

## ABSTRACT

Title of dissertation: ESSAYS ON SYSTEMIC FINANCIAL CRISES,  
DEFAULT AND PECUNIARY EXTERNALITIES

Rocio Gondo Mori, Doctor of Philosophy, 2013

Dissertation directed by: Professor Anton Korinek  
Department of Economics

The first chapter analyzes how default externalities lead to an excessive incidence of systemic private debt crises. An individual defaulting borrower does not internalize that her default leads to a depreciation in the exchange rate because international lenders will sell any seizable assets and flee the country. The exchange rate depreciation in turn reduces the value of non-tradable collateral and induces other borrowers to default, leading to a chain reaction of defaults. The inefficiency of default spillovers can be corrected by strengthening the enforcement of creditor rights, so that private individual borrowers have less incentives to default, reducing the incidence of systemic default episodes.

The second chapter analyzes the implications of developing financial markets for contingent assets on the degree of risk sharing, the incidence of systemic financial crises and credit externalities through collateral prices in emerging economies with limited access to international capital markets. We find that, in an environment with persistent shocks and collateral constraints, even though agents cannot engage in full risk sharing, access to state contingent assets improves the degree of hedging,

reduces the need for precautionary savings and lowers the incidence of financial crises. In addition, it further reduces the spillover effect of credit externalities by dampening the effect of an individual's borrowing on the valuation of collateral. In this way, borrowers face financial crises less frequently and are less debt-constrained in states where they need to borrow the most, which improves risk sharing.

ESSAYS ON SYSTEMIC FINANCIAL CRISES,  
DEFAULT AND PECUNIARY EXTERNALITIES

by

Rocio Mercedes Gondo Mori

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Advisory Committee:  
Professor Anton Korinek, Chair/Advisor  
Professor Pablo D'Erasmus  
Professor Sebnem Kalemli-Ozcan  
Professor Carlos Végh  
Professor Philip Swagel

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## Chapter 1: Overview

This dissertation analyzes the positive and normative implications of systemic financial crises in emerging economies, in terms of the amplification mechanism created by the effect of individual agent's borrowing decisions as a source of spillover effects to other agents in the economy, through its effect on relative prices. Chapter 2 focuses on the behavior of capital flows, the exchange rate and private default in a small open economy with access to foreign borrowing using collateralized debt. A default spillover effect arises because private agents fail to internalize the effect of individual default on the default incentives of other agents, by reducing the value of collateral.

Chapter 3 discusses the benefits of having access to state contingent debt in terms of risk sharing and the frequency of financial crises in emerging economies with limited access to international capital markets. It also analyzes the normative implications in terms of the spillover effects of individual debt on other borrowers' collateral constraints.

In Chapter 2, we analyze the behavior of capital flows, the exchange rate and the frequency of default during systemic private debt crises in a small open economy where private agents take collateralized debt in an environment with imperfect

enforceability of creditor rights. It differs from previous work by analyzing the amplification mechanism created by the existence of default spillover effects during episodes of private debt crises. Individual default generates capital outflows due to the inability of debt rollover. Capital outflows lead to a real exchange rate depreciation, which lowers the value of non-tradable collateral of other borrowers. This creates a debt overhang problem, as the value of collateral becomes smaller relative to the value of debt service, which triggers default of other agents. Default spillover effects generate an amplification mechanism, as all agents have higher incentives to default, increasing the incidence of default and the default risk premium and reducing the ability to borrow from abroad.

In addition, this work analyzes the normative implications of default spillover effects. This mechanism creates an inefficiency in the borrowing and default behavior of private agents because, by taking prices as given, they ignore the effect of their decisions on the valuation of collateral and through it on the default incentives of other agents. Individual borrowers default more frequently than socially optimal by ignoring that their individual default creates a chain reaction of defaults. A defaulting agent leads to a real depreciation because international lenders seize any collateral they can and flee the country. The exchange rate depreciation lowers the value of non-tradable collateral, which induces other agents to default as well.

In order to correct this distortion, optimal policy is targeted to align default incentives with the socially optimal default set, by imposing a higher default penalty or reducing the cost of repayment, so that private agents have lower incentives to default. Higher enforcement of creditor rights under the private equilibrium is needed

to align default incentives to the socially efficient. This differs from previous work, which focuses on the optimal enforcement of creditor rights from the perspective of a social planner who can choose the level of enforcement. Due to borrowers' impatience, it is optimal to fully enforce creditor rights and increase the ability to borrow more each period.

In Chapter 3, we present an analysis of the effects of developing financial markets for contingent assets on the degree of risk sharing and their effect on reducing the size of spillover effects due to credit externalities. It differs from previous work by presenting a detailed analysis of the risk sharing properties of this financial instrument in an environment where an amplification mechanism is triggered through the spillover effects of individual borrowing on other agents' borrowing through the valuation of collateral.

The results in this chapter show that, in an environment with persistent shocks and collateral constraints, having access to state contingent assets allows for partial hedging against income fluctuations, which reduces the need for precautionary savings and lowers the incidence of financial crises. Each borrower trades current tradable income for its expected value, which is positively correlated to the actual shock but less volatile. Partial exposure to the endowment shock creates a need for accumulating precautionary savings, but in a smaller magnitude than in an environment with bonds only. This result is consistent with previous literature, where adding access to state contingent bonds reduce the size of precautionary savings and lead to large transitional gains in terms of welfare.

Additionally, state contingent debt provides partial hedging against the col-

lateral constraint. Pro-cyclical interest payments loosen the collateral constraint in bad states and tightens it in good states of nature, which improves risk transfer across time. Agents are better able to smooth consumption by increasing their ability to borrow more in bad states, when the marginal benefit of debt is higher, at the expense of lowering the collateral limit in good times when the marginal benefit of debt is lower.

Given the need for precautionary savings, there is a spillover effect as private agents do not internalize that individual borrowing decisions reduces the price of non-tradable collateral and therefore tightens the collateral constraint of other agents. Therefore, private agents take excessive total debt. In addition, they take too little state contingent debt by ignoring the hedging properties of this instrument against the tightening of the collateral constraint in bad states.

## Chapter 2: Default Externalities and Systemic Private Debt Crises in Emerging Markets

### 2.1 Introduction

Default episodes in emerging market countries are characterized by repossession of collateral, large capital outflows and real exchange rate depreciation. When default takes place, lenders immediately seize whatever collateral they can and convert it into tradable goods. The repossession of collateral in terms of tradable goods leads to capital outflows because lenders seize collateral and repatriate it, whereas borrowers are excluded from international capital markets and cannot rollover their debt. Lenders convert their repossessed assets in domestic currency into foreign currency so an exchange rate depreciation takes place, which in turn affects the valuation of domestic currency collateral of other loans.

This work analyzes the behavior of capital flows, the exchange rate and the frequency of default in a small open economy where private agents borrow from international capital markets using collateralized debt in an environment with imperfect enforceability of creditor rights. Individual default generates capital outflows because they cannot rollover their debt. Capital outflows lead to a real exchange

rate depreciation, which lowers the value of non-tradable collateral of other borrowers. This creates a debt overhang problem, as the value of collateral becomes smaller relative to the value of previously taken debt, which triggers default of other agents. Default spillover effects generate an amplification mechanism, as all agents have higher incentives to default, increasing the incidence of default and the default risk premium of other borrowers and reducing the ability of private agents to borrow from abroad.

Default spillover effects create an inefficiency in the borrowing behavior of private agents because, by taking prices as given, they ignore the effect of their decisions on the valuation of collateral and through it on the default incentives of other agents. Individual borrowers default more frequently than socially optimal. An individual defaulting borrower does not internalize that her default leads to a depreciation in the exchange rate because international lenders will sell any seizable assets and flee the country. The exchange rate depreciation in turn reduces the value of non-tradable collateral and induces other borrowers to default, leading to a chain reaction of defaults.

In order to correct this distortion, optimal policy should focus on aligning default incentives with the socially optimal default set. Stronger enforcement of creditor rights through a larger default penalty in terms of repossessed collateral increases the cost of default and reduces the frequency of default in the decentralized equilibrium. Likewise, this could also be achieved by policies aimed at lowering the cost of debt repayment, by reducing the benefit of defaulting in states where the misalignment takes place.

The inefficiency created by the distortion in default incentives is completely different to the one in models with pecuniary externalities in collateral constraints on the level of debt. To our knowledge, this is the first paper that focuses on the existence of pecuniary externalities in default incentive constraints, which generates a chain reaction in defaults, a feature that has been documented during episodes of systemic debt crises.

The model environment is a two-good endowment economy where lenders seize a fraction of the individual borrower's total income in the case of default<sup>1</sup>. Each individual borrower has two choices: whether to pay back or default on previously contracted debt, and if previous debt has been paid back, how much new debt to take. We derive explicitly the financial contract of collateralized debt with the possibility of default, which is reflected on an interest rate schedule for each level of individual debt and analyze whether the debt and default choices in this model are socially efficient. Once we find the existence of a pecuniary externality effect on the default cost-benefit analysis, we introduce a policy instrument to correct this distortion, through a default penalty that is proportional to the total value of collateral.

The qualitative results show that the model accounts for the three stylized facts previously mentioned. During an episode of default, lenders partially seize the borrower's collateral and repatriate tradable goods. This results in large capital outflows and real exchange rate depreciation due to a 'transfer problem.'<sup>2</sup>On the

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<sup>1</sup>Following Uribe (2006), we need a two-good economy for a pecuniary externality to lead to a socially inefficient equilibrium, as the debt choice is socially efficient in the case of a one-good economy despite the existence of financial frictions.

<sup>2</sup>The transfer problem is a term used by Keynes (1929) to refer to the fact that a large transfer

normative side, private borrowers have higher incentives to default than socially optimal. As mentioned before, individual default has spillover effects as it leads to a real depreciation which creates a debt overhang problem for other borrowers by reducing the value of collateral and increasing the incentives to default for all borrowers in the economy.

The distortion in default incentives is an additional mechanism to the inefficiency created by the imperfect enforceability problem in the sovereign default literature. If the social planner could choose the degree of enforceability, she would choose one that leads to repayment in all states. In our case, even for a fixed degree of enforcement of creditor rights, there is a distortion in default incentives of private borrowers relative to the constrained efficient case. Private borrowers do not internalize the effect of their borrowing on the default risk premium of other agents through the effect on the valuation of collateral. In order to correct this distortion, decentralized agents should lower their incentives to default, which can be achieved through a stronger enforcement of creditor rights. A higher cost of default makes private agents internalize the effect of their borrowing in triggering a chain reaction of other agents' default, achieve a lower incidence of debt crises and improve risk sharing and consumption smoothing.

We use quantitative methods to solve for the optimal debt and default choices in the decentralized equilibrium and the constrained social planner's problem and the default penalty to correct the distortion in default incentives in the infinite

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between two countries not only has a direct effect in terms of a capital outflow but also an indirect effect through a change in the terms of trade, especially in the case of a small open economy.



horizon model. We simulate the economy to analyze the business cycle properties of the model and the behavior of real variables during default episodes. The results are consistent with the qualitative analysis, where the decentralized equilibrium shows higher incentives to default than socially efficient. This translates into a higher interest rate schedule, due to the effect of the default risk premium, and lower levels of debt compared to the case with no distortion in default incentives.

The default penalty takes positive values in the set of states where an individual borrower chooses to default whereas the social planner would not. This occurs at intermediate levels of debt, as private borrowers would efficiently choose to pay back low levels of debt and default on high ones. A higher endowment shock reduces the optimal value of the penalty, as the default penalty is proportional to tradable endowment. On the debt dimension, higher levels of debt in this set of states require a higher penalty to increase the cost of default, as the size of the default externality is increasing in debt.

**Literature Review.** The theoretical framework in this work is related to the literature on pecuniary externalities in incentive constraints (Greenwald and Stiglitz, 1986). In models of imperfect information and incomplete markets, the market equilibrium in economies with constraints that depend on market prices is not constrained efficient because the second order welfare loss from reducing default is smaller than the first order gain from relaxing the default incentives of other agents. In this model, the market equilibrium is not efficient because of the distortion that arises in the incentives to default due to the fact that private agents take the real exchange rate as given.

The optimal debt contract in this work is also related to the literature on optimal contract arrangements under the existence of commitment problems, such as Kehoe and Levine (1993) and Alvarez and Jermann (2000). A more restrictive version for the financial constraint is used in Zhang (1997), where the maximum level of debt is determined by the worst case scenario in terms of the exogenous shock. However, this class of contracts characterize an equilibrium which rules out default, with debt levels that satisfy an incentive compatibility constraint where paying back is strictly preferred to default in all states. In this model, I allow for default to be preferred in a subset of states and define the participation constraint for risk neutral international lenders that are willing to engage in risky lending.

This work is also related to the vast literature on sovereign default, such as Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), in the sense that we analyze an equilibrium where default does take place. Popov and Wiczler (2010) present a model with centralized default in a two good environment, where sovereign default episodes occur during periods of currency crises. A closer quantitative model of sovereign default to this work is the model with no trend shocks in Aguiar and Gopinath (2006), extended to a two good framework. However, the model differs in two important features: this paper focuses on private default, where both debt and default decisions are taken by decentralized borrowers, and debt is collateralized, whereas in the sovereign debt literature the cost of default is in terms of trade exclusion or reputation loss.

We can compare the implications of the constrained social planner's solution with models of sovereign default. The use of collateral that is subject to valuation

effects create an additional amplification mechanism through the chain reaction of defaults. For a given level of debt, individual default creates a real depreciation that lowers the valuation of non-tradable collateral and triggers a chain reaction of default, which amplifies the frequency of default.

Other models of private capital flows and default include Wright (2006) and Daniel (2012). Wright(2006) presents a model of private external debt with resident default risk, where private agents choose to borrow in both domestic and foreign capital markets. Daniel (2012) uses a model with private sector risky borrowing for emerging markets where the interaction between negative productivity shocks and financial market imperfections lead to a widespread of default and triggers a severe contraction in external borrowing. Our model would be similar to a representative agent version of agents who can borrow from domestic and foreign capital markets and face resident default risk in foreign markets, as the net supply of domestic debt is zero. We expand this model to allow for an imperfect degree of collateralization, where the real exchange rate amplifies the default externality.

This work also relates to others that analyze the normative implications in models with default. Tirole (2003) presents a single good model where a negative externality arises through a government policy that is set in terms of aggregate debt, which creates a mis-alignment in default incentives of private agents that take policy as given and of a government that chooses that policy. Jeske (2006) considers a default externality in a model where private agents can engage in both domestic and foreign borrowing and can default on foreign debt but not on domestic debt, which arises from the possibility of substituting access to foreign markets with access

to domestic ones. Wright(2006) presents a similar framework with domestic and external borrowing, where the optimal policy is to subsidize capital flows repayment.

It is also closely related to the model with pecuniary externalities and equilibrium default in Kim and Zhang (2012), where they show that decentralized borrowing and centralized default leads to the existence of a pecuniary externality through the effect of individual debt on the bond price schedule. The distortion in default incentives in our work is closely related to their bond price schedule effect, where private agents ignore that higher individual debt leads to a higher default risk premium. We extend this model by allowing for decentralized default and collateralized debt, which create an additional distortion on the default incentives of individual agents through the effect of the real exchange rate on the valuation of collateral.

The normative results in our model can be related to the literature on credit externalities and financial crises in models with endogenous borrowing constraints. When default is an off-equilibrium outcome, the pecuniary externality arises as private agents take excessive debt because they fail to consider the effect of debt on relative prices, as higher debt lowers the value of collateral and tightens the financial constraint in the following period. Therefore, the optimal policy, as shown by Bianchi (2011), Jeanne and Korinek (2010) and Korinek (2008), is one that reduces the amount of debt. A similar result is also obtained in Jeanne and Korinek (2010, 2011) and Bianchi and Mendoza (2010), where total borrowing is higher than optimal if uncontingent debt is the only financial instrument available.

The inefficiency created by the distortion in default incentives is completely different to the one in models with inefficient debt levels due to pecuniary externali-

ties in collateral constraints. The policy recommendations to correct this distortion is also completely different from the one in models with credit constraints: the distortion in default incentives can be corrected in the period when default occurs, whereas in the model with no default distortions, optimal policy is a precautionary tax that is charged in periods before the economy hits the borrowing constraint.

This work highlights the externality that arises in default incentives through the effect of individual debt on relative prices. The reason why prices play a key role in this type of models is related to the link between individual and aggregate borrowing. As pointed out in Krugman (1999) for the case of currency crises, individual debt taking depends on the valuation of wealth. Each agent's wealth depends on aggregate borrowing, as the volume of capital inflows affects terms of trade and through it the valuation of foreign currency denominated debt, which is specially relevant in the case of a small open economy. In an episode of systemic private default, a large fraction of borrowers that default on their debt leads to a sizable real exchange rate depreciation, which exacerbates the current account reversal.

The remainder of the paper is organized as follows. Section 2 presents a model of private borrowing and default under the decentralized equilibrium and the social planner's equilibrium to show that the decentralized equilibrium is not constrained Pareto efficient. Section 3 presents the results of the quantitative analysis of the pecuniary externality and its effect on default incentives, as well as the optimal policy to correct this distortion. Section 4 concludes.

## 2.2 Model of Systemic Private Default

This section presents a small open economy model of international borrowing with collateralized debt to illustrate the interaction mechanism between debt, default and the real exchange rate in a model with infinite discrete time. There are two representative agents: a domestic private borrower and a large pool of international lenders.

**Preferences.** The preferences of the representative domestic borrower are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

where  $\mathbb{E}_t(\cdot)$  is the time  $t$  expectations operator,  $0 < \beta < 1$  is the discount factor, and  $U(\cdot)$  is a CRRA utility function. The consumption good,  $c_t$ , is defined as a CES aggregator:

$$c_t = [\omega c_{T,t}^{-\eta} + (1 - \omega) c_{N,t}^{-\eta}]^{-1/\eta}$$

where  $c_{T,t}$  and  $c_{N,t}$  are the consumption of tradable and non-tradable goods, respectively,  $\omega$  is the weight of tradable consumption in the aggregator and  $1/(1 + \eta)$  is the elasticity of substitution between tradable and non-tradable consumption.

There is a single available instrument to borrow from abroad: a one period, non-state contingent bond denominated in units of the numeraire tradable good. Every period each domestic private borrower decides whether to pay back debt contracted in the previous period,  $d_t$ , and, if she chooses to do so, she can choose to take new debt,  $d_{t+1}$ , from a large pool of risk neutral lenders. Each domestic

agent consumes two types of goods, tradable and non-tradable goods, and receives a stochastic endowment of  $y_t$  units of the tradable good with p.d.f.  $f(y_t)$  and  $y^N$  units of the non-tradable good.

At the beginning of each period, borrowers decide whether to default or not. If a private borrower decides not to default, debt contracted in the previous period is paid back and new debt is taken. Each borrower chooses consumption of tradable and non-tradable goods. The budget constraint is given by:

$$c_{T,t}^R + p_t c_{N,t}^R = y_t + p_t y^N + d_{t+1} - (1 + r_t) d_t \quad (2.1)$$

where superscript  $R$  refers to the state of repayment and  $p_t$  is the relative price of non-tradable goods, or equivalently,  $1/p_t$  is a measure of the real exchange rate. The price of tradable goods is normalized to one.

If a private borrower defaults, lenders seize a fraction  $0 < \lambda_1 < 1$  of the borrower's total income. There is a cost related to this process, so that lenders obtain a fraction  $\lambda_2 \leq \lambda_1$  of total income, convert it into tradable goods and repatriate it.<sup>3</sup> Defaulters are excluded from international capital markets and regain access with an exogenous probability  $\phi$ .<sup>4</sup> When they regain access to capital markets, agents start with a zero debt stock. Agents choose their consumption of tradable and non-tradable goods. The budget constraint in the default state, with superscript  $D$ , is

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<sup>3</sup> $\lambda_1$  and  $\lambda_2$  are exogenous in this setup, but can be loosely related to the degree of enforcement of creditors' rights. See Djankov et al (2008) for references to the cost of private default and the value recovered by lenders. For the case study of a medium sized firm, they find that, on average, 48 percent of the value is lost during the debt enforcement process.

<sup>4</sup>We need some source of dead-weight loss due to default in order to obtain a pecuniary externality, which is obtained by setting  $\lambda_2 < \lambda_1$  and/or  $\phi < 1$ . A detailed explanation of this is shown in the qualitative results.

given by:

$$c_{T,t}^D + p_t c_{N,t}^D = (1 - \lambda_1)(y_t + p_t y^N) \quad (2.2)$$

If a private borrower defaults, there is a probability  $1 - \phi$  of staying in autarky, where they consume their endowments of tradable and non-tradable goods. The budget constraint in the autarky state, with superscript  $A$ , is given by:

$$c_{T,t}^A + p_t c_{N,t}^A = y_t + p_t y^N \quad (2.3)$$

The key feature in this setup is the fact that both tradable and non-tradable goods can be used as collateral. In the case of default, when lenders repossess non-tradable collateral, they would sell it against tradable goods which can be repatriated to lenders.<sup>5</sup>

This feature reflects in the market clearing condition for tradable goods under default. The demand of tradable goods by international lenders is given by the total value of seized collateral  $\lambda_2[y_t + p_t y^N]$ . The demand of tradable goods by domestic agents is their consumption of tradable goods in the default state,  $c_{T,t}^D$ . Supply is given by the endowment of tradable goods,  $y_t$ . Therefore, the market clearing condition for tradable goods in the state of default is:

$$c_{T,t}^D + \lambda_1(y_t + p_t y^N) = y_t \quad (2.4)$$

**International lenders.** There is a large pool of risk neutral international lenders. The interest schedule is derived a participation constraint that ensures that

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<sup>5</sup>Tornell and Westermann (2005) provide empirical evidence on the use of non-tradable goods as collateral, where external financing fuels credit booms in the non-tradable sector. Korinek (2011) points out the use of real estate collateral during many capital inflow booms and busts.



lenders are indifferent between engaging in risky lending to domestic borrowers and their riskless outside option. This pins down an interest rate schedule that depends on individual borrowing,  $r(d)$ . Lenders receive  $(1+r)d$  if a borrower decides to pay back and they seize a fraction  $\lambda_2 \leq \lambda_1$  of total income if the borrower defaults. The participation constraint is given by:

$$(1+r_t)d_t \int_{\hat{y}_t}^{\bar{y}} f(y_t)dy_t + \lambda_2 \int_{\underline{y}}^{\hat{y}_t} (y_t + p_t y^N) f(y_t)dy_t = (1+\rho)d_t \quad (2.5)$$

where  $\rho$  is the world risk free interest rate,  $\bar{y}$  and  $\underline{y}$  are the upper and lower bounds for the tradable endowment distribution, respectively, and  $\hat{y}$  is the default threshold for the tradable endowment shock, which will be defined in the next section. For a given level of debt, lenders get repaid when tradable endowment is higher than the threshold,  $y_t \geq \hat{y}_t$ , and default takes place otherwise.

