ABSTRACT

Title of dissertation: ENERGY AND SECURITY ASPECTS OF WIRELESS NETWORKS: PERFORMANCE AND TRADEOFFS

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Energy and security are becoming increasingly important in the design of future wireless communication systems. This thesis focuses on these two main aspects of wireless networks and studies their tradeoffs with other performance metrics such as throughput and delay.

The first part of the thesis deals with the energy aspect of wireless networks in which we present several novel joint physical network layer techniques and either evaluate their energy efficiency or study the energy/delay/throughput tradeoffs. First, we study the energy/delay tradeoffs for the problem of reliable packet transmission over a wireless time-varying fading link and also investigate the effect of having Channel State Information on the resulting tradeoff. Then, we extend the model to a single-hop multicast time varying wireless network. We address energy/delay/throughput tradeoffs by considering the problem of streaming a real time file with fixed delay and energy constraints where the objective is to maximize the number of packets received by the destinations. Again, the effect of having Channel

State Information is studied. Also, the effect of using Random Network Coding as a transmission scheme is studied and compared to traditional transmission schemes such as simple ARQ. Next, we consider the effect of cooperation on the energy efficiency of wireless transmissions in which we propose several joint physical-network layer cooperation techniques. Also, the effect of Random Network Coding is investigated in the context of cooperation in which Random Network Coding based cooperation techniques are investigated and compared to cooperation techniques that rely on simple ARQ solely or combined with superposition Alamouti space-time codes. We then consider the particular case of cellular systems in which we design rate allocation technique that minimizes the consumption energy in a Macro cell. This technique takes into account sleep mode configuration of current base stations.

In the second part of the thesis, we focus on security and in particular on privacy. We also study the tradeoff between securing wireless transmissions and the energy/delay overhead due to security by considering the problem of information exchange among adjacent wireless node in the presence of an eavesdropper. The nodes are required to exchange their information while keeping it secret from the eavesdropper. The nodes can choose to transmit either through public channel or though more costly private channels. We express the cost of using the private channels in terms of the extra energy or delay required to transmit through the private channel. We then minimize the security cost subject to a target security level. Also, this part presents a deterministic Network Coding based transmission scheme and investigates its effect on the achieved performance.

Last, we introduce the problem of minimum energy scheduling of a group of base stations and compare this problem to the standard minimum length scheduling problem. We also discuss the complications and the challenges associated with solving the minimum energy scheduling problem.

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by

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Chapter 1

Introduction

1.1 The importance of Energy Efficiency in Wireless Networks and Energy/ Delay/ Throughput Tradeoffs

Energy efficiency has tremendously become a crucial parameter in designing communications systems especially wireless systems. The necessity of energy efficient communication systems stems from the increasing cost of energy and the concern to reduce the global CO_2 emissions to combat climate change. However, most communications systems were initially designed to be optimal in terms of other performance metrics such as throughput, reliability and delay. Thus, they are not optimal in terms of energy efficiency. For example to ensure Quality of Service (QoS) requirements for real time applications, the problem was to find the maximum throughput given strict delay requirement without considering minimizing the energy consumed.

Thus, there are numerous performance tradeoffs that arise and will be addressed in this thesis. Some of these tradeoffs: 1) what is the minimum energy consumed given certain delay constraints or rate requirements? 2) What is the achievable throughput given certain energy/delay constraints? 3) Given that the information should be transmitted reliably (with no errors) to the corresponding destinations, how long will it take to transmit the information and how much energy will it consume? The choice of selecting which tradeoff to study depends on the target application on the upper layers. For example, the first and second tradeoffs are interesting for real time applications while the third tradeoff is suitable for non real time applications that require reliable transmission such as file transfers.

Also, this thesis studies energy efficiency in wireless multicast systems. Due to the varying nature of the wireless channel, different receivers will have different channel qualities, and hence the performance of transmission will be different among users. Thus, whenever a transmitter is multicasting packets to multiple receivers in energy/delay constrained system, there is a tradeoff between ensuring reliable packet delivery to all receivers and the number of packets that can so be delivered. Thus, one should find the transmission scheme that maximizes the number of packets transmitted with target energy/delay constraints while also it ensures that an acceptable number of receivers would receive each packet.

1.2 Security Challenges in Wireless Networks and the Security/ Energy Tradeoffs

Another crucial aspect of wireless networks is security. Security is challenging in wireless networks due to the wireless multicast property i.e. wireless signals are broadcasted over the air, and hence any user that is in the communication range of the transmitter can receive the signal. This property achieves energy savings since the transmitter can transmit the information once to all receivers instead of transmitting the information multiple times to each user; however, this feature is a security bottleneck since the transmitted signals can be easily intercepted by an attacker who is within the communication range of the transmitter, which makes information "privacy" harder to attain. Hence, this thesis focuses on the privacy aspect of security.

An important issue that is not yet well addressed is the tradeoff between energy and security. This tradeoff is due to the following: In order to ensure the privacy of information, complex modulation (such as spread spectrum), coding and encryption schemes are used, which usually increases transmission rate and hence require more energy. However, most of the current designed wireless secure systems do not take into consideration this energy overhead due to secure transmissions. On the other hand, it is known that [1] it is most energy efficient to transmit with lowest feasible rate and hence designing energy efficient systems may result in higher delays which might increase the eavesdropper chances to acquire the information and thus affect the level of security achieved.

Although some of these tradeoffs and the energy/delay/throughput tradeoffs mentioned in the previous section have been addressed in prior work, this thesis presents novel joint physical and network layer techniques for wireless transmissions and studies their effect on the different stated tradeoffs. These techniques are mainly based on Network Coding, cooperation, and sleep mode methods which are used in particular in cellular systems.

1.3 Network Coding

Network Coding, as proposed by Ahlswede el al [2], is an alternative communication concept which has proved to achieve high improvements in terms of throughput and energy efficiency in wireless networks, especially in multicasting [3] [4], and thus it is important to examine its effect on the network performance and to incorporate it in the design of secure/ energy efficient systems.

The idea of Network Coding is that unlike traditional routing where the node forwards the packets as they are received by the original sender, the node forms a new packet that is a linear combination of a group of packets and sends the new packet to the intended destination. The group of packets may belong to different flows (Inter-session Network Coding) or to the same flow (Intra-session Network Coding). After the destination receives enough linearly independent combination of the packets, it recovers the original packets by solving the system of linear equations. There are two types of Network Coding. The first type is Deterministic Network Coding in which the coefficients of all the linear combinations received by the destination are deterministic and determined prior to transmission. The other type is Random Network Coding in which the coefficients are randomly selected from a uniform distribution over the symbols' alphabet. Hence, Random Network Coding can be widely used in distributed settings since the nodes do not need centralized coordination to determine the coefficients of the linear combinations.

Network Coding is also promising for security consideration and is simple to implement since each transmitted packet is a linear combination of the original packets. Hence, it is not straightforward for an intercepting eavesdropper to recover the original packets especially for the case of Deterministic Network Coding. Thus, its use results in a form of scrambling that makes it difficult for the eavesdropper to decode.

In this thesis, we investigate the effect of using Network Coding on the energy/delay/throughput tradeoffs mentioned in part 1.1. We also study the effect of using Network Coding on the security/energy/delay tradeoffs discussed in part 1.2.

1.4 Network Cooperation Techniques

It is interesting to investigate the effect of cooperation techniques on the design of energy efficient systems especially in the context of wireless multicast as cooperation has proven to achieve performance improvements in wireless networks [5], [6], [7]. Cooperation can be achieved by adding relays that have better channels qualities with the destinations than the source node and hence can assist the source in transmitting the information to the target destinations. Another form of cooperation is user cooperation. User cooperation works whenever a source node is multicasting packets to multiple destinations, the destinations that first receive the data successfully from the source can assist the source in transmitting the data to the remaining destinations. This form of cooperation is motivated by the fact that some destinations may have better channel quality than the source node due to the nature of wireless channels. Hence, this method is anticipated to decrease the total energy consumed in the network to deliver the required data.

Relay cooperation is expensive since new resources (i.e. the relays) are added to the network. In user cooperation, on the other hand, no extra resources are added, and hence it is less expensive than relay cooperation. However, obtaining better performance is not always guaranteed in user cooperation since the users that act as relays may not always have better channel quality than the source with the remaining users. Hence, it is essential to study the cases in which user cooperation can achieve performance improvement than when no cooperation is used and design techniques that decide whether the source or the users should transmit based on the channel quality between the nodes in the network.

In this work, we consider joint physical and network layer cooperation techniques in wireless settings and evaluate their energy efficiency. We present a Random Network Coding based cooperation scheme and study the benefits of using Network Coding in achieving energy reductions. Also, we consider cooperative techniques that include using both Network Coding at the network layer and/or Alamouti space-time codes at the physical layer.

1.5 Sleep Mode Techniques

In cellular networks, the energy consumed by base stations accounts for a significant percentage of the total consumed energy. Hence, there have been several attempts to design energy efficient base stations by using advanced technologies for the RF power amplifier, the baseband processing circuits, and the cooling systems. Furthermore, system-level algorithms have been designed to reduce the power of base stations when no users are active in the network. The base station's power is reduced by using either "Micro" sleep mode or "Deep" sleep mode. In "Micro" sleep mode, only some of the components of the base station are turned off, and hence the base station can be turned on again relatively quickly (in the order of microseconds). In "Deep" sleep mode, however, most of the base station components are turned off, and hence a long time is needed for the base station to be turned on. The "Micro" sleep mode is beneficial in the case of "bursty traffic" where the base station can adapt its power efficiently based on the cell traffic on a micro-time scale, while the "Deep" sleep mode is more useful in situations when traffic displays longer-term activity or inactivity patterns.

In this thesis, we consider "Micro" sleep mode. First, we present a rate allocation algorithm that takes into account the sleep mode feature of the base station to minimize the consumed energy in a Macro cell. Then, we consider a network composed of several cells and present the problem of scheduling the base stations in order to minimize the total consumed energy in the network.

1.6 Thesis Outline

This thesis is organized as follows. In the first problem, we consider transmission over a wireless link in which packets are transmitted from source to a destination over time varying Rayleigh fading wireless link, and the source has knowledge about the Channel State Information (channel statistics). We address the tradeoff between the energy consumed and the delay spent to deliver each packet successfully by investigating two problems: In the first problem, we assume that each packet has a delay constraint and minimize the energy spent to successfully deliver each packet. In the second problem, we assume that each packet has a finite energy budget and minimize the time spent to deliver the packet successfully. Rate control and power control techniques are investigated respectively to obtain the minimum energy and delay values.

The second problem similarly addresses tradeoffs between energy and other performance metrics. We consider the problem of finding the optimal power policy of multicasting a group of packets by a transmitter to a set of receivers in a single hop network over independent time varying channels, where the packets should be delivered within a delay constraint and with a limited amount of energy. The objective is to maximize the multicast throughput. We investigate the effect of using Random Network Coding (RNC) on the achieved throughput.

The third problem also investigates the effect of Random Network Coding on the performance of wireless transmissions in particular on energy efficiency but now in simple cooperative networks. We consider different cooperative strategies for packets transmission in a simple wireless fading network where the channel statistics do not change over time. We then find the optimal power values that minimize the transmission energy consumed per successfully delivered packet. We consider both cases when simple Automatic Repeat Request (ARQ) and Random Network Coding (RNC) are used as transmission schemes. Some techniques considered also incorporate Alamouti space-time codes at the physical layer.

The fourth problem investigates energy efficient sleep mode based techniques for wireless networks but in particular for cellular networks. We consider the downlink scenario in a Macro cell in which the base station should satisfy its users' demands within a strict delay constraint. We assume that the consumed power of the base station is a linear function of the transmission power, and that the base station can go to "Micro" sleep mode when there are no active users. We start by considering the simple case when there is only one active user. Then, we consider the case when multiple users are active in the cell. In this case, we consider both time division multiplexing and frequency division multiplexing. For each case, we find the optimal rate value the base station should use to each active user in order to minimize the overall consumed energy.

The fifth problem deals with the second aspect of this thesis which is ensuring secure wireless transmissions in the presence of an eavesdropper. It also studies the tradeoff between achieving a certain security level and the energy/delay costs due to security. In the first part, we start by considering the single link case where a file is residing at a source node and should be delivered to the intended receiver. Then, we consider the case where the file is distributed among multiple nodes, and the nodes are required to exchange their packets until all they receive the file successfully. In either cases, the nodes can chose to transmit through public channels in which the eavesdropper has access to or through private channels that are not accessible for the eavesdropper. We define two security cost: the extra energy spent and the extra delay incurred due to using the private channels. The objective is then to minimize each of the security costs respectively subject to a certain security level. The parameters of the security level are the maximum number of packets that the eavesdropper is allowed to receive, and the upper bound on the probability that the number of packets the eavesdropper receives is greater than or equal to the maximum value.

In the last part, we extend the fourth problem for the case when multiple cells are present in the network and introduce the problem of scheduling the base stations to minimize the consumed energy in the network. We compare the problem to a previous work that considers a similar scheduling problem but where the objective is to minimize the emptying time of the network. However due to time limitation, the problem is not fully developed and it is interesting to consider it for future work.

Chapter 2

Energy/Delay Tradeoffs in Data Transmission over a Time Varying Wireless Link

2.1 Overview

The primary focus of this chapter is to investigate the effect of Channel State Information (CSI) on the design of energy efficient transmission schemes over time varying wireless channels. Designing energy efficient wireless systems must cope with the time varying property of the wireless channel. Furthermore, it is necessary to also meet quality of service requirement of applications such as delay.

We consider two related questions: (i) Given CSI what is the minimum energy spent to deliver a certain amount of data while maintaining Quality of Service requirements such as delay? (ii) What is the minimum delay that can be achieved given a certain energy budget?

The availability of CSI can be of considerable help in addressing these two questions. In [8], a distributed protocol is developed for energy efficient transmission in wireless sensor networks. The protocol uses CSI at the sensor nodes, and selects the users with the best channel state for transmission. In [9], dynamic control algorithms are developed. These algorithms minimize energy in a time varying wireless network by varying transmission rates. Also, techniques that maximize the stable throughput subject to power constraints have been considered. Similar to the approach in [9], stochastic control methods are used in [10] to minimize energy of data transmission but with a deadline constraints in a time varying wireless transmission. A flow based model of the system is considered and a continuous time system is used to model the evolution of the channel characteristics. Then, a transmission policy is developed to obtain energy efficient transmission for a packet arrivals system.

The problem of energy efficient delay constraint data transmission over time varying wireless channels has been further addressed in [11], [12], [13], and [14]. In [11], dynamic programming is used to find an optimal energy allocation strategy in a wireless fading channel for two problems: the first is to maximize throughput given an energy constraint; the second is to minimize energy given a minimum acceptable throughput. In both cases, strict delay constraints are imposed on transmission. Again, a flow based model is used for data transmission, and it is assumed that CSI is available at the transmitter. In [12], data transmission over a block fading channel is considered, and it is assumed that data arrive to according to a stochastic process and then are stored in bits in a buffer. Then, the transmission rate and power are dynamically adjusted based on CSI in order to regulate the average transmission power and the average buffer delay. Perfect CSI is assumed to be available at the transmitter and the receiver. In [13], delay constrained data transmission over block fading channel is considered and CSI is used to find the optimal power value that meets the target QoS and the energy cost. QoS in this case is measured in terms of the outage probability. Further, the problem of power allocation to maximize throughput subject to an average power constraint and a delay constraint is considered in [14]. Both cases of full and partial CSI are considered.

In this chapter, we consider a discrete time system in which packets are transmitted from source to a destination over a time varying Rayleigh-fading wireless link, where the source has knowledge about partial Channel State Information i.e. not actual channel state but rather only channel statistics. We address the tradeoff between the energy consumed and the delay achieved to deliver each packet successfully by investigating two problems: In the first problem, we assume that each packet must meet a delay constraint and we minimize the energy spent to successfully deliver each packet. In the second problem, we assume that each packet has a finite energy budget and minimize the time spent to deliver the packet successfully. Rate control and power control techniques are investigated respectively to obtain the optimal energy and delay values. Each of the problems is formulated as a Constrained Markov Decision Problem, and a Linear Programming method is provided to obtain the optimal solution.

2.2 System Model

Consider a wireless link with a single source and a destination. Time is slotted. In each time slot, the source can transmit a packet to the destination. We assume that the channel between the source and the destination is slow Rayleigh fading where the channel characteristics do not change within one time slot, and the fading coefficient h_k at time slot k is a complex, zero-mean Gaussian random variable with variance s_k. We consider the case when the variance s_k is time varying and is modeled as a two-state Markov chain that changes between a high value s_H and a low value s_L.

Using a discrete-time Markov chain to model flat fading channels has been widely studied (e.g. see [15]-[17]). Also, recent measurements done in [18] show that a two state Markov chain is suitable to model the wireless channel in many applications.

It is also assumed that additive white Gaussian noise (AWGN) of variance N_0 is present at the destination and that the transmitted packet is received successfully by the destination in the k^{th} time slot with probability p_k.

Note that the probability p_k of packet successful reception by the destination at time slot k is dependent on the channel quality as well as on the selected power value P and the selected rate value r, and these are related according to the SNR model:

$$p_k = P(SNR(P) \geq \theta(r)) \tag{2.1}$$

where SNR is the Signal to Noise Ratio at the receiver and is given by:

$$SNR(P) = \frac{|h_k|^2 P}{N_0} \tag{2.2}$$

where $\theta(r)$ is the required threshold at the destination. Note that $\theta(r)$ is an increasing function of the selected rate value r. In this work, we will assume that the rate and the threshold are related by Shannon's formula i.e.

$r = log_2(1 + \theta)$ hence $\theta(r) = 2^r - 1$

Although this is somewhat an approximation, it offers valid insight into the problem and is widely used. It is desirable to use different $\theta(r)$ if the modulation and coding schemes are specified.

Since the fading coefficient is Rayleigh distributed, it can be shown that the probability of success p_k is given by:

$$p_k = e^{\frac{-(2^r-1)N_0}{s_k P}} \tag{2.3}$$

Based on the above expression, a good quality channel corresponds to the case when the variance s_k has high value, and a bad quality channel corresponds to the case when the variance s_k has low value.

We assume that the the source uses simple Automatic Repeat Request (ARQ) to transmit each packet reliably. Also in each time slot k, the source knows whether the channel has good or bad quality and transmits following one of two cases:

1. Power Control: In each time slot k, the transmitter transmits with a power value $P_i \in \{P_1, P_2, ..., P_n\}$ (assuming without loss of generality that the power value P_1 is zero) with a probability $q_{iG}(k)$ or $q_{iB}(k)$ if the channel has good or bad quality respectively while keeping the transmission rate fixed. In this case, energy is defined as the expected total energy spent to successfully deliver a packet. The delay is defined as the expected total number of time slots successfully needed deliver a packet.

2. Rate Control: In each time slot k, the transmitter transmits with a rate $r_k \in \{r_1, r_2, ..., r_n\}$ ($r_i > 0$, $i = 1, 2, ..., n$) with a probability $q_{iG}(k)$ or $q_{iB}(k)$ when

the channel has good or bad quality respectively while keeping the transmission power value fixed at P.

In this problem, it is assumed that the transmission rate r, the packet size M, and the time slot duration T are related as follows:

$$r = \frac{M}{T} \tag{2.4}$$

Hence, the rate value can be controlled in either one of the following two ways:

- The packet size is kept fixed at M bits while the time slot duration T_i is varied according to the selected rate value r_i. In this case, the energy metric is defined as the expected total energy spent to successfully deliver a packet; the delay is defined as the expected total time (in seconds) to successfully deliver a packet.

- The time slot duration is kept fixed at T seconds while the packet size M_i is varied according to the selected rate r_i. In this case, the total energy spent to transmit a packet is independent of the rate value (since the power value P and the time slot duration T are fixed), the energy metric is defined as the expected energy per bit spent to successfully deliver a packet; the delay is defined as the expected total number of time slots needed to successfully deliver a packet.

For the case of power control, let p_{iG} and p_{iB} to be the probabilities of success when the channel has good or bad quality respectively, when power value P_i is selected. Similarly for the case of rate control, let p_{iG} and p_{iB} be the probabilities of success

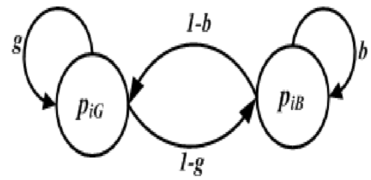


Figure 2.1: *Markov Chain Model of the Probability of Success.*

when the channel has good and bad quality respectively, when the rate value r_i is selected $(i = 1, 2, ..., n)$. Also, let $p_i(k)$ be the probability of success at time slot k when power P_i or rate r_i is selected. Since the variance s_k of the Rayleigh fading distribution of the channel evolves according to a Markov chain, the probability of success $p_i(k)$ will also evolve according to a Markov chain. Figure 1 shows this Markov chain $(g, b > 0)$,

The objective is to find the optimum probabilities $q^*_{iG}(k)$ and $q^*_{iB}(k)$ of transmission powers P_i and transmission rates r_i $(i = 1, 2, .., n)$ respectively for the following two problems: In the first problem, we minimize the energy spent to successfully deliver a packet to the destination, subject to a delay constraint (i.e. the expected time spent to deliver the packet successfully shouldn't exceed a certain value K). In the second problem, we minimize the delay (time spent to successfully deliver the packet) while the energy spent should not exceed a certain value E. We will later show that the optimal probabilities can be independent of the time slot k, i.e. the $q^*_{iG}(k) = q^*_{iG}$ and $q^*_{iB}(k) = q^*_{iB}$.

2.3 Energy/Delay Minimization with Power Control

2.3.1 Problem Formulation

Let the random variable y_k be an indicator whether the packet has failed or not to be received successfully by the destination in time slot k. It takes the following values:

$$y_k = \begin{cases} 0, & \textit{with probability } \sum_{i=1}^{n} q_i(k) p_i(k) \\ \\ 1, & \textit{otherwise} \end{cases} \tag{2.5}$$

where $q_i(k) \in \{q_{iG}(k), q_{iB}(k)\}$ and $p_i(k) \in \{p_{iG}, p_{iB}\}$

Also, let the random variable W_k indicate whether the packet has not been received successfully by the destination up to time slot k. W_k is defined in terms of y_k as follows:

$$W_0 = 1 \tag{2.6}$$

$$W_k = W_{k-1} \bullet y_k \tag{2.7}$$

where $\bullet$ is the binary AND operation

Next, let the random variable x_k be the energy spent in each time slot k. Note that:

$$x_k = P_i T W_{k-1} \textit{ with probability } q_i(k) \tag{2.8}$$

Hence, the energy spent to deliver the packet successfully is given by:

$$\xi(q_{iG}(k), q_{iB}(k)) = \sum_{k=1}^{\infty} x_k \tag{2.9}$$

Also, the number of time slots spent to deliver the packet is given by:

$$D(q_{iG}(k), q_{iB}(k)) = \sum_{k=1}^{\infty} W_{k-1} \tag{2.10}$$

Thus, the energy minimization problem with power control is formulated as follows:

$Min_{q_{iG}(k),q_{iB}(k)} E[\xi(q_{iG}(k), q_{iB}(k))]$

Subject to:

$$E[D(q_{iG}(k), q_{iB}(k))] \leq K \tag{2.11}$$

Similarly, the delay minimization problem can be formulated as follows:

$Min_{q_{iG}(k),q_{iB}(k)} E[D(q_{iG}(k), q_{iB}(k))]$

Subject to:

$$E[\xi(q_{iG}(k), q_{iB}(k))] \leq E \tag{2.12}$$

These are Constrained Markov Decision Problems.

2.3.2 Solution

2.3.2.1 MDP Model

Constrained Markov Decision Problems (CMDP) constitute a mathematical framework for dynamically optimizing constrained systems that evolve as a Markov process.

In general, a constrained MDP is composed of:

- The state space S
- The action space A

- For each state $x \in S$, the set of actions $A(x)$ pertaining to x.
- The transition probabilities P_{xay} from state x to state y $(x, y \in S)$ when action $a \in A(x)$ is taken.
- The immediate cost values $c(x, a)$ (which are used in the objective cost function) starting from state x and using action $a \in A(x)$.
- The immediate cost values $d_j(x, a)$, $j = 1, 2, ..., m$ (which are used in the constraint cost functions where m is the number of constraints)
- The class of possible policies U. In general, a policy $u \in U$ is a sequence $u = (u_1, u_2, ...)$ where each entry u_k specifies to any history of length k the probability that the action r_k taken at time slot k is action $a \in A(s_k)$; where s_k is the current state at time slot k i.e.

  $u_k(a|h_k) = P(r_k = a|h_k)$, $a \in A(s_k)$

The history h_k at time k is the sequence of previous states and actions up to the current state s_k, i.e. $h_k = (s_1, r_1, , s_{k-1}, r_{k-1}, s_k)$. One special class of policies is the class of stationary policies U_S. In a stationary policy u_s, the probability $u_k(a|h_k)$ that the action r_k taken at time slot k is $a \in A(s_k)$ if the state s_k at time k has value x is the same in all time slots and independent of the history h_k and, hence, is given by $u_x(a)$.

Now, using a policy u and starting from an initial state distribution β, the objective cost function (known as the total cost criterion) is defined as:

$C(\beta, u) = \sum_{k=1}^{\infty} E_{\beta}^{u}[c(s_k, r_k)]$ where:

- The pair (s_k, r_k) corresponds to the values of the state and the action taken at time k.
- $E_{\beta}^{u}[.]$ corresponds to the expectation over the policy u given that the initial distribution is β.

The cost functions related to the constraints are defined similarly as follows:

$$D_j(\beta, u) = \sum_{k=1}^{\infty} E_{\beta}^{u}[d_j(s_k, r_k)],\ j = 1, 2, ..., m$$

For a real vector $(V_1, ..., V_m)$, the Constrained Markov Decision Problem (CMDP) with total cost criterion can be stated as:

Find a policy $u \in U$ that minimizes $C(\beta, u)$ subject to $D_j(\beta, u) \leq V_j$, $j = 1, 2, ..., m$.

Now, we define the MDP pertaining to our problem.

- The state space S is the following finite set:

  $S = \{(0, G), (0, B), (1, G), (1, B)\}$

  where states $(1, G)$ and $(0, G)$ correspond respectively to the destination having received the packet successfully or not, when channel quality is good. Similarly, states $(1, B)$ and $(0, B)$ indicate respectively whether the destination has received the packet successfully or not when channel quality is bad.

- The action space is composed of the set $A = \{1, 2, ..., n\}$

  where the action $a = i$ $(i = 1, 2, ..., n)$ corresponds to the case when the transmitter decides to transmit with power P_i.

Also, we define the action sets pertaining to every state as follows:

$A(0, G) = A(0, B) = \{0, 1, ..., n\}$

$A(1, G) = A(1, B) = \phi$

- The transition probabilities between any two states in S for the case when action taken is $a = i$ are shown in figure 4.3.

- The immediate costs $c(x, a)$ correspond to the energy spent in each time slot and they are given by: follows:

  $c((0, G), i) = c((0, B), i) = P_i T$, $i = 1, 2, ..., n$.

  Note that the immediate costs $c(x, a)$ are used as part of the objective function in the energy minimization problem, and as part of the constraint function for the delay minimization problem.

- The immediate costs $d(x, a)$ correspond to an additional time slot spent to deliver the packet successfully.

  $d((0, G), i) = d((0, B), i) = 1$, $i = 1, 2, ..., n$.

  Note that the immediate costs $d(x, a)$ are used as part of the constraint function for the energy minimization problem, and as part of the objective function for the delay minimization problem.

- For this problem, each policy $u \in U$ is defined as follows: the source transmits with power value P_i with probability $q_{iB}(k)$ when the current state is $(0, B)$, and transmits with probability $q_{iG}(k)$ in time slot k if the current state is $(0, G)$.

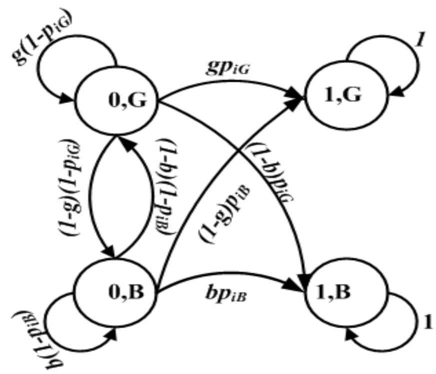


Figure 2.2: *Transition Probabilities when the action a=i.*

In the following, the conditions under which a stationary policy is an optimal solution for solving a CMDP are presented. Also, the Linear Programming approach that finds the optimal stationary policy is provided.

2.3.2.2 Linear Programming Approach

In [19], the authors show that stationary policies are optimal for solving CMDP with the total cost criterion under two conditions:

- The immediate costs $c(x, a)$ and $d_j(x, a)$ $(j = 1, 2, ..., m)$ are non negative.
- The MDP has the transient property i.e. for any initial state distribution β and for any policy $u \in U$, the state space S can be decomposed into two sets S' and M where:
  - Every state $x \in S'$ is transient i.e. the expected time to stay in state x is finite.
  - Every state $y \in M$ is absorbing i.e. any state $x \in S'$ is not reachable

once reaching any state $y \in M$.

Under these two conditions, the Constrained Markov Decision Problem (CMDP) is equivalent to solving the following linear program:

$Min_{\rho(x,a)} \sum_{x \in S} \sum_{a \in A(x)} c(x,a)\rho(x,a)$

Subject to:

$\sum_{x \in S} \sum_{a \in A(x)} d_j(x,a)\rho(x,a) \leq V_j,\ j = 1, 2, ..., m$

$\sum_{y \in S} \sum_{a \in A(y)} \rho(y,a)\delta_x(y) - P_{yax} I(x \in S')) = \beta(x) \quad \forall x \in S$

$\rho(x,a) \geq 0\ \forall x \in S, a \in A(x)$

where:

- $\beta(x)$ is the initial distribution of the state x
- $\delta_x(.)$ is the delta function centered at state x.
- $\rho(x,a)$ is the occupation measure i.e. the total expected time spent in state x when action a is chosen.

Also, according to [19], the stationary policy that minimizes the original CMDP is defined as follows: the probability $u_x(a)$ of choosing action $a \in A(s_k)$ if the current state s_k is $x \in S$ is given by:

$$u_x(a) = \rho(x,a)(\sum_{a \in A(x)} \rho(x,a))^{-1} \tag{2.13}$$

In our problem, the MDP (which is composed of finite state space and finite action space) has non negative immediate costs (since the costs are energy and delay costs). As for the transient property, the MDP is not transient if we allow the class of policies U to include all possible policies, since in this case there exist some policies in which the MDP is not transient. An example of such policies is the policy of always not transmitting with probability one. In this case, the packet will never be received successfully by the destination, and hence the state space can not be decomposed into transient states and absorbing states (states $(1, G)$ and $(1, B)$ are not reachable starting from states $(0, G)$ and $(0, B)$). One possible approach is to restrict the class of policies U to include only stationary policies but excluding the stationary policy of always not transmitting with probability 1). In this case, the optimal policy for the CMDP is stationary and the linear programming method finds the optimal solution. Including only stationary policies in the class of possible policies is somewhat restrictive and hence it is desirable to alter the class of policies U to include non-stationary policies as well. However, in order to guarantee that the MDP is transient we define each policy $u \in U$ as follows: The source transmits with power value P_i with probability $q_{iB}(k)$ when the current state is $(0, B)$, and transmits with probability $q_{iG}(k)$ in time slot k if the current state is $(0, G)$ where:

- $0 \leq q_{1G} < 1$
- $0 \leq q_{iG} \leq 1$, $i = 2, 3, ..., n$

The first condition states that the source doesn't transmit (i.e. with power $P_1 = 0$) with probability strictly less than one. Note that this mathematical restriction makes perfect sense from the practical point of view since the packet would never be received successfully. Under these conditions, the MDP is transient because for any policy $u \in U$, states $(0, G)$ and $(0, B)$ are transient and states $(1, B)$ or $(1, G)$ are absorbing . Hence, the stationary policy is optimal for the CMDP and the Linear Programming method finds the optimal solution. The importance of the second approach is that it shows that there exist a stationary policy that is optimal to the problem even if the class of possible policies is expanded to include non stationary policies.

