This dissertation studies the optimal regulatory response to financial crises. The first two chapters focus on prevention of financial crises, and the third chapter focuses on resolution of financial crises.

Chapter 1 develops a quantitative theory of overborrowing based on a systemic risk externality in an emerging market economy. In the model, debt denominated in foreign currency and balance sheet constraints cause depreciations of the real exchange rate to be contractionary. The externality arises because when private agents take debt in good times, they do not internalize that during bad times, the reduction in demand for consumption causes a higher depreciation of the real exchange rate and a further tightening of balance sheet constraints across the economy. The quantitative analysis suggests that there is an important role for policies that “throw sand in the wheels of international finance.”

Chapter 2 analyzes an externality that arises because of a feedback loop between asset prices and collateral constraints in a dynamic stochastic general equilib-
rium model calibrated to US data. In the model, a collateral constraint limits private agents not to borrow more than a fraction of the market value of their collateral assets, which take the form of an asset in fixed aggregate supply (e.g. land). When the collateral constraint binds, fire-sales of assets cause a Fisherian debt-deflation spiral that causes asset prices to decline and the economy’s borrowing ability to shrink in an endogenous feedback loop. The externality produces deeper recessions and a larger collapse in asset prices compared to the constrained efficient allocations.

Chapter 3 studies the macroeconomic and welfare effects of government intervention in credit markets during financial crises. A DSGE model to assess the interaction between ex-post interventions in credit markets and the build-up of risk ex ante is developed. During a systemic crisis, the central bank finds it beneficial to bail out the financial sector to relax balance sheet constraints across the economy. Ex ante, this leads to an increase in risk-taking, making the economy more vulnerable to a financial crisis. We ask whether the central bank should commit to avoiding a bailout of the financial sector during a systemic crisis. We find that bailouts can improve welfare by providing insurance against systemic financial crises.
Essays on Financial Crises and Financial Regulation

by

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Dedication

To Anita and Francesca.
This dissertation could not have been possible without the advice and encouragement of my advisors at the University of Maryland.

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1.1 Introduction

In the wake of the 2008 international financial crisis, there have been intense debates about reform of the international financial system that emphasize the need to address the problem of “overborrowing.” The argument typically relies on the observation that periods of sustained increases in borrowing are often followed by a devastating disruption in financial markets. This raises the question of why the private sector becomes exposed to the dire consequences of financial crises and what the appropriate policy response should be to reduce the vulnerability to these episodes. Without a thorough understanding of the underlying inefficiencies that arise in the financial sector, it seems difficult to evaluate the merit of proposals that aim to reform the current international financial architecture.

This paper presents a formal welfare-based analysis of how optimal borrowing decisions at the individual level can lead to overborrowing at the social level in a dynamic stochastic general equilibrium (DSGE) model, where financial constraints give rise to amplification effects. As in the theoretical literature (e.g. Lorenzoni, 2008), we analyze constrained efficiency by considering a social planner that faces the same financial constraints as the private economy, but internalizes the price effects of its borrowing decisions. Unlike the existing literature, we conduct a quantitative
analysis to evaluate the macroeconomic and welfare effects of overborrowing. We study how overborrowing affects the incidence and severity of financial crises, the magnitude of welfare losses, and the features of policy measures that aim to correct the externality. In a nutshell, we investigate whether overborrowing is in fact a macroeconomic problem and what should be the optimal policy response.

Our model’s key feature is an occasionally binding credit constraint that limits borrowing, denominated in the international unit of account (i.e., tradable goods), to the value of collateral in the form of output from the tradable and nontradable sector, as in Mendoza (2002). Because debt is partially leveraged in income generated in the nontradable sector, changes in the relative price of nontradable goods can induce sharp and sudden adjustments in access to foreign financing. Due to incomplete markets, agents can only imperfectly insure against adverse shocks. As a result, when agents have accumulated a large amount of debt and a typical adverse shock hits, the economy suffers the typical dislocation associated with an emerging market crisis. Demand for consumption goods falls, putting downward pressure on the price of nontradables, which drags down the real exchange rate. This leads to a further tightening of the credit constraint, setting in motion Fisher’s debt deflation channel by which declines in consumption, the real exchange rate, and access to foreign financing mutually reinforce one another, as in Mendoza’s work.

In the model, private agents form rational expectations about the evolution of macroeconomic variables—in particular the real exchange rate—and correctly perceive the risks and benefits of their borrowing decisions. Nevertheless, they fail to internalize the general equilibrium effects of their borrowing decisions on prices.
This is a pecuniary externality that would not impede market efficiency in the absence of the credit constraint linked to market prices. However, by reducing the amount of borrowing ex-ante, a social planner mitigates the decrease in demand for consumption during crises. This mitigates the real exchange rate depreciation and prevents a further tightening of financial constraints, making everyone better off.

Our quantitative analysis shows that the macroeconomic effects of the systemic credit externality are significant. The externality increases the long-run probability of a financial crisis from 0.4 percent to 5.5 percent and has important effects on the severity of these episodes. In the decentralized equilibrium, consumption drops 17 percent, capital inflows fall 8 percent, and the real exchange rate drops by 19 percent in a typical crisis. In the constrained-efficient allocations, by contrast, consumption drops 10 percent, capital inflows barely fall, and the real exchange rate drops by 1 percent. Moreover, the externality allows the model to account for two salient features of the data: procyclicality of capital inflows and the high variability of consumption.

We study a variety of policy measures that can restore constrained efficiency, all of which involve restricting the amount of credit in the economy: taxes on debt, tightening of margins, and capital and liquidity requirements. These measures are imposed before a crisis hits so that private agents internalize the external costs of borrowing and the economy becomes less vulnerable to future adverse shocks. In the calibrated version of our model, the increase in the effective cost of borrowing necessary to implement the constrained-efficient allocations is about 5 percent on average, increasing with the level of debt and with the probability of a future finan-
cial crisis. We also study simple forms of interventions and ascertain that a fixed
tax on debt can also achieve sizable welfare gains.

Our paper is related to the large literature on the macroeconomic role of financial frictions. Following the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), various studies have presented dynamic models where financial frictions can amplify macroeconomic shocks compared to a first-best benchmark where these frictions are absent.\(^1\) Our contribution to this literature is twofold. First, we study the volatility and the level of amplification of the competitive equilibrium relative to a second-best benchmark where these frictions are also present. Second, we investigate several policy measures that can significantly reduce the level of financial instability and improve welfare by making agents internalize an externality due to financial accelerator effects.

Our paper is related to the theoretical literature that investigates the role of pecuniary externalities in generating excessive financial fragility, and we borrow extensively from their insights (see for example Auernheimer and Garcia-Saltos (2000), Caballero and Krishnamurthy (2001,2003), Lorenzoni (2008), Farhi, Golosov, and Tsyvinski (2009), and Korinek (2009ab)).\(^2\) In all of these studies, however, the analysis is qualitative in nature. Our contribution to this literature is to provide a quantitative assessment of the macroeconomic, policy and welfare implications of overborrowing. This is an important first step in the evaluation of the potential benefits from reg-


\(^2\)The inefficiency result of these studies is related to the idea that economies with endogenous borrowing constraint and multiple goods are constrained inefficient (Kehoe and Levine, 1993) and to the generic inefficiency result in economies with incomplete markets (Geneakoplos and Polemarchakis, 1986; Stiglitz, 1982).
ulatory measures to correct these externalities and in the study of their practical implementation.

There is a growing macroeconomic literature that studies optimal policy in a financial crisis.\(^3\) This literature typically takes as given that the economy is in a high leverage situation and analyzes the role of policies that can moderate the impact of a large adverse shock. While this literature provides important insights on how to respond to crises once they erupt, it does not study how the economy experiences the surge in debt that leads to the crisis in the first place. This paper complements this literature by studying how an economy can become vulnerable to a financial crisis due to excessive borrowing during normal times. We model crises as infrequent episodes nested within regular business cycles and analyze the role of policies in reducing an economy’s vulnerability to financial crises, therefore placing macroprudential policy at the center of the stage. We acknowledge, however, that because our analysis requires global non-linear solution methods, we abstract from important real-world features present in larger scale DSGE models.

A related paper that allows for policy intervention during normal times and crisis times is Benigno et al. (2009). They consider the role of a subsidy on nontradable goods, which the Ramsey planner uses ex-post to mitigate the real exchange depreciation during crises, but not ex-ante since it is not effective to make agents internalize the full social costs of borrowing. We focus instead on a constrained planner who directly makes borrowing decisions and show that the decentralization

\(^3\)Notable contributions include Christiano, Gust, and Roldos (2004), Kiyotaki and Moore (2008), Gertler and Karadi (2009), and Gertler and Kiyotaki (2010).
requires ex-ante intervention to prevent excessive risk exposure.

Finally, there are a number of other theories of overborrowing that have been investigated. One theory is moral hazard: banks may lend excessively to take advantage of some form of government bailout.\footnote{See e.g. McKinnon and Pill (1996), Corsetti, Pesenti, and Roubini (1999), Schneider and Tornell (2004), and Farhi and Tirole (2010).} Uribe (2006) has also studied whether an economy with an aggregate debt limit tends to overborrow relative to an economy with debt limits imposed at the level of each individual agent, and found that borrowing decisions coincide. Our focus is on the comparison between competitive equilibrium and constrained-efficient equilibrium when financial constraints that are linked to market prices generate amplification effects.

1.2 Analytical Framework

Consider a representative-agent DSGE model of a small open economy (SOE) with a tradable goods sector and a nontradable goods sector. Tradable goods can be used for consumption, external borrowing and lending transactions; nontradable goods have to be consumed in the domestic economy. The economy is populated by a continuum of identical, infinitely-lived households of measure unity with preferences given by:

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \]  

(1.1)

In this expression, \( E_t(\cdot) \) is the time \( t \) expectation operator, and \( \beta \) is the discount factor. The period utility function \( u(\cdot) \) has the constant-relative-risk-aversion (CRRA) form. The consumption basket \( c_t \) is an Armington-type CES aggregator with elas-
ticity of substitution $1/(\eta + 1)$ between tradable $c^T$ and nontradable goods $c^N$ given by:

$$c_t = \left[ \omega (c^T_t)^{-\eta} + (1 - \omega) (c^N_t)^{-\eta} \right]^{-\frac{1}{\eta}}, \quad \eta > -1, \omega \in (0, 1)$$

In each period $t$, households receive an endowment of tradable goods $y^T_t$ and an endowment of nontradable goods $y^N_t$. We assume that the vector of endowments given by $y \equiv (y^T, y^N) \in Y \subseteq R^2_{++}$ follows a first-order Markov process. These endowment shocks are the only source of uncertainty in the model.

The menu of foreign assets available is restricted to a one period, non-state-contingent bond denominated in units of tradables that pays a fixed interest rate $r$, determined exogenously in the world market.\(^5\) Normalizing the price of tradables to 1 and denoting the price of nontradable goods by $p^N$ the budget constraint is:

$$b_{t+1} + c^T_t + p^N_t c^N_t = b_t (1 + r) + y^T_t + p^N_t y^N_t$$

where $b_{t+1}$ denotes bond holdings that households choose at the beginning of time $t$. We maintain the convention that positive values of $b$ denote assets. As there is only one asset, gross and net bond holdings (NFA) coincide.

We assume that creditors restrict loans so that the amount of debt does not exceed a fraction $\kappa^T$ of tradable income and a fraction $\kappa^N$ of nontradable income.

\(^{5}\)To have a well-defined stochastic steady state, we assume that the discount factor and the world interest rate are such that $\beta(1 + r) < 1$. If $\beta(1 + r) \geq 1$, assets will diverge to infinity in equilibrium by the supermartingale convergence theorem (see Chamberlain and Wilson (2000), 2000). See Schmitt-Grohe and Uribe (2003) for other methods to induce stationarity.
Specifically, the credit constraint is given by:

\[ b_{t+1} \geq - (\kappa^N p_t^N y_t^N + \kappa^T y_t^T) \]  

(1.3)

This credit constraint can be seen as arising from informational and institutional frictions affecting credit relationships (such as monitoring costs, limited enforcement, asymmetric information, and imperfections in the judicial system), but we do not model these frictions explicitly. Our focus is on how financial policies can be welfare improving, taking as given the frictions that lead to these debt contracts, i.e., we will assume that the social planner is a constrained social planner that is also subject to this credit constraint.

Discussion of market incompleteness.— A few comments are in order about the two deviations from complete markets that we introduce here. First, we have assumed that assets are restricted to a one-period non-state-contingent bond denominated in tradable goods. While agents typically have a richer set of assets available, this assumption is made for numerical tractability and is meant to capture the observation that debt in emerging markets is generally short term and denominated in foreign currency. In turn, these features of debt contracts are generally seen as an important source of vulnerability in emerging markets (see e.g. Calvo, Izquierdo, and Loo-Kung, 2006).

The second form of market incompleteness is given by the credit constraint. In the absence of a credit constraint, households will increase borrowing in bad times to smooth consumption. This will imply a counterfactual reaction of the current
account, which is well-known to rise during recessions in emerging markets. The credit constraint we have specified has two main features. One crucial feature is that nontradable goods are part of the collateral. At the empirical level, this is consistent with evidence that credit booms in the nontradable sector are fueled by external credit (see e.g. Tornell and Westermann, 2005). At the theoretical level, this could result because foreign borrowers can seize nontradable goods from a defaulting borrower, sell them in the domestic market, and repatriate the funds abroad. A positive gap between $\kappa^T$ and $\kappa^N$ would reflect an environment where creditors have a higher preference for tradable income as collateral. A case where $\kappa^T = \kappa^N$ would reflect an environment where creditors request and aim to verify information on total income of individual borrowers, i.e., they do not document the sectoral sources of their income.

The second feature is that the collateral is given by current income. At the empirical level, this assumption is supported by evidence that current income is a major determinant of credit market access (see e.g., Jappelli, 1990). At the theoretical level, borrowing limits could depend on current income if households can engage in fraud in the period they contract debt obligations and prevent creditors from seizing any future income. If creditors detect the fraud, however, they could seize household’s current income (see Korinek, 2009a).

An additional argument that our formulation of the credit constraint is suitable for a quantitative assessment of the externality is that our model can account

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6For more evidence on credit constraints on households, see Jappelli and Pagano (1989), Zeldes (1989).
reasonably well for the main macro features of emerging market crises, as shown by Mendoza (2002).

1.3 Equilibrium

1.3.1 Optimality Conditions

The household’s problem is to choose stochastic processes \( \{c_t^T, c_t^N, b_{t+1}\}_{t \geq 0} \) to maximize the expected present discounted value of utility (2.1) subject to (1.2) and (1.3), taking \( b_0 \) and \( \{p_t^N\}_{t \geq 0} \) as given. The household’s first-order conditions require:

\[
\lambda_t = u_T(t) \quad (1.4)
\]

\[
p_t^N = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{c_t^T}{c_t^N} \right)^{\eta+1} \quad (1.5)
\]

\[
\lambda_t = \beta (1 + r) \mathbb{E}_t \lambda_{t+1} + \mu_t \quad (1.6)
\]

\[
b_{t+1} + (\kappa^N p_t^N y_t^N + \kappa^T y_t^T) \geq 0, \quad \text{with equality if } \mu_t > 0 \quad (1.7)
\]

where \( \lambda \) is the non-negative multiplier associated with the budget constraint and \( \mu \) is the non-negative multiplier associated with the credit constraint. The optimality condition (1.4) equates the marginal utility of tradable consumption to the shadow value of current wealth. Condition (1.5) equates the marginal rate of substitution of the two goods, tradables and nontradables, to their relative price. Equation (1.6) is the Euler equation for bonds. When the credit constraint is binding, there is a wedge between the current shadow value of wealth and the expected value of reallocating
wealth to the next period, given by the shadow price of relaxing the credit constraint \( \mu_t \). Equation (1.7) is the complementary slackness condition.

Since households are identical, market clearing conditions are given by:

\[ c_t^N = y_t^N \quad (1.8) \]

\[ c_t^T = y_t^T + b_t(1 + r) - b_{t+1} \quad (1.9) \]

Notice that equation (1.5) implies that a reduction in \( c_t^T \) generates in equilibrium a reduction in \( p_t^N \), which by equation (1.3) reduces the collateral value. Besides amplification, the credit constraint produces asymmetric responses in the economy: a binding credit constraint amplifies the consumption drop in response to a negative income shock, but no amplification effects occur when the credit constraint is slack. Because of consumption-smoothing effects, the demand for borrowing generally decreases with current income, and when current income is sufficiently low, the credit constraint becomes binding.

### 1.3.2 Equilibrium Definition

We consider the optimization problem of a representative household in recursive form, which includes, as a crucial state variable, the aggregate bond holdings of the economy. Households need to forecast future aggregate bond holdings that are beyond their control to form expectations of the price of nontradables. We denote by \( \Gamma(\cdot) \) the forecast of aggregate bond holdings for every cur-
rent aggregate state \((B, y)\), i.e., \(B' = \Gamma(B, y)\). Combining equilibrium conditions (1.5), (1.8), and (1.9), the forecast price function for nontradables can be expressed as 
\[
p^N(B, y) = \frac{(1 - \omega)}{(\omega)} \left( \left( y^T + B(1 + r) - \Gamma(B, y) \right) / y^N \right)^{\eta + 1}.
\]
The other relevant state variables for the individual household are its bond holdings and the vector of endowment shocks. The problem of a representative household can then be written as:

\[
V(b, B, y) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}_{y'|y} V(b', B', y')
\]

subject to

\[
\begin{align*}
\dot{b} + p^N(B, y)c^N + c^T &= y^T + b(1 + r) + p^N(B, y)y^N \\
\dot{b} &\geq - (\kappa^N p^N(B, y)y^N + \kappa^Ty^T) \\
B' &= \Gamma(B, y)
\end{align*}
\]

where we have followed the convention of denoting current variables without subscript and denoting next period variables with the prime superscript. The solution to the household problem yields decision rules for individual bond holdings \(\hat{b}(b, B, y)\), tradable consumption \(\hat{c}^T(b, B, y)\) and nontradable consumption \(\hat{c}^N(b, B, y)\). The household optimization problem induces a mapping from the perceived law of motion for aggregate bond holdings to an actual law of motion, given by the representative agent’s choice \(\hat{b}(B, B, y)\). In a rational expectations equilibrium, as defined below, these two laws of motion must coincide.

**Definition 1 (Decentralized Recursive Competitive Equilibrium)**
A decentralized recursive competitive equilibrium for our SOE is defined by a pricing function $p^N(B, y)$, a perceived law of motion $\Gamma(B, y)$ and decision rules $\{\hat{b}(b, B, y), \hat{c}^T(b, B, y), \hat{c}^N(b, B, y)\}$ with associated value function $V(b, B, y)$ such that the following conditions hold:

1. **Household optimization:** \(\{\hat{b}(b, B, y), \hat{c}^N(b, B, y), \hat{c}^N(b, B, y), V(b, B, y)\}\) solve the recursive optimization problem of the household for given $p^N(B, y)$ and $\Gamma(B, y)$.

2. **Rational expectation condition:** the perceived law of motion is consistent with the actual law of motion: $\Gamma(B, y) = \hat{b}(B, B, y)$.

3. **Markets clear:** $y^N = \hat{c}^N(B, B, y)$ and $\Gamma(B, y) + \hat{c}^T(B, B, y) = y^T + B(1 + r)$.

1.4 Efficiency

1.4.1 Social Planner’s Problem

We previously described the equilibrium achieved when agents take aggregate variables as given, particularly the price of nontradables. Consider now a benevolent social planner with restricted planning abilities. We assume that the social planner can directly choose the level of debt subject to the credit constraint, but allows goods markets to clear competitively. That is, the planner (a) performs credit operations and rebates back to households all the proceeds in a lump sum fashion, and (b) lets households choose their allocation of consumption between tradable goods and nontradable goods in a competitive way.