### 2.2.1 Decentralized equilibrium

We present the problem faced by a representative borrower in recursive form. The state at the beginning of the period is given by  $(d, y)$ , where  $d$  is the stock of previously contracted debt and  $y$  is the tradable endowment shock. In states of repayment, labeled with superscript R, a private borrower chooses new debt,  $d'$ , and pays back debt contracted on the previous period.

$$v^R(d, y) = \max_{d', c_T^R, c_N^R} U(c_T^R, c_N^R) + \beta \mathbb{E}V(d', y') \quad (2.6)$$

subject to:

$$c_T^R + pc_N^R = y + d' - (1 + r)d + py^N \quad (2.7)$$

$$(1 + r)d \int_{\hat{y}}^{\bar{y}} f(y)dy + \lambda_2 \int_{\underline{y}}^{\hat{y}} (y + py^N)f(y)dy = (1 + \rho)d \quad (2.8)$$

where  $c_T^R$  and  $c_N^R$  are the consumption of tradable and non-tradable goods, respectively,  $p = p(d, y)$  is the price of non-tradable goods and  $r = r(d)$  is the interest rate schedule.<sup>6</sup>  $\hat{y}(d, y)$  is the threshold for the tradable endowment shock below which private borrowers default.  $V(d, y)$  is the welfare value of an agent with debt stock  $d$  and tradable endowment  $y$ .

Individual borrowers repay their debt if welfare under repayment,  $v^R$ , is higher than under default,  $v^D$ . Therefore, the repayment condition is given by:

$$v^R(d, y) \geq v^D(d, y) \quad (2.9)$$

We can show that there exists a threshold for the tradable endowment shock,  $\hat{y}$ , such that agents choose to pay back debt for realizations of  $y \geq \hat{y}$  and default otherwise.<sup>7</sup> The repayment condition shows that each borrower defaults under low realizations of the tradable income shock because the cost of default is increasing in  $y$ . The threshold is defined by:

$$v^R(d, \hat{y}) = v^D(d, \hat{y}) \quad (2.10)$$

If a private agent defaults, lenders seize a fraction  $0 < \lambda_1 < 1$  of total income.

There is a dead-weight cost of default as lenders repatriate a fraction  $0 < \lambda_2 \leq \lambda_1$

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<sup>6</sup>The interest rate schedule depends on both aggregate and individual debt because default risk is calculated for each individual borrower depending on her individual debt level but the recovery value of collateral depends on the price of non-tradable goods and therefore on aggregate debt.

<sup>7</sup>See Appendix A.1.

of total income in terms of tradable goods. Agents who default are banned from borrowing in international capital markets and go into autarky. They regain access to financial markets with a probability  $\phi$  and re-enter capital markets with no initial debt. We label this state with superscript D for default.

$$v^D(d, y) = \max_{c_T^D, c_N^D} U(c_T^D, c_N^D) + \beta(1 - \phi)\mathbb{E}v^A(y') + \beta\phi\mathbb{E}v^R(0, y') \quad (2.11)$$

subject to:

$$c_T^D + pc_N^D = (1 - \lambda_1)y + (1 - \lambda_1)py^N \quad (2.12)$$

As long as private agents stay in autarky, they consume their income. We label this state A for autarky.

$$v^A(y) = \max_{c_T^A, c_N^A} U(c_T^A, c_N^A) + \beta(1 - \phi)\mathbb{E}v^A(y') + \beta\phi\mathbb{E}v^R(0, y') \quad (2.13)$$

subject to:

$$c_T^A + pc_N^A = y + py^N \quad (2.14)$$

In every state, a private borrower's welfare is defined by  $V(d, y)$ , where repayment is chosen if welfare under repayment,  $v^R(d, y)$ , is higher than under default,  $v^D(d, y)$ , and default is chosen otherwise.

$$V(d, y) = \max\{v^R(d, y), v^D(d, y)\} \quad (2.15)$$

**Definition 1** *A recursive decentralized competitive equilibrium for a small open economy (SOE) is a pricing function,  $p(d, y)$ , an interest rate schedule,  $r(d)$ , and decision rules  $\{d'(d, y), c_T^R(d, y), c_T^D(d, y), c_T^A(y), c_N^R(d, y), c_N^D(d, y), c_N^A(y), \hat{y}(d, y)\}$  such that the following conditions hold:*

- *Household's problem:* Taking  $p(d, y)$  and  $r(d)$  as given, decision rules  $d'(d, y)$ ,  $c_T^R(d, y)$ ,  $c_N^R(d, y)$  and  $\hat{y}(d, y)$  maximize (2.6) subject to (2.7), (2.8) and (2.10); decision rules  $c_T^D(d, y)$  and  $c_N^D(d, y)$  maximize (2.11) subject to (2.12); and decision rules  $c_T^A(y)$  and  $c_N^A(y)$  maximize (2.13) subject to (2.14).
- *Market clearing:*  $c_N^R(d, y) = y^N$ ,  $c_N^D(d, y) = y^N$ ,  $c_N^A(y) = y^N$ ,  $c_T^R(d, y) = y + d'(d, y) - (1 + r(d))d$ ,  $c_T^D(d, y) = (1 - \lambda_1)y - \lambda_1 p(d, y)y^N$ ,  $c_T^A(y) = y$

Assuming that the interest rate schedule is differentiable everywhere, the intertemporal Euler equation is given by:<sup>8</sup>

$$U_1(c_t) = \beta \frac{1 + \rho + [(1 + r_{t+1})d_{t+1} - \lambda_2 (y_{t+1} + p_{t+1}y^N)] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \times \int_{\hat{y}_{t+1}}^{\bar{y}} U_1(c_{t+1}) f(y_{t+1}) dy_{t+1} \quad (2.16)$$

The optimal debt choice considers the effect of debt on the risk premium, which can be decomposed in two parts. Similar to models with uncollateralized debt, the first term is given by the probability of repayment in the denominator. Higher debt reduces the probability of repayment, as it increases default incentives because more resources are needed to pay back debt. The second term is related to the marginal loss of default due to the repatriation of collateral by lenders. The square bracket represents the marginal loss of default, as lenders get the repatriation of collateral instead of the interest rate payment. Higher debt increases the value of repayment relative to collateral, which increases the probability of default, shown

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<sup>8</sup>The infinite horizon problem is solved numerically using value function iteration in Section 3. We find that it is differentiable almost everywhere, except at the threshold where the probability of default becomes positive. The optimal choice of borrowing at this level is discussed in the section on the quantitative analysis. and shows the same result for the default externality.

by the term  $\partial \hat{y}_{t+1} / \partial d_{t+1}$ . Lenders must receive a higher interest rate in order to be willing to engage in risky lending.

## 2.2.2 Social Planner

Consider now a benevolent social planner who faces the same financial contract with limited enforcement in international capital markets. As opposed to private borrowers, the constrained social planner does internalize default spillover effects, through the effect of individual default on the valuation of non-tradable collateral and through it on the default incentives of other agents in the economy. As a result, we can show that the decentralized equilibrium is not constrained Pareto efficient.

Under repayment, the planner gets a similar pay-off to the one described for private agents. The planner's welfare under repayment  $w^R$  at initial state  $(d, y)$  is given by:

$$w^R(d, y) = \max_{d', c_T^R, c_N^R} U(c_T^R, c_N^R) + \beta \mathbb{E}W(d', y') \quad (2.17)$$

subject to:

$$c_T^R = y + d' - (1 + r)d \quad (2.18)$$

$$c_N^R = y^N \quad (2.19)$$

$$(1 + r)d \int_{\hat{y}}^{\bar{y}} f(y) dy + \lambda_2 \int_{\underline{y}}^{\hat{y}} \left( y + \frac{U_2}{U_1} y^N \right) f(y) dy = (1 + \rho)d \quad (2.20)$$

$$w^R(d, \hat{y}) = w^D(\hat{y}) \quad (2.21)$$

where  $U_2/U_1$  is the marginal rate of substitution between tradable and non-tradable goods.

Under default, the planner's welfare is defined as:

$$w^D(d, y) = \max_{c_T^D, c_N^D} U(c_T^D, c_N^D) + \beta(1 - \phi)\mathbb{E}w^A(y') + \beta\phi\mathbb{E}w^R(0, y') \quad (2.22)$$

subject to:

$$c_T^D = (1 - \lambda_1)y - \lambda_1 \frac{U_2(c_T, y_N)}{U_1(c_T, y_N)} y^N \quad (2.23)$$

$$c_N^D = y_N \quad (2.24)$$

If the planner stays in autarky, welfare is given by:

$$w^A(y) = \max_{c_T^A, c_N^A} U(c_T^A, c_N^A) + \beta(1 - \phi)\mathbb{E}w^A(y') + \beta\phi\mathbb{E}w^R(0, y') \quad (2.25)$$

subject to:

$$c_T^A = y \quad (2.26)$$

$$c_N^A = y_N \quad (2.27)$$

In each state, the planner's welfare is defined by  $W(d, y)$ , where repayment is chosen if welfare under repayment,  $w^R(d, y)$ , is higher than under default,  $w^D(y)$ , and default is chosen otherwise.

$$W(d, y) = \max\{w^R(d, y), w^D(y)\} \quad (2.28)$$

**Definition 2** *A socially efficient allocation for the small open economy (SOE) is a set of decision rules  $\{d'(d, y), c_T^R(d, y), c_T^D(d, y), c_T^A(d, y), c_N^R(d, y), c_N^D(d, y), c_N^A(d, y), \hat{y}(d, y)\}$  and an interest rate schedule  $r(d, y)$  such that decision rules  $c_T^R(d, y), c_N^R(d, y), d'(d, y)$  and  $\hat{y}(d, y)$  and the interest rate schedule maximize (2.17) subject to (2.18)-(2.21); decision rules  $c_T^D(d, y)$  and  $c_N^D(d, y)$  maximize (2.22) subject to (2.23) and*

(2.24); and decision rules  $c_T^A(d, y)$  and  $c_N^A(d, y)$  maximize (2.25) subject to (2.26) and (2.27).

The socially efficient level of debt is defined by the following Euler equation:

$$U_1(c_t) = \beta \frac{1 + \rho + [(1 + r_{t+1})d_{t+1} - \lambda_2 (\hat{y}_{t+1} + \hat{p}_{t+1}y^N)] f(\hat{y}_{t+1}) \frac{d\hat{y}_{t+1}}{dd_{t+1}} + \lambda_2 y^N \int_{\underline{y}}^{\hat{y}_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}} f(y_{t+1}) dy_{t+1}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \times \int_{\hat{y}_{t+1}}^{\bar{y}} U_1(c_{t+1}) f(y_{t+1}) dy_{t+1} \quad (2.29)$$

where

$$\frac{d\hat{y}_{t+1}}{dd_{t+1}} = \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} + \frac{\partial \hat{y}_{t+1}}{\partial p_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}}$$

**Proposition 1** *For a given level of debt, private borrowers in the decentralized equilibrium face a higher marginal effect of debt on the incentives to default than the constrained social planner, i.e.,*

$$\frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}^{CE} > \frac{d\hat{y}_{t+1}}{dd_{t+1}}^{SP}$$

**Proof.** Using the results in Appendix 1, we can derive the analytical expression for the marginal effect of debt on the default threshold under the decentralized equilibrium (CE) and the social planner's solution (SP):

CE:

$$\frac{\partial \hat{y}}{\partial d} = \frac{(1 + r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1 - \lambda_1)U_1(\hat{c}^D)} \quad (2.30)$$

SP:

$$\frac{\partial \hat{y}}{\partial d} = \frac{(1 + r)U_1(\hat{c}^R) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}_T} y^N U_1(\hat{c}^D)}{U_1(\hat{c}^R) - (1 - \lambda_1)U_1(\hat{c}^D) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}_T} y^N U_1(\hat{c}^D)} \quad (2.31)$$

For a given  $d$ , comparing equations (2.30) and (2.31), we obtain that

$$\frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}^{CE} > \frac{d\hat{y}_{t+1}}{dd_{t+1}}^{SP}$$

■

Let us analyze the debt and default decisions of a constrained social planner. The optimality condition for debt considers the same relationship as the one obtained by decentralized individuals, where a larger stock of debt leads to higher incentives to default, given the increase in the cost of repayment relative to the value of collateral. In addition, it considers the effect of individual debt and default on the valuation of collateral. There are two effects on the valuation of collateral: the first one related to the value of collateral for a given default set and the second effect related to the distortion in incentives to default. These effects are similar to the ones labeled 'over-borrowing' and bond price schedule effect in Kim and Zhang (2012), but extended to the case of collateralized debt and decentralized default.

The first effect is given by the additional term in the Euler equation,  $\lambda_2 y^N \int \partial p_{t+1} / \partial d_{t+1} f(y_{t+1}) dy_{t+1}$ , which shows that, for a given default set, higher debt leads to a lower valuation of collateral and therefore, lenders must increase the cost of borrowing to compensate for lost resources. This leads to a social planner choosing a lower level of debt than private borrowers.

The second effect is the one related to the distortion in default incentives, which is measured by the effect of individual default on others borrower's default, given by  $\frac{d\hat{y}_{t+1}}{dd_{t+1}} = \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} + \frac{\partial \hat{y}_{t+1}}{\partial \hat{p}_{t+1}} \frac{\partial \hat{p}_{t+1}}{\partial d_{t+1}}$ . Appendix 1 presents the analytical solution for the private borrower's problem in the decentralized equilibrium and the social planner's problem. A social planner that internalizes the effect of debt and default on the price of non-tradable goods has lower incentives to default because individual default leads to a chain reaction of defaults by affecting the valuation of other agents'



collateral, shown by the term containing  $\partial \hat{p}_{t+1} / \partial d_{t+1}$ .

The result on the distortion in default incentives implies that private borrowers face a higher default risk premium, which results in a lower level of debt compared to the case where the default incentives are perfectly aligned to the socially optimal. Clearly, the total effect on the level of debt depends on the relative magnitude of the two previously mentioned effects.<sup>9</sup>

A key assumption to obtain a distortion in default incentives is that default creates a dead-weight loss, given by the difference between  $(1+r)d$  and  $\lambda_2 p y^N$ . This can be achieved by lenders having to incur in a dead-weight cost to seize collateral, convert it to tradable goods and repatriate it,  $\lambda_2 < \lambda_1$ , and/or loss of access to international capital markets,  $\phi < 1$ . Appendix A.3 shows that in a model with no dead-weight cost of default, private agents face the same default incentives as the socially optimal. If there is no marginal loss of default, agents would face the same interest rate schedule and choose the same optimal level of debt.

In order to correct the distortion in default incentives, it would be welfare improving to have policies that enforce debt repayment and reduce the frequency of default, as it would increase the benefits of risk sharing by allowing private agents to borrow more under bad states of the economy.

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<sup>9</sup>If we consider the case where default is exogenous, we would have no distortion in default incentives, so that this is the only effect that remains in place. For a derivation of that case, see Appendix A.4: Exogenous Default. It is not just allowing for default to occur in equilibrium which results in lower borrowing but the distortion in default incentives between the socially efficient set of states and the ones chosen by decentralized agents.

### 2.2.3 Optimal Policy to Correct Default Distortion

In this section, we analyze the type of policy needed to correct the distortion on private agents' incentives to default. By taking prices as given, borrowers do not internalize the effect of individual default on the valuation of other borrower's collateral. We show that the constrained efficient equilibrium can be achieved by including a default penalty which increases the cost of default and reduces incentives to default so as to match the socially optimal default set. We introduce a default penalty,  $\tau(d, y)$ , that is proportional to the value of collateral and the income from the default penalty is given back to all agents as a lump sum transfer,  $T$ .<sup>10</sup>In this way, we are only directly affecting the default incentives for borrowers but not the amount of collateral seized by lenders. In practice, this could be enforced by taking away any additional assets to further penalize private agents who choose to default.

The policy instruments affect the budget constraint under the default state to include both the default penalty and the lump sum transfer.

$$c_T^D + pc_N^D = (1 - \lambda_1(1 + \tau))y + (1 - \lambda_1(1 + \tau))py^N + T \quad (2.32)$$

**Definition 3** *A recursive decentralized competitive equilibrium with default penalties for the small open economy (SOE) is a pricing function  $p(d, y)$ , an interest rate schedule  $r(d)$ , a default penalty  $\tau(d, y)$ , a lump-sum transfer,  $T(d, y)$  and decision rules  $\{d'(d, y), c_T^R(d, y), c_T^D(d, y), c_T^A(y), c_N^R(d, y), c_N^D(d, y), c_N^A(y), \hat{y}(d, y)\}$  such that the following conditions hold:*

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<sup>10</sup>For the full derivation of this result, see Appendix A.2.

- *Household's problem:* Taking  $\{p(d, y), r(d), \tau(d, y), T(d, y)\}$  as given, decision rules  $\{d'(d, y), c_T^R(d, y), c_N^R(d, y), \hat{y}(d, y)\}$  maximize (2.6) subject to (2.7), (2.8) and (2.10); decision rules  $\{c_T^D(d, y), c_N^D(d, y)\}$  maximize (2.11) subject to (2.32); and decision rules  $\{c_T^A(y), c_N^A(y)\}$  maximize (2.13) subject to (2.14).
- *Market clearing:*  $c_N^R(d, y) = y^N, c_N^D(d, y) = y^N, c_N^A(y) = y^N, c_T^R(d, y) = y + d'(d, y) - (1 + r(d))d, c_T^D(d, y) = (1 - \lambda_1)y - \lambda_1 p(d, y)y^N, c_T^A(y) = y, T(d, y) = \tau(d, y)\lambda_1(y + p(d, y)y^N)$

The value of the default penalty,  $\tau(d, y)$ , that corrects the distortion in default incentives is characterized by:

$$\frac{(1 + r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1 - \lambda_1(1 + \tau))U_1(\hat{c}^D)} = \frac{(1 + r)U_1(\hat{c}^R) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}^T} y^N U_1(\hat{c}^D)}{U_1(\hat{c}^R) - (1 - \lambda_1)U_1(\hat{c}^D) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}^T} y^N U_1(\hat{c}^D)} \quad (2.33)$$

where  $\hat{c}$  and  $\hat{p}$  are consumption and prices at initial state  $(d, \hat{y})$ .

## 2.3 Quantitative Analysis

This section presents the quantitative implications of the default externality. We solve the decentralized equilibrium and the constrained social planner's problem numerically using global non-linear methods, which are described in Appendix A.6. In order to analyze the quantitative properties of the model, we obtain the policy rules, default decisions and the price of debt under each state. We simulate the model to compute business cycle statistics and make an event analysis on default episodes.

Parameter	Value	Description	
$\sigma$	2	CRRA coefficient	
$\rho$	0.01	Risk-free interest rate	Aguiar and Gopinath (2006)
$\omega$	0.3	Tradable consumption coefficient	Bianchi (2011)
$\frac{1}{1+\eta}$	0.8	Elasticity of substitution between tradable and non-tradable goods	Bianchi (2011)
$\phi$	0.125	Access to capital markets	Mendoza and Yue (2011)
$\lambda_1 = \lambda_2$	0.1	Income seized under default	AG (2006)
$\beta$	0.93	Discount factor	Target: Debt-to-GDP ratio of 20 percent
$\rho_y$	0.9	AR(1) coefficient	AG (2006)
$\xi$	0.034	Standard deviation	AG (2006)

**Table 2.1:** Parameter Values

### 2.3.1 Parameter Values and Functional Forms

The numerical solution takes parameters from calibrations based on data for Argentina, following other models that analyze credit externalities, such as Bianchi (2011), and on-equilibrium default, such as Aguiar and Gopinath (2006, AG hereafter). A period in our model represents a quarter. The values of the parameters used in this exercise are listed in Table (2.1).

Agents preferences are given by a CRRA utility in terms of the composite consumption good ( $c$ ), which is a CES aggregator of the consumption of tradable ( $c_T$ ) and non-tradable goods ( $c_N$ ):

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

$$c = [\omega c_T^{-\eta} + (1-\omega)c_N^{-\eta}]^{-1/\eta}$$

The risk free interest rate,  $\rho$ , is set to 1 percent, which is a standard value used in the open macroeconomics literature for quarterly risk-free interest rate. The risk

aversion coefficient,  $\sigma$ , is set at a value of 2. The probability of re-entering the credit market after default,  $\phi$ , is set at 0.125, which implies an average exclusion period of about 10 quarters, consistent with an average exclusion of 2.5 years for Argentina. The weight on tradable consumption in the consumption aggregator,  $\omega$ , is set at 0.3 and the elasticity of substitution between tradable and non-tradable consumption,  $1/(1 + \eta)$  is set at 0.8, following Bianchi (2011).

We should note that the elasticity of substitution is of key importance in the size of the default externality, as it drives the size of the real exchange rate depreciation when default takes place. For a given reduction in tradable consumption, a higher elasticity implies a smaller exchange rate depreciation, and therefore we should expect weaker spillover effects from the default externality.

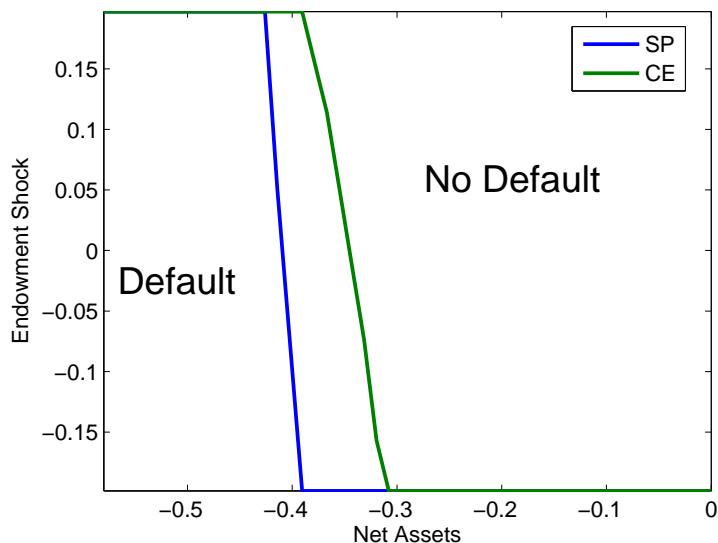
Another key parameter in the model is the fraction of seized collateral,  $\lambda_1$ . This parameter differs from other models with default as it is related to a loss in terms of collateral. We use a value that is consistent with Aguiar and Gopinath's value of 2 percent loss for every period. The discount factor,  $\beta$ , is set to match a ratio of debt-to-GDP of 20 percent.  $\lambda_2$  is set equal to  $\lambda_1$  which is the case with the smallest dead-weight loss and would therefore lead to a smaller default externality case. This gives a value for the fraction of income seized by the court when the borrower defaults,  $\lambda_1$ , of 0.1 and a discount rate  $\beta$  of 0.93.