For the MDP pertaining to the energy minimization problem, we get the following linear program:

$LP1:$

$Min_{\rho((0,G),i),\rho((0,B),i),i=1,2,...,n} \sum_{i=1}^{n} P_i T(\rho((0,G),i) + \rho((0,B),i))$

Subject to:

$\sum_{i=1}^{n} \rho((0,G),i) + \rho((0,B),i) \leq K$

$\sum_{i=1}^{n} \rho((0,G),i)(1 - P_{(0,G)i(0,G)}) + \rho((0,B),i)(-P_{(0,B)i(0,G)}) = \beta(0,G)$

$\sum_{i=1}^{n} \rho((0,G),i)(-P_{(0,G)i(0,B)}) + \rho((0,B),i)(1 - P_{(0,B)i(0,B)}) = \beta(0,B)$

$\rho((0,G),i) \geq 0 \; \rho((0,B),i) \geq 0, \forall i \in \{1,2,...,n\}$

Similarly, for this MDP we get the following linear program:

$LP2:$

$Min_{\rho((0,G),i),\rho((0,B),i),i=1,2,...,n} \sum_{i=1}^{n} \rho((0,G),i) + \rho((0,B),i)$

Subject to:

$\sum_{i=1}^{n} P_i T(\rho((0,G),i) + \rho((0,B),i)) \leq E$

$\sum_{i=1}^{n} \rho((0,G),i)(1 - P_{(0,G)i(0,G)}) + \rho((0,B),i)(-P_{(0,B)i(0,G)}) = \beta(0,G)$

$\sum_{i=1}^{n} \rho((0,G),i)(-P_{(0,G)i(0,B)}) + \rho((0,B),i)(1 - P_{(0,B)i(0,B)}) = \beta(0,B)$

$\rho((0,G),i) \geq 0 \; \rho((0,B),i) \geq 0, \forall i \in \{1,2,...,n\}$

Using the simplex method, each of the above linear programs can be solved. If the problem is feasible, the optimal values of $\rho((0,G),i)$ and $\rho((0,B),i)$ $(i = 1, 2, ..., n)$ are used according to equation 2.13 to find the values of the probabilities $u_{(0,G)}(i)$ and $u_{(0,B)}(i)$, and hence the probabilities q^*_{iG} and q^*_{iB} $(i = 1, 2, ..., n)$ are obtained (since $u_{(0,G)}(i) = q^*_{iG}$ and $u_{(0,B)}(i) = q^*_{iB}$).

The simplex method is an iterative procedure that initially selects a feasible solution to the linear program and tries to improve the solution in every step until the optimal solution is reached. Due to the iterative nature of the simplex method, it is not straightforward to find expression for the optimal policy (i.e. the optimal probabilities of selecting power values) and hence the optimal policy will be obtained through numerical computation.

2.4 Energy/Delay Minimization with Rate Control

2.4.1 Rate Control via Varying the Packet Size

2.4.1.1 Problem Formulation

Here, the transmitter is varying the transmission rate by varying the packet size. For a given rate value r_i, the corresponding packet size M_i is:

$$M_i = r_i T \tag{2.14}$$

where T is the time slot duration.

The random variables y_k and W_k are defined in the same way as in part 2.3.1.

Let the random variable z_k be the size of the packet sent at time slot k. It is given by:

$$z_k = r_i T W_{k-1} \textit{ with probability } q_i(k) \tag{2.15}$$

where $q_i(k)$ is the probability of selecting rate value r_i at time slot k.

Next, let the random variable v_k be the energy per bit spent in each time slot k; given by:

$$v_k = \frac{P}{r_i} W_{k-1} \tag{2.16}$$

Hence, the energy per bit spent to deliver the packet successfully is given by the following expression:

$$\xi(q_{iG}(k), q_{iB}(k)) = \sum_{k=1}^{\infty} v_k \tag{2.17}$$

Also, the number of time slots spent to deliver the packet successfully is given by:

$$D(q_{iG}(k), q_{iB}(k)) = \sum_{k=1}^{\infty} W_{k-1} \tag{2.18}$$

Hence, the energy minimization problem in this case is formulated as:

$Min_{q_{iG}(k),q_{iB}(k)} E[\xi(q_{iG}(k), q_{iB}(k))]$

Subject to:

$$E[D(q_{iG}(k), q_{iB}(k))] \leq K \tag{2.19}$$

Similarly, the delay minimization problem can be formulated in this case as:

$Min_{q_{iG}(k),q_{iB}(k)} E[D(q_{iG}(k), q_{iB}(k))]$

Subject to:

$$E[\xi(q_{iG}(k), q_{iB}(k))] \leq E \tag{2.20}$$

Both problems are formulated as Constrained Markov Decision Problems (CMDP) as follows.

2.4.1.2 Solution

First, we define the MDP arising from this problem.

- The state space is again the same set S (defined in part 2.3.1), where $S = \{(0, G), (0, B), (1, G), (1, B)\}$
- The action space is again composed of the set $A = \{1, 2, ..., n\}$ Where the action $a = i$ $(i = 1, 2, ..., n)$ corresponds to the case when the transmitter decides to transmit with rate r_i (i.e. the packet size used is M_i bits). Also, we define the action sets pertaining to every state:

  $A(0, G) = A(0, B) = \{0, 1, 2, ..., n\}$

  $A(1, G) = A(1, B) = \phi$
- Transition probabilities between any two states in S for the case when action taken is $a = i$ are the same as for the case of power control and are shown in Figure 4.3.

  The only difference is that in this case p_{iG} corresponds to the probability of success when rate r_i is selected for transmission and the channel has good quality and p_{iB} corresponds to the probability of success when rate r_i is selected when the channel has a bad quality.
- The immediate costs $c(x, a)$ correspond to the energy per bit spent in every time slot and they are defined as follows:

  $c((0, G), i) = c((0, B), i) = \frac{PT}{M_i}$
- In this problem, immediate costs $d(x, a)$ correspond to an additional time slot

spent to deliver the packet.

$d((0,G),i) = d((0,B),i) = 1,\ i = 1,2,...,n$

- For this problem, we define each policy $u \in U$ as follows: each policy u is a sequence $(u_1, u_2, ...)$ where the entry u_k assigns at time slot k the probabilities $q_{iG}(k)$ $(0 \leq q_{iG}(k) \leq 1)$ and $q_{iB}(k)$ $(0 \leq q_{iB}(k) \leq 1)$ for selecting each rate r_i when the current state is $(0,G)$ and $(0,B)$ respectively.

For this MDP, the immediate costs are nonnegative since they correspond to energy and delay costs. Also, the MDP is transient since for any policy u, states $(0,G)$ and $(0,B)$ are transient and states $(1,G)$ and $(1,B)$ are absorbing. This is because all rate values are strictly positive and hence under any policy there is a positive probability of moving from states $(0,G)$ and $(0,B)$ to states $(1,G)$ and $(1,B)$. Hence, there exist a stationary policy that is optimal for solving the CMDP, and hence the linear programming approach can be used to find the minimizing stationary policy.

Hence, the linear program for the energy minimization problem in this case is:

$LP3:$

$Min_{\rho((0,G),i),\rho((0,B),i),i=1,2,...,n} \sum_{i=1}^{n} \frac{PT}{M_i}(\rho((0,G),i) + \rho((0,B),i))$

Subject to:

$\sum_{i=1}^{n} \rho((0,G),i) + \rho((0,B),i) \leq K$

$\sum_{i=1}^{n} \rho((0,G),i)(1 - P_{(0,G)i(0,G)}) + \rho((0,B),i)(-P_{(0,B)i(0,G)}) = \beta(0,G)$

$\sum_{i=1}^{n} \rho((0,G),i)(-P_{(0,G)i(0,B)}) + \rho((0,B),i)(1 - P_{(0,B)i(0,B)}) = \beta(0,B)$

$\rho((0,G),i) \geq 0 \ \rho((0,B),i) \geq 0, \forall i \in \{1,2,...,n\}$

Similarly, the linear program corresponding to the delay minimization problem in this case is:

$LP4:$

$Min_{\rho((0,G),i),\rho((0,B),i),i=1,2,...,n} \sum_{i=1}^{n} \rho((0,G),i) + \rho((0,B),i)$

Subject to:

$\sum_{i=1}^{n} \frac{PT}{M_i} \rho((0,G),i) + \rho((0,B),i) \leq E$

$\sum_{i=1}^{n} \rho((0,G),i)(1 - P_{(0,G)i(0,G)}) + \rho((0,B),i)(-P_{(0,B)i(0,G)}) = \beta(0,G)$

$\sum_{i=1}^{n} \rho((0,G),i)(-P_{(0,G)i(0,B)}) + \rho((0,B),i)(1 - P_{(0,B)i(0,B)}) = \beta(0,B)$

$\rho((0,G),i) \geq 0 \ \rho((0,B),i) \geq 0, \forall i \in \{1,2,...,n\}$

The linear programs $LP3$ and $LP4$ can be solved using the Simplex method, and hence the optimum solution (using equation 2.13) can be obtained.

2.4.2 Rate Control via Varying the Time Slot Duration

2.4.2.1 Problem Formulation

Here, the transmitter is varying the transmission rate by varying the time slot duration. For a given rate value r_i, the corresponding time slot duration T_i is:

$$T_i = \frac{M}{r_i} \tag{2.21}$$

where M is the packet size. To formulate the minimization problems for this case, the following variables are defined.

The random variables y_k and W_k are defined in the same way as in part 2.3.1

Let the random variable l_k be the duration (in seconds) of the time slot k (based on the selected rate r_i at time slot k, $i = 1, 2, ..., n$).

It is given by:

$$l_k = T_i W_{k-1} \textit{ with probability } q_i(k) \tag{2.22}$$

where $q_i(k)$ is the probability of selecting rate value r_i at time slot k.

Next, we define the random variable a_k be the energy spent in each time slot k, that is:

$$a_k = PT_i W_{k-1} \textit{ with probability } q_i(k) \tag{2.23}$$

Hence, the energy per packet spent to deliver the packet successfully is given by:

$$\xi(q_{iG}(k), q_{iB}(k)) = \sum_{k=1}^{\infty} a_k \tag{2.24}$$

Also, the delay (i.e. the total duration in seconds to deliver the packet successfully) is given by:

$$D(q_{iG}(k), q_{iB}(k)) = \sum_{k=1}^{\infty} l_k \tag{2.25}$$

Hence, the energy minimization problem in this case is formulated as:

$Min_{q_{iG}(k), q_{iB}(k)} E[\xi(q_{iG}(k), q_{iB}(k))]$

Subject to:

$$E[D(q_{iG}(k), q_{iB}(k))] \leq K \tag{2.26}$$

Similarly, the delay minimization problem can be formulated as follows:

$Min_{q_{iG}(k), q_{iB}(k)} E[D(q_{iG}(k), q_{iB}(k))]$

Subject to:

$$E[\xi(q_{iG}(k), q_{iB}(k))] \leq E \tag{2.27}$$

These problems are also formulated as Constrained Markov Decision Processes (CMDP) as follows.

2.4.2.2 Solution

First, we define the MDP pertaining for this problem.

- The state space is again the set:

  $S = \{(0, G), (0, B), (1, G), (1, B)\}$

- The action space is again composed of the set $A = \{1, ..., n\}$

  Where the action $a = i$ ($i = 1, 2, ..., n$) corresponds to the case when the transmitter decides to transmit with rate r_i (i.e. the time slot duration is T_i).

  Also, we define the action sets pertaining to every state as follows:

  $A(0, G) = A(0, B) = \{0, 1, ..., n\}$

  $A(1, G) = A(1, B) = \phi$

- Transition probabilities between any two states S for the case when action taken is $a = i$ are the same as for the case of power control are shown in figure 2.

- The immediate costs $c(x, a)$ corresponding to the energy spent to transmit the packet in every time slot and they are defined as follows:

  $c((0, G), i) = c((0, B), i) = PT_i$, $i = 1, 2, ..., n$.

  Where T_i corresponds to the time slot duration when rate r_i is used.

- In this problem, immediate costs $d(x, a)$ corresponding to the duration of the current time slot (in seconds)

$d((0,G),i) = d((0,B),i) = T_i,\ i = 1,2,...,n.$

- For this problem, we define each policy $u \in U$ as follows: each policy u is a sequence $(u_1, u_2, ...)$ where the entry u_k assigns at time slot k the probabilities $q_{iG}(k)$ $(0 \le q_{iG}(k) \le 1)$ and $q_{iB}(k)$ $(0 \le q_{iB}(k) \le 1)$ to select each rate r_i when the current state is $(0,G)$ and $(0,B)$ respectively.

This MDP also satisfies the conditions stated in [19], and hence the Linear Programming approach will be used to find a minimizing stationary policy.

Thus, the linear program for the energy minimization problem in this case is:

$LP5:$

$Min_{\rho((0,G),i),\rho((0,B),i),i=1,2,...,n} \sum_{i=1}^{n} PT_i(\rho((0,G),i) + \rho((0,B),i))$

Subject to:

$\sum_{i=1}^{n} T_i(\rho((0,G),i) + \rho((0,B),i)) \le K$

$\sum_{i=1}^{n} \rho((0,G),i)(1 - P_{(0,G)i(0,G)}) + \rho((0,B),i)(-P_{(0,B)i(0,G)}) = \beta(0,G)$

$\sum_{i=1}^{n} \rho((0,G),i)(-P_{(0,G)i(0,B)}) + \rho((0,B),i)(1 - P_{(0,B)i(0,B)}) = \beta(0,B)$

$\rho((0,G),i) \ge 0\ \rho((0,B),i) \ge 0, \forall i \in \{1,2,...,n\}$

Similarly, the linear program corresponding to the delay minimization problem in this case is:

$LP6:$

$Min_{\rho((0,G),i),\rho((0,B),i,)i=1,2,...,n} \sum_{i=1}^{n} T_i(\rho((0,G),i) + \rho((0,B),i))$

Subject to:

$\sum_{i=1}^{n} PT_i(\rho((0,G),i) + \rho((0,B),i)) \le E$

$\sum_{i=1}^{n} \rho((0,G),i)(1 - P_{(0,G)i(0,G)}) + \rho((0,B),i)(-P_{(0,B)i(0,G)}) = \beta(0,G)$

$\sum_{i=1}^{n} \rho((0,G),i)(-P_{(0,G)i(0,B)}) + \rho((0,B),i)(1 - P_{(0,B)i(0,B)}) = \beta(0,B)$

$\rho((0,G),i) \geq 0 \ \rho((0,B),i) \geq 0 \ \forall i \in \{1,2,...,n\}$

Both $LP5$ and $LP6$ can be solved using the Simplex method, and hence the optimum solution can be obtained (using equation 2.13).

2.5 Numerical Results

2.5.1 Energy Minimization with Power Control

In order to investigate the effect of the availability of the Channel State Information on the minimum energy consumed to deliver the packet successfully when the channel is time varying, we compute the minimum energy consumed when the channel is time varying and modeled by two state Markov chain as described in the system model. We also compute the minimum energy obtained for the case when the channel is modeled to be time invariant and has an average quality compared to the quality of the time varying channel i.e. the value of the variance of the fading coefficient for the time invariant channel s_c is constant and equal to the average value of the variance under the Markovian channel model i.e.

$$s_c = \frac{g}{g+b} s_H + \frac{b}{g+b} s_L \tag{2.28}$$

The time invariant channel is a special case of the time varying channel (since the probability of success is the same over all time slots, the channel can be modeled by a Markov chain composed of one absorbing state). Hence, the proposed method for the time varying channel (based on the Constrained Markov Decision Problems) can still be used.

In this part, it is assumed in the following that power control is binary i.e. the source can transmit with power P or remain silent. (i.e. $n = 2$)

The following values for the power and the parameters of the time varying channel are considered:

$P = 100mW$, $T = 10msec$, $s_h = 20$, $s_L = 11$, $g = 0.2$, $b = 0.8$, $N_0 = 1W$, $r = 1bits/sec$ and assuming the starting state is $(0, G)$.

Based on the above parameters values, the probabilities of success when transmitting with power P when the channel has good and bad quality respectively are: $p_G = 0.6065$, $p_B = 0.4029$.

For the time invariant channel, the probability of packet successful reception is: $p = 0.4436$.

Table 2.1 shows the optimal probabilities q_G and q_B of transmitting with power P when the channel has good and bad quality for the cases when the channel is time varying, and the optimal probability q of transmitting with power P when the channel is time invariant. Figure 4.2 shows the minimum energy consumed (in millijoules) to deliver the packet successfully when the channel is time varying and when the channel is time invariant for different values of the maximum allowable delay (i.e. maximum value of expected number of time slots). For a delay constraint value of one time slot, the minimum energy has zero value because the problem is infeasible (i.e. the packet cannot be delivered successfully with the available delay constraint).

Table 2.1: The probabilities q_G, q_B, and q for different values of the maximum allowable delay

	Time Varying		Time Invariant
Max Delay	q_G	q_B	q
2	1.0000	0.8445	0.9007
3	1.0000	0.2341	0.7948
4	1.0000	0.0306	0.7112
5	0.9244	0.0000	0.6436
6	0.8936	0.0000	0.5876
7	0.8582	0.0000	0.5407

Based on the results, we observe that the minimum energy decreases as the value of the maximum allowable delay increases in the case when the channel is time varying. This is because as more time slots are available to transmit the packet successfully, there is a higher chance for the transmitter to exploit the time varying property of the channel by transmitting with high probability when the channel has good quality and with low probability when the channel has bad quality, this results in reducing the energy consumed which is lower than the energy consumed when the channel is time invariant, and this shows the performance improvement gained by using the Channel State Information. The values of probabilities of transmitting q_G and q_B with power P in table 2.1 verify this analysis. As for the time invariant channel, the decrease in the optimal probability of transmitting with power P is just due to the fact that as the value of the delay constraint increases, the transmitter

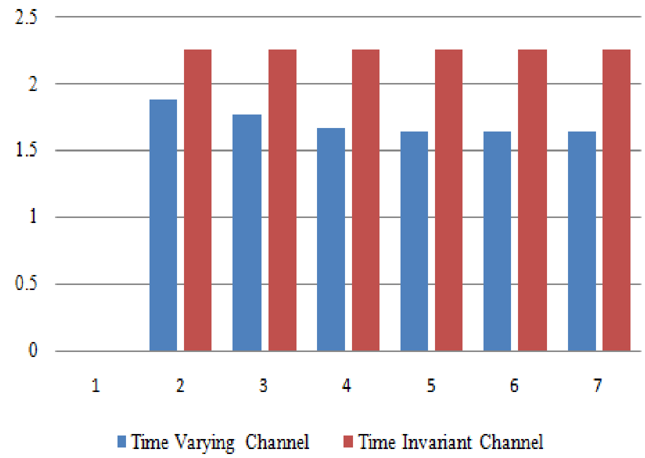


Figure 2.3: *Minimum Energy Consumed vs Maximum Allowable Delay*

can decrease the probability of transmitting with power P. However, this decrease does not have an effect on the minimum energy consumed.

Hence, based on the above values we can can reduce the minimum energy spent by 35% to 51% (depending on the target delay constraint) by knowing the Channel State Information of the time varying channel than the minimum energy spent for the case when the channel is modeled to be time invariant, which shows the advantage of using CSI when the channel is modeled as time varying.

2.5.2 Energy Minimization with Rate Control

The objective of this part are two. The first is to investigate the effect of the availability of the Channel State Information on the optimal rate values and on the minimum energy consumed to deliver the packet successfully when the channel is time varying. The second is to find out whether rate control by varying the packet size or rate control by varying the time slot duration achieve better performance.

Hence in order to make fair comparison, we compute the minimum energy consumed per bit and the delay is computed in seconds. Again, we consider both cases when the channel is time varying and time invariant. We assume in the following that rate control policy is binary i.e. the source can select the value of the rate between two values r_1 and r_2.

We consider the following values for the rates, power and the parameters of the channel: $r_1 = 5bits/sec$, $r_2 = 7bits/sec$, $P = 0.1W$, $g = 0.2$, $b = 0.8$, $s_1 = 300$, $s_2 = 200$, $N_0 = 1W$ For the rate control problem by varying the packet size, we compute the minimum energy obtained for each of the following values of the time slot duration: $T = 1$, 5, and 10 milliseconds.

For the rate control problem by varying the time slot duration, we compute the minimum energy obtained for each of the following values of the packet size: $M = 5$, 10 and 20 bits.

As for the starting state, we consider both cases when each of the states $(0, G)$ and $(0, B)$ is the starting state respectively.

Based on the results, we find that for the case of rate control by varying the time slot duration, the selected packet size value affects whether the problem is feasible or not. Similarly for the case of rate control by varying the packet size, the selected time slot duration affects whether the problem is feasible or not. This is because the values of the packet size and time slot duration directly affects the value of the delay but not the value of the energy per bit (it is rather determined by the selected rate value). However, once the problem is feasible, the optimum policy is deterministic and the transmitter decides to transmit with rate value $r_1 = 5$ bits/sec with probability one for the case when the channel has good quality and decides to transmit with rate value $r_2 = 7$ bits/sec when the channel has bad quality. As for the time invariant channel, the optimal rate value is 5 bits/sec.

Finally, table 2.4 shows the minimum energy spent per bit for the time invariant channel and for the time varying channel for both cases when the starting state is $(0, G)$ and $(0, B)$ respectively.

Table 2.2: The value of the packet size used versus the minimum value of the delay constraint at which the problem becomes feasible

M (bits)	Min Delay (Time Varying Channel)	Min Delay (Time Invariant Channel)
5	2	2
10	3	3
15	4	4

Table 2.3: The value of the time slot duration used versus the minimum value of the delay constraint at which the problem becomes feasible

T (msec)	Min Delay (Time Varying Channel)	Min Delay (Time Invariant Channel)
1	2	2
5	6	6
10	12	12

The values in table 2.4 show the effect of the starting state on the value of the mini-

Table 2.4: Minimum Energy Consumed per bit in MilliJoules

	Minimum Energy
Time Invariant	0.5685 millijoules
Time Varying/Starting state (0,G)	0.3143 millijoules
Time Varying/Starting state (0,B)	0.9163 millijoules

mum energy obtained, and hence in order to reduce the energy spent the transmitter should always start transmitting when the channel has good quality.

2.6 Summary

In this chapter, we have considered the problems of minimizing energy and delay spent to successfully deliver each packet over a time varying wireless link. We have assumed the availability of Channel State Information at the transmitter side. The problems are formulated as Constrained Markov Decision Problems, and a Linear Programming approach is provided to obtain the optimum solution. The results show the advantage of using Channel State Information; however, this advantage in some cases is dependent on the initial state conditions. Although the method is applied only over a simple wireless link, it provides a new modeling approach in optimizing performance metrics such as energy and delay for time varying wireless transmissions and it leads to exact solutions.

Chapter 3

Energy Constrained Real Time Wireless Multicasting

3.1 Overview

Similar to the previous chapter, this chapter addresses the tradeoff between energy and other important performance metrics which are throughput and delay but for a single-hop multicast network. In this chapter, we consider the problem of transmitting a file composed of a finite number of packets under energy and delay constraints over erasure channels. We are interested in finding the maximum throughput that can be achieved under such constraints in multicasting the packets by a transmitter to multiple receivers over independent time varying channels in a single hop network. This problem is motivated by the challenge of delivering real time applications that usually have strict delay constraints through wireless devices that are energy limited. Also, this problem captures the challenge in wireless multicast between reliable packet delivery to all receivers and the number of packets that can be so delivered within the specified energy and delay constraints.

Also in this chapter, we investigate the effect of using Random Network Coding (RNC) on the achieved performance. It is anticipated that RNC might result in performance improvement due to the following: Using traditional simple ARQ, the transmitter keeps transmitting each packet until it is received by every receiver. Thus, the number of packets delivered to every receiver will degrade considerably if there is only a small number of receivers with significantly worse channel than the other receivers in the system. With RNC, however, users with better channel quality can receive sooner enough linear combinations of the group of the transmitted packets to decode them, even if some receivers in the multicast session are experiencing poor channel quality.

The problem of optimizing data throughput under energy and delay constraints is considered in [20] and [21], but only for a unicast session. In [22], rate and power control techniques are considered for multiple multicast sessions to maximize the average throughput at every receiver, but without strict energy and delay constraints. In [23], the problem of power control is considered for a multicast session where a strict delay constraint is imposed on every packet, and dynamic programming is used to find the optimum power policy that maximizes the number of received packets by every receiver within the required deadline. However, none of the above approaches have considered the use of RNC and its effect on the achieved throughput. The advantages of RNC in multicasting have been amply demonstrated in [5], [6], and [7].

3.2 System Model

Consider a transmitter multicasting T packets to M receivers over a wireless single hop network. Time is slotted. The transmitter is required to deliver the T packets in N time slots and consuming no more than E units of energy. In addition, perfect channel feedback is assumed which means that acknowledgements from the receivers are guaranteed to reach the transmitter instantaneously and error-free. Also, the channel between the transmitter and each receiver is modeled as an erasure channel where the probability of successful reception for receiver i in time slot k is given by p_{ik}. We consider the case when the probability of successful reception is time varying and is modeled by a two state Markov chain. In this model, the channel changes between a good state and a bad state. Figure 3.1 shows the channel model where:

- p_{iG} is the probability of successful reception for receiver i when the channel is in the good state.
- p_{iB} is the probability of successful reception for receiver i when the channel is in the bad state.
- b_i is the transition probability of the channel at receiver i from the good to the bad state.
- g_i is the transition probability of the channel at receiver i from the bad to the good state.

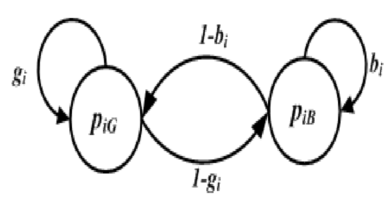


Figure 3.1: Markovian Channel Model

We also consider for comparison the case when the probability of successful reception for every receiver p_{ik} is constant in time and is given by the average value of p_{ik} under the Markovian model. The value of p_i is:

$$p_i = \frac{g}{g+b} p_{iG} + \frac{b}{g+b} p_{iB} \tag{3.1}$$

In this study, the power control policy is binary, which means that in every time slot the transmitter decides on transmitting the packet with maximum power P or not transmitting at all.

The two transmission schemes that will be considered are simple ARQ and RNC.

The objective is to find the optimal power control policy that maximizes the total number of packets successfully received by all the receivers, while satisfying the energy and delay constraints imposed on the T packets. However, the interdependence of the transmission times and energy usage from packet to packet, while the constraint applies only globally to the entire set of T packets, renders the problem truly formidable. Thus, we provide a suboptimal approach in which we distribute the global constraints among all individual packets for ARQ or groups of L packets for RNC. The details are described for the cases of ARQ and RNC in the following section.

3.3 Proposed Approach

3.3.1 ARQ Case

3.3.1.1 Problem Formulation

To simplify the problem, we translate the energy and delay constraints for the T packets into constraints for every delivered packet, and then solve the optimization problem for each packet individually. We realize that this is not optimal but it does simplify the otherwise formidably complex scheduling problem. In this approach, the transmitter should deliver the current packet t in N_t time slots and E_t amount of energy. The value of N_t is given by :

$$N_t \doteq \left\lceil \frac{N_r}{T_r} \right\rceil \tag{3.2}$$

where:

- $N_r = N - \sum_{k=1}^{t-1} n_k$ is the remaining number of time slots.
- n_k is the number of time slots consumed by the k^{th} packet.
- $T_r = T - (t-1)$ is the remaining number of packets in the system.

Similarly, E_t is given by:

$$E_t \doteq \left\lceil \frac{E_r}{T_r} \right\rceil \tag{3.3}$$

where:

- $E_r = E - \sum_{k=1}^{t-1} e_k$ is the remaining amount of energy.
- e_k is the amount of energy consumed by the k^{th} packet.

- $T_r = T - (t-1)$ is the remaining number of packets in the system.

Thus, the problem can be reduced to the problem of finding the optimum power control policy that maximizes the number of receivers who receive every packet individually. We start by solving the problem for the first packet. Then, the problem will be repeated for each of the T packets where the energy and delay constraints will be updated according to equations 3.2 and 3.3 respectively.

Since the energy expenditure for every packet t ($1 \leq t \leq T$) is constrained by the value of E_t and the maximum number of time slots required to deliver packet t by N_t, the problem is formulated as an optimization problem where the transmitter should decide on the optimal sequence of actions (whether to transmit with maximum power P or not, in every time slot) in order to maximize the expected number of receivers who receive the packet t within the time constraint of N_t time slots.

In this model, the energy expenditure during each time slot k ($1 \leq k \leq N_t$) is modeled by a variable u_k such that:

- $u_k = 1,$ if the sender is transmitting with maximum power P
- $u_k = 0,$ otherwise

In every time slot k, the variable $W_k = (w_{1k}, w_{2k}, ..., w_{Mk})$ is a random vector where every entry w_{ik} is a binary random variable that indicates whether receiver i ($1 \leq i \leq M$) sent an acknowledgement during time slot k. Each entry w_{ik} takes values as follows.

- If $u_k = 1$,

- $w_{ik} = 1$, if acknowledgement is received (with probability p_{ik})
- $w_{ik} = 0$, otherwise (with probability $1 - p_{ik}$)

- If $u_k = 0$, $w_{ik} = 0$

Also, we define the variable X_k= (x_{1k},x_{2k},...,x_{Mk}) to be another random vector where every entry x_{ik} is a binary random variable that indicates whether receiver i sends an acknowledgement up to and including time slot k. The variable x_{ik} takes the following assignment:

$$x_{ik} = x_{i(k-1)} \oplus w_{ik}$$

where $\oplus$ is the logical binary "or" operation.

Since the channel between the transmitter and each receiver i is modeled as a two state Markov chain, we define the variable Y_k to be a random vector where every entry y_{ik} takes the following values:

- $y_{ik} = 1$, if the channel between the transmitter and receiver i is in the good state

- $y_{ik} = 0$, if the channel between the transmitter and receiver i is in the bad state

The objective is to find the optimum energy allocation (u_1^*,...,$u_{N_t}^*$) that maximizes the expected number of receivers who receive packet t up to time slot N_t.

In other words, the problem is described by:

$$Max_{u_1,u_2,...,u_{N_t}} E\big[\textstyle\sum_{i=1}^{M}(x_{iN_t})\big]$$

Subject to:

$\sum_{k=1}^{N_t} u_k \leq E_t$

where E_t is the value of the maximum allowable energy to spend per packet t.

This problem can be solved using standard dynamic programming as follows.

3.3.1.2 Solution

The objective function (which is the maximum expected number of receivers who receive packet t successfully within N_t time slots under maximum energy consumed is E_t) is a function of the stochastic vector X_{N_t}. It is also dependent on the channel state Y_k ($1 \leq k \leq N_t$). Thus, we define $Z_k = (X_k, Y_k)$ to be a vector in $\{0,1\}^{2M}$ where the first M entries correspond to the entries of X_k and the remaining M entries corresponds to the entries of Y_k. Note that Z_1,Z_2,...,Z_{N_t} forms a Markov chain since both X_k and Y_k are Markov chains. Also, Z_1,Z_2,...,Z_{N_t} is a Markov chain that depends on chosen value of the variable u_k in time slot k. Then, the evolution of Z_i, $i = 1, ..., N_t$, is a Markov decision process, where each state S is a distinct vector in $\{0,1\}^{2M}$ (M is the number of receivers) and each state S has the form of $S_0S_1...S_{2M-1}$ where for $0 \leq i \leq M-1$, S_i is the entry bit that has value equal to one if receiver i received the packet successfully and zero otherwise, and for $M \leq i \leq 2M-1$, S_i is the entry bit that has value equal to one if the channel between the transmitter and receiver i is in the good state and zero otherwise. The transition probabilities $P_{SS'}^{u_k}$ from S to S' following the action u_k are given by:

- If $u_k = 0$,

  $P_{SS'}^{0} = \prod_{i=0}^{M-1} I(S'_i = S_i) \prod_{i=M}^{2M-1}[I(S_i = 0, S'_i = 1)(1 - b_i)$

$$+I(S_i = 1, S_i' = 0)(1 - g_i)$$

$$+I(S_i = 0, S_i' = 0)b_i$$

$$+I(S_i = 1, S_i' = 1)g_i]$$

- If $u_k = 1$,

$$P^1_{SS'} = \prod_{i=1}^{M-1}[I(S_i = 0, S_i' = 0)(1 - p_{ik}) + I(S_i = 0, S_i' = 1)p_{ik}$$

$$+I(S_i = 1, S_i' = 1)]$$

$$\prod_{i=M}^{2M-1}[I(S_i = 0, S_i' = 1)(1 - b_i) + I(S_i = 1, S_i' = 0)(1 - g_i)$$

$$+I(S_i = 0, S_i' = 0)b_i + I(S_i = 1, S_i' = 1)g_i]$$

where $I(.)$ is the indicator function.

Note that the value of p_{ik} depends on whether the channel between the transmitter and receiver i is in the good or bad state. Thus, the value of p_{ik} is obtained as follows:

- If $S_{i+M} = 0$, then $p_{ik} = p_{iB}$
- If $S_{i+M} = 1$, then $p_{ik} = p_{iG}$

Also, a reward $R(S, S')$ is associated with every transition from S to S' in every time slot. The reward is the number of receivers who receive the packet successfully in the current time slot; that is:

$$R(S, S') = \sum_{i=0}^{M-1} I(S_i = 0, S_i' = 1)$$

The objective is to maximize the expected sum of rewards up to time slot N_t. The expected sum of rewards is the expected number of receivers who successfully receive the packet up to time slot N_t. The problem is to find the optimum energy allocation u_k^* within N_t time slots subject to a constraint of maximum spent energy E_t to maximize the expected number of receivers that receive packet t successfully. It can be solved using dynamic programming.

The principle of dynamic programming says that if we consider the problem of finding the optimum energy allocation u_k^* within N_t time slots subject to a constraint of maximum spent energy is E_t to maximize the expected number of receivers who successfully receive the packet, then the sub problem must be solved of finding the optimum energy allocation within k time slots to maximize the expected number of receivers who successfully receive the packet subject to a constraint that the total energy spent is w, where $1 \leq k \leq N_t$ and $1 \leq w \leq E_t$.