As opposed to the representative agent, a social planner internalizes the effects
of borrowing decisions on the price of nontradables. Critically, the social planner realizes that a lower debt level mitigates the reduction in the price of nontradables and prevents a larger drop in borrowing ability when the credit constraint binds. As a result, we will show that the decentralized equilibrium allocation is not a constrained Pareto optimum, as defined below.

**Definition 2 (Constrained Efficiency)**

Let \( \{c^T_t, c^N_t, b_{t+1}\}_{t \geq 0} \) be the allocations of the competitive equilibrium yielding utility \( \hat{V} \). The competitive equilibrium is constrained efficient if a social planner that chooses directly \( \{b_{t+1}\}_{t \geq 0} \) subject to the credit constraint, but lets the goods markets clear competitively, cannot improve the welfare of households above \( \hat{V} \).

The social planner’s optimization problem consists of maximizing (2.1) subject to (1.3), (1.5), (1.8), and (1.9). Substituting for the equilibrium price in (1.3), we can express the social planner’s optimization problem in recursive form as:

\[
V(b, y) = \max_{b', c^T} \left[ u(c(c^T, y^N)) + \beta \mathbb{E}_{y'|y} V(b', y') \right] \\
\text{subject to} \\
b' + c^T = y^T + b(1 + r) \\
b' \geq - \left( \kappa N \frac{1 - \omega}{\omega} \left( \frac{c^T}{y^N} \right)^{\eta+1} y^N + \kappa^T y^T \right)
\]

Using sequential notation and the superscript “sp” to distinguish the Lagrange multipliers of the social planner’s problem from the decentralized equilibrium, the first-
order conditions for the social planner require:

\[ \lambda_{sp}^t = u_T(t) + \mu_{sp}^t \Psi_t \]  
(1.12)

\[ \lambda_{sp}^t = \beta(1 + r)E_t\lambda_{sp}^{t+1} + \mu_{sp}^t \]  
(1.13)

\[ b_{t+1} + \left( \kappa N \frac{1 - \omega}{\omega} \left( \frac{c_t^T}{y_t^N} \right) ^{\eta + 1} y_t^N + \kappa^T y_t^T \right) \geq 0, \text{ with equality if } \mu_{sp}^t > 0 \]  
(1.14)

where \( \Psi_t \equiv \kappa N (p_t^N c_t^N)/(c_t^T) (1 + \eta) > 0 \) indicates how much the collateral value changes at equilibrium when there is a change in tradable consumption. Notice that this term is directly proportional to the fraction of nontradable output that agents can pledge as collateral, the relative size of the nontradable sector and the inverse of the elasticity of substitution between tradables and nontradables. We will return to this expression in the sensitivity analysis.

The key difference between the optimization problem of the social planner relative to households follows from examining (1.12) compared with the corresponding equation for the decentralized equilibrium (1.4). The social planner’s marginal benefits from tradable consumption include the direct increase in utility \( u_T(t) \) and also the indirect increase in utility \( \mu_{sp}^t \Psi_t \). This indirect benefit, not considered by private agents, represents how an increase in tradable consumption increases the price of nontradables and relaxes the credit constraint of all agents by \( \Psi_t \), which has a shadow value of \( \mu_{sp}^t \). Thus, (1.4) and (1.12) yield the key result that, for given initial states and allocations at which the credit constraint binds, private agents value
wealth less than the social planner, which we highlight in the following remark.

Remark 1 When the credit constraint binds, private agents undervalue wealth.

To see more clearly why this different ex-post valuation generates overborrowing ex-ante, suppose that at time $t$ the constraint is not currently binding. Using (1.4) and (1.6), the Euler equation for consumption in the decentralized equilibrium becomes:

$$ u_T(t) = \beta (1 + r) \mathbb{E}_t u_T(t + 1) $$

(1.15)

Using (1.12) and (1.13), the Euler equation for consumption for the social planner becomes:

$$ u_T(t) = \beta (1 + r) \mathbb{E}_t \left[ u_T(t + 1) + \mu_{sp}^{op} \Psi_{t+1} \right] $$

(1.16)

Consider now a reallocation of wealth by the social planner starting from the privately optimal allocations in the decentralized equilibrium. In particular, consider the welfare effects of a reduction of one unit of borrowing. Because decentralized agents are at the optimum, (1.15) shows that the first-order private welfare benefits $\beta (1 + r) E_t u_T(t + 1)$ are equal to the first-order private welfare costs $u_T(t)$. Using (1.16), the social planner has a marginal cost of reducing borrowing equal to the private marginal cost, but faces higher marginal benefits: a one unit decrease in borrowing relaxes next-period ability to borrow by $(1 + r) \Psi_{t+1}$, which has a marginal utility benefit of $\mu_{sp}^{op}$. The uninternalized external benefits from savings, or equivalently the uninternalized external marginal cost of borrowing, is then given by the discounted expected marginal utility cost of the resulting tightening of the credit
constraint $\beta(1 + r)\mathbb{E}_t\mu_{t+1}^s \Psi_{t+1}$. Notice that if the credit constraint does not bind for any pair $(b, y)$ in the two equilibria, the conditions characterizing both environments are identical and therefore the allocations coincide.

**Proposition 1** *(Constrained Inefficiency)* The decentralized equilibrium is not, in general, constrained efficient.

*Proof:* See Appendix A

1.4.2 Decentralization

We study the use of various financial policies in the implementation of the constrained-efficient allocations. We start by showing how a tax on debt can restore constrained efficiency and then show the equivalence between the tax on debt and more standard forms of intervention in the financial sector (e.g., capital requirements).

Letting $\tau_t$ be the tax charged on debt issued at time $t$, the Euler equation for bonds in the regulated decentralized equilibrium (1.6) becomes:

$$u_T(t) = \beta(1 + r)(1 + \tau_t)\mathbb{E}_t u_T(t + 1) + \mu_t$$  \hspace{1cm} (1.17)

**Proposition 2** *(Optimal tax on debt)* The constrained-efficient allocations can be implemented with an appropriate state contingent tax on debt, with tax revenue rebated as a lump sum transfer.

*Proof:* See Appendix A
When the credit constraint is not binding in the constrained-efficient allocations, the tax must be set to \( \tau_t^* = \left( \sum_{t=1}^{T} \mu_{t+1}^{sp} \Psi_{t+1} \right) / \left( \sum_{t=1}^{T} u_T(t+1) \right) \) (variables are evaluated at the constrained-efficient allocations). This expression represents the uninternalized marginal cost of borrowing analyzed above, normalized by the expected marginal utility. As we will see in the quantitative analysis, this tax increases with the current level of debt, since a higher current level of debt implies a higher choice of debt, which increases the probability and the marginal utility cost of a binding constraint next period. Notice also that if the credit constraint has a zero probability of binding in the next period, the tax is set to zero.

When the credit constraint is binding, the tax does not generally influence the level of borrowing since the choice of debt is given by the credit constraint (1.3) and not by the Euler equation (1.17). Setting the tax to \( \tau_t^* = \left( \sum_{t=1}^{T} \mu_{t+1}^{sp} \Psi_{t+1} \right) / \left( \sum_{t=1}^{T} u_T(t+1) \right) - \left( \mu_t^{sp} \Psi_t \right) / \left( \beta(1+r) \sum_{t=1}^{T} u_T(t+1) \right) \) achieves constrained efficiency and equalizes the private and social shadow values from relaxing the constraint. Notice that an extra term arises because the social planner internalizes that relaxing the credit constraint today would have positive effects on the current price of nontradables. This term is negative so that the tax causes the private shadow value of relaxing the constraint to rise to the social value. As we will show in the quantitative analysis, when the planner is borrowing up to the limit, the level of borrowing desired by private agents is also the maximum available. As a result, we find that setting \( \tau_t^* = 0 \) when the constraint binds also implements the constrained-efficient allocations, and since this results in a simpler policy we set this tax to zero when we turn to describe its
quantitative features.\textsuperscript{7}

In practice, much of prudential financial regulation is implemented through banks. To take this into consideration, we develop in Appendix B a simple model of financial intermediaries, and show that our benchmark economy, in which the planner sets a tax on debt on borrowers, is equivalent to a economy where the planner sets capital requirements or reserve requirements on financial institutions. Throughout the paper, we will refer to the implied tax on debt as the increase in the cost of debt induced by the use of any of these equivalent policy measures.

Alternatively, the planner could implement the constrained-efficient allocations using margin requirements by choosing an adjustment $\theta_t \geq 0$ such that the credit constraint becomes $b_{t+1} \geq - (1 - \theta_t) \left( \kappa N p_t N y_t N + \kappa T y_t T \right)$. If the socially optimal amount of borrowing is $b^p_s$, by setting $\theta^*_t = 1 - (b^p_s) / \left( \kappa N p_t N y_t N + \kappa T y_t T \right)$ the social planner can restrict the quantity of borrowing and restore constrained efficiency.\textsuperscript{8}

\textbf{Remark 2 (Decentralization)} The constrained-efficient allocations can be implemented with appropriate capital requirements, reserve requirements, or margin requirements.

\textsuperscript{7}This result held for the wide range of parameters explored in our quantitative analysis.

\textsuperscript{8}For this policy to restore constrained efficiency, it must be the case that $\hat{b}^{de}(B, B, y) \leq \hat{b}^p(B, y) \forall (B, y)$, which we will show is the case in the numerical analysis. We assume for simplicity that $\theta^*$ is such that the credit constraint always holds with equality in the regulated economy. In the regulated economy where the adjustment of margins is given by $\theta^*$, the constraint only binds in the constrained region and in the tax-region; hence setting $\theta_t = 0$ in the no-tax region would deliver the same allocations.
1.5 Quantitative Analysis

In this section, we describe the calibration of the model and evaluate the quantitative implications of the externality. We solve the competitive equilibrium and the social planner’s problems numerically using global non-linear methods (described in detail in the online Appendix).

1.5.1 Calibration

A period in the model represents a year. The baseline calibration uses data from Argentina, an example of an emerging market with a business cycle that has been studied extensively. The risk aversion is set at $\sigma = 2$, a standard value. The interest rate is set at $r = 4$ percent, which is a standard value for the world risk-free interest rate in the DSGE-SOE literature.

We model endowment shocks as a first-order bivariate autoregressive process:

$$\log y_t = \rho \log y_{t-1} + \epsilon_t \quad \text{where} \quad y = [y^T \ y^N]' \quad \rho \text{ is a } 2\times2 \text{ matrix of autocorrelation coefficients, and } \epsilon_t = [\epsilon_t^T \ \epsilon_t^N]' \text{ follows a bivariate normal distribution with zero mean and contemporaneous variance-covariance matrix } V. \text{ This process is estimated with the HP-filtered cyclical components of tradables and nontradables GDP from the World Development Indicators (WDI) for the 1965-2007 period, the longest time series available from official sources. Following the standard methodology, we classify manufacturing and primary products as tradables and classify the rest of the components of GDP as nontradables. The estimates of } \rho \text{ and } V \text{ are:}$$
\[ \rho = \begin{bmatrix} 0.901 & 0.495 \\ -0.453 & 0.225 \end{bmatrix}, \quad V = \begin{bmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{bmatrix} \]

The standard deviations of tradable and nontradable output in the data are \( \sigma_{yT} = 0.058 \) and \( \sigma_{yN} = 0.057 \), the first-order autocorrelations are \( \rho_{yT} = 0.53 \) and \( \rho_{yN} = 0.61 \), and the correlation between the two is \( \rho_{yT,yN} = 0.81 \). Thus cyclical fluctuations in the two sectors have similar volatility and persistence, and are positively correlated with each other. We discretize the vector of shocks into a first-order Markov process, with four grid points for each shock, using the quadrature-based procedure of Tauchen and Hussey (1991). The mean of the endowments are set to one without loss of generality.

<table>
<thead>
<tr>
<th>Table 1.1: Calibration</th>
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<tbody>
<tr>
<td><strong>Value</strong></td>
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<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>Interest rate</td>
</tr>
<tr>
<td>Risk aversion</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
</tr>
<tr>
<td>Stochastic structure</td>
</tr>
<tr>
<td>Relative credit coefficients</td>
</tr>
<tr>
<td>Weight on tradables in CES</td>
</tr>
<tr>
<td>Discount factor</td>
</tr>
<tr>
<td>Credit coefficient</td>
</tr>
</tbody>
</table>

The intratemporal elasticity of substitution \( 1/(\eta + 1) \) is a crucial parameter because it affects the magnitudes of the price adjustment. For a given reduction in tradable consumption, a higher elasticity implies a smaller change in the price of nontradables, and therefore we should expect weaker effects from the externality. The range of estimates for the elasticity of substitution is between 0.40 and 0.83.\(^9\)

As a conservative benchmark, we set \( \eta \) such that the elasticity of substitution equals the upper bound of this range and then show how the externality changes with this parameter.

The ratio \( \kappa^N / \kappa^T \) determines the relative quality of nontradable and tradable output as collateral. It is difficult, however, to derive a direct mapping from the data to this ratio. We therefore take a pragmatic approach: we begin by setting \( \kappa^N = \kappa^T \) and then perform extensive sensitivity analysis.

The three remaining parameters are \( \{ \beta, \omega, \kappa^T \} \), which are set so that the long-run moments of the decentralized equilibrium match three historical moments of the data. The parameter \( \omega \) governs the tradable share in the CES aggregator and is calibrated to match a 32 percent share of tradable production.\(^{10}\) This approach is a reasonable one to calibrate \( \omega \) since given the relative endowment and consumption ratios, \( \omega \) determines the equilibrium price of nontradables by (1.5) and the share of tradables in the total value of production. This calibration results in a value of \( \omega \) of 0.32.

The discount factor \( \beta \) is set so that the average net foreign asset position-to-GDP ratio in the model equals its historical average in Argentina, which is equal to -29 percent in the dataset constructed by Lane and Milesi-Ferretti (2001). This calibration results in a value of \( \beta = 0.91 \), a relatively standard value for annual frequency in the literature.

The parameter \( \kappa^T \) is calibrated to match the observed frequency of “Sudden

\(^{10}\)Garcia (2008) reports an average share of tradables of 32 percent using almost a century of data from Argentina. The average share of tradables is also 32 percent in the data from WDI for the period 1980-2007.
Stops”, which is about 5.5 percent in the cross-country data set of Eichengreen, Gupta, and Mody (2006).\textsuperscript{11} To be consistent with their definition of Sudden Stops, we define Sudden Stops in our model as events where the credit constraint binds and where this leads to an increase in net capital outflows that exceeds one standard deviation. This calibration results in a value of $\kappa^T$ equal to 0.32, which is in the range of those used in the literature (see Mendoza, 2002).

1.5.2 Borrowing Decisions

We first show how the bond accumulation decisions of the social planner differ from those of private agents and then simulate the model to analyze how this difference affects the long-run distribution of debt, the crisis dynamics, and the unconditional second moments.

Figure 1 shows the bond decision rules in the decentralized equilibrium and in the constrained-efficient equilibrium as a function of current bond holdings when both tradable and nontradable shocks are one-standard-deviation below trend. Since the mean value of tradable output is 1, we can interpret all results as ratios with respect to the average output of tradables.

Without the endogenous borrowing constraint, the policy function for next period’s bond holdings would be monotonically increasing in current bond holdings. Instead, the policy functions are non-monotonic. The change in the sign of the slope of the policy function indicates the point at which the credit constraint is satisfied.

\textsuperscript{11}For the case of Argentina, Eichengreen et al. includes 1989 and 2001 as Sudden Stop events, yielding a similar frequency to the cross-country average over the sample period.
with equality, but is not binding. To the right of this point, the credit constraint is slack, and bond decision rules display the usual upward-sloping shape. To the left of this point, next-period bond holdings decrease in current bond holdings. To see why, notice that a decrease in the current bond position implies a reduction in tradable consumption for a given choice of next-period bond holdings by equation (1.9). This in turn lowers the price of nontradables by equation (1.5), which means that the level of borrowing must be reduced further to satisfy the credit constraint. Comparing the policy functions against the 45-degree line also shows that for relatively low current levels of bond holdings, the economy reduces the level of borrowing, which results in capital outflows.

We distinguish three regions for all pairs of \((b,y)\) according to the actions taken by the planner in the regulated economy: a “constrained region,” a “tax region”, and a “no-tax region.” The constrained region in Figure 1 is given by the range of \(b\) with sufficiently high initial debt such that the credit constraint binds in the constrained-efficient equilibrium. In this region, both private agents and the social planner borrow up to the limit, and decision rules coincide. The long-run probability of this region is 6.2 percent in the decentralized equilibrium, about twice as much as for the social planner.

The tax region appears shaded in Figure 1 and corresponds to the pairs of \((b,y)\) where the social planner would impose a tax on debt in the regulated economy. As explained above, this is the region where households borrow enough so that the credit constraint will bind with a strictly positive probability in the next period. Here, the social planner accumulates uniformly higher bond holdings than households. In fact,
households continue to borrow up to the limit, over some range of current bonds for which the social planner would choose a lower borrowing level that monotonically increases in current bond holdings. The economy spends about 80 percent of its time in the tax region in both equilibria.

The no-tax region is located to the right of the tax region and corresponds to the pairs of \((b,y)\) where the credit constraint is slack and the social planner would not impose a tax on debt. Intuitively, the economy is relatively well-insured in this region, and the amount of borrowing chosen does not make the economy vulnerable to a binding constraint. Here, the differences in the bond decision rules become quantitatively smaller, but are non-zero since different future choices of bond holdings affect current optimal choices. The social planner spends 16 percent of the time in this region, 2 percent more than the decentralized equilibrium.

Figure 1.1: Bond decision rules for negative one-standard-deviation shocks
While both the social planner and private households self-insure against the risk of financial crises, the social planner accumulates extra precautionary savings above and beyond what households consider privately optimal. As Figure 2 shows, this implies that the ergodic distribution of bond holdings in the decentralized equilibrium assigns a higher probability to higher levels of debt. In fact, the decentralized equilibrium has a 15 percent chance of carrying a larger amount of debt than the maximum held by the social planner, illustrated by the shaded region in Figure 2.

![Figure 1.2: Ergodic Distribution of Bond Holdings](image)

Notice that the large differences in the left tail distribution of debt are not translated into large differences in average debt levels: the average debt-to-GDP ratio is 29.2 percent for the private economy and 28.6 percent for the social planner. What is crucial is that the social planner reduces the exposure to debt levels that makes the economy vulnerable to a severe financial crisis when the economy is hit by an adverse shock.
1.5.3 Policy Instruments

Figure 1.3 shows the two types of policy measures that achieve the constrained-efficient allocations when shocks are one standard deviation below trend for different levels of current bond holdings: the left panel shows the effective increase in the cost of borrowing from tax-like measures (taxes, reserve or capital requirements); the right panel shows the adjustment in margin requirements $\theta$.

As explained above, for sufficiently low values of debt, the implied tax on borrowing is zero. The tax then increases with the level of debt in the tax region, until the credit constraint becomes binding for the social planner and the tax is set to zero. On average, the implied tax on debt is 5.2 percent.

Figure 1.3: Policy Instruments for negative one-standard-deviation shocks

The adjustment to the margin requirement is also zero when the constraint is already binding, but unlike the tax-like measures the adjustment decreases with the level of debt outside the constrained region. This arises because, as the level of debt increases, the excess debt capacity is reduced, thereby requiring a smaller adjustment in margin requirements to reduce the gap and socially desired amount.
of borrowing. On average, margins are tightened by 9 percent in the tax-region, which implies that the effective fraction that agents can borrow from their income is reduced from 0.32 to 0.29 in the regulated economy.

1.5.4 Financial Crises: Incidence and Severity

In this section, we establish that overborrowing in the decentralized equilibrium leaves the economy vulnerable to more frequent and more severe financial crises. Using the policy functions of the model, we perform an 80,000-period stochastic time series simulation of the decentralized and constrained-efficient equilibrium and use the resulting data to study the incidence and severity of financial crises. A financial crisis event is defined as a period in which the credit constraint binds, and in which this leads to an increase in net capital outflows that exceeds one standard deviation of net capital outflows in the ergodic distribution of the decentralized economy. (Results are similar with alternative definitions of a crisis event.)