The stochastic process for tradable output follows a log-normal AR(1) process,  $\log(y_t) = \rho_y \log(y_{t-1}) + \varepsilon_t^y$ , where  $E[\varepsilon^y] = 0$  and  $E[\varepsilon^{y^2}] = \xi^2$ . The parameters are set to  $\rho_y = 0.9$  and  $\xi = 0.034$ . It is necessary to create a large number of values for the discretized representation of the shock in order to get default as an equilibrium

outcome, as mentioned in Aguiar and Gopinath (2006) and Arellano (2008). Therefore, the shock is discretized to a 25-state Markov chain, following the procedure proposed by Tauchen and Hussey (1991).

### 2.3.2 Results

We present the default choice of private borrowers to show that they differ from those of the social planner and then simulate it to analyze the business cycle properties and crisis dynamics of this model.



**Figure 2.1:** Default decision under the decentralized equilibrium and constrained planner’s problem

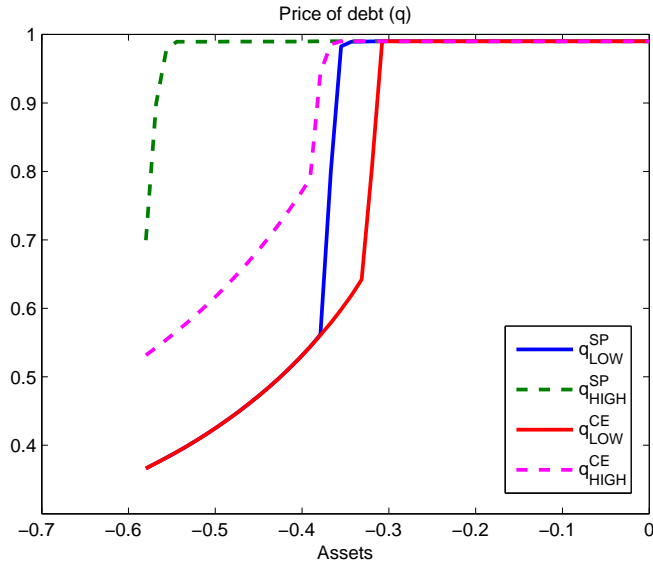
Figure (2.1) presents the default decisions for private individuals and the social planner for a set of states. The x-axis shows different values of debt (net assets) and the y-axis shows different values of the tradable endowment shock. The blue and green lines depict the default threshold for the social planner and decentralized

borrowers, respectively. The area on the left of the default threshold is the default region, where default is more likely to occur under high levels of debt (or low levels of net assets) and low levels of tradable endowment. For a given value of the endowment shock, agents have higher incentives to default under high levels of debt because higher debt increases the cost of repayment relative to default. For a given level of debt, agents have higher incentives to default under low levels of the endowment shock, as the value of seized collateral is increasing in tradable endowment.

By comparing the default sets, we observe that a distortion arises in the middle region because private agents ignore that default creates a spillover effect through the valuation of collateral. This is depicted by the region between the blue and green lines where decentralized agents default, whereas the constrained social planner repays debt. Default spillover effects are only relevant for intermediate levels of debt because, for high levels of debt, the cost of repayment is very large so that, even taking into account spillover effects, it is optimal to default. Likewise, for very low levels of debt, the cost of repayment is very small so that both private borrowers and constrained social planner choose not to default.

Figure (2.2) shows the price of new debt for the social planner (SP) and for individual borrowers (CE) under different states. The low state (LOW) refers to one where agents are highly indebted and get a low endowment shock. The high state (HIGH) refers to one where borrowers have low initial debt and get a high endowment shock. Consistent with default occurring more frequently under a low endowment shock and a high level of debt, as shown in Figure (2.1), the default risk premium is lower so that the price of debt is higher in the high state.

Given the higher probability of default for decentralized borrowers, international lenders charge a higher interest rate to be willing to engage in riskier lending, which translates into a lower price of debt for private borrowers. This result is consistent with the bond price schedule effect in Kim and Zhang (2012). Notice that because private borrowers have higher incentives to default, the maximum value of debt that they would repay is lower than the socially optimal, which is shown at the debt level where there is a change in the concavity of the price of debt. At values above this maximum debt limit, there are no values of tradable income at which borrowers repay, so they always get the value of their collateral, given by a fraction  $\lambda_2$  of total income.

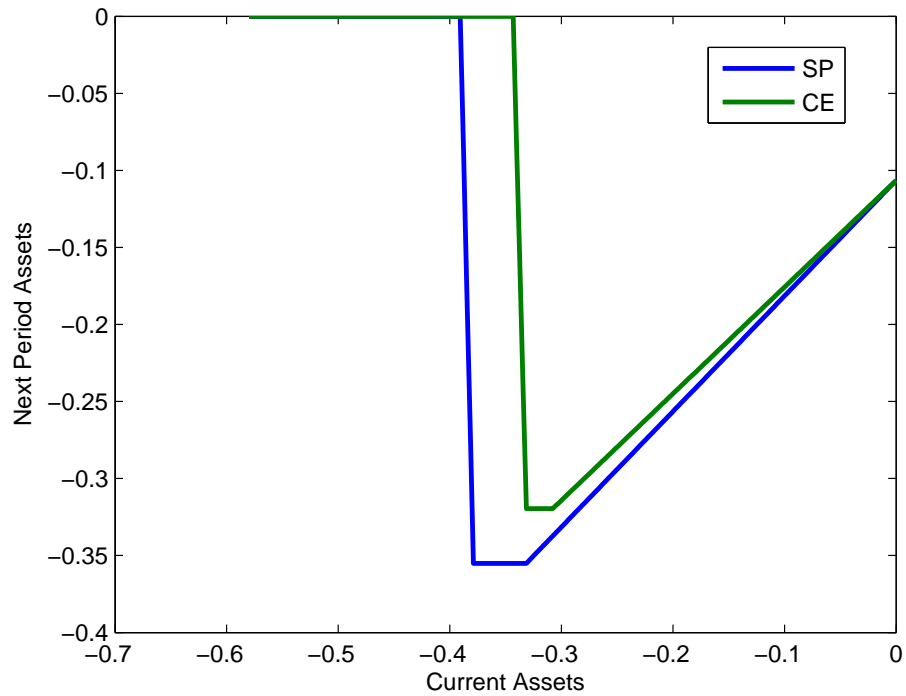


**Figure 2.2:** Price of debt

The debt choice in the private equilibrium considers two effects related to the valuation of collateral. The first effect is the one given by the recovery value of collateral on the interest rate schedule. If default incentives were perfectly aligned,



such as in models with an exogenous frequency of default, private borrowers would take more debt than the social planner.<sup>11</sup>The second effect is the one introduced by the distortion in default incentives. The decentralized equilibrium has higher default incentives and a lower price of debt, which reduce the marginal benefit of taking new debt. This leads to lower debt in the decentralized equilibrium than in the social planner’s problem. The final effect on the level of debt depends on the relative magnitude of these two effects.



**Figure 2.3:** Policy rule for a negative 1 s.d. tradable endowment shock

Figure (2.3) presents the policy rule for new debt for a tradable shock that is one standard deviation below the mean. It shows that the social planner (SP)

<sup>11</sup>A qualitative analysis of the pecuniary externality with exogenous default is presented in Appendix A.4. This result is also consistent with models with endogenous borrowing constraints and no default such as Bianchi (2011) and Korinek (2010).

chooses a higher level of debt than decentralized agents (CE), which is the case where the bond price schedule effect is larger than the over-borrowing effect. Note that, under high levels of debt, agents would choose to default, which is shown by the value of 0 debt for high levels of debt. Higher default incentives for private borrowers is shown by the fact that the policy rule goes back to a value of 0 under a lower level of debt than the social planner.

In contrast to an economy with no financial frictions, this economy faces an upper limit on the level of optimal debt, as there is a level above which it is always optimal to default, regardless of the value of the tradable income shock. Figure (2.3) shows that this limit is also lower for decentralized agents. The lack of commitment to pay back debt in the optimal set of states translates into a lower price of debt and to lower levels of debt than in an environment with no distortion on default incentives.

Standard Deviation	SP	CE
Output	0.0450	0.0457
Interest Rate	0.0012	0.0013
Trade Balance	0.002	0.008
Consumption	0.0459	0.0469
Correlations with Output		
Interest Rate	0.1295	-0.0455
Trade Balance	-0.2145	-0.1662
Default Frequency	0.0706	0.1514
Debt to GDP	0.2123	0.20

**Table 2.2:** Business cycle statistics

In order to analyze the behavior of real aggregates and the default externality

effects during systemic private default episodes, we simulate the economy for 10,000 periods for the same path of the income shock for the decentralized equilibrium and the social planner's equilibrium. We obtain some descriptive statistics on the behavior of real aggregates under the two settings. Table (2.2) summarizes the main results.

The simulation results show some standard stylized facts for business cycles in small open economies. Consumption volatility is higher than output volatility due to the financial friction, as agents face limited risk sharing due to higher default risk in the states where they need to borrow to smooth consumption. The volatility of the interest rate is extremely low because of the low frequency of default in the model.

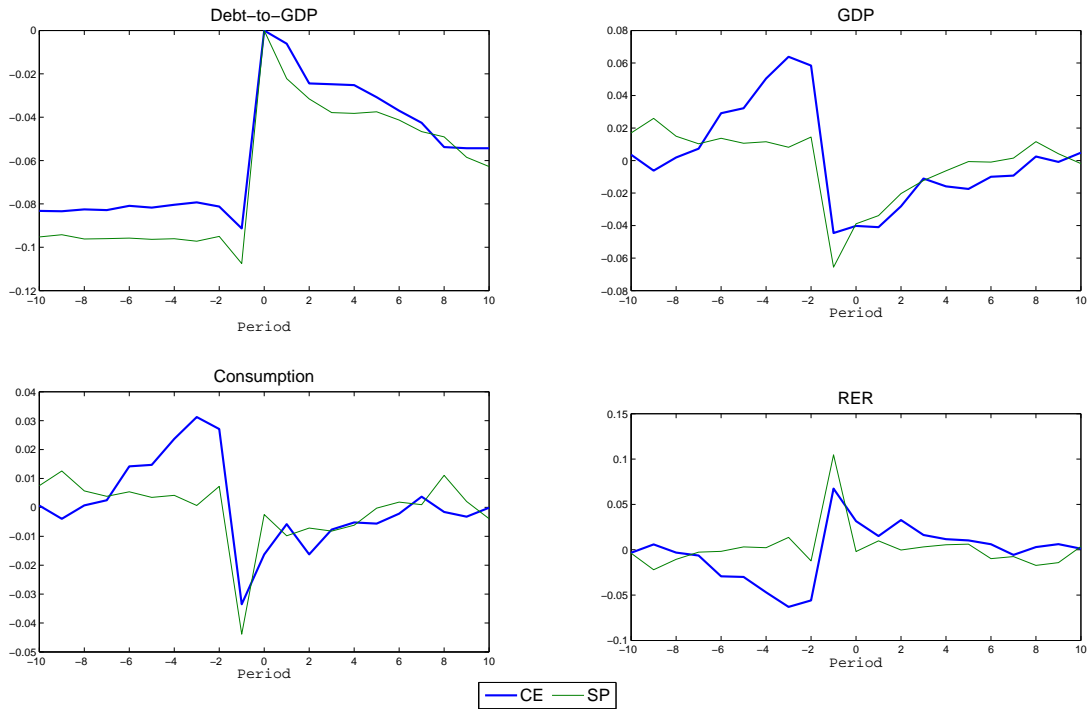
<sup>12</sup>There is a negative correlation between output and the interest rate, suggesting that default risk increases the most under low states of the tradable income shock.

Comparing the debt levels under the decentralized equilibrium and the socially optimal, we get higher average debt for the social planner, consistent with the policy rules shown in Figure (2.3), as well as higher frequency of default in the private equilibrium. Comparing the numerical results with Aguiar and Gopinath (2006), the amplification mechanism created by the relative price of non-tradable goods allows us to obtain a higher frequency of default (0.15 compared to 0.02 percent). Compared to models with centralized default, the chain reaction of defaults creates an amplification mechanism in the default frequency. If we compare them with our results for the social planner, we still find some amplification in the default frequency

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<sup>12</sup>As mentioned before, proportional costs of default do not allow to sustain high levels of debt and high frequencies of default in equilibrium.

(0.07 compared to 0.02 percent), because default triggers a chain reaction in default by lowering the valuation of collateral of all agents in the economy. Decentralized default amplifies this effect even more because the private cost of default is smaller than the social cost of default.

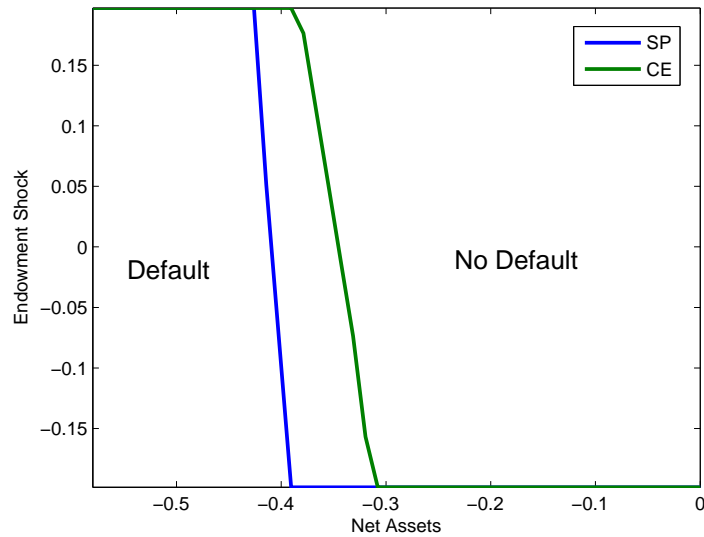


**Figure 2.4:** Default Event

Figure (2.4) shows the behavior of some key real aggregates under the decentralized equilibrium and social planner's problem. Each line represents the average value of all default events in the simulation under each setting. Prior to default, there is an increase in the ratio of debt-to-GDP, which triggers default due to a combination of high debt and a low endowment shock. This is also shown in the sharp fall in the figure labeled 'GDP', which is consistent with a fall in the total value of collateral. A default event leads to capital outflows and a real exchange rate

depreciation, consistent with the transfer problem. Private borrowers who default lose a fraction of their collateral and access to international capital markets, which is shown as a reduction in consumption in the period when default takes place and a lower possibility of consumption smoothing in the following periods.

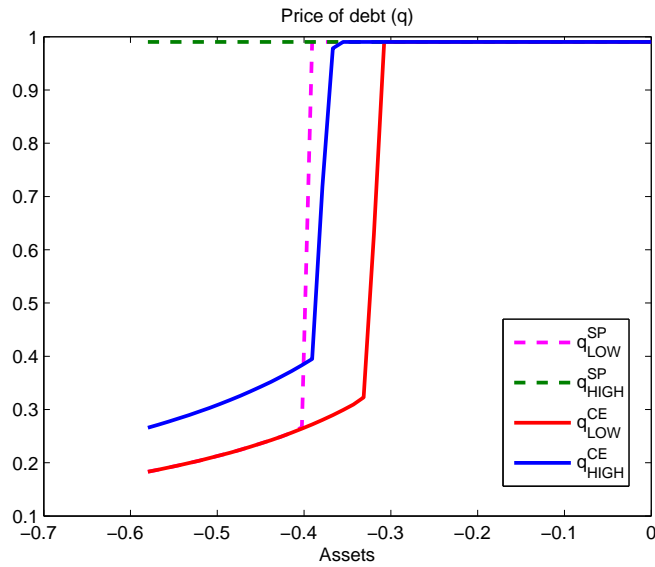
### 2.3.3 Sensitivity Analysis



**Figure 2.5:** Default Set for  $\lambda_2 = 0.9\lambda_1$

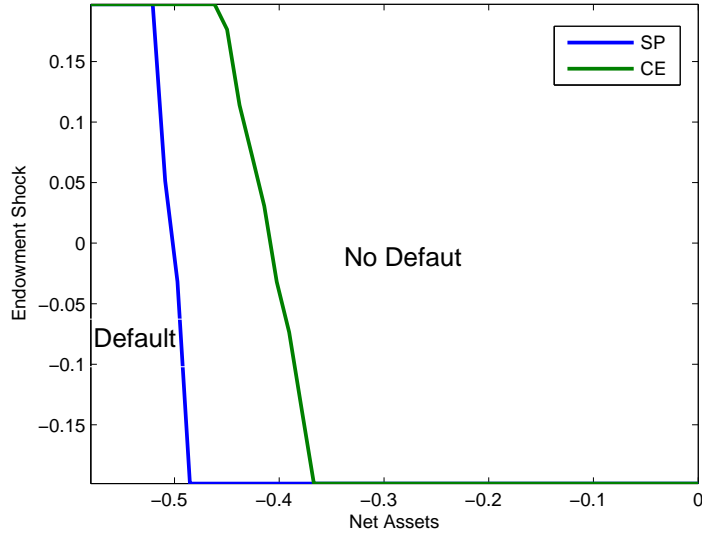
In this section, we analyze the behavior of the default externality under alternative calibrations. One of the key features of this model is to include a cost of default in terms of the value of collateral seized by lenders. Therefore, we present an analysis on different parametrizations of the value of  $\lambda_1$  and  $\lambda_2$ . The first experiment is to increase the size of the dead-weight cost of default, by creating a wedge between  $\lambda_1$  and  $\lambda_2$  of 10 percent of the value of  $\lambda_1$ . Intuitively, an increase in the deadweight-loss reduces the amount of collateral repatriated by lenders so

they would compensate it by charging a higher interest rate. An increase in the cost of borrowing has a second order effect on increasing incentives to default and on default spillover effects. However, it does have a first order spillover effect on the interest rate faced by all agents to compensate for the dead-weight loss. Figure (2.5) shows a very small effect on the default set, whereas Figure (2.6) shows a lower price of debt to compensate for the larger dead-weight loss.



**Figure 2.6:** Price of Debt for  $\lambda_2 = 0.9\lambda_1$

Figure (2.7) presents the results for a second experiment, where we consider a higher value of  $\lambda_1 = \lambda_2 = 0.12$ . A higher level of enforcement increases the cost of default for both decentralized borrowers and a constrained social planner, so that they choose to default less often than under the benchmark scenario. This is reflected in a shift in the default sets to the left, so that default is preferred only under very high levels of debt and/or very low values of tradable endowment. In terms of the default externality, the default distortion occurs under higher levels of

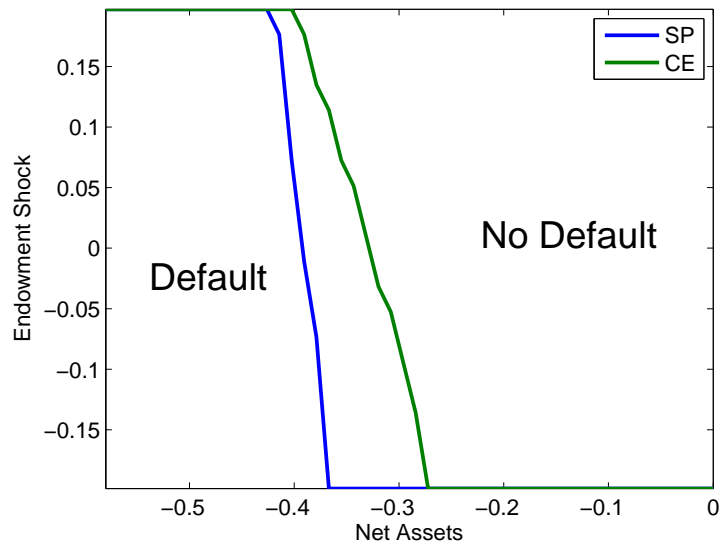


**Figure 2.7:** Default Set for  $\lambda_1 = \lambda_2 = 0.12$

debt and/or lower levels of tradable endowment, so that the default penalty is used less frequently.

There are several other parameters that are relevant in the magnitude of the default externality. By looking at the Euler conditions in the decentralized equilibrium and the social planner's problem, (2.16) and (2.29), the key parameters affecting the size of the default spillover effect are the ones that affect  $\partial \hat{y} / \partial p$   $\partial p / \partial d$ . The first term measures the impact of a real exchange rate depreciation on the incentives to default. The second one is the impact of an increase in individual debt on the exchange rate.

There are three key parameters affecting the size of the spillover effect: the elasticity of substitution between tradable and non-tradable goods,  $1/(1 + \eta)$ , the weight of non-tradable consumption in the consumption aggregator,  $1 - \omega$  and the discount factor,  $\beta$ . Our results show that the default externality is qualitatively un-

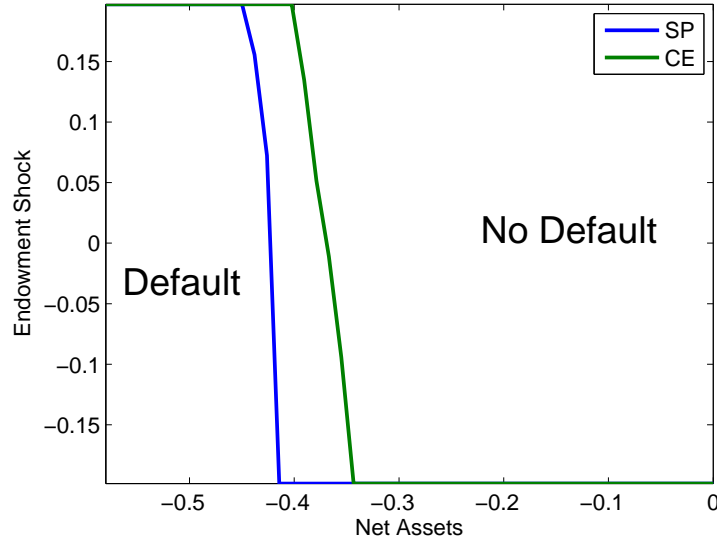


**Figure 2.8:** Default Set for  $1/(1 + \eta) = 0.4$

changed under the alternative scenarios, where internalizing the effect of default on the valuation of collateral leads to lower default incentives under the social planner’s equilibrium.