Let $J_k(S, w)$ be the expected sum of rewards (expected number of receivers who receive packet t successfully) starting from time slot k and from state S and let the value of remaining energy be w; then $J_k(S, w)$ is defined recursively as follows:

- $If w = 0,$

  $J_k(S, 0) = J_{k-1}(S, 0) = 0$

- $If k = N_t + 1$

  $J_{N_t+1}(S, w) = 0, \qquad \forall S, w$

- $If w > 0, k < N_t + 1,$

  $J_k(S, w) = Max_{u_k} \sum_{allS'} P_{SS'}^{u_k}(J_{k+1}(S', w - u_k) + R(S, S'))$

where $J_0("0000..0", E_t)$ is the objective function since the system starts from the state where none of the receivers have the packet at time slot 0 and having E_t units of energy.

As with all discrete optimization problems, here too, the issue of complexity needs to be dealt with. Typically, heuristics are developed that are less complex or the action space is restricted and the complete optimization within the restricted set is carried out. Additionally, there are methods to convert the discrete problem to continuous formulations and convexify the optimization problem. The literature is replete with such techniques (see e.g. [25], [26] and [27]). However, such a line of investigation is outside the scope of this work.

3.3.2 RNC Case

3.3.2.1 Problem Formulation

In this case, the constraint is simplified by translating the energy and delay constraints for the group of the T packets into constraints for every sub-group of L packets (L is the coding parameter) and therefore solve the optimization problem for the current t^{th} batch of L packets, the delay constraints for the current t^{th} group of L packets will be:

$$N_t = \left\lceil \frac{N_r}{T_r} * L \right\rceil \tag{3.4}$$

where:

- $N_r = N - \sum_{k=1}^{t-1} n_k$ is the remaining number of time slots.

- n_k is the number of time slots consumed by the k^{th} group of L packets.

- $T_r = T - L \cdot (t-1)$ is the remaining number of packets in the system

and the corresponding energy constraint is:

$$E_t = \left\lceil \frac{E_r}{T_r} * L \right\rceil \tag{3.5}$$

where:

- E_r=$E - \sum_{k=1}^{t-1} e_k$ is the remaining amount of energy.

- e_k is the amount of energy consumed by the k^{th} group of L packets.

- $T_r = T - L \cdot (t-1)$ is the remaining number of packets in the system.

As in the case of ARQ, the energy expenditure during each time slot k is modeled by a variable u_k such that:

- $u_k = 1$, if the sender is transmitting with maximum power P

- $u_k = 0$, otherwise

Since RNC is considered in this case, each receiver i should keep a matrix that stores the coefficients of every received coded packet. Let the variable W_k be a random vector where every entry w_{ik} is a random variable which stores the current value of the rank of the matrix of receiver i. The random variable $w_{ik} \in \{0, 1, 2, ..., L\}$ where L is the number of coded packets. As we know from RNC, to successfully decode all L packets in a batch, the rank of the matrix of the coefficients of the successfully received packets must reach the value of L.

However, the number of linearly independent coded packets received by the receivers are correlated since the source is transmitting in every time slot the same random linear combinations to all the receivers.

Hence, in order to track the evolution of the number of linearly independent packets received by every receiver, we need to track the correlations between them. This is achieved by also tracking the number of linearly independent packets received by every subgroup of the receivers; however, the number of variables that we need to track is exponential function of the number of receivers. Hence from now on, we will restrict our analysis here to the case of two receivers.

For the case of two receiver, we will define the vector $W_k = (w_{1k}, w_{2k}, w_{12k})$ where the entries w_{1k} and w_{2k} be the number of linearly independent packets received by receivers 1 and 2 respectively up to time slot k. Also, the entry w_{12k} is the number of linearly independent packets that are received by both receivers.

At every time slot k, we define the variable $X_k = (x_{1k}, x_{2k})$ is a random vector where every entry x_{ik} is a random variable that indicates if receiver i ($i = 1, 2$) successfully decodes the t^{th} group of L coded packets during time slot k. Each entry x_{ik} takes the following assignment:

- $x_{ik} = 1, \quad \text{if } w_{ik} = L$

- $x_{ik} = 0, \quad \text{otherwise}$

Also, as in the case of ARQ, we define the variable Y_k to be a random vector where every entry y_{ik}, $(i = 1, 2)$ takes the following values:

- $y_{ik} = 1$, if the channel between the transmitter and receiver i is in the good state

- $y_{ik} = 0$, if the channel between the transmitter and receiver i is in the bad state

The objective is to find the optimum energy allocation $(u_1^*,..,u_{N_t}^*)$ that maximizes the number of receivers who are able to decode the current delivered L packets up to time slot N_t. In other words, the problem is formulated as follows:

$Max_{u_1,u_2,...,u_{N_t}} E[\sum_{i=1}^{2}(x_{iN_t})]$

Subject to:

$\sum_{k=1}^{N_t} u_k \leq E_t$

where E_t is the value of the maximum allowable energy to spend by the t^{th} batch of L coded packets.

This problem can also be solved using standard dynamic programming.

3.3.2.2 Solution

As in the case of ARQ, the objective function(which is the maximum expected number of receivers who successfully decode the t^{th} group of L coded packets successfully within N_t time slots under maximum energy consumed is E_t) is a function of the stochastic vector W_{N_t}. It is also dependent on the channel state Y_k $(1 \leq k \leq N_t)$. Thus, we also define $Z_k = (W_k, Y_k)$ to be a vector in $\{0,1\}^5$. Note that Z_1,Z_2,...,Z_{N_t} forms a Markov chain since both W_k and Y_k are Markov chains. Also, Z_1,Z_2,...,Z_{N_t}

is a Markov chain that depends on the chosen value for the variable u_k in time slot k. Then, the evolution of Z_i, $i = 1, ..., N_t$ is a Markov decision process where each state S is a distinct vector in $\{0,1\}^5$ and each state S has the form of $S_0 S_1 ... S_5$ where S_1, S_2 and S_3 are the number of linearly independent packets received by receivers 1, 2 and by both of them respectively. S_4 and S_5 indicates if the channel between the transmitter and receivers 1 and 2 respectively is in the good state.

The expressions for the transition probabilities $P_{SS'}^{u_k}$ from S to S', following the action u_k, are found in Appendix 3.4.

The reward function $R(S, S')$ is the number of receivers who can successfully decode the L coded packets in the current time slot. It is defined as follows.

$R(S, S') = \sum_{i=1}^{2} I(S_i = L - 1, S'_i = L)$.

The objective is to maximize the expected sum of rewards up to time slot N_t. The expected sum of rewards is the expected number of receivers who successfully decode the L coded packets up to time slot N_t. The problem of optimum energy allocation u_k^* within N_t time slots subject to a constraint of maximum energy spent is E_t to maximize the expected number of receivers who successfully decode the L coded packets can be solved using dynamic programming and following the same solution that is presented in the case of ARQ.

3.3.3 Numerical Evaluation

In this section, we illustrate the performance of the power control policy for both ARQ and RNC. The evaluation criteria are the expected number of packets received by each receiver. For the evaluation, we use T=6 packets, E=12 units, N=18 time slots and M=2 receivers. For RNC, we chose the coding parameter L=2,4 and 6 respectively. The alphabet size is chosen to be q =997. The evaluation is done for the cases when the channel between the transmitter and each receiver is modeled by the Markovian channel model as well as by time invariant channel model respectively. The parameters chosen for the Markovian channel model are:

For receiver 1: $p_{1G} = 0.8$, $p_{1B} = 0.4$, $b_1 = 0.2$, $g_1 = 0.8$

For receiver 2: $p_{2G} = 0.5$, $p_{2B} = 0.2$, $b_2 = 0.4$, $g_2 = 0.5$

The corresponding probability of successful reception of the packet in the case of the time-invariant channel for receiver 1 is p_1=0.72 and for receiver 2 is p_2=0.37 according to equation 3.1.

Figures 3.2 and 3.3 plot the expected number of packets received per time slot as a function of the coding parameter L for receivers 1 and 2 respectively. We assume that L=1 corresponds to the case when the transmitter is using ARQ. As shown, the expected number of packets received by receiver 1 increases as the coding parameter L increases; however for receiver 2, that has worse channel quality than receiver 1, the expected number of packets successfully received per time slot decreases as the coding parameter L increases.

Also, for both receivers 1 and 2, the performance is better when the channel is time varying than when it is constant. This is because the transmitter can exploit the statistics of the time varying channel and transmit more frequently when the channel has good quality.

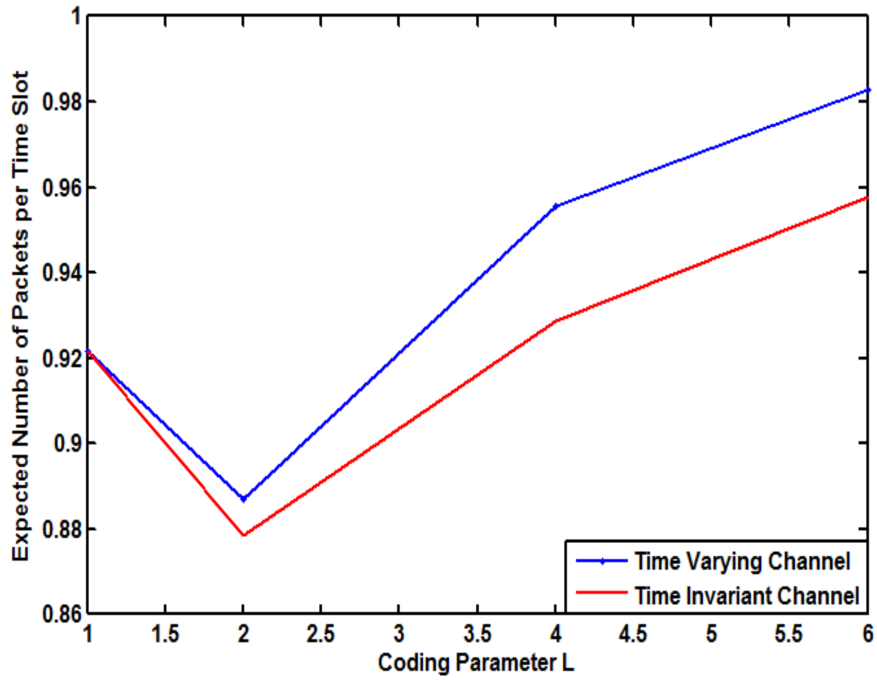


Figure 3.2: *Throughput at Receiver 1*

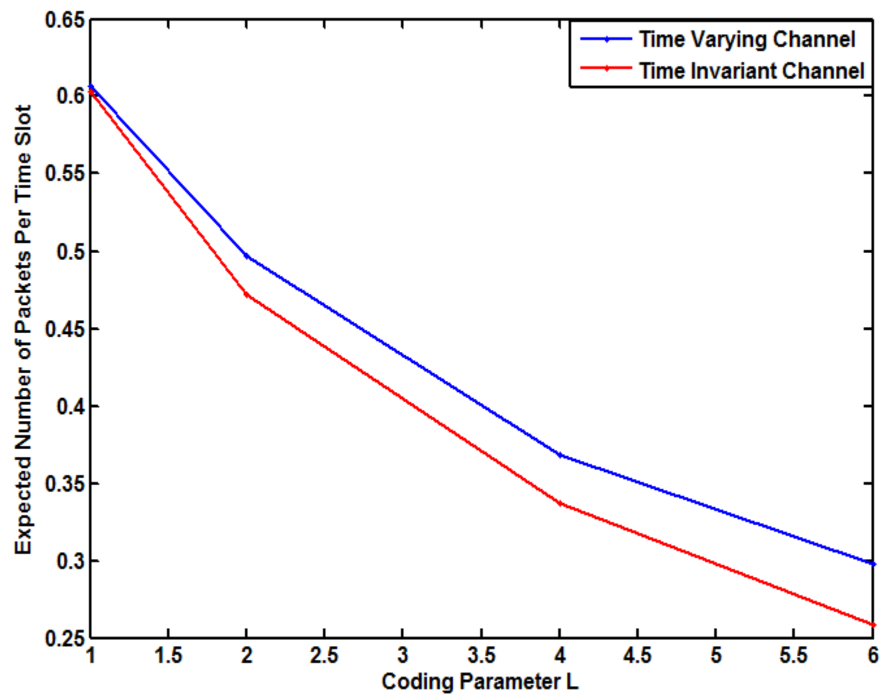


Figure 3.3: *Throughput at Receiver 2*

This is a remarkable illustration that RNC combat the effect of fading more successfully than ARQ for good channels but not for poor channels.

3.4 Summary

In this chapter, we have used dynamic programming to maximize the multicast throughput in a finite delay constraint and within an energy budget over a time varying channel. We have modeled the time varying channel as a two state markov chain where the channel switches between a good quality state and a bad quality state. We have considered two transmission schemes: ARQ and RNC. Our results show that the performance of the transmission schemes is dependent on the channel quality for every receiver. We have only considered single hop multicasting, and we simplified the constraint application. However, the solution is rigorous and exact and introduces the physical layer aspects of fading and channel variation (as it should) to the constrained multicast problem. It can serve as a spring board for extending it to multiple sources and more general multihop topologies.

3.5 Appendix: Transition Probabilities for the Markov Chain Model considered in section 3.2.2

- From state (i, j, k, l, m) to state (i, j, k, p, q)

  $(1 \leq i < L,\ 1 \leq j < L,\ 1 \leq k < min(i, j),\ 0 \leq l, m, p, q \leq 1\ )$

  $P^1_{(i,j,k,l,m),(i,j,k,p,q)} = p_{gb}((1 - p_1)(1 - p_2) + (1 - p_1)p_2u^{j-L})$

- From state (i, j, k, l, m) to state $(i + 1, j, k, p, q)$

$$P^1_{(i,j,k,l,m),(i+1,j,k,p,q)} = p_{gb}(p_1(1-p_2)(1-u^{i-L}) + p_1p_2(u^j - u^k)u^{-L})$$

- From state (i, j, k, l, m) to state $(i, j+1, k, p, q)$

$$P^1_{(i,j,k,l,m),(i,j+1,k,p,q)} = p_{gb}(p_2(1-p_1)(1-u^{j-L} + p_1p_2(u^i - u^k)u^{-L})$$

- From state (i, j, k, l, m) to state $(i+1, j+1, k+1, p, q)$

$$P^1_{(i,j,k,l,m),(i+1,j+1,k+1,p,q)} = p_{gb}(p_2p_1(1-u^i + u^j - u^k)u^{-L})$$

- From state (i, L, k, l, m) to state (i, L, k, p, q)

$$P^1_{(i,L,k,l,m),(i,L,k,p,q)} = p_{gb}((1-p_1) + p_1u^{i-L})$$

- From state (i, L, k, l, m) to state $(i+1, L, k+1, p, q)$

$$P^1_{(i,L,k,l,m),(i+1,L,k+1,p,q)} = p_{gb}(p_1(L-k)u^{-L}))$$

- From state (L, j, k, l, m) to state (L, j, k, p, q)

$$P^1_{(L,j,k,l,m),(L,j,k,p,q)} = P_{gb}((1-p_2) + p_2u^{j-L})$$

- From state (L, j, k, l, m) to state $(L, j+1, k, p, q)$

$$P^1_{(L,j,k,l,m),(L,j+1,k,p,q)} = p_{gb}(p_2(1-(u^j + L - k)u^{-L}))$$

- From state (L, j, k, l, m) to state $(L, j+1, k+1, p, q)$

$$P^1_{(L,j,k,l,m),(L,j+1,k+1,p,q)} = p_{gb}(p_2(L-k)u^{-L})$$

- From state (L, L, k, l, m) to state (L, L, k, p, q)

$$P^1_{(L,L,k,l,m),(L,L,k,p,q)} = p_{gb}$$

The probabilities p_{gb}, p_1, and p_2 are given by the following expressions:

$$p_{gb} = I(l=0, p=0)b_1 + I(l=0, p=1)(1-b_1) + I(l=1, p=1)g_1$$
$$+I(l=1, p=0) + (1-g_2)I(m=0, q=0)b_2 + I(m=0, q=1)(1-b_2)$$

$$+I(m=1,q=1)b_2+I(m=1,q=0)(1-g_2)$$

$$p_1 = I(l=1)p_{1G} + I(l=0)p_{1B}$$

$$p_2 = I(m=1)p_{2G} + I(m=0)p_{2B}$$

Chapter 4

The Effect of Cooperation and Network Coding on the Energy Efficiency of Wireless Transmissions

4.1 Overview

The objective of this chapter is to present different physical and network layer cooperative techniques for wireless fading transmissions and to evaluate their energy efficiency. Both user and relay cooperation are captured by considering two models for wireless transmissions. The first model considers transmissions over a wireless link, and a relay is used to assist the source node to deliver its data to the destination node. The second model considers multicast transmissions in which the source node is multicasting its data to two destinations. In this case, user cooperation is utilized i.e. the destination node that first receives the data successfully can assist the source in transmitting the data to the remaining destination. To evaluate the energy efficiency of each transmission scheme, the minimum energy will be computed by finding the optimal transmission powers. Then, the tradeoff between energy efficiency and the maximum stable throughput is studied by using the optimal transmission powers resulting from minimizing energy to compute the maximum stable throughput.

Early work has considered cooperation at the physical layer. One of the main techniques used in physical layer cooperation is the use of Space-time codes and in particular Alamouti code [28]. Some of the cooperative algorithms that use Alamouti code are studied in [29] and [30]. Then, due to the growing importance of energy efficiency, there has been recently much attention in finding which cooperative schemes are energy efficient. In [34], a wireless fading network consisting of a single source, a single destination and N relays is considered, and it is shown the tradeoff between decreasing the overhead of obtaining the Channel State Information (CSI) by using less relays and decreasing the energy consumption. In [35], energy efficient cooperative scheme is proposed in a wireless sensor network where the cooperating nodes employ Alamouti codes, and it is shown show that under certain distance ranges between the nodes, the energy of the cooperative scheme is reduced compared to non cooperative schemes.

Also, this chapter investigates the effect of using Random Network Coding on the energy efficiency of cooperative transmission. In [36] and [37], it is shown that cooperation using Network Coding increases the maximum stable throughput. More recent work has considered cooperative techniques that use Alamouti codes combined with Network Coding such as in [38] and [39] and evaluates their performance in terms of outage probability. However, there is no work that evaluates the use of Alamouti codes and Network Coding in terms of energy efficiency, which will be considered in this chapter.

4.2 Relay Cooperation in Single Link Wireless Transmission

4.2.1 System Model

Consider a wireless network as shown in figure 4.1. Packets arrive at the source according to a Bernoulli process with rate λ. Each packet is composed of N symbols (N is fixed for all packets). Time is slotted; each time slot corresponds to the transmission duration of a single packet. The nodes cannot send and receive at the same time. The channels between each pair of nodes are independent Rayleigh fading with constant fading level during each slot; however, the value of the fading level changes from one time slot to another. We denote by h_{it} the gain of channel i at every time slot t. The channel gains are independent Rayleigh distributed with pdf given by:

$$f_{h_{it}}(h) = \frac{2h}{s_i} e^{-\frac{h^2}{s_i}} \tag{4.1}$$

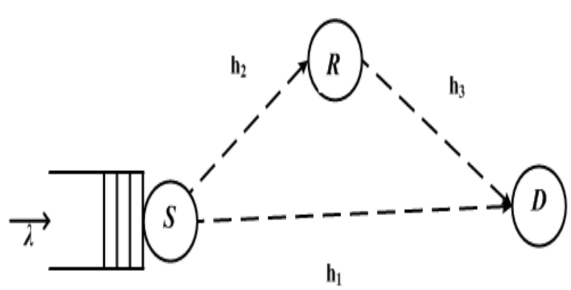


Figure 4.1: Schematic diagram that shows the system model

It is assumed that the network nodes do not have full Channel State Information instead they have only knowledge of the channels' statistics. It is also assumed that AWGN noise is present at each receiver. The packet erasure model is used i.e. the packet is received successfully with a probability; otherwise it is discarded. Due to the assumption of flat fading in each time slot, all the symbols of the packet are subject to the same level of fading (i.e. the value of the channel gain is the same during the transmission of all symbols), and hence the probability of successful transmission is given by the probability that the Signal to Noise Ratio (SNR) of the received symbols exceeds the threshold γ required at the receiving node and hence it is given by:

$$p_{success} = P(SNR \geq \gamma) \tag{4.2}$$

.The threshold γ depends on communication parameters such as the transmission rate, the target error probability, the modulation and coding scheme, etc. Although we could track the dependence of γ on these parameters, we choose for simplicity to consider a value of γ that may encompass all of these parameters. We denote by p_i the probability of packet successful transmission on channel i. These probabilities are constant in every time slot t. It is assumed that channel 3 has better quality than channel 1, and hence the probability p_1 has higher value than the probability p_3. The source S can use either:

- Simple Automatic Repeat Request (ARQ)
- Random Network Coding (RNC) .i.e. in every time slot, the transmitter selects randomly L coefficients from u-ary alphabet (where u is the alphabet size) and forms random linear combination of a group of L packets in its buffer and keeps transmitting random linear combinations of the same group of L packets in

every time slot. Once the destination receives L linearly independent random combinations, it sends an acknowledgement to the source.

Acknowledgements from the receivers are assumed to reach the transmitter instantaneously and error-free. At the relay, the Store and Forward Protocol is used i.e. the relay forwards a packet to the destination after it decodes it successfully. Also, the source and the relay transmit with powers P_1 and P_2 respectively where $P_i \in [0, P_{max}]$. To transmit the data, one of the following cooperation protocols are used.

4.2.1.1 Plain Relaying (PR) using ARQ

The source transmits each packet using ARQ until either the destination or the relay receives the packet. If the destination receives the packet successfully, transmission is completed and the source starts transmitting the next packet. If the relay successfully receives the packet before the destination, the relay transmits the packet using ARQ to the destination until the destination receives the packet successfully. Using this scheme, the received SNR values are given by:

- From the source to the destination:

$$SNR_{SD} = \frac{|h_{1t}|^2}{N_0} P_1 \tag{4.3}$$

- From the source to the relay:

$$SNR_{SR} = \frac{|h_{2t}|^2}{N_0} P_1 \tag{4.4}$$

- From the relay to the destination:

$$SNR_{RD} = \frac{|h_{3t}|^2}{N_0} P_2 \tag{4.5}$$

Based on the above expressions, the SNR variables are exponentially distributed with means $\frac{s_1 P_1}{N_0}$, $\frac{s_2 P_1}{N_0}$ and $\frac{s_3 P_2}{N_0}$ respectively, and hence the probabilities of success are given by:

- From the source to the destination:

$$p_1 = e^{\frac{-\gamma N_0}{s_1 P_1}} \tag{4.6}$$

- From the source to the relay:

$$p_2 = e^{\frac{-\gamma N_0}{s_2 P_1}} \tag{4.7}$$

- From the relay to the destination:

$$p_3 = e^{\frac{-\gamma N_0}{s_3 P_2}} \tag{4.8}$$

4.2.1.2 Relaying with Alamouti Coding (AC) using ARQ

The first stage of this protocol is similar to Plain Relaying i.e. the source transmits the packet using ARQ until either the destination or the relay receives the packet. If the relay receives the packet successfully before the destination, it forms an encoded packet by applying Alamouti Coding to every pair of consecutive symbols of the original packet.

After the relay forms the encoded packet, both source and relay transmit in the next time slot where the source transmits the original packet, and the relay transmits the encoded packet until the destination receives the packet successfully. In this case, perfect synchronization is assumed between the source and the relay. Although both the sender and the relay transmit simultaneously, they do not interfere with each other. This is because Alamouti Coding constructs a packet that is orthogonal to the original packet. Assuming channel estimation is performed at the receiver, the decoding process of the transmitted signals using Alamouti Coding is similar to Maximum Ratio Combining (MRC) as shown in [28]. Hence, the SNR at the destination in the cooperation phase is:

$$SNR_{AC} = \frac{|h_{1t}|^2 P_1 + |h_{3t}|^2 P_2}{N_0} \tag{4.9}$$

Based on the above expression, SNR_{AC} has a hypoexponential distribution with mean $\frac{s_1 P_1}{N_0} + \frac{s_3 P_2}{N_0}$. The probability p_{AC} is then given by:

$$p_{AC} = \frac{P_1 s_1 e^{-\frac{N_0 \gamma}{P_2 s_3}} - P_2 s_3 e^{-\frac{N_0 \gamma}{P_1 s_1}}}{P_2 s_3 - P_1 s_1} \tag{4.10}$$

The expressions for the Signal to Noise Ratios SNR_{SD} and SNR_{SR} and the probabilities of success p_1 and p_2 are as shown in equations 4.3, 4.4, 4.6, and 4.7 respectively.

4.2.1.3 Plain Relaying with Random Network Coding

In this case, the source transmits random linear combinations of every group of L packets until either the destination or the relay successfully decodes the L packets. If the destination decodes successfully the L packets before the relay, transmission is successful and the source starts transmitting the next group of L packets. If the relay successfully decodes the L packets before the destination, it starts transmitting the

L packets to the destination using RNC until the destination successfully decodes the L packets. The destination uses the previously successfully received random linear combinations directly from the source along with the new ones generated by the relay to perform its decoding. Using this scheme, the received SNR expressions and the expressions for the probabilities of success are identical to the case of Plain Relaying using ARQ but this time they apply to the coded packets.

4.2.1.4 Relaying using Alamouti Coding using Pseudo Random Network Coding

Under Alamouti Coding, in the cooperation phase, the relay transmits the Alamouti coded version of the packet transmitted by the source. So using Alamouti Coding in conventional Random Network Coding is not feasible because the source and the relaying node select independently different random linear combination in every time slot. Hence in order to be able to use Alamouti Coding with Random Network Coding, we assume that: the source starts transmitting random linear combinations of every group of L packets until either the relay or the destination decodes the L packets successfully. If the relay decodes the L packets before the destination, in every subsequent time slot the source forms a new random linear combination and sends the coefficients of the formed linear combination to the relay in order to form the same linear combination. Then, the relay forms the Alamouti coded version of the linear coded packet, and subsequently the source and the relay transmit simultaneously to the destination. This process is repeated in every time slot until the destination node decodes the L packets successfully. Again, perfect synchronization between the source and the relay is assumed. The SNR values and the success probabilities are again given by equations 4.3, 4.4, 4.9, 4.6, 4.7 and 4.10.

4.2.2 Energy Cost Functions

The distance between the nodes is considered sufficiently large to make the transmission energy the major contributor to the total energy consumed. Thus, the cost is defined as the expected transmission energy consumed per successfully transmitted packet. The cost expressions for each cooperation protocol are obtained as follows:

4.2.2.1 Plain Relaying(PR) using (ARQ)

In this case, the energy cost $C_{ARQ}(PR)$ is given by:

$$C_{ARQ}(PR) = E[\xi_{ARQ}(PR)] \tag{4.11}$$

where $\xi_{ARQ}(PR)$ is the energy spent per packet using Plain Relaying with ARQ, which is given by:

$$\xi_{ARQ}(PR) = \begin{cases} P_1 T_{SR} + P_2 T_{RD}, & T_{SR} < T_{SD} \\ P_1 T_{SD}, & otherwise \end{cases} \tag{4.12}$$

where T_{SR}, T_{SD}, and T_{RD} are the number of time slots needed for the successful transmission of the current delivered packet from source to relay, from source to destination, and from relay to destination respectively. Based on our assumption, the random variables T_{SD}, T_{SR}, and T_{RD} are geometrically distributed with parameters

p_1, p_2 and p_3 respectively. Hence,

$$\begin{aligned}
&\mathrm{E}[\xi_{ARQ}(PR)] \\
&= \Pr(T_{SR} < T_{SD}) \times (P_1 \times \mathrm{E}[T_{SR}|T_{SR} < T_{SD}] + P_2 \times \mathrm{E}[T_{RD}]) \\
&\quad + \Pr(T_{SR} \geq T_{SD}) \times (P_1 \times \mathrm{E}[T_{SD}|T_{SR} \geq T_{SD}])
\end{aligned} \tag{4.13}$$

The probability $\Pr(T_{SR} \geq T_{SD})$ is given by:

$$\Pr(T_{SR} \geq T_{SD}) = \frac{p_1}{1-(1-p_2)(1-p_1)} \tag{4.14}$$

Thus, we obtain the probability $P(T_{SR} < T_{SD})$ as:

$$\Pr(T_{SR} < T_{SD}) = 1 - P(T_{SR} \geq T_{SD}) \tag{4.15}$$

The expected value $E[T_{SD}|T_{SR} \geq T_{SD}]$ is then:

$$\begin{aligned}
E[T_{SD}|T_{SR} \geq T_{SD}] &= \frac{1}{p_1} - \frac{p_2(1-p_1)}{p_1(1-(1-p_1)(1-p_2))} \\
&\quad - \frac{p_2(1-p_1)}{(1-(1-p_1)(1-p_2))^2}
\end{aligned} \tag{4.16}$$

Also, the expected value $E[T_{SR}|T_{SR} < T_{SD}]$ is similarly derived as:

$$\begin{aligned}
E[T_{SR}|T_{SR} < T_{SD}] &= \frac{1-p_1}{p_2} - \frac{p_1}{p_2}\frac{(1-p_1)(1-p_2)}{1-(1-p_1)(1-p_2)} \\
&\quad - p_1\frac{(1-p_1)(1-p_2)}{(1-(1-p_1)(1-p_2))^2}
\end{aligned} \tag{4.17}$$

Since T_{RD} follows a geometric distribution with parameter p_3, its expected value $E(T_{RD})$ is given by:

$$E[T_{RD}] = \frac{1}{p_3} \tag{4.18}$$

4.2.2.2 Relaying with Alamouti Coding (AC) using ARQ

The cost is similarly given by:

$$C_{ARQ}(AC) = \mathrm{E}[\xi_{ARQ}(AC)] \tag{4.19}$$

where $\xi_{ARQ}(AC)$ is the energy spent per packet using Relaying with ARQ and AC, which is given by:

$$\xi_{ARQ}(AC) = \begin{cases} P_1 T_{SR} + (P_1 + P_2) T_{SRD}, & T_{SR} < T_{SD} \\ P_1 T_{SD}, & otherwise \end{cases} \tag{4.20}$$

where T_{SRD} is the number of time slots needed for the successful transmission of the current delivered packet simultaneously from the source and the relay using Alamouti Coding to the destination. T_{SRD} follows a geometric distribution with parameter p_{AC}. Hence,

$$\begin{aligned} &\mathrm{E}[\xi_{ARQ}(AC)] \\ &= \Pr(T_{SR} \geq T_{SD}) \times P_1 \times \mathrm{E}[T_{SD} | T_{SR} \geq T_{SD}] \\ &\quad + \Pr(T_{SR} < T_{SD}) \times \Big(P_1 \times \mathrm{E}[T_{SR} | T_{SR} < T_{SD}] \\ &\qquad + (P_1 + P_2) \times \mathrm{E}[T_{SRD}]\Big) \end{aligned} \tag{4.21}$$

The quantities $\Pr(T_{SR} \geq T_{SD})$, $\Pr(T_{SR} < T_{SD})$, $\mathrm{E}[T_{SR} | T_{SR} < T_{SD}]$, and $\mathrm{E}[T_{SD} | T_{SR} \geq T_{SD}]$ are given by equations 4.14, 4.15, 4.16, and 4.17. Since T_{SRD} is geometrically distributed, we have:

$$\mathrm{E}[T_{SRD}] = \frac{1}{p_{AC}} \tag{4.22}$$

4.2.2.3 Plain Relaying with RNC

Again, the cost is given by:

$$C_{RNC}(PR) = E[\xi_{RNC}(PR)] \tag{4.23}$$

where $\xi_{RNC}(PR)$ is the energy spent per successfully delivered packet using Plain Relaying with RNC and is given by:

$$\xi_{RNC}(PR) = \begin{cases} \dfrac{P_1 T_{SR} + P_2 T_{RD}}{L}, & T_{SR} < T_{SD} \\ \dfrac{P_1 T_{SD}}{L}, & otherwise \end{cases} \tag{4.24}$$

where T_{SR}, T_{SD}, and T_{RD} are the number of time slots needed for the successful transmission of the current L packets from the source to the relay, from the source to the destination, and from the relay to the destination respectively. The random variable T_{RD} depends on the random variable N_D which is the number of linearly independent packets received by the destination from source S transmission. Hence,

$$C_{RNC}(PR) = \frac{E_1 + E_2}{L} \tag{4.25}$$

where

$$\begin{aligned} E_1 &= \Pr(T_{SR} < T_{SD}) \times \\ &\quad \Big(\sum_{n=0}^{L-1} \Pr(N_D = n | T_{SR} < T_{SD}) \times \\ &\quad (P_1 \times \mathrm{E}[T_{SR} | T_{SR} < T_{SD}] + P_2 \times \mathrm{E}[T_{RD} | N_D = n])\Big) \end{aligned} \tag{4.26}$$

$$E_2 = \Pr(T_{SR} \geq T_{SD}) \times P_1 \times \mathrm{E}[T_{SD} | T_{SR} \geq T_{SD}] \tag{4.27}$$

In the case of RNC, T_{SR} and T_{SD} are correlated since source S transmits the same random linear combinations to both destinations, and hence the joint distribution function of T_{SR} and T_{SD} is dependent on N_R and N_D, the number of linearly independent packets received by the relay and the destination respectively from the source S. Thus, the derivation of probabilities and expected values become complicated. Hence, these computations are done through a Markov chain model that keeps track of the number of linearly independent coded packets received by the relay and the destination as well as the linearly independent packets received by both of them. The Markov chain is composed of the triplet $(L_1(k), L_2(k), L_c(k))$ where $L_1(k)$, $L_2(k)$, $L_c(k)$ are the number of linearly independent packets received by the relay, by the destination, and by both the relay and the destination respectively at time k. $(0 \leq L_1(k), L_2(k), L_c(k) \ \leq L)$

The transition probabilities for the Markov chain are presented in Appendix 4.5. The computation proceeds as follows:

- Computing $\Pr(T_{SR} < T_{SD})$ and $\mathrm{E}[T_{SR}|T_{SR} < T_{SD}]$: When $T_{SR} < T_{SD}$, the relay receives L linearly independent coded packets before the destination. This corresponds to first time passage from state $(0, 0, 0)$ to any state in set Q=$\{(L, j, k)$ where $0 \leq j < L$ and $0 \leq k \leq j$ $\}$ before the first time passage to any one of the states in set R=$\{$ (i, L, k) where $0 \leq i \leq L$ and $0 \leq k \leq i\}$. Now, the probability $\mathrm{P}(T_{SR} < T_{SD})$ is computed as follows:

$$\begin{aligned}
&\Pr(T_{SR} < T_{SD}) \\
&= \sum_{i=2}^{\infty} \Pr(T_{SR} < T_{SD}|T_{SD} = i) \times \Pr(T_{SD} = i) \\
&= \sum_{i=2}^{\infty} \sum_{j=1}^{i-1} f_{0Q}(j) \times f_{0R}(i)
\end{aligned} \tag{4.28}$$

where $f_{0Q}(j)$ is the probability of first passage from state 0 to either one of the states in the set Q at time j and $f_{0R}(i)$ is the probability of first passage from state 0 to either one of the states in the set R at time i. The expected value E$[T_{SR}|T_{SR} < T_{SD}]$ is computed as follows:

$$\begin{aligned} & E[T_{SR}|T_{SR} < T_{SD}] \\ & \quad = \sum_{i=0}^{\infty} E[T_{SR}|T_{SR} < i] \times Pr(T_{SD} = i) \\ & \quad = \sum_{i=2}^{\infty} \sum_{j=1}^{i-1} j f_{0Q}(j) \times f_{0R}(i) \end{aligned} \tag{4.29}$$

The expected values E$[T_{SD}|T_{SR} \geq T_{SD}]$ and $E[T_{RD}]$ can be computed in a similar way.