Two important results emerge from the event analysis. First, crises in the decentralized equilibrium are much more likely: the long-run probability of crises is 5.5 percent (versus 0.4 percent for the social planner). Thus, by reducing the amount of debt, the social planner cuts the long-run probability of a financial crisis more than tenfold.
Second, the magnitudes of financial crises are substantially more severe because of the externality. Figure 4 shows the distribution of the response of the consumption basket on impact during financial crises for the two equilibria, expressed as a percentage deviation from the average long-run value of consumption. This figure shows that the decentralized equilibrium assigns non-trivial probabilities to consumption drops of more than 22 percent, while such a fall in consumption is a zero probability event in the social planner’s allocations.

Drops in the real exchange rate and capital inflows are also more pronounced because of the externality. Table 2 compares the responses of these variables during the median financial crisis under the decentralized equilibrium with the economy’s response under the constrained-efficient allocations, conditional on the social planner having the same level of debt two periods before such crisis and receiving the same
sequence of shocks.\textsuperscript{12}

In this experiment, we find that while the credit constraint also becomes binding for the social planner at time $t$, the impact of the adverse shocks is less severe: consumption falls 10 percent (versus 17 percent in the decentralized equilibrium), the current account does not increase (versus an 8 percent increase in the decentralized equilibrium), and the real exchange rate depreciates 1 percent (versus 19 percent in the decentralized equilibrium).\textsuperscript{13} Notice that since the initial level of debt and the sequence of shocks are the same for the two equilibria, the difference in the impact of crises is entirely due to the more prudent behavior of the social planner during the periods preceding the crisis, which makes the required adjustment following an adverse shock less severe.

<table>
<thead>
<tr>
<th>Table 1.2: Severity of Financial Crises</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>-16.7</td>
</tr>
<tr>
<td>-10.1</td>
</tr>
<tr>
<td>Current Account-GDP</td>
</tr>
<tr>
<td>7.8</td>
</tr>
<tr>
<td>0.0</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
</tr>
<tr>
<td>19.2</td>
</tr>
<tr>
<td>1.1</td>
</tr>
</tbody>
</table>

Note: Consumption and the depreciation of the real exchange rate represent responses on impact expressed as percentage deviations from averages in the corresponding ergodic distribution.

\textsuperscript{12}More precisely, we first define the median financial crisis in the decentralized as the period with the median current account reversal considering all the crisis episodes in the decentralized equilibrium. Then we backtrack the initial level of debt two periods before this episode and the sequence of shocks that hit the economy at time $t - 2$, $t - 1$, and $t$. Given this initial level of debt at $t - 2$ and the sequence of shocks, we then use the decision rules of the constrained-efficient allocations to simulate the economy and compare the impact effects at time $t$ with the impact effects in the decentralized equilibrium median crisis.

\textsuperscript{13}We define the real exchange rate as $\left[ \omega^{1/(1+\eta)} + (1 - \omega)^{1/(1+\eta)} \left( p^N \right)^{\eta/(1+\eta)} \right]^{-\eta/(1+\eta)}$ implying a one-to-one negative relationship between the price of nontradables and the real exchange rate.
Table 3 compares the unconditional second moments for decentralized and constrained-efficient equilibria, which are computed using each economy’s ergodic distribution, and for the Argentinian data. It is apparent that the externality produces non-trivial effects on the volatility of consumption, capital flows, and especially the real exchange rate. Two reasons for this are that, first, the economy spends most of the time in the tax-region where the bond accumulation decisions differ significantly; and second, the decentralized equilibrium experiences financial crises with a 5.5 percent probability, which is more than 10 times larger than that of the social planner.

Table 1.3: Second Moments

<table>
<thead>
<tr>
<th></th>
<th>Decentralized</th>
<th>Social Planner</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard Deviations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>5.9</td>
<td>5.3</td>
<td>6.2</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>9.2</td>
<td>3.5</td>
<td>8.2</td>
</tr>
<tr>
<td>Current Account-GDP</td>
<td>2.8</td>
<td>0.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Trade Balance-GDP</td>
<td>2.9</td>
<td>0.7</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>Correlation with GDP in units of tradables</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.84</td>
<td>0.85</td>
<td>0.88</td>
</tr>
<tr>
<td>Real Exchange Rate</td>
<td>-0.79</td>
<td>-0.43</td>
<td>-0.41</td>
</tr>
<tr>
<td>Current Account-GDP</td>
<td>-0.76</td>
<td>-0.05</td>
<td>-0.63</td>
</tr>
<tr>
<td>Trade Balance-GDP</td>
<td>-0.77</td>
<td>-0.22</td>
<td>-0.84</td>
</tr>
</tbody>
</table>

*Note: Data is annual from WDI for Argentina from 1965-2007. The real exchange rate is calculated as $\left[\omega^{1/(1+\eta)} + (1 - \omega)^{1/(1+\eta)} (p^N)^{\eta/(1+\eta)}\right]^{-(1+\eta)/\eta}$ and is measured empirically using value added deflators."

Table 3 also shows that the model accounts reasonably well for observed business cycle moments for Argentina, in line with previous studies. Moreover, it is apparent that the externality is important in accounting for two key regularities in the
emerging market business cycle: the high volatility of consumption and the strong procyclicality of capital flows (see e.g. Kaminsky, Reinhart, and Vegh, 2004). The constrained-efficient equilibrium cannot account for these two stylized facts. This occurs because the social planner accumulates sufficiently large precautionary savings to make large reversals in capital flows a much lower probability event compared to the decentralized equilibrium.

1.5.6 Welfare Effects

We compute the welfare gains from correcting the externality as the proportional increase in consumption for all possible future histories in the decentralized equilibrium that would make households indifferent between remaining in the decentralized equilibrium (without government intervention) and correcting the externality. These calculations explicitly consider the costs of a lower consumption in the transition to the constrained-efficient allocations. Because of the homotheticity of the utility function, the welfare gain $\gamma$ at a state $(b, y)$ is given by:

$$\left(1 + \gamma (b, y)\right)^{1-\sigma} V^{de}(b, y) = V^{sp}(b, y)$$

The welfare gains from correcting the externality are shown in Figure 1.5 as a function of current bond holdings, for negative, one-standard-deviation endowment shocks. Notice the parallel between the welfare effects and the three regions described in Figure 1, which gives the welfare gains from correcting the externality a hump shape. In the constrained region, the borrowing decisions coincide; there-
fore, the welfare gains only arise from how future allocations will differ. In the tax region, because the social planner acts in a significantly more precautionary way, the welfare gains increase. In the no-tax region, where financial crises are less likely, borrowing decisions are similar in the two equilibria and welfare gains are smaller.

On average, the welfare gains from correcting the externality are 0.135 percent of permanent consumption, consistent with the well-known result that the welfare cost of business cycles is typically small. Even if the planner does not introduce additional securities that partially complete the market, welfare gains are still larger than the benefits from introducing asset price guarantees (Durdu and Mendoza, 2006) or the benefits from introducing indexed bonds (Durdu, 2009), often suggested as policies to address Sudden StOPS (Caballero, 2002).

Figure 1.5: Welfare gains from correcting externality for negative one-standard-deviation shocks

We see these welfare gains of correcting the externality as a lower bound. First, the supply side of the economy is the same for both equilibria. If financial crises distort the efficient use of production resources, correcting the externality
could deliver higher welfare gains. Second, the risk we have considered is only aggregate; Chatterjee and Corbae (2007) shows that the welfare gains of eliminating the possibility of a crisis state can be as large as 7 percent of permanent consumption when considering idiosyncratic risk.

1.5.7 Simple forms of intervention:

Decentralizing the social planner’s allocations requires a state-contingent policy that might be challenging to implement in practice. Therefore, we also investigate if more simple forms of intervention can take the economy close to the second-best. In particular, we find that a fixed tax on debt induces welfare gains that are quite close to the second-best solution. The optimal fixed tax is 3.6 percent, which is about 70 percent of the average of the state-contingent tax, and achieves 62 percent of the welfare gains from implementing the constrained-efficient allocations. This fixed tax cuts by more than half the probability of a crisis. Allowing the tax to drop to zero when the credit constraint binds in the regulated economy or in the constrained region, delivers about the same welfare gains as the fixed tax.

By contrast, a fixed tightening in margins across all states of nature delivers welfare losses. This is intuitive given that tightening margins when the constraint is already binding delivers significant welfare losses, which outweigh the benefits from a lower average amount of debt. Tightening margins outside the constrained region only, however, does generate welfare gains, albeit smaller gains than a fixed tax on borrowing.
1.5.8 Sensitivity Analysis

In this section, we examine the sensitivity of our results to alternative calibrations. Figure 1.6 shows how the average welfare effects and the optimal average tax on debt vary with some key parameters. The online Appendix includes results from changing all parameter values and details of the sensitivity analysis.

We can gain a better understanding of the results by analyzing the externality term $\mu^s t \Psi_t$, which is the wedge between the social shadow value of wealth and the private shadow value of wealth. Recalling that $\Psi_t \equiv \kappa^N (p_t^N c_t^N) / (c_t^T) (1 + \eta)$, we have that the fraction of nontradable output that can be collateralized, the size of the nontradable sector and the elasticity of substitution are the key parameters determining the price effects in the externality term.

To arrive to a unit free measure of how the different parameters affect the price responses, we decompose the effects of the different parameters in terms of elasticities. Two elasticities are crucial for determining the price effects on the borrowing capacity. First is the elasticity that measures how much the depreciation of nontradables tightens the borrowing constraint. Second is the elasticity that measures how much nontradables depreciate as a result of an increase in debt service. Denoting the borrowing limit by $\Delta \equiv \kappa^N p^N y^N + \kappa^T y^T$, debt service by $DS \equiv b(1 + r) - b'$, and the elasticity of $y$ with respect to $x$ by $\varepsilon_{y,x}$, we can decompose the elasticity of the borrowing limit with respect to debt service as the product of these.
two elasticities.\textsuperscript{14}

\[
\varepsilon_{\Delta, DS} = \left( \frac{p_N y_N}{y_T + p_N y_N} \right) \left( \frac{\eta + 1}{y_T - DS} \right)
\]

In the baseline calibration, $\varepsilon_{\Delta, p_N} = 0.65$ and $\varepsilon_{p_N, DS} = 0.40$ at the median crisis in the decentralized equilibrium. That is, a reduction in the debt service of 1 percent would mitigate the drop in the price of nontradables during a median crisis by 0.40 percent (and the real exchange rate by 0.22 percent) and in turn this would increase the borrowing capacity by $0.40 \times 0.65 = 0.26$ percent.

It is clear from (1.19) that the elasticity of substitution $1/(1 + \eta)$ between tradables and nontradables is a key determinant of how much the real exchange rate depreciates as a result of an increase in debt service: a lower elasticity implies that a given decrease in tradable consumption requires a greater adjustment in the real exchange rate to equilibrate the market. Moreover, increases in the level of debt service lead to a larger depreciation in the real exchange rate all else equal. While the elasticity of substitution in our baseline calibration is at the high end of the range of existing estimates, Panel A of Figure 1.6 shows that a smaller elasticity of substitution increases the optimal average tax on debt and the welfare benefits of correcting the externality. While in theory the overborrowing distortion is present for any finite value of the elasticity, Panel A also shows that in practice, the allocations of the social planner and the decentralized equilibrium become almost identical for

\textsuperscript{14}This formula yields as a result of applying the definition of $\Delta$ and $DS$, the three elasticities and using $\Psi = - (\partial \Delta)/(\partial DS)$. 

36
values of the elasticity greater than 4.

The elasticity decomposition also shows that for low shares of the nontradable sector, a depreciation of the real exchange rate has smaller effects on the borrowing capacity. By adjusting $\omega$ to reduce the equilibrium share of the nontradable sector, Panel B in Figure 1.6 shows that this is reflected in lower effects from the externality.

We next study changes in $\kappa^T$ and $\kappa^N$. It is reasonable to argue that in the presence of debt denominated in units of tradable, creditors may be less willing to accept nontradable income as collateral. We explore this idea by varying the values of $\kappa^N$ and $\kappa^T$ such that $\kappa^N = c\kappa^T$ for $c < 1$ and such that the long-run average of $\kappa^N p^N y^N + \kappa^T y^T$ remains as in the baseline calibration. As Panel (D) in Figure 1.6 shows, the effects of the externality remain significant even if the quality of nontradable collateral is half of the quality of tradable collateral, i.e. when $c = 0.5$. In fact, crises in the decentralized equilibrium are 3.5 times more likely and significant differences remain in the severity of these episodes.

In Panel (C) we set $\kappa^T = \kappa^N = \kappa$ and vary the value of $\kappa$. Notice that there are two opposing effects from an increase in $\kappa$. On one hand, since $\kappa$ scales up the price effects in the externality term, we should expect higher effects from the externality. On the other hand, increasing $\kappa$ makes the credit constraint less likely to bind, thereby reducing the externality. As Panel (C) shows, a relatively higher $\kappa$ increases the externality.

The other component of the externality term is the shadow value from relaxing the credit constraint $\mu$. While it is not possible to derive an analytical expression for $\mu$, for a given state $\mu$ should be positively related to the household’s share of
tradables in the utility function, and the inverse of the intratemporal and intertemporal elasticity of substitution, because these parameters affect the utility cost from a large drop in tradable consumption; $\mu$ should also depend on the discount factor and the interest rate, because these parameters affect the household’s impatience and its willingness to borrow.

Figure 1.6: Sensitivity Analysis: Welfare and Implied Tax (in Percentage)

We extend the model by allowing for production in the nontradable sector with intermediate inputs, as in Durdu, Mendoza, and Terrones (2009). Specifically, firms use intermediate inputs $m$ to produce nontradables with a technology such that $y_t = A_t m_t^\alpha$. Firms maximize $p^N_t A_t m_t^\alpha p m_t$ and redistribute profits to households, whose income is now given by the endowment of tradables plus profits. Since a binding credit constraint induces a depreciation of nontradables, this feature generates a drop in the value of the marginal product of imported inputs, and therefore a drop in the
production of nontradables during financial crises. As a result, since crises in the
decentralized equilibrium generate a larger depreciation, the externality generates a
larger drop in both tradable and nontradable consumption. Setting $\alpha = 0.10$ in line
with Goldberg and Campa (2010) and re-calibrating the rest of the parameters, we
find that the effects of the externality remain very similar overall.\footnote{We continue to assume that in the constrained-efficient allocations the social planner makes borrowing decisions, while households choose consumption allocations and firms choose intermediate inputs. This yields an identical decentralization to the endowment economy model where a tax on debt is sufficient to achieve constrained efficiency.}

Overall, the sensitivity analysis suggests that overborrowing creates significant
distortions for plausible parameterizations. Only when the probability of a binding
credit constraint becomes negligible or when debt deflation effects are very weak do we find that the effects of overborrowing are insignificant.

1.6 Policy Remarks

The new paradigm in financial regulation stresses the need for a macropruden-
tial approach to consider how actions of individual market participants can destabi-
lize macroeconomic conditions with adverse effects over the whole economy (see e.g.
Borio, 2003). The analysis presented here suggests that overborrowing externalities
have a large enough quantitative impact on welfare to justify macroprudential reg-
ulation. It is worth noting that correcting these externalities does not eliminate the
possibility of financial crises in our simulations, but the incidence and severity of
crises are considerably reduced under regulation. This is consistent with the con-
strained notion of efficiency that we consider in our analysis: the social planner is
subject to the same financial frictions as the decentralized economy, so that regulation does not fully eliminate the financial accelerator effects that arise when a negative shock triggers the credit constraint.

In the context of the debate on financial globalization, there is a view that a Tobin-style tax can help smooth the boom-bust cycle caused by sharp changes in access to credit in emerging markets. A recent “IMF Staff Position Note” by Ostry et al. (2010) emphasizes the benefits experienced by emerging markets from the recent use of reserve requirements, although some controversy remains in the literature. Our paper contributes to this debate by undertaking a quantitative investigation of how curbing external finance can deliver a reduction in the vulnerability to financial crises while still allowing an economy to reap the benefits of access to global capital markets. At the same time, fostering the development of financial markets could also generate significant welfare gains by improving risk sharing and addressing the root of the externality, i.e., the credit constraint. To the extent that the degree of financial development remains incomplete, our results suggest that there is a scope for “throwing sand in the wheels of international finance.”

1.7 Conclusions

This paper investigates a systemic credit externality that magnifies the incidence and severity of financial crises. Households accumulate precautionary savings to smooth consumption during the cycle, but they fail to internalize the systemic feedback effects between borrowing decisions, the real exchange rate, and financial
constraints. By reducing the amount of debt ex-ante, a social planner mitigates the downward spiral in the exchange rate and in the borrowing capacity during a crisis, thereby improving social welfare.

The key contribution of this paper is its quantitative analysis of this externality: we analyze the effects on financial crisis dynamics and welfare, and the policy measures needed to correct this externality. Our main conclusion is that there is much to gain from introducing macroprudential regulation. Correcting the credit externality reduces the long-run probability of a financial crisis more than ten times (from 5.5 percent to 0.4 percent) and reduces the consumption drop during a typical crisis by 7 percentage points (from 17 percent to 10 percent).

On the policy side, we show that several regulatory measures commonly used to maintain financial stability can achieve the constrained-efficient allocations. These measures effectively impose an increase in the cost of borrowing whenever there is a positive probability of a crisis, but before the crisis materializes so that the economy becomes less vulnerable to future adverse shocks. While these policies are equivalent in the model, in practice there are different costs and benefits associated with their actual implementation. We also acknowledge that the actual implementation of these policies is a challenging task, but we also show that simple interventions such as a fixed tax on debt yields sizable welfare gains.

Within our framework, incorporating capital accumulation and specifying a richer supply side of the economy would be important to extend the quantitative analysis. There are also other natural extensions of our work. While our externality stems from a feedback loop between the real exchange rate and financial constraints,
our results suggest that pecuniary externalities resulting from a similar mechanism involving other relative prices might play a quantitatively important role as well. For example, it would be interesting to study a similar externality involving asset prices and economic activity. Another direction for future research would be to study the role for macroprudential regulation in a setup with an explicit role for financial intermediation, as in for example Gertler and Karadi (2009). These issues remain for future research.
Chapter 2

Overborrowing, Financial Crises and Macro-Prudential Policy

(coauthored with Enrique G. Mendoza)

2.1 Introduction

A common argument in narratives of the causes of the 2008 global financial crisis is that economic agents “borrowed too much.” The notion of “overborrowing,” however, is often vaguely defined or presented as a value judgment on borrowing decisions, in light of the obvious fact that a prolonged credit boom ended in collapse. This lack of clarity makes it difficult to answer two key questions: First, is overborrowing a significant macroeconomic problem, in terms of causing financial crises and playing a central role in driving macro dynamics during both ordinary business cycles and crises episodes? Second, are the so-called “macro-prudential” policy instruments effective to contain overborrowing and reduce financial fragility, and if so what are their main quantitative features?

In this paper, we answer these questions using a dynamic stochastic general equilibrium model of asset prices and business cycles with credit frictions. We provide a formal definition of overborrowing and use quantitative methods to determine how much overborrowing the model predicts and how it affects business cycles, financial crises, and social welfare. We also compute a state-contingent schedule of
taxes on debt and dividends that can solve the overborrowing problem.

Our definition of overborrowing is in line with the one used in the academic literature (e.g. Lorenzoni, 2008, Korinek, 2009, Bianchi, 2010): The difference between the amount of credit that an agent obtains acting atomistically in an environment with a given set of credit frictions, and the amount obtained by a social planner who faces the same frictions but internalizes the general-equilibrium effects of its borrowing decisions. In the model, the credit friction is in the form of a collateral constraint on debt that has two important features. First, it drives a wedge between the marginal costs and benefits of borrowing considered by individual agents and those faced by a social planner. Second, when the constraint binds, it triggers Irving Fisher’s classic debt-deflation financial amplification mechanism, which causes a financial crisis.