The elasticity of substitution between tradable and non-tradable goods,  $1/(1 + \eta)$ , plays a key role in determining the size of the real depreciation when default takes place. Figure (2.8) shows the results for an elasticity of substitution of 0.4, on the lower end of the estimates in Bianchi (2011), and Figure (2.9) for a higher elasticity of substitution of 1.25, as in Benigno et al (2010), where tradable and non-tradable goods are gross substitutes. A lower elasticity increases the level of complementarity in the consumption of tradable and non-tradable goods. Therefore, a reduction in tradable consumption due to the inability of debt rollover sharply reduces the demand for non-tradable goods as well, leading to a larger real exchange rate depreciation. Therefore, the size of the default externality increases as tradable



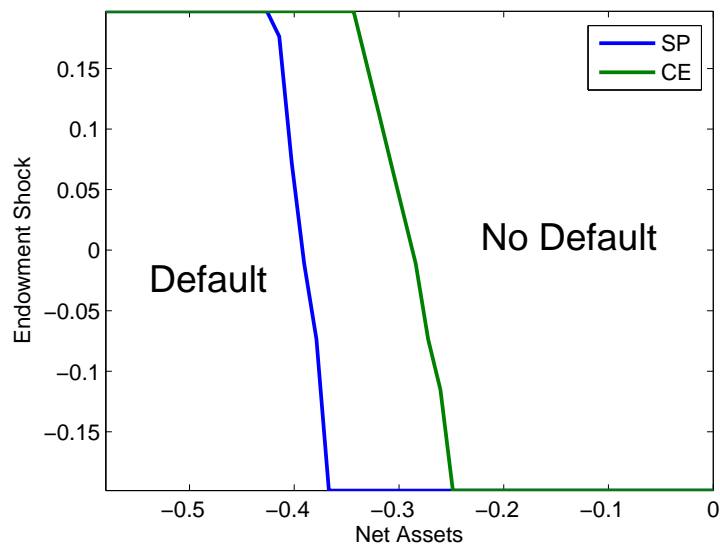


**Figure 2.9:** Default Set for  $1/(1 + \eta) = 1.2$

and non-tradable consumption become closer complements, which occurs at lower values of the elasticity of substitution.

From a positive perspective, the size of the elasticity of substitution affects the relative cost of repayment. In bad states, repayment becomes more costly, as it implies a sharper fall in tradable consumption, so that default is preferred in a larger set of states. This leads to larger incentives to default under a lower elasticity of substitution, which is shown by comparing the default thresholds for the social planner, depicted by the blue lines in Figures (2.8) and (2.9).

Similarly, a higher weight of tradable collateral,  $\omega$ , increases the size of the default externality, as it directly affects the incidence of a real exchange rate depreciation. When default takes place, supply of tradable goods to domestic agents after collateral repatriation falls. Figure (2.10) shows the size of the default externality for an alternative scenario with a value of  $\omega = 0.5$ . A higher weight of tradable col-



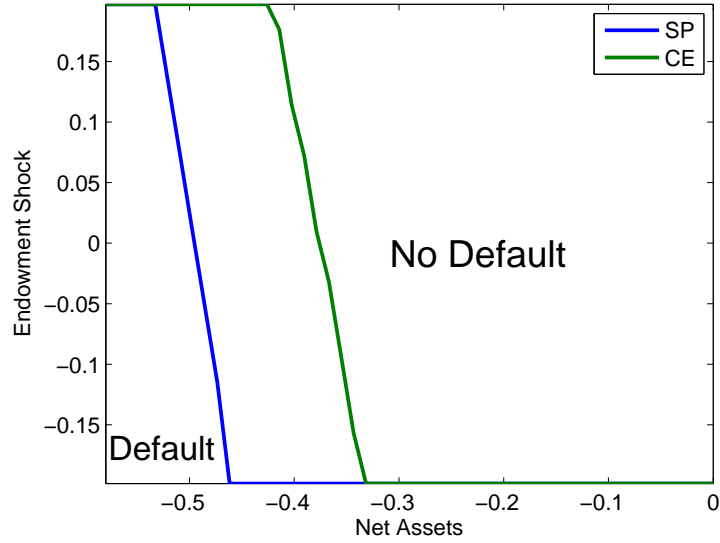
**Figure 2.10:** Default Set for  $\omega = 0.5$

lateral sharply reduces the relative price of non-tradable goods, which results in a larger amplification mechanism through the default spillover effect. From a positive perspective, a higher weight on tradable consumption increases the cost of repayment in bad states, so that agents default more often, widening the set of states where default takes place.

Another key parameter is the relative size of non-tradable and tradable collateral,  $y^N$ , as it is only the fraction of non-tradable collateral which is affected by changes in valuation through the real exchange rate depreciation.<sup>13</sup> We repeat the numerical exercise by reducing the size of non-tradable endowment in the composition of collateral to half.<sup>14</sup> Figure (2.11) shows the reduction in the size of the default exter-

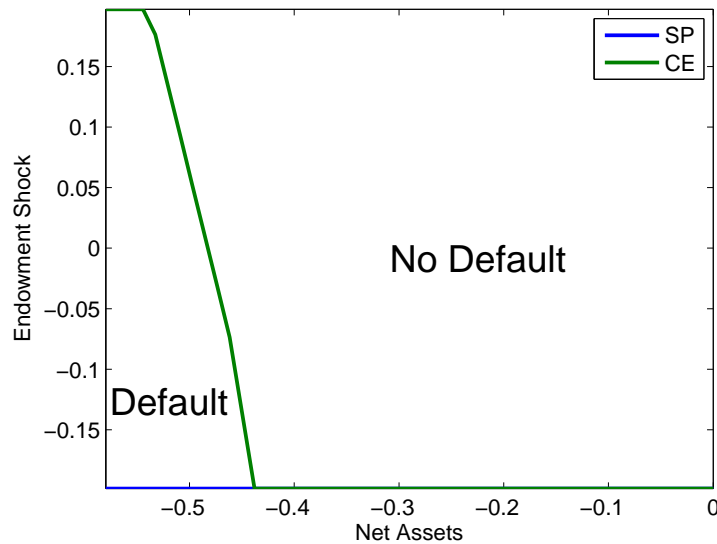
<sup>13</sup>Similar to the result obtained in Uribe (2006), if there is only tradable collateral, the pecuniary externality disappears. If only tradable goods can be used as collateral, there is no valuation effect on the default incentives and the financial contract faced by borrowers, so the decentralized equilibrium is Pareto efficient.

<sup>14</sup>We change the value of  $y^N$  so that the steady state value of total endowment remains the same, but with a lower ratio of  $y^N$  to  $y$ .



**Figure 2.11:** Default Set for  $y^N = 0.5$

nality for a lower share of non-tradable collateral in the total composition. A lower share of non-tradable collateral decreases the magnitude of the default externality, as the real exchange rate depreciation has a smaller impact on the total value of collateral, reducing the spillover effects on incentives to default. The last alternative scenario considers a lower discount factor,  $\beta$ . On one hand, agents want to borrow more due to the impatience factor. However, on the other hand, higher debt increases incentives to default, which amplify the default spillover effect. Figure (2.12) shows the size of default externality with a lower value of  $\beta = 0.9$ . The impatience effect dominates so that agents choose to default in a smaller set of states and face a smaller default externality. By comparing the default decisions of private borrowers and the social planner, we observe a smaller default externality.

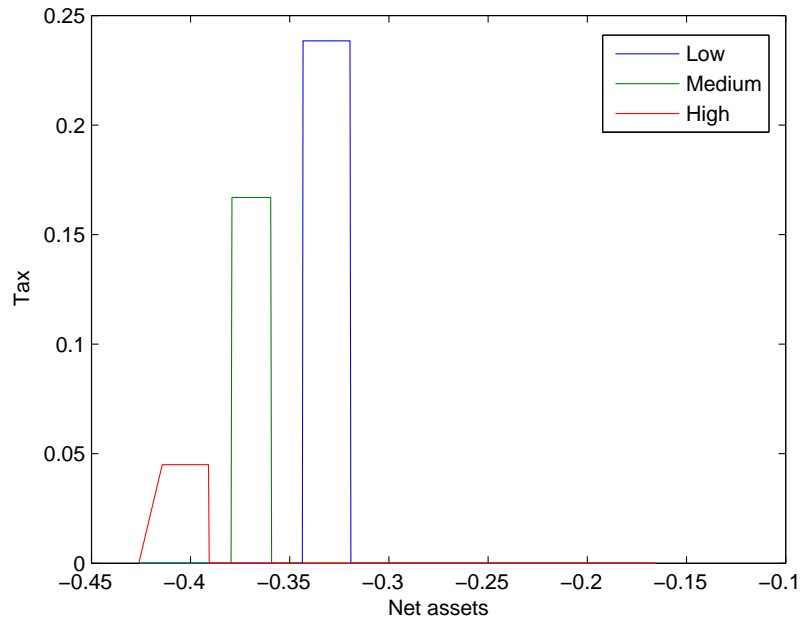


**Figure 2.12:** Default Set for  $\beta = 0.9$

### 2.3.4 Default Penalty

We solve numerically for the default penalty in the infinite horizon model. The default penalty is the additional cost paid by individual defaulters that is proportional to the value of their own collateral. Figure (2.13) depicts the optimal default penalty for each state, where only the states with a distortion in default incentives show a positive default penalty value.<sup>15</sup>The x-axis shows different values of initial net assets, whereas each line corresponds to a different value of the tradable income shock. As previously mentioned, the distortion in default incentives occur in the middle range of debt values. Under high levels of debt, both the social planner and private individuals choose to repay, whereas under low levels of debt both choose to default.

<sup>15</sup>We could impose a positive penalty value on those states where both private agents and the social planner would choose to repay but it has no effect on the final outcome.



**Figure 2.13:** Optimal Policy

On those states where a default penalty is needed to correct the externality, a higher endowment shock reduces the optimal value of the default penalty, as the cost of defaulting is increasing in tradable endowment. On the debt dimension, higher levels of debt require a higher tax to increase the cost of default and reduce incentives to default, as the size of the externality is increasing in debt.

Note that in order to correct default incentives, we could also use a policy instrument that reduces the marginal benefit of defaulting. Policy measures such as credit refinancing could also correct the externality by increasing the marginal benefit of repayment relative to defaulting. Another policy consistent with correcting the distortion due to default spillover effects is the introduction of capital flow subsidies as in Wright (2006).

Let us compare this result with models with endogenous borrowing constraints

but no default. In that case, the optimal policy measure is to impose a tax in the states where the constraint is not binding to prevent individual borrowers from taking excessive levels of debt, which would tighten the constraint through its effect on collateral prices. In periods where borrowers are already facing a binding constraint the tax does not affect their debt decision. In contrast, in the model with distortions in the default choice, the optimal policy instrument needs to address only the states where default takes place. Private borrowers can now take higher levels of debt than in models with no default, but at the expense of higher risk and lower price of debt. Therefore, the optimal policy instrument affects a different subset of states, at higher levels of debt than the ones implied by the binding borrowing constraint.

## 2.4 Conclusions

This paper analyzes the distortion in incentives to default on collateralized debt due to the existence of a default spillover externality in a two-good endowment small open economy. Individuals fail to internalize the effect of their default decision on the exchange rate and through it on the incentives to default of other borrowers in the economy. An individual defaulter generates an exchange rate depreciation and capital outflows that reduce the value of non-tradable collateral and induces other borrowers to default. Therefore, by taking prices as given, private borrowers default more frequently and face a higher risk premium, which limits optimal borrowing and consumption smoothing.

Enforcement of creditor rights should be strengthened in order to align default

incentives of individual agents with the socially optimal, so that individual agents default less often. This leads private borrowers to increase their borrowing capacity, face lower costs of taking debt and engage in better consumption smoothing. In addition, in aggregate terms, it also leads to a lower probability of default and a lower frequency of sharp current account reversals and real depreciations.

The model uses a simplified way of modeling the costs of the default, which has the benefit of making it easy in terms of tractability and implementation to illustrate the interaction mechanism between default and the value of collateral. However, we could extend these results to a model that considers a more realistic approach on the punishment of the debt contract under default. Even though this paper is related to private default, similar conditions to the ones for sovereign default in Arellano (2008) and Mendoza and Yue (2012) can be applied for the costs of private debt default in order to obtain a more realistic default frequency and level of debt.

A further analysis of the optimal policy measures should focus on a wider variety of policies that affect the incentives to default in order to align the default choice of private borrowers to the socially optimal. This work only considers a default penalty that affects the cost of default, but other policy measures that reduce the marginal benefit of default such as credit refinancing could also correct the externality.

## Chapter 3: State Contingent Assets, Financial Crises and Pecuniary Externalities in Models with Collateral Constraints

### 3.1 Introduction

There has been great interest in the benefits and policy implications of financial innovation as a possible source of insurance against the vulnerabilities faced by emerging countries and the likelihood of financial crises and default episodes. Different types of state contingent financial instruments and their theoretical benefits have been discussed in the literature, as a way to stabilize capital flows to emerging market economies and to provide a better source of hedging against macroeconomic risks.<sup>1</sup>

This work analyzes the implications of diversifying the external liability portfolio of private agents by having access to financial instruments with state contingent payments, in the spirit of instruments such as GDP-linked bonds and future contracts. We focus on its effect in terms of risk sharing, the frequency of financial crises and its amplification mechanism through the effect on the valuation of collateral for

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<sup>1</sup>Shiller (1993) presents an analysis of the benefits of developing financial markets for bonds with payments linked to the level of GDP as a way to hedge against macroeconomic risks in a closed economy setup. For references in the open economy literature, Williamson (2005) analyzes the benefits of having access to growth indexed bonds that could be traded in international financial markets as a way to reduce the probability of sharp reversals in capital flows.



other agents in the economy.

In an environment with a single bond with a fixed interest rate, private agents have only one instrument to engage in risk sharing in two margins: inter-temporally between current and future consumption and intra-temporally between consumption in periods of high and low endowment. Higher debt allows for inter-temporal risk sharing for agents with high impatience, but does not allow for risk sharing across states. When there is additional access to a state contingent, private agents can improve risk sharing. In an environment with i.i.d. shocks, they can engage in full risk sharing across states by using the state contingent bond to perfectly hedge against uncertainty in the income shock and use the regular bond to engage in inter-temporal risk sharing. With persistent shocks, this result is relaxed, as it is only possible to partially hedge against the income shock, so that agents find it optimal to accumulate precautionary savings, but in a smaller magnitude compared to the case with bonds only.

In terms of the frequency of financial crises, access to state contingent debt lowers the likelihood of default and financial crises, as repayment becomes less costly in bad states. When a country faces a bad shock, a GDP-linked bond reduces the amount of resources needed for debt rollover. As financial crises are related to high levels of debt, there is a lower probability of crisis and smoother drops in consumption. Lower frequency of financial crises benefits the borrower because crises are extremely costly in terms of the reduction in output and consumption and loss of access to financial markets.<sup>2</sup> On the lender's side, lower frequency of crises

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<sup>2</sup>See Eichengreen (2004)

is also optimal because debt crises are usually resolved by costly renegotiation or enforcement process.

The main contribution of this work is to analyze the normative implications of the interaction between the risk sharing properties of introducing contingent repayment and the spillover effects of excessive debt to other agents in the economy. State contingent debt service provides additional insurance through its effect on the valuation of collateral, where lower interest payments in bad states loosens the collateral constraint. This translates into smaller capital outflows, milder real exchange rate depreciation and therefore a smaller impact of individual debt on the valuation of collateral of other agents in the economy.

We present a qualitative analysis of the effects of having access to state contingent instruments in a two-good endowment small open economy with limited access to international financial markets. In an environment with i.i.d. shocks, agents can fully insure against income shocks by using the state contingent asset and borrow up to the collateral constraint using the non-state contingent bond to increase current consumption. There is no additional spillover effect, as it is optimal to borrow at the binding collateral constraint due to the impatience factor.

However, if the tradable endowment shock is persistent, agents cannot fully hedge against this shock. Each private borrower exchanges current tradable income for its expected value at the previous period, which is positively correlated to the actual shock but less volatile. Partial exposure to the endowment shock creates a need for accumulating precautionary savings, but in a smaller magnitude than in an environment with bonds only. A constrained social planner considers the

spillover effect of diversifying the debt portfolio towards the state contingent bond, as it provides additional insurance by relaxing the collateral constraint in bad states without tightening them in good states. State contingent debt service reduces capital outflows, dampens the real exchange depreciation and therefore reduces the size of the spillover effect.

From a normative perspective, private agents do not internalize the insurance properties of the state contingent instrument in terms of relaxing the collateral constraint. Higher individual borrowing reduces the price of non-tradable collateral and therefore tightens the collateral constraint of other agents. A private agent would therefore accumulate less precautionary savings than a constrained social planner. Compared to the environment with bonds only, partial insurance through the state contingent asset dampens the size of the pecuniary externality.

We also present a quantitative analysis of the benefits of having access to financial instruments with state contingent interest payments and its effect on risk sharing, the frequency of financial crises and the reduction in the pecuniary externality that arises through the valuation of collateral. We calculate the optimal holdings of both types of assets by private agents and the frequency of financial crises and compare them to the solution of a constrained social planner who does internalize the effect of individual borrowing on prices. We calculate the size of the spillover effects on other agents borrowing in an environment with access to non-state contingent bonds only and in one where agents have access to both non-state contingent and state contingent bonds.

Our quantitative results show that private agents take higher levels of total

debt and face a lower frequency of financial crises when they have access to both state contingent and non-state contingent bonds. Comparing the decentralized equilibria with and without state contingent bonds shows that, consistent with the previously mentioned argument in Borenzstein and Mauro (2004), more stable debt service reduces the requirement for new borrowing in bad times, hence hitting the borrowing constraint less often. Partial insurance through state contingent debt holdings lowers the need for precautionary savings.<sup>3</sup>

By comparing the size of the spillover effects on the valuation of collateral, we find that the difference in the distribution of debt and the frequency of financial crises between the decentralized equilibrium and the constrained social planner's problem is smaller in an environment with pro-cyclical debt service.<sup>4</sup>This is related to the milder incidence of exchange rate depreciations on the collateral constraint in bad states.

**Related Literature.** This work is related to the literature on the effects of having access to indexed bonds. Among models of the benefits of indexed bonds, Durdu (2009) studies the effect of indexing debt to GDP and solves for the optimal degree of indexation, as there is a trade-off between income fluctuations and interest rate fluctuations. We extend this model to consider the effect of pecuniary externalities in this environment with endogenous collateral constraints, where a mismatch

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<sup>3</sup>Caballero and Panageas (2008) show that having access to future contracts lowers the size of precautionary savings by roughly 10 percent of GDP.

<sup>4</sup>Given the presence of financial frictions, as mentioned in Korinek (2010), even a complete set of state contingent bonds would not help to overcome the collateral constraint and achieve a socially optimal outcome. However, there might be certain types of instruments that reduce the likelihood of hitting the collateral constraint, and are able to get a smaller distortion in the debt choice compared to the one of a credit constrained social planner.

in the valuation of total income and debt denominated in tradable goods creates an additional channel that amplifies the cost and frequency of financial crises.

Borensztein and Mauro (2004) present a detailed study on the benefits of indexed bonds and a discussion on the concerns around choosing the optimal variable for indexing debt, as well as the implications of developing markets for liquid trade of this type of instrument. Among the advantages of GDP-indexed bonds is that it reduces the volatility of the debt-to-GDP ratio and hence reduces the likelihood of financial crises. During a period of low GDP, debt repayments fall and a lower value of debt is required to rollover previously contracted debt. As financial crises are related to high levels of debt, this financial instrument lowers the probability of crises and reduces the amplification effect on consumption, the real exchange rate depreciation and the reversal in capital flows.

On the theoretical side, this work is related to models of incomplete markets and limited enforcement, such as Kehoe and Perri (2002), where agents have access to a full set of state contingent assets. However, due to the existence of an enforcement constraint, where borrowing is limited to prevent default, agents are not able to engage in full risk sharing. This work shows a similar model where the collateral constraint is similar to the enforcement constraint in Kehoe and Perri (2002), but agents only have access to one additional asset that has a state contingent interest payment, which is closer in nature to a GDP-linked bond. Bai and Zhang (2010) present a model of limited enforcement and limited spanning for a production economy, where the financial constraint comes from an incentive compatibility constraint that ensures repayment even under the worst case scenario, and agents have access

to non-state contingent bonds only. This work uses a similar setup in an endowment economy, expands it to the two-good case and analyzes the features of the model when a wider range of state contingent bonds is available.

A closely related strand of the literature analyzes the quantitative implications of contingent financial instruments as a substitute to large accumulation of precautionary savings. Caballero and Panageas (2008) present a quantitative model to analyze the gains of using contingent financial instruments to reduce the probability of sudden stops. They present a model with a borrowing constraint with limited pledgeability of future income to finance current account deficits. Using VIX futures as hedging instruments, a country can reduce the need for precautionary savings and have large transitional gains. A similar quantitative analysis is presented in Borenzstein et al (2009), where borrowers use precautionary savings jointly with commodity futures as contingent hedging instruments for commodity exporter countries and calculate the welfare gains.

This work is also related to models where a pecuniary externality arises through the effect of valuation of collateral in credit constraints. Bianchi (2011) and Korinek (2010) analyze the presence of an externality effect and the over-borrowing result associated with it. By ignoring the effect of their debt choice on relative prices and on the tightness of the collateral constraint of other agents, each private borrower takes excessive debt compared to the debt level of a constrained social planner. Thus, decentralized agents face a higher probability of financial crises, described as an episode of large current account reversals.

The optimal policy to correct this externality is the use of Pigouvian taxes

to achieve the second best solution, given by the outcome of the constrained social planner's problem. We show that having access to hedging instruments with state contingent interest payments reduce the size of the externality in this type of models, as they provide partial hedging against shocks, reducing the need for precautionary savings and have an asymmetric effect on the collateral constraint, by loosening it in bad states.

The chapter is organized as follows. Section 2 presents a description of the model. Section 3 presents the analytical results on the benefits of having access to contingent financial instruments with pro-cyclical debt service. Section 4 presents an analysis of the quantitative results. Section 5 concludes.

