_j])\right]$$

0

One round game under single user i;

Rewriting the profit and marginal profit functions when only player i is allowed to use the resource;

$$\text{NPsing}[ai\_, wi\_, xi\_, aj\_, wj\_, xj\_] = ai\, wi - \frac{wi^2}{2} - c\, wi\left(h - \frac{xi + (1-s)(xi - wi) + s(xj - 0)}{2}\right);$$

$$\text{SMsing}[wi\_, xi\_, wj\_, xj\_] = (1-s)(xi - wi) + s(xj - 0) + R;$$

$$\text{DNPsing}[ai\_, wi\_, xi\_, aj\_, wj\_, xj\_] = ai - wi - c\left(h - \frac{xi + (1-s)(xi - 2\, wi) + s\, xj}{2}\right);$$

$$\text{FullSimplify}[\{\partial_{w_i} \text{NPsing}[a_i, w_i, x_i, a_j, w_j, x_j] - \text{DNPsing}[a_i, w_i, x_i, a_j, w_j, x_j]\}]$$

{0}

Solving for the extraction decision with a single user;

$$\text{sols} = \text{Solve}[\text{DNPsing}[a_i, w_i, x_i, a_j, w_j, x_j] == 0, w_i]; \{w_{i,sing}\} = \text{Simplify}[\{w_i\} /. \text{sols}[[1]]];$$

$$\text{FullSimplify}\left[\left\{w_{i,sing} - \left(\frac{-c}{2 + 2c(1-s)}(2h + (-2+s)x_i - s\, x_j) + \frac{2a_i}{2 + 2c(1-s)}\right)\right\}\right]$$

{0}

Rewriting the extraction decision as linear functions of stock levels;

$$\alpha_{1\,sing} = -\frac{c(2-s)}{2 + 2c(1-s)};\ \alpha_{2\,sing} = -\frac{c\, s}{2 + 2c(1-s)};\ \alpha_{3\,i,sing} = \frac{2a_i}{2 + 2c(1-s)};$$

$$\text{FullSimplify}[\{w_{i,sing} - (\alpha_{1\,sing}(h - x_i) + \alpha_{2\,sing}(h - x_j) + \alpha_{3\,i,sing})\}]$$

{0}

$$\alpha_{3\,sing}[ai\_, aj\_] = \frac{ai}{1 + c(1-s)};$$

$$w_{sing}[ai\_, xi\_, aj\_, xj\_] = \alpha_{1\,sing}(h - xi) + \alpha_{2\,sing}(h - xj) + \alpha_{3\,sing}[ai, aj];$$

FullSimplify[{$w_{sing}[a_i, x_i, a_j, x_j] - w_{i,sing}$}]

{0}

Defining the single user valuation of stocks based on his individual profit ;

V_{sing}[ai_, xi_, aj_, xj_] = NPsing[ai, w_{sing}[ai, xi, aj, xj], xi, aj, w_{sing}[aj, xj, ai, xi], xj];

FullSimplify[{ $V_{sing}[a_i, x_i, a_j, x_j] - \frac{1}{2}(1 + c - c\,s)\left(w_{i,sing}\right)^2$ }]

{0}

Deriving the stock valuation by the single user as quadratic function of stock levels;

$\overset{sing}{\Delta}_1 = \frac{c^2(-2+s)^2}{8+8c(1-s)}$; $\overset{sing}{\Delta}_2 = \frac{c^2 s^2}{8+8c(1-s)}$; $\overset{sing}{\Delta}_3 = \frac{c^2(2-s)s}{4+4c(1-s)}$;

$\overset{sing}{\Delta}_4$[ai_, aj_] = $-\frac{ai\,c\,(2-s)}{2+2c(1-s)}$; $\overset{sing}{\Delta}_5$[ai_, aj_] = $-\frac{ai\,c\,s}{2+2c(1-s)}$; $\overset{sing}{\Delta}_6$[ai_, aj_] = $\frac{ai^2}{2+2c(1-s)}$;

$\overset{sing}{\mathcal{V}}$[ai_, xi_, aj_, xj_, s_] = $\overset{sing}{\Delta}_1 (h - xi)^2 + \overset{sing}{\Delta}_2 (h - xj)^2 +$
$\overset{sing}{\Delta}_3 (h - xi)(h - xj) + \overset{sing}{\Delta}_4$[ai, aj] (h − xi) + $\overset{sing}{\Delta}_5$[ai, aj] (h − xj) + $\overset{sing}{\Delta}_6$[ai, aj];

FullSimplify[$\overset{sing}{\mathcal{V}}[a_i, x_i, a_j, x_j, s]$ − (NPsing[a_i, $w_{sing}[a_i, x_i, a_j, x_j]$, x_i, a_j, $w_{sing}[a_j, x_j, a_i, x_i]$, x_j])]

0

Theoretical results;

Under Competition;

Rewriting the aggregate profit function as function the level of inequality ϵ;

$$\overset{\text{comp}}{\mathcal{V}}\epsilon[a\_, x\_, \epsilon\_, s\_] := 4(1+c-c\,s)\left(\frac{\epsilon^2}{(2+2c-3c\,s)^2}+\frac{(a-c(h-x))^2}{(2+2c-c\,s)^2}\right);$$

$$\text{FullSimplify}\left[\left\{\left(\overset{\text{comp}}{\mathcal{V}}\epsilon[a, x, \epsilon, s]\right)-\overset{\text{comp}}{\mathcal{V}}[a+\epsilon, x, a-\epsilon, x, s]\right\}\right]$$

{0}

Observation1;

inequality increases the sum of all profits from the CPR;

$$\text{FullSimplify}\left[\left\{\left(\overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, s]-\overset{\text{comp}}{\mathcal{V}}\left[\frac{a_i+a_j}{2}, x, \frac{a_i+a_j}{2}, x, s\right]\right)-\left(\frac{(1+c-c\,s)}{(2+2c-3c\,s)^2}(a_i-a_j)^2\right),\right.\right.$$
$$\left.\left.D\left[\overset{\text{comp}}{\mathcal{V}}\epsilon[a, x, \epsilon, s], \epsilon\right]-\left(\frac{8(1+c-c\,s)\epsilon}{(2+2c-3c\,s)^2}\right)\right\}\right]$$

{0, 0}

Observation2;

Transmissivity decreases the sum of all profits from the CPR for players with low inequality;

$$\text{FullSimplify}\left[\left\{D\left[\overset{\text{comp}}{\mathcal{V}}[a, x, a, x, s], s\right]-\left(-\frac{4c^2 s(a-c(h-x))^2}{(2+2c-c\,s)^3}\right),\right.\right.$$
$$\left.\left.\overset{\text{comp}}{\mathcal{V}}[a, x, a, x, 1/2]-\overset{\text{comp}}{\mathcal{V}}[a, x, a, x, 0]-\left(-\frac{c^2(a-c(h-x))^2}{(1+c)(4+3c)^2}\right)\right\}\right]$$

{0, 0}

Observation3;

in the case of highly unequal players transmissivity has an increasing effect on the sum of all profits;

$$\text{FullSimplify}\left[D\left[\overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, s], s\right]-\left((a_i-a_j)^2\frac{c(4+4c-3c\,s)}{(2+2c-3c\,s)^3}-\frac{4c^4 s}{(2+2c-c\,s)^3}\left(\frac{(a_i+a_j)}{2c}-(h-x)\right)^2\right)\right]$$

0

$$\text{FullSimplify}\Big[\text{D}\Big[\overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, s], s\Big] - \frac{(4+4c-3cs)}{(2+2c-3cs)^3} c \left((a_i - a_j) - \left(\frac{cs}{4+4c-3cs}\right)^{1/2} \left(\frac{2+2c-3cs}{2+2c-cs}\right)^{3/2} (a_i + a_j - 2c(h-x))\right) \left((a_i - a_j) + \left(\frac{cs}{4+4c-3cs}\right)^{1/2} \left(\frac{2+2c-3cs}{2+2c-cs}\right)^{3/2} (a_i + a_j - 2c(h-x))\right)\Big]$$

0

$$\text{FullSimplify}\Big[\text{D}\Big[\overset{\text{comp}}{\mathcal{V}}\epsilon[a, x, \epsilon, s], s\Big] - \left(\frac{4c(4+4c-3cs)}{(2+2c-3cs)^3} \epsilon^2 - \frac{4c^2 s}{(2+2c-cs)^3} (a - c(h-x))^2\right)\Big]$$

0

$$\text{FullSimplify}\Big[\text{D}\Big[\overset{\text{comp}}{\mathcal{V}}\epsilon[a, x, \epsilon, s], s\Big] - \frac{4(4+4c-3cs)}{(2+2c-3cs)^3} c \left(\epsilon - \left(\frac{cs}{4+4c-3cs}\right)^{1/2} \left(\frac{2+2c-3cs}{2+2c-cs}\right)^{3/2} (a - c(h-x))\right) \left(\epsilon + \left(\frac{cs}{4+4c-3cs}\right)^{1/2} \left(\frac{2+2c-3cs}{2+2c-cs}\right)^{3/2} (a - c(h-x))\right)\Big]$$

0

`Solving for the minimum acceptable value of al for the less efficient player;`

$\text{sols} = \text{Solve}\big[w_{\text{comp}}[a_l, x, a_h, x] == 0, a_l\big]; \{a_{l,\text{MinCompS}}\} = \text{Simplify}[\{a_l\} /. \text{sols}[[1]]];$

$$\text{FullSimplify}\Big[a_{l,\text{MinCompS}} - \left(c\, s \frac{a_h - c(h-x)}{2+2c(1-s)} + c(h-x)\right)\Big]$$

0

$\text{sols} = \text{Solve}\big[w_{\text{comp}}[a - \epsilon, x, a + \epsilon, x] == 0, \epsilon\big]; \{\epsilon_{\text{MaxCompS}}\} = \text{Simplify}[\{\epsilon\} /. \text{sols}[[1]]];$

$$\text{FullSimplify}\Big[\epsilon_{\text{MaxCompS}} - \frac{(2+2c-3cs)}{(2+2c-cs)} (a - c(h-x))\Big]$$

0

Showing the positive derivative with the minimum acceptable efficiency;

$$\overset{\text{comp}}{\mathcal{V}}_{\mathbf{S}}[ai\_, x\_, aj\_, x\_, s\_] = (ai - aj)^2 \frac{c(4+4c-3cs)}{(2+2c-3cs)^3} - \frac{4c^4 s}{(2+2c-cs)^3} \left(\frac{(ai + aj)}{2c} - (h-x)\right)^2;$$

$$\text{FullSimplify}\Big[\text{D}\Big[\overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, s], s\Big] - \overset{\text{comp}}{\mathcal{V}}_{\mathbf{S}}[a_i, x, a_j, x, s]\Big]$$

0

$$\text{FullSimplify}\Big[\Big\{\, w_{\text{comp}}\big[a_{\text{l, MinCompS}}, x, a_h, x\big]\ ,$$
$$\overset{\text{comp}}{\mathcal{V}_{\mathbf{S}}}\big[a_{\text{l, MinCompS}}, x, a_h, x, s\big] - \frac{c}{2}\left(1 + \frac{1+c}{2+2c-3cs} - \frac{1+c}{2+2c-cs}\right)\left(\frac{a_h - c(h-x)}{1+c-cs}\right)^2 \Big\}\Big]$$

{0, 0}

$$\overset{\text{comp}}{\mathcal{V}\epsilon_{\mathbf{S}}}[a\_, x\_, \epsilon\_, s\_] = 4\epsilon^2 \left(\frac{c(4+4c-3cs)}{(2+2c-3cs)^3} - \frac{4c^2 s}{(2+2c-cs)^3}\right)(a - c(h-x))^2;$$

$$\text{FullSimplify}\Big[D\Big[\overset{\text{comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s], s\Big] - \overset{\text{comp}}{\mathcal{V}\epsilon_{\mathbf{S}}}[a, x, \epsilon, s]\Big]$$

0

$$\text{FullSimplify}\Big[\Big\{\, w_{\text{comp}}\big[a - \epsilon_{\text{MaxCompS}}, x, a + \epsilon_{\text{MaxCompS}}, x\big]\ ,$$
$$\overset{\text{comp}}{\mathcal{V}\epsilon_{\mathbf{S}}}\big[a, x, \epsilon_{\text{MaxCompS}}, s\big] - \frac{c}{2}\left(1 + \frac{1+c}{2+2c-3cs} - \frac{1+c}{2+2c-cs}\right)\left(\frac{a + \epsilon_{\text{MaxCompS}} - c(h-x)}{1+c-cs}\right)^2 \Big\}\Big]$$

{0, 0}

$$\epsilon_{\text{CompNullDerivS}} = \left(\frac{cs}{4+4c-3cs}\right)^{1/2}\left(\frac{2+2c-3cs}{2+2c-cs}\right)^{3/2}(a - c(h-x));$$

$$\text{FullSimplify}\Big[\big(\epsilon_{\text{MaxCompS}} / \epsilon_{\text{CompNullDerivS}}\big)^2 - \left(\frac{2+2c-cs}{cs}\right)\left(\frac{4+4c-3cs}{2+2c-3cs}\right)\Big]$$

0

$$\text{FullSimplify}\Big[\Big\{\overset{\text{comp}}{\mathcal{V}\epsilon_{\mathbf{S}}}[a, x, 0, s] - \left(-\frac{4c^2 s(a - c(h-x))^2}{(2+2c-cs)^3}\right), \overset{\text{comp}}{\mathcal{V}\epsilon_{\mathbf{S}}}\big[a, x, \epsilon_{\text{CompNullDerivS}}, s\big]\Big\}\Big]$$

{0, 0}

Showing the positive derivative at low levels of transmissivity;

$$\text{FullSimplify}\Big[\Big\{\ \text{Limit}\Big[D\Big[\overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, s], s\Big], s \to 0\Big] - \left(\frac{c(a_i - a_j)^2}{2(1+c)^2}\right)\ ,$$
$$\text{Limit}\Big[D\Big[\overset{\text{comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s], s\Big], s \to 0\Big] - \frac{2c\epsilon^2}{(1+c)^2}\Big\}\Big]$$

{0, 0}

Showing the conditions for a gain from a nonmarginal change of transmissivity;

$$\text{FullSimplify}\Big[\overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, 1/2] - \overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, 0] - \frac{c^4}{(1+c)(4+3c)^2}\left((a_i - a_j)^2 \frac{(4+3c)^2(16+7c)}{4c^3(4+c)^2} - \left(\frac{(a_i + a_j)}{2c} - (h-x)\right)^2\right)\Big]$$

0

$$\text{FullSimplify}\Big[\overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, 1/2] - \overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, 0] - \frac{c}{(1+c)} \frac{(16+7c)}{4(4+c)^2}\left((a_i - a_j) - \left(\frac{c(4+c)^2}{(4+3c)^2(16+7c)}\right)^{1/2}((a_i + a_j) - 2c(h-x))\right) \left((a_i - a_j) + \left(\frac{c(4+c)^2}{(4+3c)^2(16+7c)}\right)^{1/2}((a_i + a_j) - 2c(h-x))\right)\Big]$$

0

$$\text{FullSimplify}\Big[\left\{\left(\overset{\text{comp}}{\mathcal{V}}\epsilon[a, x, \epsilon, 1/2] - \overset{\text{comp}}{\mathcal{V}}\epsilon[a, x, \epsilon, 0]\right) - \frac{c^2}{(1+c)(4+3c)^2}\left(\epsilon^2 \frac{(4+3c)^2(16+7c)}{c(4+c)^2} - (a - c(h-x))^2\right),\right.$$
$$\left.\left(\overset{\text{comp}}{\mathcal{V}}\epsilon[a, x, \epsilon, 1/2] - \overset{\text{comp}}{\mathcal{V}}\epsilon[a, x, \epsilon, 0]\right) - \frac{c(16+7c)}{(1+c)(4+c)^2}\left(\epsilon - \frac{c^{1/2}(a - c(h-x))}{(16+7c)^{1/2}} \frac{(4+c)}{(4+3c)}\right)\left(\epsilon + \frac{c^{1/2}(a - c(h-x))}{(16+7c)^{1/2}} \frac{(4+c)}{(4+3c)}\right)\right\}\Big]$$

{0, 0}

$$\epsilon_{\text{MaxCompInfS}} = \left(\frac{4+c}{4+3c}\right)(a - c(h-x));\ \text{FullSimplify}\big[\text{Limit}[\epsilon_{\text{MaxCompS}}, s \to 1/2] - \epsilon_{\text{MaxCompInfS}}\big]$$

0

$$\text{FullSimplify}\left[\left\{\left(\frac{\epsilon_{\text{MaxCompInfS}}}{\frac{c^{1/2}(a-c(h-x))}{(16+7c)^{1/2}}\frac{(4+c)}{(4+3c)}}\right)^2-\left(\frac{16+7c}{c}\right),\right.\right.$$

$$\left.\left.\mathcal{V}\epsilon^{\text{comp}}[a,x,\epsilon_{\text{MaxCompInfS}},1/2]-\mathcal{V}\epsilon^{\text{comp}}[a,x,\epsilon_{\text{MaxCompInfS}},0]-\frac{2c(8+3c)}{(1+c)(4+3c)^2}(a-c(h-x))^2\right\}\right]$$

{0, 0}

Observation4;

transmissivity has a decreasing effect on the profits of less efficient players ;

for the efficient player transmissivity has a decreasing effect at low levels of inequality and; an increasing effect when equality is high;

$$V_{\text{compS}}[\text{ai\_},\text{xi\_},\text{aj\_},\text{xj\_},\text{s\_}]=-\Big(\big((-1+c(-1+s))(4\,ai(1+c-cs)+c(-2\,aj\,s+h(-4+c(-4+6s))+(4-2s+c(4+(-6+s)s))xi+s(2-cs)xj))^2\big)\Big/\big(2(-2+c(-2+s))^2(-2+c(-2+3s))^2\big)\Big);$$

$$\text{FullSimplify}[V_{\text{compS}}[a_i,x_i,a_j,x_j,s]-(V_{\text{comp}}[a_i,x_i,a_j,x_j])]$$

0

$$\text{FullSimplify}\left[V_{\text{compS}}[a_i,x,a_j,x,s]-\frac{(1+c-cs)}{2}\left(\frac{a_i+a_j-2c(h-x)}{2+2c-cs}+\frac{a_i-a_j}{2+2c-3cs}\right)^2\right]$$

0

$$\text{FullSimplify}\left[V_{\text{compS}}[a+\epsilon,x,a-\epsilon,x,s]-2(1+c-cs)\left(\frac{a-c(h-x)}{2+2c-cs}+\frac{\epsilon}{2+2c-3cs}\right)^2\right]$$

0

$$\text{FullSimplify}\left[\partial_S V_{\text{compS}}[a_i,x,a_j,x,s]-\frac{2c^4}{(2+2c-cs)^3}\left(\frac{2a_i(1+c-cs)-a_j cs}{c(2+2c-3cs)}-(h-x)\right)\left(\frac{(a_i-a_j)(2+2c-cs)^2(4+4c-3cs)}{2c^2(2+2c-3cs)^2}-s\left(\frac{(a_i+a_j)}{2c}-(h-x)\right)\right)\right]$$

0

$$\text{FullSimplify}\left[\partial_S V_{\text{compS}}[a+\epsilon,x,a-\epsilon,x,s]-2c\left(\frac{a-c(h-x)}{2+2c-cs}+\frac{\epsilon}{2+2c-3cs}\right)\left(\frac{(4+4c-3cs)\epsilon}{(2+2c-3cs)^2}-\frac{cs(a-c(h-x))}{(2+2c-cs)^2}\right)\right]$$

0

Showing positive sign of the first term for less efficient player
(for the efficient player it is self evident since a_i is higher than a_j);
to do that we show that the term is nil for the minimum acceptable value;

$$\text{FullSimplify}\left[\frac{2\,a_{l,\text{MinCompS}}\,(1+c-c\,s)-a_h\,c\,s}{c\,(2+2\,c\,-3\,c\,s)}-(h-x)\right]$$

0

$$\text{FullSimplify}\left[\frac{a-c\,(h-x)}{2+2\,c-c\,s}+\frac{-\epsilon_{\text{MaxCompS}}}{2+2\,c-3\,c\,s}\right]$$