This paper contributes to the literature by providing a quantitative assessment of overborrowing in an equilibrium model of business cycles and asset prices. The model is similar to those examined by Mendoza and Smith (2006) and Mendoza (2010). These studies showed that cyclical dynamics in a competitive equilibrium lead to periods of expansion in which leverage ratios raise enough so that the collateral constraint becomes binding, triggering a Fisherian deflation that causes sharp declines in credit, asset prices, and macroeconomic aggregates.¹ In this paper, we study instead the efficiency properties of the competitive equilibrium, by comparing its allocations with those attained by a benevolent social planner subject to the same

¹This is also related to the classic work on financial accelerators by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) and the more recent quantitative literature on this topic as in the work of Jermann and Quadrini (2010).
credit frictions as agents in the competitive equilibrium. Thus, while those previous studies focused on the amplification and asymmetry of the responses of macro variables to aggregate shocks, we focus here on the differences between competitive equilibria and constrained social optima.

In the model, the collateral constraint limits private agents not to borrow more than a fraction of the market value of their collateral assets, which take the form of an asset in fixed aggregate supply (e.g. land). Private agents take the price of this asset as given, and hence a “systemic credit externality” arises, because they do not internalize that, when the collateral constraint binds, fire-sales of assets cause a Fisherian debt-deflation spiral that causes asset prices to decline and the economy’s borrowing ability to shrink in an endogenous feedback loop. Moreover, when the constraint binds, production plans are also affected, because working capital financing is needed in order to pay for a fraction of labor costs, and working capital loans are also subject to the collateral constraint. As a result, when the credit constraint binds output falls because of a sudden increase in the effective cost of labor. This affects dividend streams and therefore equilibrium asset prices, and introduces an additional vehicle for the credit externality to operate, because private agents do not internalize the supply-side effects of their borrowing decisions.

We conduct a quantitative analysis in a version of the model calibrated to U.S. data. The results show that financial crises in the competitive equilibrium are significantly more frequent and more severe than in the constrained-efficient equilibrium. The incidence of financial crises is about three times larger. Asset prices drop about 25 percent in a typical crisis in the decentralized equilibrium,
versus 5 percent in the constrained-efficient equilibrium. Output drops about 50 percent more, because the fall in asset prices reduces access to working capital financing. The more severe asset price collapses also generate a “fat tail” in the distribution of asset returns in the decentralized equilibrium, which causes the price of risk to rise 1.5 times and excess returns to rise by 5 times, in both tranquil times and crisis times. The social planner can replicate exactly the constrained-efficient allocations in a decentralized equilibrium by imposing taxes on debt and dividends of about 1 and -0.5 percent on average respectively.

The existing macro literature on credit externalities provides important background for our work. The externality we study is a pecuniary externality similar to those examined in the theoretical studies of Caballero and Krishnamurthy (2001), Lorenzoni (2008), and Korinek (2009). The pecuniary externality in these papers arises because financial constraints that depend on market prices generate amplification effects, which are not internalized by private agents. The literature on participation constraints in credit markets initiated by the work of Kehoe and Levine (1993) has also examined the role of inefficiencies that arise because of endogenous borrowing constraints. In particular, Jeske (2006) showed that if there is discrimination against foreign creditors, private agents have a stronger incentive to default than a planner who internalizes the effects of borrowing decisions on the domestic interest rate, which affects the tightness of the participation constraint. Wright (2006) then showed that as a consequence of this externality, subsidies on capital flows restore constrained efficiency.

Our work is also related to the quantitative studies of macro-prudential policy
by Bianchi (2010) and Benigno, Chen, Otrok, Rebucci, and Young (2009). These authors studied a credit externality at work in the model of emerging markets crises of Mendoza (2002), in which agents do not internalize the effect of their individual debt plans on the market price of nontradable goods relative to tradables, which influences their ability to borrow from abroad. Bianchi examined how this externality leads to excessive debt accumulation and showed that a tax on debt can restore constrained efficiency and reduce the vulnerability to financial crises. Benigno et al. studied how the effects of the overborrowing externality are reduced when the planner has access to instruments that can affect directly labor allocations during crises.\footnote{In a related paper Benigno et al. (2009) found that intervening during financial crisis by subsidizing nontradable goods leads to large welfare gains.}

Our analysis differs from the above quantitative studies in that we focus on asset prices as a key factor driving debt dynamics and the credit externality, instead of the relative price of nontradables. This is important because private debt contracts, particularly mortgage loans like those that drove the high household leverage ratios of many industrial countries in the years leading to the 2008 crisis, use assets as collateral. Moreover, from a theoretical standpoint, a collateral constraint linked to asset prices introduces forward-looking effects that are absent when using a credit constraint linked to goods prices. In particular, expectations of a future financial crisis affect the discount rates applied to future dividends and distort asset prices even in periods of financial tranquility. In addition, our model also differs in that we study a production economy in which working capital financing is subject to the
collateral constraint. As a result, the credit externality distorts production plans and dividend rates, and thus again asset prices.

More recently, the quantitative studies by Nikolov (2009) and Jeanne and Korinek (2010) examine other models of macro-prudential policy in which assets serve as collateral.\(^3\) Nikolov found that simple rules that impose tighter collateral requirements may not be welfare-improving in a setup in which consumption is a linear function that is not influenced by precautionary savings. In contrast, precautionary savings are critical determinants of optimal borrowing decisions in our model, because of the strong non-linear amplification effects caused by the Fisherian debt-deflation dynamics, and for the same reason we find that debt taxes are welfare improving. Jeanne and Korinek construct estimates of a Pigouvian debt tax in a model in which output follows an exogenous Markov-switching process and individual credit is limited to the sum of a fraction of aggregate, rather than individual, asset holdings plus a constant term. In their calibration analysis, this second term dominates and the probability of crises matches the exogenous probability of a low-output regime, and as result the tax cannot alter the frequency of crises and has small effects on their magnitude.\(^4\) In contrast, in our model the probability of crises and their output dynamics are endogenous, and macro-prudential policy reduces sharply the incidence and magnitude of crises.

Our results also contrast with the findings of Uribe (2006). He found that an environment in which agents do not internalize an aggregate borrowing limit

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\(^3\)Galati and Moessner (2010) conduct an exhaustive survey of the growing literature in research and policy circles on macro-prudential policy.

\(^4\)They also examined the existence of deterministic cycles in a non-stochastic version of the model.
yields identical borrowing decisions to an environment in which the borrowing limit is internalized. An essential difference in our analysis is that the social planner internalizes not only the borrowing limit but also the price effects that arise from borrowing decisions. Still, our results showing small differences in average debt ratios across competitive and constrained-efficient equilibria are in line with his findings.

The rest of the paper is organized as follows: Section 2 presents the analytical framework. Section 3 analyzes constrained efficiency. Section 4 presents the quantitative analysis. Section 5 provides conclusions.

2.2 Competitive Equilibrium

We follow Mendoza (2010) in specifying the economic environment in terms of firm-household units who make production and consumption decisions. Preferences are given by:

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t - G(n_t)) \right] \quad (2.1) \]

In this expression, \( E(\cdot) \) is the expectations operator, \( \beta \) is the subjective discount factor, \( n_t \) is labor supply and \( c_t \) is consumption. The period utility function \( u(\cdot) \) is assumed to have the constant-relative-risk-aversion (CRRA) form. The argument of \( u(\cdot) \) is the composite commodity \( c_t - G(n_t) \) defined by Greenwood, Hercowitz, and Huffman (1988). \( G(n) \) is a convex, strictly increasing and continuously differentiable function

\(^5\)He provided analytical results for a canonical endowment economy model with a credit constraint where there is an exact equivalence between the two sets of allocations. In addition, he examined a model in which the exact equivalence of his first example does not hold, but still overborrowing is negligible.
that measures the disutility of labor supply. This formulation of preferences removes
the wealth effect on labor supply by making the marginal rate of substitution be-
tween consumption and labor depend on labor only.

Each household can combine land and labor services purchased from other
households to produce final goods using a production technology such that \( y =
\varepsilon_t F(k_t, h_t) \), where \( F \) is a decreasing-returns-to-scale production function, \( k_t \)
represents individual land holdings, \( h_t \) represents labor demand and \( \varepsilon_t \) is a productivity
shock, which has compact support and follows a finite-state, stationary Markov
process. Individual profits from this production activity are therefore given by
\( \varepsilon_t F(k_t, h_t) - w_t h_t \).

The budget constraint faced by the representative firm-household is:

\[
q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + w_t n_t + [\varepsilon_t F(k_t, h_t) - w_t h_t]
\]  
(2.2)

where \( b_t \) denotes holdings of one-period, non-state-contingent discount bonds at the
beginning of date \( t \), \( q_t \) is the market price of land, \( R_t \) is the real interest rate, and
\( w_t \) is the wage rate.

The interest rate is assumed to be exogenous. This is equivalent to assuming
that the economy is a price-taker in world credit markets, as in other studies of
the U.S. financial crisis like those of Boz and Mendoza (2010), Corbae and Quintin
(2009) and Howitt (2010), or alternatively it implies that the model can be in-
terpreted as a partial-equilibrium model of the household sector. This assump-
tion is adopted for simplicity, but is also in line with the evidence indicating that
in the era of financial globalization even the U.S. risk-free rate has been significantly influenced by outside factors, such as the surge in reserves in emerging economies and the persistent collapse of investment rates in South East Asia after 1998. Warnock and Warnock (2009) provide econometric evidence of the significant effect of foreign capital inflows on U.S. T-bill rates since the mid 1980s. Mendoza and Quadrini (2009) document that about 1/2 of the surge in net credit in the U.S. economy since then was financed by foreign capital inflows, and more than half of the stock of U.S. treasury bills is now owned by foreign agents.

Household-firms are subject to a working capital constraint. In particular, they are required to borrow a fraction $\theta$ of the wages bill $w_t h_t$ at the beginning of the period and have to repay at the end of the period. In the conventional working capital setup, a cash-in-advance-like motive for holding funds to pay for inputs implies that the wages bill carries a financing cost determined by the inter-period interest rate. In contrast, here we simply assume that working capital funds are within-period loans. Hence, the interest rate on working capital is zero, as in some recent studies on the business cycle implications of working capital and credit frictions (e.g. Chen and Song (2009)). We follow this approach so as to show that the effects of working capital in our model hinge only on the need to provide collateral for working capital loans, as explained below, and not on the effect of interest rate fluctuations on effective labor costs. 6

As in Mendoza (2010), agents face a collateral constraint that limits total

6We could also change to the standard setup, but in our calibration, $\theta = 0.14$ and $R = 1.028$, and hence working capital loans would add 0.4 percent to the cost of labor implying that our findings would remain largely unchanged.
debt, including both intertemporal debt and atemporal working capital loans, not to exceed a fraction $\kappa$ of the market value of asset holdings (i.e. $\kappa$ imposes a ceiling on the leverage ratio):

$$-\frac{b_{t+1}}{R_t} + \theta w_t h_t \leq \kappa q_t k_{t+1}$$

(2.3)

Following Kiyotaki and Moore (1997) and Aiyagari and Gertler (1999), we interpret this constraint as resulting from an environment where limited enforcement prevents lenders to collect more than a fraction $\kappa$ of the value of a defaulting debtor’s assets, but we abstract from modeling the contractual relationship explicitly.

2.2.1 Private Optimality Conditions

In the competitive equilibrium, agents maximize (2.1) subject to (2.2) and (2.3) taking land prices and wages as given. This maximization problem yields the following optimality conditions for each date $t$:

$$w_t = G'(n_t)$$

(2.4)

$$\varepsilon_tF_h(k_t, h_t) = w_t [1 + \theta \mu_t/u'(t)]$$

(2.5)

$$u'(t) = \beta RE_t [u'(t + 1)] + \mu_t$$

(2.6)

$$q_t(u'(t) - \mu_t \kappa) = \beta E_t [u'(t + 1) (\varepsilon_{t+1} F_k(k_{t+1}, h_{t+1}) + q_{t+1})]$$

(2.7)

where $\mu_t \geq 0$ is the Lagrange multiplier on the collateral constraint.

Condition (2.4) is the individual’s labor supply condition, which equates the marginal disutility of labor with the wage rate. Condition (2.5) is the labor de-
mand condition, which equates the marginal productivity of labor with the effective marginal cost of hiring labor. The latter includes the extra financing cost \( \theta \mu_t/u'(t) \)
in the states of nature in which the collateral constraint on working capital binds.

The last two conditions are the Euler equations for bonds and land respectively. When the collateral constraint binds, condition (2.6) implies that the marginal utility of reallocating consumption to the present exceeds the expected marginal utility cost of borrowing in the bond market by an amount equal to the shadow price of relaxing the credit constraint. Condition (2.7) equates the marginal cost of an extra unit of land investment with its marginal gain. The marginal cost nets out from the marginal utility of foregone current consumption a fraction \( \kappa \) of the shadow value of the credit constraint, because the additional unit of land holdings contributes to relax the borrowing limit.

Condition (2.7) yields the following forward solution for land prices:

\[
q_t = E_t \left[ \sum_{j=0}^{\infty} \left( \prod_{i=0}^{j} m_{t+1+i} \right) d_{t+j+1} \right], \quad m_{t+1+i} \equiv \frac{\beta u'(t + 1 + i)}{u'(t + i) - \mu_{t+i} \kappa}, \quad d_t \equiv \varepsilon_t F_k(k_t, h_t)
\]

Thus, we obtain what seems a standard asset pricing condition stating that, at equilibrium, the date-\( t \) price of land is equal to the expected present value of the future stream of dividends discounted using the stochastic discount factors \( m_{t+1+i}, \)
for \( i = 0, \ldots, \infty \). The key difference with the standard asset pricing condition, however, is that the discount factors are adjusted to account for the shadow value of relaxing the credit constraint by purchasing an extra unit of land whenever the collateral constraint binds (at any date \( t + i \) for \( i = 0, \ldots, \infty \).
Combining (2.6), (2.7) and the definition of asset returns \( R_{t+1}^q \equiv \frac{d_{t+1} + q_{t+1}}{q_t} \), it follows that the expected excess return on land relative to bonds (i.e. the equity premium), \( R_t^{ep} \equiv E_t(R_{t+1}^q - R) \), satisfies the following condition:

\[
R_t^{ep} = \frac{\mu_t(1 - \kappa)}{(u'(t) - \mu_t \kappa)E_t[m_{t+1}]} - \frac{\text{cov}_t(m_{t+1}, R_{t+1}^q)}{E_t[m_{t+1}]}, \tag{2.9}
\]

where \( \text{cov}_t(m_{t+1}, R_{t+1}^q) \) is the date-\( t \) conditional covariance between \( m_{t+1} \) and \( R_{t+1}^q \).

Following Mendoza and Smith (2006), we characterize the first term in the right-hand-side of (2.9) as the direct (first-order) effect of the collateral constraint on the equity premium, which reflects the fact that a binding collateral constraint exerts pressure to fire-sell land, depressing its current price.\(^7\) There is also an indirect (second-order) effect given by the fact that \( \text{cov}_t(m_{t+1}, R_{t+1}^q) \) is likely to become more negative when there is a possibility of a binding credit constraint, because the collateral constraint makes it harder for agents to smooth consumption.

Given the definitions of the Sharpe ratio \( S_t \equiv \frac{R_t^{ep}}{\sigma_t(R_{t+1}^q)} \) and the price of risk \( s_t \equiv \sigma_t(m_{t+1})/E_t m_{t+1} \), we can rewrite the expected excess return and the Sharpe ratio as:

\[
R_t^{ep} = S_t \sigma_t(R_{t+1}^q), \quad S_t = \frac{\mu_t(1 - \kappa)}{(u'(t) - \mu_t \kappa)E_t[m_{t+1}]} \sigma_t(R_{t+1}^q) - \rho_t(R_{t+1}^q, m_{t+1}) s_t \tag{2.10}
\]

where \( \sigma_t(R_{t+1}^q) \) is the date-\( t \) conditional standard deviation of land returns and \( \rho_t(R_{t+1}^q, m_{t+1}) \) is the conditional correlation between \( R_{t+1}^q \) and \( m_{t+1} \). Thus, the col-

---

\(^7\)Notice that this effect vanishes when \( \kappa = 1 \), because when 100 percent of the value of land can be collateralized, the shadow value of relaxing the constraint by acquiring an extra unit of land equals the shadow value of relaxing it by reducing the debt by one unit.
lateral constraint has direct and indirect effects on the Sharpe ratio analogous to
those it has on the equity premium. The indirect effect reduces to the usual ex-
pression in terms of the product of the price of risk and the correlation between
asset returns and the stochastic discount factor. The direct effect is normalized by
the variance of land returns. These relationships will be useful later to study the
quantitative effects of the credit externality on asset pricing.

Since \( q_tE_t[R^q_{t+1}] \equiv E_t[d_{t+1} + q_{t+1}] \), we can rewrite the asset pricing condition
in this way:

\[
q_t = E_t \sum_{j=0}^{\infty} \left( \prod_{i=0}^{j} E_t^{i+1}R^q_{t+1+i} \right)^{-1} d_{t+j+1},
\]  

(2.11)

Notice that (2.9) and (2.11) imply that a binding collateral constraint at date \( t \)
implies an increase in expected excess land returns and a drop in asset prices at \( t \).
Moreover, since expected returns exceed the risk free rate whenever the collateral
constraint is expected to bind at any future date, asset prices at \( t \) are affected by
collateral constraint not just when the constraints binds at \( t \), but whenever it is
expected to bind at any future date.

2.2.2 Recursive Competitive Equilibrium

The competitive equilibrium is defined by stochastic sequences of allocations
\( \{c_t, k_{t+1}, b_{t+1}, h_t, n_t\}_{t=0}^{\infty} \) and prices \( \{q_t, w_t\}_{t=0}^{\infty} \) such that: (A) agents maximize utility
(2.1) subject to the sequence of budget and credit constraints given by (2.2) and
(2.3) for \( t = 0, ..., \infty \), taking as given \( \{q_t, w_t\}_{t=0}^{\infty} \); (B) the markets of goods, labor
and land clear at each date \( t \). Since land is in fixed supply \( \bar{K} \), the market-clearing
condition for land is \( k_t = \bar{K} \). The market clearing condition in the goods and labor markets are

\[
ct + \frac{b_{t+1}}{R} = \varepsilon_tF(\bar{K}, nt) + bt \quad \text{and} \quad h_t = nt \text{ respectively.}
\]

We now characterize the competitive equilibrium in recursive form. The state variables for a particular individual’s optimization problem at time \( t \) are the individual bond holdings \( b \), aggregate bond holdings \( B \), individual land holdings \( k \), and the TFP realization \( \varepsilon \). Aggregate land holdings are not carried as a state variable because land is in fixed supply. Denoting by \( \Gamma(B, \varepsilon) \) the agents’ perceived law of motion of aggregate bonds and \( q(B, \varepsilon) \) and \( w(B, \varepsilon) \) the pricing functions for land and labor respectively, the agents’ recursive optimization problem is:

\[
V(b, k, B, \varepsilon) = \max_{b', k', c, n, h} u(c - G(n)) + \beta E_{t+1} [V(b', k', B', \varepsilon')]
\]

\[\begin{align*}
\text{s.t.} \quad & q(B, \varepsilon)k' + c + \frac{b'}{R} = q(B, \varepsilon)k + b + w(B, \varepsilon)n + [\varepsilon F(k, h) - w(B, \varepsilon)h] \\
& B' = \Gamma(B, \varepsilon) \\
& -\frac{b'}{R} + \theta w(B, \varepsilon)h \leq \kappa q(B, \varepsilon)k'
\end{align*}\]  

The solution to this problem is characterized by the decision rules \( \hat{b}'(b, k, B, \varepsilon) \), \( \hat{k}'(b, k, B, \varepsilon) \), \( \hat{c}(b, k, B, \varepsilon) \), \( \hat{n}(b, k, B, \varepsilon) \) and \( \hat{h}(b, k, B, \varepsilon) \). The decision rule for bond holdings induces an actual law of motion for aggregate bonds, which is given by \( \hat{b}'(B, \bar{K}, B, \varepsilon) \). In a recursive rational expectations equilibrium, as defined below, the actual and perceived laws of motion must coincide.