## 3.2 Model

We present an infinite horizon model of a continuum of domestic private agents who have limited access to international capital markets from a large pool of risk neutral lenders. Domestic agents' preferences are defined on the consumption of a composite good, which depend on the consumption of tradable and non-tradable goods. They can borrow using two types of financial instruments: a non-state contingent bond, with a fixed interest payment, and a bond with state contingent interest rate payments. International borrowing is subject to limited enforcement, so that private agents can only borrow up to a fraction of the value of their total income.

### 3.2.1 Consumers

We make reference to the two-good model presented in Mendoza (2006) of a small open economy inhabited by a continuum of identical private agents that consume a composite good  $c$ . The composite good consists of tradable ( $c^T$ ) and non-tradable goods ( $c^N$ ).

$$c(c_t^T, c_t^N) = [a(c_t^T)^{-\mu} + (1-a)(c_t^N)^{-\mu}]^{-\frac{1}{\mu}} \quad (3.1)$$

where  $a$  is the CES weighting factor for tradable goods ( $0 \leq a \leq 1$ ) and  $\frac{1}{1+\mu}$  is the elasticity of substitution between tradable and non-tradable goods ( $\mu \geq -1$ ).

Each agent's preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} \quad (3.2)$$

where  $\gamma$  is the relative risk aversion coefficient.

Agents have access to two types of financial instruments that are traded in international capital markets: a non-state contingent bond ( $b_{t+1}$ ), with a gross interest rate of  $R$ , and a state contingent financial instrument ( $\hat{b}_{t+1}$ ), where the repayment of the principal is fixed but the interest rate payment ( $R_t^y$ ) is contingent on the realization of the endowment shock. Agents choose consumption of tradable ( $c_t^T$ ) and non-tradable goods ( $c_t^N$ ) and foreign asset holdings ( $b_{t+1}$  and  $\hat{b}_{t+1}$ ).

There is a large pool of risk neutral international lenders, where the participation constraint that allows for both types of contracts in equilibrium is given by:

$$\mathbb{E}_t R_{t+1}^y = R \quad (3.3)$$



### 3.2.2 Collateral Constraint

Due to imperfect enforceability of contracts, agents face a borrowing constraint in international credit markets. We can think of this as the requirement of collateral in order to obtain credit, based on the value of the current income of domestic agents. Only collateralized debt is offered by international creditors up to a fraction  $\kappa$  of the domestic agent's current income. A key feature in order to obtain an inefficiency in this type of models is the effect of relative prices on the collateral constraint. The use of non-tradable goods as collateral is related to the empirical evidence on credit booms in the non-tradable sector with external credit, as shown in Tornell and Westermann (2005). The other important feature is that collateral is calculated in terms of current income, which is consistent with the evidence found in Japelli (1990) where current income is a key factor affecting the probability of having access to credit markets.

The collateral constraint is given by:<sup>5</sup>

$$b_{t+1} + \hat{b}_{t+1} \geq -\kappa(y_t^T + p_t^N y_t^N) \quad (3.4)$$

which is more binding than the natural debt limit.  $y_t^T$  is the tradable endowment shock,  $y_t^N$  is non-tradable endowment and  $p_t$  is the relative price of non-tradable goods, which is also an indicator of the inverse of the real exchange rate.

In comparison to a model with no borrowing constraints, domestic agents reduce consumption and borrowing and accumulate precautionary savings to hedge

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<sup>5</sup>Notice that positive values of non-state contingent and state contingent assets can also be used as collateral. Going long on one asset and short on the other one by the same amount does not affect the collateral constraint.

against negative shocks that make the borrowing constraint binding.

### 3.2.3 Model Equilibrium

We solve for the allocations in the constrained social planner's problem and compare them to the ones in the competitive equilibrium in order to measure the size of the pecuniary externality, the price distortion, and the frequency of financial crises.

#### 3.2.3.1 Competitive Equilibrium

Each private agent in the domestic economy solves the following recursive problem:

$$V(b, \hat{b}, y^T) = \max_{b', \hat{b}'} \left\{ \frac{\left[ [a(c^T)^{-\mu} + (1-a)(c^N)^{-\mu}]^{-1/\mu} \right]^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}V(b', \hat{b}', y^{T'}) \right\} \quad (3.5)$$

s.t.

$$b' + \hat{b}' + c^T + p^N c^N = y^T + bR + \hat{b}R^y + p^N y^N \quad (3.6)$$

$$b' + \hat{b}' \geq -\kappa[y^T + p^N y^N] \quad (3.7)$$

where the relative price of non-tradable goods,  $p^N = p^N(b, \hat{b}, y^T)$ , is a function of aggregate borrowing and the tradable endowment shock ( $y^T$ ).

**Definition 4** *The recursive competitive equilibrium is defined by the policy functions  $\{c^T(b, \hat{b}, y^T), c^N(b, \hat{b}, y^T), b'(b, \hat{b}, y^T), \hat{b}'(b, \hat{b}, y^T)\}$ , prices  $\{p^N(b, \hat{b}, y^T)\}$  such that:*

1. *Borrowers: Taking  $p^N(b, \hat{b}, y^T)$  as given, each agent solves the borrower's problem to obtain the policy functions.*

2. *Market clearing: Markets for tradable and non-tradable goods clear:*

$$c^N = y^N \quad (3.8)$$

$$c^T(b, \hat{b}, y^T) + b'(b, \hat{b}, y^T) + \hat{b}'(b, \hat{b}, y^T) = y^T + bR + \hat{b}R^y \quad (3.9)$$

### 3.2.3.2 Constrained Social Planner Solution

**Definition 5** *The constrained social planner's equilibrium is defined by policy functions  $\{c^T(b, \hat{b}, y^T), c^N(b, \hat{b}, y^T), b'(b, \hat{b}, y^T), \hat{b}'(b, \hat{b}, y^T)\}$  that solve:*

$$W(b, \hat{b}, y^T) = \max_{b', \hat{b}'} \left\{ \frac{\left[ [a(c^T)^{-\mu} + (1-a)(c^N)^{-\mu}]^{-1/\mu} \right]^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}W(b', \hat{b}', y^{T'}) \right\} \quad (3.10)$$

s.t.

$$c^T = y^T - b' - \hat{b}' + bR + \hat{b}R^y \quad (3.11)$$

$$c^N = y^N \quad (3.12)$$

$$b' + \hat{b}' \geq -\kappa \left[ y^T + \frac{(1-a)}{a} \left( \frac{c^T}{c^N} \right)^{1+\mu} y^N \right] \quad (3.13)$$

## 3.3 Analytical Results

In this section, we present a qualitative analysis of the implications of state contingent assets in terms of risk sharing and the size of the pecuniary externality.

With pro-cyclical interest payments, where borrowers pay a higher interest rate in

good states and lower interest rate in bad ones, state contingent assets allow borrowers to partially hedge against the borrowing constraint in bad states, which are exactly the states where they need to borrow more in order to smooth consumption. By being less credit constrained, collateral prices drop less than in the case with only non-state contingent bonds and, therefore, the amplification mechanism created by the pecuniary externality is dampened.

### 3.3.1 Two bonds with i.i.d. tradable income shocks

A first step is to analyze an environment with i.i.d. shocks, where state contingent interest payments can be used to offset the fluctuations in tradable income. Debt with pro-cyclical interest payments allows agents to improve the transfer of resources from periods of high GDP to periods of low GDP, hence smoothing consumption across states.

When the tradable income shock is i.i.d., agents can perfectly hedge against uncertainty in the income shock across states with state contingent debt and use the non-state contingent bond to engage in risk sharing between periods.<sup>6</sup>

**Proposition 2** *If the tradable income shock is i.i.d., state contingent bonds are used to perfectly hedge against uncertainty in the income shock, whereas the non-state contingent bond is used for inter-temporal risk sharing. If  $\beta R < 1$ , the optimal choice for the non-state contingent bond is given by the binding collateral constraint:*

$$b' = -\kappa[y^T + p^N y^N] - \hat{b} \tag{3.14}$$

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<sup>6</sup>The derivation is presented in Appendix B.1.

**Proof.** For any two realizations of the tradable endowment shock,  $y$  and  $y'$ , it is possible to choose a level of the state contingent debt,  $\hat{b}$ , so that consumption is the same across states.

$$\begin{aligned} c(y) &= y - b' - \hat{b}' + bR + \hat{b}R^y \\ c(y') &= y' - b' - \hat{b}' + bR + \hat{b}R^{y'} \\ \hat{b} &= -\frac{y - y'}{R^y - R^{y'}} = -\frac{1}{\alpha} \end{aligned}$$

where  $\alpha$  is the degree of indexation of the interest rate payment for state contingent debt.<sup>7</sup>

If  $\beta R < 1$ , it is optimal to borrow up to the binding borrowing constraint. The constrained social planner and decentralized agents choose the same amount of debt:

$$b' = -\kappa[y^T + p^N y^N] - \hat{b}$$

Let us analyze the case of a  $-\epsilon$  shock to tradable endowment in period 0. Agents borrow up to the collateral constraint:

$$b_1 = \kappa[\epsilon + p^N(\epsilon)y^N] + \frac{1}{\alpha} \quad (3.15)$$

and from  $t=2$  onwards:

$$b_{t+1} = \frac{1}{\alpha} \quad (3.16)$$

$$c_0 = \epsilon(1 - \kappa) - \kappa p^N(\epsilon)y^N \quad (3.17)$$

$$c_1 = R\kappa[\epsilon + p^N(\epsilon)y^N] \quad (3.18)$$

and  $c_t = 0$  for  $t \geq 2$ .

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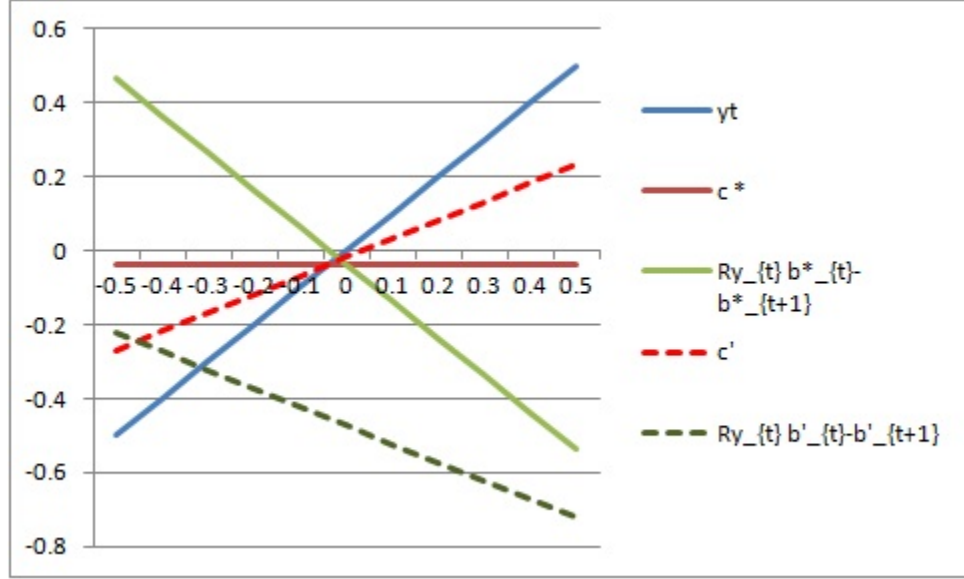
<sup>7</sup>We define the gross interest rate  $R^y = R + \alpha(y^T - \mathbb{E}\{y^T\})$ .

This result is optimal and satisfies the inter-temporal budget constraint:

$$\sum_{t=0}^{\infty} \frac{c_t}{R^t} = \epsilon(1 - \kappa) - \kappa p^N(\epsilon) y^N + \epsilon[\kappa + p^N(\epsilon) y^N] = \epsilon \quad (3.19)$$

$$\sum_{t=0}^{\infty} \frac{y_t}{R^t} = \epsilon \quad (3.20)$$

■



**Figure 3.1:** Optimal insurance across states using state contingent assets

Figure (3.1) shows the ability to smooth consumption across different states for an i.i.d. tradable endowment shock. The x-axis represents different values of the tradable endowment shock, centered at zero. Agents would optimally choose to trade the amount of the state contingent asset that allows for perfect hedging. The optimal amount of  $\hat{b}$  is such that, every period, agents would trade their income realization,  $y_t$ , for a fixed income,  $E_{t-1}[y_t] = E[y_t]$ , and achieve full consumption smoothing across states, shown by the constant consumption value at  $c^*$ . In order to achieve this result, agents need to borrow  $\hat{b} = -\frac{1}{\alpha}$ , so that the net flow in state

contingent debt,  $R_t^y \hat{b}_t - \hat{b}_{t+1}$ , perfectly offsets the effect of the tradable shock. If agents choose a lower amount of state contingent assets, like for example  $\hat{b} = -\frac{1}{2\alpha}$ , they would not be able to completely insure against the tradable income shock and would face a positive correlation between the shock and tradable consumption. This results in a positively correlated tradable consumption path,  $c'$ .

In terms of the effect on the pecuniary externality, both decentralized borrowers and a constrained social planner would find it optimal to borrow up to the binding collateral constraint, so that there is no pecuniary externality.<sup>8</sup>

### 3.3.2 Model with two bonds and persistent shocks

When shocks are persistent, state contingent debt provides partial hedging against income shocks across states, even though it is not possible to achieve full risk sharing. Partial exposure to tradable income requires agents to accumulate some precautionary savings, which makes it suboptimal to borrow up to the binding collateral constraint. State contingent debt provides two benefits: it provides partial hedging to reduce consumption fluctuations and reduces the tightness of collateral constraints in bad states.

From a normative perspective, we find that a constrained social planner accumulates more precautionary savings (or equivalently takes lower debt) as she internalizes that higher debt leads to a fall in the value of collateral through its effect on the real exchange rate, making other agents more financially constrained as

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<sup>8</sup>A pecuniary externality would arise in a special case where the coefficient of the equilibrium path is equal to one, so that the equilibrium oscillates between the constrained and unconstrained cases.

well, consistent with the results in an environment with bonds only. The pecuniary externality arises because agents fail to internalize the effect of their decisions in the valuation of collateral, and hence the whole economy faces financial crises more frequently.

However, in terms of the composition of total debt, a constrained social planner also internalizes that state contingent debt allows for partial hedging against the tightening of the collateral constraint, as it dampens the exchange rate depreciation in bad states. Lower interest payments allow for more debt rollover, which reduces the fall in tradable consumption and hence reduces the size of the real depreciation. This leads to a higher level of state contingent debt relative to the private equilibrium.

Let us analyze why the collateral constraint does not allow for perfect hedging against persistent tradable income shocks. We first present graphically the case where there is no borrowing constraint and no impatience factor,  $\beta R = 1$ , and show that agents can perfectly hedge against risk across states and time. We present the results for the effect of a one time  $-\epsilon$  shock on tradable endowment in period 0 on consumption and debt.

Tradable endowment follows an AR(1) process:

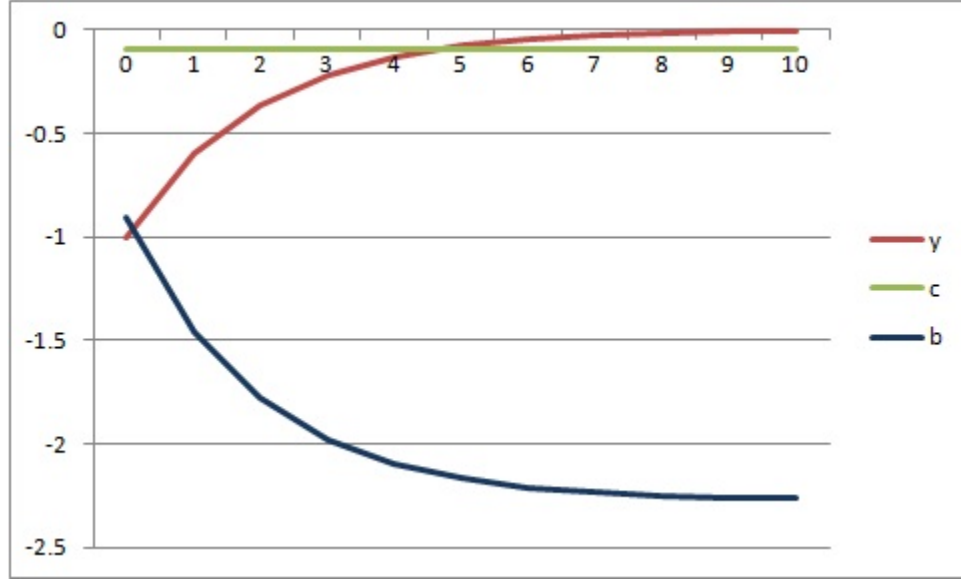
$$y_t = \rho y_{t-1} + \epsilon_t \tag{3.21}$$

For a one time shock in period 0, this becomes:

$$y_t = -\rho^t \epsilon \tag{3.22}$$

Given that  $\beta R = 1$ , the inter-temporal Euler equation establishes that agents would





**Figure 3.2:** Effect of  $-\epsilon$  shock on tradable endowment

optimally choose to engage in full consumption smoothing and accumulate debt to finance this consumption path. From the inter-temporal budget constraint:

$$c_t = -\frac{R}{R - \rho}\epsilon \quad (3.23)$$

$$b_{t+1} = -\frac{1 - \rho^t}{R - \rho}\epsilon \quad (3.24)$$

Figure (3.2) shows the time path for tradable endowment ( $y$ ), tradable consumption ( $c$ ) and assets ( $b$ ) in the case with no borrowing constraint. Agents can optimally choose to accumulate debt until they get a positive endowment shock that allows them to pay it back.

Let us compare this result to the case with the borrowing constraint and  $\beta R < 1$ . The following proposition shows that it is not possible to achieve full risk sharing as in the i.i.d. case.

**Proposition 3** *In an environment with persistent tradable endowment shocks, if*

$\beta R < 1$ , agents borrow less than the amount given by the collateral constraint,  $-\kappa[y^T + p^N y^N]$ .

**Proof.** By contradiction, let us assume that agents borrow up to the collateral constraint.

$$b_{t+1} = -\kappa y_t + \frac{1}{\alpha} \quad (3.25)$$

For a one time  $-\epsilon$  shock in period 0, this becomes:

$$b_{t+1} = -\kappa \epsilon \rho^t + \frac{1}{\alpha} \quad (3.26)$$

Tradable consumption must satisfy the budget constraint:

$$c_t = -(1 + \kappa)\rho^t \epsilon + R\kappa \epsilon \rho^{t-1} \quad (3.27)$$

The inter-temporal budget constraint does not hold:

$$\sum_{t=0}^{\infty} \frac{c_t}{R^t} = -\frac{R}{R-\rho} \epsilon \left[ \frac{R\kappa}{\rho} - \rho(1 + \kappa) \right] \quad (3.28)$$

$$\sum_{t=0}^{\infty} \frac{y_t}{R^t} = -\frac{R}{R-\rho} \epsilon \quad (3.29)$$

Therefore,  $b_{t+1}$  is less than the value given by the collateral constraint. ■

### 3.3.2.1 Decentralized Equilibrium

Optimal bond holdings in the decentralized equilibrium are given by:

$$U_1(c_t) = \beta RE_t \{U_1(c_{t+1})\} + \xi_t \quad (3.30)$$

where  $c_t = (c_t^T, c_t^N)$  and  $\xi_t$  is the shadow price of the collateral constraint. Compared to a standard model with no financial frictions, the Euler equation shows that agents borrow less if they face a borrowing constraint, given by the term  $\xi_t$ .

The inter-temporal Euler equation for the bond with state contingent interest payment,  $\hat{b}$ , is given by:

$$\begin{aligned} U_1(c_t) &= \beta E_t \{ R_{t+1}^y U_1(c_{t+1}) \} + \xi_t \\ &= \beta RE_t \{ U_1(c_{t+1}) \} + \beta cov(R_{t+1}^y, U_1(c_{t+1})) + \xi_t \end{aligned} \quad (3.31)$$

Pro-cyclical interest payments reduce the need to accumulate precautionary savings. There is a negative covariance between the marginal utility of consumption of tradable goods and the interest rate, so that debt provides partial hedging against income fluctuations. Therefore, private agents borrow more or, equivalently, accumulate less precautionary savings. By partially hedging against the tradable income shock, agents do not need to borrow as much in bad states, so that they hit the collateral constraint less frequently, which allows for better consumption smoothing and risk transfer. This is consistent with the results in Caballero and Panageas and the idea behind the rationality of introducing GDP indexed bonds to stabilize the debt service ratio in Borensztein and Mauro (2004).

Compared to the case with i.i.d. shocks, private agents are now unable to engage in perfect risk sharing across states because of the collateral constraint. There is a trade-off between the two margins: across states and across time, where higher risk sharing across states tightens the collateral constraint, so that there is less space for risk sharing across time using the non-state contingent bond. In equilibrium, the marginal benefit of increasing risk sharing across time, given by the right hand side of equation (3.30) must be equal to the marginal benefit of increasing risk sharing across states, given by the right hand side of equation (3.31).

If agents choose to borrow with bonds only ( $\hat{b}_t = 0$ ),  $cov(R^y, U_1(c_{t+1})) < 0$ . By increasing the holdings of debt in the state contingent asset, agents are able to optimally eliminate the correlation between income fluctuations, and hence interest rate fluctuations, with the marginal utility of consumption. This result is similar to the argument in Figure (3.1), even though private agents can only achieve partial consumption smoothing. The analytical result is obtained by combining equations (3.30) and (3.31):

$$cov(R_{t+1}^y, U_1(c_{t+1})) = 0 \quad (3.32)$$

### 3.3.2.2 Constrained social planner

Consider now a benevolent social planner who faces the same financial contracts as the private borrowers, but does internalize the spillover effects of agents' debt on collateral prices. As a result, we can show that the decentralized equilibrium is not constrained Pareto optimal, but the magnitude of the spillover effect is dampened when there is access to state contingent bonds.

The inter-temporal Euler equation for uncontingent bonds is given by:

$$U_1(c_t) = \beta RE_t \left\{ U_1(c_{t+1}) + \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N \right\} + \xi_t \left[ 1 - \kappa \frac{\partial p_t^N}{\partial c_{T,t}} \right] \quad (3.33)$$

By comparing equations (3.30) and (3.33), a constrained social planner chooses lower debt,  $-b_{t+1} - \hat{b}_{t+1}$ , or equivalently, accumulates more precautionary savings to insure against the fact that a higher level of debt lowers the price of non-tradable collateral, and therefore, further tightens the borrowing constraint.