0

Extending the observation to the nonmarginal change in transmissivity;

$$\text{FullSimplify}\Big[\big(V_{\text{compS}}[a_i, x, a_j, x, 1/2]-V_{\text{compS}}[a_i, x, a_j, x, 0]\big)-\frac{4\,(2+c)}{(4+c)^2}\left(\left(a_i-a_j\right)^2-\frac{c^4\,(4+c)^2}{8\,(1+c)\,(2+c)\,(4+3\,c)^2}\left((h-x)-\frac{a_i+a_j}{2\,c}+\frac{\left(a_i-a_j\right)(4+3\,c)\,(8+5\,c)}{2\,c^2\,(4+c)}\right)^2\right)\Big]$$

0

$$\text{FullSimplify}\Big[\big(V_{\text{compS}}[a_i, x, a_j, x, 1/2]-V_{\text{compS}}[a_i, x, a_j, x, 0]\big)-\frac{4\,(2+c)}{(4+c)^2}\left(\left(a_i-a_j\right)\left(1-\frac{(8+5\,c)}{4\,\sqrt{2}\,\sqrt{(1+c)\,(2+c)}}\right)+\frac{c\,(4+c)}{4\,\sqrt{2}\,\sqrt{(1+c)\,(2+c)}\,(4+3\,c)}\left(a_i+a_j-2\,c\,(h-x)\right)\right)\left(\left(a_i-a_j\right)\left(1+\frac{(8+5\,c)}{4\,\sqrt{2}\,\sqrt{(1+c)\,(2+c)}}\right)-\frac{c\,(4+c)}{4\,\sqrt{2}\,\sqrt{(1+c)\,(2+c)}\,(4+3\,c)}\left(a_i+a_j-2\,c\,(h-x)\right)\right)\Big]$$

0

$$\text{FullSimplify}\Big[\big(V_{\text{compS}}[a_i, x, a_j, x, 1/2]-V_{\text{compS}}[a_i, x, a_j, x, 0]\big)-\left(\frac{2\,c\,(16+7\,c)\left(a_i-a_j\right)}{4\,\sqrt{1+c}\,(4+c)\left(8\,\sqrt{2}\,+5\,\sqrt{2}\,c+8\,\sqrt{1+c}\,\sqrt{2+c}\right)}+\frac{c\left(a_i+a_j-2\,c\,(h-x)\right)}{2\,\sqrt{2}\,\sqrt{1+c}\,(4+3\,c)}\right)\left(\frac{\left(a_i-a_j\right)\left(5\,\sqrt{2}\,c+8\left(\sqrt{2}\,+\sqrt{1+c}\,\sqrt{2+c}\right)\right)}{4\,\sqrt{1+c}\,(4+c)}-\frac{c\left(a_i+a_j-2\,c\,(h-x)\right)}{2\,\sqrt{2}\,\sqrt{1+c}\,(4+3\,c)}\right)\Big]$$

0

$$\text{FullSimplify}\Big[\big(V_{\text{compS}}[a+\epsilon, x, a-\epsilon, x, 1/2]-V_{\text{compS}}[a+\epsilon, x, a-\epsilon, x, 0]\big)-\frac{1}{2}\left(8\,(2+c)\left(\frac{(4+c)\,(a-c\,(h-x))+(4+3\,c)\,\epsilon}{(4+c)\,(4+3\,c)}\right)^2-\frac{(a-c\,(h-x)+\epsilon)^2}{1+c}\right)\Big]$$

0

$$\text{FullSimplify}\Big[\big(V_{\text{compS}}[a+\epsilon, x, a-\epsilon, x, 1/2] - V_{\text{compS}}[a+\epsilon, x, a-\epsilon, x, 0]\big) - \frac{1}{2}\left(2\sqrt{2}\,\sqrt{2+c}\left(\frac{(4+c)(a-c(h-x))+(4+3c)\epsilon}{(4+c)(4+3c)}\right)+\frac{(a-c(h-x)+\epsilon)}{\sqrt{1+c}}\right)\left(2\sqrt{2}\,\sqrt{2+c}\left(\frac{(4+c)(a-c(h-x))+(4+3c)\epsilon}{(4+c)(4+3c)}\right)-\frac{(a-c(h-x)+\epsilon)}{\sqrt{1+c}}\right)\Big]$$

0

$$\text{FullSimplify}\Big[\big(V_{\text{compS}}[a+\epsilon, x, a-\epsilon, x, 1/2] - V_{\text{compS}}[a+\epsilon, x, a-\epsilon, x, 0]\big) - \frac{1}{2}\left(\frac{\left(2\sqrt{2}\,\sqrt{1+c}\,\sqrt{2+c}\,+4+3c\right)(a-c(h-x))}{\sqrt{1+c}\,(4+3c)}+\left(\frac{2\sqrt{2}\,\sqrt{2+c}}{4+c}+\frac{1}{\sqrt{1+c}}\right)\epsilon\right)\left(\frac{-c^2(a-c(h-x))}{\sqrt{1+c}\,(4+3c)\left(2\sqrt{2}\,\sqrt{1+c}\,\sqrt{2+c}\,+4+3c\right)}+\frac{c(16+7c)\epsilon}{(4+c)\sqrt{1+c}\left(2\sqrt{2}\,\sqrt{2+c}\,\sqrt{1+c}\,+4+c\right)}\right)\Big]$$

0

Showing that the first term is always positive for acceptable levels of efficiency;

The more efficient player; obvious; for the less efficient player; showing that the first term is increasing in players own efficiency;

$$\text{FirstTerm[ai\_, aj\_]} = \frac{2c(16+7c)(ai-aj)}{4\sqrt{1+c}\,(4+c)\left(8\sqrt{2}\,+5\sqrt{2}\,c+8\sqrt{1+c}\,\sqrt{2+c}\right)}+\frac{c(ai+aj-2c(h-x))}{2\sqrt{2}\,\sqrt{1+c}\,(4+3c)};$$

$$\text{FullSimplify}\Big[D[\text{FirstTerm}[a_l, a_h], a_l] - \frac{1}{4\sqrt{1+c}}\left(\frac{\sqrt{2}\,c}{4+3c}+\frac{2c(16+7c)}{(4+c)\left(8\sqrt{2}\,+5\sqrt{2}\,c+8\sqrt{1+c}\,\sqrt{2+c}\right)}\right)\Big]$$

0

$$\text{FirstTerm}\epsilon[\epsilon\_] := \frac{\left(2\sqrt{2}\,\sqrt{1+c}\,\sqrt{2+c}\,+4+3c\right)(a-c(h-x))}{\sqrt{1+c}\,(4+3c)}+\left(\frac{2\sqrt{2}\,\sqrt{2+c}}{4+c}+\frac{1}{\sqrt{1+c}}\right)\epsilon;$$

The derivative been positive we only need to check the sign for the minimum acceptable value of a_l corresponding to a positive extraction for the less efficient player;

$$\text{sols} = \text{Solve}\big[\{w_{\text{comp}}[a_l, x, a_h, x] == 0, s == 1/2\}, \{a_l, s\}\big]; \{a_{l,\text{MinComp}}, s_{\text{open}}\} = \text{Simplify}[\{a_l, s\} /. \text{sols}[[1]]];$$

$$\text{FullSimplify}\left[\left\{\left\{a_{l,\text{MinComp}} - \left(\frac{(4+c)\,c\,(h-x) + c\,a_h}{4+2c}\right)\right\},\right.\right.$$
$$\left.\left.\left\{\text{FirstTerm}\left[a_{l,\text{MinComp}}, a_h\right] - \left(\frac{2c^2\,(a_h - c\,(h-x))}{4\sqrt{1+c}\;(2+c)\left(4\sqrt{2}\;+3\sqrt{2}\;c+4\sqrt{1+c}\;\sqrt{2+c}\right)}\right)\right\}\right\}\right]$$

{{0}, {0}}

$$\text{FullSimplify}\left[\text{FirstTerm}\epsilon\left[-\epsilon_{\text{MaxCompInfS}}\right] - \frac{2c\,(a + c\,(-h+x))}{\sqrt{1+c}\;(4+3c)}\right]$$

0

Solving for the minimum inequality beyond which the more efficient player is better off after the non marginal shift in transmissivity;

$$\epsilon_{0\,\text{NonMarg}} = \epsilon_{\text{MaxCompInfS}}\,\frac{c}{8+5c+4\sqrt{2}\;\sqrt{1+c}\;\sqrt{2+c}};$$

$$\text{FullSimplify}\left[\left(V_{\text{compS}}\left[a+\epsilon_{0\,\text{NonMarg}}, x, a-\epsilon_{0\,\text{NonMarg}}, x, 1/2\right] - V_{\text{compS}}\left[a+\epsilon_{0\,\text{NonMarg}}, x, a-\epsilon_{0\,\text{NonMarg}}, x, 0\right]\right)\right]$$

0

with Single user;

Rewriting the aggregate profit function as function the level of inequality ϵ;

$$\overset{\text{sing}}{\mathcal{V}}\epsilon[a\_, x\_, \epsilon\_, s\_] := \left(\frac{(a + c\,(-h+x) + \epsilon)^2}{2+2c\;-2c\,s}\right);\ \text{FullSimplify}\left[\left\{\left(\overset{\text{sing}}{\mathcal{V}}\epsilon[a, x, \epsilon, s]\right) - \overset{\text{sing}}{\mathcal{V}}[a+\epsilon, x, a-\epsilon, x, s]\right\}\right]$$

{0}

Observation0a;

$$\text{FullSimplify}\left[\left\{w_{\text{sing}}\left[a_i, x, a_j, x\right] - \left(\frac{a_i - c\,(h-x)}{1+c-c\,s}\right),\ \overset{\text{sing}}{\mathcal{V}}\left[a_i, x, a_j, x, s\right] - \frac{(a_i - c\,(h-x))^2}{2\,(1+c-c\,s)}\right\}\right]$$

{0, 0}

Observation0b;

The most efficient player makes more profits from buy out ;

$$\text{FullSimplify}\left[\left(\overset{\text{sing}}{\mathcal{V}}\left[a_i, x, a_j, x, s\right] - \overset{\text{sing}}{\mathcal{V}}\left[a_j, x, a_i, x, s\right]\right) - \frac{(a_i - a_j)}{1+c\,(1-s)}\left(\frac{a_i + a_j}{2} - c\,(h-x)\right)\right]$$

0

Observation1;

inequality increases the profits with a single efficient user;
and has the opposite with a single less efficient user;

$$\text{FullSimplify}\Big[\Big\{\Big(\overset{\text{sing}}{\mathcal{V}}[a_i, x, a_j, x, s] - \overset{\text{sing}}{\mathcal{V}}\Big[\frac{a_i + a_j}{2}, x, \frac{a_i + a_j}{2}, x, s\Big]\Big) - \frac{(a_i - a_j)}{2 + 2c(1 - s)}\Big(\frac{3a_i + a_j}{4} - c(h - x)\Big),$$

$$D\Big[\overset{\text{sing}}{\mathcal{V}}\epsilon[a, x, \epsilon, s], \epsilon\Big] - \Big(\frac{a + \epsilon - c(h - x)}{1 + c - cs}\Big)\Big\}\Big]$$

{0, 0}

Observation2;

Transmissivity increases the revenues from buy out ;

$$\text{FullSimplify}\Big[\Big\{D\Big[\overset{\text{sing}}{\mathcal{V}}[a_i, x, a_j, x, s], s\Big] - \frac{c}{2}\Big(\frac{a_i - c(h - x)}{1 + c - cs}\Big)^2,$$

$$\overset{\text{sing}}{\mathcal{V}}\Big[a_i, x, a_j, x, \frac{1}{2}\Big] - \overset{\text{sing}}{\mathcal{V}}[a_i, x, a_j, x, 0] - \frac{c}{2}\Big(\frac{(a_i - c(h - x))^2}{2 + 3c + c^2}\Big)\Big\}\Big]$$

{0, 0}

Single user / competition;

Defining the gain / loss function from single use vs competition ;

$$\overset{\text{sing-comp}}{\mathcal{V}}[\text{ai\_}, \text{x\_}, \text{aj\_}, \text{x\_}, \text{s\_}] := -\frac{(2+2c-3cs)^2 - 2c^2 s^2}{2(2+2c-cs)^2(1+c-cs)}\left(\frac{\text{ai}+\text{aj}}{2} - c(h-x)\right)^2 + \frac{(\text{ai}-\text{aj})}{2(1+c-cs)}\left(\frac{\text{ai}+\text{aj}}{2} - c(h-x)\right) - \frac{\left((2+2c-3cs)^2 + 2cs(4+4c-5cs)\right)}{8(2+c(2-3s))^2(1+c-cs)}(\text{ai}-\text{aj})^2;$$

$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon}[\text{a\_}, \text{x\_}, \epsilon\text{\_}, \text{s\_}] := -\left(\frac{(2+2c-3cs)^2 - 2c^2 s^2}{2(1+c-cs)(2+2c-cs)^2}\right)(a-c(h-x))^2 + \frac{(a-c(h-x))\ \epsilon}{(1+c-cs)} - \left(\frac{(2+2c-2cs)^2 + cs(4+4c-5cs)}{2(1+c-cs)(2+2c-3cs)^2}\right)\epsilon^2;$$

$$\text{FullSimplify}\left[\left\{\left\{\overset{\text{sing-comp}}{\mathcal{V}}[a_i, x, a_j, x, s] - \left(\overset{\text{sing}}{\mathcal{V}}[a_i, x, a_j, x, s] - \overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, s]\right)\right\}, \left\{\overset{\text{sing-comp}}{\mathcal{V}}[a+\epsilon, x, a-\epsilon, x, s] - \overset{\text{sing-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s]\right\}\right\}\right]$$

{{0}, {0}}

Observation3; in the case with high cost of extraction $\left(c \text{ positive and higher than } 2\big/\left(-2+\left(3+\sqrt{2}\right)s\right)\right)$ and high transmissivity (s higher than 0.453082) it would be profitable for identical players to buy out – in all other cases the pay off under competition is higher;

$$\text{FullSimplify}\left[\left\{\overset{\text{sing-comp}}{\mathcal{V}}[a, x, a, x, s] - \left(-\frac{(2+2c-3cs)^2 - 2c^2 s^2}{2(2+2c-cs)^2(1+c-cs)}\right)(a-c(h-x))^2\ , \frac{\overset{\text{sing-comp}}{\mathcal{V}}[a, x, a, x, s]}{(a-c(h-x))^2} - \left(\frac{-\left(2+2c-3cs-cs\sqrt{2}\right)}{2(2+2c-cs)^2(1+c-cs)}\right)\left(2+2c-3cs+cs\sqrt{2}\right)\right\}\right]$$

{0, 0}

$$\text{FullSimplify}\left[\left\{\text{Solve}\left[2+2c-3cs-cs\sqrt{2} == 0, c\right]\ ,\quad \text{N}\left[\text{Solve}\left[-2+\left(3+\sqrt{2}\right)s == 0, s\right]\right]\right\}\right]$$

$$\left\{\left\{\left\{c \to \frac{2}{-2+\left(3+\sqrt{2}\right)s}\right\}\right\}, \{\{s \to 0.453082\}\}\right\}$$

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{sing-comp}}{\mathcal{V}}[a, x, a, x, 0] - \left(-\frac{(a - c\,(h - x))^2}{2\,(c + 1)}\right)\Big\},$$

$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}}[a, x, a, x, 1/2] - \left(\left(c - 4 - 4\sqrt{2}\right)\left(\frac{c + 4\left(\sqrt{2} - 1\right)}{8 + 4c}\right)\left(\frac{a - c\,(h - x)}{2 + c\,(2 - 1/2)}\right)^2\right)\Big\}\Big\}\Big]$$

{{0}, {0}}

Observation4; **Showing the effect of inequality;**

Defining the derivative of the difference function with regard to inequality;

$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon_\epsilon}[a\_, x\_, \epsilon\_, s\_] := \frac{a - c\,(h - x)}{1 + c - c\,s} - \left(\frac{(2 + 2c - 2c\,s)^2 + c\,s\,(4 + 4c - 5c\,s)}{(1 + c - c\,s)\,(2 + 2c - 3c\,s)^2}\right)\epsilon;$$

$$\text{FullSimplify}\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon_\epsilon}[a, x, \epsilon, s] - \text{D}\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s], \epsilon\Big]\Big]$$

0

$$\text{FullSimplify}\Big[\Big\{\text{D}\Big[\overset{\text{sing}}{\mathcal{V}\epsilon}[a, x, \epsilon, s], \epsilon\Big] - \frac{a + \epsilon - c\,(h - x)}{1 + c - c\,s},\ \text{D}\Big[\overset{\text{comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s], \epsilon\Big] - \frac{8\,(1 + c - c\,s)\,\epsilon}{(2 + 2c - 3c\,s)^2}\Big\}\Big]$$

{0, 0}

Solving for the roots of the difference function and the maximum and showing that they fit in the ;

Showing that the gain / loss function has two roots the first is the the maximum inequality level $\epsilon_{+\text{MaxCompS}}$ and the second / inferior root that has the same sign as $\left((2 + 2c - 3c\,s)^2 - 2c^2 s^2\right)$ and always lower than $\epsilon_{+\text{MaxCompS}}$;

$$\epsilon_{\text{Root,Inf}} = \frac{\left((2 + 2c - 3c\,s)^2 - 2c^2 s^2\right)(2 + 2c - 3c\,s)}{(2 + 2c - c\,s)\left((2 + 2c - 2c\,s)^2 + c\,s\,(4 + 4c - 5c\,s)\right)}(a - c\,(h - x));$$

$$\epsilon_{+\text{MaxCompS}} = \left(\frac{2 + 2c - 3c\,s}{2 + 2c - c\,s}\right)(a - c\,(h - x));$$

$$\text{FullSimplify}\Big[\Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon_{\text{Root,Inf}}, s],\ \overset{\text{sing-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon_{+\text{MaxCompS}}, s],$$

$$\epsilon_{+\text{MaxCompS}} - \epsilon_{\text{Root,Inf}} - \left(\frac{8c\,s\,(1 + c - c\,s)\,(2 + 2c - 3c\,s)\,(a - c\,(h - x))}{(2 + 2c - c\,s)\left((2 + 2c - 2c\,s)^2 + c\,s\,(4 + 4c - 5c\,s)\right)}\right)\Big\}\Big]$$

{0, 0, 0}

$$\text{FullSimplify}\Big[\Big\{\Big\{a-\epsilon_{Root,Inf}-c\,(h-x)-\frac{2\,c\,s\left(3\,(2+2\,c-3\,c\,s)^2+12\,c\,s\,(1+c-2\,c\,s)+8\,c^2\,s^2\right)}{(2+2\,c-c\,s)\left((2+2\,c-2\,c\,s)^2+c\,s\,(4+4\,c-5\,c\,s)\right)}\,(a-c\,(h-x)),$$

$$a+\epsilon_{Root,Inf}-c\,(h-x)-\frac{4\,(1+c-c\,s)\left((2+2\,c-2\,c\,s)^2+c^2\,s^2\right)(a-c\,(h-x))}{(2+2\,c-c\,s)\left((2+2\,c-2\,c\,s)^2+c\,s\,(4+4\,c-5\,c\,s)\right)}\Big\},$$

$$\Big\{a-\epsilon_{+MaxCompS}-c\,(h-x)-\frac{2\,c\,s\,(a-c\,(h-x))}{2+2\,c-c\,s}\Big\}\Big\}\Big]$$

{{0, 0}, {0}}

Solving for the maximum of the difference function and showing its positive sign (s);

$$\epsilon_{+MaxSingComp}=\frac{(2+2\,c-3\,c\,s)^2\;(a-c\,(h-x))}{(2+2\,c-2\,c\,s)^2+c\,s\,(4+4\,c-5\,c\,s)};$$

$$\text{FullSimplify}\Big[\Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon_{\epsilon}}[a,x,\epsilon_{+MaxSingComp},s],\ \overset{\text{sing-comp}}{\mathcal{V}\epsilon}[a,x,\epsilon_{+MaxSingComp},s]-$$

$$\left(\frac{8\,c^2\,(1+c-c\,s)\,s^2\,(a-c\,(h-x))^2}{(2+2\,c-c\,s)^2\left((2+2\,c-2\,c\,s)^2+c\,s\,(4+4\,c-5\,c\,s)\right)}\right),\ \text{FullSimplify}\Big[\left(\frac{\epsilon_{+MaxSingComp}-\epsilon_{Root,Inf}}{\epsilon_{+MaxCompS}-\epsilon_{Root,Inf}}\right)-\frac{1}{2}\Big]\Big\}\Big]$$