In order to obtain analytic expressions, we consider the special case when the alphabet size u is infinite, and when the probabilities of success p_1 and p_2 are equal to p. Given that the destination received successfully i linearly independent packets from the source, the probability that the newly received coded packet is linearly independent from the previously received linearly independent packets is $1 - u^{i-L}$. Hence as u goes to infinity, the probability becomes one, and the packet is linearly independent from the previously received packets. Thus, the number of packets received successfully by the relay and the destination are independent. The analytic expressions for the probabilities and expected values in this case are listed in Appendix 4.6.

4.2.2.4 Relaying using Alamouti Coding with Pseudo Random Network Coding

In this case, the cost is given by:

$$C_{RNC}(AC) = E[\xi_{RNC}(AC)] \tag{4.30}$$

where $\xi_{RNC}(AC)$ is the energy spent per successfully delivered packet using Alamouti Coding with pseudo RNC. It is given by:

$$\xi_{RNC}(AC) = \begin{cases} \dfrac{P_1 T_{SR} + (P_1 + P_2) T_{SRD}}{L}, & T_{SR} < T_{SD} \\ \dfrac{P_1 T_{SD}}{L}, & otherwise \end{cases} \tag{4.31}$$

where T_{SRD} is the number of time slots needed for the successful transmission of the current L packets from the simultaneous transmission of the source and the relay (using Alamouti Coding with pseudo RNC) to the destination. The random variable T_{SRD} depends on the random variable N_D which is the number of linearly independent packets received by the destination from source S transmission prior to the cooperation phase. Hence,

$$C_{RNC}(PR) = \frac{E_1 + E_2}{L} \tag{4.32}$$

where

$$\begin{aligned} E_1 \;=\; & \Pr\big(T_{SR} < T_{SD}\big) \times \\ & \Big(\sum_{n=0}^{L-1} \Pr(N_D = n | T_{SR} < T_{SD}) \times \\ & \Big(P_1 \times \mathrm{E}[T_{SR} | T_{SR} < T_{SD}] + (P_1 + P_2) \times \\ & \qquad \mathrm{E}[T_{SRD} | N_D = n]\Big)\Big) \end{aligned} \tag{4.33}$$

$$E_2 = \Pr(T_{SR} \geq T_{SD}) \times P_1 \times \mathrm{E}[T_{SD}|T_{SR} \geq T_{SD}] \quad (4.34)$$

Note that we do not distinguish between the energy needed to transmit the "payload" bits versus the "overhead" bits in each packet. The evaluation of these terms is similar to the one described in part 4.2.2.3. As in Plain relaying with RNC, when the alphabet size u goes to infinity and when the probabilities p_1, p_2, and p_3 are equal to p, the expressions for the cost functions terms have the expressions defined in Appendix 4.6.

4.2.3 Cost Optimization

The objective is now to find the optimal power values P_1^* and P_2^* for each of the cooperation strategies that minimize their corresponding cost and the conditions i.e.(channel characteristics, transmission scheme,etc) which performs better. Since the cost functions have complicated structures and in the case of RNC do not have closed form expression, numerical global optimization is performed. This is achieved by choosing closely spaced power values over the interval $[0, P_{max}]$. Then for every pair of values for the powers P_1 and P_2, the cost function for every cooperation scheme is computed based on the method presented in part 4.2.2. Finally, the power values which correspond to the lowest cost are selected. Based on the optimal power values, the maximum stable throughput achieved at the source for every cooperative protocol will be computed in the following section.

4.2.4 Stable Throughput Computation

We know [40] that for a single link system stability corresponds to:

$$\lambda_s < \mu_s \tag{4.35}$$

where λ_s is the arrival rate of the source and μ_s is the service rate. The service rate is given by the reciprocal of the expected completion time of the successful transmission of the current delivered packet when ARQ is used and is given by the ratio of the Network Coding parameter L over the completion time of the successful transmission of the current delivered L packets when RNC is used. The completion time for each of the three cooperation schemes is derived as follows:

4.2.4.1 Plain Relaying with ARQ

In this case, The completion time $T_{PR,ARQ}$ of successful delivery of a packet is given by:

$$T_{PR,ARQ} = \begin{cases} T_{SR} + T_{RD}, & T_{SR} < T_{SD} \\ T_{SD}, & otherwise \end{cases} \tag{4.36}$$

Hence, the expected completion time $\mathrm{E}[T_{PR,ARQ}]$ is given by:

$$\begin{aligned} & \mathrm{E}[T_{PR,ARQ}] \\ &= \Pr(T_{SR} < T_{SD}) \times \Big(\mathrm{E}[T_{SR}|T_{SR} < T_{SD}] + E[T_{RD}]\Big) \\ &\quad + \Pr(T_{SR} \geq T_{SD}) \times \mathrm{E}[T_{SD}|T_{SR} \geq T_{SD}] \end{aligned}$$

where the values of the above quantities are given by equations 4.14, 4.15, 4.16, 4.17, and 4.18.

4.2.4.2 Relaying with Alamouti Coding using ARQ

In this case, the completion time $T_{AC,ARQ}$ of successful delivery of a packet is given by:

$$T_{PR,ARQ} = \begin{cases} T_{SR} + T_{SRD}, & T_{SR} < T_{SD} \\ T_{SD}, & otherwise \end{cases} \tag{4.37}$$

The expected completion time $\mathrm{E}[T_{AC,ARQ}]$ is given by:

$$\begin{aligned} &\mathrm{E}[T_{AC,ARQ}] \\ &= \Pr(T_{SR} < T_{SD}) \times \Big(\mathrm{E}[T_{SR}|T_{SR} < T_{SD}] + \mathrm{E}[T_{SRD}]\Big) \\ &\quad + \Pr(T_{SR} \geq T_{SD}) \times \mathrm{E}[T_{SD}|T_{SR} \geq T_{SD}] \end{aligned} \tag{4.38}$$

The values of the above quantities are evaluated by equations 4.14, 4.15, 4.16, and 4.17, and 4.22.

4.2.4.3 Plain Relaying with Random Network Coding

Using the first Markov chain model used in the energy cost function for Plain Routing under Random Network Coding, the expected completion time $E[T_{PR,RNC}]$ of the successful delivery of the L coded packets is computed as the expected number of transitions before entering any of the absorbing states starting from state (0,0,0). The absorbing states correspond to completion of successful decoding of L packets. The expected completion time can be computed using a similar method for the case of relaying using Alamouti Coding with pseudo RNC.

4.2.5 Numerical Results

In this section, we present some numerical results that illustrate the effect of the channel conditions on the performance of each of the cooperation protocols described above.

Since there are three different channels in the network and to limit the number of variables in the analysis, we vary the channel quality between the source and the destination while fixing the quality of the remaining channels. Thus, the variance s_1 of the Rayleigh fading distribution between the source and the destination is varied between 40 and 50 dB (the higher the value, the better the channel quality) while the variances s_2 and s_3 of the other two Rayleigh fading distributions are kept fixed at 50 dB. The optimal cost for each cooperation protocol is computed for every considered value of s_1. These optimal costs are shown in figure 4.2. Also, the optimal power values obtained are used to compute the service rate for every cooperation protocol. The results are shown in figure 4.3.

In order to verify the validity of the analytic results, we simulate the process of packet loss. The simulation is performed through a Matlab program. For every value of the variance s_1 of the Rayleigh fading distribution between the source and the destination, the program takes as an input the optimal power values P_1^* and P_2^* obtained from the numerical global optimization performed in section 4.2 and substitutes the power values in equations 4.6, 4.7, 4.8 and 4.10 to compute the probabilities p_1, p_2, p_3 and p_{AC} respectively. Then, the program computes the energy spent per packet for each of the four cooperation protocols using the following procedure:

In the case of ARQ, two Bernoulli random variables with success probabilities p_1 and p_2 respectively are generated. If both of the two generated random numbers are zero, another two Bernoulli numbers are generated and the process repeats until one of the random numbers is one. Also, the program counts the number of time slots the source spends to deliver the packet to either the relay or the destination by counting the number of time the two Bernoulli numbers are zero. In the case when the first number is zero and the second number is one, a new Bernoulli number is generated with success probability p_3 in the case of plain relaying and p_{AC} in the case of Alamouti coding. If its value is zero, another Bernoulli random number is generated and the process repeats until the value of the random number is one. Finally, the value of the energy spent to deliver the packet successfully is computed for the case of plain relaying according to equation 4.13 and for the case of relaying with Alamouti coding according to equation 4.21.

In the case of RNC, a vector of L random numbers is generated from a discrete uniform random distribution with maximum u. The L random numbers correspond to the random coefficients of the coded packet. As in the case of ARQ, two Bernoulli random numbers with success probabilities p_1 and p_2 are generated. In case both numbers are zero, another two Bernoulli numbers are generated and the process repeats until either one of the random numbers is one. If the first random number is one, the vector of L random numbers is stored in a matrix M_D provided it is linear independent from the vectors previously stored in the matrix. Otherwise if the second random number is one, the vector of L random numbers is stored in a matrix M_R provided that it is linear independent from the vectors stored in the matrix. The whole process is repeated until the number of vectors stored in matrix M_D or in matrix M_R is L. During this process, the number of time slots that that source spends to deliver the packet to either the relay or the destination is obtained by counting the number of times a new vector of L random numbers is generated. In case the number of vectors stored in matrix M_R is L, a new Bernoulli number (with success probabilities p_3 and p_{AC} in cases of plain relaying and Alamouti Coding respectively) and a new vector of L uniformly distributed numbers are generated. This process is repeated until the value of the Bernoulli number is one. In this case, the vector is stored in matrix M_D if it is linear independent from the vectors stored in matrix M_D. The process repeats until the number of vectors in matrix M_D is L. During this process, the program obtains the number of time slots that the relay spends to deliver the packet to the destination by counting the number of times a new vector of L random numbers is generated. Finally, the value of the energy spent per successfully delivered packet is computed according to equations 4.25 and equation 4.32 for the cases of plain relaying and Alamouti Coding respectively. The simulation is repeated 100000 times for each cooperation protocol; and the corresponding average energy per successfully delivered packet is computed. The average energy is computed as:

$$E_{avg} = \frac{1}{N_s} \sum_{i=1}^{N_s} E_i \tag{4.39}$$

where N_s is the number of simulations, E_i is the energy consumed per successfully delivered packet during the i^{th} simulation, and E_{avg} is the average energy per

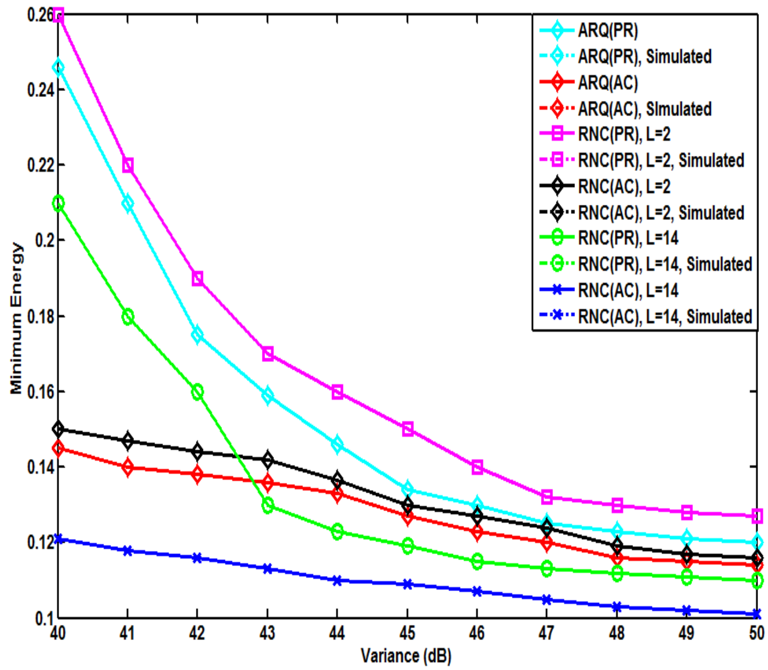


Figure 4.2: *Optimal cost for each cooperation scheme as a function of the variance*

successfully delivered packet over all the simulations. Also in each simulation, the completion time T_i (to deliver each packet in the case of ARQ, or to deliver the group of L packets in case of RNC) during the i^{th} step is computed leading to:

$$T_{avg} = \frac{1}{N_s} \sum_{i=1}^{N_s} T_i \tag{4.40}$$

The average service rate is then the the reciprocal of the average completion time T_{avg} in the case of ARQ, and the ratio of the number of linearly coded packets L over the average completion time T_{avg} in the case of RNC. These are shown in figures 4.2 and 4.3

Figures 4.2 and 4.3 first show that using Alamouti Coding with ARQ achieves higher service rate and consumes less energy per successfully transmitted packet compared to the case of ARQ with Plain Relaying; this is because under Alamouti

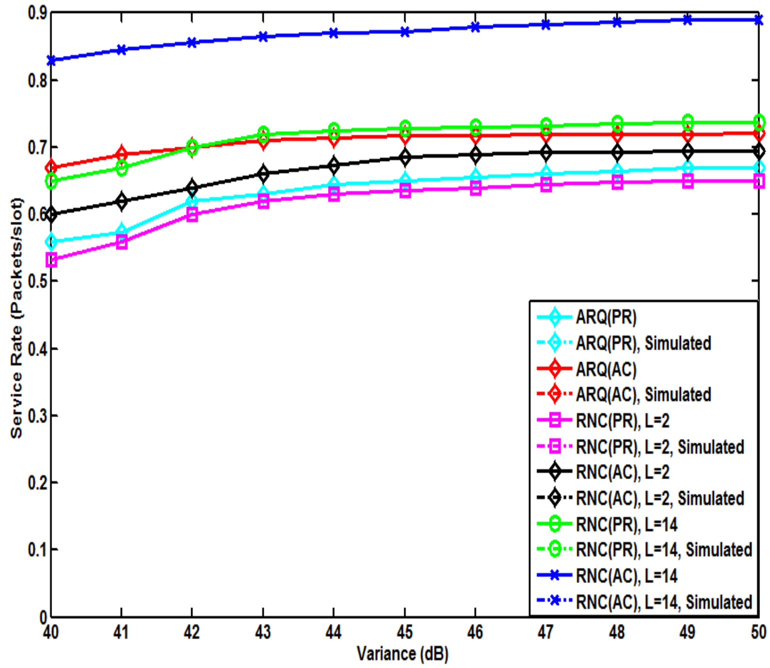


Figure 4.3: *Service rate for each cooperation scheme as a function of the variance*

Coding in the cooperation phase, the probability that the destination receives the packet successfully (from the simultaneous transmission of the source and the relay) is higher than the case of Plain Relaying with ARQ. Also, figures 4.2 and 4.3 show that as the Network Coding parameter L increases, the service rate increases and RNC becomes more energy efficient. This is because when the relay receives the packet successfully from the source's transmission under ARQ, it starts transmitting the packet again to the destination, while in the case of RNC even though the destination may not have successfully decoded the L packets (while the relay has successfully decoded the L packets), it may have successfully received linearly independent packets from the source, and thus the relay need not retransmit the L packets again but only sufficient additional random linear combinations of the currently delivered L packets until the destination. Thus, the total number of time slots required for successful delivered packet decreases, and the performance of RNC becomes better than ARQ combined with Alamouti Coding. Also, more energy reduction is observed under Alamouti Coding used with RNC. Finally, figures 3 and 4 show that simulation results confirm the results of the theoretical results. Similar conclusions can be drawn for different values used for the system parameters.

4.3 User Cooperation in a Simple Wireless Multicast Network

4.3.1 System Model

Consider source S multicasting packets to two destinations D_1 and D_2 as shown in figure 4.4. Time is slotted, and it is assumed that packets are always available at the source. The channels between the source and each destination D_i $(i = 1, 2)$ are independent Rayleigh fading with fading coefficient h_i $(i = 1, 2)$, and between both destinations are also Rayleigh fading with coefficient h_{ij} $(i, j = 1, 2$ and $j \neq i)$. The fading coefficient h_i is Rayleigh distributed with parameter s_i i.e. the pdf of h_i is given by:

$$f_{h_i} \doteq \frac{2h}{s_i} e^{\frac{-h^2}{s_i}} \tag{4.41}$$

Similarly, h_{ij} is Rayleigh distributed with parameter s_{ij} $(i, j = 1, 2$ and $j \neq i)$. Further, each of the channels is slowly fading i.e. the channel characteristics do not change within the duration of a time slot and AWGN noise is present at each destination D_i . Hence, the packet erasure model is appropriate; namely, the probability of successful transmission is given by the probability that the Signal to

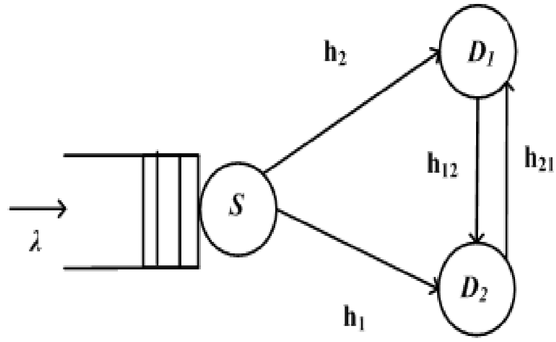


Figure 4.4: Schematic diagram that shows the system model

Noise Ratio (SNR) exceeds the threshold γ required at the destination and it is given by:

$$p_{success} = P(SNR \geq \gamma) \tag{4.42}$$

The Signal to Noise Ratio (SNR_i) at destination D_i is given by:

$$SNR_i = \frac{|h_i|^2 P}{N_0} \tag{4.43}$$

where P is the value of the power used by the transmitting node.

We denote by p_i ($i = 1, 2$) the probability of successful transmission by source S to destinations D_i, and by p_{ij} the probability of successful transmission from destination D_i to Destination D_j ($i = 1, 2$ and $j \neq i$).

In every time slot, source S can either:

- Multicast a single packet to both destinations using simple Automatic Repeat Request (ARQ)
- Multicast a group of L packets using Random Network Coding (RNC).

Also, each of the nodes can transmit with power $P \in [0, P_{max}]$ where P_{max} is the maximum allowable power for transmission. We define P_s to be the transmission power value of source S and P_i be the transmission power value of destination D_i.

4.3.2 Transmission Strategies

To transmit the packets reliably from the source to both destinations D_1 and D_2. We consider the following transmission protocols.

4.3.2.1 Plain Relaying Using ARQ

The source transmits each packet until either of destinations D_1 or D_2 receive the packet. If both destinations receive the packet at the same time slot, transmission is successful, and the source starts transmitting the next packet. If only one of the destinations receive the packet successfully, this destination transmits the packet using ARQ to the remaining destination. Using this scheme, the received SNR values are given by:

- From the source to destination D_1

$$SNR_1 = \frac{|h_1|^2}{N_0} P_s \tag{4.44}$$

- From the source to destination D_2

$$SNR_2 = \frac{|h_2|^2}{N_0} P_s \tag{4.45}$$

- From destination D_i to destination D_j $(i, j \in \{1, 2\}, i \neq j)$

$$SNR_{ij} = \frac{|h_{ij}|^2}{N_0} P_i \tag{4.46}$$

where N_0 is the power spectral density of the AWGN at both destinations D_1 and D_2.

4.3.2.2 Relaying with Alamouti Coding (AC) using ARQ

The source transmits the packet using ARQ until either of the destinations receives the packet. If both destinations receive the packet at the same time slot, transmission is successful, and the source starts transmitting a new packet. If only one of the destinations receive the packet successfully, it forms an encoded packet by applying Alamouti Coding to every pair of consecutive symbols of the original packet. Then, both the source and this destination transmit in the next time slot where the source transmits the original packet, and the destination transmits the encoded packet until the remaining destination receives the packet successfully.

The received Signal to Noise at destinations D_1 and D_2 when the source is transmitting in the non cooperative phase are the same as the expressions given by equations and

During the cooperation phase, the Signal to Noise ratio at destination D_1 is:

$$SNR_{AC} = \frac{|h_1|^2 P_s + |h_{21}|^2 P_2}{N_0} \tag{4.47}$$

The Signal to Noise ratio at destination D_2 in the cooperation phase is:

$$SNR_{AC} = \frac{|h_2|^2 P_s + |h_{12}|^2 P_1}{N_0} \tag{4.48}$$

4.3.2.3 Plain Relaying with Random Network Coding

In this case, the source transmits random linear combinations of every group of L packets (L is determined prior to transmission) until either of the destinations decode the L packets. If both destinations decode successfully the L packets at the same time slot, transmission is successful and the source starts transmitting the next group of L packets. If only one of the destinations successfully decode the L packets, it starts transmitting the L packets to the remaining destination using RNC until the remaining destination successfully decodes the L packets. The remaining destination retains the coded packets that were received successfully from the source's transmissions.

The received SNR expressions are identical to the case of Plain Relaying using ARQ but this time they apply to the coded packets.

4.3.2.4 Relaying using Alamouti Coding with Pseudo Random Random Network Coding

Similar to the case of wireless unicast transmission, we will propose a scheme that combine Alamouti Coding with Random Network Coding. The scheme works as follows: The source starts transmitting random linear combinations of every group of L packets until one of the destinations decode the L packets successfully. If both destinations decode the L packets in the same time slot, transmission is successful, and the source starts transmitting the next group of L packets. If only one of the destinations decodes successfully the L packets, in every subsequent time slot the source forms a new random linear combination and sends the coefficients to this destination node in order to form the same linear combination. Then, the destination forms the Alamouti coded version of the packet, and subsequently the source and the transmitting destination node transmit simultaneously to the remaining destination. This process is repeated until the remaining destination decodes the L packets successfully. In this case, the SNR expressions are the same as in the case of relaying using Alamouti Coding with ARQ.

4.3.2.5 No Cooperation

The source keeps transmitting until both destinations receive the data. (i.e. the individual packet in the case of ARQ or all L packets in the case of RNC). This case is used as baseline comparison and to assess under what conditions user cooperation achieve performance improvement.

The following section defines the energy cost used to evaluate each of the cooperation protocols. It also presents the method of minimizing the energy cost for each of the considered protocols.

4.3.3 Cost Functions

The cost associated with each cooperation/transmission scheme pair is defined as follows:

4.3.3.1 No Cooperation Using ARQ

Using ARQ, the cost is defined as the expected energy spent per successfully delivered packet.

For the case of no cooperation with ARQ, the cost is:

$$C_{ARQ}(NC) = E[\xi_{ARQ}(NC)] \tag{4.49}$$

where $\xi_{ARQ}(NC)$ is the energy spent per successfully delivered packet using ARQ when no coding is used. It is given by:

$$\xi_{ARQ}(NC) = P_s T_{max} \tag{4.50}$$

where T_{max} is the time required for successful transmission of the current delivered packet using ARQ to both destinations and is given, in turn, by

$$T_{max} = max(T_1, T_2) \tag{4.51}$$

where T_i is the number of time slots for source S to successfully transmit the current delivered packet to destination D_i. Using ARQ, T_i is a random variable that follows a geometric distribution with parameter p_i i.e. $T_i \sim geom(p_i)$

Hence,

$$C_{ARQ}(NC) = E[P_s T_{max}] = P_s E[T_{max}] \tag{4.52}$$

4.3.3.2 Plain Relaying with ARQ

In this case, the cost is:

$$C_{ARQ}(PR) = E[\xi_{ARQ}(PR)] \tag{4.53}$$

where $\xi_{ARQ}(PR)$ is the energy spent per successfully delivered packet using ARQ with Strategy 2. It is given by:

$$\xi_{ARQ}(PR) = \begin{cases} P_sT_1 + P_1T_{12}, & T_1 < T_2 \\ P_sT_2 + P_2T_{21}, & T_2 < T_1 \\ P_sT, & T_2 = T_1 \end{cases} \tag{4.54}$$

where

- T_{ij} is the number of time slots needed for the successful transmission of the current delivered packet from destination D_i to destination D_j ($i, j = 1, 2$ and $j \neq i$).
- T is the number of time slots needed for successful transmission to both destinations knowing that both destinations receive the packet at the same time slot.

Hence,

$$\begin{aligned} & E[\xi_{ARQ}(PR)] \\ = \;& Pr(T_1 < T_2) \times (P_s \times E[T_1|T_1 < T_2] + P_1 \times E[T_{12}]) \\ & + Pr(T_2 < T_1) \times (P_s \times E[T_2|T_2 < T_1] + P_2 \times E[T_{21}]) \\ & + Pr(T_1 = T_2) \times P_s \times E[T|T_1 = T_2] \end{aligned} \tag{4.55}$$

4.3.3.3 Relaying with Alamouti Coding (AC) using ARQ

In this section, the cost is:

$$C_{ARQ}(AC) = E[\xi_{ARQ}(AC)] \tag{4.56}$$

where $\xi_{ARQ}(AC)$ is the energy spent per successfully delivered packet using ARQ with Strategy 2. It is given by:

$$\xi_{ARQ}(AC) = \begin{cases} P_s T_1 + P_1 T_{s12}, & T_1 < T_2 \\ P_s T_2 + P_2 T_{s21}, & T_2 < T_1 \\ P_s T, & T_2 = T_1 \end{cases} \tag{4.57}$$

where T_{sij} is the number of time slots needed for the successful transmission of the current delivered packet from the simultaneous transmission and destination D_i (using Alamouti coding) to destination D_j ($i, j = 1, 2$ and $j \neq i$).

Hence,

$$\begin{aligned} & E[\xi_{ARQ}(AC)] \\ = \quad & Pr(T_1 < T_2) \times (P_s \times E[T_1|T_1 < T_2] + P_1 \times E[T_{s12}]) \\ & +Pr(T_2 < T_1) \times (P_s \times E[T_2|T_2 < T_1] + P_2 \times E[T_{s21}]) \\ & +Pr(T_1 = T_2) \times P_s \times E[T|T_1 = T_2] \end{aligned} \tag{4.58}$$

The analytic expressions for the probabilities and expected values terms in the above cost functions are presented in Appendix 4.7.

4.3.3.4 No Cooperation using RNC

The cost is:

$$C_{RNC}(NC) = E[\xi_{RNC}(NC)] \tag{4.59}$$

where $\xi_{RNC}(NC)$ is the energy spent per packet using RNC when no cooperation. It is given by:

$$\xi_{RNC}(NC) = \frac{P_s T_{max}}{L} \tag{4.60}$$

where T_{max} is the time required for successful transmission of the current delivered L packets using RNC to both destinations. Hence,

$$C_{RNC}(NC) = \frac{P_s E[T_{max}]}{L} \tag{4.61}$$

4.3.3.5 Plain Relaying with RNC

The cost is:

$$C_{RNC}(PR) = E[\xi_{RNC}(PR)] \tag{4.62}$$

where $\xi_{RNC}(PR)$ is the energy spent per packet using RNC when plain relaying is used. It is given by:

$$\xi_{RNC}(PR) = \begin{cases} \dfrac{P_s T_1 + P_1 T_{12}(n)}{L}, & T_1 < T_2 \\ \dfrac{P_s T_2 + P_2 T_{21}(n)}{L}, & T_2 < T_1 \\ \dfrac{P_s T}{L}, & T_2 = T_1 \end{cases} \tag{4.63}$$

where

- $T_{ij}(n)$ is the number of time slots needed for the successful transmission of the current L packets from destination D_i to destination D_j ($i = 1, 2$ and $j \neq i$) knowing that destination D_j has received n linearly independent combinations of the L packets from source S, where $0 \leq n < L$.

- T is the number of time slots needed for successful transmission of the current L packets to both destinations if both destinations successfully decode the L packets in the same time slot. (i.e. they receive successfully the L^{th} linearly independent combination in the same slot).

Hence, the cost is given by:

$$C_{RNC}(PR) = \frac{E_1 + E_2 + E_3}{L} \tag{4.64}$$

where

$$\begin{aligned}
E_1 &= Pr(T_1 < T_2) \times \Big(\sum_{n=0}^{L-1} Pr(N_2 = n | T_1 < T_2) \\
&\quad \times (P_s \times E[T_1 | T_1 < T_2] + P_1 \times E[T_{12}(n)])\Big) \\
E_2 &= Pr(T_2 < T_1) \times \Big(\sum_{n=0}^{L-1} Pr(N_1 = n | T_2 < T_1) \\
&\times \quad (P_s \times E[T_2 | T_2 < T_1] + P_2 \times E[T_{21}(n)])\Big) \\
E_3 &= Pr(T_1 = T_2) \times P_s \times E[T | T_1 = T_2]
\end{aligned}$$

As explained in the problem of relay cooperation over a single link, T_1 and T_2 are correlated since source S transmits the same random linear combinations to both destinations,and hence the joint distribution function of T_1 and T_2 is dependent on N_1 and N_2, the number of linearly independent packets received by destinations D_1 and D_2 respectively. Thus similar to the approach in part 4.2.3, the computation of each of these probabilities and expected values is done through a Markov chain model that keeps track of the number of linearly independent coded packets received by every destination as well as the linearly independent packets received by both of them.

4.3.4 Cost Optimization

After obtaining the expressions of the cost functions as described in the preceding section the different transmission protocols, the objective is to find the optimum power values P_s, P_1, and P_2 for each of the protocols that minimize their corresponding cost and to find under what conditions(channel characteristics, transmission scheme) each performs better. However, since the cost functions have complicated structures and in the case of RNC do not have closed form expression, the same method of optimization is performed as in the problem of relay cooperation over a single link i.e. optimization is performed by generating dense vectors of power values over the interval $[0, Pmax]$. Then for every power value, the cost function for each protocol is computed. Finally, the power values which correspond to the lowest cost are selected.

4.3.5 Numerical Results

In this section, we will investigate the effect of the channel conditions on each of the cooperation protocols. Hence, the channel qualities between the destinations D_1 and D_2 are varied simultaneously while fixing the quality of the remaining channels. The channel qualities between the destinations D_1 and D_2 are varied by varying simultaneously the values of the variances s_{12} and s_{21} for the Rayleigh distribution of the channel between destination D_1 to destination D_2 and the channel between destination D_2 to D_1. The values of the variances are varied between 40 and 50 dB while the variance values of the Rayleigh distribution of the channel between the source and destinations D_1 and D_2 (corresponding to the channels between the source and the destinations D_1 and D_2) are kept fixed at 45 dB, and the minimum energy for each cooperation protocol is computed for every considered value of s_{12}

and s_{21}. Then, the optimal costs for every transmission scheme as a function of the value of the variances are computed and are shown in figure 4.5. Also similar to the relay cooperation case, the optimal power values obtained are used to compute the service rate for every protocol. The results are shown in figure 4.6.