The inter-temporal Euler equation for state contingent assets is given by:

$$\begin{aligned}
U_1(c_t) &= \beta E_t \left\{ R_{t+1}^y [U_1(c_{t+1}) + \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N] \right\} + \xi_t (1 - \kappa \frac{\partial p_t^N}{\partial c_{T,t}} y^N) \\
&= \beta RE_t \{ U_1(c_{t+1}) + \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N \} + \xi_t (1 - \kappa \frac{\partial p_t^N}{\partial c_{T,t}} y^N) \\
&\quad + \beta cov(R_{t+1}^y, U_1(c_{t+1}) + \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N) \quad (3.34)
\end{aligned}$$

By comparing equations (3.33) and (3.34), we get that:

$$cov(R_{t+1}^y, U_1(c_{t+1})) + cov(R_{t+1}^y, \kappa \xi_{t+1} \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N) = 0 \quad (3.35)$$

Similar to the decentralized equilibrium, increasing state contingent debt provides partial hedging against income fluctuations, given by the first term in equation (3.35). In addition, it also provides partial hedging against the collateral constraint, by making it less tight in bad states. Comparing equations (3.32) and (3.35), we can show that private agents take lower state contingent debt than the constrained social planner,  $-\hat{b}_{t+1}^{CE} < -\hat{b}_{t+1}^{SP}$ .

The intuition behind this result is that the constrained social planner does internalize the fact that taking more state contingent debt today gives the additional benefit of relaxing the collateral constraint tomorrow through the lower interest rate payments which lead to a lower drop in the price of non-tradable goods. Therefore, the state contingent asset has two benefits: it gives partial insurance across states by reducing consumption fluctuations and it affects insurance across time by reducing the tightness of collateral constraints in bad times.

We analyze the size of the pecuniary externality on total debt holdings for a private agent who is currently unconstrained ( $\xi_t = 0$ ) and compare it to the case

with bonds only. The pecuniary externality is smaller because state contingent debt provides partial hedging against bad shocks and against hitting the collateral constraint in bad times, so that the amplification mechanism given by the spillover effects of debt through collateral prices is dampened. By comparing equations (3.30) and (3.33), the pecuniary externality term is:

$$\beta RE \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N \right\}$$

This result is consistent with the result in Bianchi (2011) and Korinek (2010) that a constrained social planner that internalizes the effect of debt on relative prices chooses lower levels of debt than decentralized private borrowers.

By comparing the two Euler equations for state contingent debt, we find that the pecuniary externality term for a borrower that is currently unconstrained ( $\xi_t = 0$ ) is given by:

$$\beta RE \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N \right\} + \beta cov \left( R_{t+1}^y, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N \right)$$

Using the result in equation (3.35), agents who have access to borrowing with state contingent interest rate payments get a smaller pecuniary externality effect because

$$cov \left( R_{t+1}^y, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{T,t+1}} y^N \right) < 0$$

The pecuniary externality is smaller because agents are allowed to borrow more in bad states of the economy, which is precisely when they need to borrow more to smooth consumption at the expense of facing tighter constraints in good states, where they do not need to borrow as much.

Given that agents can borrow more in bad states of nature, they face a milder drop in relative prices compared to the case where there is only access to non-state contingent bonds. The smaller drop in relative prices and smaller amplification mechanism in the tightening of the collateral constraint reduces the size of the pecuniary externality. This results in lower need for precautionary savings and the constraint is binding in a lower frequency of states as well.

### 3.4 Quantitative Results

This section presents the solution to the infinite horizon problem using value function iteration. We analyze the distribution of debt levels, the frequency of financial crises and the size of the pecuniary externality in an environment with access to non-state and state contingent bonds. The results are consistent with the qualitative results in the previous section.

In order to analyze the quantitative properties of the model, we obtain the policy rules, the distribution of debt levels, the size of the pecuniary externality and the price of non-tradable goods under each state. Then, we simulate the model for 10,000 periods, where each period represents a year, in order to get the distribution of real variables and the frequency of financial crises.

#### 3.4.1 Parameter Values

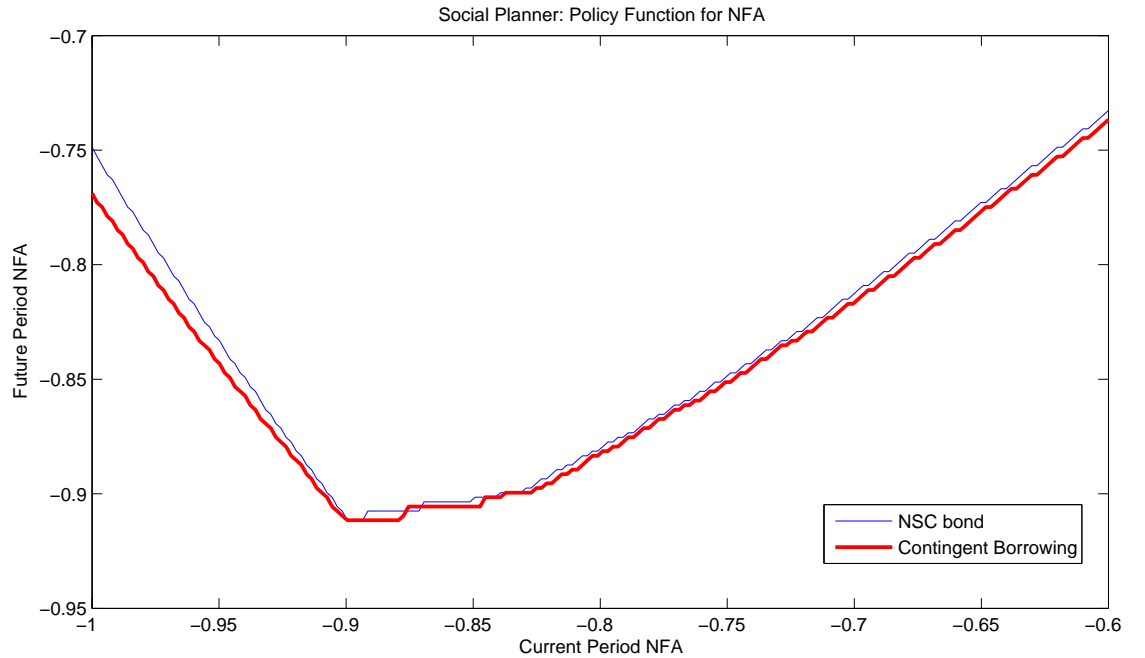
The values of the parameters are shown in Table (3.1). We use the parameter values for the quantitative exercise in Bianchi (2011), which is calibrated to the

Parameter	Value	Parameter	Value
$a$	0.31	$y^N$	1
$\beta$	0.945	$\kappa$	0.32
$\gamma$	2	$\alpha$	0.2
$\mu$	-0.205	$R$	1.045

**Table 3.1:** Parameter Values

economy of Argentina at an annual frequency. For the endowment shock process, we match a standard deviation of 0.059 and an autocorrelation value of 0.54. We use the discretization method proposed by Tauchen (1987) to approximate the first order autocorrelation process with a five grid point first-order Markov process.

### 3.4.2 Policy functions



**Figure 3.3:** Social Planner: Policy Functions for NFA

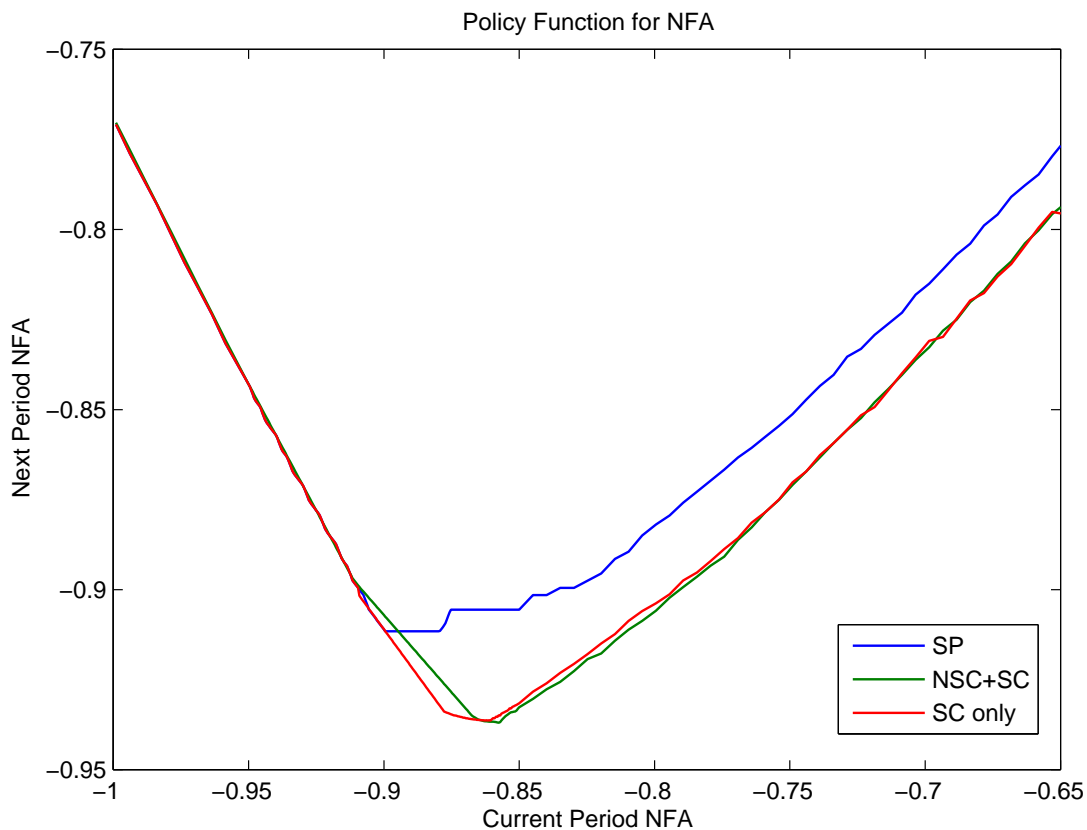
Figure (3.3) shows the policy functions for net foreign assets (NFA, defined



as  $b' + \hat{b}'$ ), chosen by a social planner who faces a tradable endowment shock that is one standard deviation below the mean, with and without access to contingent bonds. The line labeled 'NSC bond' shows the total asset position chosen by a constrained social planner with access to non-state contingent bonds only and the line labeled 'Contingent Borrowing' is the total asset position when they have access to both non-state contingent and state contingent instruments. Pro-cyclical interest payments provide partial hedging against the shock and allows the social planner to borrow more in states with a low endowment shock, as they need to accumulate lower precautionary savings.

The largest gain in terms of the ability to borrow in bad states is seen in periods where the agent is already highly indebted, due to the insurance benefit of state contingent debt which allows to relax the binding collateral constraint in bad states. A smaller drop in consumption of tradable goods dampens the effect of the real exchange rate depreciation on the valuation of collateral and therefore on the tightness of the collateral constraint.

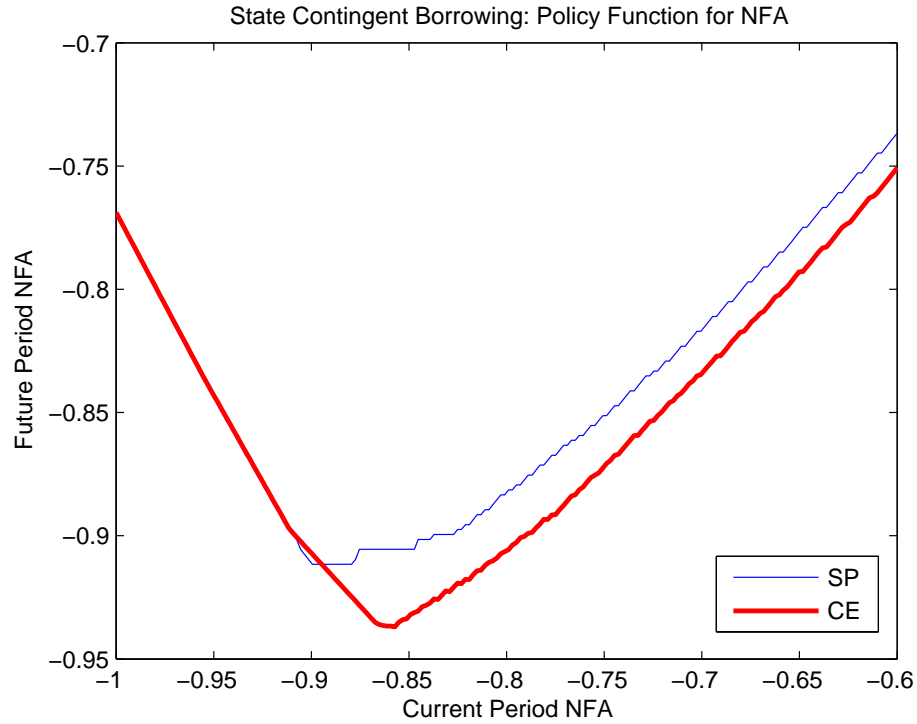
Now, let us compare the optimal debt choice in an environment with access to state contingent repayment assets only. Figure (3.4) presents the results for an environment with state contingent assets only, labeled 'SC only', and both types of assets, labeled 'NSC+SC'. State contingent repayment allows for better insurance against shocks to the tradable endowment, reducing the volatility of net income. If all debt has state contingent repayment, then borrowers need to repay less in bad states of nature, making the constraint less tight than if total debt has a combination of state contingent and non-state contingent repayment. As shown by the policy



**Figure 3.4:** Policy Functions for NFA with and without access to non-state contingent bonds

functions, this effect is especially important in intermediate levels of debt, where the pecuniary externality is the largest.

Figure (3.5) presents the policy functions for NFA in an environment with access to both types of bonds. (SP) represents the policy function for a constrained social planner, whereas (CE) represents the policy function for decentralized agents who do not internalize the effect of their decisions on the exchange rate. In this scenario, state contingent debt provides partial hedging against income fluctuations, but the social planner additionally internalizes that it also provides insurance against



**Figure 3.5:** Two Bonds: Policy Functions for NFA

hitting the collateral constraint by dampening the effect of individual debt on the real exchange rate.

The difference between the two policy functions is given by internalizing the effect of the debt choice on the valuation of collateral. Similar to the benchmark case with bonds only, decentralized agents do not consider that higher levels of debt further tightens the constraint through a sharp drop on the valuation of collateral. Highly leveraged agents are limited by the borrowing constraint, so they must lower borrowing in bad states of nature. A fall in consumption translates into a real exchange rate depreciation due to capital outflows that occur during sudden stop episodes.

However, with contingent assets, agents face the binding borrowing constraint

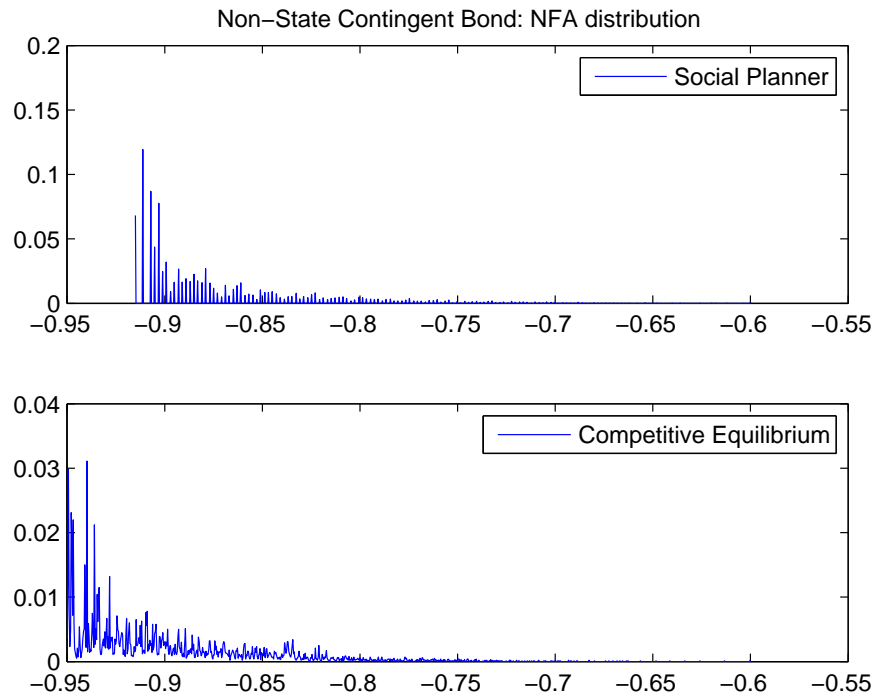
less often, as the value of repayment is much lower in bad states of nature. Given that agents do not need to reduce their tradable consumption as much, the effect of the current account reversal on the exchange rate is dampened. Decentralized agents face a smaller real exchange rate depreciation and a smaller drop in the nominal value of collateral. Therefore, the collateral constraint allows for higher amounts of debt by relaxing the borrowing constraint in low states, which are exactly the ones where agents want to borrow more.

### 3.4.3 Distribution of NFA

We compute the stationary distribution of net foreign asset holdings for the decentralized equilibrium with non-state contingent bonds, the constrained efficient solution and the equilibrium with both types of bonds. The stationary distribution of net foreign assets shows that there is a higher probability of reaching higher debt levels under an equilibrium with a single non-state contingent bond than under the constrained efficient case. This result is consistent with the policy functions, where agents in the decentralized economy borrow more than the constrained efficient level of debt.

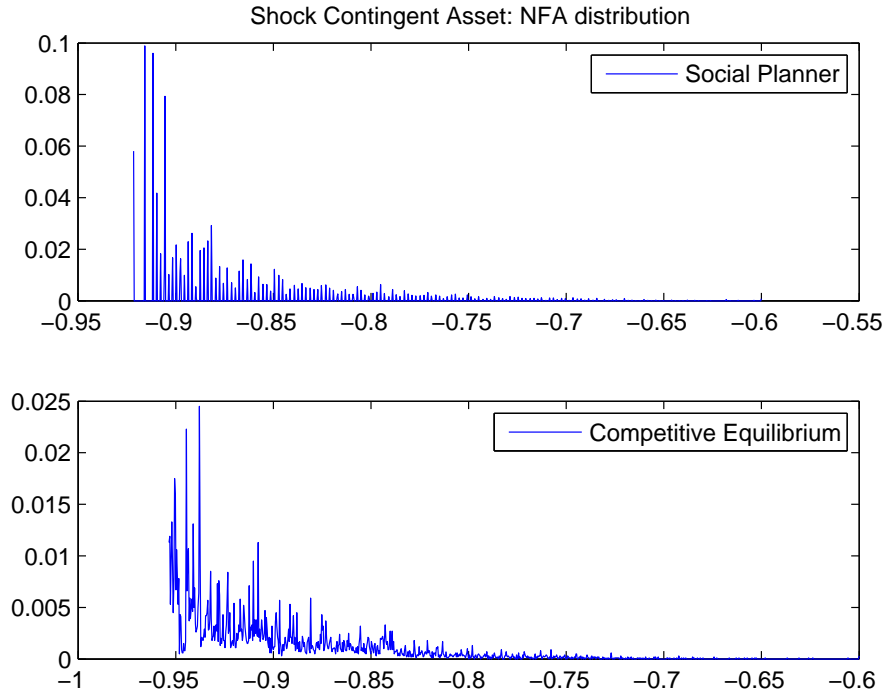
Figure (3.6) shows the distribution of NFA for the constrained efficient case (SP) in the top panel and for the decentralized equilibrium (CE) in the bottom panel, with non-state contingent bonds only. The average debt in terms of tradable income is 87 percent (27,5 percent of GDP) for the constrained efficient case, while the average debt is 90 percent (28,6 percent of GDP) for the decentralized economy.

Moreover, 49 percent of the highest debt levels for the decentralized equilibrium are not achieved under the constrained efficient equilibrium.



**Figure 3.6:** NFA distribution with Non-State Contingent Bonds

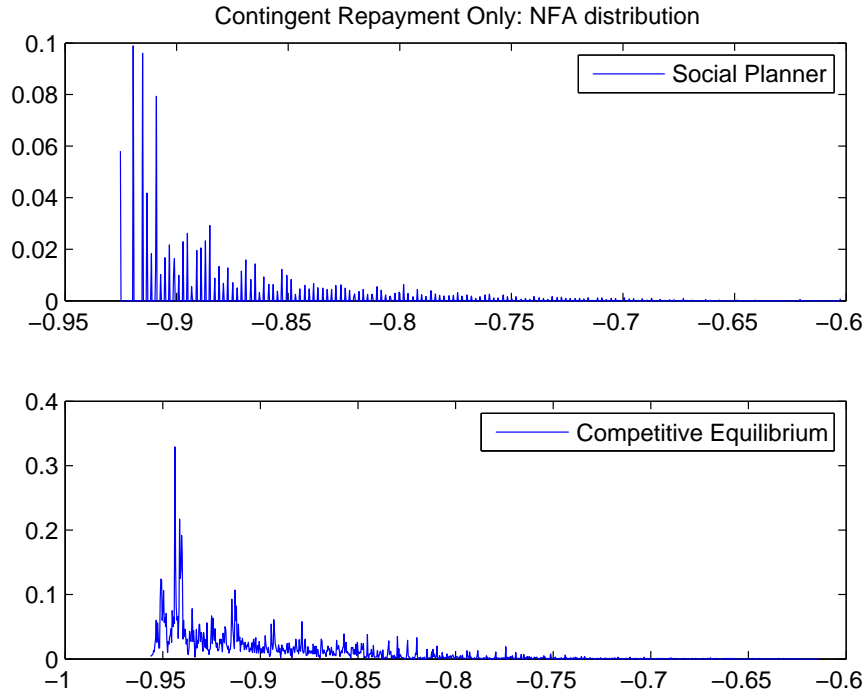
Figure (3.7) shows the distribution of NFA for the constrained efficient case (SP) and for the decentralized equilibrium (CE) in an environment with the two types of bonds. The results are consistent with the qualitative analysis in that there is a smaller difference in the distribution of total debt, and hence a smaller pecuniary externality. Also, state contingent debt reduces the need for precautionary savings and increases the average debt level in both cases. The average debt level under the decentralized equilibrium rises to 90,2 percent of tradable income (28,7 percent of GDP), whereas a constrained social planner who internalizes the effect of higher debt on the valuation of collateral has an average debt level of 88 percent of tradable



**Figure 3.7:** NFA distribution with Contingent Instruments

income (27,7 percent of GDP).