{0, 0, 0}

$$\text{FullSimplify}\Big[$$

$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon_{\epsilon}}[a,x,0,s]-\left(\frac{a-c\,(h-x)}{1+c-c\,s}\right),\ \overset{\text{sing-comp}}{\mathcal{V}\epsilon_{\epsilon}}[a,x,\epsilon_{Root,Inf},s]-\frac{4\,c\,s\,(a-c\,(h-x))}{(2+2\,c-c\,s)\,(2+2\,c-3\,c\,s)},$$

$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon_{\epsilon}}[a,x,\epsilon_{+MaxCompS},s]-4\,c\,s\left(-\frac{(a-c\,(h-x))}{(2+2\,c-c\,s)\,(2+2\,c-3\,c\,s)}\right)\Big\}\Big]$$

{0, 0, 0}

$$\text{FullSimplify}\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon_{\epsilon}}[a,x,\epsilon_{+MaxCompS},s]-\left(-\frac{4\,c\,s\,(a+c\,(-h+x))}{(2+c\,(2-s))\,(2+c\,(2-3\,s))}\right)\Big]$$

0

$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon_{\epsilon}}[a\_,x\_,\epsilon\_,s\_]:=\frac{a-c\,(h-x)}{1+c-c\,s}-\left(\frac{(2+2\,c-2\,c\,s)^2+c\,s\,(4+4\,c-5\,c\,s)}{(1+c-c\,s)\,(2+2\,c-3\,c\,s)^2}\right)\epsilon;$$

$$\text{FullSimplify}\Big[\text{Limit}\Big[\Big\{\epsilon_{\text{Root,Inf}} - \frac{(4+c)(16+(8-c)c)(a-c(h-x))}{(4+3c)(16+c(24+7c))},\ \epsilon_{\text{+MaxSingComp}} - \frac{(4+c)^2(a-c(h-x))}{16+c(24+7c)},\ \epsilon_{\text{+MaxCompS}} - \frac{(4+c)(a-c(h-x))}{4+3c}\Big\},\ s\to 1/2\Big]\Big]$$

{0, 0, 0}

$$\text{FullSimplify}\Big[\frac{(4+c)(a-c(h-x))}{4+3c}\Big/\frac{(4+c)^2(a-c(h-x))}{16+c(24+7c)}\Big]$$

$$\frac{16+c(24+7c)}{(4+c)(4+3c)}$$

$$\text{FullSimplify}[\epsilon_{\text{+MaxCompS}} / \epsilon_{\text{+MaxSingComp}}]$$

$$-\frac{1}{3} + \frac{2(1+c)}{6+c(6-9s)} - \frac{2(1+c)}{-2+c(-2+s)}$$

Observation5; Showing the effect of transmissivity;

Defining the derivative of the difference function with regard to transmissivity;

$$\overset{\text{sing-comp}}{\mathcal{V}_S}[ai\_, x\_, aj\_, x\_, s\_] := \frac{\left(c(2(1+c)-cs)^3+8c^2s(1+c-cs)^2\right)}{2(2+c(2-s))^3(1+c-cs)^2}\left(\frac{ai+aj}{2}-c(h-x)\right)^2 + c(ai-aj)\left(\frac{\frac{ai+aj}{2}+c(-h+x)}{2(1+c-cs)^2}+\frac{\left(24(1+c)^3-52c(1+c)^2s+26c^2(1+c)s^2+3c^3s^3\right)(ai-aj)}{8(1+c-cs)^2(-2+c(-2+3s))^3}\right);$$

$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a\_, x\_, \epsilon\_, s\_] := \left(\frac{c(2+2c-cs)^3+8c^2s(1+c-cs)^2}{2(2+2c-cs)^3(1+c-cs)^2}\right)(a-c(h-x))^2 + c\epsilon\frac{(a-c(h-x))}{(1+c-cs)^2} - \left(3(2+2c-2cs)^3+5cs(2+2c-3cs)^2+7c^2s^2(2+2c-3cs)+3c^3s^3\Big/\left(2(1+c-cs)^2(2+2c-3cs)^3\right)\right)c\epsilon^2;$$

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{sing-comp}}{\mathcal{V}_S}[a_i, x, a_j, x, s] - D\Big[\overset{\text{sing-comp}}{\mathcal{V}}[a_i, x, a_j, x, s], s\Big]\Big\},\ \Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, \epsilon, s] - D\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s], s\Big]\Big\}\Big\}\Big]$$

{{0}, {0}}

Observation5a;

Showing the positive effect of transmissivity on the difference function (buy out) for identical players;

Observation3ai; The derivative is actually positive indicating a gain in profits from buying out from a marginal increase in transmissivity; but the gains are only realisable when the conditions to buy out $\left(2\,c^2\,s^2 > (2+2\,c-3\,c\,s)^2\right)$ are satisfied;

$$\textbf{FullSimplify}\Big[\Big\{\Big\{\, D\Big[\overset{\text{sing}}{\mathcal{V}}[a, x, a, x, s], s\Big] - \left(\frac{c\,(a + c\,(-h + x))^2}{2\,(1 + c - c\,s)^2}\right),$$

$$D\Big[\overset{\text{comp}}{\mathcal{V}}[a, x, a, x, s], s\Big] - \frac{4\,c^2\,s\,(a + c\,(-h + x))^2}{(-2 + c\,(-2 + s))^3}\Big\},$$

$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}_s}[a, x, a, x, s] - \left(\frac{c\,(2 + 2\,c - c\,s)^3 + 8\,c^2\,s\,(1 + c - c\,s)^2}{2\,(2 + 2\,c - c\,s)^3\,(1 + c - c\,s)^2}\right)(a - c\,(h - x))^2\Big\},$$

$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}}[a, x, a, x, s] - \left(\frac{2\,c^2\,s^2 - (2 + 2\,c - 3\,c\,s)^2}{2\,(1 + c - c\,s)\,(2 + 2\,c - c\,s)^2}\right)(a - c\,(h - x))^2\Big\}\Big\}\Big]$$

{{0, 0}, {0}, {0}}

Observation3aii; Showing the positive non marginal effect of transmissivity on the gain / loss from the buy out when the when the conditionsto buy out $\left(c > 4\left(1+\sqrt{2}\right)\right)$ is satisfied;

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{sing-comp}}{\mathcal{V}}\left[a, x, a, x, \tfrac{1}{2}\right] - \frac{\left(c - 4\left(1+\sqrt{2}\right)\right)\left(c + 4\left(\sqrt{2}-1\right)\right)}{(2+c)(4+3c)^2}(a - c(h-x))^2\Big\},$$
$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}}[a, x, a, x, 0] - \left(-\frac{(a-c(h-x))^2}{2(1+c)}\right)\Big\},$$
$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}}\left[a, x, a, x, \tfrac{1}{2}\right] - \overset{\text{sing-comp}}{\mathcal{V}}[a, x, a, x, 0] - \left(\frac{c(16+c(28+11c))}{2(1+c)(2+c)(4+3c)^2}(a-c(h-x))^2\right)\Big\}\Big\}\Big]$$

{{0}, {0}, {0}}

Observation5b;

Showing the effect of transmissivity on the difference function (buy out) for unequal players;

Solving for the roots of the derivative of the difference function w.r.t transmissivity for unequal players (one positive and one negative);

$$Q1_+ = \sqrt{(2+2c-3cs)(2+2c-cs)}\sqrt{\left((2+2c-cs)^4 + cs(2+2c-3cs)^3 + 4c^2s^2(2+2c-3cs)^2 + 2c^3s^3(2+2c-4cs) + c^4s^4\right)};$$
$$\text{Term1}_+ = (2+2c-3cs)^3 \Big/ \left(3(2+2c-2cs)^3 + 20cs(1+c-2cs)^2 + 34c^2s^2(1+c-2cs) + 15c^3s^3\right)(a-c(h-x));$$
$$\text{Term2}_+ = \left((4(2+2c-3cs)(1+c-cs)Q1_+)\Big/\left(3(2+2c-2cs)^3 + 20cs(1+c-2cs)^2 + 34c^2s^2(1+c-2cs) + 15c^3s^3\right)\right)\left(\frac{a-c(h-x)}{(2+2c-cs)^2}\right);$$

$$\epsilon_{-\text{RootDerivS,Inf}} = \text{Term1}_+ - \text{Term2}_+; \qquad \epsilon_{+\text{RootDerivS,Sup}} = \text{Term1}_+ + \text{Term2}_+;$$

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, \epsilon_{-\text{RootDerivS,Inf}}, s]\Big\},$$
$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, 0, s] - \frac{c}{2}\left(\frac{(2+2c-cs)^3 + 8cs(1+c-cs)^2}{(2+2c-cs)^3(1+c-cs)^2}\right)(a-c(h-x))^2,$$
$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, \epsilon_{+\text{MaxSingComp}}, s] - \frac{8c^2s\left(16(1+c)^2(1+c-cs)^2 + c^4s^4\right)(a-c(h-x))^2}{(2+2c-cs)^3\left((2+2c-2cs)^2 + cs(4+4c-5cs)\right)^2},$$
$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, \epsilon_{+\text{RootDerivS,Sup}}, s]\Big\},$$
$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, \epsilon_{+\text{MaxCompS}}, s] - \left(-\frac{16c^2(1+c)s(a-c(h-x))^2}{(2+2c-cs)^3(2+2c-3cs)}\right)\Big\}\Big]$$

{{0}, 0, 0, {0}, 0}

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, \epsilon_{Root,Inf}, s] - \Big(16 c^2 (1+c) s \Big(48 (1+c-2 c s)^4 + 192 c s (1+c-2 c s)^3 + 264 c^2 s^2 (1+c-2 c s)^2 + 144 c^3 s^3 (1+c-2 c s) + 23 c^4 s^4\Big) (a - c (h-x))^2\Big) \Big/ \Big((2+2c-c s)^3 (2+2c-3 c s) \big(4+8c+4c^2-4 c s-4 c^2 s-c^2 s^2\big)^2\Big)\Big\}\Big\}\Big]$$

{{0}}

$$\overset{\text{sing-comp}}{\mathcal{V}\epsilon_{S\epsilon}}[a\_, x\_, \epsilon\_, s\_] := c \frac{(a - c (h-x))}{(1+c-c s)^2} + \frac{c\left(24 (1+c)^3 - 52 c (1+c)^2 s + 26 c^2 (1+c) s^2 + 3 c^3 s^3\right)}{(1+c-c s)^2 (-2+c(-2+3 s))^3} \epsilon;$$

$$\text{FullSimplify}\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon_{S\epsilon}}[a, x, \epsilon, s] - D\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, \epsilon, s], \epsilon\Big]\Big]$$

0

$$\text{FullSimplify}\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon_{S\epsilon}}[a, x, \text{Term1}_+, s]\Big]$$

0

$$\text{FullSimplify}\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon_S}[a, x, \text{Term1}_+, s] - \Big(8 c \Big(16 (1+c)^4 - 24 c (1+c)^3 s + 4 c^2 (1+c)^2 s^2 + 2 c^3 (1+c) s^3 + 3 c^4 s^4\Big) (a - c (h-x))^2\Big) \Big/ \Big((2+c(2-s))^3 \Big(24 (1+c)^3 - 52 c (1+c)^2 s + 26 c^2 (1+c) s^2 + 3 c^3 s^3\Big)\Big)\Big]$$

0

Showing the shape of the graph for the effects of transmissivity on the difference function for unequal players; positive at low and moderate inequalities and negative when inequality higher than $\epsilon_{+RootDerivS,Sup}$ and lower than $\epsilon_{+MaxCompS}$;

$$Q2_+ = 3 (2+2c-3 c s)^4 + 12 c s (2+2c-3 c s)^3 + 12 c^2 s^2 (2+2c-4 c s)^2 + 24 c^3 s^3 (2+2c-4 c s) + 8 c^4 s^4;$$

$$Q4_+ = 3 (2+2c-2 c s)^3 + 5 c s (2+2c-3 c s)^2 + 7 c^2 s^2 (2+2c-3 c s) + 3 c^3 s^3;$$

$$Q6_+ = 16 (1+c-c s)^7 + c (1+c) s \big(31 (1+c-2 c s)^5 + 152 c s (1+c-2 c s)^4 + 305 c^2 s^2 (1+c-2 c s)^3 + 274 c^3 s^3 (1+c-2 c s)^2 + 93 c^4 s^4 (1+c-2 c s) + 2 c^5 s^5\big);$$

$$Q7_+ = 64 (1+c-c s)^5 + 72 c s (1+c-2 c s)^4 + 263 c^2 s^2 (1+c-2 c s)^3 + 319 c^3 s^3 (1+c-2 c s)^2 + 133 c^4 s^4 (1+c-2 c s) + 4 c^5 s^5;$$

$$Q8_+ = Q1_+ \big((2+2c-2 c s)^2 + c s (4+4c-5 c s)\big) + 32 (1+c-c s)^5 + 72 c s (1+c-c s)^4 + 31 c^2 s^2 (1+c-2 c s)^3 + 75 c^3 s^3 (1+c-2 c s)^2 + 53 c^4 s^4 (1+c-2 c s) + 9 c^5 s^5;$$

$$\text{FullSimplify}\Big[\Big\{\epsilon_{-\text{RootDerivS,Inf}} - (-\text{Term1}_{+})\left(\frac{(2+2c-cs)^3 + 8cs(1+c-cs)^2}{\epsilon_{+\text{RootDerivS,Sup}}(2+2c-cs)^3}\right)(a-c(h-x)),\ \epsilon_{\text{Root,Inf}} - \epsilon_{-\text{RootDerivS,Inf}} - (a-c(h-x))\left(\frac{4(1+c-cs)(2+2c-3cs)}{Q4_{+}(2+2c-cs)^2}\right)\left(\frac{c^3 s^3 Q6_{+} + Q7_{+} Q8_{+}}{(2+2c-2cs)^2 + cs(4+4c-5cs)}\frac{1}{Q8_{+}}\right),$$

$$\epsilon_{+\text{MaxSingComp}} - \epsilon_{\text{Root,Inf}} - \frac{4c(1+c-cs)s(2+2c-3cs)}{(2+2c-3cs)^2 + 2cs(4+4c-5cs)}\frac{a-c(h-x)}{2+2c-cs},$$

$$\epsilon_{+\text{RootDerivS,Sup}} - \epsilon_{+\text{MaxSingComp}} - \left(\frac{(a-c(h-x))}{(2+2c-cs)\left((2+2c-2cs)^2 + cs(4+4c-5cs)\right)}\right)\Big(\big(4cs(1+c-cs)(2+2c-3cs)^2\left((2+2c-2cs)^2(2+2c)^2 + c^4 s^4\right)\big)\Big/\Big(Q1_{+}\left((2+2c-2cs)^2 + cs(4+4c-5cs)\right) + 4(1+c)(1+c-cs)(2+2c-3cs)(2+2c-cs)^2\Big)\Big),$$

$$\left(\epsilon_{+\text{MaxCompS}} - \epsilon_{+\text{RootDerivS,Sup}}\right) - \left(\frac{a-c(h-x)}{(2+2c-cs)}\right)\left(\frac{8c(1+c)(1+c-cs)s(2+2c-3cs)}{(2+2c-cs)\left((2+2c-cs)^2 + 2cs(1+c-2cs)\right) + Q1_{+}}\right)\Big\}\Big]$$

{0, 0, 0, 0, 0}

Observation3bi; Showing that a marginal increase in transmissivity has no effect when buy out is not a profitable option; in the case where buy out is the rational economic decision transmissivity is shown to have a positive effect under low inequalities and a negative effect under a high asymmetry; Showing that the derivative with regard to s has two roots one negative and the second root positive and lower than the maximum inequality level $\epsilon_{\text{MaxcompS}}$;

Showing the effect of transmissivity on the shape of the graph and the different markers including roots for the difference function and the maximum;

Showing the effect of transmissivity on the shape of the graph and the different markers including roots for the difference function and the maximum;

$$Q9_+ = 1664\,(1 + c - s\,c)^8 + 5824\,c\,s\,(1 + c - s\,c)^7 + 6880\,c^2\,s^2\,(1 + c - s\,c)^6 + 2224\,c^3\,s^3\,(1 + c - c\,s)^5 + 120\,c^4\,s^4\,(1 + c - c\,s)^4 + 364\,c^5\,s^5\,(1 + c - 2\,c\,s)^3 + 1042\,c^6\,s^6\,(1 + c - 2\,c\,s)^2 + 995\,c^7\,s^7\,(1 + c - 2\,c\,s) + 316\,c^8\,s^8;$$

$$\text{FunctRootDerivS}[c\_, s\_] := -768\,(1 + c)^{10} + 21504\,c\,(1 + c)^9\,s - 109824\,c^2\,(1 + c)^8\,s^2 + 258048\,c^3\,(1 + c)^7\,s^3 - 341664\,c^4\,(1 + c)^6\,s^4 + 278016\,c^5\,(1 + c)^5\,s^5 - 152720\,c^6\,(1 + c)^4\,s^6 + 68768\,c^7\,(1 + c)^3\,s^7 - 28583\,c^8\,(1 + c)^2\,s^8 + 8052\,c^9\,(1 + c)\,s^9 - 828\,c^{10}\,s^{10};$$

$$\text{FullSimplify}\Big[\Big\{\Big\{ D[(\epsilon_{-\text{RootDerivS,Inf}}), s] - (a - c\,(h - x))\left(\frac{4\,c\,(1 + c)\,(2 + 2\,c - 3\,c\,s)}{(2 + 2\,c - c\,s)^2\,Q1_+}\right)\Big(\text{FunctRootDerivS}[c, s] \Big/ \Big(4\,(1 + c - c\,s)\,(2 + 2\,c - 3\,c\,s)\,(7 + 7\,c - 6\,c\,s)\,(2 + 2\,c - c\,s)^2\,Q1_+ + Q9_+\Big)\Big)\Big\},$$
$$\{\text{FunctRootDerivS}[1, 0], \text{FunctRootDerivS}[1, 0.1], \text{FunctRootDerivS}[1, 0.2], \text{FunctRootDerivS}[1, 0.3], \text{FunctRootDerivS}[1, 0.4], \text{FunctRootDerivS}[1, 0.5]\}\Big\}\Big]$$

{{0}, {−786432, 64353.4, 522932., 720883., 755265., 695686.}}

$$\text{FullSimplify}\Big[\Big\{\Big\{\partial_s \epsilon_{\text{Root,Inf}} - 4\,c\,(1 + c)\left(-\frac{Q2_+}{\left((2 + 2\,c - 3\,c\,s)^3 + 4\,c\,s\,(1 + c - c\,s)\,(6 + 6\,c - 7\,c\,s)\right)^2}\right)(a - c\,(h - x))\Big\},$$
$$\Big\{\partial_s \epsilon_{+\text{MaxSingComp}} - 16\,c\,(1 + c)\left(-\frac{(1 + c - c\,s)\,(2 + 2\,c - 3\,c\,s)}{\left((2 + 2\,c - 2\,c\,s)^2 + c\,s\,(4 + 4\,c - 5\,c\,s)\right)^2}\right)(a - c\,(h - x))\Big\},$$
$$\Big\{\partial_s \epsilon_{+\text{RootDerivS,Sup}} - \Big(-\Big(4\,c\,(1 + c)\,(2 + 2\,c - 3\,c\,s)\Big(Q9_+ + 4\,Q1_+\,(2 + 2\,c - 3\,c\,s)\,(2 + 2\,c - c\,s)^2\,(1 + c - c\,s)\,(7 + 7\,c - 6\,c\,s)\Big)\Big)\Big/\Big(Q1_+\,(2 + 2\,c - c\,s)^2\,(Q4_+)^2\Big)\Big)(a - c\,(h - x))\Big\}, \Big\{\partial_s \epsilon_{+\text{MaxCompS}} - 4\,c\,(1 + c)\left(-\frac{(a - c\,(h - x))}{(2 + 2\,c - c\,s)^2}\right)\Big\}\Big\}\Big]$$

{{0}, {0}, {0}, {0}}

$$\text{FullSimplify}\Big[D\Big[\overset{\text{sing-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon_{+\text{MaxSingComp}}, s], s\Big] - \left(\frac{8\,c^2\,s\left(c^4\,s^4 + 16\,(1 + c)^2\,(1 + c - c\,s)^2\right)(a - c\,(h - x))^2}{(2 + 2\,c - c\,s)^3\left((2 + 2\,c - 2\,c\,s)^2 + c\,s\,(4 + 4\,c - 5\,c\,s)\right)^2}\right)\Big]$$