Figures 4.5 and 4.6 show as in the previous problem using Alamouti Coding with ARQ achieves higher service rate and consumes less energy per successfully transmitted packet compared to the other strategies. As the channel quality between the destinations becomes higher and as the Network Coding parameter L increases, the service rate increases and using RNC becomes more energy efficient even than the case when ARQ is used with Alamouti Coding. Also in the case of wireless multicast, for certain values of the coding parameter L, Random Network Coding combined with Alamouti coding achieves the best performance. The results were always verified by the simulation setup similar to the one described in section 4.2; however, the simulation curves are removed for the clarity of the figure.

4.4 Summary

We have considered several joint physical/network layer cooperative schemes that use either Automatic Repeat Request(ARQ) or Random Network Coding(RNC). For each of the proposed protocols, we have obtained the energy needed for successful packet delivery and the optimum power values that minimize that energy. Based on the optimal power values, we have obtained the stable throughput achieved at the source. We find that for certain values of the Network Coding parameter L, coop-

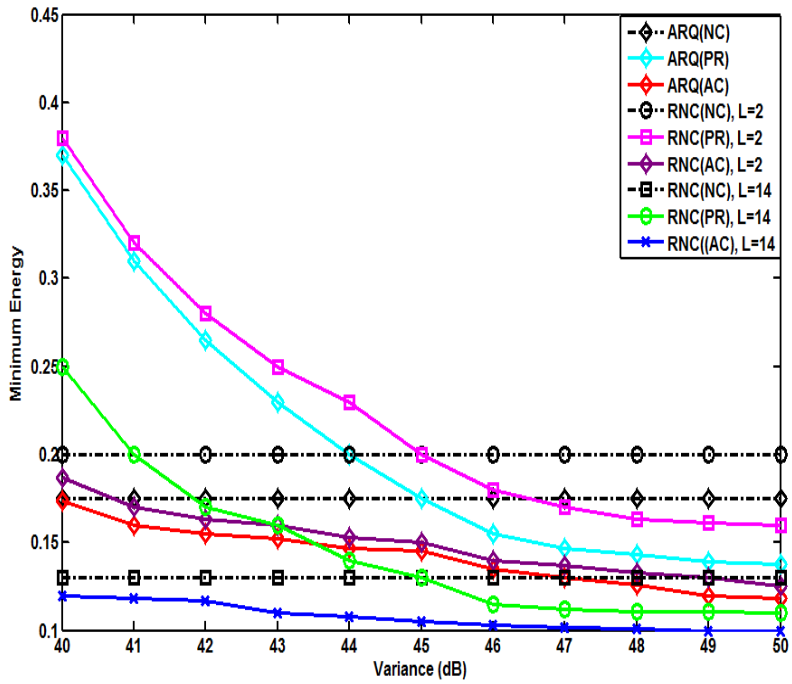


Figure 4.5: Optimal cost for each cooperation scheme as a function of s_{12}/s_{21}

eration using RNC combined with Alamouti Coding achieves the best performance among the considered cooperation protocols.

4.5 Appendix: Transition Probabilities for the Markov Chain Model considered in section 4.2

- From state (i, j, k) to state (i, j, k)

  $(1 \leq i < L,\ 1 \leq j < L,\ 1 \leq k < min(i, j))$

  $P_{(i,j,k),(i,j,k)} = (1 - p_1)(1 - p_2) + (1 - p_1)p_2 u^{j-L} + (1 - p_2)p_1 u^{i-L}$

- From state (i, j, k) to state $(i + 1, j, k)$

  $P_{(i,j,k),(i+1,j,k)} = p_1(1 - p_2)(1 - u^{i-L}) + p_1 p_2 \frac{(u^j - u^k)}{u^L}$

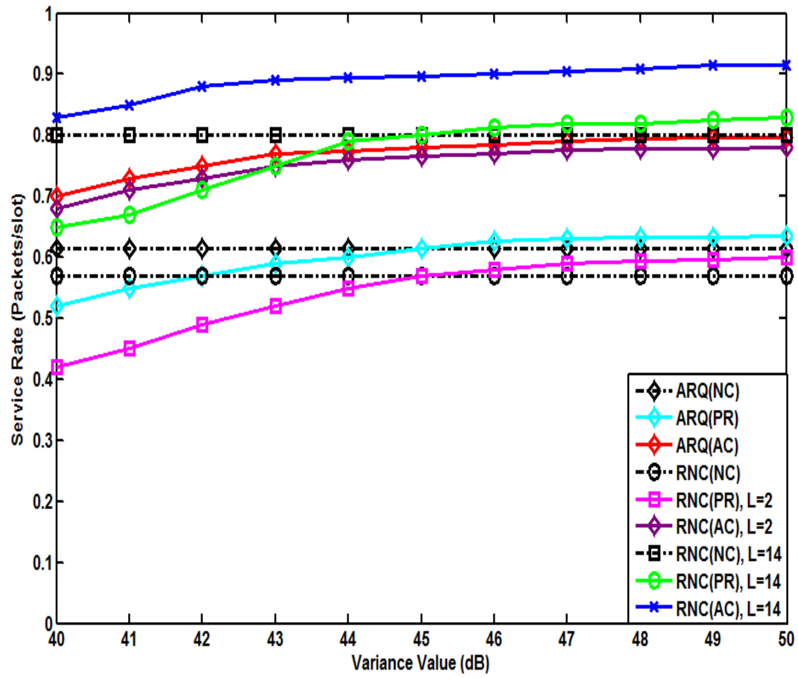


Figure 4.6: Service Rate (in packets/slot) for each cooperation scheme as a function of s_{12}/s_{21}

- From state (i, j, k) to state $(i, j+1, k)$

  $P_{(i,j,k),(i,j+1,k)} = p_2(1-p_1)(1-u^{j-L}) + \frac{p_1 p_2(u^i - u^k)}{u^L}$

- From state (i, j, k) to state $(i+1, j+1, k+1)$

  $P_{(i,j,k),(i+1,j+1,k+1)} = p_2 p_1(1-(u^i+u^j-u^k)u^{-L})$

- From state (i, L, k) to state (i, L, k)

  $P_{(i,L,k),(i,L,k)} = 1$

- From state (L, j, k) to state (L, j, k)

  $P_{(L,j,k),(L,j,k)} = (1-p_3) + p_3 u^{j-L}$

- From state (L, j, k) to state $(L, j+1, k)$

$P_{(L,j,k),(L,j+1,k)} = p_3(1 - (u^j + L - k)u^{-L})$

- From state (L, j, k) to state $(L, j+1, k+1)$

  $P_{(L,j,k),(L,j+1,k+1)} = p_3(L - k)u^{-L}$

- From state (L, L, k) to state (L, L, k)

  $P_{(L,L,k),(L,L,k)} = 1$

4.6 Appendix: Analytic Expressions for the cost for a special case of plain relaying using RNC

For the case when the alphabet size u is infinite and when the probabilities of success p_1, p_2 and p_3 are equal to p, we get the following expressions:

$$\begin{aligned}\Pr[T_{SR} = i] &= \Pr[T_{SD} = i] \\ &= \binom{i}{L-1}(1-p)^{i-L+1}p^L\end{aligned}$$

$$\begin{aligned}&E[T_{RD}|N_D = n] \\ &= \sum_{i=L}^{\infty} i\binom{i}{L-1-n}(1-p)^{i-L+1+n}p^{L-n}\end{aligned}$$

$$\begin{aligned}&\Pr[T_{SR} \geq T_{SD}] \\ &= \sum_{i=L}^{\infty}\sum_{j=i}^{\infty}\binom{j}{L-1}\binom{i}{L-1}(1-p)^{i+j-2L+2}p^{2L}\end{aligned}$$

$$\begin{aligned}&\Pr[N_D = n|T_{SR} < T_{SD}] \\ &= \sum_{i=L}^{\infty}\binom{i}{n-1}\binom{i}{L-1}(1-p)^{2i-n-L+1}p^{n+L}\end{aligned}$$

$$
\begin{aligned}
& E[T_{SR}|T_{SR} < T_{SD}] \\
= & \sum_{i=L+1}^{\infty} \sum_{j=L}^{i-1} \binom{j}{L-1} j \binom{i}{L-1} (1-p)^{i+j-2L+2} p^{2L}
\end{aligned}
$$

The expected value $E[T_{SRD}|N_D = n]$ has the same expression as the above equation with p replaced by p_{AC}.

$$
\begin{aligned}
& E[T_{SD}|T_{SR} \geq T_{SD}] \\
= & \sum_{i=L}^{\infty} \sum_{j=L}^{i} j \binom{j}{L-1} \binom{i}{L-1} (1-p)^{i+j-2L+2} p^{2L}
\end{aligned}
$$

4.7 Appendix: Analytic Expressions for the Probabilities and Expected Values Terms in the ARQ Cost functions Presented in section 4.3

$$E[T_{12}] = \frac{1}{p_{12}}$$

$$E[T_{21}] = \frac{1}{p_{21}}$$

$$E[T_{s12}] = \frac{1}{p_{s12}}$$

$$E[T_{s21}] = \frac{1}{p_{s21}}$$

$$Pr(T_1 < T_2) = 1 - \frac{p_2}{1-(1-p_1)(1-p_2)}$$

$$Pr(T_2 < T_1) = 1 - \frac{p_1}{1-(1-p_1)(1-p_2)}$$

$$E[T_1|T_1 < T_2] = \frac{E_[cond1]}{P(T_1 < T_2)}$$

$$E[T_2|T_2 < T_1] = \frac{E_[cond2]}{P(T_2 < T_1)}$$

$$Pr(T_1 = T_2) = \frac{p_1 p_2}{1-(1-p_1)(1-p_2)}$$

$$E[T|T_1 = T_2] = \frac{(1-p_1)(1-p_2)}{1-(1-p_1)(1-p_2)}$$

$$\begin{aligned} E_[cond1] \quad &= \quad \frac{1-p_2}{p_1} - \frac{p_2}{p_1}\frac{(1-p_1)(1-p_2)}{1-(1-p_1)(1-p_2)} \\ &\quad - \frac{p_2(1-p_1)^2(1-p_2)^2}{(1-(1-p_1)(1-p_2))^2} \\ &\quad - \frac{p_2(1-p_1)(1-p_2)}{(1-(1-p_1)(1-p_2))} \end{aligned}$$

$$
\begin{aligned}
E_[cond2] \quad = \quad & \frac{1-p_1}{p_2} - \frac{p_1}{p_2}\frac{(1-p_1)(1-p_2)}{1-(1-p_1)(1-p_2)} \\
& - \frac{p_1(1-p_1)^2(1-p_2)^2}{(1-(1-p_1)(1-p_2))^2} \\
& - \frac{p_1(1-p_1)(1-p_2)}{(1-(1-p_1)(1-p_2))}
\end{aligned}
$$

Chapter 5

Optimal Rate Allocation for Minimization of the Consumed Energy of Base Stations with Sleep Mode

This chapter deals with another technique used to reduce the consumed energy in particular in cellular systems. This technique is based on exploiting the sleep mode feature of current base stations. During sleep mode, the base station is allowed to reduce its power when no users are active in the cell, which may result in considerable energy savings.

In addition, the value of the transmission rate affects energy efficiency. When base stations are required to be in the "ON" mode, it has been shown (see [1]) that transmitting at the lowest acceptable rate is most energy efficient. However, this may not be the case when base stations are allowed to switch to a sleep mode. The reason is that although the base station will consume more energy by transmitting at higher rates, it will satisfy the users' demands in a shorter time, and hence it can stay in the sleep mode for a longer period of time. Hence, it is not clear what rate values should be used in conjunction with sleep modes.

Furthermore, when multiple users are active in the cell, it is anticipated that the base station scheduling technique will affect the sleep mode duration of the base station as well as the energy efficiency of the system. Prior work has focused on the effect of the scheduling method on networks throughput. In [41] and [42], it has been proven that Time Division Multiplexing (TDM) achieves capacity when transmitting over fading channels. In [43] and [44], it has been proven that TDM achieves the best downlink system throughput for the case when users have bursty traffic. In [45], rate and power control algorithms are used in the downlink of CDMA network to maximize the system throughput. In [46] and [47], power control is used to minimze the transmission energy in a CDMA cell, and it is shown that time division scheduling is most energy efficient. However, none of these methods have considered the case when the base station is allowed to reduce its power when no users are active in the network.

We consider in this chapter the downlink scenario in a Macro cell in which the base station should satisfy its users' demands within a strict delay constraint. We assume that the consumed power of the base station is a linear function of the transmission power, and that the base station can go to "Micro" sleep mode when there are no active users. We start by considering the simple case when there is only one active user. Then, we consider the case when multiple users are active in the cell. In this case, we consider both time division multiplexing and frequency division multiplexing. For each case, we find the optimal rate value the base station should use to each active user in order to minimize the overall consumed energy. Although there is a prior work [48] that considers the uplink problem and has a very similar formulation for the case of time division, this work provides a formulation for the frequency division case (which is to our knowledge not yet provided). Also, we provide a comparison between the performance of time division and frequency division scheduling.

5.1 Single User

5.1.1 Problem Formulation

We consider a Macro cell in which there is only one active user. The user has a demand of B bits to be delivered within T seconds. In the active mode, the consumed power P_C of the base station is a linear function of the transmission power P_T. Measurements done in [49] on various base station models show that a linear function of the transmission power is a good approximation to the consumed power. Also, it is assumed that the base station can reduce its consumed power and switch to a sleep mode when the user is not active. Hence, the consumed power P_C at the base station follows a piecewise linear model and is given by the following expression:

$$P_C = \begin{cases} s, & P_T = 0 \\ \Delta_P P_T + P_0, & 0 < P_T \leq P_{max} \end{cases} \tag{5.1}$$

where the values of the linear model parameters Δ_P and P_0 depend on the base station type, P_{max} is the maximum transmission power, and the parameter s is the consumed power value when the base station is in the sleep mode ($s \leq P_0$). The received power P_R at the user follows the path-loss power model. Also, it is assumed that the value of the channel gain H between the base station and the user is known at the base station

Hence, the received power value is given by:

$$P_R = A|H|^2 P_T |d|^{-\alpha} \tag{5.2}$$

where α is the path-loss exponent, d is the distance from the user to the base station, and A is a constant which accounts for system losses. Further, it is assumed that there is a receiver noise of power spectral density N_R.

The base station transmits to the user over a bandwidth of W Hz at a rate of R bits/sec. It is assumed that the achievable rate and the transmission power P_T are related through Shannon's capacity formula, and hence we have:

$$R = Wlog\left(1 + \frac{A|H|^2 P_T |d|^{-\alpha}}{N_R W}\right) \tag{5.3}$$

The energy spent by the base station is given by:

$$E = ((\Delta_P P_T + P_0)\tau + s(1-\tau))T \tag{5.4}$$

where τ is the fraction of time the base station is in the active mode and given by:

$$\tau = \frac{B}{RT} \tag{5.5}$$

Combining (5.3), (5.4), and (5.5), we have:

$$E(R) = \left(\Delta_P \frac{2^{R/W} - 1}{\varphi(d)} + P_0\right)\frac{B}{R} + s\left(T - \frac{B}{R}\right) \tag{5.6}$$

where

$$\varphi(d) = \frac{A|H|^2 |d|^{-\alpha}}{N_R W} \tag{5.7}$$

The objective is to find the optimal rate value that minimizes the consumed energy. Note that since $0 < \tau \leq 1$, we have $R \geq \frac{B}{T}$. Also, since $0 \leq P_T \leq P_{max}$ and by using (5.3), we obtain:

$$0 \leq R \leq Wlog\left(1 + \frac{A|H|^2 P_{max} |d|^{-\alpha}}{N_R W}\right)$$

Hence, the objective can be stated as:

$min_R E(R)$

$$\text{s.t. } \frac{B}{T} \leq R \leq W log\left(1 + \frac{A|H|^2 P_{max}|d|^{-\alpha}}{N_R W}\right) \tag{5.8}$$

5.1.2 Solution

It can be easily seen that the energy $E(R)$ given in (5.6) is a convex function of R (since it is differentiable, it suffices to show that the second derivative with respect to R is nonnegative for every value of R in the constraint set), and the constraints are only bound constraints. Hence, any local minimizer is a global minimizer. However, the function is nonlinear in R. Hence, the optimal value is obtained by numerical nonlinear optimization methods. We use the so-called-standard "Interior Point" method to solve it. The "Interior Point" method is of interest because of its polynomial complexity, and it provides solution to a wide range of nonlinear optimization problems.

The details of the "Interior Point" method can be found in [50]. The complete proof of the convexity of $E(R)$ is shown in appendix 5.4.

5.1.3 Numerical Results

To evaluate the rate control algorithm, the following values of the parameters for the Macro base station considered in [51] are used: $P_{max} = 20W$, $W = 10MHz$

The values of the user's demand and the delay constraint are: $T = 10sec$ and

$B = 15Mbits$

The values of the path-loss model parameters are taken from 3GPP simulation scenarios and are given by: $A = 0.03$, $\alpha = 3.76$. Without loss of generality, the value of the channel gain H is: $H = 1$.

The values of the receiver's noise used are: $N_R = 8.1875 \times 10^{-15} W/MHz$. In addition to the thermal noise, external interference of value $I = 1.77 \times 10^{-11} W$ is added in order the base station operates in its typical range of transmitted rate.

In order to investigate the effect of system parameters (such as the distance of the user, the values of the parameters of the power model used, etc), we compute the minimum energy consumed using the rate control algorithm for the following two cases:

Case 1: The value of the sleep mode power value s is varied between $0.1P_0$ and P_0 while the values of Δ_P and P_0 are kept fixed at: $\Delta_P = 5$ and $P_0 = 118.7$ [?].

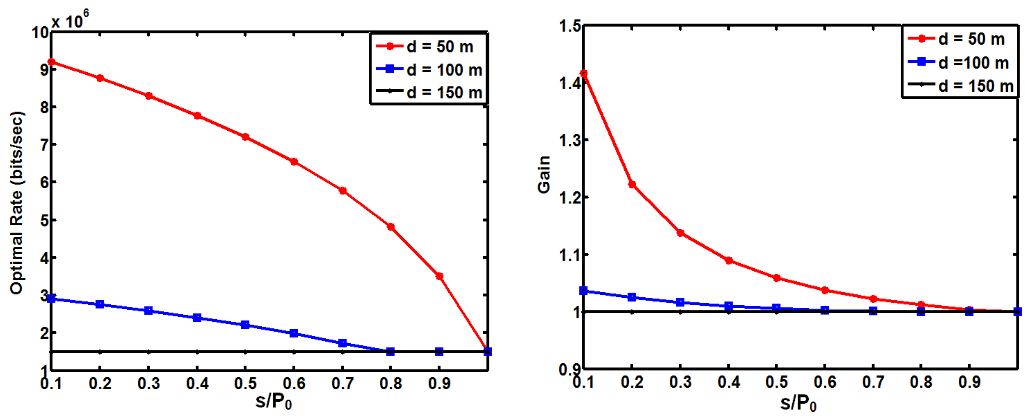
The minimum energy is computed for each value of s. Also, the minimum energy value is compared to the energy spent using the non-optimal rate allocation method that uses the lowest feasible rate to deliver the required load to the user. In the non-optimal rate allocation method, the energy is computed by substituting the value of the lowest feasible rate in (5.6). Figure 5.1(a) plots the optimal rate value obtained versus the value of the sleep mode power value used for the cases when the distance between the user and the base station is given by: d =50, 100 and 150 meters respectively. Figure 5.1(b) plots the gain of using the optimal rate allocation algorithm over the "Lowest Feasible Rate" allocation algorithm versus the sleep mode power value. The gain is defined as:

$$Gain = \frac{Energy_{Nonoptimal}}{Energy_{Optimal}} \tag{5.9}$$

Case 2: The value of Δ_P is varied from 1 to 5 while the value of P_0 is varied from 0 to 119. Both values are varied in steps of one. For every value of P_0, the value of s is kept fixed at $0.4P_0$, and the minimum energy is computed for every pair of values of Δ_P and P_0. Figures 5.2(a) and 5.2(b) plot the minimum energy consumed (in Joules) and the optimal rate (in bits/sec) respectively versus the different values of the pair (Δ_P, P_0). In this case, the value of the distance of the user from the base station used is: $d = 50m$.

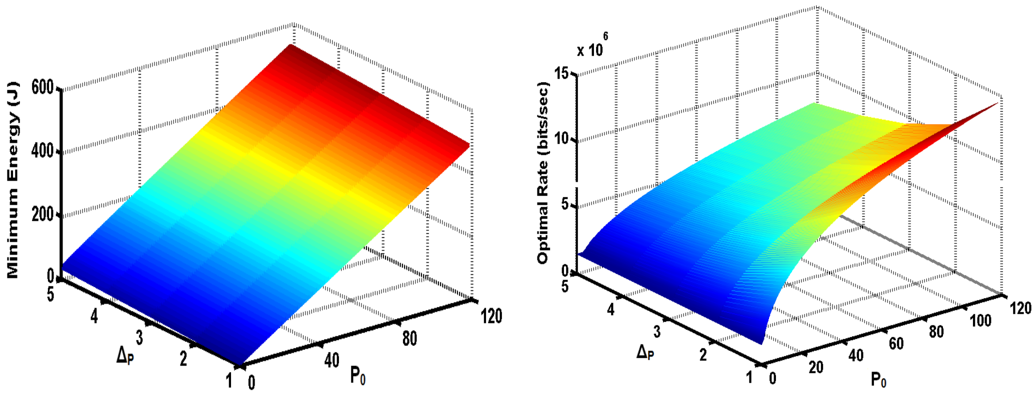
Figure 5.1(a) shows that as the distance between the user and the base station increases, the gain of the optimal rate allocation method decreases. This is because as the distance between the user and the base station increases, the optimal rate decreases and this is shown in figure 5.1(b). Also, Figure 5.1(a) and figure 5.1(b) show that the gain of using the optimal rate allocation decreases as the sleep mode power value increases until it reaches unity for the case when the value of the sleep mode power is equal to the power when the base station is active (i.e. $s = P_0$). The reason the gain decreases is that the optimal rate decreases with increasing sleep mode power value as shown in figure 5.1(b) until the optimal rate value is equal to the lowest feasible rate. This agrees with the previous studies that prove that for the case when there is no sleep mode, the optimal rate to minimize energy is the lowest feasible rate.

Furthermore, figure 5.2(a) shows that the minimum energy decreases slightly with decreasing the value of the parameter Δ_P and decreases considerably with decreasing the value of the parameter P_0. Also, figure 5.2(b) shows that for low values of P_0 the base station transmits with lowest rate; however, as the value of P_0 increases, the base station transmits with higher rate and hence tries to maximize the duration of the sleep mode. Similar conclusions are drawn when considering other values of system parameters.



(a) Optimal Rate Value (b) The gain of the rate allocation method

Figure 5.1: The optimal rate values and the gain of the optimal rate allocation versus the sleep mode power value s



(a) Minimum Energy Consumed (b) Optimal Rate Value

Figure 5.2: The minimum energy consumed and the optimal rate values respectively versus the base station parameters Δ_P and P_0

5.2 Multiple Users

5.2.1 System Model

Now, we consider the case when multiple users are active in the Macro cell. Let M be the number of users in the cell. Each user i is located at a distance d_i meters from the base station and has a demand of B_i bits. The base station should satisfy the demands of every user within time T seconds. Also, the base station transmits to each user with power value P_{iT}. The consumed power P_C by the base station follows the same piecewise linear model as in the preceding section. Also, the received power at each user follows the path-loss model as before. Further, it is assumed that the value H_i of the channel gain between the base station and user i is known at the base station. Also, there is receiver noise of power spectral density N_R.

The base station uses one of the following:

- Time Division, where it transmits using the total bandwidth W with rate R_i to user u_i during a fraction τ_i of the time duration T, and the rate value R_i is varied by varying P_{iT}.

- Frequency division, where it allocates to each user u_i a bandwidth w_i of the total bandwidth W and delivers to each user i load with rate R_i during the time duration of T seconds, and the rate value R_i is varied by varying the bandwidth value w_i.

5.2.2 Time Division Scheduling

5.2.2.1 Problem Formulation

Let τ_i be the fraction of time that the base station is transmitting to user i; we have:

$$\tau_i = \frac{B_i}{R_i T} \tag{5.10}$$

Thus, the fraction of time the base station is active is given by:

$$\tau = \sum_{i=1}^{M} \tau_i \tag{5.11}$$

Hence, the energy spent by the base station is given by:

$$E = \left(\sum_{i=1}^{M} (\Delta_P P_{iT} + P_0)\tau_i + s(1 - \sum_{i=1}^{M} \tau_i) \right) T \tag{5.12}$$

Also, by substituting in (5.12) the value of the time fractions τ_i and the power P_{iT} as given by (5.3) and (5.10), we obtain:

$$E(R_1, ..., R_M) = \sum_{i=1}^{M} \left(\Delta_P \frac{2^{\frac{R_i}{W}} - 1}{\varphi(d_i)} + P_0 \right) \frac{B_i}{R_i} + s\left(T - \sum_{i=1}^{M} \frac{B_i}{R_i} \right) \tag{5.13}$$

However, and the rate values $R_1, ..., R_M$ are constrained by:

$\sum_{i=1}^{M} \frac{B_i}{R_i T} \leq 1$ since $0 \leq \sum_{i=1}^{M} \tau_i \leq 1$

Since $0 \leq P_{iT} \leq P_{max}$ and by (5.3), we obtain the following constraint on R_i:

$$0 \leq R_i \leq Wlog\left(1 + \frac{A|H_i|^2 P_{max}|d_i|^{-\alpha}}{N_R W}\right)$$

Also, the rate value R_i is constrained by: $R_i \geq \frac{B_i}{T}$

Hence, the objective can be stated as:

$$\begin{aligned}
&min_{R_1,..R_M} E(R_1, .., R_M) \\
&\quad s.t. \sum_{i=1}^{M} \frac{B_i}{R_i T} \leq 1 \\
&\quad \quad \frac{B_i}{T} \leq R_i \leq Wlog\left(1 + \frac{A|H_i|^2 P_{max}|d_i|^{-\alpha}}{N_R W}\right) \qquad \forall i = 1, 2, ..., M
\end{aligned} \tag{5.14}$$

5.2.2.2 Solution

Similar to the single user case, it can be shown that the energy function $E(R_1, .., R_M)$ described in (5.13) is convex. Also, the constraints are either bound constraints or convex functions of R_i. Hence, any local minimizer for the problem is a global minimizer. But since the objective function and some of the constraints are nonlinear, numerical nonlinear optimization methods such as the standard “Interior Point” method will be used to obtain the optimal solution.

The proof of the convexity of the objective function as well as for the constraint functions are presented in appendix 5.4.

5.2.3 Frequency Division Scheduling

5.2.3.1 Problem Formulation

Again, let τ_i to be the fraction of time the base station is serving user i. It is given by:

$$\tau_i = \frac{B_i}{R_i T} \tag{5.15}$$

The fraction of time the base station is active is given by:

$$\tau = max_{1 \leq i \leq M} \tau_i \tag{5.16}$$

The rate R_i depends on the bandwidth w_i and the transmission power value P_{iT} assigned to user i; we assume the dependence that corresponds to the Shannon's formula, that is:

$$R_i = w_i log\left(1 + \frac{A|H_i|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right) \tag{5.17}$$

The rate R_i depends on the bandwidth w_i while the value of the transmission power P_{iT} stays fixed.

Combining (5.15) and (5.17), we obtain:

$$\tau_i = \frac{B_i}{w_i log\left(1 + \frac{A|H_i|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right) T} \tag{5.18}$$

Thus, the energy spent to satisfy the users' demands is given by:

$$E = \sum_{i=1}^{M} (\Delta_P P_{iT} + P_0)\tau_i + s(1-\tau)T \tag{5.19}$$

By substituting the value of τ_i (given by (5.18)) in (5.19), we obtain:

$$E = \sum_{i=1}^{M} (\Delta_P P_{iT} + P_0) \frac{B_i}{w_i log\left(1 + \frac{A P_{iT}|d|^{-\alpha}}{N_R w_i}\right)} + s(1-\tau)T \tag{5.20}$$

The bandwidth value w_i assigned to user i is constrained by:

$$w_i log\left(1 + \frac{A|H_i|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right) \geq \frac{B_i}{T}, \text{ since } R_i \geq \frac{B_i}{T}$$

Also, the total assigned bandwidth should not exceed W i.e.

$$\sum_{i=1}^{M} w_i \leq W$$

Hence, the objective can be stated as:

$$\begin{aligned}
&min_{w_1,..w_M} E(w_1, .., w_M) \\
&\quad s.t. \sum_{i=1}^{M} w_i - W \leq 0 \\
&\quad \frac{B_i}{T} - w_i log\left(1 + \frac{A|H_i|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right) \leq 0 \forall i = 1, 2, ..., M
\end{aligned}$$

5.2.3.2 Solution

From (5.16), it follows that the fraction of time the base station is active is the maximum of the fractions of time the base station is serving each of its users. Hence, the energy function $E(w_1, ..., w_M)$ described in (5.20) is not differentiable. In order to solve the minimization problem, we rewrite it as:

$$\begin{aligned}
&min_{w_1,..w_M} E(w_1, .., w_M) \\
&= min_{w_1,..w_M} \sum_{i=1}^{M} \left((\Delta_P P_{iT} + P_0) \frac{B_i}{w_i log\left(1 + \frac{A|H_i|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right)} + s(1 - max_{1 \leq j \leq M} \tau_j) T \right) \\
&= min_{w_1,..w_M} \sum_{i=1}^{M} \left((\Delta_P P_{iT} + P_0) \frac{B_i}{w_i log\left(1 + \frac{A|H_i|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right)} + s(1 + min_{1 \leq j \leq M} \ (-\tau_j)) T \right) \\
&= min_{w_1,..w_M} min_{1 \leq j \leq M} \sum_{i=1}^{M} \left((\Delta_P P_{iT} + P_0) \frac{B_i}{w_i log\left(1 + \frac{A|H_i|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right)} + s(1 - \tau_j) T \right)
\end{aligned} \tag{5.21}$$

But we have that

$$
\begin{aligned}
E_j(w_1,...,w_M) &= \sum_{i=1}^{M}\Bigg((\Delta_P P_{iT}+P_0)\frac{B_i}{w_i log\Big(1+\frac{A|H_i|^2P_{iT}|d|^{-\alpha}}{N_R w_i}\Big)}+s(1-\tau_j)T\Bigg) \\
&= \sum_{i=1}^{M}\Bigg((\Delta_P P_{iT}+P_0)\frac{B_i}{w_i log\Big(1+\frac{A|H_i|^2P_{iT}|d|^{-\alpha}}{N_R w_i}\Big)} \\
&\qquad +s(1-\frac{B_j}{w_j log\Big(1+\frac{AP_{jT}|d|^{-\alpha}}{N_R w_j}\Big)})T\Bigg) \qquad (5.22)
\end{aligned}
$$

Hence,

$$
min_{w_1,..w_M}E(w_1,..,w_M) = min_{1\leq j\leq M} min_{w_1,..w_M}E_j(w_1,...,w_M) \qquad (5.23)
$$

It can be shown that the function $E_j(w_1,...,w_M)$ is a convex function of the bandwidth values w_1,...,w_M. Also, both constraint functions of the optimization problem in (5.21) are convex. Hence according to (5.23), the solution to the minimization problem is obtained as the minimum value of the minima of M convex functions. That is, the minimum value of $E(w_1,...,w_M)$ is found by minimizing each of the M functions $E_j(w_1,...,w_M)$ $(1 \leq j \leq M)$ and then selecting the minimum value of the function with the lowest value among the minima of the M convex functions. The proof of the convexity of $E_j(w_1,...,w_M)$ and of the constraints of the optimization problem in (5.21) are presented in appendix 5.4.

5.2.4 Numerical Results

In this part, we investigate the performance of the rate allocation method for both cases of time division scheduling and frequency division scheduling.

The same values of the parameters of the base station and the power consumption model used in section 5.1 are used here as well. To keep things simple, we assume that there are two users in the cell.

The values considered for the distances of users 1 and 2 from the base station are: $d_1 = 100m$ and $d_2 = 50m$. The values of the channel gains between the base station and the users are chosen to be: $H_1 = H_2 = 1$. As for users' loads, we consider both cases when B_1 is kept fixed at 10 and 40 Mbits respectively while user 2 load is varied between 10 to 40 Mbits in steps of 10 Mbits. Also, we consider three cases of the sleep mode power value s: in the first case we let $s = 0.4P_0$, and in the second case when $s = P_0$ (i.e. there is no sleep mode). For the case of frequency division, the transmission powers assigned to the users are: $P_{1T} = P_{2T} = \frac{P_{max}}{2}$.

Then, for both cases of time division scheduling and frequency division scheduling, the minimum energy is computed for the different values of the user load combinations B_1 and B_2, and for both cases when there is sleep mode and when there is no sleep mode. Figure 5.4 and figure 5.3 show the optimal rate values of users 1 and 2 respectively for both cases of time division scheduling and frequency division scheduling. In the case of frequency division, the optimal rate values are obtained by substituting the optimal bandwidth values in (5.17).

To evaluate the energy savings achieved by using a lower sleep mode power value, we compute the gain which is defined as the ratio of the minimum energy consumed when there is no sleep mode to the minimum energy consumed when there is sleep mode. This gain is computed for both cases of time division scheduling and frequency division scheduling. Figures 5.5 and 5.6 show the gain achieved when time division scheduling and frequency division scheduling are used respectively.