If we compare these results with an environment with access to state contingent repayment only, shown in Figure (3.8), we find that 44,9 percent of the highest debt levels for the decentralized equilibrium are not achieved under the constrained efficient equilibrium, compared to 46,5 percent in the case with access to both types of assets. This result shows that there is a smaller pecuniary externality in an environment where agents have access only to bonds with state contingent repayment. The intuition behind this result is that, with state contingent repayment only, the pecuniary externality only affects the total level of debt, while benefiting from the insurance properties of the state contingent repayment. In contrast, in an environment with both types of bonds, the pecuniary externality affects the total



**Figure 3.8:** NFA distribution with Contingent Instruments Only

amount and the composition of debt, creating a larger distortion from the second best solution.

### 3.4.4 Crisis Probability

A financial crisis is defined in our setup as a state where the economy is constrained by the collateral requirement and where the current account suffers a reversal with a magnitude larger than one standard deviation. There are two channels that create a wedge between the frequency of financial crises under the constrained social planner's problem and the decentralized equilibrium. The first one is related to the higher proportion of state contingent debt chosen by a constrained social planner. Higher state contingent debt provides insurance by reducing the tightness

of the collateral constraint in bad times so that binding collateral constraints are less frequent. The second channel is related to the fact that the social planner chooses a lower amount of total debt, as they internalize that higher debt tightens the collateral constraint through the effect on the valuation of collateral. Lower borrowing translates into lower probability of facing a binding collateral constraint as well.

In the environment with bonds only, the probability of a financial crisis is 4,6 percent, compared to 0,8 percent for the constrained social planner. However, if agents have access to the two types of bonds, the probability of a financial crisis in the decentralized equilibrium falls to 4,2 percent.

### 3.5 Conclusions

In this paper we analyze the effect of using alternative hedging instruments with contingent interest payments on risk sharing, the probability of a financial crisis and the size of the price externality in a two-good endowment economy subject to an endogenous collateral constraint.

Access to state contingent bonds allows agents to obtain partial hedging against income fluctuations and therefore engage in better consumption smoothing. In addition, state contingent debt creates an asymmetric effect on the collateral constraint, where pro-cyclical interest payments relaxes the collateral constraint in bad states, when agents need to borrow more, at the cost of tightening it in good states, when they do not need to borrow as much. Lower volatility of consumption dampens the fall in the price of non-tradable collateral in bad states, as agents would be facing



a less tight collateral constraint and would not need to reduce their borrowing and consumption as much.

Having access to state contingent financial instruments reduce the probability of experiencing a financial crisis and dampens the amplification effect created by the spillover effect of individual debt on the valuation of collateral of other agents. However, it is not possible to fully correct the pecuniary externality. As shown in Bianchi (2011) and Korinek (2010), the use of a Pigouvian tax that depends on the level of individual debt is able to align the private equilibrium to the one obtained by a constrained social planner. The optimal tax level is higher in states of higher debt levels, as there is a higher probability that the economy could face a binding collateral constraint in future periods. A financial instrument that is only contingent on the income shock does not allow to differentiate its interest payments between agents with high and low leverage, who face different marginal benefits of hedging against the income shock.

## Chapter A: Appendix for Chapter 2

### A.1 Pecuniary externality, optimal default and debt choices in the decentralized equilibrium and the social planner's solution

The intertemporal Euler equation is given by:

$$U_1(c_t) = \beta \int_{\hat{y}_{t+1}}^{\bar{y}} \frac{d[(1+r_{t+1})d_{t+1}]}{dd_{t+1}} U_1(c_{t+1}) f(y_{t+1}) dy_{t+1} \quad (\text{A.1})$$

#### A.1.1 Intertemporal Euler equations

##### A.1.1.1 Decentralized Equilibrium

Total differentiation on the lenders' participation constraint:

$$\begin{aligned} \frac{\partial[(1+r_{t+1})d_{t+1}]}{\partial d_{t+1}} \int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1} dd_{t+1} - [(1+r_{t+1})d_{t+1} - \lambda_2 (y_{t+1} + p_{t+1}y^N)] f(\hat{y}_{t+1}) \frac{d\hat{y}_{t+1}}{dd_{t+1}} &= (1+\rho) \\ \frac{\partial[(1+r_{t+1})d_{t+1}]}{\partial d_{t+1}} &= \frac{1+\rho + [(1+r_{t+1})d_{t+1} - \lambda_2 (y_{t+1} + p_{t+1}y^N)] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \end{aligned} \quad (\text{A.2})$$

The first order condition becomes:

$$U_1(c_t) = \beta \frac{1+\rho + [(1+r_{t+1})d_{t+1} - \lambda_2 (y_{t+1} + p_{t+1}y^N)] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \int_{\hat{y}_{t+1}}^{\bar{y}} U_1(c_{t+1}) f(y_{t+1}) dy_{t+1} \quad (\text{A.3})$$

### A.1.1.2 Social Planner

Total differentiation on the lenders' participation constraint:

$$\begin{aligned} & \frac{\partial[(1+r_{t+1})d_{t+1}]}{\partial d_{t+1}} \int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1} + \lambda_2 y^N \int_{\underline{y}}^{\hat{y}_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}} f(y_{t+1}) dy_{t+1} \\ & - [(1+r_{t+1})d_{t+1} - \lambda_2 (y_{t+1} + p_{t+1} y^N)] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} = (1+\rho) \end{aligned}$$

$$\frac{\partial[(1+r_{t+1})d_{t+1}]}{\partial d_{t+1}} = \frac{1+\rho + [(1+r_{t+1})d_{t+1} - \lambda_2 (y_{t+1} + p_{t+1} y^N)] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} - \lambda_2 y^N \int_{\underline{y}}^{\hat{y}_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}} f(y_{t+1}) dy_{t+1}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \quad (\text{A.4})$$

The first order condition becomes:

$$\begin{aligned} U_1(c_t) = \beta & \frac{1+\rho + [(1+r_{t+1})d_{t+1} - \lambda_2 (y_{t+1} + p_{t+1} y^N)] f(\hat{y}_{t+1}) \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}} - \lambda_2 y^N \int_{\underline{y}}^{\hat{y}_{t+1}} \frac{\partial p_{t+1}}{\partial d_{t+1}} f(y_{t+1}) dy_{t+1}}{\int_{\hat{y}_{t+1}}^{\bar{y}} f(y_{t+1}) dy_{t+1}} \\ & \times \int_{\hat{y}_{t+1}}^{\bar{y}} U_1(c_{t+1}) f(y_{t+1}) dy_{t+1} \end{aligned} \quad (\text{A.5})$$

## A.1.2 Effect of individual debt on default incentives

### A.1.2.1 Decentralized equilibrium

Total differentiation on the definition of the default threshold:

$$\frac{\partial v^R(d, D, \hat{y})}{\partial d} dd + \frac{\partial v^R(d, D, \hat{y})}{\partial \hat{y}} d\hat{y} = \frac{\partial v^D(D, \hat{y})}{\partial \hat{y}} d\hat{y} \quad (\text{A.6})$$

$$-(1+r)U_1(\hat{c}^R)dd + U_1(\hat{c}^R)d\hat{y} = (1-\lambda_1)U_1(\hat{c}^D)d\hat{y}$$

$$\frac{\partial \hat{y}}{\partial d} = \frac{(1+r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1-\lambda_1)U_1(\hat{c}^D)} \quad (\text{A.7})$$

### A.1.2.2 Social planner

Total differentiation on the definition of the default threshold:

$$\frac{\partial v^R(d, \hat{y})}{\partial d} dd + \frac{\partial v^R(d, \hat{y})}{\partial \hat{y}} d\hat{y} = \frac{\partial v^D(d, \hat{y})}{\partial d} dd + \frac{\partial v^D(d, \hat{y})}{\partial \hat{y}} d\hat{y} \quad (\text{A.8})$$

$$-(1+r)U_1(\hat{c}^R)dd + U_1(\hat{c}^R)d\hat{y} = (1-\lambda_1)U_1(\hat{c}^D)d\hat{y} - \lambda_1 \frac{\partial p}{\partial \hat{y}} y^N U_1(\hat{c}^D)d\hat{y} - \lambda_1 \frac{\partial p}{\partial d} y^N U_1(\hat{c}^D)dd$$

$$\frac{\partial \hat{y}}{\partial d} = \frac{(1+r)U_1(\hat{c}^R) + \lambda_1 \frac{\partial p}{\partial \hat{c}_T} y^N U_1(\hat{c}^D)}{U_1(\hat{c}^R) - (1-\lambda_1)U_1(\hat{c}^D) + \lambda_1 \frac{\partial p}{\partial \hat{c}_T} y^N U_1(\hat{c}^D)} \quad (\text{A.9})$$

By comparing (A.7) and (A.9), we find that:

$$\frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}^{CE} > \frac{\partial \hat{y}_{t+1}}{\partial d_{t+1}}^{SP} \quad (\text{A.10})$$

## A.2 Optimal default penalty

We solve for the competitive equilibrium conditions when the default penalty instrument is available. The problem of a representative private borrower under repayment is the same, except for the new definition of welfare under the default state,  $v^D$ , which affects the default threshold,  $\hat{y}$ .

$$v^D(d, \hat{y}) = \max U(c_T^R, c_N^R) + \beta \phi E v^R(0, y') + \beta(1-\phi) E v^A(y') \quad (\text{A.11})$$

subject to:

$$c_T^D + p c_N^D = (1 - \lambda_1(1 + \tau))(\hat{y} + p y^N) + T \quad (\text{A.12})$$

Taking total differentiation on the definition of the default threshold:

$$\frac{\partial v^R(d, D, \hat{y})}{\partial d} dd + \frac{\partial v^R(d, D, \hat{y})}{\partial \hat{y}} d\hat{y} = \frac{\partial v^D(D, \hat{y})}{\partial \hat{y}} d\hat{y} \quad (\text{A.13})$$

$$-(1+r)U_1(\hat{c}^R)dd + U_1(\hat{c}^R)d\hat{y} = (1-\lambda_1(1+\tau))U_1(\hat{c}^D)d\hat{y}$$

$$\frac{\partial \hat{y}}{\partial d} = \frac{(1+r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1 - \lambda_1(1+\tau))U_1(\hat{c}^D)} \quad (\text{A.14})$$

In order to align default incentives to the social planner's equilibrium,  $\tau$  must satisfy:

$$\frac{(1+r)U_1(\hat{c}^R)}{U_1(\hat{c}^R) - (1 - \lambda_1(1+\tau))U_1(\hat{c}^D)} = \frac{(1+r)U_1(\hat{c}^R) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}^T} y^N U_1(\hat{c}^D)}{U_1(\hat{c}^R) - (1 - \lambda_1)U_1(\hat{c}^D) + \lambda_1 \frac{\partial \hat{p}}{\partial \hat{c}^T} y^N U_1(\hat{c}^D)} \quad (\text{A.15})$$

### A.3 Deadweight cost of default

This section shows that if there is no deadweight cost created by default, then it leads to efficient debt and default decisions. We show it in a simple two period model, where we impose that  $\lambda_1 = \lambda_2$ , which is the case where all seized assets under default are transferred to the lender in terms of tradable goods.

#### A.3.1 Decentralized equilibrium

Debt follows the following optimality condition:

$$U_1(c_1) = \beta(1+\rho) \left\{ \int_{\hat{y}}^{\bar{y}} f(y_2) dy_2 - [(1+r_1)d_1 - \lambda_1(\hat{y} + \hat{p}y^N)] \frac{1}{\lambda_1} f(\hat{y}) \right\}^{-1} \\ \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) f(y_2) dy_2$$

The default threshold is defined as:

$$U(\hat{y} - (1+r_1)d_1, y_N) = U(\hat{c}_T^{D,CE}, \hat{c}_N^{D,CE})$$

where  $\hat{c}_T^{D,CE}, \hat{c}_N^{D,CE}$  solve

$$\max U(\hat{c}_T^{D,CE}, \hat{c}_N^{D,CE}) \\ \text{s.t. } \hat{c}_T^{D,CE} + p\hat{c}_N^{D,CE} = (1 - \lambda_1)(\hat{y} + \hat{p}y_N)$$

To simplify the exercise, assume  $U(c_T, c_N) = \log c_T + \log c_N$ .

$$\begin{aligned}\hat{c}_T^{D,CE} &= \frac{1 - \lambda_1}{2}(\hat{y} + \hat{p}y^N) \\ \hat{c}_N^{D,CE} &= \frac{1 - \lambda_1}{2\hat{p}}(\hat{y} + \hat{p}y^N)\end{aligned}$$

and the default threshold condition is

$$\begin{aligned}\log(\hat{y} - (1 + r_1)d_1) + \log(y_N) &= \log\left(\frac{1 - \lambda_1}{2}(\hat{y} + \hat{p}y^N)\right) \\ &+ \log\left(\frac{1 - \lambda_1}{2\hat{p}}(\hat{y} + \hat{p}y^N)\right)\end{aligned}$$

Market clearing in the non-tradable sector implies that  $\hat{c}_N^{D,CE} = y_N$ . Therefore, the market clearing price becomes

$$\hat{p} = \frac{(1 - \lambda_1)\hat{y}}{(1 + \lambda_1)y_N}$$

Replacing the market clearing price in the default condition it becomes

$$\begin{aligned}\log(\hat{y} - (1 + r_1)d_1) + \log(y_N) &= \log\left(\frac{1 - \lambda_1}{1 + \lambda_1}\hat{y}\right) + \log(y^N) \\ (1 + r_1)d_1 &= \frac{2\lambda_1}{1 + \lambda_1}\hat{y} \\ (1 + r_1)d_1 &= \lambda_1(\hat{y} + \hat{p}y_N)\end{aligned}$$

Plugging this result in the optimality condition, it simplifies to: CE

$$U_1(c_1) = \frac{\beta(1 + \rho)}{\int_{\hat{y}}^{\bar{y}} f(y_2)dy_2} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) f(y_2)dy_2$$

### A.3.2 Social Planner

Optimal debt chosen by the social planner follows:

$$\begin{aligned}U_1(c_1) &= \beta(1 + \rho) \left\{ \int_{\hat{y}}^{\bar{y}} f(y_2)dy_2 - [(1 + r_1)d_1 - \lambda_1(\hat{y} + \hat{p}y^N)] \frac{1}{\lambda_1} \left(1 - \frac{1 - \lambda_1}{2}\right) f(\hat{y}) \right\}^{-1} \\ &\int_{\hat{y}}^{\bar{y}} U_1(c_2^R) f(y_2)dy_2\end{aligned}$$

The default threshold is given by:

$$\begin{aligned}\log(\hat{y} - (1 + r_1)d_1) + \log(y_N) &= \log\left((1 - \lambda_1)\hat{y} - \lambda_1\frac{\hat{U}_2}{\hat{U}_1}y^N\right) + \log(y_N) \\ (1 + r_1)d_1 &= \lambda_1\left(\hat{y} + \frac{\hat{U}_2}{\hat{U}_1}y^N\right)\end{aligned}$$

Plugging this result in the optimality condition, it simplifies to: SP:

$$U_1(c_1) = \frac{\beta(1 + \rho)}{\int_{\hat{y}}^{\bar{y}} f(y_2)dy_2} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R)f(y_2)dy_2$$

## A.4 Exogenous default

Agents consume only tradable goods in the first period (to simplify the problem) and tradable and non-tradable goods in the second period. The tradable endowment in the second period can take 2 values:  $y_L$  with probability  $\pi$  and  $y_H$  with probability  $1 - \pi$ .

To analyze the size of the pecuniary externality in a model with exogenous default, we must choose parameter values such that agents choose to repay in the high state ( $y_H$ ) and default in the low state ( $y_L$ ).

Therefore, the parameters must satisfy the condition for default under  $y_2 = y_L$ :

$$\lambda_1(y_L + \frac{U_2}{U_1}y^N) < (1 + r)d$$

and for repayment under  $y_2 = y_H$

$$\lambda_1(y_H + \frac{U_2}{U_1}y^N) \geq (1 + r)d$$

### A.4.1 Decentralized equilibrium

Private agents choose debt to maximize expected utility:

$$U(y + d) + \beta\pi U(c_{T2}^L, c_{N2}^L) + \beta(1 - \pi)U(c_{T2}^H, c_{N2}^H)$$

subject to the budget constraints

$$c_2^L + p_2 c_{N2}^L = (1 - \lambda_1)y_L + (1 - \lambda_1)p_2 y^N$$

$$c_2^H + p_2 c_{N2}^H = y_H - (1 + r)d + p_2 y^N$$

and the interest rate schedule:

$$1 + \rho = (1 - \pi)(1 + r) + \pi \lambda_2 \frac{y_L + p_2 y^N}{d}$$

The optimal choice of debt satisfies:

$$U_1(c_1) = \beta(1 + \rho)U_1(c_2^H)$$

## A.4.2 Social Planner

Let us assume log-utility  $U(c_T, c_N) = \log(c_T) + \log(c_N)$  to simplify the problem. The social planner chooses debt to maximize expected utility:

$$U(y + d) + \beta \pi U(c_{T2}^L, c_{N2}^L) + \beta(1 - \pi)U(c_{T2}^H, c_{N2}^H)$$

subject to the resource constraints:

$$c_{T2}^L = (1 - \lambda_1)y_L - \lambda_1 \frac{U_2}{U_1} y^N$$

$$c_{T2}^H = y_H - (1 + r)d$$

$$c_{N2}^L = c_{N2}^H = y^N$$

and the interest rate schedule:

$$1 + \rho = (1 - \pi)(1 + r) + \pi \lambda_2 \frac{y_L + \frac{U_2}{U_1} y^N}{d}$$

The optimal level of debt satisfies:

$$U_1(c_1) = \beta \left[ 1 + r + \frac{d(1 + r)}{dd} d \right] (1 - \pi) U_1(c_2^H)$$



$$U_1(c_1) = \beta(1 + \rho) \frac{1 - \pi}{1 - (1 + \lambda_2)\pi} U_1(c_2^H)$$

Comparing the debt choice under the decentralized equilibrium and the socially efficient level, we see that:

$$U_1(c_1^{SP}) > U_1(c_1^{CE})$$

$$d_1^{SP} < d_1^{CE}$$

Increase in debt reduces  $p_2$ . Given an exogenous default threshold, the probability of default does not change. However, the relative price does affect the amount of resources recovered by the lender under default, given by  $\frac{y^L + p_2 y^N}{d}$ . Lower prices and higher debt means that they get a lower amount of resources under the default scenario, so they charge a higher interest rate schedule to compensate for this loss. As the interest rate schedule in the social planner's problem is steeper, the social planner chooses a lower level of debt.

## A.5 Optimal Policy in the Two-Period Model

We introduce new tax on default ( $\tau$ ), which is an additional cost faced by borrowers in case of default and is charged in terms of total income. The revenue is given back as a lump sum transfer  $T$ .

Under this new setup, the new default threshold  $\hat{y}$  is given by

$$\hat{y} - (1 + r)d = (1 - \lambda_1 - \tau)\hat{y} - (\lambda_1 + \tau)\hat{p}y^N + \hat{T} \quad (\text{A.16})$$

In equilibrium,  $\hat{T} = \tau(\hat{y} + \hat{p}y^N)$ , and the default set becomes:

$$\hat{y} - (1 + r)d = (1 - \lambda_1)\hat{y} - \lambda_1\hat{p}y^N \quad (\text{A.17})$$

which is consistent with the default threshold under the social planner's problem. The

interest rate schedule is unchanged.

$$(1+r)d \int_{\hat{y}}^{\bar{y}} f(y_2) dy_2 + \lambda_2 \int_{\underline{y}}^{\hat{y}} (y + py^N) f(y_2) dy_2 = 1 + \rho \quad (\text{A.18})$$

The problem faced by each decentralized borrower is:

$$\max_{c_{1T}, c_{1N}, c_{2T}, c_{2N}, d} U(c_{1T}, c_{1N}) + \beta \int_{\hat{y}}^{\bar{y}} U(c_{2T}^R, c_{2N}^R) f(y_2) dy_2 + \beta \int_{\underline{y}}^{\hat{y}} U(c_{2T}^D, c_{2N}^D) f(y_2) dy_2$$

s.t.

$$c_{1T} + p_1 c_{1N} = y_1 + d + p_1 y_N$$

$$c_{2T}^R + p_2 c_{2N}^R = y_2 - (1+r)d + p_2 y^N$$

$$c_{2T}^D + p_2 c_{2N}^D = (1 - \lambda_1 - \tau) y_2 - (\lambda_1 + \tau) p_2 y^N + T$$

$$\hat{y} - (1+r)d = (1 - \lambda_1 - \tau) \hat{y} - (\lambda_1 + \tau) \hat{p} y^N + \hat{T}$$

$$(1+r)d \int_{\hat{y}}^{\bar{y}} f(y_2) dy_2 + \lambda_2 \int_{\underline{y}}^{\hat{y}} (y + py^N) f(y_2) dy_2 = 1 + \rho$$

The intertemporal optimality condition for debt is:

$$U_1(y_1 + d) = \beta \frac{d(1+r)d}{dd} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) f(y_2) dy_2$$

Taking total differentiation on the default threshold and the interest rate equation, we get:

$$\frac{d\hat{y}}{dd} = \frac{1}{\lambda_1 + \tau} \frac{d(1+r)d}{dd}$$

Total differentiation on zero profit condition:

$$\frac{d(1+r)d}{dd} \left\{ \int_{\hat{y}}^{\bar{y}} f(y_2) dy_2 - [(1+r)d - \lambda_2(\hat{y} + \hat{p}y^N)] \frac{f(\hat{y})}{\lambda_1 + \tau} \right\} = 1 + \rho$$

The optimality condition becomes:

$$U_1(y_1 + d) = \frac{\beta(1+\rho) \int_{\hat{y}}^{\bar{y}} U_1(y_2 - (1+r)d) f(y_2) dy_2}{\int_{\hat{y}}^{\bar{y}} f(y_2) dy_2 - [(1+r)d - \lambda_2(\hat{y} + \hat{p}y^N)] \frac{f(\hat{y})}{\lambda_1 + \tau}}$$

Plugging the value of the default threshold:

$$U_1(y_1 + d) = \frac{\beta(1 + \rho) \int_{\hat{y}}^{\bar{y}} U_1(y_2 - (1 + r)d) f(y_2) dy_2}{\int_{\hat{y}}^{\bar{y}} f(y_2) dy_2 - [(\lambda_1 + \tau - \lambda_2)(\hat{y} + \hat{p}y^N) - \hat{T}] \frac{f(\hat{y})}{\lambda_1 + \tau}}$$

Using the equilibrium value of  $\hat{T}$

$$U_1(y_1 + d) = \frac{\beta(1 + \rho) \int_{\hat{y}}^{\bar{y}} U_1(y_2 - (1 + r)d) f(y_2) dy_2}{\int_{\hat{y}}^{\bar{y}} f(y_2) dy_2 - [(\lambda_1 - \lambda_2)(\hat{y} + \hat{p}y^N)] \frac{f(\hat{y})}{\lambda_1 + \tau}} \quad (\text{A.19})$$

In order to get the same allocations as in the social planner's problem, we choose a tax that makes this equation equivalent to the one in the social planner's problem. From the previous section, the optimal debt choice under the social planner's problem is given by

$$U_1(c_1) = \frac{\beta(1 + \rho) \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) f(y_2) dy_2}{\int_{\hat{y}}^{\bar{y}} f(y_2) dy_2 - (\lambda_1 - \lambda_2) \frac{1 + \lambda_1}{2\lambda_1} f(\hat{y})} \quad (\text{A.20})$$

Comparing it to the social planner's optimality condition to get the value of  $\tau$  :

$$\begin{aligned} \frac{(\lambda_1 - \lambda_2)(\hat{y} + \hat{p}y^N)}{\lambda_1 + \tau} &= \frac{(\lambda_1 - \lambda_2)(\hat{y} + \hat{p}y^N)(1 + \lambda_1)}{2\lambda_1} \\ 2\lambda_1 &= (1 + \lambda_1)\lambda_1 + (1 + \lambda_1)\tau \\ \tau &= \frac{\lambda_1 - \lambda_1^2}{1 + \lambda_1} = \frac{\lambda_1(1 - \lambda_1)}{1 + \lambda_1} \end{aligned} \quad (\text{A.21})$$

## A.6 Algorithm for the Numerical Solution

### A.6.1 Social Planner's Problem

1. Start with some guess for the price of bonds  $q^0(d, y) = \frac{1}{1 + \rho}$  for all  $d$  and  $y$ .
2. Given the interest rate schedule, solve for the optimal consumption  $c(d, y)$ , debt holdings  $d'(d, y)$ , and default set  $\delta(d, y)$  using value function iteration. Iterate on the value function until convergence is achieved.