0

$$\text{FullSimplify}\Big[\frac{8\,c^2\,s\left((2 + 2\,c - 2\,c\,s)^2\,(2 + 2\,c)^2 + c^4\,s^4\right)(a - c\,(h - x))^2}{(2 + 2\,c - c\,s)^3\left((2 + 2\,c - 2\,c\,s)^2 + c\,s\,(4 + 4\,c - 5\,c\,s)\right)^2}\Big]$$

$$\frac{8\,c^2\,s\left(c^4\,s^4 + 16\,(1 + c)^2\,(1 + c - c\,s)^2\right)(a + c\,(-h + x))^2}{(2 - c\,(-2 + s))^3\left(-4 + 4\,c\,(-2 + s) + c^2\,(-4 + s\,(4 + s))\right)^2}$$

$$\text{Factor}\Big[16\,(1 + c)^4 - 32\,c\,(1 + c)^3\,s + 16\,c^2\,(1 + c)^2\,s^2 + c^4\,s^4 - \left((2 + 2\,c - c\,s)^4 - 2\,c^2\,s^2\,(2 + 2\,c - c\,s)^2 + 2\,c^4\,s^4\right)\Big]$$

0

$$\text{Factor}\left[16\,(1+c)^4 - 32\,c\,(1+c)^3\,s + 16\,c^2\,(1+c)^2\,s^2 + c^4\,s^4 - \left(\left((2+2\,c-c\,s)^2 - c^2\,s^2\right)^2 + c^4\,s^4\right)\right]$$

0

Observation5d; Showing the effect of a non marginal shift in transmissivity on the difference between single user with an open resource and maximizing outcome with a closed resource;

Showing the effect of transmissivity on the shape of the graph and the different markers including roots for the difference function and the maximum;

$$\overset{\text{sing-comp}}{\mathcal{V}}_{\text{O}_p\,\text{-C}\iota}[\text{a\_}, \text{x\_}, \epsilon\text{\_}] := -\left(\frac{\epsilon^2 - 2\,(1+c)\,(a-c\,(h-x))\,\epsilon + (a-c\,(h-x))^2}{2+3\,c+c^2}\right);$$

$$\text{FullSimplify}\left[\left\{\left\{\overset{\text{sing-comp}}{\mathcal{V}}_{\text{O}_p\,\text{-C}\iota}[a, x, \epsilon] - \left(\overset{\text{sing}}{\mathcal{V}\epsilon}\left[a, x, \epsilon, \frac{1}{2}\right] - \left(\overset{\text{comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, 0]\right)\right)\right\},\right.\right.$$
$$\left.\left.\left\{\overset{\text{sing-comp}}{\mathcal{V}}_{\text{O}_p\,\text{-C}\iota}[a, x, \epsilon] - \left(\overset{\text{sing}}{\mathcal{V}\epsilon}\left[a, x, \epsilon, \frac{1}{2}\right] - \left(\overset{\text{sing}}{\mathcal{V}\epsilon}[a, x, \epsilon, 0] + \overset{\text{sing}}{\mathcal{V}\epsilon}[a, x, -\epsilon, 0]\right)\right)\right\}\right\}\right]$$

{{0}, {0}}

$$\epsilon_{\text{+MaxCl}} = a - c\,(h-x);$$
$$\text{FullSimplify}\left[\left\{\left\{\text{FullSimplify}\left[\text{Limit}\left[\epsilon_{\text{+MaxCompS}}, s \to 0\right] - \epsilon_{\text{+MaxCl}}\right]\right\},\right.\right.$$
$$\left.\left.\left\{\overset{\text{sing-comp}}{\mathcal{V}}_{\text{O}_p\,\text{-C}\iota}[a, x, \epsilon_{\text{+MaxCl}}] - \frac{2\,c\,(a-c\,(h-x))^2}{2+3\,c+c^2}\right\}, \left\{\overset{\text{sing-comp}}{\mathcal{V}}_{\text{O}_p\,\text{-C}\iota}[a, x, 0] - \left(-\frac{(a+c\,(-h+x))^2}{2+3\,c+c^2}\right)\right\}\right\}\right]$$

{{0}, {0}, {0}}

$$\epsilon_{\text{+RootSingOpCl}} = \frac{a-c\,(h-x)}{1+c+\sqrt{c\,(2+c)}};$$

$$\text{FullSimplify}\left[\left\{\left\{\overset{\text{sing-comp}}{\mathcal{V}}_{\text{O}_p\,\text{-C}\iota}\left[a, x, \epsilon_{\text{+RootSingOpCl}}\right]\right\}, \left\{\left(\epsilon_{\text{+MaxCl}} - \epsilon_{\text{+RootSingOpCl}}\right) - \frac{2\,c\,(a-c\,(h-x))}{c+\sqrt{c\,(2+c)}}\right\}\right\}\right]$$

{{0}, {0}}

Showing the effect of transmissivity on the shape of the graph and the different markers including roots for the difference function and the maximum;

$$\overset{\text{sing-comp}}{\partial_\epsilon \mathcal{V}}_{\mathrm{O_p\,\_C}\iota}[a\_, x\_, \epsilon\_] = 2\left(-\frac{\epsilon}{2+3c+c^2} + \frac{a - c(h-x)}{2+c}\right);$$

$$\text{FullSimplify}\left[\overset{\text{sing-comp}}{\partial_\epsilon \mathcal{V}}_{\mathrm{O_p\,\_C}\iota}[a, x, \epsilon] - \partial_\epsilon\left(\overset{\text{sing-comp}}{\mathcal{V}}_{\mathrm{O_p\,\_C}\iota}[a, x, \epsilon]\right)\right]$$

0

$$\epsilon_{\text{+HighRoot}} = (1+c)(a - c(h-x));\ \text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{sing-comp}}{\partial_\epsilon \mathcal{V}}_{\mathrm{O_p\,\_C}\iota}[a, x, \epsilon_{\text{+HighRoot}}]\Big\},$$
$$\Big\{\overset{\text{sing-comp}}{\partial_\epsilon \mathcal{V}}_{\mathrm{O_p\,\_C}\iota}[a, x, 0] - 2\left(\frac{a-c(h-x)}{2+c}\right)\Big\}, \Big\{\overset{\text{sing-comp}}{\partial_\epsilon \mathcal{V}}_{\mathrm{O_p\,\_C}\iota}[a, x, \epsilon_{\text{+MaxCl}}] - 2c\left(\frac{a-c(h-x)}{2+3c+c^2}\right)\Big\}\Big\}\Big]$$

{{0}, {0}, {0}}

$$\epsilon_{\text{RootInfOp}} = -\frac{(4+c)(c^2-8c-16)}{(4+3c)(16+24c+7c^2)}(a - c(h-x));\ \ \epsilon_{\text{+MaxCompOp}} = \frac{(4+c)}{4+3c}(a - c(h-x));$$

$$\text{FullSimplify}\Big[\Big\{\text{Limit}\big[\{\epsilon_{\text{+MaxCompOp}} - \epsilon_{\text{+MaxCompS}},\ \epsilon_{\text{RootInfOp}} - \epsilon_{\text{Root,Inf}}\},\ s \to 1/2\big],$$
$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon}\left[a, x, \epsilon_{\text{RootInfOp}}, \frac{1}{2}\right], \overset{\text{sing-comp}}{\mathcal{V}\epsilon}\left[a, x, \epsilon_{\text{+MaxCompOp}}, \frac{1}{2}\right]\Big\}\Big\}\Big]$$

{{0, 0}, {0, 0}}

$$\text{FullSimplify}\Big[\Big\{\frac{\epsilon_{\text{+RootSingOpCl}}}{\epsilon_{\text{+MaxCompOp}}} - \frac{4+3c}{(4+c)\left(1+c+\sqrt{c(2+c)}\right)},$$
$$\text{Solve}\left[\frac{\epsilon_{\text{+RootSingOpCl}}}{\epsilon_{\text{+MaxCompOp}}} == 1\right], \text{Limit}\left[\frac{\epsilon_{\text{+RootSingOpCl}}}{\epsilon_{\text{+MaxCompOp}}}, c \to \text{Infinity}\right]\Big\}\Big]$$

{0, {{c → −2}, {c → 0}}, 0}

$$\text{FullSimplify}\Big[$$
$$\Big\{\overset{\text{sing-comp}}{\mathcal{V}\epsilon}\left[a, x, \epsilon_{\text{+RootSingOpCl}}, \frac{1}{2}\right] - \left(2c\left(-128 + c(4+c)\left(4\left(-7+\sqrt{c(2+c)}\right) + c\left(5+c+\sqrt{c(2+c)}\right)\right)\right)\right)\Big/$$
$$\left((4+c)^2(4+3c)^2\left(1+c+\sqrt{c(2+c)}\right)^2\right)(a - c(h-x))^2,$$
$$\text{Solve}\left[\frac{\overset{\text{sing-comp}}{\mathcal{V}\epsilon}\left[a, x, \epsilon_{\text{+RootSingOpCl}}, \frac{1}{2}\right]}{(a-c(h-x))^2} == 0, c\right]\Big\}\Big]$$

$$\Big\{0, \Big\{\Big\{c \to -\frac{4}{\sqrt{3}}\Big\}, \Big\{c \to \frac{4}{\sqrt{3}}\Big\}, \Big\{c \to \text{Root}\left[128 + 176\,\#1 + 72\,\#1^2 + 7\,\#1^3\ \&, 2\right]\Big\},$$
$$\Big\{c \to \text{Root}\left[128 + 176\,\#1 + 72\,\#1^2 + 7\,\#1^3\ \&, 3\right]\Big\}, \{c \to 0\}\Big\}\Big\}$$

$$\text{FullSimplify}\Big[\Big\{\frac{\epsilon_{\texttt{RootInfOp}}}{\epsilon_{+\text{RootSingOpCl}}}-\frac{(4+c)\left(-c^2+8c+16\right)\left(1+c+\sqrt{c(2+c)}\right)}{(4+3c)(16+c(24+7c))},$$
$$\text{N}\Big[\text{Solve}\Big[\frac{\epsilon_{\texttt{RootInfOp}}}{\epsilon_{+\text{RootSingOpCl}}}==1\Big]\Big],\ \text{Limit}\Big[\frac{\epsilon_{\texttt{RootInfOp}}}{\epsilon_{+\text{RootSingOpCl}}},\ c\to\text{Infinity}\Big]\Big\}\Big]$$

{0, {{c → −2.}, {c → 0.}, {c → −2.3094}, {c → 2.3094}, {c → −1.58689 − 0.230017 i}, {c → −1.58689 + 0.230017 i}}, −∞}

$$\overset{\text{comp}}{\mathbf{F}}[a\_, x\_, \epsilon\_, s\_, c\_, h\_] := 4(1+c-cs)\left(\frac{(a-c(h-x))^2}{(2+2c-cs)^2}+\frac{\epsilon^2}{(2+2c-3cs)^2}\right);$$

$$\overset{\text{sing}}{\mathbf{F}}[a\_, x\_, \epsilon\_, s\_, c\_, h\_] := \frac{(a+\epsilon-c(h-x))^2}{2+2c-2cs};$$

$$\text{FullSimplify}\Big[$$
$$\Big\{\overset{\text{comp}}{\mathcal{V}}[a+\epsilon, x, a-\epsilon, x, s]-\overset{\text{comp}}{\mathbf{F}}[a, x, \epsilon, s, c, h],\ \overset{\text{sing}}{\mathcal{V}}[a+\epsilon, x, a-\epsilon, x, s]-\overset{\text{sing}}{\mathbf{F}}[a, x, \epsilon, s, c, h]\Big\}\Big]$$

{0, 0}

$$\text{Plot}\Big[\Big\{\overset{\text{comp}}{\mathbf{F}}[15, 100, \epsilon, 1/4, 10, 101],\ \overset{\text{sing}}{\mathbf{F}}[15, 100, \epsilon, 1/4, 10, 101]\Big\}, \{\epsilon, -5, 5\}\Big];$$

$$\text{Plot}\Big[\Big\{\overset{\text{comp}}{\mathbf{F}}[15, 100, \epsilon, 1/2, 6, 101],\ \overset{\text{sing}}{\mathbf{F}}[15, 100, \epsilon, 1/2, 6, 101]\Big\}, \{\epsilon, -10, 10\}\Big];$$

$$\text{FullSimplify}\Big[\Big\{\overset{\text{sing}}{\mathcal{V}}[a+\epsilon, x, a-\epsilon, x, s]-\frac{(a+\epsilon-c(h-x))^2}{2+2c-2cs},$$
$$\text{Limit}\Big[\frac{\overset{\text{sing}}{\mathcal{V}}[a+\epsilon, x, a-\epsilon, x, s]}{\overset{\text{comp}}{\mathcal{V}}[a+\epsilon, x, a-\epsilon, x, s]}, \epsilon\to\text{Infinity}\Big]-\left(\frac{(2+2c-3cs)^2}{8(1+c-cs)^2}\right)\Big\}\Big]$$

{0, 0}

$$\text{Plot}\left[4(1+c-cs)\left(\frac{(a-c(h-x))^2}{(2+2c-cs)^2}+\frac{\epsilon^2}{(2+2c-3cs)^2}\right),\{\epsilon,0,\epsilon_{+\text{Max}}\}\right];$$

Plot::plln : Limiting value ϵ_{Max} in $\{\epsilon, 0, \epsilon_{+\text{Max}}\}$ is not a machine−sized real number. ≫

$$\text{Plot}\left[\frac{\epsilon^2}{4}+4,\{\epsilon,-4,4\}\right];$$

Observation6;

Defining the individual and aggregate use of water under single user and comparison to the competitive case;

transmissivity has an increasing effect on the profits of the efficient player; for the less efficient players the effect is positive under low levels of inequality and negative under high levels;

$$\text{AggW}_{\text{comp}}[\text{ai\_},\text{x\_},\text{aj\_},\text{x\_}]:=\frac{2\,\text{ai}+2\,\text{aj}-4c(h-x)}{2+2c-cs};$$

$$\text{FullSimplify}\Big[\Big\{\text{AggW}_{\text{comp}}[a_i,x,a_j,x]-(w_{\text{comp}}[a_i,x,a_j,x]+w_{\text{comp}}[a_j,x,a_i,x]),$$
$$\Big\{w_{\text{comp}}[a_i,x,a_j,x]-\left(\frac{a_i+a_j-2c(h-x)}{2+2c-cs}+\frac{a_i-a_j}{2+2c-3cs}\right)\Big\},\Big\{w_{\text{sing}}[a_i,x,a_j,x]-\frac{a_i-c(h-x)}{1+c-cs}\Big\}\Big\}\Big]$$

{0, {0}, {0}}

$$\text{DiffW}_{\text{comp-sing}}[\text{ai\_},\text{x\_},\text{aj\_},\text{x\_}]:=\frac{(2+2c-3cs)(\text{ai}+\text{aj}-2c(h-x))}{2(2+2c-cs)(1+c-cs)}-\frac{\text{ai}-\text{aj}}{2+2c-2cs};$$

$$\text{FullSimplify}\Big[\Big\{\Big\{(\text{AggW}_{\text{comp}}[a_i,x,a_j,x]-w_{\text{sing}}[a_i,x,a_j,x])-\text{DiffW}_{\text{comp-sing}}[a_i,x,a_j,x]\Big\},$$
$$\Big\{D[\text{DiffW}_{\text{comp-sing}}[a+\epsilon,x,a-\epsilon,x],\epsilon]-\left(\frac{-1}{1+c-cs}\right)\Big\},$$
$$\Big\{\text{DiffW}_{\text{comp-sing}}[a+\epsilon_{+\text{MaxCompS}},x,a-\epsilon_{+\text{MaxCompS}},x]\Big\}\Big\}\Big]$$

{{0}, {0}, {0}}

$$\text{FullSimplify}\Big[$$
$$\Big\{w_{\text{comp}}[a+\epsilon,x,a-\epsilon,x]-w_{\text{sing}}[a+\epsilon,x,a-\epsilon,x]-\frac{cs}{(1+c-cs)}\left(\frac{\epsilon}{2+2c-3cs}-\frac{a-c(h-x)}{2+2c-cs}\right),$$
$$w_{\text{comp}}[a+\epsilon_{+\text{MaxCompS}},x,a-\epsilon_{+\text{MaxCompS}},x]-w_{\text{sing}}[a+\epsilon_{+\text{MaxCompS}},x,a-\epsilon_{+\text{MaxCompS}},x]\Big\}\Big]$$

{0, 0}

$$\textbf{FullSimplify}\Big[\Big\{\Big\{\textbf{D}\Big[\textbf{AggW}_{\textbf{comp}}[a+\epsilon, x, a-\epsilon, x], s\Big] - 4c\left(\frac{a-c(h-x)}{(2+2c-cs)^2}\right)\Big\},$$
$$\Big\{\textbf{D}\Big[\textbf{w}_{\textbf{sing}}[a+\epsilon, x, a-\epsilon, x], s\Big] - c\left(\frac{a+\epsilon-c(h-x)}{(1+c-cs)^2}\right)\Big\}\Big\}\Big]$$

{{0}, {0}}

$$\textbf{FullSimplify}\Big[\textbf{D}\Big[\textbf{DiffW}_{\textbf{comp-sing}}[a+\epsilon, x, a-\epsilon, x], s\Big] - \left(-\frac{c^2 s(4+4c-3cs)(a-c(h-x))}{(2+2c-cs)^2(1+c-cs)^2} - \frac{c\epsilon}{(1+c-cs)^2}\right)\Big]$$

0

$$\textbf{FullSimplify}\Big[\Big\{\textbf{Limit}\Big[\textbf{DiffW}_{\textbf{comp-sing}}[a+\epsilon, x, a-\epsilon, x], s\to 0\Big] - \frac{a-ch+cx-\epsilon}{1+c},$$
$$\textbf{Limit}\Big[\textbf{DiffW}_{\textbf{comp-sing}}[a+\epsilon, x, a-\epsilon, x], s\to 1/2\Big] - \frac{2(4+c)(a+c(-h+x))-2(4+3c)\epsilon}{(2+c)(4+3c)},$$
$$\Big(\textbf{Limit}\Big[\textbf{DiffW}_{\textbf{comp-sing}}[a+\epsilon, x, a-\epsilon, x], s\to 1/2\Big] - \textbf{Limit}\Big[\textbf{DiffW}_{\textbf{comp-sing}}[a+\epsilon, x, a-\epsilon, x], s\to 0\Big]\Big) -$$
$$\left(-\frac{c\epsilon}{2+3c+c^2} - \frac{c^2(a-c(h-x))}{(1+c)(2+c)(4+3c)}\right)\Big\}\Big]$$

{0, 0, 0}

$$\textbf{FullSimplify}\Big[\textbf{Limit}\Big[\Big\{\frac{\epsilon_{\text{-RootDerivS,Inf}}}{a-c(h-x)}, \frac{\epsilon_{\text{Root,Inf}}}{a-c(h-x)}, \frac{\epsilon_{\text{+MaxSingComp}}}{a-c(h-x)}, \frac{\epsilon_{\text{+RootDerivS,Sup}}}{a-c(h-x)}, \frac{\epsilon_{\text{+MaxCompS}}}{a-c(h-x)}\Big\}, s\to 0\Big], c>0\Big]$$

$$\left\{-\frac{1}{3}, 1, 1, 1, 1\right\}$$

$$\textbf{FullSimplify}\Big[\textbf{Limit}\Big[\Big\{\frac{\epsilon_{\text{-RootDerivS,Inf}}}{a-c(h-x)}, \frac{\epsilon_{\text{Root,Inf}}}{a-c(h-x)}, \frac{\epsilon_{\text{+MaxSingComp}}}{a-c(h-x)}, \frac{\epsilon_{\text{+RootDerivS,Sup}}}{a-c(h-x)}, \frac{\epsilon_{\text{+MaxCompS}}}{a-c(h-x)}\Big\}, s\to 1/2\Big], c>0\Big]$$

$$\Big\{\Big((4+c)\Big(-(4+c)^2 + 1\Big/(4+3c)^2 \quad 4(2+c)\sqrt{((4+c)(4+3c)(256+c(832+c(976+c(484+87c)))))}\Big)$$
$$(-a+c(h-x))\Big)\Big/((192+c(368+c(212+39c)))(a+c(-h+x))),$$
$$-\frac{(4+c)(-16+(-8+c)c)}{(4+3c)(16+c(24+7c))}, \frac{(4+c)^2}{16+c(24+7c)},$$
$$\Big((4+c)\Big(-(4+c)^2 - 1\Big/(4+3c)^2 \quad 4(2+c)\sqrt{((4+c)(4+3c)(256+c(832+c(976+c(484+87c)))))}\Big)$$
$$(-a+c(h-x))\Big)\Big/((192+c(368+c(212+39c)))(a+c(-h+x))), \frac{4+c}{4+3c}\Big\}$$

$$\textbf{FullSimplify}\Big[$$
$$\textbf{N}\Big[\textbf{Limit}\Big[\textbf{Limit}\Big[\Big\{\frac{\epsilon_{\text{-RootDerivS,Inf}}}{a-c(h-x)}, \frac{\epsilon_{\text{Root,Inf}}}{a-c(h-x)}, \frac{\epsilon_{\text{+MaxSingComp}}}{a-c(h-x)}, \frac{\epsilon_{\text{+RootDerivS,Sup}}}{a-c(h-x)}, \frac{\epsilon_{\text{+MaxCompS}}}{a-c(h-x)}\Big\}, s\to 1/2\Big], c\to 1\Big]\Big]\Big]$$