Also to investigate whether time division scheduling or frequency division scheduling is more energy efficient, the optimal power values $P^*(1)$ and $P^*(2)$ for both users 1 and 2 obtained from the case of time division (computed by substituting the optimal rate values in (5.3)) are used to compute the minimum energy consumed for the case of frequency division. However, the sum of the optimal power values may exceed the value of the maximum power allowed. This is because in the case of time division, the power value of each user should not exceed P_{max}, while in the case of frequency division the sum of power values of both users should not exceed P_{max}. Hence, the optimal power values are normalized so that their sum does not exceed P_{max}. The normalized power values P_i' for each user i are given by:

$$P_i' = \frac{P_{max} P^* i}{P^*(1) + P^*(2)} \tag{5.24}$$

where $i = 1, 2$

Then, we compute the gain which is defined as the ratio of the minimum energy consumed when frequency division scheduling is used to the minimum energy consumed when time division scheduling is used. We consider the case when the power is reduced during sleep mode ($s = 0.4P_0$). Figure 5.7 plots the gain versus the different values of users loads combinations.

Figure 5.5 and figure 5.6 show that the gain of reducing the power during sleep mode, increases as the users' loads decreases. This is because when there is no sleep mode, the optimal rate values that the base station uses are the lowest feasible rates; however when there is sleep mode, the base station transmits at higher rates. This is verified in figure 5.4 and figure 5.3. Hence, as the load decreases, the base station completes transmitting the loads faster, and hence it can stay in the sleep mode for a longer time, which results in energy savings.

Figure 5.7 shows that using time division scheduling is always more energy efficient than using frequency division scheduling. This is because when using time division scheduling the base station uses the full bandwidth to transmit to each user, and hence the base station can transmit with much higher rates than the case of frequency division, which allows the base station to finish delivering both users' loads faster. Thus when using time division scheduling, the base station can stay in sleep mode for a longer period of time, which results in energy savings. Also due to the higher rates used in time division scheduling, figure 5.7 shows that the gain achieved by time division scheduling over frequency division scheduling increases as the users' loads increases. The values of the optimal rates displayed in figure 5.4 and figure 5.3 verify this analysis.

In order to compare the performances of time division scheduling and frequency division scheduling, the minimum energy is computed for both cases when the number of active users in the cell are 2,4, 6, 8 and 10 respectively. In each case, each user has a load of size 5 Mbits, and the users loads should be delivered in a duration T of 10 sec. Also, the value of the distance d_i of user i from the base station is given by (5.25), namely,

$$d_i = \frac{80}{M} i \tag{5.25}$$

where M is the number of users. The gain of time division scheduling over frequency division scheduling for the different number of active users is plotted in figure 5.8.

Figure 5.8 shows that the energy efficiency of time division scheduling compared to frequency division increases as the number of users increases. This is because as the number of users increases, the optimal rates of frequency division scheduling decreases and becomes considerably lower than the rates used in time division scheduling. Hence, the sleep mode duration of the base station when using frequency division scheduling becomes considerably shorter than the sleep mode duration using time division scheduling, which results in more energy savings when time division scheduling is used.

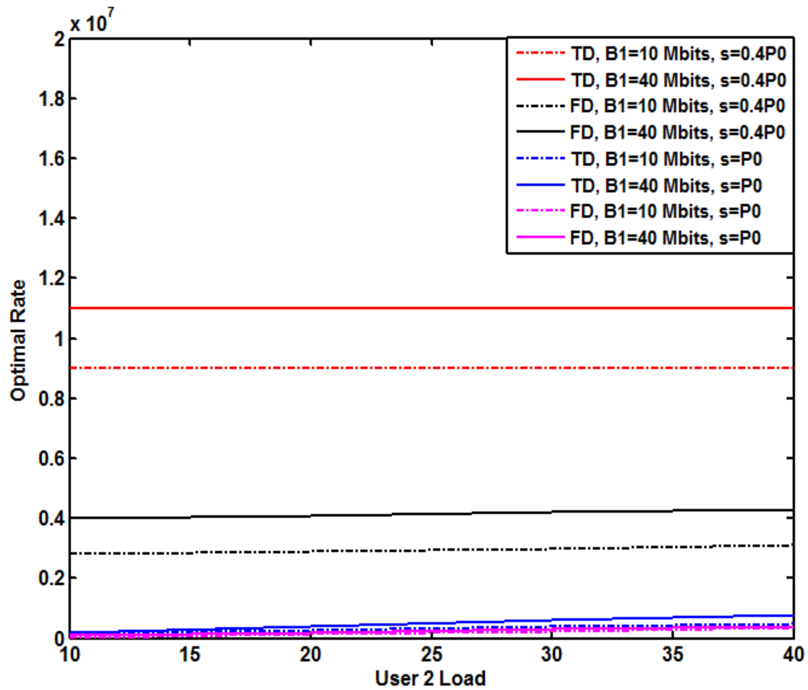


Figure 5.3: Optimal Rate Value for User 1

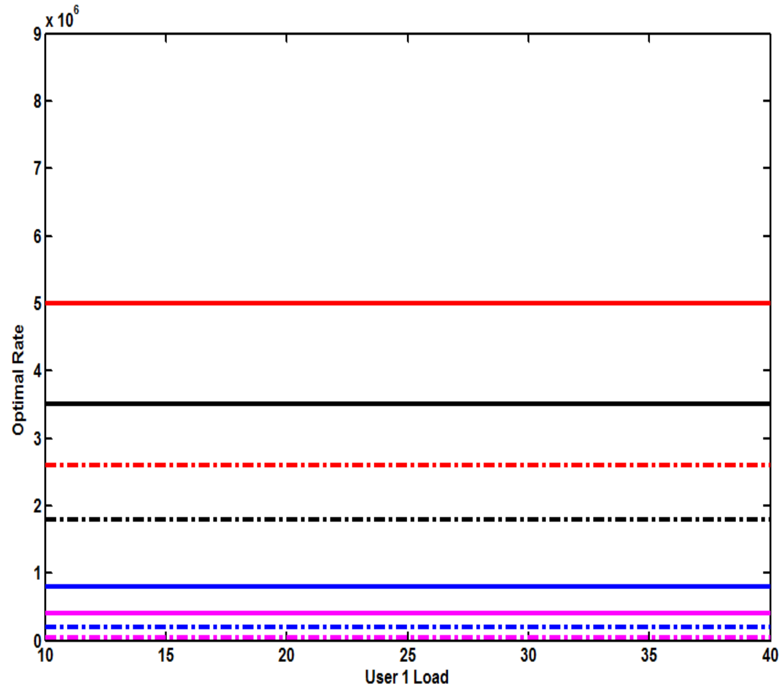


Figure 5.4: Optimal Rate Value for User 1

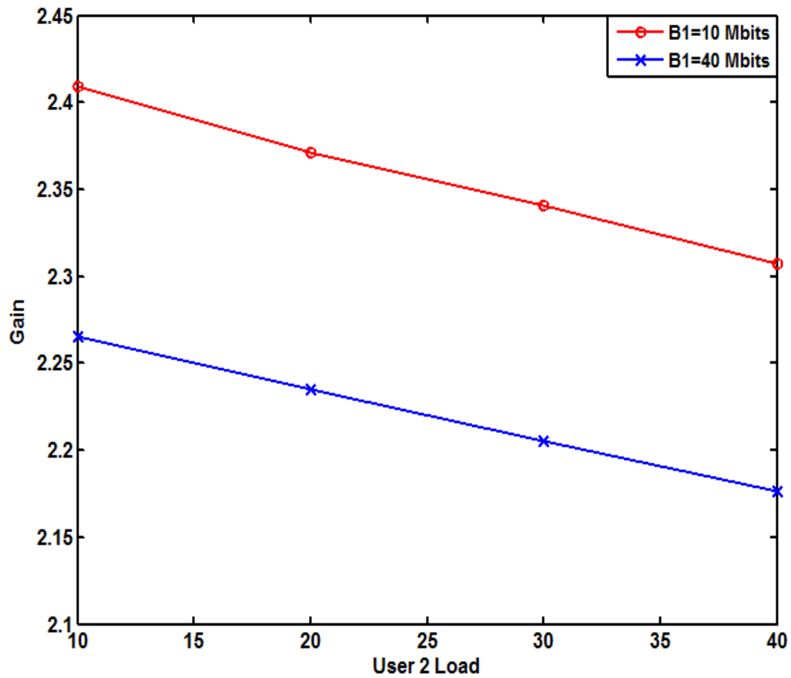


Figure 5.5: The gain of time division scheduling

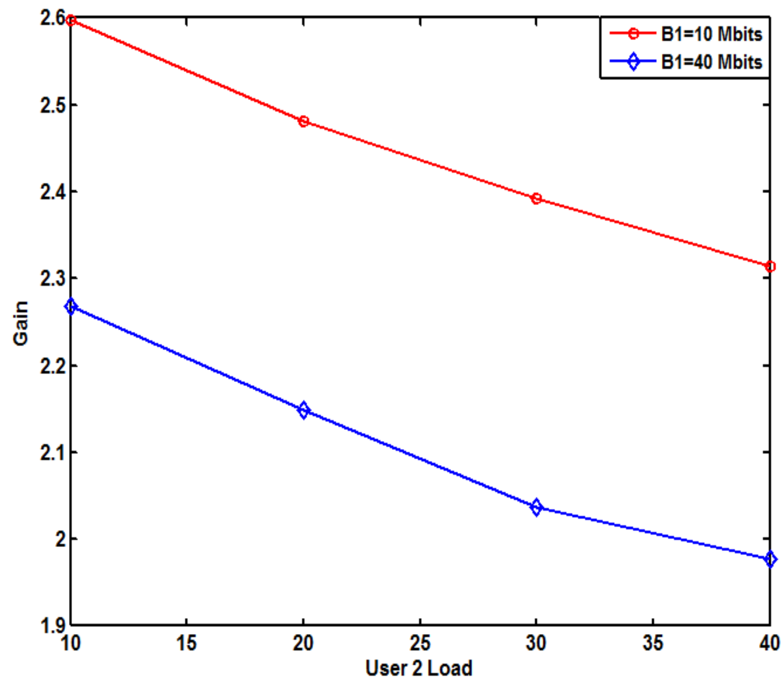


Figure 5.6: The gain of frequency division scheduling

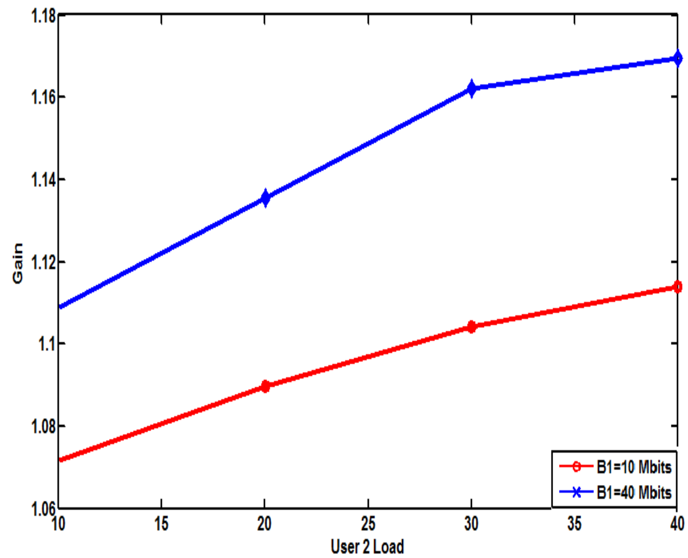


Figure 5.7: *The gain of time division scheduling over frequency division scheduling.*

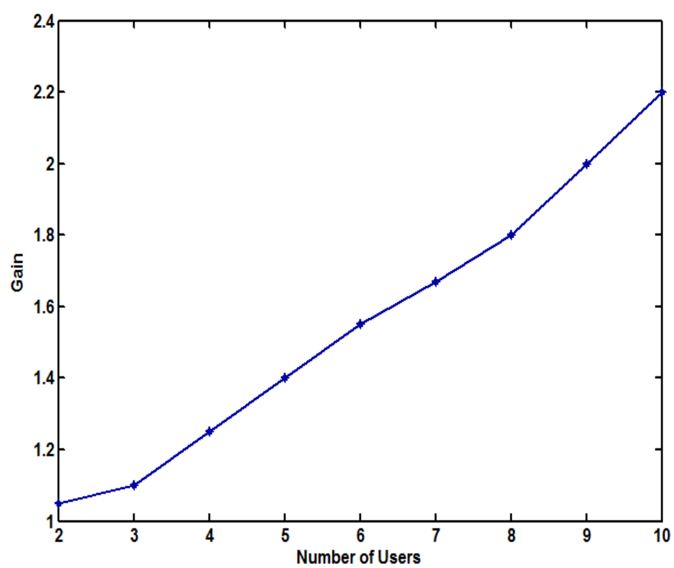


Figure 5.8: *The gain of time division scheduling over frequency division scheduling.*

5.3 Summary

In this chapter, we have addressed rate allocation problems to minimize the consumed energy of a Macro base station. The rate allocation problem takes into account the linear model of the consumed power as a function of the transmission power, and the reduced sleep mode power value used by the base station when no users are active in the cell. Also, we have considered time division and frequency division scheduling methods to satisfy the users' demands. For each scheduling method, the rate allocation problem is formulated as a nonlinear convex optimization problem. This chapter constitutes a natural extension of the rate allocation techniques previously considered for cellular networks. However while the previous rate allocation techniques take into account the transmission power of the base stations, the rate allocation problem considered in this chapter takes into account the total consumed power of the base station and the sleep mode capability of the modern base stations. Also, it considers consumed energy as the performance metric. It remains of interest to extend the rate allocation problem to the case when multiple base stations can serve the users and to incorporate into the problem the fading nature of the wireless channel.

5.4 Appendix: Proof of the Convexity of the Energy Functions

5.4.1 Single User Case

For the case of a single user, the second derivative of the energy function E given by (5.6) with respect to the rate R is given by:

$$\frac{\partial^2 E}{\partial R^2} = \frac{aB((R^2 ln(2)^2 + 2w^2 - 2Rwln(2))2^{R/w} - 2w^2)}{R^3 w^2 \varphi(d)} + \frac{2B}{R^3}(P_0 - s) \quad (5.26)$$

where ln(.) is the natural logarithm.

Since $P_0 \geq s$, we have that $\frac{2B}{R^3}(P_0 - s) \geq 0$. Also, the denominator $R^3 w^2 \varphi(d)$ is positive since R is positive. Hence, it remains to check the sign of the function $N(R) = (R^2 ln(2)^2 + 2w^2 - 2Rwln(2))2^{R/w} - 2w^2$. Since the function $N(R)$ is zero when R is zero, it suffices to show that $N(R)$ is increasing function of R, in order to prove that $N(R)$ is positive. By computing the derivative of $N(R)$ with respect to R, we obtain:

$$w\frac{\partial N}{\partial R} = (ln(2)^2 R^2 + 2wln(2)(ln(2) - 1)R + 2w^2(1 - ln(2)))2^{R/w} \quad (5.27)$$

Since the exponential function $2^{R/w}$ is positive, there remains to check the sign the quadratic function: $ln(2)^2 R^2 + 2wln(2)(ln(2) - 1)R + 2w^2(1 - ln(2))$. The discriminant Δ of the quadratic function is given by:

$$\Delta = (4ln(2)^4 - 4ln(2)^2)w^2 < 0 \quad (5.28)$$

Hence, since the discriminant is negative, the roots of the quadratic equation are complex, which implies that for real values of R, the function is either positive or negative. By selecting $R = 0$, the value of the quadratic function at R is equal to $2w^2(1 - ln(2))$, and, hence, positive.

5.4.2 Multiple Users

5.4.2.1 Time Division Case

By arranging the terms of the energy function given by (5.13), we have:

$$E(R_1, ..., R_M) = \sum_{i=1}^{M} \left(\left(a \frac{2^{\frac{R_i}{w}} - 1}{\varphi(d_i)} + P_0 \right) \frac{B_i}{R_i} - s \frac{B_i}{R_i} \right) + sT \quad (5.29)$$

In this expression, each i^{th} term of the summation is a convex function of R_i. This can be verified since the second derivative of each term i with respect to R_i has the same expression as in (5.26). The only difference is that the term is a function of R_i instead of R. Hence, the hessian H_E of the energy function is an $M \times M$ diagonal matrix, where each entry $H_E(i,i)$ is nonnegative. Hence, the function is convex. As for the constraint function, the hessian matrix H_C is also a diagonal matrix where each entry $H_C(i,i)$ is given by:

$$H_C(i,i) = \frac{2B_i}{R_i^3 T} \quad (5.30)$$

In the above expression, $H_C(i,i)$ is positive since the rate R_i is positive, and hence the constraint is convex.

5.4.2.2 Frequency Division Case

We start by proving that the function $L(w_i) = w_i log\left(1 + \frac{A|H|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right)$ is concave. The second derivative of $L(w_i)$ is given by:

$$\frac{\partial^2 L}{\partial w_i^2} = -\frac{A|H|^2 P_{iT}|d|^{-\alpha}(N_R w_i)}{(N_R w_i)^2 (A|H|^2 P_{iT}|d|^{-\alpha} + N_R w_i)^2} \quad (5.31)$$

The second derivative $\frac{\partial^2 L}{\partial w_i^2}$ is negative since all of the parameters and the variables in (5.31) are positive. Hence $L(w_i)$ is concave.

We then show that the function $G(w_i) = \frac{1}{w_i log\left(1+\frac{A|H|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right)}$ is convex by showing that under appropriate conditions the inverse of a concave function is convex. Let $f(x)$ be a concave function of x, and let $g(x) = \frac{1}{f(x)}$. The second derivative of $g(x)$ with respect to x is given by:

$$\frac{\partial^2 g}{\partial x^2} = \frac{-\frac{\partial^2 f}{\partial x^2}(f(x))^2 + 2f(x)(\frac{\partial f}{\partial x})^2}{f(x)^4} \tag{5.32}$$

The first term in the numerator in (5.32) is positive since $\frac{\partial^2 f}{\partial x^2}$ is negative; however, the sign of the second term depends on whether the value of $f(x)$ is positive or negative. If the value of $f(x)$ is positive for all values of x, the second term of the numerator is positive, and $\frac{\partial^2 g}{\partial x^2}$ is positive. Hence in this case, $g(x)$ is convex.

The function $L(w_i) = w_i log\left(1 + \frac{A|H|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right)$ is concave (as proved earlier). Also, $L(w_i)$ is positive for all positive values of w_i. This is because for positive values of w_i, the argument of the logarithm function is always greater than one, and thus the value of $L(w_i)$ for any positive value w_i is positive.

Hence, the function $G(w_i) = \frac{1}{w_i log\left(1+\frac{A|H|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right)}$ is convex.

In the function $E_j(w_1, w_2, ..., w_M)$ (described by (5.22)), each term i in the sum is a positive multiple of the function $G(w_i) = \frac{1}{w_i log\left(1+\frac{A|H|^2 P_{iT}|d|^{-\alpha}}{N_R w_i}\right)}$ and, hence, is convex function of w_i. Thus, the hessian matrix H_E of the function $E_j(w_1, w_2, ..., w_M)$ is a $M \times M$ diagonal matrix, where each entry $H_E(i,i)$ $(i \neq j)$ is given by:

$$H_E(i,i) = (\Delta_P P_{iT} + P_0) B_i \frac{\partial^2 G(w_i)}{\partial w_i^2} \tag{5.33}$$

Also, the entry $H_E(j,j)$ is given by:

$$H_E(j,j) = (\Delta_P P_{jT} + P_0 - s) B_j \frac{\partial^2 G(w_j)}{\partial w_j^2} \tag{5.34}$$

Since the function $G(w_i)$ is convex, each entry $H_E(i,i)$ is positive and hence the function $E_j(w_1, w_2, ..., w_M)$ is convex. Furthermore, the first constraint function in the optimization problem in (5.14) is a convex function of the bandwidth values w_1, w_2,..,w_M since it is a linear function of w_1, w_2,...,w_M. Also, the second constraint function is convex since its second derivative with respect to w_i is given by $-\frac{\partial^2 L(w_i)}{\partial w_i^2}$ and $L(w_i)$ is concave.

Chapter 6

Secure Distributed Information Exchange

6.1 Introduction

This chapter considers the problem of information exchange under secrecy requirements in wireless systems. Usually, most security techniques achieve the required level of security at the expense of extra consumed energy or higher delays. Thus, it is important to study the tradeoff between achieving the security requirement and the energy and delay costs. The overall problem combines the issues of communication complexity, security, and energy/delay performance cost.

More specifically, this work considers a wireless system in which it is required to deliver securely a file composed of a finite number of packets. First, the single link case is considered where the file is residing at one source node and must be delivered to one intended receiver over a wireless link. Then, the case where the file is distributed among multiple nodes is considered, where the nodes are required to exchange their portions of the file until all nodes possess the entire file. In either case, the nodes can chose to transmit through public channels to which an eavesdropper may have direct access or through private secure channels that are not accessible to the eavesdropper. It is understood that the costs in the private channel are higher. Two security costs are defined: the extra energy spent and the extra delay incurred for using the private channel. The objective is to minimize each of the security costs respectively subject to a certain security requirement. The security requirement is that the probability that the number of packets the eavesdropper is allowed to receive must not exceed a specific threshold.

There have been earlier attempts to address the above tradeoffs. In [52], the problem of information exchange between two nodes in the presence of an eavesdropper is considered. The channels between each pair of nodes are considered to be error free. Then, the authors minimize the security cost of the data exchange by computing the minimum number of bits that should be transmitted over the private channel. The same problem in [53] is considered; however, the channels are assumed to be noisy. Then, it is shown that the minimum number of bits that should be exchanged over the private channel are fewer in the case when the channels are noisy than when the channels are error free. These works follow on the pioneering communication complexity work of A. Yao [54].

This work considers a more complex version of the problem. It also employs Network Coding and examines its effects on achieving more secure and efficient systems. Network Coding, [2], has previously proved to achieve throughput performance improvements, and mostly in multicast networks. It is however also promising for security considerations and is simple to implement where each transmitted packet is a linear combination of the original packets. Note that here we will *NOT* consider random linear coding. Thus, the use of Network Coding results in a form of scrambling that makes it difficult for the eavesdropper to decode. Network Coding security was initially studied using Information Theoric notions as in [55]-[60]. For each of the considered problems, the maximum achievable capacity to achieve secure transmission using Network Coding is computed. However, Information Theory considers a strict notion of security and demands that information should be delivered securely in a way that any potential eavesdropper does not obtain any information about the secure message. In [61], the problem of secure transmission against wiretapping in a multihop network is considered. The definition of security is relaxed where the eavesdropper is allowed to receive a part of the message, and the tradeoff between the security level and the cost incurred due to security is investigated. Two Network Coding-based heuristics are proposed that construct coded subgraphs with low network cost and high security level. It was shown that Network Coding achieves lower costs than traditional routing.

Our approach is different than the work in [61] in that we do not consider flow-based models and Random Network Coding as was done in [61]. Also, the network costs and other system parameters were just defined as constants while in our work the network costs are related to physical layer parameters such as channel fading parameters and transmission power.

6.2 Single Link Case

6.2.1 System Model

Consider transmissions over a wireless link in which the source S is required to deliver a file of M packets to destination D in the presence of an eavesdropper E. Each packet is composed of a finite number of symbols belonging to an alphabet A of finite field size F. Time is slotted with slot duration equal to μ seconds, and in each time slot the source can transmit a packet. Also in every time slot, the source S can chose to transmit through a private channel, which the eavesdropper does not have access to, or over a public channel that is accessible to the eavesdropper.

In every time slot, the private and public channels between source S and destination D are independent slow Rayleigh Fading (i.e. the values of the fading coefficients do not change within one time slot). Also, the channel between source S and eavesdropper E is slow Rayleigh fading and independent of the private and public channel between source S and the destination. Further, all channels are independent across time slots. We denote by $h_{private,D}$ and $h_{public,D}$ the values of the fading coefficients for the private channel and the public channel respectively between source S and destination D and by h_E be the value of the fading coefficient between source S and the eavesdropper E. It is assumed that the channels' statistics are time-invariant (i.e. the distributions of the fading coefficients do not change from one time slot to another). Also, AWGN noise of variance N_0 is present at each node and at the eavesdropper.

Each node can receive the transmitted packet if the received Signal to Noise (SNR) ratio exceeds a threshold. Due to fading, this is a random event. We define $\gamma_{private,D}$ and $\gamma_{public,D}$ to be the required threshold values at destination D, when source S is transmitting through the private and public channels respectively, and γ_E to be the required threshold at eavesdropper E. The Signal to Noise Ratio is given by:

$$SNR(P) = \frac{|h|^2 P}{N_0} \tag{6.1}$$

where h is the value of the fading coefficient and P is the transmission power.

The reason that the thresholds at destination D are different when the source is transmitting through the public channel or the private channel is that the SNR threshold is an increasing function of the rate, and in order to use the private channel, the source S must use appropriate, usually more complex modulation coding and encryption schemes, to guarantee the privacy and thus the transmission rate must increase if the protected packet is to "fit" using one time slot. Hence, the threshold must increase.

The assumed fading structure permits the use of a packet-erasure channel model. Let $p_{private,D}$ be the probability of successful packet reception by destination D when the source is transmitting over the private channel, and let $p_{public,D}$ and p_E be the probabilities of successful packet reception by the destination and the eavesdropper respectively when the source is transmitting over the public channel. Since the fading coefficients are Rayleigh distributed, the probabilities of success are given by:

$$p_{private,D} = e^{-\frac{\gamma_{private,D} N_0}{\sigma^2_{private,D} P}} \tag{6.2}$$

$$p_{public,D} = e^{-\frac{\gamma_{public,D} N_0}{\sigma^2_{public,D} P}} \tag{6.3}$$

$$p_E = e^{-\frac{\gamma_E N_0}{\sigma_E^2 P}} \tag{6.4}$$

where $\sigma^2_{private,D}$, $\sigma^2_{public,D}$, and σ^2_E are the variances of the fading coefficients $h_{private,D}$, $h_{public,D}$, and h_E respectively.

It is required that the source deliver the file to the destination while keeping it secret from the eavesdropper. The secrecy requirement is that the probability that the eavesdropper receives successfully n or more packets is less than a target value λ (where $0 \leq n \leq M$).

To transmit the packet reliably we assume that the source can use either:

- Simple Automatic Repeat Request (ARQ), or
- Deterministic Network Coding (DNC), where in each time slot, the source forms M linearly independent combinations of the M packets and then uses simple ARQ to transmit each linear combination reliably to the destination.

Instant error-free acknowledgements are assumed in all cases as is usually assumed in similar investigations.

The objective is to find the optimal (i.e. minimum) number of packets that the source should transmit through the private channel in order to minimize an appropriate cost subject to the secrecy requirement. Two types of costs are considered:

- The extra energy spent to transmit through the private channel
- The extra delay required to transmit through the private channel

In what follows, the problem is explained in detail for both cases when simple ARQ and Deterministic Network Coding (DNC) are used respectively.

6.2.2 Problem Formulation

6.2.2.1 ARQ Case

Let the random variables $T_{private}$ and T_{public} be the number of time slots spent to deliver a packet successfully though the private channel and the public channel respectively from the source to destination. Since the channels are time invariant and independent, the random variables $T_{private}$ and T_{public} are geometrically distributed with probabilities $p_{private,D}$ and $p_{public,D}$ respectively.

Let $Pr(E|T_{public} = k)$ be the probability that eavesdropper E receives a particular, lets call it "current", packet successfully from the public channel given that destination D receives the packet successfully at time slot k. This reception may occur in any one or more of the k slots. This probability is given by the following expression:

$$Pr(E|T_{public} = k) = \sum_{t=1}^{k}(1 - p_E)^{t-1}p_E = 1 - (1 - p_E)^k \tag{6.5}$$

Then, the probability that eavesdropper E receives the packet successfully is given by:

$$\begin{aligned}
p(E) &= \sum_{k=1}^{\infty} Pr(E|T_{public}=k)Pr(T_{public}=k) \\
&= \sum_{k=1}^{\infty}(1-(1-p_E)^k)(1-p_{public,D})^{k-1}p_{public,D} \\
&= \sum_{k=1}^{\infty}(1-p_{public,D})^{k-1}p_{public,D} - \frac{p_{public,D}}{1-p_{public,D}}\sum_{k=1}^{\infty}(1-p_E)^k(1-p_{public,D})^k \\
&= 1-\frac{(1-p_E)p_{public,D}}{1-(1-p_E)(1-p_{public,D})} \qquad (6.6)
\end{aligned}$$

Also, let the random variables M_D and M_E be the numbers of packets received successfully by destination D and eavesdropper E respectively over the public channel. Since the channels are independent and time invariant, the conditional probability the the eavesdropper receives i packets given that the destination has received successfully m packets is binomially distributed with m number of trials and probability of success $p(E)$. Hence, it is given by the following expression:

$$Pr(M_E=i|M_D=m) = \binom{m}{i}(1-p(E))^{m-i}p(E)^i \qquad (6.7)$$

Then, the probability that the eavesdropper receives n or more packets given that the destination receives m ($m \geq n$) packets over the public channel is given by:

$$Pr(M_E \geq n|M_D=m) = \sum_{i=n}^{m}\binom{m}{i}(1-p(E))^{m-i}p(E)^i \qquad (6.8)$$

This probability can be rewritten as:

$$\begin{aligned}
Pr(M_E \geq n|M_D=m) &= 1-Pr(M_E \leq n-1|M_D=m) \\
&= 1-\sum_{i=0}^{n-1}\binom{m}{i}(1-p(E))^{m-i}p(E)^i \qquad (6.9)
\end{aligned}$$

In general, the cumulative distribution function of a binomially distributed random variable Z with number of trials n and success probability p can be expressed in terms of the regularized incomplete beta function (see [62]). The regularized incomplete beta function $I_x(a,b)$ is defined as:

$$I_x(a,b) = \frac{B(x,a,b)}{B(a,b)} \tag{6.10}$$

where $B(x,a,b)$ is the incomplete beta function and $B(a,b)$ is the beta function as follows:

$$B(x,a,b) = \int_0^x t^{a-1}(1-t)^{b-1}\mathrm{d}t \tag{6.11}$$

$$B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\mathrm{d}t \tag{6.12}$$

Then, the relationship between the cumulative distribution function of the random variable Z and the regularized incomplete beta function (see [**?**]) is given by:

$$Pr(Z \leq k) = I_{1-p}(n-k, k+1) \tag{6.13}$$

Using equation (6.13), we obtain:

$$Pr(M_E \leq n-1|M_D = m) = I_{1-p(E)}(m-n+1, n) \tag{6.14}$$

$$Pr(M_E \geq n|M_D = m) = 1 - I_{1-p(E)}(m-n+1, n) \tag{6.15}$$

Hence,

$$\begin{aligned} Pr(M_E \geq n|M_D = m) &= 1 - Pr(M_E \leq n-1|M_D = m) \\ &= 1 - \frac{\int_0^{1-p(E)} t^{m-n}(1-t)^{n-1}\,\mathrm{d}t}{\int_0^1 t^{m-n}(1-t)^{n-1}\,\mathrm{d}t} \end{aligned} \tag{6.16}$$

Also, we define ξ_{public} and $\xi_{private}$ to be the energy spent to deliver a packet successfully through the public channel and the private channel respectively.

Since $T_{private}$ is geometrically distributed, the expected number of time slots and the expected energy spent to deliver a packet over the private channel are given by:

$$E[T_{private}] = \frac{1}{p_{private,D}} = \frac{1}{e^{-\frac{\gamma_{private,D} N_0}{\sigma^2_{private,D} P}}} \tag{6.17}$$

$$E[\xi_{private}] = P\mu \times E[T_{private}] = \frac{P\mu}{e^{-\frac{\gamma_{private,D} N_0}{\sigma^2_{private,D} P}}} \tag{6.18}$$

Similarly, the expected number of time slots and the expected energy spent to deliver a packet over the public channel are given by:

$$E[T_{public}] = \frac{1}{p_{public,D}} = \frac{1}{e^{-\frac{\gamma_{public,D} N_0}{\sigma^2{public,D} P}}} \tag{6.19}$$

$$E[\xi_{public} = P\mu \times E[T_{public}] = \frac{P\mu}{e^{-\frac{\gamma_{public,D} N_0}{\sigma^2{public,D} P}}} \tag{6.20}$$

Given that the source transmits m packets over the public channel, the extra delay required to deliver the remaining $M - m$ packets over the private channel is given by:

$$\begin{aligned} C_{Delay}(m) &= (M-m)(E[T_{private}] - E[T_{public}]) \\ &= (M-m)\left(\frac{1}{e^{-\frac{\gamma_{private,D} N_0}{\sigma^2_{private,D} P}}} - \frac{1}{e^{-\frac{\gamma_{public,D} N_0}{\sigma^2{public,D} P}}}\right) \end{aligned} \tag{6.21}$$

The function $\frac{1}{e^{-\frac{a}{x}}}$ is increasing in a when both x and a are positive. Hence in order for the security cost to be positive, we need to have that $\frac{\gamma_{private,D} N_0}{\sigma^2_{private,D}}$ is greater than $\frac{\gamma_{public,D} N_0}{\sigma^2{public,D}}$.