3. Using the default set and repayment set, compute the new price of bonds  $q^1(d, y)$  that satisfies the zero profit condition for the risk neutral lender and compare it with the one used in the previous iteration  $q^0(d, y)$ . If a convergence criterion is met,  $\max |q^1(d, y) - q^0(d, y)| < \epsilon_r$ , go to the next step. If not, update the bond price using a Gauss - Seidel algorithm and go back to the previous step.

### A.6.2 Decentralized Equilibrium

1. Start with some guess for the relative price of non-tradable goods in each state  $p^0(D, y)$ . An initial guess is the price obtained from the social planner's problem.
2. Start with some guess for the price of bonds  $q^0(d, D, y) = q^{SP}(d, y)$  for all  $D$  from the social planner's problem.
3. Start with some guess for the law of motion of aggregate debt  $\Gamma^0(D, y) = d'(d, y)$  from the social planner's problem.
4. Given the interest rate schedule, solve for the optimal consumption  $c(d, D, y)$ , debt holdings  $d'(d, D, y)$  and default set  $\delta(d, D, y)$  using value function iteration. For every iteration, update the law of motion for aggregate debt  $\Gamma^i(D, y) = d'^i(D, D, y)$ . Iterate on the value function until convergence is achieved.
5. Using the default and repayment sets, compute the bond price  $q^1(d, D, y)$  that satisfies the zero profit condition for the risk neutral lender and compare it with the one used in the previous iteration  $q^0(d, D, y)$ . If a convergence criterion is met,  $\max |q^1(d, D, y) - q^0(d, D, y)| < \epsilon_r$ , go to the next step. If not, update the interest rate schedule using a Gauss - Seidel algorithm and go back to the previous step.

6. Using optimal debt holdings, compute the price of non- tradable goods  $p^1(D, y)$  and compare it with the previous guess  $p^0(D, y)$ . If a convergence criterion is met,  $\max |p^1(D, y) - p^0(D, y)| < \epsilon_p$ , go to the next step. If not, update the price using a Gauss - Seidel algorithm and go back to step 2.

## A.7 Default as a source of insurance in a two-state model

We present a two period model with two states for the exogenous endowment shock: state H, with probability  $\pi$ , where repayment takes place, and state L, with probability  $1 - \pi$ , where default takes place. There are two instruments available, default and debt. In this simple environment we switch off the deadweight cost of default, such that  $\lambda_1 = \lambda_2 = \lambda$ . We have two instruments and two states, such that full insurance is attainable by choosing the value of  $\lambda$  that allows for full risk sharing.

The problem is given by:

$$\max_{d, \lambda} U(y + d, y^N) + \beta\pi U(y^H - (1 + r)d, y^N) + \beta(1 - \pi)U((1 - \lambda)y^L - \lambda p^L y^N, y^N) \quad (\text{A.22})$$

s.t.

$$\pi(1 + r)d + (1 - \pi)\lambda(y^L + p^L y^N) = (1 + \rho)d \quad (\text{A.23})$$

Intertemporal optimality condition:

$$U_1(y^L + d, y^N) = \beta(1 + \rho)U_1(y^H - (1 + r)d, y^N) \quad (\text{A.24})$$

Full insurance: same consumption under both states

$$y^H - (1 + r)d = (1 - \lambda)y^L - \lambda p^L y^N \quad (\text{A.25})$$

Plugging (A.23) into (A.25):

$$y^H - \frac{(1 + \rho)d}{\pi} + \frac{1 - \pi}{\pi}\lambda(y^L + p^L y^N) = y^L - \lambda(y^L + p^L y^N)$$

$$d = \frac{1}{1+\rho} [\pi(y^H - y^L) + \lambda(y^L + p^L y^N)] \quad (\text{A.26})$$

Plugging (A.26) into (A.24)

$$\begin{aligned} U_1 \left( y^L + \frac{1}{1+\rho} [\pi(y^H - y^L) + \lambda(y^L + p^L y^N)], y^N \right) &= \beta(1+\rho) U_1 \left( (1-\lambda)y^L - \lambda p^L y^N, y^N \right) \\ U_1 \left( y^L + \frac{1}{1+\rho} \left[ \pi(y^H - y^L) + \frac{2\lambda}{1+\lambda} y^L \right], y^N \right) &= \beta(1+\rho) U_1 \left( \frac{1-\lambda}{1+\lambda} y^L, y^N \right) \end{aligned} \quad (\text{A.27})$$

The value of  $\lambda$  that solves for equation (A.27) allows for perfect insurance under this setup.

## A.8 Optimal enforcement of creditor rights

### A.8.1 Trade off with no deadweight loss ( $\lambda_1 = \lambda_2 = \lambda$ )

$$\max_{d,\lambda} U(y+d, y^N) + \beta \int_{\underline{y}}^{\hat{y}} U[(1-\lambda)y - \lambda p y^N, y^N] dF(y) + \beta \int_{\hat{y}}^{\bar{y}} U[y - (1+r)d, y^N] dF(y) \quad (\text{A.28})$$

s.t.

$$(1+r)d \int_{\hat{y}}^{\bar{y}} dF(y) + \lambda \int_{\underline{y}}^{\hat{y}} (y + p y^N) dF(y) = (1+\rho)d \quad (\text{A.29})$$

$$\hat{y} = \frac{(1+r)d}{\lambda} - p y^N \quad (\text{A.30})$$

Optimality condition with respect to debt:

$$U_1(c_1) = \frac{\beta(1+\rho)}{\int_{\hat{y}}^{\bar{y}} dF(y)} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) dF(y) \quad (\text{A.31})$$

Optimality condition with respect to  $\lambda$ :  $\lambda$ :

$$- \int_{\underline{y}}^{\hat{y}} U_1(c_2^D)(y + p y^N) dF(y) = \frac{d[(1+r)d]}{d\lambda} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) dF(y) \quad (\text{A.32})$$

Differentiate (A.29) with respect to  $\lambda$ :

$$\frac{d[(1+r)d]}{d\lambda} \int_{\hat{y}}^{\bar{y}} dF(y) - \frac{d\hat{y}}{d\lambda} [(1+r)d - \lambda(\hat{y} + p y^N)] f(\hat{y}) + \int_{\underline{y}}^{\hat{y}} (y + p y^N) dF(y) = 0 \quad (\text{A.33})$$

Using (A.30),  $(1+r)d = \lambda(\hat{y} + \hat{p}y^N)$

$$\frac{d[(1+r)d]}{d\lambda} = -\frac{\int_{\underline{y}}^{\hat{y}} (y + py^N) dF(y)}{\int_{\hat{y}}^{\bar{y}} dF(y)} \quad (\text{A.34})$$

The optimality condition becomes:

$$\int_{\underline{y}}^{\hat{y}} U_1(c_2^D)(y + py^N) dF(y) = \frac{\int_{\underline{y}}^{\hat{y}} (y + py^N) dF(y)}{\int_{\hat{y}}^{\bar{y}} dF(y)} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) dF(y) \quad (\text{A.35})$$

### A.8.2 Trade off with deadweight loss ( $\lambda_1 > \lambda_2$ )

$$\begin{aligned} \max_{d, \lambda_2} U(y + d, y^N) + \beta \int_{\underline{y}}^{\hat{y}} U[(1 - \lambda_2(1 + \lambda_2))y - \lambda_2(1 + \lambda_2)py^N, y^N] dF(y) \\ + \beta \int_{\hat{y}}^{\bar{y}} U[y - (1+r)d, y^N] dF(y) \end{aligned} \quad (\text{A.36})$$

s.t.

$$(1+r)d \int_{\hat{y}}^{\bar{y}} dF(y) + \lambda_2 \int_{\underline{y}}^{\hat{y}} (y + py^N) dF(y) = (1+\rho)d \quad (\text{A.37})$$

$$\hat{y} = \frac{(1+r)d}{\lambda_2(1 + \lambda_2)} - py^N \quad (\text{A.38})$$

Optimality condition with respect to debt:

$$U_1(c_1) = \frac{\beta(1+\rho)}{\int_{\hat{y}}^{\bar{y}} dF(y) - \frac{\lambda_2}{1+\lambda_2}(\hat{y} + \hat{p}y^N)f(\hat{y})} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) dF(y) \quad (\text{A.39})$$

Optimality condition with respect to  $\lambda_2$ :

$$-(1+2\lambda_2) \int_{\underline{y}}^{\hat{y}} U_1(c_2^D)(y + py^N) dF(y) = \frac{d[(1+r)d]}{d\lambda_2} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) dF(y) \quad (\text{A.40})$$

Differentiate (A.37) with respect to  $\lambda_2$ :

$$\frac{d[(1+r)d]}{d\lambda} \int_{\hat{y}}^{\bar{y}} dF(y) - \frac{d\hat{y}}{d\lambda_2} [(1+r)d - \lambda_2(\hat{y} + \hat{p}y^N)] f(\hat{y}) + \int_{\underline{y}}^{\hat{y}} (y + py^N) dF(y) = 0 \quad (\text{A.41})$$

Differentiate (A.38) with respect to  $\lambda_2$ :

$$\frac{d\hat{y}}{d\lambda_2} = \frac{1}{\lambda_2(1+\lambda_2)} \frac{d[(1+r)d]}{d\lambda_2} - \frac{(1+2\lambda_2)}{\lambda_2^2(1+\lambda_2)^2} (1+r)d \quad (\text{A.42})$$

Combining the two equations:

$$\begin{aligned} & \frac{d[(1+r)d]}{d\lambda} \left[ \int_{\hat{y}}^{\bar{y}} dF(y) - \frac{[(1+r)d - \lambda_2(\hat{y} + \hat{p}y^N)]}{\lambda_2(1+\lambda_2)} f(\hat{y}) \right] \\ &= -\frac{(1+2\lambda_2)}{\lambda_2^2(1+\lambda_2)^2} (1+r)d [(1+r)d - \lambda_2(\hat{y} + \hat{p}y^N)] f(\hat{y}) \\ & \quad - \int_{\underline{y}}^{\hat{y}} (y + py^N) dF(y) \\ & \frac{d[(1+r)d]}{d\lambda} \left[ \int_{\hat{y}}^{\bar{y}} dF(y) - \frac{\lambda_2}{(1+\lambda_2)} (\hat{y} + \hat{p}y^N) f(\hat{y}) \right] = -\frac{(1+2\lambda_2)}{(1+\lambda_2)^2} (1+r) df(\hat{y}) - \int_{\underline{y}}^{\hat{y}} (y + py^N) dF(y) \\ & \frac{d[(1+r)d]}{d\lambda} = -\frac{\int_{\underline{y}}^{\hat{y}} (y + py^N) dF(y) + \frac{(1+2\lambda_2)}{(1+\lambda_2)^2} (1+r) df(\hat{y})}{\int_{\hat{y}}^{\bar{y}} dF(y) - \frac{\lambda_2}{1+\lambda_2} (\hat{y} + \hat{p}y^N) f(\hat{y})} \quad (\text{A.43}) \end{aligned}$$

The optimality condition becomes:

$$\frac{d\lambda_1}{d\lambda_2} \int_{\underline{y}}^{\hat{y}} U_1(c_2^D)(y + py^N) dF(y) = \frac{\int_{\underline{y}}^{\hat{y}} (y + py^N) dF(y) + \frac{(\lambda_1 - \lambda_2)}{\lambda_2^2} (1+r) df(\hat{y}) \frac{d\lambda_1}{d\lambda_2}}{\int_{\hat{y}}^{\bar{y}} dF(y) - \frac{\lambda_1 - \lambda_2}{\lambda_1} (\hat{y} + \hat{p}y^N) f(\hat{y})} \int_{\hat{y}}^{\bar{y}} U_1(c_2^R) dF(y) \quad (\text{A.44})$$



## Chapter B: Appendix for Chapter 3

### B.1 Pecuniary externalities in an environment with i.i.d. shocks and with bonds with state contingent interest payments

When shocks are i.i.d., there is no pecuniary externality, as both the constrained social planner and private agents are able to perfectly hedge against the income shock using the state contingent asset and take non-state contingent debt up to the limit given by the borrowing constraint given the impatience factor.

For any two states,  $y$  and  $y'$ , tradable consumption is given by:

$$c_{T,t} = y + Rb_t + R_t^y \hat{b}_t - b_{t+1} - \hat{b}_{t+1} \quad (\text{B.1})$$

$$c'_{T,t} = y' + Rb_t + R_t^{y'} \hat{b}_t - b_{t+1} - \hat{b}_{t+1} \quad (\text{B.2})$$

The interest rate is given by

$$R_t^y = R + \alpha(y_t^T - \mathbb{E}_{t-1}\{y_t^T\}) \quad (\text{B.3})$$

It is possible to choose  $\hat{b}$  to make consumption constant across states

$$\hat{b}_t = -\frac{y' - y}{R^{y'} - R^y} = -1/\alpha \quad (\text{B.4})$$

In a more general setup, we can think about  $\hat{b}$  as a synthetic of a non-state contingent asset and a swap instrument where borrowers exchange their tradable income endowment

and get paid the expected value of the tradable income distribution. For a borrower who goes short on  $\hat{b}_t$  units of the synthetic instrument in period t-1, he pays  $\alpha y_t + R$  and gets  $\alpha \mathbb{E}_{t-1}\{y_t\}$  per unit in period t. Therefore, the budget constraint is the same as before:

$$c_{T,t} + p_t c_{N,t} = y_t + Rb_t - b_{t+1} - \hat{b}_{t+1} + [R + \alpha(y_t - \mathbb{E}_{t-1}y_t)]\hat{b}_t \quad (\text{B.5})$$

## B.2 Pecuniary externalities with and without access to debt with state contingent interest payments

### B.2.1 Competitive equilibrium

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{Tt}, c_{Nt})$$

s.t.

$$c_{Tt} + p_t^N c_{Nt} = y_t^T - b_{t+1} + Rb_t - \hat{b}_{t+1} + R_t^y \hat{b}_t + p_t^N y^N \quad (\text{B.6})$$

$$b_{t+1} + \hat{b}_{t+1} \geq -\kappa [y_t^T + p_t^N y^N] \quad \text{with multiplier } \xi_t \quad (\text{B.7})$$

Euler equation for regular debt ( $b_{t+1}$ )

$$U_1(c_t) = \beta R \mathbb{E}_t \{U_1(c_{t+1})\} + \xi_t \quad (\text{B.8})$$

Euler equation for debt with state contingent interest payments ( $\hat{b}_{t+1}$ )

$$\begin{aligned} U_1(c_t) &= \beta \mathbb{E}_t \{R_{t+1}^y U_1(c_{t+1})\} + \xi_t \\ &= \beta R \mathbb{E}_t \{U_1(c_{t+1})\} + \beta \text{cov}(R_{t+1}^y, U_1(c_{t+1})) + \xi_t \end{aligned} \quad (\text{B.9})$$

The interest payment for  $\hat{b}$  is given by:

$$R_t^y = R + \alpha(y_t - \mathbb{E}_{t-1}y_t) \quad (\text{B.10})$$

Plugging it into the previous equation, we get:

$$U_1(c_t) = \beta R \mathbb{E}_t \{U_1(c_{t+1})\} + \alpha \beta \text{cov}(y_{t+1}, U_1(c_{t+1})) + \xi_t \quad (\text{B.11})$$

Lower precautionary savings with access to  $\hat{b}$  are shown by the fact that:

$$\text{cov}(y_{t+1}, U_1(c_{t+1})) < 0 \quad (\text{B.12})$$

Procyclical interest payments makes it less likely to hit the collateral constraint, so agents need to accumulate less precautionary savings to insure against this risk. Higher individual debt increases the probability of being financially constrained next period if borrowers get a low income shock, but due to lower interest payments in the procyclical case, the tightening of the constraint is less severe and it allows for more debt rollover compared to the case with regular bonds only.

## B.2.2 Social Planner

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{Tt}, c_{Nt})$$

s.t.

$$c_{Tt} = y_t^T - b_{t+1} + Rb_t - \hat{b}_{t+1} + R_t^y \hat{b}_t \quad (\text{B.13})$$

$$c_{Nt} = y^N \quad (\text{B.14})$$

$$b_{t+1} + \hat{b}_{t+1} \geq -\kappa \left[ y_t^T + \frac{U_2(c_t)}{U_2(c_t)} y^N \right] \quad \text{with multiplier } \xi_t \quad (\text{B.15})$$

Euler equation for regular debt ( $b_{t+1}$ )

$$U_1(c_t) = \beta R \mathbb{E}_t \{U_1(c_{t+1})\} + \xi_t \left[ 1 - \kappa \frac{\partial p_t^N}{\partial c_{Tt}} y^N \right] + \beta R \mathbb{E}_t \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} \quad (\text{B.16})$$

Euler equation for debt with state contingent interest payments ( $\hat{b}_{t+1}$ )

$$\begin{aligned}
U_1(c_t) &= \beta \mathbb{E}_t \{ R_{t+1}^y U_1(c_{t+1}) \} + \xi_t \left[ 1 - \kappa \frac{\partial p_t^N}{\partial c_{Tt}} y^N \right] + \beta \mathbb{E}_t \left\{ R_{t+1}^y \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} \\
&= \beta R \mathbb{E}_t \{ U_1(c_{t+1}) \} + \beta \text{cov} (R_{t+1}^y, U_1(c_{t+1})) + \xi_t \left[ 1 - \kappa \frac{\partial p_t^N}{\partial c_{Tt}} y^N \right] \\
&\quad + \beta R \mathbb{E}_t \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} + \beta \text{cov} \left( R_{t+1}^y, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right) \tag{B.17}
\end{aligned}$$

Using (B.10), we can simplify this to:

$$\begin{aligned}
U_1(c_t) &= \beta R \mathbb{E}_t \{ U_1(c_{t+1}) \} + \xi_t \left[ 1 - \kappa \frac{\partial p_t^N}{\partial c_{Tt}} y^N \right] + \beta R \mathbb{E}_t \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} \\
&\quad + \beta \alpha \text{cov} (y_{t+1}, U_1(c_{t+1})) + \beta \alpha \text{cov} \left( y_{t+1}, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right) \tag{B.18}
\end{aligned}$$

By comparing equations (B.16) and (B.18), we can show that access to  $\hat{b}$  lowers precautionary savings because:

$$\text{cov} (y_{t+1}, U_1(c_{t+1})) + \text{cov} \left( y_{t+1}, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right) < 0 \tag{B.19}$$

Procyclical interest payments leads to higher interest payments in good times with higher consumption (and lower marginal utility of consumption). The second term is also negative because it is more likely for agents to be financially constrained in bad times, as they need to borrow more to smooth consumption, hence the shadow price of the collateral constraint is higher in periods of low income.

Comparing equations (B.8) and (B.16), we observe that when agents only have access to non-state contingent bonds, the pecuniary externality is given by the term:

$$\beta R \mathbb{E}_t \left\{ \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} > 0$$

Similar to the results in Bianchi (2011) and Korinek (2010), in an environment with a single non-state contingent bond, private agents who ignore the effect of individual

borrowing on the valuation of collateral of other agents borrow more than socially optimal and therefore face financial crises more frequently.

$$\begin{aligned}
 U_1(c_t^{CE}) &< U_1(c_t^{SP}) \\
 c_t^{CE} > c_t^{SP} &\rightarrow -b_{t+1}^{CE} > -b_{t+1}^{SP}
 \end{aligned} \tag{B.20}$$

When we add access to debt with state contingent interest payments, the pecuniary externality term becomes

$$\beta \mathbb{E}_t \left\{ R_{t+1}^y \xi_{t+1} \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right\} > 0 \tag{B.21}$$

Similarly, private borrowers ignore the effect of their individual borrowing on the valuation of collateral of other agents. Higher individual debt leads to a higher probability of being financially constrained next period, which results in inability to rollover debt and leads to capital outflows and real depreciation. This in turn lowers the value of collateral for other agents in the economy and hence leads to larger capital outflows.

However, by comparing the two pecuniary externality terms, we can see that the feedback effect through the valuation of collateral is milder when there is access to  $\hat{b}$ . Higher individual debt leads to a higher probability of being financially constrained next period if borrowers get a low income shock. However, it is less likely to be financially constrained if the interest payment is procyclical, as a low income shock is partially offset by the lower debt service, so that there is more space for debt rollover. This in turn makes the capital outflows and the real depreciation milder, which is given by the term:

$$cov \left( y_{t+1}, \xi_{t+1} \kappa \frac{\partial p_{t+1}^N}{\partial c_{Tt+1}} y^N \right) < 0 \tag{B.22}$$

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