{−0.30439, 0.349544, 0.531915, 0.612651, 0.714286}

FullSimplify[
N[Limit[Limit[{$\frac{\epsilon_{-\text{RootDerivS,Inf}}}{a - c(h - x)}$, $\frac{\epsilon_{\text{Root,Inf}}}{a - c(h - x)}$, $\frac{\epsilon_{+\text{MaxSingComp}}}{a - c(h - x)}$, $\frac{\epsilon_{+\text{RootDerivS,Sup}}}{a - c(h - x)}$, $\frac{\epsilon_{+\text{MaxCompS}}}{a - c(h - x)}$}, s → 1/4], c → 1]]]

{−0.330329, 0.649028, 0.757848, 0.809448, 0.866667}

Under Cooperation;

$$\overset{\text{coop}}{\mathcal{V}}\epsilon[a\_, x\_, \epsilon\_, s\_] := \left(\frac{(a - c(h - x))^2}{1 + c} + \frac{\epsilon^2}{1 + c - 2cs}\right);$$

$$\text{FullSimplify}\left[\left(\overset{\text{coop}}{\mathcal{V}}\epsilon[a, x, \epsilon, s]\right) - \overset{\text{coop}}{\mathcal{V}}[a + \epsilon, x, a - \epsilon, x, s]\right]$$

0

Defining the individual profit functions for cooperative users;

$$V_{\text{coopS}}[ai\_, xi\_, aj\_, xj\_, s\_] = \frac{1}{8(1 + c)(-1 + c(-1 + 2s))}(2ai + c(-2h + 2xi - s\,xi + s\,xj))$$
$$(2ai(-1 + c(-1 + s)) + c(2h(1 + c - 2cs) - 2(1 + c)xi + s(2aj + xi + 3c\,xi + (-1 + c)xj)));$$

$$\text{FullSimplify}[V_{\text{coopS}}[a_i, x_i, a_j, x_j, s] - \text{NP}[a_i, w_{\text{coop}}[a_i, x_i, a_j, x_j], x_i, a_j, w_{\text{coop}}[a_j, x_j, a_i, x_i], x_j]]$$

0

$$\text{FullSimplify}\left[\left\{\left\{\overset{\text{coop}}{\mathcal{V}}[a_i, x, a_j, x, s] - \frac{1}{4}\left(\frac{(a_i + a_j - 2c(h - x))^2}{1 + c} + \frac{(a_i - a_j)^2}{1 + c - 2cs}\right)\right\},\right.\right.$$
$$\left.\left.\left\{\overset{\text{coop}}{\mathcal{V}}[a + \epsilon, x, a - \epsilon, x, s] - \left(\frac{\epsilon^2}{1 + c - 2cs} + \frac{(a - c(h - x))^2}{1 + c}\right)\right\}\right\}\right]$$

{{0}, {0}}

Observation0;

Under cooperation;
the efficient player uses more water and makes more profits;

$$\text{FullSimplify}\left[\left\{w_{\text{coop}}[a_h, x, a_l, x] - w_{\text{coop}}[a_l, x, a_h, x] - \left(\frac{a_h - a_l}{1 + c - 2cs}\right),\right.\right.$$
$$\left.\left.V_{\text{coopS}}[a_h, x, a_l, x, s] - V_{\text{coopS}}[a_l, x, a_h, x, s] - \frac{(a_h - a_l)}{(1 + c)}\left(\frac{a_h + a_l}{2} - c(h - x)\right)\left(\frac{1 + c - cs}{1 + c - 2cs}\right)\right\}\right]$$

{0, 0}

Observation1;

inequality has an increasing effect on aggregate revenues
under cooperation; the increase is increasing in transmisivity;

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{coop}}{\mathcal{V}}[a_i, x, a_j, x, s] - \overset{\text{coop}}{\mathcal{V}}\Big[\frac{a_i + a_j}{2}, x, \frac{a_i + a_j}{2}, x, s\Big] - \frac{(a_i - a_j)^2}{4(1 + c - 2 c s)},$$

$$\overset{\text{coop}}{\mathcal{V}}[a_i, x, a_j, x, 0] - \overset{\text{coop}}{\mathcal{V}}\Big[\frac{a_i + a_j}{2}, x, \frac{a_i + a_j}{2}, x, 0\Big] - \frac{(a_i - a_j)^2}{4 + 4 c}\Big\},$$

$$\Big\{\partial_\epsilon \overset{\text{coop}}{\mathcal{V}}[a + \epsilon, x, a - \epsilon, x, s] - \frac{2\epsilon}{1 + c - 2 c s}, \partial_{\epsilon,s} \overset{\text{coop}}{\mathcal{V}}[a + \epsilon, x, a - \epsilon, x, s] - \frac{4 c \epsilon}{(1 + c - 2 c s)^2}\Big\}\Big\}\Big]$$

{{0, 0}, {0, 0}}

Observation2;

Revenues under cooperation are strictly
increasing in transmissivity for inequal players;

$$\text{FullSimplify}\Big[\Big\{\Big\{\partial_s \overset{\text{coop}}{\mathcal{V}}[a, x, a, x, s]\Big\}, \Big\{\partial_s \overset{\text{coop}}{\mathcal{V}}[a_i, x, a_j, x, s] - \frac{c(a_i - a_j)^2}{2(1 + c - 2 c s)^2},$$

$$\partial_s \overset{\text{coop}}{\mathcal{V}}[a + \epsilon, x, a - \epsilon, x, s] - \frac{2\epsilon^2 c}{(1 + c - 2 c s)^2}\Big\}\Big\}\Big]$$

{{0}, {0, 0}}

Checking for the non marginal shift in transmissivity;

$$\text{FullSimplify}\Big[\Big\{\Big\{\Big(\overset{\text{coop}}{\mathcal{V}}\Big[a_i, x, a_j, x, \frac{1}{2}\Big] - \overset{\text{coop}}{\mathcal{V}}[a_i, x, a_j, x, 0]\Big) - \frac{c(a_i - a_j)^2}{4(1 + c)},$$

$$\Big(\overset{\text{coop}}{\mathcal{V}}\Big[a + \epsilon, x, a - \epsilon, x, \frac{1}{2}\Big] - \overset{\text{coop}}{\mathcal{V}}[a + \epsilon, x, a - \epsilon, x, 0]\Big) - \frac{c\epsilon^2}{1 + c}\Big\}\Big\}\Big]$$

{{0, 0}}

Observation3;

Under cooperation;
transmissivity has an increasing effect on the profits of the efficient player;
and a decreasing effect for the less efficient;

$$\text{FullSimplify}\Big[\Big\{V_{\text{coopS}}[ai, x, aj, x, s] - \frac{(ai - c\,(h - x))}{2\,(1 + c)}\left(ai - c\,(h - x) + \frac{(ai - aj)\,c\,s}{(1 + c - 2\,c\,s)}\right),\; V_{\text{coopS}}[a_i, x, a_j, x, s] - \left(\left(4\,(1 + c - 2\,c\,s)\left(\frac{a_i + a_j}{2} - c\,(h - x)\right)^2 + 4\,(a_i - a_j)\,(1 + c - c\,s)\left(\frac{a_i + a_j}{2} - c\,(h - x)\right) + (1 + c)\,(a_i - a_j)^2\right)\Big/(8\,(1 + c)\,(1 + c - 2\,c\,s))\right),\; V_{\text{coopS}}[a + \epsilon, x, a - \epsilon, x, s] - \left(\left((1 + c - 2\,c\,s)\,(a - c\,(h - x))^2 + 2\,\epsilon\,(1 + c - c\,s)\,(a - c\,(h - x)) + (1 + c)\,\epsilon^2\right)\Big/(2\,(1 + c)\,(1 + c - 2\,c\,s))\right)\Big\}\Big]$$

{0, 0, 0}

$$\text{FullSimplify}\Big[\Big\{\partial_s V_{\text{coopS}}[a_i, x, a_j, x, s] - c\,(a_i - a_j)\,\frac{(a_i - c\,(h - x))}{2\,(1 + c - 2\,c\,s)^2},\; \partial_s V_{\text{coopS}}[a + \epsilon, x, a - \epsilon, x, s] - \frac{c\,\epsilon\,(a + \epsilon - c\,(h - x))}{(1 + c - 2\,c\,s)^2}\Big\}\Big]$$

{0, 0}

cooperation / Competition;

Defining the gain / loss function from cooperation vs competition and its derivative ;

$$\overset{\text{coop-comp}}{\mathcal{V}}[ai\_, x\_, aj\_, x\_, s\_] := \frac{c^2\,s^2}{4}\left(\frac{1}{(1 + c)}\left(\frac{ai + aj - 2\,c\,(h - x)}{2 + 2\,c - c\,s}\right)^2 + \frac{1}{(1 + c - 2\,c\,s)}\left(\frac{ai - aj}{2 + 2\,c - 3\,c\,s}\right)^2\right);$$

$$\overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a\_, x\_, \epsilon\_, s\_] := c^2\,s^2\left(\frac{(a - c\,(h - x))^2}{(1 + c)\,(2 + 2\,c - c\,s)^2} + \frac{\epsilon^2}{(2 + 2\,c - 3\,c\,s)^2\,(1 + c - 2\,c\,s)}\right);$$

$$\text{FullSimplify}\Big[\Big\{\overset{\text{coop-comp}}{\mathcal{V}}[a_i, x, a_j, x, s] - \left(\overset{\text{coop}}{\mathcal{V}}[a_i, x, a_j, x, s] - \overset{\text{comp}}{\mathcal{V}}[a_i, x, a_j, x, s]\right),\; \overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s] - \left(\overset{\text{coop}}{\mathcal{V}}[a + \epsilon, x, a - \epsilon, x, s] - \overset{\text{comp}}{\mathcal{V}}[a + \epsilon, x, a - \epsilon, x, s]\right)\Big\}\Big]$$

{0, 0}

Observation4;

inequality has an increasing effect on the difference between aggregate profits under cooperation and those under competition;

$$\text{FullSimplify}\Big[\Big\{\partial_\epsilon \overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s] - \frac{2\,c^2\,s^2\,\epsilon}{(2 + 2\,c - 3\,c\,s)^2\,(1 + c - 2\,c\,s)},\; \partial_\epsilon \overset{\text{coop}}{\mathcal{V}\epsilon}[a, x, \epsilon, s] - \frac{2\,\epsilon}{1 + c - 2\,c\,s},\; \partial_\epsilon \overset{\text{comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s] - \frac{8\,(1 + c - c\,s)\,\epsilon}{(2 + 2\,c - 3\,c\,s)^2}\Big\}\Big]$$

{0, 0, 0}

$$\text{FullSimplify}\Big[\Big\{\Big(\overset{\text{coop-comp}}{\mathcal{V}}[a_i, x, a_j, x, s] - \overset{\text{coop-comp}}{\mathcal{V}}\Big[\frac{a_i + a_j}{2}, x, \frac{a_i + a_j}{2}, x, s\Big]\Big) - \frac{c^2 s^2}{4(1 + c - 2 c s)}\Big(\frac{a_i - a_j}{2 + 2 c - 3 c s}\Big)^2,$$
$$\Big(\overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s] - \overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, 0, s]\Big) - \frac{c^2 s^2}{(1 + c - 2 c s)}\Big(\frac{\epsilon}{2 + 2 c - 3 c s}\Big)^2\Big\}\Big]$$

{0, 0}

$$\text{FullSimplify}\Big[\Big\{\partial_{\epsilon,s}\overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s] - \frac{4 c^2 s\big(2(1 + c - 2 c s)^2 + 6 c s(1 + c - 2 c s) + c^2 s^2\big)\epsilon}{(1 + c - 2 c s)^2(2 + 2 c - 3 c s)^3}\Big\}\Big]$$

{0}

Observation5;

Transmissivity has an increasing effect on the difference between aggregate profits under cooperation and those under competition;

$$\text{FullSimplify}\Big[\Big\{\partial_s\overset{\text{coop-comp}}{\mathcal{V}}[a_i, x, a_j, x, s] - 2 c^2 s\Big(\frac{(a_i + a_j - 2 c(h - x))^2}{2(2 + 2 c - c s)^3} + \frac{\big(2(1 + c - 2 c s)^2 + 6 c s(1 + c - 2 c s) + c^2 s^2\big)}{(1 + c - 2 c s)^2(2 + 2 c - 3 c s)^3}\Big(\frac{a_i - a_j}{2}\Big)^2\Big),$$
$$\partial_s\overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s] - 2 c^2 s\Big(\frac{2(a - c(h - x))^2}{(2 + 2 c - c s)^3} + \frac{\big(2(1 + c - 2 c s)^2 + 6 c s(1 + c - 2 c s) + c^2 s^2\big)}{(1 + c - 2 c s)^2(2 + 2 c - 3 c s)^3}\epsilon^2\Big)\Big\}\Big]$$

{0, 0}

for identical players; revenues under cooperation are higher than revenues under competition; the difference is positive and increasing in transmissivity ;

for unequal players transmissivity has an increasing effect on the gain attributed to inequality as players move to cooperation ;

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{coop-comp}}{\mathcal{V}}[a, x, a, x, s] - \Big(\frac{c^2 s^2(a - c(h - x))^2}{(1 + c)(2 + 2 c - c s)^2}\Big),$$
$$\partial_s\overset{\text{coop-comp}}{\mathcal{V}}[a, x, a, x, s] - \Big(\frac{4 c^2 s(a - c(h - x))^2}{(2 + 2 c - c s)^3}\Big)\Big\},$$
$$\Big\{\partial_{\epsilon,s}\overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, s] - \frac{4 c^2 s\big(2(1 + c - 2 c s)^2 + 6 c s(1 + c - 2 c s) + c^2 s^2\big)\epsilon}{(1 + c - 2 c s)^2(2 + 2 c - 3 c s)^3}\Big\}\Big\}\Big]$$

{{0, 0}, {0}}

The difference is null when transmissivity is null and the resource closed;

$$\text{FullSimplify}\Big[\Big\{\overset{\text{coop-comp}}{\mathcal{V}}[a_i, x, a_j, x, 0], \overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, 0]\Big\}\Big]$$

{0, 0}

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{coop-comp}}{\mathcal{V}}[a_i, x, a_j, x, 1/2] - \left(\frac{c^2}{(1+c)(4+3c)^2}\left(\frac{a_i+a_j}{2} - c(h-x)\right)^2 + \frac{(a_i-a_j)^2 c^2}{4(4+c)^2}\right),$$

$$\overset{\text{coop-comp}}{\mathcal{V}\epsilon}[a, x, \epsilon, 1/2] - c^2\left(\frac{(a-c(h-x))^2}{(1+c)(4+3c)^2} + \frac{\epsilon^2}{(4+c)^2}\right)\Big\}\Big\}\Big]$$

{{0, 0}}

Observation6;

The move to cooperation improves the efficient player revenues;
for the less efficient user;
the increase only occurs at low levels of inequality;
at higher levels of inequality;
the less efficient user individual earnings suffer from social optimum;

Defining the individual difference function with interior solution and its derivatives ;

$$V_{\text{CoopComp}}[ai\_, xi\_, aj\_, xj\_, s\_] := \frac{1}{8(1+c)(-1+c(-1+2s))}(2ai + c(-2h + 2xi - s\,xi + s\,xj))$$
$$(2ai(-1+c(-1+s)) + c(2h(1+c-2cs) - 2(1+c)xi + s(2aj + xi + 3c\,xi + (-1+c)xj))) +$$
$$\Big(\big((-1+c(-1+s))(4ai(1+c-cs) + c(-2aj\,s + h(-4+c(-4+6s)) + (4-2s+c(4+(-6+s)s))xi +$$
$$s(2-cs)xj))^2\big)\Big/\Big(2(-2+c(-2+s))^2(-2+c(-2+3s))^2\Big)\Big);$$

$$dV_{\text{CoopComp}_S}[a\_, x\_, \epsilon\_, s\_] := \frac{2c^2 s(a-c(h-x))^2}{(2+2c-cs)^3} + \Big(3c^2 s\big((2+2c-3cs)^3 + 2cs(2+2c-3cs)^2 +$$
$$c^2 s^2(2+2c-4cs)\big)(a-c(h-x))\epsilon\Big)\Big/\Big((2+2c-cs)^2(1+c-2cs)^2(2+2c-3cs)^2\Big) +$$
$$\frac{c^2 s\left(2(1+c-2cs)^2 + 6cs(1+c-2cs) + c^2 s^2\right)\epsilon^2}{(1+c-2cs)^2(2+2c-3cs)^3};$$

$$dV_{\text{CoopComp}_\epsilon}[a\_, x\_, \epsilon\_, s\_] := \frac{3c^2(1+c-cs)s^2(a-c(h-x))}{(1+c)(2+2c-cs)(2+2c-3cs)(1+c-2cs)} +$$
$$\frac{c^2 s^2 \epsilon}{(2+2c-3cs)^2(1+c-2cs)};$$

$$\text{FullSimplify}\Big[\Big\{V_{\text{CoopComp}}[a_i, x_i, a_j, x_j, s] - \left(V_{\text{coopS}}[a_i, x_i, a_j, x_j, s] - V_{\text{compS}}[a_i, x_i, a_j, x_j, s]\right),$$
$$\partial_S V_{\text{CoopComp}}[a+\epsilon, x, a-\epsilon, x, s] - dV_{\text{CoopComp}_S}[a, x, \epsilon, s],$$
$$\partial_\epsilon V_{\text{CoopComp}}[a+\epsilon, x, a-\epsilon, x, s] - dV_{\text{CoopComp}_\epsilon}[a, x, \epsilon, s]\Big\}\Big]$$

{0, 0, 0}

$$\text{FullSimplify}\Big[V_{\text{CoopComp}}[a+\epsilon, x, a-\epsilon, x, s] - \Big(\frac{c^2 s^2 (a - c(h-x))^2}{2(1+c)(2+2c-\ cs)^2} + \frac{3c^2(1+c-cs)s^2(a-c(h-x))\epsilon}{(1+c)(2+2c-\ cs)(1+c-2cs)(2+2c-3cs)} + \frac{1}{2}\frac{c^2 s^2 \epsilon^2}{(2+2c-3cs)^2(1+c-2cs)}\Big)\Big]$$

0

$$\text{FullSimplify}\Big[\Big\{\ V_{\text{CoopComp}}[a_i, x, a_j, x, s] - \frac{c^2 s^2}{2(1+c)}\Big(\frac{(a_i+a_j-2c(h-x))^2}{4(2+2c-\ cs)^2} + \frac{3(1+c-cs)(a_i+a_j-2c(h-x))(a_i-a_j)}{2(2+2c-cs)(1+c-2cs)(2+2c-3cs)} + \frac{(1+c)(a_i-a_j)^2}{4(2+2c-3cs)^2(1+c-2cs)}\Big),$$

$$V_{\text{CoopComp}}[a+\epsilon, x, a-\epsilon, x, s] - c^2 s^2\Big(\frac{1}{2(1+c)}\frac{(a-c(h-x))^2}{(2+2c-cs)^2} + \frac{3(1+c-cs)(a-c(h-x))\epsilon}{(1+c)(2+2c-cs)(1+c-2cs)(2+2c-3cs)} + \frac{\epsilon^2}{2(2+2c-3cs)^2(1+c-2cs)}\Big)\ \Big\}\ \Big]$$