**Definition 3 (Recursive Competitive Equilibrium)**

A recursive competitive equilibrium is defined by an asset pricing func-
tion \( q(B, \varepsilon) \), a pricing function for labor \( w(B, \varepsilon) \), a perceived law of motion for aggregate bond holdings \( \Gamma(B, \varepsilon) \), and a set of decision rules
\[
\left\{ \hat{b}'(b, k, B, \varepsilon), \hat{k}'(b, k, B, \varepsilon), \hat{c}(b, k, B, \varepsilon), \hat{n}(b, k, B, \varepsilon), \hat{h}(b, k, B, \varepsilon) \right\}
\]
with associated value function \( V(b, k, B, \varepsilon) \) such that:

1. \( \left\{ \hat{b}'(b, k, B, \varepsilon), \hat{k}'(b, k, B, \varepsilon), \hat{c}(b, k, B, \varepsilon), \hat{n}(b, k, B, \varepsilon), \hat{h}(b, k, B, \varepsilon) \right\} \) and \( V(b, k, B, \varepsilon) \)
solve the agents’ recursive optimization problem, taking as given \( q(B, \varepsilon) \), \( w(B, \varepsilon) \)
and \( \Gamma(B, \varepsilon) \).

2. The perceived law of motion for aggregate bonds is consistent with the actual law of motion: \( \Gamma(B, \varepsilon) = \hat{b}'(B, \bar{K}, B, \varepsilon) \).

3. Wages satisfy \( w(B, \varepsilon) = G'(\hat{n}(B, \bar{K}, B, \varepsilon)) \) and land prices satisfy \( q(B, \varepsilon) = \)

\[
E_{\varepsilon'|\varepsilon} \left\{ \frac{\beta u'(\bar{c}(\Gamma(B, \varepsilon), \bar{K}, \Gamma(B, \varepsilon), \varepsilon'))}{u'(\bar{c}(B, K, B, \varepsilon)) - \beta \max \left[ 0, u'(\bar{c}(B, K, B, \varepsilon)) - \beta \varepsilon \right]} + q(\Gamma(B, \varepsilon), \varepsilon') \right\}
\]

4. Goods, labor and asset markets clear: \( \frac{\hat{b}(B, \bar{K}, B, \varepsilon)}{R} + \hat{c}(B, \bar{K}, B, \varepsilon) = \varepsilon F(\bar{K}, \hat{n}(B, \bar{K}, B, \varepsilon)) + \)
\( B, \hat{n}(B, \bar{K}, B, \varepsilon) = \hat{h}(B, \bar{K}, B, \varepsilon) \) and \( \hat{k}(B, \bar{K}, B, \varepsilon) = \bar{K} \)

2.3 Constrained-Efficient Equilibrium

2.3.1 Equilibrium without collateral constraint

We start studying the efficiency properties of the competitive equilibrium by briefly characterizing an efficient equilibrium in the absence of the collateral constraint (2.3). The allocations of this equilibrium can be represented as the solution
to the following standard planning problem:

$$H(B, \varepsilon) = \max_{B', \varepsilon, c, n} u(c - G(n)) + \beta E_{\varepsilon' | \varepsilon} [H(B', \varepsilon')]$$  \hspace{1cm} (2.13)

s.t. \hspace{0.5cm} c + \frac{B'}{R} = \varepsilon F(K, n) + B$$

and subject also to either this problem’s natural debt limit, which is defined by $B' > \frac{\varepsilon^{\text{min}} F(K, n^*(\varepsilon^{\text{min}}))}{R-1}$, where $\varepsilon^{\text{min}}$ is the lowest possible realization of TFP and $n^*(\varepsilon^{\text{min}})$ is the optimal labor allocation that solves $\varepsilon^{\text{min}} F_{n}(K, n) = G'(n)$, or to a tighter ad-hoc time- and state-invariant borrowing limit.

The common strategy followed in quantitative studies of the macro effects of collateral constraints (see, for example, Mendoza and Smith, 2006 and Mendoza, 2010) is to compare the allocations of the competitive equilibrium with the Fisherian collateral constraint with those arising from the above benchmark case. The competitive equilibria with and without the collateral constraint differ in that in the former private agents borrow less (since the collateral constraint limits the amount they can borrow and also because they build precautionary savings to self-insure against the risk of the occasionally binding credit constraint), and there is financial amplification of the effects of the underlying exogenous shocks (since binding collateral constraints produce large recessions and drops in asset prices). Compared with the constrained-efficient equilibrium we define next, however, we will show that the competitive equilibrium with collateral constraints displays overborrowing (i.e. agents borrow more than in the constrained-efficient equilibrium).
2.3.2 Recursive Constrained-Efficient Equilibrium

We study a benevolent social planner who maximizes the agents’ utility subject to the resource constraint, the collateral constraint and the same menu of assets of the competitive equilibrium. In particular, we consider a social planner that is constrained to have the same “borrowing ability” (the same market-determined value of collateral assets $\kappa q(B, \varepsilon) \bar{K}$) at every given state as agents in the decentralized equilibrium, but with the key difference that the planner internalizes the effects of its borrowing decisions on the market prices of assets and labor.

The recursive problem of the social planner is defined as follows:

$$ W(B, \varepsilon) = \max_{B', c, n} u(c - G(n)) + \beta E_{\varepsilon' | \varepsilon} [W(B', \varepsilon')] $$ (2.14)

subject to

$$ c + \frac{B'}{R} = \varepsilon F(\bar{K}, n) + B $$

$$ -\frac{B'}{R} + \theta w(B, \varepsilon)n \leq \kappa q(B, \varepsilon) \bar{K} $$

where $q(B, \varepsilon)$ is the equilibrium pricing function obtained in the competitive equilibrium. Wages can be treated in a similar fashion, but it is easier to decentralize the planner’s allocations as as competitive equilibrium if we assume that the plan-

---

8We refer to the social planner’s equilibrium and constrained-efficient equilibrium interchangeably. Our focus is on second-best allocations, so when we refer to the social planner’s choices it should be understood that we mean the constrained social planner.

9We could also allow the social planner to manipulate the borrowing ability state by state (i.e., by allowing the planner to alter $\kappa q(B, \varepsilon) \bar{K}$). Allowing for this possibility can potentially increase the welfare losses resulting from the externality but the macroeconomic effects are similar. In addition, since asset prices are forward-looking, this would create a time-inconsistency problem in the planner’s problem. Allowing the planner to commit to future actions would lead the planner to internalize not only how today’s choice of debt affects tomorrow’s asset prices but also how it affects asset prices and the tightness of collateral constraints in previous periods.
ner takes wages as given and wages need to satisfy \( w(B, \varepsilon) = G'(n) \).\(^{10}\) Under this assumption, we impose the optimality condition of labor supply as a condition that the constrained-efficient equilibrium must satisfy, in addition to solving problem (2.14) for given wages.

Using the envelope theorem on the first-order conditions of problem (2.14) and imposing the labor supply optimality condition, we obtain the following optimality conditions for the constrained-efficient equilibrium:

\[
 u'(t) = \beta R_{t+1} [u'(t+1) + \mu_{t+1}\psi_{t+1}] + \mu_t, \quad \psi_{t+1} \equiv \kappa K \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial w_{t+1}}{\partial b_{t+1}}
\]  

\[
\varepsilon_t F_n(\bar{K}, n_t) = G'(n_t) [1 + \theta \mu_t / u'(t)]
\]  

The key difference between the competitive equilibrium and the constrained-efficient allocations follows from examining the Euler equations for bond holdings in both problems. In particular, the term \( \mu_{t+1}\psi_{t+1} \) in condition (2.15) represents the additional marginal benefit of savings considered by the social planner at date t, because the planner takes into account how an extra unit of bond holdings alters the tightness of the credit constraint through its effects on the prices of land and labor at \( t + 1 \). Note that, since \( \frac{\partial q_{t+1}}{\partial b_{t+1}} > 0 \) and \( \frac{\partial w_{t+1}}{\partial b_{t+1}} \geq 0 \), \( \psi_{t+1} \) is the difference of two opposing effects and hence its sign is in principle ambiguous. The term \( \frac{\partial q_{t+1}}{\partial b_{t+1}} \) is positive, because an increase in net worth increases demand for land and land is in fixed supply.

---

\(^{10}\)This implies that the social planner does not internalize the direct effects of choosing the contemporaneous labor allocation on contemporaneous wages. We have also investigated the possibility of having the planner internalize these effects but results are very similar. This occurs again because our calibrated interest rate and working capital requirement are very small.
The term $\frac{\partial w_{t+1}}{\partial b_{t+1}}$ is positive, because the effective cost of hiring labor increases when the collateral constraint binds, reducing labor demand and pushing wages down. We found, however, that the value of $\psi_{t+1}$ is positive in all our quantitative experiments with baseline parameter values and variations around them, and this is because $\frac{\partial q_{t+1}}{\partial b_{t+1}}$ is large and positive when the credit constraint binds due the effects of the Fisherian debt-deflation mechanism.

**Definition 4 (Recursive Constrained-Efficient Equilibrium)**

The recursive constrained-efficient equilibrium is given by a set of decision rules $\{\hat{B}'(B, \varepsilon), \hat{c}(B, \varepsilon), \hat{n}(B, \varepsilon)\}$ with associated value function $W(B, \varepsilon)$, and wages $w(B, \varepsilon)$ such that:

1. $\{\hat{B}'(B, \varepsilon), \hat{c}(B, \varepsilon), \hat{n}(B, \varepsilon)\}$ and $W(B, \varepsilon)$ solve the planner’s recursive optimization problem, taking as given $w(B, \varepsilon)$ and the competitive equilibrium’s asset pricing function $q(B, \varepsilon)$.

2. Wages satisfy $w(B, \varepsilon) = G'(\hat{n}(B, \varepsilon))$.

### 2.3.3 Comparison of Equilibria & ‘Macro-prudential’ Policy

Using a simple variational argument, we can show that the allocations of the competitive equilibrium are inefficient, in the sense that they violate the conditions that support the constrained-efficient equilibrium. In particular, private agents undervalue net worth in periods during which the collateral constraint binds. To see this, consider first the marginal utility of an increase in individual bond holdings. By the envelope theorem, in the competitive equilibrium this can be written as
For the constrained-efficient economy, however, the marginal benefit of an increase in bond holdings takes into account the fact that prices are affected by the increase in bond holdings, and is therefore given by $\frac{\partial V}{\partial b} = u'(t) + \psi_t \mu_t$. If the collateral constraint does not bind, $\mu_t = 0$ and the two expressions coincide. If the collateral constraint binds, the social benefits of a higher level of bonds include the extra term given by $\psi_t \mu_t$, because one more unit of aggregate bonds increases the inter-period ability to borrow by $\psi_t$ which has a marginal value of $\mu_t$.

The above argument explains why bond holdings are valued differently by the planner and the private agents “ex post,” when the collateral constraint binds. Since both the planner and the agents are forward looking, however, it follows that those differences in valuation lead to differences in the private and social benefits of debt accumulation “ex ante,” when the constraint is not binding. Consider the marginal cost of increasing the level of debt at date $t$ evaluated at the competitive equilibrium in a state in which the constraint is not binding. This cost is given by the discounted expected marginal utility from the implied reduction in consumption next period $\beta RE[u'(t + 1)]$. In contrast, the social planner internalizes the effect by which the larger debt reduces tomorrow’s borrowing ability by $\psi_{t+1}$, and hence the marginal cost of borrowing at period $t$ that is not internalized by private agents is given by $\beta RE_t \left[ \mu_{t+1} \left( \kappa K \frac{\partial \eta_{t+1}}{\partial n_{t+1}} - \theta n_{t+1} \frac{\partial \omega_{t+1}}{\partial b_{t+1}} \right) \right]$.

We now show that the planner can implement the constrained-efficient allocations as a competitive equilibrium in the decentralized economy by introducing a macro-prudential policy that taxes debt and dividends (the latter can turn into a
subsidy too, as we show in the next Section, but we refer to it generically as a tax).\textsuperscript{11} In particular, the planner can do this by constructing state-contingent schedules of taxes on bond purchases ($\tau_i$) and on land dividends ($\delta_i$), with the total cost (revenues) financed (rebated) as lump-sum taxes (transfers). The tax on bonds ensures that the planner’s optimal plans for consumption and bond holdings are consistent with the Euler equation for bonds in the competitive equilibrium. This requires setting the tax to $\tau_t = E_t \mu_{t+1} \psi_{t+1} / E_t u'(t + 1)$. The tax on land dividends ensures that these optimal plans and the pricing function $q(B, \varepsilon)$ are consistent with the private agents’ Euler equation for land holdings.

The Euler equations of the competitive equilibrium with the macro-prudential policy in place become:

\begin{align*}
    u'(t) &= \beta R(1 + \tau_t) E_t \left[ u'(t + 1) \right] + \mu_t \quad (2.17) \\
    q_t(u'(t) - \mu_t \kappa) &= \beta E_t \left[ u'(t + 1) \right] \left( \varepsilon_{t+1} F_k(k_{t+1}, n_{t+1})(1 + \delta_{t+1}) + q_{t+1} \right) \quad (2.18)
\end{align*}

By combining these two Euler equations we can derive the expected excess return on land paid in the land market under the macro-prudential policy. In this case, after-tax returns on land and bonds are defined as $\tilde{R}_{t+1}^q \equiv \frac{d_{t+1}(1+\delta_{t+1})+q_{t+1}}{q_t}$ and $\tilde{R}_{t+1} \equiv R(1 + \tau_t)$ respectively, and the after-tax expected equity premium reduces to an expression analogous to that of the decentralized equilibrium:

\begin{equation}
    \tilde{R}_t^{ep} = \frac{\mu_t(1 - \kappa)}{E_t [u'(t) - \mu_t \kappa] m_{t+1}} - \frac{\text{Cov}_t(m_{t+1}, \tilde{R}_{t+1}^q)}{E_t [m_{t+1}]} \quad (2.19)
\end{equation}

\textsuperscript{11}See Bianchi (2010) for other decentralizations using capital and liquidity requirements and loan-to-value ratios.
This excess return also has a corresponding interpretation in terms of the Sharpe ratio, the price of risk, and the correlation between land returns and the pricing kernel as in the case of the competitive equilibrium without macro-prudential policy.

It follows from comparing the expressions for $R_{t}^{cp}$ and $\tilde{R}_{t}^{cp}$ that differences in the after-tax expected equity premia of the competitive equilibria with and without macro-prudential policy are determined by differences in the direct and indirect effects of the credit constraint in the two environments. As shown in the next Section, these effects are stronger in the decentralized equilibrium without policy intervention, in which the inefficiencies of the credit externality are not addressed. Intuitively, higher leverage and debt in this environment imply that the constraint binds more often, which strengthens the direct effect. In addition, lower net worth implies that the stochastic discount factor covaries more strongly with the excess return on land, which strengthens the indirect effect. Notice also that dividends in the constrained-efficient allocations are discounted at a rate which depends positively on the tax on debt. This premium is required by the social planner so that the excess returns reflect the social costs of borrowing.

2.4 Quantitative Analysis

2.4.1 Calibration

We calibrate the model to annual frequency using data from the U.S. economy. The functional forms for preferences and technology are the following:
The real interest rate is set to $R - 1 = 0.028$ per year, which is the ex-post average real interest rate on U.S. three-month T-bills during the period 1980-2005. We set $\sigma = 2$, which is a standard value in quantitative DSGE models. The parameter $\varkappa$ is inessential and is set so that mean hours are equal to 1, which requires $\varkappa = 0.64$. Aggregate land is normalized to $\bar{K} = 1$ without loss of generality and the share of labor in output $\alpha_h$ is equal to 0.64, the standard value. The Frisch elasticity of labor supply ($1/\omega$) is set equal to 1, in line with evidence by Kimball and Shapiro (2008).

We follow Schmitt-Grohe and Uribe (2007) in taking M1 money balances in possession of firms as a proxy for working capital. Based on the observations that about two-thirds of M1 are held by firms (Mulligan, 1997) and that M1 was on average about 14 percent of annual GDP over the period 1980 to 2009, we calibrate the working capital-GDP ratio to be $(2/3)0.14 = 0.093$. Given the 64 percent labor share in production, and assuming the collateral constraint does not bind, we obtain $\theta = 0.093/0.64 = 0.146$.

The value of $\beta$ is set to 0.96, which is also a standard value but in addition it supports an average household debt-income ratio in a range that is in line with U.S. data from the Federal Reserve’s *Flow of Funds* database. Before the mid-1990s this
ratio was stable at about 30 percent. Since then and until just before the 2008 crisis, it rose steadily to a peak of almost 70 percent. By comparison, the average debt-income ratio in the stochastic steady-state of our baseline calibration is 38 percent. A mean debt ratio of 38 percent is sensible because 70 percent was an extreme at the peak of a credit boom and 30 percent is an average from a period before the substantial financial innovation of recent years.

Table 2.1: Calibration

<table>
<thead>
<tr>
<th>Source / target</th>
<th>Interest rate $R - 1 = 0.028$</th>
<th>Risk aversion $\sigma = 2$</th>
<th>Share of labor $\alpha_n = 0.64$</th>
<th>Labor disutility coefficient $\chi = 0.64$</th>
<th>Frisch elasticity parameter $\omega = 1$</th>
<th>Supply of land $\bar{K} = 1$</th>
<th>Working capital coefficient $\theta = 0.14$</th>
<th>Discount factor $\beta = 0.96$</th>
<th>Collateral coefficient $\kappa = 0.36$</th>
<th>Share of land $\alpha_K = 0.05$</th>
<th>TFP process $\sigma_e = 0.014, \rho_e = 0.53$</th>
</tr>
</thead>
</table>

The values of $\kappa, \alpha_K$ and the TFP process are calibrated to match targets from U.S. data by simulating the model. We set $\alpha_K$ so as to match the average ratio of housing to GDP at current prices, which is equal to 1.35. The value of housing is taken from the *Flow of Funds*, and is measured as real state tangible assets owned by households (reported in Table B.100, row 4). The model matches the 1.35 ratio when we set $\alpha_K = 0.05$.\(^\text{12}\)

\(^{12}\alpha_K\) represents the share of fixed assets in GDP, and not the standard share of capital income in GDP. There is little empirical evidence about the value of this parameter, with estimates that vary depending, for example, on whether we consider land used for residential or commercial purposes, or owned by government at different levels. We could also calibrate $\alpha_K$ using the fact that, in
TFP shocks follow a log-normal AR(1) process $\log(\varepsilon_t) = \rho \log(\varepsilon_{t-1}) + \eta_t$. We construct a discrete approximation to this process using the quadrature procedure of Tauchen and Hussey (1991) using 15 nodes. The values of $\sigma_\varepsilon$ and $\rho$ are set so that the standard deviation and first-order autocorrelation of the output series produced by the model match the corresponding moments for the cyclical component of U.S. GDP in the sample period 1947-2007 (which are 2.1 percent and 0.5 respectively). This procedure yields $\sigma_\varepsilon = 0.014$ and $\rho = 0.53$.

Finally, we set the value of $\kappa$ so as to match the frequency of financial crises in U.S. data. We define a financial crisis as an event in which both the credit constraint binds and there is a decrease in credit of more than one standard deviation. Then, we set $\kappa$ so that financial crises in the baseline model simulation occur about 3 percent of the time, which is consistent with the fact that the U.S. has experienced three major financial crises in the last hundred years. This yields the value of $\kappa = 0.36$.