The extra energy spent by the source to deliver the remaining $M - m$ packets over the private channel is given by:

$$\begin{aligned} C_{Energy}(m) &= (M-m)(E[\xi_{private}] - E[\xi_{public}]) \\ &= (M-m)\left(\frac{P\mu}{e^{-\frac{\gamma_{private,D}N_0}{\sigma^2_{private,D}P}}} - \frac{P\mu}{e^{-\frac{\gamma_{public,D}N_0}{\sigma^2{public,D}\,P}}}\right) \end{aligned} \quad (6.22)$$

Based on equations 6.21 and 6.22, the energy cost is a multiple of the delay cost. Hence, it suffices to study any one of them.

The objective is to find m^*, the optimal number of packets the source should transmit through the public channel, in order to minimize each of the security costs C_{Delay} and C_{Energy} subject to the probability that the eavesdropper receives n or more packets is less than λ. The problems can be stated as follows:

(P1) $Min_m C_{Delay}(m)$

Subject to:

$Pr(M_E \geq n | M_D = m) \leq \lambda$

(P2) $Min_m C_{Energy}(m)$

Subject to:

$Pr(M_E \geq n | M_D = m) \leq \lambda$

In order to solve these problems, we will first prove the following lemma:

Lemma 5.1:

1. **The probability** $Pr(M_E \geq n | M_D = m) \leq \lambda$ **is increasing function of** m

2. **The security costs** C_{Delay} **and** C_{Energy} **are decreasing functions of** m

3. **The optimal solution to both problems is** $m^* = m_\lambda$ **where** m_λ **is the**

highest integer ($0 \leq m_\lambda \leq M$) that satisfies:

$$Pr(M_E \geq n | m_\lambda) \leq \lambda \tag{6.23}$$

Proof of 1:

Let m_1 and m_2 be two integers such that $m_1 < m_2$.

Let X be binomially distributed random variable with number of trials m_1 and success probability p.

Let Y be binomially distributed random variable with number of trials m_2 and success probability p.

Then, the probability $Pr(X \geq n)$ is given by:

$$Pr(X \geq n) = 1 - Pr(X \leq n-1) \tag{6.24}$$

Hence, using equation (6.13), we obtain:

$$Pr(X \leq n-1) = I_{1-p}(m_1 - n + 1, n) \tag{6.25}$$

$$Pr(X \geq n) = 1 - I_{1-p}(m_1 - n + 1, n) \tag{6.26}$$

Similarly, we get the following expression for the random variable Y:

$$Pr(Y \geq n) = 1 - I_{1-p}(m_2 - n + 1, n) \tag{6.27}$$

The regularized incomplete beta function $I_x(a, b)$ has the following property [**?**]:

$$I_x(a+1, b) = I_x(a, b) - \frac{x^a(1-x)^b}{aB(a, b)} \tag{6.28}$$

By iteratively applying equation (6.28), it can be shown that for any integer $k > 0$, the quantity $I_x(a, b)$ has the following property:

$$I_x(a+k, b) = I_x(a, b) - \sum_{i=0}^{k-1} \frac{x^{a+i}(1-x)^b}{(a+i)B(a+i, b)} \tag{6.29}$$

Using equation (6.29), the probability $Pr[Y \geq n]$ in equation (6.27) can be rewritten as:

$$\begin{aligned} Pr(Y \geq n) &= 1 - I_{1-p}(m_2 - n + 1, n) = 1 - I_{1-p}(m_2 - m_1 + m_1 - n + 1, n) \\ &= 1 - I_{1-p}(m_1 - n + 1, n) + \sum_{i=0}^{m_2-m_1-1} \frac{(1-p)^{m_1-n+1+i} p^n}{(m_1 - n + 1 + i) B(m_1 - n + 1 + i, n)} \end{aligned} \tag{6.30}$$

Hence using equation (6.26), we obtain:

$$Pr(Y \geq n) = Pr(X \geq n) + \sum_{i=0}^{m_2-m_1-1} \frac{(1-p)^{m_1-n+1+i} p^n}{(m_1 - n + 1 + i) B(m_1 - n + 1 + i, n)} \tag{6.31}$$

In equation (6.31), each term in the summation is positive, which results in the probability $Pr(Y \geq n)$ being greater than the probability $Pr(X \geq n)$. Since the probability $Pr(M_E|M_D = m)$ is binomially distributed with number of trials m, $Pr(M_E \geq n|M_D = m)$ increases with m, which is the number of packets transmitted through the public channel.

Proof of 2: The derivatives of cost functions C_{Delay} and C_{Energy} (given by equations (6.21) and (6.22)) are both -1, and hence C_{Delay} and C_{Energy} are decreasing functions of m.

Proof of 3: The optimal solution to both problems is $m^* = m_\lambda$ where m_λ is the highest integer that satisfies:

$$Pr(M_E \geq n|M_D = m_\lambda) \leq \lambda \tag{6.32}$$

This is because any value of m higher than m_λ is not feasible since it will violate the security level constraint in the minimization problems $P1$ and $P2$. Also, any value less than m_λ is not optimal because the security costs are decreasing function of m, and hence the value of the security costs will be higher than the value of the security costs $C_{Delay}(m_\lambda)$ and $C_{Energy}(m_\lambda)$.

And this completes the proof of the lemma.

Since the probability $Pr(M_E \geq n | M_D = m)$ is non-linear in m, the optimal number of packets m^* is found by searching iteratively over the range of values of m i.e. start from $m = 0$ and increase the value of m each time the probability $Pr(M_E \geq n | M_D = m)$ is less than or equal λ until the probability exceeds λ. The optimal value of m^* will be the value of m obtained from the iterative method minus one.

6.2.2.2 DNC Case

In this case, the source S constructs a system of M linearly independent combinations of the M packets such that the eavesdropper can not recover the value of any of the M packets except if it receives successfully all M linearly coded packets.

In what follows, the conditions under which a linear system of equations satisfies the above property are presented, and a method is provided to construct a linear system satisfying this property.

Conditions

Consider n linear independent equations in m variables $x_1, ..., x_m$, $(n < m)$ of the form:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + ... + a_{1m}x_m &= b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2m}x_m &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nm}x_m &= b_n \end{aligned} \tag{6.33}$$

Since $n < m$, this system has infinitely many solutions. In order for the value of *any* variable x_i not to be recoverable, the following conditions must be satisfied:

(a) **For any equation with non zero coefficient of the variable x_i, the coefficient vector of the remaining variables must not be the all-zero vector.**

(b) **For all equations with non-zero coefficient of the variable x_i, the coefficients vectors of the remaining variables must be linearly independent.**

In what follows, the stated conditions are proved to be necessary and sufficient.

Proof of Sufficiency: In any equation with non-zero coefficient of the variable x_i, the coefficients vector of the remaining variables is not the all zeros vector (according to condition a). Hence, it is not possible to recover the value of the variable x_i from that equation. Also since in all equations with non-zero coefficients of the variable x_i, the coefficient vectors of the remaining variables are linearly independent (according to condition b), performing elementary row operations on the equations with non-zero coefficients of the variable x_i cannot convert all the coefficients of the remaining variables to zero. Hence the value of the variable x_i cannot not be recovered.

Proof of Necessity: Assume there exists a linear system that satisfies the desired property (i.e. it is not possible to recover the value of any variable from any subset of equations) but does not satisfy the above conditions. Hence in this linear system, there exist either some equations with non-zero coefficient of a variable x_i in which the coefficient vectors of the remaining variables are the all-zero vector or there exist some equations in which the coefficient vectors of the remaining variables are linearly dependent. In either case, the value of the variable x_i can be recovered, which is a contradiction.

The following part presents a simple method of constructing a linear network code that satisfies these properties.

Method of Construction

In order to obtain a linear network code of m variables where the first $m-1$ equations satisfy the above property, we proceed as follows:

- Construct the first equation composed of any two variables with non-zero coefficients.
- Construct each subsequent equation using two variables with non-zero coefficients as follows: one variable has been used in a previous equation and one variable has not been used.
- Repeat step 2 until $m-1$ equations have been constructed.

- Construct the m^{th} equation composed of any one of the variables.

  The above method satisfies condition (a) since each of the first $m-1$ equations is constructed of two variables with non-zero coefficients. Also in every equation, a new variable with a non-zero coefficient is introduced. Hence, the first $m-1$ equations are linearly independent, and for all equations with non-zero coefficient of any variable x_i, the coefficient vectors are linearly independent. Thus, the value of none of the variables will be recovered from the first $m-1$ equations. Furthermore, the m^{th} equation is linearly independent of the first $m-1$ equations. This is because each of the first $m-1$ equation contains at least an additional variable than the m^{th} equation. Hence, all of the m equations are linearly independent.

Example

Consider a linear system of five equations in five unknowns is constructed as follows:

$$\begin{aligned} x_1 + x_2 &= b_1 \\ x_1 + x_3 &= b_2 \\ x_2 + x_4 &= b_3 \\ x_3 + x_5 &= b_4 \\ x_1 &= b_5 \end{aligned} \tag{6.34}$$

Hence, the eavesdropper can recover the value of all the packets only if it receives successfully the 5 linearly coded packets. Otherwise, it can not recover the value of any of the x_i's from the b_i's.

Thus, the probability that the eavesdropper receives n or more packets given that the destination receives successfully m packets through the public channel is:

$$Pr(M_E \geq n|M_D = m) = \begin{cases} 0, & 0 \leq m \leq M-1 \\ p(E)^M, & m = M \end{cases} \tag{6.35}$$

where $p(E)$ is the probability that the eavesdropper receives a packet successfully and is given by equation (6.6).

Given that the source transmits m coded packets over the public channel, the extra time and the extra energy spent by the source to deliver the remaining $M-m$ coded packets over the private channel are given by equations (6.21) and (6.22) respectively.

The objective is then to find m^* the optimal number of packets the source should transmit through the public channel in order to minimize each of the security costs C_{Delay} and C_{Energy} subject to the probability that the eavesdropper receives n or more packets is less than λ. These problems can be stated as follows:

(P3) $Min_m C_{Delay}(m)$

Subject to:

$Pr(M_E \geq n|M_D = m) \leq \lambda$

(P4) $Min_m C_{Energy}(m)$

Subject to:

$Pr(M_E \geq n|M_D = m) \leq \lambda$

It was shown in the case of ARQ, that the security costs C_{Delay} and C_{Energy} are decreasing in m. Hence, the optimal solution is $m^* = M$ if $Pr(M_E \geq n|M_D = M) \leq \lambda$; otherwise the optimal solution is $m^* = M-1$. This is because when $Pr(M_E \geq n|M_D = M) \leq \lambda$, any value $m' < M$ will incur security cost higher than the cost incurred when M packets are transmitted through the public channel. Also for the case when $Pr(M_E \geq n|M_D = M) > \lambda$, any value $m' < M - 1$ used will incur security cost higher than the cost incurred when $M - 1$ packets are transmitted through the public channel. Also, any value $m \leq M - 1$ is feasible since $Pr(M_E \geq n|M_D = m) = 0$ for any $m \leq M - 1$, which proves the optimality of the proposed solution.

6.2.3 Numerical Results

In this part, we compare the performance of the secure transmission using ARQ to the case when Deterministic Network Coding (DNC) is used. Also, we investigate the effect of the security level parameters n and λ on the minimum security cost.

The following values for the system parameters are used:

$P = 2W$, $M = 7$, $N_0 = 0$ dB, $\sigma_E^2 = 9$ dB, $\gamma_E = 10$ dB, $\sigma^2_{private,D} = 10$ dB, $\gamma_{private,D} = 13$ dB, $\sigma^2_{public,D} = 10$ dB, $\gamma_{public,D} = 10$ dB.

Since the energy cost is a constant multiple of the delay cost, we will consider only in this section the delay cost and compute the minimum delay for the following cases:

- The value of the security level parameter n is varied between 0 and 7. Two values of λ are considered respectively: $\lambda = 0.04, 0.5$.
- The value of the security level parameter n is kept fixed at 3 while the value of λ is varied between 0.01 and 0.1 in steps of 0.01.
- The value of the security level parameter n is kept fixed at 3 while the value

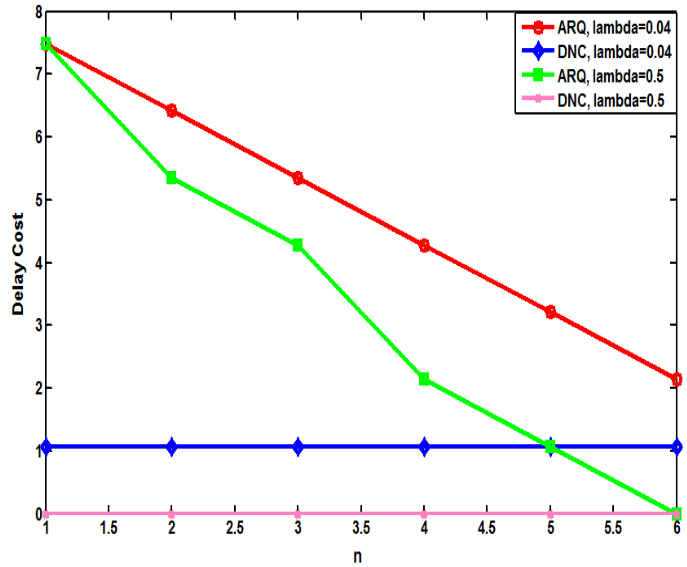


Figure 6.1: *Security cost as a function of the security level parameter n*

of λ is varied between 0.1 and 1 in steps of 0.1.

For each case, the minimum security cost and the optimal number of packets that should be transmitted through the public channel are shown in figures 6.1, 6.2, 6.3, 6.4, 6.5 and 6.6.

Figures 6.1 and 6.2 show that as the security level parameters n and λ increases, the optimal number of packets that the source transmits through the public channel increases and hence the security cost decreases. Also for any value of the security level parameters n and λ, the security cost, when using Network Coding, is considerably lower than when using ARQ. This is because due to the structure of the network coded packets, the receiver can not decode any of the packets unless it receives successfully the M coded packets. Hence the probability that the eavesdropper decodes successfully n or more packets when the source transmits all of the

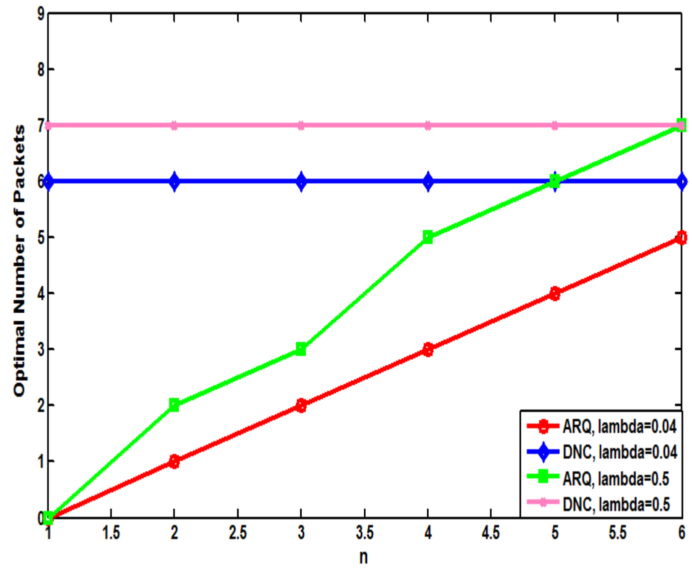


Figure 6.2: *Optimal number of packets m as a function of the security level parameter n*

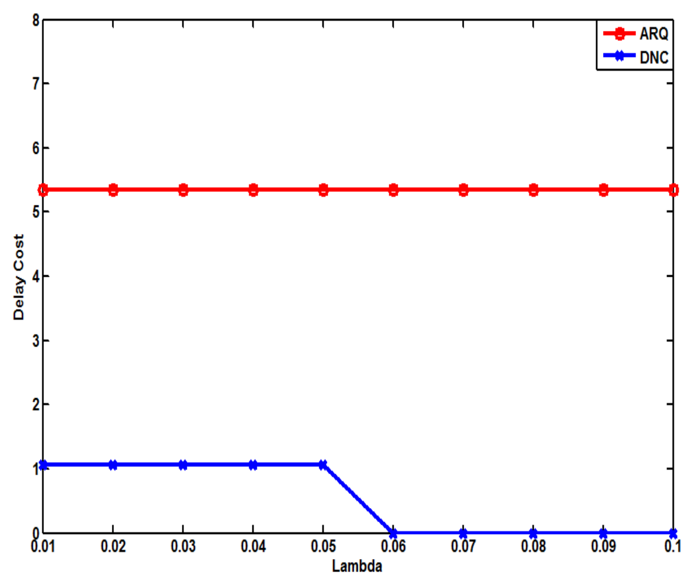


Figure 6.3: *Security cost as a function of the security level parameter lambda* ($0.01 \leq \lambda \leq 0.1$)

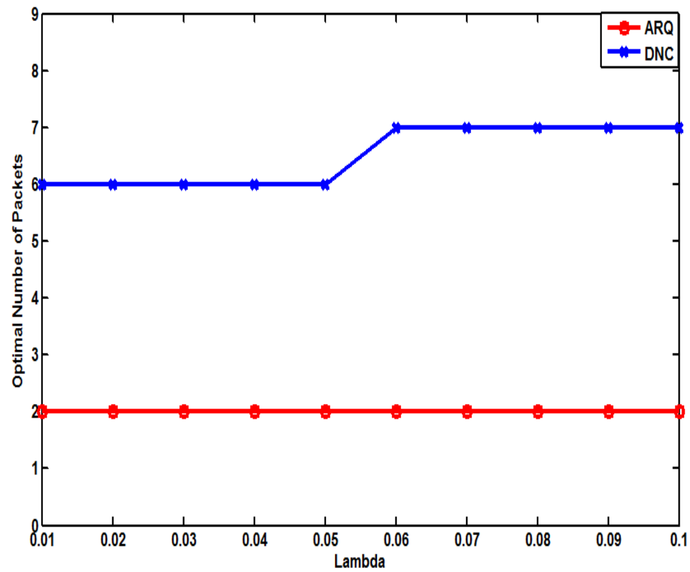


Figure 6.4: *Optimal number of packets m as a function of the security level parameter lambda* ($0.01 \leq \lambda \leq 0.1$)

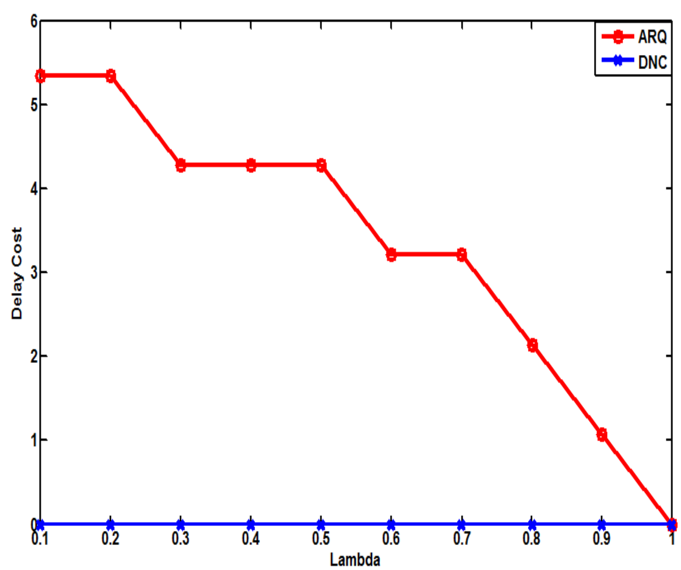


Figure 6.5: *Optimal cost for every strategy/transmission scheme pair* ($0.1 \leq \lambda \leq 1$)

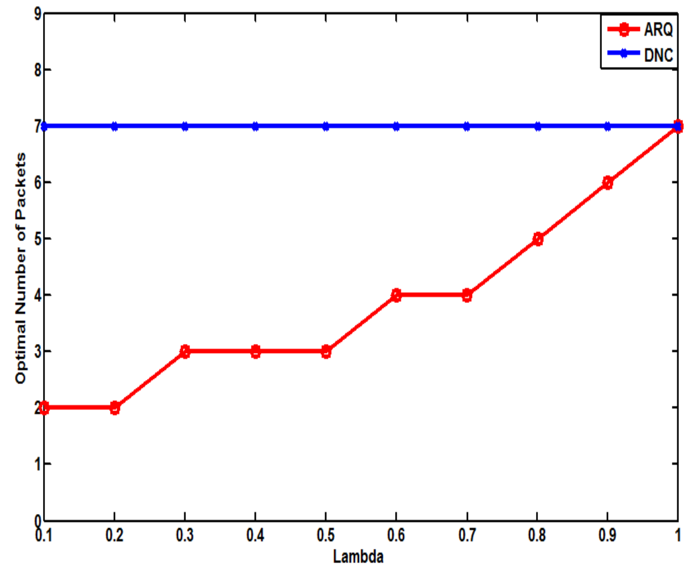


Figure 6.6: *Optimal number of packets m as a function of the security level parameter lambda* ($0.1 \leq \lambda \leq 1$)

M network coded packets is considerably lower than the the case when ARQ is used. Also for any security level parameter n, the transmitter can at most send one packet over the private channel when using DNC and even send no packets through the private channel when the value of λ is high, which results in considerable security costs savings when using DNC compared to ARQ.

Figures 6.3, 6.4, 6.5 and 6.6 investigate further the effect of the security level parameter λ on the security cost. Figures 6.3 and 6.4 show that when the security level parameter λ is very low, the source can transmit very few packets through the public channel when ARQ is used, and thus the security cost when using ARQ is much higher than when using Network Coding.

For the case when the values of λ increases as in figures 6.5 and 6.6, the source can transmit more packets through the public channel when ARQ is used, but there is a still security cost gap between ARQ and Network Coding, and hence Network Coding still outperforms ARQ.

Similar conclusions can be drawn for different values of the system parameters.

6.3 Multiple Nodes Case

6.3.1 System Model

Now, we consider the more general case in which the file is distributed among several nodes in a wireless network. We consider a wireless network composed of a set of K nodes that we designate as G, and an eavesdropper E. It is required that a file X of M packets belonging to an alphabet A with a finite field size F be delivered to all K nodes. Initially, each node i $(1 \leq i \leq K)$ has a distinct subset X_i of the M packets and all the M packets are available somewhere in the network i.e $\bigcup_{1 \leq i \leq K} X_i = X$. In the general case, some of the nodes may possess common packets amongst themselves. There are two ways to deal with this issue. The first one is to find the best node that should transmit each common packet, which may not be easy, especially for the case when the number of nodes that have the same packet is large. The other approach is to assume a rather pessimistic procedure in that many of the packets may be redundantly transmitted. Namely, it is not assumed that each node knows what packets of the file the other nodes possess. Hence, it must transmit everything it has, and thus multiple transmissions of the same packet are possible, which results to the benefit of the eavesdropper. In order to simplify the problem, we assume the simpler case in which the packets available at each node are distinct from the packets available at other nodes i.e. $X_i \bigcap X_j = \emptyset \ \forall i, j$. Also, relaying is not assumed, because if nodes are allowed to transmit packets that are successfully received from other nodes' transmissions, the problem of having common packets among the nodes to transmit will again arise. Furthermore, a fully connected network is assumed in which the K nodes share private broadcast channels amongst each other and public broadcast channels amongst each other and with the eavesdropper. Time is slotted where each time slot is of duration μ seconds, and in each time slot a node can transmit a packet to the remaining nodes. In every time slot k, the private and public channels between each pair of nodes i and j $(1 \leq i, j \leq K)$ or between any node i and the eavesdropper E are independent slow Rayleigh Fading (i.e. the value of the fading coefficients do not change within one time slot). We denote by $h_{private,ij}$ and $h_{public,ij}$ the values of the fading coefficients for the private channel and the public channel respectively between nodes i and j, and by h_{iE} be the value of the fading coefficient between each node i and the eavesdropper E. Also, AWGN noise of variance N_0 is present at each node and at the eavesdropper.

Furthermore in every time slot, one of the nodes transmits to the remaining nodes with power value P, and decides if it should transmit through the public channels or the private channels.

Due to fading, each node can successfully receive the transmitted packet with a certain probability. This probability is the probability that the received Signal to Noise Ratio (SNR) at the receiving node exceeds its required threshold. Let $\gamma_{private,i}$ and $\gamma_{public,i}$ be the required threshold at node i when transmission occurs through the private and public channels respectively. Also, we define γ_E to be the required threshold at eavesdropper E.

Also, let $p_{private,ij}$ and $p_{public,ij}$ be the probability of packet successful reception by node j when node i is transmitting through the private and public channels respectively. Also, let p_{iE} be the probability of successful reception by eavesdropper E when node i is transmitting.

Once a node receives a new transmitted packet, it sends an acknowledgment (ACK) packet. Acknowledgements are received by *all* nodes instantaneously and error free.

As in the single-source single-destination case, the nodes will use either simple ARQ or deterministic linear Network Coding (DNC) respectively to deliver the packets reliably to the remaining nodes.

It is required that the nodes exchange their packets while keeping the file secret from the eavesdropper. The secrecy requirement is that the probability that the eavesdropper successfully receives n or more packets of the file is less than a target value λ ($0 \leq n \leq M$, $0 \leq \lambda \leq 1$).

Since in this case the file is distributed among the nodes in the network, it is more complicated to find the number of packets that each node should transmit through the public channel to solve the security minimization problem. Instead, we consider finding which nodes should transmit through the public channels and through the private channels respectively in order to minimize the security cost subject to the secrecy requirement. Similarly to the single-source case, two costs are defined which are the extra energy spent to transmit through the private channel and the extra delay required to transmit through the private channel.

6.3.2 Problem Formulation

6.3.2.1 ARQ Case

In order to formulate the security minimization problem, it is necessary to define a number of variables as follows:

- Let S be the set of nodes transmitting through the public channel, and let $N_s = |S|$
- Let n_i be the number of packets available at node i i.e. $n_i = |X_i|$
- Let Y_i be the set of integers such that $Y_i = \{t, 1 \leq t \leq n_i\}$
- Let $\phi : S \rightarrow Y_{N_s}$ be a bijection on the indices of the nodes in S. It is defined as follows:

  $\phi(i) = j$ if node i has the j^{th} lowest index value in S
- Let $R = (r_1, r_2, ..., r_{N_s})$ be a vector such that $R \in \prod_{j=1}^{N_s} Y_{\phi^{-1}(j)}$.

  where $\prod$ denotes Cartesian product, and vector R represents the number of the packets that the eavesdropper receives successfully from each node in S where entry r_j corresponds to the number of packets that the eavesdropper receives successfully from node $\phi^{-1}(j)$.
- Let χ be the set of vectors such that $\chi = \{R \; s.t. \sum_{j=1}^{N_s} r_j = n\}$

  where r_j is the j^{th} entry of the vector R.

- Let M_{iE} be a random variable that represents the number of of packets that the eavesdropper receives successfully from node i transmission.

- Let w_i be the security cost of node i if it's transmitting through the private channel. Depending whether delay or energy costs are considered, cost w_i is given by either:

  - Delay cost: It is given by:

$$w_i = m_i(E[T_{private,i}] - E[T_{public,i}]) \tag{6.36}$$

    where $T_{private,i}$ and $T_{public,i}$ are the number of time slots spent by node i to deliver the current packet to the remaining nodes through the private and public channels respectively. The random variable $T_{private,i}$ can be written as:

$$T_{private,i} = max_{1 \leq j \leq K, j \neq i} T_{private,ij} \tag{6.37}$$

    where the random variable $T_{private,ij}$ is the number of time slots spent by node i to deliver the current packet to node j. Since the channels are time invariant and independent, the random variable $T_{private,ij}$ is geometrically distributed with probability of success $p_{private,ij}$.

Then, the probability $Pr(T_{private,i} \leq k)$ is given by:

$$\begin{aligned}
Pr(T_{private,i} \leq k) &= Pr(max_{1\leq j\leq K, j\neq i} T_{private,ij} \leq k) \\
&= \prod_{j=1,j\neq i}^{j=K} Pr(T_{private,ij} \leq k) \\
&= \prod_{j=1,j\neq i}^{j=K} \sum_{t=1}^{k} (1-p_{private,ij})^{t-1} p_{private,ij} \\
&= \prod_{j=1,j\neq i}^{j=K} (1-(1-p_{private,ij})^{k}) \\
&= 1 + \sum_{X\in\{0,1\}^K} \prod_{j=1,j\neq i}^{K} -I(x_j)(1-p_{private,ij})^{k}
\end{aligned} \tag{6.38}$$

where $I(.)$ is the indicator function, x_j is the j^{th} entry of the binary vector X. Also, the fifth line of equation (6.38) is obtained by expanding the product term of the forth line. The probability $Pr(T_{public,i} \leq k)$ can be derived similarly as equation 6.38 and thus has the following expression:

$$Pr(T_{public,i} \leq k) = 1 + \sum_{X\in\{0,1\}^K} \prod_{j=1,j\neq i}^{K} -I(x_j)(1-p_{public,ij})^{k} \tag{6.39}$$

Hence, the expected value $E[T_{private,i}]$ is given by:

$$
\begin{aligned}
E[T_{private,i}] &= \sum_{k=1}^{\infty} Pr(T_{private,i} \geq k) \\
&= \sum_{k=1}^{\infty} \Big(1 - Pr(T_{private,i} \leq k-1)\Big) \\
&= \sum_{k=1}^{\infty} \Big(1 - \prod_{j=1,j\neq i}^{j=K} (1-(1-p_{private,ij})^{k-1})\Big) \\
&= \sum_{k=1}^{\infty} \sum_{X\in\{0,1\}^K} \prod_{j=1,j\neq i}^{K} -I(x_j)(1-p_{private,ij})^{k-1} \\
&= \sum_{X\in\{0,1\}^K} \frac{\prod_{j=1,j\neq i}^{K} -I(x_j)}{1-\prod_{j=1,j\neq i}^{K}(1-p_{private,ij})} \qquad (6.40)
\end{aligned}
$$

The expected value $E[T_{public,i}]$ can be similarly derived and has the following expression:

$$E[T_{public,i}] = \sum_{X\in\{0,1\}^K} \frac{\prod_{j=1,j\neq i}^{K} -I(x_j)}{1-\prod_{j=1,j\neq i}^{K}(1-p_{public,ij})} \qquad (6.41)$$

– Energy cost: It is given by:

$$w_i = m_i(E[\xi_{private,i}] - E[\xi_{public,i}]) \qquad (6.42)$$

where $\xi_{private,i}$ and $\xi_{public,i}$ are the energy spent by node i to deliver a packet through the private and public channel respectively. Thus, we have the following expressions:

$$
\begin{aligned}
E[\xi_{private,i}] &= E[P\mu T_{private,i}] \\
&= P\mu E[T_{private,i}] \qquad (6.43)
\end{aligned}
$$

$$
\begin{aligned}
E[\xi_{public,i}] &= E[P\mu T_{public,i}] \\
&= P\mu E[T_{public,i}] \qquad (6.44)
\end{aligned}
$$

where $E[T_{private,i}]$ and $E[T_{public,i}]$ are given by equations 6.40 and 6.41 respectively.

To compute the probability that the number M_E of packets that the eavesdropper receives successfully is exactly n packets, we proceed by considering all possible combinations of the number of packets that the eavesdropper can receive successfully from the nodes in S such that the total number of packets is n i.e. by considering all the vectors in χ. Then since the channels among the nodes are independent, the probability that number of packets received from the nodes in S is n is computed by multiplying the probabilities that eavesdropper receives r_j packets from node $\phi^{-1}(j)$ $(1 \leq j \leq N_s)$. Hence, we obtain the following:

$$Pr(M_E = n) = \sum_{R \in \chi} \prod_{i \in S} Pr(M_{iE} = r_{\phi(i)}) \tag{6.45}$$

Similarly to equation (6.7), the probability $Pr(M_{iE} = r_{\phi(i)})$ is computed as:

$$Pr(M_{iE} = r_{\phi(i)}) = \binom{n_i}{r_{\phi(i)}} (1 - p_i(E))^{n_i - r_{\phi(i)}} p_i(E)^{r_{\phi(i)}} \tag{6.46}$$

where $p_i(E)$ is the probability that the eavesdropper E receives the packet transmitted by node i given that it has been received successfully by all the remaining nodes.

To derive $p_i(E)$, we proceed as follows.