{0, 0}

Solving for the maximum level of inequality
for the less efficient user to continue under cooperation
and showing that it is lower than that for competition ;

$$\epsilon_{+\text{MaxCoopS}} = \frac{(1+c-2cs)(a-c(h-x))}{1+c};\quad \text{FullSimplify}\Big[\Big\{\Big\{\ w_{\text{coop}}[a-\epsilon_{+\text{MaxCoopS}}, x, a+\epsilon_{+\text{MaxCoopS}}, x]\Big\},\ \Big\{(\epsilon_{+\text{MaxCompS}} - \epsilon_{+\text{MaxCoopS}}) - \Big(\frac{2c\ s\ (1+c-cs)}{(1+c)(2+2c-cs)}\Big)(a-c(h-x))\Big\}\Big\}\Big]$$

{{0}, {0}}

Showing that the difference is always
increasing in inequality with an interior solution;
we do it by solving for the null derivative and showing the
root is outside the domain with an interior solution probing
that the derivative does not change sign and has the same
positive sign as shown on the borders and at zero inequality;

Solving for the level of inequality where the derivative wrt inequality equals zero;

$$\epsilon_{-\text{coopcomp}\epsilon} = \left(-\frac{3\,(1+c-c\,s)\,(2+2\,c-3\,c\,s)}{(1+c)\ (2+2\,c-c\,s)}\right)(a-c\,(h-x));\ \textbf{FullSimplify}\left[dV_{\text{CoopComp}_\epsilon}[a, x, \epsilon_{-\text{coopcomp}\epsilon}, s]\right]$$

0

Showing that the solution is higher in absolute value than the maximum inequality for in interior solution and confirming the negative extraction under cooperation;

$$\textbf{FullSimplify}\Big[\Big\{\left(\frac{\epsilon_{+\text{MaxCoopS}} - (-\epsilon_{-\text{coopcomp}\epsilon})}{\epsilon_{+\text{MaxCoopS}}}\right) - \left(-\left(\frac{(2+2\,c-3\,c\,s)^2 + c\,s\,(2+2\,c-2\,c\,s)}{(1+c-2\,c\,s)\,(2+2\,c-c\,s)}\right)\right),$$
$$w_{\text{coop}}[a+\epsilon_{-\text{coopcomp}\epsilon}, x, a-\epsilon_{-\text{coopcomp}\epsilon}, x] - \left(-\left(\frac{(2+2\,c-3\,c\,s)^2 + c\,s\,(2+2\,c-2\,c\,s)}{(1+c)\,(1+c-2\,c\,s)\,(2+2\,c-c\,s)}\right)\right)(a-c\,(h-x))\Big\}\Big]$$

{0, 0}

Showing that the derivative is positive on the borders of the domain of an interior solution;

$$\text{Qd}\epsilon_+ = \frac{3\,c^2\,s^2\,(a-c\,(h-x))}{(1+c)\,(2+2\,c-c\,s)\,(2+2\,c-3\,c\,s)^2\,(1+c-2\,c\,s)};$$

$$\textbf{FullSimplify}\Big[\Big\{\Big\{dV_{\text{CoopComp}_\epsilon}[a, x, -\epsilon_{+\text{MaxCoopS}}, s] - \left(\frac{\text{Qd}\epsilon_+}{3}\left((2+2\,c-3\,c\,s)^2 + c\,s\,(2+2\,c-2\,c\,s)\right)\right)\Big\},$$
$$\Big\{dV_{\text{CoopComp}_\epsilon}[a, x, 0, s] - \text{Qd}\epsilon_+\,(2+2\,c-3\,c\,s)\,(1+c-c\,s)\Big\},$$
$$\Big\{dV_{\text{CoopComp}_\epsilon}[a, x, \epsilon_{+\text{MaxCoopS}}, s] - \frac{\text{Qd}\epsilon_+}{3}\left(2\,(2+2\,c-3\,c\,s)^2 + 4\,c\,s\,(1+c-2\,c\,s) + c^2\,s^2\right)\Big\}\Big\}\Big]$$

{{0}, {0}, {0}}

Showing that the difference has a unique negative root in the area with an interior solution;

Showing that the solution to the zero diffrences has two roots of which only one would produces an interior solution;

$$\epsilon_{-\text{coopcomp}} = -\frac{(2+2c-3cs)}{(2+2c-cs)} \frac{(1+c-2cs)}{3(1+c-cs)+\sqrt{8(1+c-cs)^2+c^2s^2}}(a-c(h-x));$$

$$\epsilon_{-\text{coopcomp2}} = -\frac{(2+2c-3cs)}{(2+2c-cs)} \frac{3(1+c-cs)+\sqrt{8(1+c-cs)^2+c^2s^2}}{1+c}(a-c(h-x));$$

$$\text{FullSimplify}\Big[\Big\{\{V_{\text{CoopComp}}[a+\epsilon_{-\text{coopcomp}}, x, a-\epsilon_{-\text{coopcomp}}, x, s], \; V_{\text{CoopComp}}[a+\epsilon_{-\text{coopcomp2}}, x, a-\epsilon_{-\text{coopcomp2}}, x, s]\},$$

$$\Big\{\left(\frac{\epsilon_{+\text{MaxCoopS}} - (-\epsilon_{-\text{coopcomp}})}{\epsilon_{+\text{MaxCoopS}}}\right) - \left(\left((2+2c-cs)\left(cs+\sqrt{c^2s^2+8(1+c-cs)^2}\right) + (2+2c-2cs)^2\right)\Big/\left((2+2c-cs)\left(3+3c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}\right)\right)\right),$$

$$\Big\{\left(\frac{(-\epsilon_{-\text{coopcomp2}}) - \epsilon_{+\text{MaxCoopS}}}{\epsilon_{+\text{MaxCoopS}}}\right) - \left(\left((2+2c-3cs)^2 + 2cs(1+c-cs) + (2+2c-3cs)\sqrt{c^2s^2+8(1+c-cs)^2}\right)\Big/((2+2c-cs)(1+c-2cs))\right),$$

$$\left(\frac{(-\epsilon_{-\text{coopcomp2}}) - \epsilon_{+\text{MaxCompS}}}{\epsilon_{+\text{MaxCompS}}}\right) - \left(\frac{2+2c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}}{1+c}\right)\Big\}\Big\}\Big\}\Big]$$

{{0, 0}, {0, {0, 0}}}

Confirming with the extraction under cooperation;

$$\text{FullSimplify}\Big[\Big\{w_{\text{coop}}[a+\epsilon_{-\text{coopcomp}}, x, a-\epsilon_{-\text{coopcomp}}, x] - \left(\frac{\sqrt{c^2s^2+8(1+c-cs)^2}}{(1+c)} + \frac{(2+2c-2cs)^2 + cs(2+2c-cs)}{(1+c)(2+2c-cs)}\right) \frac{(a-c(h-x))}{\left(3+3c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}\right)}, \; w_{\text{coop}}[a+\epsilon_{-\text{coopcomp2}}, x, a-\epsilon_{-\text{coopcomp2}}, x] -$$

$$\left(-\left((2+2c-3cs)^2 + cs(2+2c-2cs) + (2+2c-3cs)\sqrt{c^2s^2+8(1+c-cs)^2}\right)\Big/((1+c)(2+2c-cs)(1+c-2cs))\right)(a-c(h-x))\Big\}\Big]$$

{0, 0}

Showing that the difference is negative for the less efficient at higher inequality levels; increases and reaches zero at the solution; and increases afterwards;

$$Qc1_{+} = \frac{2\,c^2\,s^2\,(1 + c - c\,s)\,(a - c\,(h - x))^2}{(1 + c)^2\,(2 + 2\,c - 3\,c\,s)\,(2 + 2\,c - c\,s)^2};$$

$$\text{FullSimplify}\Big[\Big\{\Big\{V_{\text{CoopComp}}[a - \epsilon_{+\text{MaxCoopS}}, x, a + \epsilon_{+\text{MaxCoopS}}, x, s] - \left(\frac{-4\,(1 + c - c\,s)^2}{2 + 2\,c - 3\,c\,s}\right) Qc1_{+}\Big\},$$
$$\Big\{V_{\text{CoopComp}}[a, x, a, x, s] - \left(\frac{(1 + c)\,(2 + 2\,c - 3\,c\,s)}{4\,(1 + c - c\,s)}\right) Qc1_{+}\Big\},$$
$$\Big\{V_{\text{CoopComp}}[a + \epsilon_{+\text{MaxCoopS}}, x, a - \epsilon_{+\text{MaxCoopS}}, x, s] -$$
$$\left(\frac{2\,(2 + 2\,c - 3\,c\,s)^2 + 8\,c\,s\,(1 + c - 2\,c\,s) + 3\,c^2\,s^2}{2 + 2\,c - 3\,c\,s}\right) Qc1_{+}\Big\}\Big\}\Big]$$

{{0}, {0}, {0}}

Observation7;

$$Qc2_{+} = \sqrt{\left(8\,(2 + 2\,c - 3\,c\,s)^6 + 32\,c\,s\,(2 + 2\,c - 4\,c\,s)^5 + 220\,c^2\,s^2\,(2 + 2\,c - 4\,c\,s)^4 + \right.}$$
$$\left. 588\,c^3\,s^3\,(2 + 2\,c - 4\,c\,s)^3 + 720\,c^4\,s^4\,(2 + 2\,c - 4\,c\,s)^2 + 384\,c^5\,s^5\,(2 + 2\,c - 4\,c\,s) + 73\,c^6\,s^6\right);$$

$$Qc3_{+} = 3\,(2 + 2\,c - 3\,c\,s)^3 + 6\,c\,s\,(2 + 2\,c - 3\,c\,s)^2 + 6\,c^2\,s^2\,(1 + c - 2\,c\,s);$$

$$Qc4_{+} = 16\,(1 + c - 2\,c\,s)^3 + 24\,c\,s\,(1 + c - 2\,c\,s)^2 + 8\,c^2\,s^2\,(1 + c - 2\,c\,s) + 3\,c^3\,s^3;$$

$$Qc5_{+} = 2\,(2 + 2\,c - 3\,c\,s)^4 + 10\,c\,s\,(2 + 2\,c - 3\,c\,s)^3 +$$
$$18\,c^2\,s^2\,(2 + 2\,c - 4\,c\,s)^2 + 39\,c^3\,s^3\,(2 + 2\,c - 4\,c\,s) + 15\,c^4\,s^4;$$

$$Qc6_{+} = 4\,(2 + 2\,c - 3\,c\,s)^4 + 11\,c\,s\,(2 + 2\,c - 3\,c\,s)^3 + 14\,c^2\,s^2\,(2 + 2\,c - 4\,c\,s)^2 +$$
$$67\,c^3\,s^3\,(1 + c - 2\,c\,s) + 12\,c^4\,s^4;$$

Solving for the (negative) levels of inequality
where the derivative wrt TRANSMISSIVITY equals zero;

$$\epsilon_{-\text{CoopComp}_S} = -\frac{4\,(1 + c - 2\,c\,s)^2\,(2 + 2\,c - 3\,c\,s)^2\,(a - c\,(h - x))}{(2 + 2\,c - c\,s)\,(Qc2_{+} + Qc3_{+})};\quad \epsilon_{-\text{CoopComp}_{S2}} = -(2 + 2\,c - 3\,c\,s)$$
$$(a - c\,(h - x))\left((Qc3_{+} + Qc2_{+}) \Big/ \left(2\,(2 + 2\,c - c\,s)^2\left(2\,(1 + c - 2\,c\,s)^2 + 3\,c\,s\,(2 + 2\,c - 4\,c\,s) + c^2\,s^2\right)\right)\right);$$

$$\text{FullSimplify}\Big[\Big\{dV_{\text{CoopComp}_S}[a, x, \epsilon_{-\text{CoopComp}_S}, s],\ dV_{\text{CoopComp}_S}[a, x, \epsilon_{-\text{CoopComp}_{S2}}, s]\Big\}\Big]$$

{0, 0}

Showing that only one solution belongs in the area with interior solution;

$$\text{FullSimplify}\Big[\Big\{\left(\frac{(-\epsilon_{-\text{CoopComp}_S}) - \epsilon_{+\text{MaxCoopS}}}{\epsilon_{+\text{MaxCoopS}}}\right) - (1 + c)\left(-\frac{(2 + 2\,c - c\,s)\,Qc2_{+} + Qc5_{+}}{(1 + c)\,(2 + 2\,c - c\,s)\,(Qc3_{+} + Qc2_{+})}\right),$$
$$\left(\frac{(-\epsilon_{-\text{CoopComp}_{S2}}) - \epsilon_{+\text{MaxCompS}}}{\epsilon_{+\text{MaxCompS}}}\right) - \frac{1}{2 + 2\,c - c\,s}\left(\frac{Qc4_{+} + Qc2_{+}}{4\,(1 + c - 2\,c\,s)^2 + 12\,c\,s\,(1 + c - 2\,c\,s) + 2\,c^2\,s^2}\right)\Big\}\Big]$$

{0, 0}

Showing that the actual root for the derivative wrt TRANSMISSIVITY is higher (lower in absolute value) than the root wrt INEQUALITY;

$$\text{FullSimplify}\Big[\left(-\epsilon_{-\text{coopcomp}} - \left(-\epsilon_{-\text{CoopComp}_S}\right)\right) - \left(\frac{(2+2c-3cs)(a-c(h-x))}{(2+2c-cs)}\right)\left(\frac{2cs(1+c-2cs)}{\left(3+3c-3cs+\sqrt{c^2 s^2+8(1+c-cs)^2}\right)}\right)$$

$$\Big(\Big(8(2+2c-3cs)^3+17cs(2+2c-3cs)^2+8c^2s^2(2+2c-4cs)+2c^3s^3+3\left(8(1+c-2cs)^2+6cs(2+2c-4cs)+3c^2s^2\right)\sqrt{c^2s^2+8(1+c-cs)^2}\Big)\Big/$$

$$\Big(\text{Qc6}_+ + \left(3(2+2c-3cs)^3+6cs(2+2c-3cs)^2+6c^2s^2(1+c-2cs)\right)\sqrt{c^2s^2+8(1+c-cs)^2}+\left(3+3c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}\right)\text{Qc2}_+\Big)\Big)\Big]$$

0

Double checking for the extracted water;

$$\text{Qc7}_+ = 32(1+c)^5 - 176c(1+c)^4 s + 384c^2(1+c)^3 s^2 - 394c^3(1+c)^2 s^3 + 163c^4(1+c)s^4 - 12c^5 s^5;$$

$$\text{FullSimplify}\Big[\Big\{w_{\text{coop}}\left[a+\epsilon_{-\text{CoopComp}_S}, x, a-\epsilon_{-\text{CoopComp}_S}, x\right] - \left(\frac{\text{Qc5}_+ + (2+2c-cs)\text{Qc2}_+}{(1+c)(2+2c-cs)(\text{Qc3}_+ + \text{Qc2}_+)}\right)(a-c(h-x)),$$

$$w_{\text{coop}}\left[a+\epsilon_{-\text{CoopComp}_{S2}}, x, a-\epsilon_{-\text{CoopComp}_{S2}}, x\right] - \Big(-((1+c)(2+2c-3cs)\text{Qc2}_+ + \text{Qc7}_+)\Big/\Big(2(1+c)(2+2c-cs)^2(1+c-2cs)\left(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\right)\Big)\Big)(a-c(h-x))\Big\}\Big]$$

{0, 0}

Checking for the sign and value of derivatives at selected locations;

$$\text{Qcx}_+ = 16(1+c-2cs)^3 + 56cs(1+c-2cs)^2 + 48c^2s^2(1+c-2cs) + 9c^3s^3;$$

$$\text{Qc8}_+ = (2+2c-3cs)^6 + 7cs(2+2c-3cs)^5 + 19c^2s^2(2+2c-3cs)^4 + 25c^3s^3(2+2c-3cs)^3 + 12c^4s^4(2+2c-4cs)^2 + 20c^5s^5(2+2c-4cs) + 2c^6s^6;$$

$$\text{Qc9}_+ = 8(2+2c-3cs)^3 + 17cs(2+2c-3cs)^2 + 8c^2s^2(2+2c-4cs) + 2c^3s^3;$$

$$\text{Qc10}_+ = (2+2c-3cs)^6 + 7cs(2+2c-3cs)^5 + 16c^2s^2(2+2c-3cs)^4 + \frac{23}{2}c^3s^3(2+2c-4cs)^3 + 30c^4s^4(2+2c-4cs)^2 + 37c^5s^5(1+c-2cs) + 3c^6s^6;$$

$$\text{FullSimplify}\Big[\Big\{ dV_{\text{CoopComp}_S}[a, x, -\epsilon_{+\text{MaxCoopS}}, s] - \left(-\frac{c^2\, s\, Qc8_+ \,(a - c\,(h - x))^2}{(1+c)^2\,(2+2c-c\,s)^3\,(1+c-2\,c\,s)\,(2+2c-3\,c\,s)^3}\right),$$

$$dV_{\text{CoopComp}_S}[a, x, \epsilon_{-\text{coopcomp}}, s] - c\,s\left(-\frac{c^2\,s\,(a - c\,(h-x))^2}{(2+2c-c\,s)^3}\right)$$

$$\left(\left(Qc9_+ + 3\left(8\,(1+c-2\,c\,s)^2 + 12\,c\,s\,(1+c-2\,c\,s) + 3\,c^2\,s^2\right)\sqrt{c^2\,s^2 + 8\,(1+c-c\,s)^2}\right)\Big/\right.$$

$$\left.\left((2+2c-3\,c\,s)\,(1+c-2\,c\,s)\left(3+3c-3\,c\,s+\sqrt{c^2\,s^2+8\,(1+c-c\,s)^2}\right)^2\right)\right),$$

$$dV_{\text{CoopComp}_S}[a, x, 0, s] - \frac{2\,c^2\,s\,(a-c\,(h-x))^2}{(2+2c-c\,s)^3},\; dV_{\text{CoopComp}_S}[a, x, \epsilon_{+\text{MaxCoopS}}, s] -$$

$$\left(\frac{2\,c^2\,s\;Qc10_+}{(1+c)^2\,(2+2c-c\,s)^3\,(1+c-2\,c\,s)\,(2+2c-3\,c\,s)^3}\right)(a-c\,(h-x))^2\Big\}\Big]$$

{0, 0, 0, 0}

$$\text{FullSimplify}\left[\left\{\left\{V_{\text{coop comp}_S}[a, x, \epsilon, 0]\right\},\left\{V_{\text{coop comp}_S}[a, x, 0, s] - \frac{2\,c^2\,s\,(a-c\,(h-x))^2}{(2+2c-c\,s)^3}\right\}\right\}\right]$$

$$\left\{\left\{V_{\text{coop comp}_S}[a, x, \epsilon, 0]\right\},\left\{\frac{2\,c^2\,s\,(a+c\,(-h+x))^2}{(-2+c\,(-2+s))^3} + V_{\text{coop comp}_S}[a, x, 0, s]\right\}\right\}$$

$$\text{FullSimplify}\Big[64\,(1+c)^4 - 296\,c\,(1+c)^3\,s + 524\,c^2\,(1+c)^2\,s^2 - 427\,c^3\,(1+c)\,s^3 + 129\,c^4\,s^4 - \Big(4\,(2+2c-3\,c\,s)^4 +$$

$$11\,c\,s\,(2+2c-3\,c\,s)^3 + 14\,c^2\,s^2\,(2+2c-4\,c\,s)^2 + 67\,c^3\,s^3\,(1+c-2\,c\,s) + 12\,c^4\,s^4\Big)\Big]$$

0

$$\text{FullSimplify}\left[\text{Limit}\left[\text{Limit}\left[N\left[\frac{1}{(a-c\,(h-x))}\left\{\epsilon_{-\text{coopcomp}}, \epsilon_{-\text{CoopComp}_S}, \epsilon_{\text{MaxCoopS}}, \epsilon_{\text{MaxCompS}}\right\}\right], s\to 0.0\right], c\to 1\right]\right]$$

{−0.171573, −0.171573, 1, 1.}

$$\text{FullSimplify}\left[\text{Limit}\left[\text{Limit}\left[N\left[\frac{1}{(a-c\,(h-x))}\left\{\epsilon_{-\text{coopcomp}}, \epsilon_{-\text{CoopComp}_S}, \epsilon_{\text{MaxCoopS}}, \epsilon_{\text{MaxCompS}}\right\}\right], s\to 0.25\right], c\to 1\right]\right]$$