We recognize that several of the parameter values are subject of debate (e.g. there is a fair amount of disagreement about the Frisch elasticity of labor supply), or relate to variables that do not have a clear analog in the data (as is the case with $\kappa$ or $\theta$). Hence, we will perform sensitivity analysis to examine the robustness of our results to changes in the model’s key parameters.

---

a deterministic steady state where the collateral constraint does not bind, the value-of-land-GDP ratio is equal to $\alpha_K/(R-1)$, which would imply $\alpha_K = 1.35(0.028) = 0.038$. This yields very similar results as $\alpha_K = 0.05$.

13 The three crises correspond to the Great Depression, the Savings and Loans Crisis and the Great Recession (see Reinhart and Rogoff (2008)). While a century may be a short sample for estimating accurately the probability of a rare event in one country, Mendoza (2010) estimates a probability of about 3.6 percent for financial crises using a similar definition but applied to all emerging economies using data since 1980.
2.4.2 Borrowing decisions

We start the quantitative analysis by exploring the effects of the credit externality on optimal borrowing plans. The solution method is described in the Appendix. Since mean output is normalized to 1, all quantities can be interpreted as fractions of mean output.

The two panels of Figure 1 show the bond decision rules \( b' \) of private agents and the social planner as a function of \( b \) (left panel) as well as the pricing function for land (right panel), both for a negative two-standard-deviations TFP shock. The key point is to note that the Fisherian deflation mechanism generates non-monotonic bond decision rules, instead of the typical monotonically increasing decision rules. The point at which bond decision rules switch slope corresponds to the value of \( b \) at which the collateral constraint holds with equality but does not bind. To the right of this point, the collateral constraint does not bind and the bond decision rules are upward-sloping. To the left of this point, the bond decision rules are decreasing in \( b \), because a reduction in current bond holdings results in a sharp reduction in the price of land, as can be seen in the right panel, and tightens the borrowing constraint, thus increasing \( b' \).

As in Bianchi (2010), we can separate the bond decision rules in the left panel of Figure 1 into three regions: a “constrained region,” a “high-externality region” and a “low-externality region.” The “constrained region” is given by the range of \( b \) in the horizontal axis with sufficiently high initial debt (i.e. low \( b \)) such that the collateral constraint binds in the constrained-efficient equilibrium. This is the range
with $b \leq -0.385$. In this region, the collateral constraint binds in both constrained-efficient and competitive equilibria, because the credit externality implies that the constraint starts binding at higher values of $b$ in the latter than in the former, as we show below.

By construction, the total amount of debt (i.e. the sum of bond holdings and working capital) in the constrained region is the same under the constrained-efficient allocations and the competitive equilibrium. If working capital were not subject to the collateral constraint, the two bond decision rules would also be identical. But with working capital in the constraint the two can differ. This is because the effective cost of labor differs between the two equilibria, since the increase in the marginal financing cost of labor when the constraint binds, $\theta \mu_t/u'(t)$, is different. These differences, however, are very small in the numerical experiments, and thus the bond decision rules are approximately the same in the constrained region.\textsuperscript{14}

The high-externality region is located to the right of the constrained region, and it includes the interval $-0.385 < b < -0.363$. Here, the social planner chooses uniformly higher bond positions (lower debt) than private agents, because of the different incentives driving the decisions of the two when the constrained region is near. In fact, private agents hit the credit constraint at $b = -0.383$, while at that initial $b$ the social planner still retains some borrowing capacity. Moreover, this region is characterized by “financial instability,” in the sense that the levels of

\textsuperscript{14}The choice of $b'$ becomes slightly higher for the social planner as $b$ gets closer to the upper bound of the constrained region, because the deleveraging that occurs around this point is small enough for the probability of a binding credit constraint next period to be strictly positive. As a result, for given allocations, conditions (2.15) and (2.6) imply that $\mu$ is lower in the constrained-efficient allocations.
debt chosen for $t + 1$ are high enough so that a negative TFP shock of standard magnitude in that period can lead to a binding credit constraint that leads to large falls in consumption, output, land prices and credit. We will show later that this is also the region of the state space in which the planner uses actively its macro-prudential policy to manage the inefficiencies of the competitive equilibrium.

The low-externality region is the interval for which $b \geq -0.363$. In this region, the probability of a binding constraint next period is zero for both the social planner and the competitive equilibrium. The bond decision rules still differ, however, be-
cause expected marginal utilities differ for the two equilibria. But the social planner
does not set a tax on debt, because negative shocks cannot lead to a binding credit
constraint in the following period.

The long-run probabilities with which the constrained-efficient (competitive)
economy visits the three regions of the bond decision rules are 2 (4) percent for
the constrained region, 69 (70) percent for the high-externality region, and 29 (27)
percent for the low-externality region. Both economies spend more than 2/3rds
of the time in the high-externality region, but the prudential actions of the social
planner reduce the probability of entering in the constrained region by a half. Later
we will show that this is reflected also in financial crises that are much less frequent
and less severe than in the competitive equilibrium.

The larger debt (i.e. lower bond) choices of private agents relative to the social
planner, particularly in the high-externality region, constitute our first measure of
the overborrowing effect at work in the competitive equilibrium. The social planner
accumulates extra precautionary savings above and beyond what private individuals
consider optimal in order to self-insure against the risk of financial crises. This
effect is quantitatively small in terms of the difference between the two decision
rules, but this does not mean that its macroeconomic effects are negligible. Later in
this Section we illustrate this point by comparing financial crises events in the two
economies. In addition, the fact that small differences in borrowing decisions lead
to major differences when a crisis hits can be illustrated using Figure 2 to study
further the dynamics implicit in the bond decision rules.

Figure 2.2 shows bond decision rules for the social planner and the competitive
Figure 2.2: Comparison of Debt Dynamics

equilibrium over the range (-0.39,-0.36) for two TFP scenarios: average TFP and TFP two-standard-deviations below the mean. The ray from the origin is the $b' = b$ line. We use a narrower range than in Figure 1 to “zoom in” and highlight the differences in decision rules. Assume both economies start at a value of $b$ such that at average TFP the debt of agents in the competitive equilibrium would remain unchanged (this is point $A$ with $b = -0.389$). If the TFP realization is indeed the average, private agents in the decentralized equilibrium keep that level of debt. On
the other hand, the social planner builds precautionary savings and reduces its debt to point $B$ with $b = -0.386$. Hence, the next period the two economies start at the debt levels in $A$ and $B$ respectively. Assume now that at this time TFP falls by two standard deviations. Now we can see the large dynamic implications of the small differences in the bond decision rules of the two economies: The competitive equilibrium suffers a major correction caused by the Fisherian deflation mechanism. The collateral constraint becomes binding and the economy is forced to a large deleveraging that results in a sharp reduction in debt (an increase in $b$ to -0.347 at point $A'$). Consumption falls leading to a drop in the the stochastic discount factor and a drop in asset prices. In contrast, the social planner, while also facing a binding credit constraint, adjusts it debt marginally to just about $b = -0.379$ at point $B'$. This was possible for the social planner because, taking into account the risk of a Fisherian deflation and internalizing its price dynamics, the planner chose to borrow less than agents in the decentralized equilibrium a period earlier.

Overborrowing can also be assessed by comparing the long-run distributions of debt and leverage across the competitive and constrained-efficient equilibria. The fact that the planner accumulates more precautionary savings implies that its ergodic distribution concentrates less probability at higher leverage ratios than in the competitive equilibrium. Figure 2.3 shows the ergodic distributions of leverage ratios (measured as $\frac{-b_{t+1} + \theta w_t n_t}{q_t K}$) in the two economies. The maximum leverage ratio in both economies is given by $\kappa$ but notice that the decentralized equilibrium concentrates higher probabilities in higher levels of leverage. Comparing averages across these ergodic distributions, however, mean leverage ratios differ by less than
1 percent. Hence, overborrowing is relatively small again if measured by comparing differences in unconditional long-run averages of leverage ratios.\footnote{Measuring “ex ante” leverage as $\frac{-b_{t+1} + \theta w_t h_t}{q_t K}$, we find that leverage ratios in the competitive equilibrium can exceed the maximum of those for the planner 3 percent of the time and by up to 12 percentage points.}

2.4.3 Asset Returns

Overborrowing has important quantitative implications for asset returns and their determinants. Figure 2.4 shows the long-run distributions of land returns for the competitive equilibrium and the social planner. The key difference in these distributions is that the one for the competitive equilibrium features fatter tails. In particular, there is a sharply fatter left tail in the competitive equilibrium, for
which the 99th percentile of returns is about -17.5 percent, v. -1.6 percent in the constrained-efficient equilibrium. The fatter left tail in the competitive equilibrium corresponds to states in which a negative TFP shock hits when agents have a relatively high level of debt. Intuitively, as a negative TFP shock hits, expected dividends decrease and this puts downward pressure on asset returns. In addition, if the collateral constraint becomes binding, asset fire-sales lead to a further drop in asset prices.

We show below that the fatter tails of the distribution of asset returns, and the associated time-varying risk of financial crises, have substantial effects on the risk premium. These features of our model are similar to those examined in the literature on asset pricing and “disasters” (see Barro, 2009). Note, however, that this literature generally treats financial disasters as resulting from exogenous stochastic processes with fat tails and time-varying volatility, whereas in our setup financial crises and their time-varying risk are both endogenous. The underlying shocks driving the model are standard TFP shocks, even in periods of financial crises. In our model, as in Mendoza (2010), financial crises are endogenous outcomes that occur when shocks of standard magnitudes trigger a Fisherian deflation. Table 2.2 reports statistics that characterize the main properties of asset returns in the constrained-efficient and competitive equilibria. We also report statistics for a competitive equilibrium

16Similarly, the fatter right-tail in the distribution of returns of the competitive equilibrium corresponds to periods with positive TFP shocks, which were preceded by low asset prices due to fire sales.

17The literature on disasters typically uses Epstein-Zin preferences so as to be able to match the large observed equity premia. Here we use standard CRRA preferences with a risk aversion coefficient of 2, and as we show later, we can obtain larger risk premia than in the typical CRRA setup without credit frictions. Moreover, we obtain realistically large risk premia when the credit constraint binds.
Figure 2.4: Ergodic Cumulative Distribution of Land Returns

in which land in the collateral constraint is valued at a fixed price equal to the average price across the ergodic distribution $\bar{q}$ (i.e. the credit constraint becomes $-\frac{b_{t+1}}{\bar{K}_t} + \theta w_t n_t \leq \kappa \bar{q} k_{t+1}$). This fixed-price scenario allows us to compare the properties of asset returns in the competitive and social planner equilibria with a setup in which a collateral constraint exists but the Fisherian deflation channel and the credit externality are removed.

Table 2.2 lists expected excess returns (i.e. the equity risk premia), the direct and indirect (covariance) effects of the credit constraint on excess returns, the (log) standard deviation of returns, the price of risk, and the Sharpe ratio. These moments are reported for the unconditional long-run distributions of each model economy, as

---

18 Because the asset is in fixed supply, these allocations would be the same if we use instead an ad-hoc borrowing limit such that $-\frac{b_{t+1}}{\bar{K}_t} + \theta w_t n_t \leq \kappa \bar{q} K$. The price of land, however, would be lower since with the ad-hoc borrowing constraint land does not have collateral value.
well as for distributions conditional on the collateral constraint being binding and not binding.

The mean unconditional excess return is 1.09 percent in the competitive equilibrium v. only 0.17 percent in the constrained-efficient equilibrium and 0.86 percent in the fixed-price economy. The risk premium in the competitive equilibrium is large, about half as large as the risk-free rate. The fact that the other two economies produce lower premia indicates that the high premium of the competitive equilibrium is the combined result of the Fisherian deflation mechanism and the inefficiencies induced by the credit externality. Note also that the high premium produced by our model contrasts sharply with the findings of Heaton and Lucas (1996), who found that credit frictions without the Fisherian deflation mechanism do not produce large premia, unless transactions costs are very large.\(^{19}\)

The excess returns conditional on the collateral constraint not binding in the constrained-efficient and fixed-price economies are in line with those obtained in classic asset pricing models that display the “equity premium puzzle.” The equity premia we obtained in these two scenarios are driven only by the covariance effect, as in the classic models, and they are negligible: 0.03 percent in the fixed-price economy and 0.06 percent in the constrained-efficient economy. This is natural because, without the constraint binding and with the effects of the credit externality and the Fisherian deflation removed or weakened, the model is in the same class as those that display the equity premium puzzle. In contrast, our baseline competitive

\(^{19}\)The unconditional premium in the fixed price economy, at 0.86 percent, is not trivial, but note that it results from the fact that the constraint binds with very high probability, given the smaller incentives to accumulate precautionary savings. The risk premium in the unconstrained region of the fixed-price model is only 0.03 percent, v. 0.23 in our baseline model.
economy yields a 0.23 percent premium conditional on the constraint not binding, which is small relative to data estimates that range from 6 to 18 percent, but 4 to 8 times larger than in the other two economies.

Conditional on the collateral constraint being binding, mean excess returns in the competitive equilibrium are nearly 14 percent, 4.86 percent for the social planner, and 1.29 percent in the fixed-price economy. Interestingly, the lowest unconditional premium is the one for the constrained-efficient economy (0.17 percent), but conditional on the constraint binding, the lowest premium is the one for the fixed-price economy (1.29 percent). This is because on one hand the Fisherian deflation effect is still at work when the collateral constraint binds in the constrained-efficient economy, but not in the fixed-price economy, while on the other hand the constrained-efficient economy has a lower probability of hitting the collateral constraint (so that the higher premium when the constraint binds does not weigh heavily when computing the unconditional average). In turn, the probability of hitting the collateral constraint is higher for the fixed-price economy, because the incentive to build precautionary savings is weaker when there is no Fisherian amplification.

The unconditional direct and covariance effects of the collateral constraint on excess returns are significantly stronger in the competitive equilibrium than in the constrained-efficient and fixed-price economies, and even more so if we compare them conditional on the constraint being binding. Again, the direct and covariance effects are larger in the competitive equilibrium because of the effects of the overborrowing externality and the Fisherian deflation mechanism.

In terms of the decomposition of excess returns based on condition (2.10),
Table 2.2: Asset Pricing Moments

<table>
<thead>
<tr>
<th></th>
<th>Excess Direct Covariance Effect</th>
<th>Covariance Effect</th>
<th>$s_t$</th>
<th>$\sigma_t(R^q_{t+1})$</th>
<th>$S_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Decentralized Equilibrium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional</td>
<td>1.09</td>
<td>0.87</td>
<td>0.22</td>
<td>5.22</td>
<td>3.05</td>
</tr>
<tr>
<td>Constrained</td>
<td>13.94</td>
<td>13.78</td>
<td>0.16</td>
<td>4.05</td>
<td>2.71</td>
</tr>
<tr>
<td>Unconstrained</td>
<td>0.23</td>
<td>0.00</td>
<td>0.23</td>
<td>5.3</td>
<td>3.08</td>
</tr>
<tr>
<td><strong>Constrained-Efficient Equilibrium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional</td>
<td>0.17</td>
<td>0.11</td>
<td>0.06</td>
<td>2.88</td>
<td>1.85</td>
</tr>
<tr>
<td>Constrained</td>
<td>4.86</td>
<td>4.80</td>
<td>0.06</td>
<td>3.02</td>
<td>2.07</td>
</tr>
<tr>
<td>Unconstrained</td>
<td>0.06</td>
<td>0.00</td>
<td>0.06</td>
<td>2.86</td>
<td>1.84</td>
</tr>
<tr>
<td><strong>Fixed Price Equilibrium</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional</td>
<td>0.86</td>
<td>0.82</td>
<td>0.04</td>
<td>2.59</td>
<td>1.69</td>
</tr>
<tr>
<td>Constrained</td>
<td>1.29</td>
<td>1.23</td>
<td>0.05</td>
<td>2.81</td>
<td>1.84</td>
</tr>
<tr>
<td>Unconstrained</td>
<td>0.03</td>
<td>0.00</td>
<td>0.03</td>
<td>2.16</td>
<td>1.39</td>
</tr>
</tbody>
</table>

Note: The table reports averages of the conditional excess return after taxes, the direct effect, the covariance effect, the price of risk $s_t$, the (log) volatility of the return of land denoted $\sigma_t(R^q_{t+1})$, and the Sharpe ratio. All numbers except the Sharpe ratios are in percentage.

Table 2.2 shows that the unconditional average of the price of risk is about twice as large in the decentralized equilibrium than in the constrained-efficient and fixed-price economies. This reflects the fact that consumption, and therefore the pricing kernel, fluctuate significantly more in the decentralized equilibrium. The Sharpe ratio and the variability of land returns are also much larger in the competitive equilibrium. The increase in the former indicates, however, that the mean excess return rises significantly more than the variability of returns, which indicates that risk-taking is “overcompensated” in the competitive equilibrium (relative to the compensation it receives when the social planner internalizes the credit externality). Note also that the correlations between land returns and the stochastic discount factor, not shown in the Table, are very similar under the three equilibria and very close to 1. This is
important because it implies that the differences in excess returns and Sharpe ratios cannot be attributed to differences in this correlation.

2.4.4 Incidence and Magnitude of Financial Crises

We show now that overborrowing in the competitive equilibrium increases the incidence and severity of financial crises. To demonstrate this result we construct an event analysis of financial crises with simulated data obtained by performing long (100,000-period) stochastic time-series simulations of the competitive, constrained-efficient and fixed-price economies, removing the first 1,000 periods. A financial crisis episode is defined as a period in which the credit constraint binds and this causes a decrease in credit that exceeds one standard deviation of the first-difference of credit in the corresponding ergodic distribution.

The event analysis exercise is important also because it sheds light on whether the model can produce financial crises with realistic features, which is a key first step in order to make the case for treating the normative implications of the model as relevant. We show here that, while we did not aim to build a rich equilibrium business cycle model so we could keep the analysis of the externality tractable, and hence our match to the data is not perfect, the model does produce financial crises with realistic features in terms of abrupt, large declines in allocations, credit, and land prices, and it supports non-crisis output fluctuations in line with observed U.S. business cycles. Moreover, studies more focused on matching data from financial crisis events have shown that the Fisherian deflation mechanism can do well at explaining
crisis dynamics nested within realistic long-run business cycle co-movements (see Mendoza (2010)).

The first important result of the event analysis is that the incidence of financial crises is significantly higher in the competitive equilibrium. We calibrated $\kappa$ so that the competitive economy experiences financial crises with a long-run probability of 3.0 percent. But with the same $\kappa$, financial crises occur in the constrained-efficient economy only with 0.9 percent probability in the long run. Thus, the credit externality increases the frequency of financial crises by a factor of 3.33.\(^{20}\)

The second important result is that financial crises are more severe in the competitive equilibrium. This is illustrated in the event analysis plots shown in Figure 2.5. The event windows are for total credit, consumption, labor, output, TFP and land prices, all expressed as deviations from long-run averages. These event dynamics are shown for the decentralized, constrained-efficient, and fixed-price economies.

We construct comparable event windows for the three scenarios following this procedure: First we identify financial crisis events in the competitive equilibrium, and isolate five-year event windows centered in the period in which the crisis takes place. That is, each event window includes five years, the two years before the crisis, the year of the crisis, and the two years after. Second, we calculate the median TFP shock across all of these event windows in each year $t - 2$ to $t + 2$, and the median initial debt at $t - 2$. This determines an initial value for bonds and a

\(^{20}\) We could also define crises in the constrained-efficient equilibrium by using the value of the credit threshold obtained in the competitive equilibrium. However, with this criterion we would obtain an even lower probability of crises, because credit declines equal to at least one standard deviation of the first-difference of credit in the decentralized equilibrium are zero-probability events in the constrained efficient equilibrium.
five-year sequence of TFP realizations. Third, we feed this sequence of shocks and initial value of bonds to the decisions rules of each model economy and compute the corresponding endogenous variables plotted in Figure 2.5. By proceeding in this way, we ensure that the event dynamics for the three equilibria are simulated using the same initial state and the same sequence of shocks.\textsuperscript{21}

The features of financial crises at date $t$ in the competitive economy are in line with the results in Mendoza (2010): The debt-deflation mechanism produces financial crises characterized by sharp declines in credit, consumption, asset prices and output.