The probability that node i successfully transmits the packet to all of the other nodes at time slot k through the public channel is the probability that the nodes receive the packet successfully up to time slot k minus the probability that they receive the packet successfully up to time slot $k-1$:

$$Pr(T_{public,i} = k) = Pr(T_{public,i} \leq k) - Pr(T_{public,i} \leq k-1) \tag{6.47}$$

By substituting the value of $Pr(T_{public,i} \leq k)$ from equation 6.39, we get the following:

$$Pr(T_{public,i} = k) = \sum_{X \in \{0,1\}^K} \prod_{j=1, j \neq i}^{K} -I(x_j)(1-p_{public,ij})^k - \sum_{X \in \{0,1\}^K} \prod_{j=1, j \neq i}^{K} -I(x_j)(1-p_{public,ij})^{k-1} \tag{6.48}$$

Given that node i successfully delivers the packet to the remaining nodes in exactly k time slots, the eavesdropper can receive the packet at any of the k time slots, and hence the conditional probability that the eavesdropper receives the packet knowing that node i successfully delivers the packet to the remaining nodes in exactly k time slots is given by:

$$\begin{aligned} Pr(E|T_{public,i} = k) &= \sum_{t=1}^{k} (1-p_{iE})^{t-1} p_{iE} \\ &= 1-(1-p_{iE})^k \end{aligned} \tag{6.49}$$

Thus, the probability $p_i(E)$ is given by:

$$\begin{aligned}
p_i(E) &= \sum_{k=1}^{\infty} Pr(E|T_{public,i}=k) \times Pr(T_{public,i}=k) \\
&= \sum_{k=1}^{\infty}(1-(1-p_{iE})^k) \times \Bigg(\sum_{X\in\{0,1\}^K} \prod_{j=1,j\neq i}^{K} -I(x_j)(1-p_{public,ij})^k \\
&- \sum_{X\in\{0,1\}^K} \prod_{j=1,j\neq i}^{K} -I(x_j)(1-p_{public,ij})^{k-1} \Bigg) \\
&= \sum_{X\in\{0,1\}^K} \frac{\prod_{j=1,j\neq i}^{K} -I(x_j)(1-p_{public,ij})}{1-\prod_{j=1,j\neq i}^{K}(1-p_{public,ij})} - \sum_{X\in\{0,1\}^K} \frac{\prod_{j=1,j\neq i}^{K} -I(x_j)}{1-\prod_{j=1,j\neq i}^{K}(1-p_{public,ij})} \\
&- \sum_{X\in\{0,1\}^K} \frac{\prod_{j=1,j\neq i}^{K} -I(x_j)(1-p_{public,ij})(1-p_{iE})}{1-\prod_{j=1,j\neq i}^{K}(1-p_{public,ij})(1-p_{iE})} \\
&+ \sum_{X\in\{0,1\}^K} \frac{\prod_{j=1,j\neq i}^{K} -I(x_j)(1-p_{iE})}{1-\prod_{j=1,j\neq i}^{K}(1-p_{public,ij})(1-p_{iE})} \qquad (6.50)
\end{aligned}$$

Then, the probability that the eavesdropper receives n or more packets given that the nodes in S transmit through the public channel is given by:

$$Pr(M_E \geq n) = \sum_{t=n}^{n_{total}} Pr(M_E = t) \qquad (6.51)$$

where n_{total} is the total number of packets available at the nodes in S i.e. $n_{total} = \sum_{i\in S} n_i$

Then, the total security cost $Scost_{ARQ}$ is computed by adding the security costs of all the nodes transmitting through the private channel. Hence,

$$Scost_{ARQ} = \sum_{l\in G\setminus S} w_l \qquad (6.52)$$

where $G \setminus S$ is the complement of S in G.

The optimization problem becomes:

(P5) $Min_{S\in Part(G)} Scost_{ARQ}$

Subject to:

$Pr(M_E \geq n) \leq \lambda$

where $Part(G)$ is the partition set of G i.e. the set of all possible subsets of G

6.3.2.2 DNC Case

To formulate the problem for this case, we define the following:

- Let n_i be the number of packets available at node i.
- Let S be the set of nodes transmitting all their packets through the public channel.
- Let S' be the set of subsets of S such that every subset V of S must satisfy: $\sum_{j \in V} n_j \geq n$
- Let M_{iE} be a random variable that represents the number of of packets that the eavesdropper receives successfully from node i.

As was discussed in the single-link case, the network-coded packets are constructed such that the eavesdropper can not decode any of the packets except if the eavesdropper receives successfully all the network coded packets from the transmitting node. Hence, the probability that the eavesdropper receives n or more packets, given that the nodes in S are transmitting through the public channel, is computed by considering all the possible subset of nodes such that the sum of their transmitted packets is greater than or equal to n and thus the probability is given by:

$$Pr(M_E \geq n) = \sum_{V \in S'} \prod_{j \in V} Pr(M_{jE} = n_j) \tag{6.53}$$

The probability that the eavesdropper decodes successfully the n_j packets from node j is given by:

$$Pr(M_{jE} = n_j) = p_j(E)^{n_j} \tag{6.54}$$

where $p_j(E)$ is the probability that the eavesdropper E receives successfully the packet transmitted by node j, given that it has been received successfully by all the remaining nodes. The probability $p_j(E)$ has the same expression as that in equation (6.50).

In the case of DNC, each node has to transmit at most one packet through the private channel. Hence, the security cost of node i transmitting through the private channel is given by:

- Delay cost:

$$w_i = E[T_{private,i}] - E[T_{public,i}] \tag{6.55}$$

- Energy cost:

$$w_i = E[\xi_{private,i}] - E[\xi_{public,i}] \tag{6.56}$$

where the expressions $E[T_{private,i}]$, $E[T_{public,i}]$, $E[\xi_{private,i}]$, $E[\xi_{public,i}]$ are the same given by equations 6.40, 6.41, 6.43, and 6.44 respectively.

Then, the total security cost $Scost_{DNC}$ is computed by adding the security costs of all the nodes transmitting through the private channel. Hence,

$$Scost_{DNC} = \sum_{l \in G \setminus S} w_l \tag{6.57}$$

The optimization problem becomes:

(P6) $Min_{S \in Part(G)} Scost_{DNC}$

Subject to:

$Pr(M_E \geq n) \leq \lambda$

6.3.3 Solution

For both cases of ARQ and DNC, the expression of the probability $Pr[M_E \geq n]$ in equations (6.51) and (6.53) is dependent on S and is quite complicated. Hence, in order to find the optimal set of nodes that solves the minimization problem, it is necessary to consider all possible subsets of the set of nodes G which of course has exponential computational complexity.

In order to simplify the analysis, we analyze the following special case in which there is a lot of symmetry.

6.3.4 Special Case

All nodes have the same public channel quality amongst each other and with the eavesdropper, but not necessarily the same private channel quality amongst each other. Also, we assume that the nodes have equal number of packets. Let I be the number of packets available at each node.

We denote by σ_E^2 the variance of the channel between each node and the eavesdropper, γ_E the SNR threshold at the eavesdropper, σ_{public}^2 the variance of the channel between each pair of nodes, and γ_{public} the SNR threshold at each node when receiving through the public channel.

Thus for any node transmitting through the public channel, the probability

$p(E)$ that the eavesdropper receives successfully a packet is the same irrespective of which node is transmitting and can be derived as in equation (6.50).

6.3.4.1 ARQ Case

Similar to equation (6.8), the probability that the eavesdropper receives n or more packets given that the number of packets M_{public} transmitted through the public channel is m, is given by:

$$Pr(M_E \geq n | M_{public} = m) = \sum_{i=n}^{m} \binom{m}{i} (1 - p(E))^{m-i} p(E)^i \tag{6.58}$$

The security cost w_i of node i if it's transmitting through the private channel is given by equations (6.36) and (6.42).

In this case, the solution to the optimization problem $P5$ is given by the following proposition.

Proposition:

- The optimal number of nodes K^* that should transmit through the private channel is given by:

  $K^* = K - \lfloor M^*_{public}/I \rfloor$ where M^*_{public} is the maximum number of packets that can be transmitted through the public channel. Also, M^*_{public} is the highest integer that satisfies.

$$Pr(M_E \geq n | M_{public} = M^*_{public}) \leq \lambda \tag{6.59}$$

- The optimal set of nodes that should transmit through the private channel are the nodes that have the lowest security cost w_i.

Proof:

Based on equation (6.52), the security cost is additive (it is the sum of the security costs of each node transmitting through the private channel). Hence, the cost decreases as the number of nodes transmitting through the private channel decreases. Also, the cost decreases as the individual security costs of the nodes transmitting through the private channel are lower.

However, the number of nodes that should transmit through the private channel is lower bounded by:

$K_L = K - K^*_{public}$

where K^*_{public} is the maximum number of nodes that can transmit through the public channel. Since the public channel among the nodes are identical and each node has I packets, K^*_{public} can be expressed as:

$K^*_{public} = \lfloor \frac{M^*_{public}}{I} \rfloor$

where M^*_{public} is the maximum number of packets that can be transmitted through the public channel. Also, the probability that the eavesdropper receives n or more packets in equation (6.58) has the same expression as equation (6.7), and it is shown in section 6.2 that this probability is an increasing function of the number of packets that can be transmitted through the public channel. Hence, M^*_{public} is the highest integer that satisfies:

$$Pr(M_E \geq n | M_{public} = M^*_{public}) \leq \lambda \tag{6.60}$$

The value of M^*_{public} can be calculated using the same iterative method proposed in part 6.2.2.

6.3.4.2 DNC Case

In this case, the probability that the eavesdropper receives successfully the I coded packets transmitted by node i through the public channel is given by:

$$Pr(M_{iE} = I) = p(E)^I \tag{6.61}$$

The probability that eavesdropper receives n or more packets given that k nodes are transmitting through the public channel is given by:

$$Pr(M_E \geq n) = \sum_{j=t}^{k} \binom{k}{t} (1 - p(E)^I)^{k-t} p(E)^{It} \tag{6.62}$$

where t is the lowest integer such that $tI \geq n$

In this case, the solution to the optimization problem $P6$ is given by the following proposition.

Proposition:

- The optimal number of nodes K^* that should transmit through the public channel is the highest integer that satisfies:

  $Pr(M_E \geq n) \leq \lambda$

- The optimal set of nodes that should transmit through the private channel are the nodes that have the lowest security cost w_i.

Proof:

Based on equation (6.57), the security cost is additive (it is the sum of the nonnegative security costs of each node transmitting through the private channel). Hence, the cost decreases as the number of nodes transmitting through the private

channel decreases. Also, the cost decreases as the individual security costs of the nodes transmitting through the private channel are lower. However, the number of nodes that should transmit through the private channel is lower bounded by $G - K^*$ where K^* is the maximum number of nodes that should transmit through the public channel. Also, it can be easily shown (based on equation (6.62)) that the probability that the eavesdropper receives n or more packets through the public channel is an increasing function of k, that is the number of nodes that transmit through the public channel (since the cost has similar expression as equation 6.8 which has been proven in section 6.2 to be increasing). Hence, the maximum number of nodes K^* that should transmit through the public channel is the highest integer that satisfies:
$Pr(M_E \geq n) \leq \lambda$
The value of K^* can be found using the same iterative technique explained in part 6.2.2.

6.3.5 Numerical Results

In this part, we compare for the distributed case the performance of the secure transmission using ARQ to the case when Deterministic Network Coding is used. Also in this part, we consider the symmetric case in which the nodes have same public channel quality among each others and with the eavesdropper, and that the nodes have equal number of packets. Furthermore, we consider that the nodes have the same private channel quality among each others.

As for the values, we consider 7 nodes in the network where each node has 3

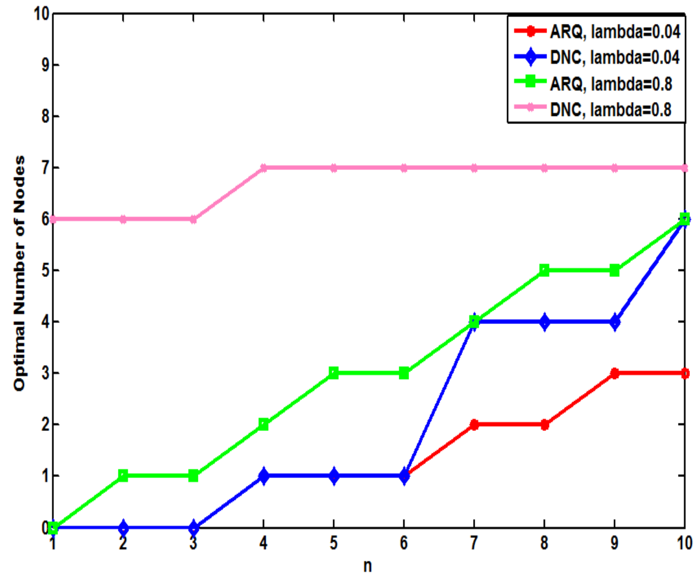


Figure 6.7: *Optimal number of nodes as a function of the security level parameter n.*

packets. As for the other system's parameters, we consider the following values:

$P = 1W$, $N_0 = 0$ dB, $\sigma_E^2 = 9$ dB, $\gamma_E = 10$ dB, $\sigma_{private}^2 = 10$ dB, $\gamma_{private} = 13$ dB, $\sigma_{public}^2 = 10$ dB, $\gamma_{public} = 7$ dB.

For both ARQ and Network Coding, the optimal number of nodes that should transmit through the public channel, and the minimum delay cost is computed for the case when the security parameter n is varied between 1 and 10 packets. Two values of λ are considered respectively: $\lambda = 0.04, 0.8$.

The optimal number of nodes and the minimum security cost for each case are shown in figures 6.7 and 6.8 respectively.

Figure 6.7 shows that for both cases of ARQ and Network Coding, the optimal number of nodes that transmit through the public channel increases as the security parameter n increases. Also for both considered values of λ, the optimal number

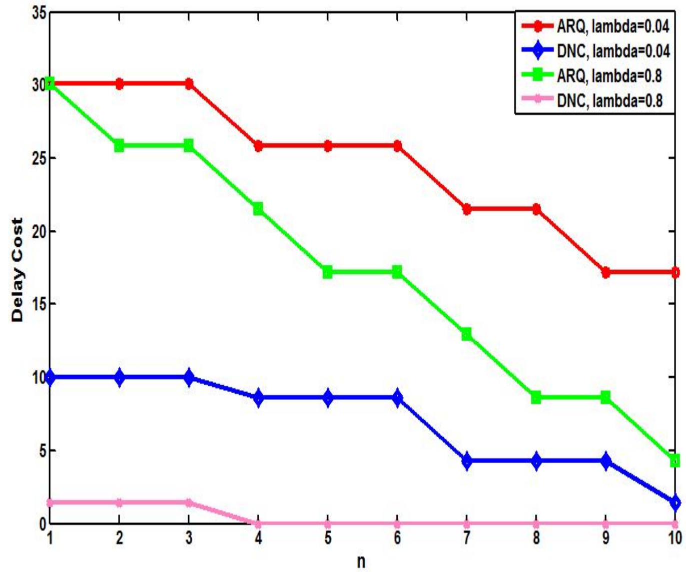


Figure 6.8: *Security cost as a function of the security level parameter n.*

of nodes is higher for the case of Network Coding than the case of ARQ. This is because in the case of Network Coding, the eavesdropper should decode all the packets transmitted by each node. This consequently decreases the probability that the eavesdropper receives the target number of packets, and hence for high values of λ all the nodes can transmit using Network Coding through the public channel. Also, figure 6.8 shows for both considered values of λ, the security cost using Network Coding is lower than the case of ARQ. There are two main reasons for that. The first one is that the number of nodes that should transmit through the public channel is higher for the case of Network Coding than ARQ as shown in figure 6.7. The second reason is that each node needs to transmit only one packet through the private channel in the case of Network Coding while in the case of ARQ each node has to transmit all its packets through the private channel, which again emphasizes the gains achieved by using Network Coding for secure transmissions. Similar conclusions can be drawn for different values of the systems parameters.

6.4 Summary

We considered the issue of secure transmissions in a wireless fading network in which a file is required to be delivered while keeping it secure from an eavesdropper. Two transmissions schemes were proposed: the first was based on simple ARQ while the second was based on deterministic Network Coding. The results show the tradeoff between achieving a certain security level and the cost incurred to achieve a required level of security. Also, the results show that Network Coding considerably reduces the security cost compared to the case when simple ARQ is used. Overall, this work constitutes a modest but important step towards resolving any important problem of wireless networking that combines security issues with performance costs and distributed operation.

Chapter 7

Minimum Energy Scheduling of Base Stations with Sleep Modes

7.1 Overview

This chapter extends the problem of minimizing the consumed energy in a Macro cell discussed in chapter five to the case of a network of multiple cells. The case of multiple cells is of a higher complexity due to the following: The received rate at each user is affected by the interference caused by other base stations transmitting simultaneously with the base station serving that user. Hence, the problem in this case will be transformed to a scheduling problem which should select the groups of base stations to be activated simultaneously in order to minimize the total consumed energy in the network. Also similar to the single cell case, we assume that each base station reduces its power and switches to Micro sleep mode when it completes serving the users in its cell, and hence it is anticipated that the sleep mode power values of the base stations will affect the optimal activation groups and the minimum energy consumed in the network.

The problem of scheduling multiple transmitters and its complexity has been extensively studied in the past (see e.g. [63]-[66]) where the optimal solution is usually found for special cases of the general scheduling problem, but most of these works minimize the time required to serve the users. The work in [67] considers a very similar model to the one that we consider in this chapter. However, the objective in [67] is to minimize the emptying time of the network i.e. the total time needed to serve all the users in the network, and hence it does not take the energy consumed into consideration. Also in [67], the conditions that make particular scheduling techniques optimal (such as the optimality of time division) are presented. Hence, the interesting question that arises: Is the optimal solution to the minimum energy problem the same as for the minimum time problem? If not, under what conditions are the optimal solutions the same?

Thus in this chapter, we will first present the minimum time scheduling problem studied in [67] . Based on the minimum time problem, we will then formulate the problem of minimum energy scheduling of base stations. Due to time limitations, the problem of minimum energy scheduling is not fully developed and is interesting to pursue for future work.

7.2 Minimum Time Scheduling Problem

The problem studied in [67] considers a wireless network composed of N transmitter-receiver pairs. Each transmitter is required to deliver the requested data to its receiver, and it is assumed that the data is available at each of the transmitters. It is also assumed that rate at each receiver depends on which group of transmitter-receiver pairs are activated simultaneously. Two models of the rate are considered: the first one assumes the rate to be a function of the Signal to Interference Noise Ratio (SINR). The second model is a special case of the first model, and it assumes that the rate depends on the cardinality of the activation group. It is then required to find the optimal activation groups and the optimal activation time of each group that minimize the time required the receivers' demands. In order to formulate the problem, the following lemma is used:

Lemma 7.1: There exists an optimal schedule such that, before reaching the end of the time duration of a group, none of the link queues in the group is empty.

The full proof of the lemma can be found in [67].

Using this lemma, the problem has a linear programming formulation as follows:

$$\begin{aligned}
&min \sum_{S \in H} \tau_S \\
&s.t. \sum_{S \in H} r_{iS}\tau_S = B_i \qquad\qquad \forall i = 1, 2, ..., N \\
&\tau_S \geq 0
\end{aligned} \tag{7.1}$$

where S is an activation group, τ_S is the activation duration of group S, H is the set of all possible activation groups, B_i is the demand of receiver i, r_{iS} is the rate at receiver i when group S is activated.

In general, the cardinality of the activation group set H is exponential in the number of transmitter-receiver pairs and hence it is complex to solve the linear program. Thus in [67], the authors present the conditions under which some of the basic scheduling strategies are optimal. The scheduling strategies considered are: the first strategy is time division (i.e. each of the links is activated solely) while the second strategy is when all links are activated simultaneously. Here, we will present the condition under which time division is optimal through the following theorem.

Theorem 7.1: For any group $S \in H$, the sum of the ratios between the members' rates in S and their respective rates of being served individually, is at most 1, that is,

$$\sum_{i \in S} \frac{r_{iS}}{r_{ii}} \leq 1 \tag{7.2}$$

7.3 Single User Cells

7.3.1 System Model

We consider a cellular network K composed of N cells. In each cell i, base station station i is required to deliver B_i bits to a user i. It is assumed that the queue at each base station is saturated. Also, base station i is transmitting with power value P_{iT} to user i. The consumed power P_{iC} by base station i is a linear function of the transmission power P_{iT}. Also, it is assumed that the base station can reduce its consumed power and switch to macro sleep mode during the time it is deactivated and not delivering the load to user i. Hence, the consumed power P_{iC} at base station i follows a piecewise linear model and is given by the following expression:

$$P_{iC} = \begin{cases} s_i, & P_{iT} = 0 \\ \\ \triangle_{iP} P_{iT} + P_{i0}, & 0 < P_{iT} \leq P_{max} \end{cases} \tag{7.3}$$

where the values of the linear model parameters $\triangle_{iP}$ and P_{i0} depend on the base station type, P_{max} is the maximum transmission power, and the parameter s_i is the consumed power value when base station i is in the sleep mode ($s_i \leq P_{i0}$). The received power P_{ijR} at user j from base station i follows the pathloss power model given by:

$$P_{ijR} = AP_{iT}|d_{ij}|^{-\alpha} \tag{7.4}$$

where α is the pathloss exponent, d_{ij} is the distance from user j to base station i, and A is a constant which accounts for system losses. Also, AWGN is present at each user with power spectral density N_R.

In order to deliver the target load to each user, it is required to find which group of base stations should be activated simultaneously, and the time duration that the group should be activated in order to minimize the total consumed energy of the network.

The problem formulation is explained in the following section.

7.3.2 Problem Formulation

In order to formulate the problem, we define the following entities.

Let S be the current activated set of base stations and let H be the set of all possible activation sets. It is assumed that the rate r_{iS} of each base station $i \in S$ follows the SINR model i.e.

$$r_{iS} = Wlog(1 + SINR_{iS}) \tag{7.5}$$

where W is the transmission bandwidth value (in Hz) and $SINR_{iS}$ is the signal to interference noise ratio from base station i to user i when group S is activated. It is given by:

$$SINR_{iS} = \frac{AP_{iT}|d_{ii}|^{-\alpha}G_{ii}}{\sum_{k\in S, k\neq i} AP_{kT}|d_{ki}|^{-\alpha}G_{ki} + N_R} \tag{7.6}$$

where G_{ki} is the channel gain between base station k and user i and N_R is the variance of the receiver's noise. It is assumed that the values of the channel gains are known at each base station.

Let τ_S be the duration of time group S is activated. The energy spent during the activation period of group S is:

$$E_S = (\sum_{i \in S}(\triangle_{iP} P_{iT} + P_{i0}) + \sum_{j \in K \setminus S} s_j)\tau_S \tag{7.7}$$

Define the quantity e_S such that:

$$e_S = \sum_{i \in S}(\triangle_{iP} P_{iT} + P_{i0}) + \sum_{j \in K \setminus S} s_j \tag{7.8}$$

In order to formulate the energy minimization problem, we will formulate a lemma similar to lemma 7.1 (which is used in [67]). The lemma states the following:

Lemma 7.2: There exists an optimal schedule such that before reaching the end of the time duration of a group, none of the base stations has completed delivering the load to the designated user.

The proof of the lemma is similar to the proof of lemma 7.1 and is as follows: Assume the opposite. Then, there exist a group S with activation period τ and a cell i such that base station i has completed delivering the load to user i. Let t' be the completion time of delivery of user i load. Consider spitting the time duration τ into two time durations such that the first is of period t' and group S is activated while the second is of period $\tau - t'$ such that group $S \setminus i$ is activated.

Then, the energy minimization problem has the following LP formulation:

$$\begin{aligned}
&min \sum_{S \in H} e_S \tau_S \\
&s.t. \sum_{S \in H} r_{iS} \tau_S = B_i \qquad\qquad \forall i = 1, 2, ..., N \\
&\quad \tau_S \geq 0
\end{aligned} \tag{7.9}$$

Since the linear program depends on the piecewise power model parameters and the users load, it is anticipated that the optimal schedule will also depend on these values. In the following theorem, we derive the conditions (as was done for the minimum length scheduling problem in [67]) in which time division is the optimal solution.

Theorem 7.2: Time division is optimal if and only if the following condition is satisfied:

$$\sum_{i \in S} \frac{e_i r_{iS}}{r_{ii}} \leq 1 \tag{7.10}$$

For all $S \in H$ where e_i is the total consumed power when base station i is activated.

The proof follows the same argument done [67], the only difference is that the vector of costs of the basic variables will be $(e_1, e_2, ..., e_N)$ where e_i is the total consumed power when base station i is transmitting.

Assuming that the consumed power at each base station i is greater than one, we can easily see based on theorems 7.1 and 7.2 that when time division is optimal for the minimum energy problem, it is also optimal for the minimum time problem.

7.4 Multiple Users Cells

7.4.1 System Model

We consider a cellular network K composed of N cells. Each cell i is composed of base station i and M_i users. Base station i is required to deliver B_{ij} bits to user j ($1 \leq j \leq M_i$). Also, base station i can transmit with power P_{iT} and assigns transmission power value P_{ijT} to user j (such that $\sum_{j=1}^{M_i} P_{ijT} = P_{iT}$). It is assumed that the power values are integers. As in section 7.3, the consumed power P_{iC} by base station i is a linear function of the transmission power P_{iT} and is given by equation 7.3. The received power P_{ijR} at user j from base station i follows the pathloss power model given by:

$$P_{ijR} = AP_{iT}|d_{ij}|^{-\alpha} \tag{7.11}$$

where α is the pathloss exponent, d_{ij} is the distance from user j to base station i, and A is a constant which accounts for system losses. Also, AWGN is present at each user with power spectral density N_R.

In order to deliver the target load to each user, it is required to find which group of base stations should be activated simultaneously, the power values that each base station assigns to each user, and the time duration that the group with a specific power assignment should be activated in order to minimize the total consumed energy of the network.

7.4.2 Problem Formulation

In order to formulate the problem, we define the following variables:

Let S be the activated group of base stations and let H be the the set of all possible activation groups.

Let π_i be a power assignment vector of base station i to its M_i users. Let Ψ_i be the set of all possible power assignment vectors of base station i.

Let π_S be the vector of power assignment vectors of the base stations in the activation set S and let Ψ_S be the set of all possible power assignment vectors of activation group S.

Let Ψ_H be the set of all possible power assignment vectors of all possible activation groups i.e.

$\Psi_H = \bigcup_{S \in H} \Psi_S$

The received rate at user j $(1 \leq j \leq M_i)$ of base station i when group S is activated with power assignment π_S follows the SINR criterion and is given by:

$$r_{j(S,\pi_S)} = Wlog(1 + SINR_{j(S,\pi_S)}) \tag{7.12}$$

where the SINR at user j of base station i when group S is activated with power assignment π_S is given by the following expression:

$$SINR_{j(S,\pi_S)} = \frac{AP_{ijT}|d_{ij}|^{-\alpha}G_{ij}}{\sum_{k \in S, k \neq i} AP_{kjT}|d_{kj}|^{-\alpha}G_{kj} + N_R} \tag{7.13}$$

where G_{ij} is the channel gain between base station i and user j and d_{ij} is the distance between base station i and user j.

let $\tau_{(S,\pi_S)}$ be the activation duration of group S with power assignment vector

π_S. The energy spent during the activation time of group S with power assignment vector π_S is independent of π_S and is given by equation 7.7.

Thus and using the same argument as lemma 7.2, the energy minimization problem has the following LP formulation:

$$\begin{aligned} min \quad & \sum_{(S,\pi_S)\in H\times\Psi_H} e_S \tau_{(S,\pi_S)} \\ s.t. \quad & \sum_{(S,\pi_S)\in H\times\Psi_H} r_{i(S,\pi_S)}\tau_{(S,\pi_S)} = B_i \qquad \forall i = 1,2,...,N \\ & \tau_{(S,\pi_S)} \geq 0 \end{aligned} \tag{7.14}$$

where the couple (S, π_S) corresponds to a particular activation set and a particular power assignment of the base stations in set S and e_S is given by equation 7.8. This problem is of high complexity because the number of possible power allocations that base station i can assign to its M_i users is exponential in the number of users. Also, the number of possible groups of base stations that can be activated together is exponential in the number of base stations.

In order to reduce the complexity of the problem, one possibility is to restrict the power allocation policies that each base station can use to distribute its power on its users. These policies are:

- Equal Power Policy: In which each base station serves its users simultaneously and divide its transmission power equally among its users.

- Distance-Based Policy: In which each base station serves its users simultaneously; however unlike the equal power policy, each base station assigns higher power value to the user with the lower distance. Hence under this policy, base station i assigns power value to user j according to the following method. First, the indices of the users are arranged in ascending order of their distance where the new index t corresponds to the user with the t^{th} lowest distance. Then, the power assigned to user t by base station i is:

$$P_{itT} = \frac{d_{it'} P_{iT}}{\sum_{t=1}^{M_i} d_{it}} \tag{7.15}$$

where $t' = M_i - t$

Hence under each of the presented policies, the complexity of the problem will be only exponential in the number of base stations, and the problem will be formulated as in 7.9.

7.5 Summary

In this chapter, we have formulated the minimum energy scheduling problem in a wireless network first for the case when a user is present in each cell and then for the general case when multiple users are present in each cell. Following the steps done in [67], both problems were formulated as linear programs, and the complexity of the number of variables of each problem is demonstrated. It remains interesting to design efficient techniques to solve the linear programs associated with the minimum energy problems.

Chapter 8

Conclusion

8.1 Summary of Contributions

The main contributions of this thesis are the proposed novel joint physical-network layer techniques, evaluating their performance while taking energy into consideration and studying the tradeoff between energy and other performance metrics such as throughput and delay. In the second chapter, we considered discrete time packetized transmissions over a wireless link and studied the tradeoff between energy and delay. We considered the time varying nature of the wireless channel by representing the channel by the markov chain model. Also, we proposed rate and power control techniques respectively where the transmitter changed its rate/power based on Channel State Information, and we showed the performance gain achieved by using Channel State Information compared to the case when the transmitter knows the average channel quality. These results are presented in [68].

In the third chapter, we studied the tradeoff between throughput, energy, and delay but for a single hop multicast network. We considered the problem of streaming a real-time file with finite energy and delay constraints while the objective was to maximize the number of packets received by the receivers. Also as in chapter two, markov chain model was used to represent the time variations of the wireless channel. Moreover, we studied the effect of using Random Network Coding as a transmission method on the achieved performance. Our results showed that Random Network Coding benefited receivers with good channel quality while it harmed receivers with bad channel quality. Also, this chapter reinforced the findings of chapter two of the importance of having Channel State Information. The results are presented in [69].

In chapter four, we focused on the effect of cooperation on the energy efficiency of wireless transmissions, and we proposed several joint physical-network cooperative techniques. Some of these techniques incorporated Random Network Coding at the network layer and/or Alamouti space-time codes at the physical layer. We considered two system models: the first was the simple relay network while the second was the simple multicast network, and we evaluated the cooperative protocols by computing the minimum energy required per successfully delivered packet. We then studied the tradeoff between the minimum energy consumed and the maximum stable throughput achieved. We showed that Random Network Coding can achieve performance gains compared to Alamouti-only-based cooperation techniques. Further, we showed that techniques that combined both Random Network Coding and Alamouti codes can achieve the best performance. The results are shown in [70] and [71].

In chapter five, we switched to studying techniques for energy efficient cellular systems, and in particular we considered a downlink scenario and we proposed a rate allocation method to minimize the energy consumed in a Macro cell. This method takes into account the "Micro" sleep mode feature of current base stations. We proposed both time division scheduling and frequency division scheduling. Although there is an earlier work that considered the uplink scenario and that considered a similar problem formulation for the case of time division, our frequency division formulation and the comparison between the time and frequency scheduling techniques were not considered earlier. The results are presented in [72].

Another main contributions of this thesis are investigating the effect of using of Network Coding on securing wireless transmissions and studying the tradeoff between security and energy/delay. In chapter six, we considered the problem of distributed information exchange of a file among a group of adjacent wireless nodes in the presence of an eavesdropper and with the choice of the nodes transmitting through public channels or more expensive private channels. We addressed the tradeoff between security and energy/delay by expressing the cost of using the private channel by energy/delay costs, and we minimized the total security cost while achieving the required level of security. We proposed a deterministic Network Coding transmission strategy and showed the vast performance gains achieved by using Network Coding. The results are presented in [73]

Last, we introduced the problem of minimum energy scheduling for a cellular network of several cells. Similar to chapter four, the scheduling technique assumed that the base stations had "Micro" sleep mode. We compared the minimum energy scheduling problem to a previous work that the considered a similar scheduling problem but minimized the time required to serve the users. Also, we extended the model considered in the previous work by considering several users in each cell, and we showed the additional complexity associated with this case.

8.2 Future Work

There are several ideas that are associated with the problems considered in this thesis and are beneficial to pursue in the future. The first issue is fairness. In chapter three, we considered the problem of maximizing the total number of packets received by all receivers under energy and delay constraints; however, we did not offer any guarantees on the minimum number of packets that each receiver should receive. In chapter six, we minimized the total energy/delay security costs by selecting which nodes should transmit through the private channel; however, we did not consider individual energy constraints on the nodes. In other words, nodes that transmit through private channels consume more energy than nodes that transmit through public channel. Hence, it is essential to rethink these problem while taking fairness into account.

For the distributed exchange problem discussed in chapter six, the nodes were assumed to be transmitting with fixed power. Hence, one could incorporate power control techniques and investigate their effect on the security cost. Also for the general case of multiple nodes, it is of interest to develop heuristics that are close to optimal and that allow the file to be delivered successfully to all the nodes in the network without special symmetry assumptions and without the restriction that the parts of the file residing at each node are distinct.

As pointed out in chapter seven, the problem of minimum energy scheduling was introduced but not fully addressed. It remains important to design efficient heuristics that reduce the complexity the problem and are close to optimality. Further, it is again essential to consider fairness in this problem. Our model minimizes the energy consumed in the network while did not put constraints on the time each user is served.

Also, some of the problems considered in this thesis were based on simple single hop networks as we were more focused on assessing the performance of the different proposed physical-network layer techniques. Thus, it is beneficial to consider these problems, as a next step, in more general multihop networks.

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