{−0.127375, −0.109103, 0.75, 0.866667}

$$\text{FullSimplify}\left[\text{Limit}\left[\text{Limit}\left[N\left[\frac{1}{(a-c\,(h-x))}\left\{\epsilon_{-\text{coopcomp}}, \epsilon_{-\text{CoopComp}_S}, \epsilon_{\text{MaxCoopS}}, \epsilon_{\text{MaxCompS}}\right\}\right], s\to 0.5\right], c\to 1\right]\right]$$

{−0.0814279, −0.0543372, 0.5, 0.714286}

s =.; c =.;

$$\mathbf{FullSimplify}\Big[\epsilon_{\mathrm{CoopComp}_{S1}} - (2+2c-3cs)\ (a-c(h-x))\Big(\Big(4(1+c-2cs)^2(2+2c-3cs)\Big)\Big/\Big((2+2c-cs)\Big(\Big(3(2+2c-3cs)^3+6cs(2+2c-3cs)^2+6c^2s^2(1+c-2cs)\Big)+\sqrt{\mathrm{CoopCompDerS}}\Big)\Big)\Big)\Big]$$

$$-\Big(4(2+c(2-3s))^2(1+c-2cs)^2(a+c(-h+x))\Big)\Big/\Big((2-c(-2+s))\Big(24+\sqrt{\mathrm{CoopCompDerS}}+3c\Big(8(3+c(3+c))-28(1+c)^2s+32c(1+c)s^2-13c^2s^3\Big)\Big)\Big)+\epsilon_{\mathrm{CoopComp}_{S1}}$$

$$\mathbf{FullSimplify}\Big[\epsilon_{\mathrm{coopcomp1}}-\epsilon_{\mathrm{CoopComp}_{S1}}-\frac{(2+2c-3cs)(a-c(h-x))}{2(2+2c-cs)^2}\left(\frac{2(2+2c-cs)(1+c-2cs)}{3+3c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}}-\Big(\Big(3(2+2c-3cs)^3+6cs(2+2c-3cs)^2+6c^2s^2(1+c-2cs)-\sqrt{\mathrm{CoopCompDerS}}\Big)\Big/\Big(2(1+c-2cs)^2+3cs(2+2c-4cs)+c^2s^2\Big)\Big)\right)\Big]$$

$Aborted

$$\mathbf{CoopCompNumerator}=-64(1+c)^4+296c(1+c)^3s-524c^2(1+c)^2s^2+427c^3(1+c)s^3-129c^4s^4+\Big(3+3c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}\Big)\Big(\sqrt{\mathrm{CoopCompDerS}}\Big)-\sqrt{c^2s^2+8(1+c-cs)^2}\ \Big(3(2+2c-3cs)^3+6cs(2+2c-3cs)^2+6c^2s^2(1+c-2cs)\Big);$$

$$\mathbf{FullSimplify}\Big[\epsilon_{\mathrm{coopcomp1}}-\epsilon_{\mathrm{CoopComp}_{S1}}-\left(\frac{(2+2c-3cs)(a-c(h-x))}{2(2+2c-cs)^2}\right)\Big(\mathrm{CoopCompNumerator}\Big/\Big(\Big(3+3c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}\Big)\Big(2(1+c-2cs)^2+3cs(2+2c-4cs)+c^2s^2\Big)\Big)\Big)\Big]$$

$Aborted

$$\mathbf{CoopCompNumeratorPositif}=\Big(64(1+c-2cs)^4+216cs(1+c-2cs)^3+284c^2s^2(1+c-2cs)^2+165c^3s^3(1+c-2cs)+27c^4s^4\Big)+\Big(3+3c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}\Big)\Big(\sqrt{\mathrm{CoopCompDerS}}\Big)+\sqrt{c^2s^2+8(1+c-cs)^2}\ \Big(3(2+2c-3cs)^3+6cs(2+2c-3cs)^2+6c^2s^2(1+c-2cs)\Big);$$

$$\mathbf{PositifCoopComp1}=4c(2+2c-cs)s\Big(64(1+c-2cs)^3+164cs(1+c-2cs)^2+132c^2s^2(1+c-2cs)+27c^3s^3\Big)(1+c-2cs)\Big(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\Big);$$

$$\mathbf{PositifCoopComp2}=12c(2+2c-cs)s(1+c-2cs)\sqrt{c^2s^2+8(1+c-cs)^2}\Big(8(1+c-2cs)^2+12cs(1+c-2cs)+3c^2s^2\Big)\Big(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\Big);$$

FullSimplify[
(CoopCompNumerator CoopCompNumeratorPositif) – (PositifCoopComp1 + PositifCoopComp2)]

0

$$\text{FullSimplify}\Big[\epsilon_{\text{coopcomp2}} - \epsilon_{\text{CoopComp}_{S2}} - \frac{(2+2c-3cs)(a-c(h-x))}{2(2+2c-cs)^2}\left(\frac{2(2+2c-cs)\left(3+3c-3cs+\sqrt{c^2s^2+8(1+c-cs)^2}\right)}{1+c} - \left(\left(\left(24(1+c-2cs)^3+60cs(1+c-2cs)^2+48c^2s^2(1+c-2cs)+9c^3s^3\right)+\sqrt{\text{CoopCompDerS}}\right)\Big/\left(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\right)\right)\right)\Big]$$

0

$$\text{CoopCompNumerator2} = 3cs(1+c-2cs)\left(8(1+c-2cs)^2+20cs(1+c-2cs)+11c^2s^2\right)+2(2+2c-cs)\left(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\right)\sqrt{c^2s^2+8(1+c-cs)^2}-(1+c)\sqrt{\text{CoopCompDerS}};$$

$$\text{FullSimplify}\left[\epsilon_{\text{coopcomp2}} - \epsilon_{\text{CoopComp}_{S2}} - \frac{(2+2c-3cs)(a-c(h-x))}{2(2+2c-cs)^2}\left(\frac{\text{CoopCompNumerator2}}{(1+c)\left(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\right)}\right)\right]$$

0

$$\text{CoopCompNumerator2Positif} = 3cs(1+c-2cs)\left(8(1+c-2cs)^2+20cs(1+c-2cs)+11c^2s^2\right)+2(2+2c-cs)\left(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\right)\sqrt{c^2s^2+8(1+c-cs)^2}+(1+c)\sqrt{\text{CoopCompDerS}};$$

$$\text{PositifCoopComp12} = 4cs(2+2c-cs)(1+c-2cs)\left(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\right)\left(64(1+c-2cs)^3+220cs(1+c-2cs)^2+244c^2s^2(1+c-2cs)+93c^3s^3\right);$$

$$\text{PositifCoopComp22} = 12c(2+2c-cs)s(1+c-2cs)\sqrt{c^2s^2+8(1+c-cs)^2}\left(8(1+c-2cs)^2+20cs(1+c-2cs)+11c^2s^2\right)\left(2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2\right);$$

$$\text{FullSimplify}[(\text{CoopCompNumerator2}\ \text{CoopCompNumerator2Positif}) - (\text{PositifCoopComp12} + \text{PositifCoopComp22})]$$

0

$$V_{\text{coop comp}_\epsilon}[a\_, x\_, \epsilon\_, s\_] := \frac{3c^2(1+c-cs)s^2(a-c(h-x))}{(1+c)(2+2c-cs)(2+2c-3cs)(1+c-2cs)} + \frac{c^2s^2\epsilon}{(2+2c-3cs)^2(1+c-2cs)};$$

$$\text{FullSimplify}\left[\partial_\epsilon V_{\text{coopS-compS}}[a+\epsilon, x, a-\epsilon, x, s] - V_{\text{coop comp}_\epsilon}[a, x, \epsilon, s]\right]$$

0

$$\text{FullSimplify}\left[V_{\text{coop comp}_\epsilon}\left[a, x, -\left(\frac{3(1+c-cs)(2+2c-3cs)}{(1+c)(2+2c-cs)}(a-c(h-x))\right), s\right]\right]$$

0

$$\text{FullSimplify}\left[V_{\text{coop comp}_\epsilon}[a, x, -\epsilon_{\text{MaxCoopS}}, s] - \frac{c^2 s^2 (a-c(h-x))}{(1+c)(2+2c-3cs)^2}\left(\frac{(2+2c-4cs)^2+3cs(2+2c-3cs)}{(2+2c-cs)(1+c-2cs)}\right)\right]$$

0

$$\text{FullSimplify}\left[\left(V_{\text{coop comp}_S}[a, x, 0, s] - \mathcal{V}\epsilon^{\text{coop-comp}}[a, x, 0, s]\right) - c^2 s (a-c(h-x))^2 \left(\frac{2(c-cs)+2(1-s)+cs^2}{(1+c)(2+2c-cs)^3}\right)\right]$$

0

$$V_{\text{coop comp}_{S\epsilon}}[a\_, x\_, \epsilon\_, s\_] := c^2 s\left(2\,\epsilon\left(\frac{2(1+c-2cs)^2+6cs(1+c-2cs)+c^2s^2}{(1+c-2cs)^2(2+2c-3cs)^3}\right) + \left(\left(8(1+c-2cs)^3+20cs(1+c-2cs)^2+16c^2s^2(1+c-2cs)+3c^3s^3\right)\Big/\left((2+2c-3cs)^2(2+2c-cs)^2(1+c-2cs)^2\right)\right)3(a-c(h-x))\right);$$

$$\text{FullSimplify}\left[D\left[V_{\text{coop comp}_S}[a, x, \epsilon, s], \epsilon\right] - V_{\text{coop comp}_{S\epsilon}}[a, x, \epsilon, s]\right]$$

0

$$\text{FullSimplify}\left[V_{\text{coop comp}_{S\epsilon}}[a, x, -\epsilon_{\text{MaxCompS}}, s] - c^2 s\left(\left(2(2+2c-4cs)^3+6cs(2+2c-4cs)^2+4c^2s^2(2+2c-4cs)+3c^3s^3\right)\Big/\left((2+2c-3cs)^2(2+2c-cs)^2(1+c-2cs)^2\right)\right)(a-c(h-x))\right]$$

0

$$\text{FullSimplify}\left[V_{\text{coop comp}_{S\epsilon}}[a, x, a+c(-h+x), s] - c^2 s\left(\left(4(2+2c-3cs)^4+14cs(2+2c-4cs)^3+62c^2s^2(2+2c-4cs)^2+83c^3s^3(2+2c-4cs)+23c^4s^4\right)\Big/\left((2+2c-3cs)^3(2+2c-cs)^2(1+c-2cs)^2\right)\right)(a-c(h-x))\right]$$

0

$$\text{FullSimplify}\left[\text{Limit}\left[\frac{1}{(a-c(h-x))}\{\epsilon_{\text{CoopComp}_{S1}}, \epsilon_{\text{coopcomp1}}, \epsilon_{\text{MaxCoopS}}, \epsilon_{\text{MaxCompS}}, \epsilon_{\text{CoopComp}_{S2}}, \epsilon_{\text{coopcomp2}}\}, s \to 0\right], c > 0\right]$$

$$\{3-2\sqrt{2}, 3-2\sqrt{2}, 1, 1, 3+2\sqrt{2}, 3+2\sqrt{2}\}$$

$$\text{FullSimplify}\Big[\text{Limit}\Big[\frac{1}{(a-c(h-x))}\{\epsilon_{\text{CoopComp}_{S1}},\epsilon_{\text{coopcomp1}},\epsilon_{\text{MaxCoopS}},\epsilon_{\text{MaxCompS}},\epsilon_{\text{CoopComp}_{S2}},\epsilon_{\text{coopcomp2}}\}, s\to\frac{1}{2}\Big], c>0\Big]$$

$$\Big\{\Big((4+c)\big(192+3c(4+c)(20+3c)-\sqrt{(32768+c(81920+c(87040+c(47872+3c(4480+9c(64+3c))))))}\big)\Big)\Big/\Big(2(4+3c)^2(8+c(12+c))\Big), \frac{2(4+c)}{(4+3c)\left(6+3c+\sqrt{32+c(32+9c)}\right)}, \frac{1}{1+c}, \frac{4+c}{4+3c},$$
$$\Big((4+c)\big(192+3c(4+c)(20+3c)+\sqrt{(32768+c(81920+c(87040+c(47872+3c(4480+9c(64+3c))))))}\big)\Big)\Big/\Big(2(4+3c)^2(8+c(12+c))\Big), \frac{(4+c)\left(6+3c+\sqrt{32+c(32+9c)}\right)}{(1+c)(8+6c)}\Big\}$$

$$\text{FullSimplify}\Big[D[\epsilon_{\text{CoopComp}_{S1}}, s]\frac{\left(2(2+2c-cs)^3(2+4c+2c^2-2cs-2c^2s-3c^2s^2)^2\right)}{4c(1+c)(-a+c(h-x))(1+c-2cs)} - \Big(3\left(48(1+c)^4-184c(1+c)^3s+234c^2(1+c)^2s^2-99c^3(1+c)s^3-3c^4s^4\right)\Big) - \Big(\Big(-\big(3072(1+c)^7-22912c(1+c)^6s+71840c^2(1+c)^5s^2-121664c^3(1+c)^4s^3+116872c^4(1+c)^3s^4-58974c^5(1+c)^2s^5+10953c^6(1+c)s^6+729c^7s^7\big)\Big)\Big/\Big(\sqrt{\text{CoopCompDerS}}\Big)\Big)\Big]$$

0

$$\text{TempQtityPositif1} = 18(2+2c-3cs)^8+120cs(2+2c-3cs)^7+290c^2s^2(2+2c-3cs)^6+300c^3s^3(2+2c-4cs)^5+\frac{5985}{4}c^4s^4(2+2c-4cs)^4+2803c^5s^5(2+2c-4cs)^3+\frac{9933}{4}c^6s^6(2+2c-4cs)^2+1083c^7s^7(2+2c-4cs)+193c^8s^8;$$

$$\text{TempQtityPositif2} = 48(1+c-2cs)^4+200cs(1+c-2cs)^3+282c^2s^2(1+c-2cs)^2+165c^3s^3(1+c-2cs)+31c^4s^4;$$

$$\text{TempQtityPositif3} = 24(2+2c-3cs)^7+146c(2+2c-3cs)^6s+337c^2(2+2c-3cs)^5s^2+421c^3(2+2c-4cs)^4s^3+1815c^4(2+2c-4cs)^3s^4+\frac{5385}{2}c^5(2+2c-4cs)^2s^5+3167c^6(1+c-2cs)s^6+312c^7s^7;$$

$$\text{FullSimplify}\Big[\partial_s\epsilon_{\text{CoopComp}_{S1}}+\frac{32c(1+c)(1+c-2cs)(2+2c-3cs)(a-c(h-x))}{(2+2c-cs)^2\sqrt{\text{CoopCompDerS}}}\left(\text{TempQtityPositif1}\Big/3\,\text{TempQtityPositif2}\sqrt{\text{CoopCompDerS}}+\text{TempQtityPositif3}\right)\Big]$$

0

$$\text{FullSimplify}\Big[\partial_s\epsilon_{\text{MaxCoopS}}-\left(-\frac{2c}{1+c}(a-c(h-x))\right)\Big]$$

0

$$\text{FullSimplify}\Big[\Big(\partial_s \epsilon_{\text{coopcomp2}} - \frac{-c\,(a - c\,(h - x))}{(2 + 2c - c\,s)^2\,(1 + c)}\Big(6\,(2 + 2c - 4c\,s)^2 + 30\,c\,s\,(2 + 2c - 4c\,s) + 33\,c^2\,s^2 + \Big(8\,(2 + 2c - 2c\,s)^3 + 7\,c\,s\,(2 + 2c - 3c\,s)^2 + 12\,c^2\,s^2\,(2 + 2c - 3c\,s) + 10\,c^3\,s^3\Big) \Big/ \sqrt{c^2\,s^2 + 8\,(1 + c - c\,s)^2}\Big)\Big)\Big]$$

0

$$\text{FullSimplify}\Big[\partial_s \epsilon_{\text{CoopComp}_{s2}} - \frac{-2\,c\,(1 + c)\,(1 + c - 2c\,s)\,(a - c\,(h - x))}{(2 + 2c - c\,s)^3\,\big(-2 + 2c\,(-2 + s) + c^2\,(-2 + s\,(2 + 3s))\big)^2}\Big(3\,\big(48\,(1 + c)^4 - 184\,c\,(1 + c)^3\,s + 234\,c^2\,(1 + c)^2\,s^2 - 99\,c^3\,(1 + c)\,s^3 - 3\,c^4\,s^4\big) + \frac{1}{\sqrt{\text{CoopCompDerS}}}\big(3072\,(1 + c)^7 - 22\,912\,c\,(1 + c)^6\,s + 71\,840\,c^2\,(1 + c)^5\,s^2 - 121\,664\,c^3\,(1 + c)^4\,s^3 + 116\,872\,c^4\,(1 + c)^3\,s^4 - 58\,974\,c^5\,(1 + c)^2\,s^5 + 10\,953\,c^6\,(1 + c)\,s^6 + 729\,c^7\,s^7\big)\Big)\Big]$$

0

$$\text{FullSimplify}\Big[\partial_s \epsilon_{\text{CoopComp}_{s2}} - (-2\,c\,(1 + c)\,(1 + c - 2c\,s)\,(a - c\,(h - x)))\Big/\Big((2 + 2c - c\,s)^3\,\big(2\,(1 + c - 2c\,s)^2 + 6\,c\,s\,(1 + c - 2c\,s) + c^2\,s^2\big)^2\Big)\Big(3\,\text{TempQtityPositif2} + \frac{\text{TempQtityPositif3}}{\sqrt{\text{CoopCompDerS}}}\Big)\Big]$$

0

Observation8;

Defining the individual and aggregate use of water under cooperation and comparison to the competitive case;

transmissivity has an increasing effect on the profits of the efficient player; for the less efficient players the effect is positive under low levels of inequality and negative under high levels;

$$\text{FullSimplify}\Big[\Big\{\Big\{w_{\text{comp}}[a_i, x, a_j, x] - \Big(\frac{a_i + a_j - 2\,c\,(h - x)}{2 + 2c - c\,s} + \frac{a_i - a_j}{2 + 2c - 3c\,s}\Big)\Big\},\ \Big\{w_{\text{comp}}[a + \epsilon, x, a - \epsilon, x] - 2\Big(\frac{a - c\,(h - x)}{2 + 2c - c\,s} + \frac{\epsilon}{2 + 2c - 3c\,s}\Big)\Big\}\Big\}\Big]$$

{{0}, {0}}

$$\mathbf{FullSimplify}\Big[\Big\{\Big\{ w_{coop}[a_i, x, a_j, x] - \Big(\frac{a_i + a_j - 2c(h-x)}{2+2c} + \frac{a_i - a_j}{2+2c-4cs}\Big)\Big\}, \Big\{ w_{coop}[a+\epsilon, x, a-\epsilon, x] - \Big(\frac{a - c(h-x)}{1+c} + \frac{\epsilon}{1+c-2cs}\Big)\Big\}\Big\}\Big]$$

{{0}, {0}}

$$\mathrm{AggW}_{comp}[ai\_, x\_, aj\_, x\_] := \frac{2ai + 2aj - 4c(h-x)}{2+2c-cs}; \quad \mathrm{AggW}_{coop}[ai\_, x\_, aj\_, x\_] := \frac{ai + aj - 2c(h-x)}{1+c};$$

$$\mathbf{FullSimplify}\Big[\Big\{\Big\{ \mathrm{AggW}_{comp}[a_i, x, a_j, x] - (w_{comp}[a_i, x, a_j, x] + w_{comp}[a_j, x, a_i, x]) \Big\}, \Big\{ \mathrm{AggW}_{coop}[a_i, x, a_j, x] - (w_{coop}[a_i, x, a_j, x] + w_{coop}[a_j, x, a_i, x]) \Big\}\Big\}\Big]$$

{{0}, {0}}

$$\mathbf{FullSimplify}\Big[\Big\{\Big\{(\mathrm{AggW}_{coop}[a_i, x, a_j, x] - \mathrm{AggW}_{comp}[a_i, x, a_j, x]) - \frac{cs}{1+c}\Big(-\frac{a_i + a_j - 2c(h-x)}{2+2c-cs}\Big)\Big\}, \Big\{(\mathrm{AggW}_{coop}[a-\epsilon, x, a+\epsilon, x] - \mathrm{AggW}_{comp}[a-\epsilon, x, a+\epsilon, x]) - \frac{2cs}{1+c}\Big(-\frac{a - c(h-x)}{2+2c-cs}\Big)\Big\}\Big\}\Big]$$