The five macro variables illustrated in the event windows show similar dynamics across the three economies in the two years before the financial crisis. When the crisis hits, however, the collapses observed in the competitive equilibrium are much larger. Credit falls about 20 percentage points more, and two years after the crisis the credit stock of the competitive equilibrium remains 10 percentage points below that of the social planner.\textsuperscript{22} Consumption, asset prices, and output also fall much more sharply in the competitive equilibrium than in the planner’s equilibrium. The declines in consumption and asset prices are particularly larger (-16 percent v. -5 percent for consumption and -24 percent v. -7 percent for land prices). The asset price collapse also plays an important role in explaining the more pronounced de-

\textsuperscript{21}The sequence of TFP shocks is 0.9960, 0.9881, 0.9724, 0.9841, 0.9920 and the initial level of debt is 1.6 percent above the average.

\textsuperscript{22}The model overestimates the drop in credit relative to what we have observed so far in the U.S. crisis (which as of the third quarter of 2010 reached about 7 percent of GDP) . One reason for this is that in the model, credit is in the form of one-period bonds, whereas in the data, loans have on average a much larger maturity. In addition, our model does not take into account the strong policy intervention that took place with the aim to prevent what would have been a larger credit crunch.
cline in credit in the competitive equilibrium, because it reflects the outcome of the Fisherian deflation mechanism. Output falls by 2 percentage points more, and labor falls almost 3 percentage points more, because of the higher shadow cost of hiring labor due to the effect of the tighter binding credit constraint on access to working capital.

Figure 2.5: Event Analysis: percentage differences relative to unconditional averages
The event analysis results can also be used to illustrate the relative significance of the wage and land price components in the externality term \( \psi_t \equiv \kappa \tilde{K} \frac{\partial q_t}{\partial b_t} - \theta n_t \frac{\partial w_t}{\partial b_t} \) identified in condition (2.15). Given the unitary Frisch elasticity of labor supply, wages decrease one-to-one with labor (and hence the event plot for wages would be identical to the one shown for employment in Figure 2.5). As a result, the extent to which the drop in wages can help relax the collateral constraint is very limited. Wages and employment fall about 6 percent at date \( t \), and with a working capital coefficient of \( \theta = 0.14 \), this means that the effect of the drop in wages in the borrowing capacity is \( 0.14(1 - 0.06)0.06 = 0.79 \) percent. On the other hand, given that \( \tilde{K} = 1 \) and that asset prices fall about 25 percent below trend at date \( t \), and since \( \kappa = 0.36 \), the effect of land fire sales on the collateral constraint is \( 0.36(0.25) = 9 \) percent. Thus, the land price effect of the externality is about 10 times bigger than the wage effect. This finding will play an important role in our quantitative analysis of the features of the macro-prudential policy later in this section.

The fixed-price economy displays very little amplification given that the economy is free from the Fisherian deflation mechanism. Credit increases slightly at date \( t \) in order to smooth consumption and remains steady in the following periods. The fact that land is valued at the average price, and not the market price, contributes to mitigate the drop in the price of land, since it remains relatively more attractive as a source of collateral.

To gain more intuition on why land prices drop more because of the credit externality, we plot in Figure 2.6 the projected conditional sequences of future divi-
dends and land returns on land up to 30 periods ahead of a financial crisis that occurs at date \( t = 0 \) (conditional on information available on that date). These are the sequences of dividends and returns used to compute the present values of dividends that determine the equilibrium land price at \( t \) in the event analysis of Figure 2.5. The expected land returns start very high when the crisis hits in both competitive and constrained-efficient equilibria, but significantly more for the former (at about 40 percent) than the latter (at 10 percent). On the other hand, expected dividends do not differ significantly, and therefore we conclude that the sharp change in the pricing kernel reflected in the surge in projected land returns when the crisis hits is what drives the large differences in the drop of asset prices.

![Figure 2.6: Forecast of expected dividends and land returns.](image)

The large deleveraging that takes places when a financial crisis occurs in the competitive equilibrium implies that projected land returns for the immediate future (i.e. the first 6 periods after the crisis) drop significantly. Returns are also projected to fall for the social planner, but at a lower pace, so that in fact the planner projects
higher land returns than agents in the competitive equilibrium for a few periods. Projected dividends for the same immediate future after the crisis are slightly smaller than the long-run average of 0.05 in both economies because of the persistence of the TFP shock. In the long-run, expected dividends are slightly higher for the social planner, because the marginal productivity of land drops less during the financial crisis as a result of the lower amount of debt. Notice also that the planner projects to discount dividends with a slightly higher land return in the long run, because the tax on debt more than offsets the fact that the risk premium of the planner is lower (recall that we are comparing after-tax returns as defined in Section 2). This arises because the tax on debt makes bonds relatively more attractive and this leads in equilibrium to a higher required return on land.

2.4.5 Long-Run Business Cycles

Table 2.3 reports the long-run business cycle moments of the competitive, constrained-efficient and fixed-price equilibria, which are computed using each economy’s ergodic distribution. The credit externality at work in the competitive equilibrium produces higher business cycle variability in output and labor, and especially in consumption, compared with the constrained-efficient and fixed-price economies. The high variability of consumption and credit are consistent with the results in Bianchi (2010), but we find in addition that the credit externality produces a moderate increase in the variability of labor and a substantial increase in the variability of land prices and leverage. Notice that the variability in consumption is higher
than the variability of output in the decentralized equilibrium which is not the case in U.S. data. However, if we exclude the crisis periods, the ratio of the variability of consumption to the variability of GDP would be 0.87 (compared with 0.88 in annual U.S. data from 1960 to 2007).

It may seem puzzling that we can obtain non-trivial differences in long-run business cycle moments even though financial crises are a low probability event in the competitive equilibrium. To explain this result, it is useful to go back to Figure 1. This plot shows that even during normal business cycles the optimal plans of the competitive and constrained-efficient equilibria differ, and this is particularly the case in the high-externality region. Because the economy spends about 70 percent of the time in this region, where private agents borrow more and are more exposed to the risk of financial crises, long-run business cycle moments differ. In addition, the larger effects that occur during crises have a non-trivial effect on long-run moments. This is particularly noticeable in the case of consumption where the variability drops from 2.7 to 1.7 in the decentralized equilibrium when we exclude the crises episodes.

The business cycle moments for consumption, output and labor in the constrained-efficient economy are about the same as those of the fixed-price economy. This occurs even though the constrained-efficient economy is subject to the Fisherian deflation mechanism and the fixed-price economy is not. The reason for this is because the social planner accumulates extra precautionary savings, which compensate for the sudden change in the borrowing ability when the credit constraint binds. The constraint binds less often and when it does it has weak effects on macro variables. On the other hand, the constrained-efficient economy does display lower variability in
leverage and land prices that the fixed-price economy, and this occurs because the social planner internalizes how a drop in the price tightens the collateral constraint.

The output correlations of leverage, credit, and land prices also differ significantly across the model economies. The GDP correlations of leverage and credit are significantly higher in the competitive equilibrium, while the correlation between the price of land and GDP is lower. The model without credit frictions would have a natural tendency to produce countercyclical credit because consumption-smoothing agents want to save in good times and borrow in bad times. This effect still dominates in the constrained-efficient and fixed-price economies, but in the competitive equilibrium the collateral constraint and the Fisherian deflation hamper consumption smoothing enough to produce procyclical credit and a higher GDP-leverage correlation. Similarly, the GDP-land price correlation is nearly perfect when the Fisherian deflation mechanism is weakened (constrained-efficient case) or removed (fixed-price case), but falls to about 0.8 in the competitive equilibrium. Because of the strong procyclicality of land prices, leverage is countercyclical with a GDP correlation of -0.57. This is in line with the countercyclicality of household leverage in U.S. data, although the correlation is lower than in the data (the correlation between the ratio of net household debt to the value of residential land and GDP is -0.25 at the business cycle frequency).

In terms of the first-order autocorrelations, the competitive equilibrium dis-

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Two caveats on this point. First, at lower frequencies the correlation is positive. As Boz and Mendoza (2010) report, the household leverage ratio rose together with GDP, land prices and debt between 1997 and 2007. Second, the countercyclical of leverage for the household sector differs sharply from the strong procyclicality of leverage in the financial sector (see Adrian and Shin (2010)).
plays lower autocorrelations in all its variables compared to both constrained-efficient and fixed-price equilibria. This occurs because crises in the competitive equilibrium are characterized by deep but not very prolonged recessions.

Table 2.3: Long Run Moments

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Correlation with GDP</th>
<th>Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DE</td>
<td>SP</td>
<td>FP</td>
</tr>
<tr>
<td>Output</td>
<td>2.10</td>
<td>1.98</td>
<td>1.97</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.71</td>
<td>1.87</td>
<td>1.85</td>
</tr>
<tr>
<td>Employment</td>
<td>1.25</td>
<td>1.02</td>
<td>0.98</td>
</tr>
<tr>
<td>Leverage</td>
<td>3.92</td>
<td>2.72</td>
<td>3.80</td>
</tr>
<tr>
<td>Total Credit</td>
<td>3.55</td>
<td>0.95</td>
<td>0.76</td>
</tr>
<tr>
<td>Land Price</td>
<td>3.95</td>
<td>2.24</td>
<td>3.48</td>
</tr>
<tr>
<td>Working capital</td>
<td>2.48</td>
<td>2.04</td>
<td>1.97</td>
</tr>
</tbody>
</table>

Note: ‘DE’ represents the decentralized equilibrium, ‘SP’ represents the social planner, ‘FP’ represents an economy with land valued at a fixed price equal to the average of the price of land in the competitive equilibrium.

2.4.6 Properties of Macro-prudential Policies

Table 4 shows the statistical moments that characterize the state-contingent schedules of taxes on debt and dividends by which the social planner decentralizes the constrained-efficient allocations as a competitive equilibrium. To make the two comparable, we express the dividend tax as a percent of the price of land.

The unconditional average of the debt tax is 1.07 percent, v. 0.09 when the constraint binds and 1.09 when it does not. The tax remains positive, albeit small, on average when the collateral constraint binds, because in some these states the social planner wants to allocate borrowing ability across bonds and working capital in a way that differs from the competitive equilibrium. If there is a positive
probability that the credit constraint will bind again next period, the social planner allocates less debt capacity to bonds and more to working capital. As a result, a tax on debt remains necessary in a subset of the constrained region. Note, however, that these states are not associated with financial crisis events in our simulations. They correspond to events in which the collateral constraint binds but the deleveraging that occurs is not strong enough for a crisis to occur.

The debt tax fluctuates about 2/3rds as much as GDP and is positively correlated with leverage, i.e. $-\frac{b_{t+1} + \theta \omega \epsilon_{t+1}}{q_t K}$. This is consistent with the macro-prudential rational behind the tax: The tax is high when leverage is building up and low when the economy is deleveraging. Note, however, that since leverage itself is negatively correlated with GDP, the tax also has a negative GDP correlation. When the constraint binds, the correlation between the tax and leverage is zero by construction, because leverage remains constant at the value of $\kappa$.

<table>
<thead>
<tr>
<th>Table 2.4: Long Run Moments of Macro-prudential Policies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average</strong></td>
</tr>
<tr>
<td>Debt Tax</td>
</tr>
<tr>
<td>Unconditional</td>
</tr>
<tr>
<td>Constrained</td>
</tr>
<tr>
<td>Unconstrained</td>
</tr>
</tbody>
</table>

The unconditional average of the dividend tax is negative (i.e. it is a subsidy), and it is very small at about -0.46 percent: when the constraint binds it is on average about 0.52 percent (v. a -0.49 percent average when the constraint does not bind).
The fact that on average the planner requires a subsidy on dividends may seem puzzling, given that land is less risky in the regulated decentralized equilibrium, as we have shown above. There is another effect at work, however, because the debt tax puts downward pressure on land prices by making bonds relatively more attractive than land, and this effect turns out to be quantitatively larger. Thus, since by definition the constrained-efficient allocations are required to support the same land pricing function of the competitive economy without policy intervention, the planner calls for a dividend subsidy on average in order to offset the effect of the debt tax on land prices. The variability of the tax on dividends is 0.62 percentage points, less than 1/3rds the variability of GDP. The correlation between this tax on dividends and leverage is negative in the unconstrained region reflecting the negative correlation between the tax on debt and the tax on land explained above.

The dynamics of the debt and dividend taxes around crisis events are shown in Figure 2.7. The debt tax is high relative to its average, at about 2.7 percent, at $t - 2$ and $t - 1$, and this again reflects the macro-prudential nature of these taxes: Their goal is to reduce borrowing so as to mitigate the magnitude of the financial crisis if bad shocks occur. At date $t$ the debt tax falls to zero, and it rises again at $t + 1$ and $t + 2$ to about 2 percent. The latter occurs because this close to the crisis the economy still remains financially fragile (i.e. there is still a non-zero probability of agents becoming credit constrained next period). The tax on dividends follows a similar pattern. Dividends are subsidized at a similar rate before and after financial crises events, but they are actually taxed when crises occur. The reason is again that the social planner needs to support the same pricing function of the competitive
equilibrium that would arise without policy intervention. Hence, with the tax on
debt falling to almost zero, there is pressure for land prices to be higher than what
that pricing function calls for, and hence dividends need to be taxed to offset this
effect.

The macro-prudential behavior of the debt tax is very intuitive and follows
easily from the precautionary behavior of the planner we have described. On the
other hand, the tax on dividends and its dynamic behavior seem less intuitive and
harder to sell as a policy rule (i.e. the notion of proposing to tax dividends at the
through of a financial crisis is bound to be unpopular). The two policy instruments
are required, however, in order to implement exactly the allocations of the con-
strained social planner as a decentralized competitive equilibrium. Moreover, the
planner's allocations are guaranteed to attain a level of welfare at least as high as
that of the competitive equilibrium without macro-prudential policy, since this equi-
librium remains feasible to the social planner. If one takes the debt tax and not the
tax on dividends, one cannot guarantee this Pareto improvement. Indeed, we solved
a variant of the model in which we introduced the optimal schedule of debt taxes
but left the tax on dividends out, and found that average welfare is actually lower
than without policy intervention by -0.02 percent. This occurs because welfare in
the states of nature in which the constraint is already binding is lower than without
policy intervention.\textsuperscript{24} Hence, while our results may provide a justification for the
use of macro-prudential policies, they also provide a warning because selective use

\textsuperscript{24}If we reduce the debt tax we can obtain again average welfare gains, which again illustrates
the interdependence of macroprudential policies.
of macro-prudential policies (i.e. partial implementation of the policy instruments indicated by the model) can reduce welfare in some states of nature. In this experiment this happens because the selective use of the debt tax without the tax on dividends lowers asset prices in some states of nature, and reduces welfare in those states by reducing the value of collateral.

Jeanne and Korinek (2010) also compute a schedule of macroprudential taxes on debt to correct a similar externality that arises because of a collateral constraint that depends on asset prices. In their constraint, however, the agents’ borrowing capacity is determined by the aggregate level of assets and by a linear state- and time-invariant term (i.e. their borrowing constraint is defined as \( b_{t+1}^R \geq -\kappa q_t \bar{K} - \psi \)). The fact that their constraint depends on aggregate rather than individual asset holdings, as in our model, matters because it implies that agents do not value additional asset holdings as a mechanism to manage their borrowing ability.\(^{25}\) But more im-

\(^{25}\)To illustrate this point, we recomputed our model assuming that the borrowing constraint depends on the aggregate value of assets, as in their setup. Because assets do not have individual
portantly, leaving aside this difference, they calibrate parameter values to $\kappa = 0.046$, $\psi = 3.07$ and $q_t\bar{K} = 4.8$, which imply that the effects of the credit constraint are driven mainly by $\psi$, and only less than 7 percent ($0.07 = 0.046*4.8/(0.046*4.8+3.07)$) of the borrowing ability depends on the value of asset holdings. As a result, the Fisherian deflation effect and the credit externality are weak, and hence they find that macroprudential policy lessens the macro effects of financial crises much less than in our setup. The asset price drop is reduced from 12.3 to 10.3 percent, and the consumption drop is reduced from 6.2 to 5.2 percent (compared with declines from 24 to 7 and 16 to 5 percent respectively in our model). Moreover, they model the stochastic process of dividends as an exogenous, regime-switching Markov chain such that the probability of a crisis (i.e. binding credit constraint) coincides with the probability of a bad realization of dividends, implying that the probability of busts is unaffected by macroprudential regulation. Thus, in their setup macro-prudential policy is much less effective at reducing the magnitude of financial crises and has no impact on their incidence.

2.4.7 Welfare Effects

We move next to explore the welfare implications of the credit externality. To this end, we calculate welfare costs as compensating consumption variations for each state of nature that make agents indifferent between the allocations of the competitive equilibrium and the constrained-efficient allocations. Formally, for a value as collateral, asset prices drop even more during crises, and this leads private agents to accumulate more precautionary savings, which results in crises having zero-probability in the long-run under both competitive and constrained-efficient equilibria for our baseline calibration.
given initial state \((B, \varepsilon)\) at date 0, the welfare cost is computed as the value of \(\gamma\) such that the following condition holds:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{DE}(1 + \gamma) - G(n_t^{DE})) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{SP} - G(n_t^{SP}))
\]  

(2.22)

where the superscript \(DE\) denotes allocations in the decentralized competitive equilibrium and the superscript \(SP\) denotes the social planner’s allocations. Note that these welfare costs reflect also the welfare gains that would be obtained by introducing the social planner’s optimal debt and dividend tax policies, which by construction implement the constrained-efficient allocations as a competitive equilibrium.

The welfare losses of the DE arise from two sources. The first source is the higher variability of consumption, due to the fact that the credit constraint binds more often in the DE, and when it binds it induces a larger adjustment in asset prices and consumption. The second is the efficiency loss in production that occurs due to the effect of the credit friction on working capital. Without the working capital constraint, the marginal disutility of labor equals the marginal product of labor. With the working capital constraint, however, the shadow cost of employing labor rises when the constraint binds, and this drives a wedge between the marginal product of labor and its marginal disutility. Again, since the collateral constraint binds more often in the DE than in the SP, this implies a larger efficiency loss.

Figure 6 plots the welfare costs of the credit externality as a function of \(b\) for a negative, two-standard-deviations TFP shock. These welfare costs approximate a bell shape skewed to the left. This is due to the differences in the optimal plans of the
social planner vis-a-vis private agents in the decentralized equilibrium. Recall than in the constrained region, the current allocations of the decentralized equilibrium essentially coincide with those of the constrained-efficient economy, as described in Figure 1. Therefore, in this region the welfare gains from implementing the constrained-efficient allocations only arise from how future allocations will differ. On the other hand, in the high-externality region, the constrained-efficient allocations differ sharply from those of the decentralized equilibrium, and this generally enlarges the welfare losses caused by the credit externality. Notice that, since the constrained-efficient allocations involve more savings and less current consumption, there are welfare losses in terms of current utility for the social planner, but these are far outweighed by less vulnerability to sharp decreases in future consumption during financial crises. Finally, as the level of debt is decreased further and the economy enters the low-externality region, financial crises are unlikely and the welfare costs of the inefficiency decrease.

The unconditional average welfare cost over the decentralized equilibrium’s ergodic distribution of bonds and TFP is 0.046 percentage points of permanent consumption. This contrasts with Bianchi (2010) who found welfare costs about 3 times larger. Note, however, that our results are in line with his if we express the welfare costs as a fraction of the variability of consumption. Consumption was more volatile in his setup because he examined a calibration to data for emerging economies, which are more volatile than the United States.