{{0}, {0}}

$$\mathbf{FullSimplify}\Big[D[\mathrm{AggW}_{coop}[a_i, x, a_j, x] - \mathrm{AggW}_{comp}[a_i, x, a_j, x], s] + 2\frac{c(a_i + a_j - 2c(h-x))}{(2+2c-cs)^2}\Big]$$

0

$$\mathbf{FullSimplify}\Big[\Big\{\Big\{(w_{coop}[a_i, x, a_j, x] - w_{comp}[a_i, x, a_j, x]) - \frac{cs}{2}\Big(\frac{a_i - a_j}{(1+c-2cs)(2+2c-3cs)} - \frac{a_i + a_j - 2c(h-x)}{(1+c)(2+2c-cs)}\Big)\Big\}, \Big\{(w_{coop}[a+\epsilon, x, a-\epsilon, x] - w_{comp}[a+\epsilon, x, a-\epsilon, x]) - cs\Big(\frac{\epsilon}{(1+c-2cs)(2+2c-3cs)} - \frac{a - c(h-x)}{(1+c)(2+2c-cs)}\Big)\Big\}\Big\}\Big]$$

{{0}, {0}}

$$\mathbf{FullSimplify}\Big[\mathrm{Solve}\Big[cs\Big(\frac{\epsilon}{(1+c-2cs)(2+2c-3cs)} - \frac{a - c(h-x)}{(1+c)(2+2c-cs)}\Big) == 0, \epsilon\Big]\Big]$$

$$\Big\{\Big\{\epsilon \to \frac{(-1 + c(-1+2s))(-2 + c(-2+3s))(-a + c(h-x))}{(1+c)(-2 + c(-2+s))}\Big\}\Big\}$$

$$\mathbf{FullSimplify}\Big[\Big(\frac{(1+c-2cs)(2+2c-3cs)(a - c(h-x))}{(1+c)(2+2c-cs)}\Big) - \Big(\frac{2+2c-3cs}{2+2c-cs}\Big)\epsilon_{MaxCoopS}\Big]$$

0

FullSimplify[3 (2 + c (2 − s)) − 4 (1 + c)]

$2 + c\,(2 - 3\,s)$

f1[s_] = (1 + c − c s) (28 (1 + c − 2 c s)^3 + 17 c s (2 + 2 c − 4 c s)^2 + 57 c^2 s^2 (1 + c − 2 c s) + 9 c^3 s^3);

f2[s_] = (1 + c − 2 c s)^2 (5 (2 + 2 c − 2 c s)^2 + 4 c s (2 + 2 c − 2 c s) + 3 c^2 s^2);

$$\mathbf{FullSimplify}\Big[\partial_s V_{\text{coopS}-\text{compS}}[a_i, x, a_j, x, s] - s\,c^2\left(\frac{2\left(\frac{a_i+a_j}{2} - c\,(h-x)\right)^2}{(2+2c-c\,s)^3} + (a_i - a_j)\left(\frac{f1[s]\,(a_i - c\,(h-x)) + f2[s]\,(a_j - c\,(h-x))}{2\,(2+2c-c\,s)^2\,(1+c-2c\,s)^2\,(2+2c-3c\,s)^3}\right)\right)\Big]$$

0

$$\mathbf{FullSimplify}\Big[V_{\text{coopS}-\text{compS}}[a_i, x, a_j, x, 1/2] - c^2\left(\frac{\left(\frac{a_i+a_j}{2} - c\,(h-x)\right)^2}{2\,(1+c)\,(4+3c)^2} + (a_i - a_j)\,\big((28 + c\,(25 + 6c))\,(a_i - c\,(h-x)) + (20 + 11c)\,(a_j - c\,(h-x))\big)\Big/\big(8\,(1+c)\,(4+c)^2\,(4+3c)\big)\right)\Big]$$

0

$$a_{l,\,\text{MinCoopS}} = \left(1 - \frac{c\,s}{1+c-c\,s}\right)c\,(h-x) + \frac{c\,s}{1+c-c\,s}\,a_h;$$

$$\mathbf{FullSimplify}\big[\{\{w_{\text{comp}}[a_{l,\,\text{MinCompS}}, x, a_h, x]\},\ \{w_{\text{coop}}[a_{l,\,\text{MinCoopS}}, x, a_h, x]\}\}\big]$$

{{0}, {0}}

$$\mathbf{FullSimplify}\Big[\Big\{\Big\{w_{\text{comp}}[a_{l,\,\text{MinCoopS}}, x, a_h, x] - \left(\frac{2\,c\,s\,(a_h - c\,(h-x))}{(2+2c-c\,s)\,(2+c\,(2-3s))}\right)\Big\},$$
$$\Big\{w_{\text{coop}}[a_{l,\,\text{MinCompS}}, x, a_h, x] - \left(-\frac{c\,s\,(a_h - c\,(h-x))}{2\,(1+c)\,(1+c\,(1-2s))}\right)\Big\},$$
$$\Big\{a_{l,\,\text{MinCoopS}} - a_{l,\,\text{MinCompS}} - \left(\frac{c\,s}{2+2c-2c\,s}\,(a_h - c\,(h-x))\right)\Big\}\Big\}\Big]$$

{{0}, {0}, {0}}

$$\text{FullSimplify}\left[\;V_{coopS-compS}[a_{l,MinCoopS}, x, a_h, x, s] - 2c^2 s^2 \left(-\frac{1+c-cs}{(2+2c-cs)^2}\left(\frac{a_h - c(h-x)}{2+2c-3cs}\right)^2\right)\right]$$

0

camparing cooperation / Single user;

$$\text{FullSimplify}\left[\left\{\frac{\epsilon_{-RootDerivS,Inf}}{a-c(h-x)}, \frac{\epsilon_{Root,Inf}}{a-c(h-x)}, \frac{\epsilon_{+MaxSingComp}}{a-c(h-x)}, \frac{\epsilon_{+RootDerivS,Sup}}{a-c(h-x)}, \frac{\epsilon_{+MaxCompS}}{a-c(h-x)}\right\}\right]$$

$$\Bigg\{\Bigg((2+c(2-3s))\Bigg((2+c(2-3s))^2 + \frac{1}{(-2+c(-2+s))^2} 4\sqrt{(2+c(2-3s))(2-c(-2+s))}\,(-1+c(-1+s)) \sqrt{\left(16(1+c)^4 - 24c(1+c)^3 s + 4c^2(1+c)^2 s^2 + 2c^3(1+c)s^3 + 3c^4 s^4\right)}\Bigg)\Bigg) \Big/ \left(24(1+c)^3 - 52c(1+c)^2 s + 26c^2(1+c)s^2 + 3c^3 s^3\right), \frac{(2+c(2-3s))\left((2+c(2-3s))^2 - 2c^2 s^2\right)}{(2-c(-2+s))(4-c(4(-2+s)+c(-4+s(4+s))))},$$

$$-\frac{(2+c(2-3s))^2}{-4+4c(-2+s)+c^2(-4+s(4+s))},$$

$$\Bigg((2+c(2-3s))\Bigg((2+c(2-3s))^2 - \frac{1}{(-2+c(-2+s))^2} 4\sqrt{(2+c(2-3s))(2-c(-2+s))}\,(-1+c(-1+s)) \sqrt{\left(16(1+c)^4 - 24c(1+c)^3 s + 4c^2(1+c)^2 s^2 + 2c^3(1+c)s^3 + 3c^4 s^4\right)}\Bigg)\Bigg) \Big/ \left(24(1+c)^3 - 52c(1+c)^2 s + 26c^2(1+c)s^2 + 3c^3 s^3\right), 3 + \frac{4(1+c)}{-2+c(-2+s)}\Bigg\}$$

$$\text{FullSimplify}\left[\epsilon_{MaxCoopS} - \epsilon_{+MaxSingComp} - \left(-\frac{2c^2(1+c-cs)s^2(a-c(h-x))}{(1+c)\left((2+2c-2cs)^2 + cs(4+4c-5cs)\right)}\right)\right]$$

0

$$\text{FullSimplify}\left[4 - c(4(-2+s)+c(-4+s(4+s))) - \left((2+2c-2cs)^2 + cs(4+4c-5cs)\right)\right]$$

0

camparing cooperation / Single user;

Defining the gain from cooperation Vs. single use and its derivative ;

$$\overset{\text{coop-sing}}{\mathcal{V}}[\text{ai\_}, \text{x\_}, \text{aj\_}, \text{x\_}, \text{s\_}] := \frac{((1 + c - c\,s)\,(aj - c\,(h - x)) - c\,s\,(ai - c\,(h - x)))^2}{2\,(1 + c)\,(1 + c - c\,s)\,(1 + c\,(1 - 2\,s))};$$

$$\text{FullSimplify}\left[\overset{\text{coop-sing}}{\mathcal{V}}[a_i, x, a_j, x, s] - \left(\overset{\text{coop}}{\mathcal{V}}[a_i, x, a_j, x, s] - \overset{\text{sing}}{\mathcal{V}}[a_i, x, a_j, x, s]\right)\right]$$

0

$$\overset{\text{coop-sing}}{\Delta\mathcal{V}}[\text{ai\_}, \text{x\_}, \text{aj\_}, \text{x\_}, \text{s\_}] :=$$

$$-\frac{c\left(\frac{ai+aj}{2} - c\,(h - x)\right)^2}{2\,(1 + c - c\,s)^2} + \left(c\,(ai - aj)\left(\left(1 + c\left(2 + c + 4\,(1 + c)\,s - 8\,c\,s^2\right)\right)(ai - c\,(h - x)) - \left(5 + c\left(5\,(2 + c) - 12\,(1 + c)\,s + 8\,c\,s^2\right)\right)(aj - c\,(h - x))\right)\right) \Big/ \left(8\,(1 + c - c\,s)^2\,(1 + c\,(1 - 2\,s))^2\right);$$

$$\text{FullSimplify}\left[\overset{\text{coop-sing}}{\Delta\mathcal{V}}[a_i, x, a_j, x, s] - \left(D\left[\overset{\text{coop}}{\mathcal{V}}[a_i, x, a_j, x, s] - \overset{\text{sing}}{\mathcal{V}}[a_i, x, a_j, x, s], s\right]\right)\right]$$

0

Observation0;

the difference between the cooperative outcome and the single user outcome is always positive;

Observation1;

inequality decreases the difference between the cooperative outcome and the single user outcome;

$$\text{FullSimplify}\left[\left(\overset{\text{coop-sing}}{\mathcal{V}}[a_i, x, a_j, x, s] - \overset{\text{coop-sing}}{\mathcal{V}}\left[\frac{a_i + a_j}{2}, x, \frac{a_i + a_j}{2}, x, s\right]\right) - (a_i - a_j)\left(-\left((1 + c - 4\,c\,s)\,(a_i - c\,(h - x)) + (3 + 3\,c - 4\,c\,s)\left(a_j - c\,(h - x)\right)\right) \Big/ (8\,(1 + c - c\,s)\,(1 + c\,(1 - 2\,s)))\right)\right]$$

0

Observation2a;

in the case of identical users,
transmissivity decreases the difference between the cooperative outcome and the single user outcome;

$$\text{FullSimplify}\Big[\Big\{\Big\{\overset{\text{coop-sing}}{\mathcal{V}}[a, x, a, x, s] - \left(\frac{1+c(1-2s)}{1+c(1-s)}\frac{(a-c(h-x))^2}{2(1+c)}\right),$$

$$D\Big[\overset{\text{coop-sing}}{\mathcal{V}}[a, x, a, x, s], s\Big] - \left(-\frac{c(a-c(h-x))^2}{2(1+c-cs)^2}\right)\Big\},$$

$$\Big\{\overset{\text{coop-sing}}{\mathcal{V}}[a, x, a, x, 0] - \frac{(a-c(h-x))^2}{2(1+c)}, \overset{\text{coop-sing}}{\mathcal{V}}\Big[a, x, a, x, \frac{1}{2}\Big] - \frac{(a-c(h-x))^2}{(2+c)(1+c)}\Big\}\Big\}\Big]$$

{{0, 0}, {0, 0}}

Observation2b;

in the case of unequal users, transmissivity decreases the difference between the cooperative outcome and the single user outcome;

showing that the derivative function is negative;
to do so we show that the derivative is decreasing in aj THEN we show that the derivative at the lowest acceptable value for aj is nil (and that it is strictly negative at equal efficiencies);

$$\text{FullSimplify}\Big[\Big\{\Big\{D\Big[\overset{\text{coop-sing}}{\Delta\mathcal{V}}[a_i, x, a_j, x, s], a_j\Big] - \left(-\frac{c(a_i - a_j)}{(1+c-2cs)^2}\right)\Big\},$$

$$\Big\{\overset{\text{coop-sing}}{\Delta\mathcal{V}}[a_h, x, a_{l,\text{MinCoopS}}, x, s]\Big\}, \Big\{\Big\{\overset{\text{coop-sing}}{\Delta\mathcal{V}}[a_h, x, a_h, x, s] - \left(-\frac{c(a_h - c(h-x))^2}{2(1+c-cs)^2}\right)\Big\}\Big\}\Big\}\Big]$$

{{0}, {0}, {{0}}}

Checking that the derivative is negative for the extreme values of transmissivity;

$$\text{FullSimplify}\Big[$$

$$\Big\{\overset{\text{coop-sing}}{\Delta\mathcal{V}}[a_i, x, a_j, x, 0] - \left(-\frac{c(a_j - c(h-x))(2a_i - a_j - c(h-x))}{2(1+c)^2}\right), \Big\{\overset{\text{coop-sing}}{\Delta\mathcal{V}}\Big[a_i, x, a_j, x, \frac{1}{2}\Big] -$$

$$\left(\frac{1}{2(2+c)^2}c\big(c(a_i - c(h-x)) - (2+c)(a_j - c(h-x))\big)\big(2(a_i - c(h-x)) + (2+c)(a_i - a_j)\big)\right), \text{Limit}\Big[$$

$$\overset{\text{coop-sing}}{\Delta\mathcal{V}}[a_h, x, a_{l,\text{MinCoopS}}, x, s], s \to \frac{1}{2}\Big], \overset{\text{coop-sing}}{\Delta\mathcal{V}}\Big[a_h, x, a_h, x, \frac{1}{2}\Big] - \left(-\frac{2c(a_h - c(h-x))^2}{(2+c)^2}\right)\Big\}\Big\}\Big]$$

{0, {0, 0, 0}}

Checking that the derivative is positive for the extreme values of transmissivity and checking the effect of a non marginal ;

$$\text{FullSimplify}\left[\left\{\left\{\overset{\text{coop-sing}}{\mathcal{V}}[a_i, x, a_j, x, 0] - \frac{(a_j - c(h-x))^2}{2(1+c)}\right\}, \left\{\overset{\text{coop-sing}}{\mathcal{V}}\left[a_i, x, a_j, x, \frac{1}{2}\right] - \frac{(a_j(2+c) - c(a_i + 2h - 2x))^2}{4(2+3c+c^2)}\right\}\right\}\right]$$

{{0}, {0}}

Checking the effect of a non marginal change in transmissivity calculated as proportional to the multiplication of two functions both positive for any acceptable value of ai and aj for an open resource and a closed one;

$$\text{f1sc[ai\_, aj\_]} := \frac{c(ai - c(h-x)) - (aj - c(h-x))\left(2 + c - \sqrt{2}\sqrt{2+c}\right)}{\sqrt{2}\sqrt{2+c}};$$

$$\text{f2sc[ai\_, aj\_]} := \frac{-c(ai - c(h-x)) + (aj - c(h-x))\left(2 + c + \sqrt{2}\sqrt{2+c}\right)}{\sqrt{2}\sqrt{2+c}};$$

$$\text{FullSimplify}\left[\left(\overset{\text{coop-sing}}{\mathcal{V}}[a_i, x, a_j, x, 0] - \overset{\text{coop-sing}}{\mathcal{V}}\left[a_i, x, a_j, x, \frac{1}{2}\right]\right) - \frac{1}{2(1+c)}\,\text{f1sc}[a_i, a_j]\,\text{f2sc}[a_i, a_j]\right]$$

0

$$\text{FullSimplify}\Big[\left\{\text{Limit}\left[\left\{\text{f1sc}[a_h, a_{l,\text{MinCoopS}}] - \frac{c(a_h - c(h-x))}{2+c},\ \text{f2sc}[a_h, a_{l,\text{MinCoopS}}] - \frac{c(a_h - c(h-x))}{2+c}\right\}, s \to \frac{1}{2}\right],\right.$$
$$\left.\left\{\text{f1sc}[a_h, a_h] - \left(\frac{\sqrt{2+c} - \sqrt{2}}{\sqrt{2+c}}\right)(a_h - c(h-x)),\ \text{f2sc}[a_h, a_h] - \left(\frac{\sqrt{2+c} + \sqrt{2}}{\sqrt{2+c}}\right)(a_h - c(h-x))\right\}\right\}\Big]$$

{{0, 0}, {0, 0}}

Numerical example;

$a_i = 900;\ a_j = 800;\ x = 98;\ x = 98;\ c = 40;\ s = 0.5;\ h = 100;$

$w_{icomp} = w_{comp}[a_i, x, a_j, x];\ w_{jcomp} = w_{comp}[a_j, x, a_i, x];\ \text{FullSimplify}[\{w_{icomp}, w_{jcomp}\}]$

{29.3842, 20.2933}

$$\text{FullSimplify}\Big[\Big\{\Big\{\Big\{a_i\, w_{icomp} - \frac{w_{icomp}{}^2}{2}\Big\}, \Big\{-c\, w_{icomp}\left(h - \frac{x + (1-s)\left(x - w_{icomp}\right) + s\left(x - w_{jcomp}\right)}{2}\right)\Big\},$$

$$\Big\{a_i\, w_{icomp} - \frac{w_{icomp}{}^2}{2} - c\, w_{icomp}\left(h - \frac{x + (1-s)\left(x - w_{icomp}\right) + s\left(x - w_{jcomp}\right)}{2}\right)\Big\}\Big\},$$

$$\{\}, \Big\{\Big\{a_j\, w_{jcomp} - \frac{w_{jcomp}{}^2}{2}\Big\}, \Big\{-c\, w_{jcomp}\left(h - \frac{x + (1-s)\left(x - w_{jcomp}\right) + s\left(x - w_{icomp}\right)}{2}\right)\Big\},$$

$$\Big\{a_j\, w_{jcomp} - \frac{w_{jcomp}{}^2}{2} - c\, w_{jcomp}\left(h - \frac{x + (1-s)\left(x - w_{jcomp}\right) + s\left(x - w_{icomp}\right)}{2}\right)\Big\}\Big\},$$

$$\{\}, \Big\{a_i\, w_{icomp} - \frac{w_{icomp}{}^2}{2} - c\, w_{icomp}\left(h - \frac{x + (1-s)\left(x - w_{icomp}\right) + s\left(x - w_{jcomp}\right)}{2}\right) +$$

$$a_j\, w_{jcomp} - \frac{w_{jcomp}{}^2}{2} - c\, w_{jcomp}\left(h - \frac{x + (1-s)\left(x - w_{jcomp}\right) + s\left(x - w_{icomp}\right)}{2}\right)\Big\}\Big\}\Big]$$

{{{26 014.}, {−16 948.}, {9066.01}}, {}, {{16 028.7}, {−11 704.6}, {4324.07}}, {}, {13 390.1}}

$$w_{ising} = w_{sing}[a_i, x, a_j, x]$$

39.0476

$$\text{FullSimplify}\Big[\Big\{\Big\{a_i\, w_{ising} - \frac{w_{ising}{}^2}{2}\Big\}, \Big\{-c\, w_{ising}\left(h - \frac{x + (1-s)\left(x - w_{ising}\right) + s\,(x - 0)}{2}\right)\Big\},$$

$$\Big\{a_i\, w_{ising} - \frac{w_{ising}{}^2}{2} - c\, w_{ising}\left(h - \frac{x + (1-s)\left(x - w_{ising}\right) + s\,(x - 0)}{2}\right)\Big\}\Big\}\Big]$$

{{34 380.5}, {−18 371.}, {16 009.5}}