The fact that welfare losses from the externality are small although the differences in consumption variability are large is related to the well-known Lucas result
that models with CRRA utility, trend-stationary income, and no idiosyncratic uncertainty produce low welfare costs from consumption fluctuations. Moreover, the efficiency loss in the supply-side when the constraint binds produces low welfare costs on average because those losses have a low probability in the ergodic distribution.

2.4.8 Sensitivity Analysis

We examine now how the quantitative effects of the credit externality change as we vary the values of the model’s key parameters. Table C.1 shows the main model statistics for different values of $\sigma$, $\kappa$, $\omega$ and $\theta$. The Table shows the unconditional averages of the tax on debt and the welfare loss, the covariance effect on excess
returns, the probability of financial crises, and the impact effects of a financial crisis on key macroeconomic variables. In all of these experiments, only the parameter listed in the first column changes and the rest of the parameters remain at their baseline calibration values.

The results of the sensitivity analysis reported in Table C.1 can be understood more easily by referring to the externality term derived in Section 2: The wedge between the social and private marginal costs of debt that separate competitive and constrained-efficient equilibria, \( \beta RE_t \left[ \mu_{t+1} \left( \kappa K^e \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial \mu_{t+1}}{\partial b_{t+1}} \right) \right] \). For given \( \beta \) and \( R \), the magnitude of the externality is given by the expected product of two terms: the shadow value of relaxing the credit constraint, \( \mu_{t+1} \), and the associated price effects \( \kappa K^e \frac{\partial q_{t+1}}{\partial b_{t+1}} - \theta n_{t+1} \frac{\partial \mu_{t+1}}{\partial b_{t+1}} \) that determine the effects of the externality on the ability to borrow when the constraint binds. As explained earlier, the price effects are driven mostly by \( \frac{\partial q_{t+1}}{\partial b_{t+1}} \), because of the documented large asset price declines when the collateral constraint binds. It follows therefore, that the quantitative implications of the credit externality must depend mainly on the parameters that affect \( \mu_{t+1} \) and \( \frac{\partial q_{t+1}}{\partial b_{t+1}} \), as well as those that affect the probability of hitting the constraint.

The coefficient of relative risk aversion \( \sigma \) plays a key role because it affects both \( \mu_{t+1} \) and \( \frac{\partial q_{t+1}}{\partial b_{t+1}} \). A high \( \sigma \) implies a low intertemporal elasticity of substitution in consumption, and therefore a high value from relaxing the constraint since a binding constraint hinders the ability to smooth consumption across time. A high \( \sigma \) also makes the stochastic discount factors more sensitive to changes in consumption, and therefore makes the price of land react more to changes in bond holdings. Accordingly, rising \( \sigma \) from 2 to 2.5 rises the welfare costs of the credit externality.
by a factor of 5, and widens the differences in the covariance effects across the competitive and constrained-efficient equilibria. In fact, the covariance effect in the decentralized equilibrium increases from 0.22 to 0.37 whereas for the constrained efficient allocations the increase is from 0.06 to 0.08. Stronger precautionary savings reduce the probability of crises in the competitive equilibrium, and financial crises become a zero-probability event in the constrained-efficient equilibrium. Conversely, reducing $\sigma$ to 1 makes the externality extremely small, measured either by differences in the incidence or severity of financial crises.\footnote{Notice that the probability of a crisis in the competitive equilibrium becomes 10 percent, more than three times larger than the target employed in the baseline calibration due to the reduction in the level of precautionary savings.}

The collateral coefficient $\kappa$ also plays an important role because it alters the effect of land price changes on the borrowing ability. A higher $\kappa$ implies that, for a given price response, the change in the collateral value becomes larger. Thus, this effect makes the externality term larger. On the other hand, a higher $\kappa$ has two additional effects that go in the opposite direction. First, a higher $\kappa$ implies that the direct effect of the collateral constraint on the land price is weaker, leading to a lower fall in the price of land during fire sales. Second, a higher $\kappa$ makes the constraint less likely to bind, reducing the externality. The effects of changes in $\kappa$ are clearly non-monotonic. If $\kappa$ is equal to zero, there is no effect of prices on the borrowing-ability. At the same time, for high enough values of $\kappa$, the constraint never binds. In both cases, the externality does not play any role. Quantitatively, Table C.1 shows that small changes in $\kappa$ are positively associated with the size of the inefficiency. In particular, an increase in $\kappa$ from the baseline value of 0.36 to

\footnote{Notice that the probability of a crisis in the competitive equilibrium becomes 10 percent, more than three times larger than the target employed in the baseline calibration due to the reduction in the level of precautionary savings.}
0.40 increases the welfare cost of the inefficiency by a factor of 6 and financial crises again become a zero-probability event in the constrained-efficient equilibrium.

The above results have interesting policy implications. In particular, they suggest that while increasing credit access by rising $\kappa$ may increase welfare relative to a more financially constrained environment, rising $\kappa$ can also strengthen the effects of credit externalities and hence make macro-prudential policies more desirable (since the welfare cost of the externality also rises).

A high Frisch elasticity of labor supply ($1/\omega = 1.2$) implies that output drops more when a negative shock hits. If the credit constraint binds, this implies that consumption falls more, which increases the marginal utility of consumption and raises the return rate at which future dividends are discounted.\footnote{The increase in leisure mitigates the decrease in the stochastic discount factor but does not compensate for the fall in consumption} Moreover, everything else constant, a higher elasticity makes the externality term higher by weakening the effects of wages on the borrowing capacity. Hence, a higher elasticity of labor supply is associated with higher effects from the credit externality, captured especially by larger differences in the severity of financial crises, a higher probability of crises, and a larger welfare cost of the credit externality.

The fraction of wages that have to be paid in advance $\theta$ plays a subtle role. On one hand, a larger $\theta$ increases the shadow value of relaxing the credit constraint, since this implies a larger rise in the effective cost of hiring labor when the constraint binds. On the other hand, a larger $\theta$ implies, ceteris paribus, a weaker effect on borrowing ability, since the reduction of wages that occurs when the collateral constraint binds
has a positive effect on the ability to borrow. Quantitatively, increasing (decreasing) $\theta$ by 5 percent increases (decreases) slightly the effects that reflect the size of the externality.

Changes in the volatility and autocorrelation of TFP do not have significant effects. Increasing the variability of TFP implies that financial crises are more likely to be triggered by a large shock. This results in larger amplification and a higher benefit from internalizing price effects. In general equilibrium, however, precautionary savings increase too, resulting in a lower probability of financial crises for both equilibria. Therefore, the overall effects on the externality of a change in the variability of TFP depend on the relative change in the probability of financial crisis in both equilibria and the change in the severity of these episodes. An increase in the autocorrelation of TFP leads to more frequent financial crises for given bond decision rules. Again, in general equilibrium, precautionary savings increase making ambiguous the effect on the externality.
### Table 2.5: Sensitivity Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average Welfare</th>
<th>Covariance Effect</th>
<th>Crisis Probability</th>
<th>Consumption</th>
<th>Credit</th>
<th>Land Price</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tax</td>
<td>Loss</td>
<td>DE</td>
<td>SP</td>
<td>DE</td>
<td>SP</td>
<td>DE</td>
</tr>
<tr>
<td>benchmark</td>
<td>1.1</td>
<td>0.05</td>
<td>0.22</td>
<td>0.06</td>
<td>3.0</td>
<td>0.9</td>
<td>-15.7</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>0.6</td>
<td>0.001</td>
<td>0.03</td>
<td>0.03</td>
<td>9.3</td>
<td>7.8</td>
<td>-3.8</td>
</tr>
<tr>
<td>$\sigma = 1.5$</td>
<td>1.0</td>
<td>0.01</td>
<td>0.08</td>
<td>0.04</td>
<td>5.7</td>
<td>2.5</td>
<td>-6.5</td>
</tr>
<tr>
<td>$\sigma = 2.5$</td>
<td>1.2</td>
<td>0.24</td>
<td>0.37</td>
<td>0.08</td>
<td>2.0</td>
<td>0.0</td>
<td>-15.9</td>
</tr>
<tr>
<td>$\kappa = 0.32$</td>
<td>1.0</td>
<td>0.02</td>
<td>0.14</td>
<td>0.06</td>
<td>4.6</td>
<td>2.1</td>
<td>-8.8</td>
</tr>
<tr>
<td>$\kappa = 0.4$</td>
<td>1.2</td>
<td>0.29</td>
<td>0.34</td>
<td>0.06</td>
<td>2.2</td>
<td>0.0</td>
<td>-17.2</td>
</tr>
<tr>
<td>$1/\omega = 0.83$</td>
<td>1.0</td>
<td>0.03</td>
<td>0.16</td>
<td>0.05</td>
<td>3.7</td>
<td>1.1</td>
<td>-8.6</td>
</tr>
<tr>
<td>$1/\omega = 1.2$</td>
<td>1.1</td>
<td>0.12</td>
<td>0.27</td>
<td>0.06</td>
<td>3.9</td>
<td>2.3</td>
<td>-18.5</td>
</tr>
<tr>
<td>$\theta = 0.13$</td>
<td>1.1</td>
<td>0.03</td>
<td>0.18</td>
<td>0.06</td>
<td>1.7</td>
<td>1.6</td>
<td>-15.3</td>
</tr>
<tr>
<td>$\theta = 0.15$</td>
<td>1.1</td>
<td>0.05</td>
<td>0.21</td>
<td>0.06</td>
<td>2.8</td>
<td>1.2</td>
<td>-17.5</td>
</tr>
<tr>
<td>$\sigma_\epsilon = 0.010$</td>
<td>1.1</td>
<td>0.06</td>
<td>0.19</td>
<td>0.04</td>
<td>3.23</td>
<td>0.00</td>
<td>-13.9</td>
</tr>
<tr>
<td>$\sigma_\epsilon = 0.018$</td>
<td>1.0</td>
<td>0.05</td>
<td>0.26</td>
<td>0.08</td>
<td>2.51</td>
<td>0.16</td>
<td>-17.3</td>
</tr>
<tr>
<td>$p_\kappa = 0.43$</td>
<td>1.1</td>
<td>0.05</td>
<td>0.22</td>
<td>0.05</td>
<td>2.78</td>
<td>0.00</td>
<td>-15.6</td>
</tr>
<tr>
<td>$p_\kappa = 0.63$</td>
<td>1.1</td>
<td>0.05</td>
<td>0.23</td>
<td>0.06</td>
<td>2.74</td>
<td>1.17</td>
<td>-16.1</td>
</tr>
<tr>
<td>Stochastic $\kappa$</td>
<td>1.1</td>
<td>0.04</td>
<td>0.25</td>
<td>0.06</td>
<td>3.02</td>
<td>1.22</td>
<td>-12.0</td>
</tr>
</tbody>
</table>

Note: ‘DE’ represents the decentralized equilibrium; ‘SP’ represents the social planner. The average tax on debt corresponds to the average value of the state contingent tax on debt to decentralize the constrained-efficient allocations. The covariance effect represents the unconditional average of the covariance effect. Consumption, credit, land price and output are responses of these variables on impact during a financial crisis (see section 4.4 for a definition of the event analysis). The baseline parameter values are: $R - 1 = 0.028, \beta = 0.96, \sigma = 2, \alpha_h = 0.64, \chi = 0.64, \omega = 1\bar{K} = 1, \theta = 0.14, \kappa = 0.36, \alpha_\kappa = 0.05, \sigma_\epsilon = 0.014, p_\kappa = 0.53$.
In terms of the optimal debt on tax, the results of the sensitivity analysis produce an important finding: The average debt tax of about 1.1 percent is largely robust to the parameter variations we considered. Except for the scenario that approximates logarithmic utility ($\sigma = 1$), in all other scenarios included in Table C.1 the mean tax ranges between 1.01 and 1.2 percent.

We consider now shocks that affect directly the collateral constraint by affecting the extent to which agents can pledge assets as collateral. We consider a stochastic process for $\kappa$ that follows a symmetric two-state Markov chain independent from shocks to TFP. In line with evidence from Mendoza and Terrones (2008) on the mean duration of credit booms in industrial countries, we calibrate the probabilities of the Markov chain so that the average duration of each state is 6 years. We keep the average value of $\kappa$ as in our benchmark model and consider fluctuations of $\kappa$ of 10 percent which is meant to be suggestive. As shown in Table C.1, the effects of the externality remain largely unchanged to this modification.

Overall, the results of the sensitivity analysis show that parameter changes that weaken the model’s financial amplification mechanism also weaken the magnitude of the externality. This results in smaller average taxes, smaller welfare costs and smaller differences in the incidence and severity of financial crises. The coefficient of risk aversion is particularly important also because it influences directly the price elasticity of asset demand, and hence it determines how much asset prices can be affected by the credit externality. This parameter plays a role akin to that of to the elasticity of substitution in consumption of tradables and non-tradables in Bianchi (2010), because in his model this elasticity drives the response of the price...
at which the collateral is valued. Accordingly, he found that the credit externality has significant effects only if the elasticity is sufficiently low.

2.5 Conclusion

This paper examined the positive and normative effects of a credit externality in a dynamic stochastic general equilibrium model in which a collateral constraint limits access to debt and working capital loans to a fraction of the market value of an asset in fixed supply (e.g. land). We compared the allocations and welfare attained by private agents in a competitive equilibrium in which agents face this constraint taking prices as given, with those attained by a constrained social planner that faces the same borrowing limits but takes into account how current borrowing choices affect future asset prices and wages. This planner internalizes the debt-deflation process that drives macroeconomic dynamics during financial crises, and hence borrows less in periods in which the collateral constraint does not bind, so as to weaken the debt-deflation process in the states in which the constraint becomes binding. Conversely, private agents overborrow in periods in which the constraint does not bind, and hence are exposed to the stronger adverse effects of the debt-deflation mechanism when a financial crisis occurs.

The novelty of our analysis is in that it quantifies the effects of the credit externality in a setup in which the credit friction has effects on both aggregate demand and supply. The effects on demand are well-known from models with credit constraints: consumption drops as access to debt becomes constrained, and this
induces an endogenous increase in excess returns that leads to a decline in asset prices. Because collateral is valued at market prices, the drop in asset prices tightens the collateral constraint further and leads to fire-sales of assets and a spiraling decline in asset prices, consumption and debt. On the supply side, production and labor demand are affected by the collateral constraint because firms buy labor using working capital loans that are limited by the collateral constraint, and hence when the constraint binds the effective cost of labor rises, so the demand for labor and output drops. This affects dividend rates and hence feeds back into asset prices. Previous studies in the macro/finance literature have shown how these mechanisms can produce financial crises with features similar to actual financial crises, but the literature had not conducted a quantitative analysis comparing constrained-efficient v. competitive equilibria in an equilibrium model of business cycles and asset prices.

We conducted a quantitative analysis in a version of the model calibrated to U.S. data. This analysis showed that, even though the credit externality results in only slightly larger average ratios of debt and leverage to output compared with the constrained-efficient allocations (i.e. overborrowing is not large), the credit externality does produce financial crises that are significantly more severe and more frequent than in the constrained-efficient equilibrium, and produces higher long-run business cycle variability. There are also important asset pricing implications. In particular, the credit externality and its associated higher macroeconomic volatility in the competitive equilibrium produce equity premia, Sharpe ratios, and market price of risk that are much larger than in the constrained-efficient equilibrium. We also found that the degree of risk aversion plays a key role in our results, because it is a key
determinant of the response of asset prices to volatility in dividends and stochastic
discount factors. For the credit externality to be important, these price responses
need to be nontrivial, and we found that they are nontrivial already at commonly
used risk aversion parameters, and larger at larger risk aversion coefficients that are
still in the range of existing estimates.

This analysis has important policy implications. In particular, the social plan-
ner can decentralize the constrained-efficient allocations as a competitive equilibrium
by introducing an optimal schedule of state-contingent taxes on debt and dividends.
By doing so, it can neutralize the adverse effects of the credit externality and pro-
duce an increase in social welfare. In our calibrated model, the tax on debt necessary
to attain this outcome is about 1 percent on average. The tax is higher when the
economy is building up leverage and becoming vulnerable to a financial crisis, but
before a crisis actually occurs, so as to induce private agents to value more the
accumulation of precautionary savings than they do in the competitive equilibrium
without taxes.

These findings are relevant for the ongoing debate on the design of new finan-
cial regulation to prevent financial crises, which emphasizes the need for “macro-
prudential” regulation. Our results lend support to this approach by showing that
credit externalities associated with fire-sales of assets have large adverse macroeco-
nomic effects. At the same time, however, we acknowledge that actual implementa-
tion of macro-prudential policies in financial markets remains a challenging task. In
particular, the optimal design of these policies requires detailed information on a va-
riety of credit constraints that private agents and the financial sector face, real-time
data on their leverage positions, and access to a rich set of state-contingent policy instruments. Moreover, as we showed in this paper, implementing only a subset of the optimal policies because of these limitations (or limitations of the political process) can reduce welfare in some states.
Chapter 3
Efficient Bailouts?

3.1 Introduction

The recent financial crisis has led to massive government intervention in credit markets. The initial Troubled Assets Relief Program (TARP), for example, required 700 billion dollars to provide credit assistance to financial and non-financial institutions. These measures have triggered an intense debate on the desirability of such interventions. Supporters argue that these bailouts were necessary to avoid a complete meltdown of the financial sector that would have brought an extraordinary contraction in output and employment. Critics argue that the presence of bailouts generate incentives for investors to take even more risk ex ante, sowing the seeds of future crises. Such critics, therefore, propose regulations to limit the ability of central banks to conduct bailouts.

In this paper, we seek an answer to the following questions: What are the implications of bailout expectations for the stability of the financial sector? Is it desirable to prohibit the use of public funds to conduct bailouts? If the central bank could commit to a bailout policy, under what states of nature is it optimal to conduct a bailout? How large should these bailouts be?

This paper answers these questions based on a non-linear DSGE model where credit frictions generate scope for government credit intervention during a financial
crisis, but where such effects generate more risk-taking before the crisis actually hits. Recent research (see e.g. Gertler and Kiyotaki, 2010) has developed a quantitative framework to analyze how governmental intervention can mitigate the credit crunch and moderate the recession ex post. At the same time, a growing theoretical literature has emphasized the moral hazard implications of such interventions (see e.g. Farhi and Tirole, 2010), but little is known about their quantitative implications. In this paper, we develop a quantitative DSGE model to assess within a unified framework the interaction between ex-post interventions in credit markets and the build-up of risk ex ante.

The model features a representative corporate entity that finances investment using equity and debt. There are two key frictions affecting the capacity of firms to finance investment. First, debt contracts are not fully enforceable, giving rise to a collateral constraint that limits the amount that firms can borrow. Second, there is a constraint on minimum dividends that firms must make each period. Therefore, firms balance the desire to increase borrowing and investment now with the risk of becoming financially constrained in the future. When leverage is sufficiently high and an adverse financial shock hits the economy, firms hit their balance sheet constraints, generating a fall in investment and a protracted recession.

In this environment, we consider a social planner that engages in direct credit policy with the purpose of relaxing the balance sheet constraints of firms, but faces two types of costs in this intervention. First, there is the static cost of transferring resources from workers to firms. Second, there is the dynamic effect (or a moral hazard effect) that arises because firms anticipate the planner’s intervention and
therefore take more risk. As a result, there is a trade-off involved in the design of a bailout policy. On the one hand, a commitment to bailing out the entire corporate sector in some adverse states of nature, i.e., a systemic financial crisis, provides a form of insurance against such future episodes, which the market would otherwise not provide. On the other hand, as firms adjust their leverage choices, the economy becomes more exposed to the deadweight losses produced by aggregate financial distress.

Our answer to the question of whether it is desirable or undesirable to prohibit the use of public funds to conduct bailouts is that public funds can in fact be used effectively to conduct bailouts. The quantitative analysis shows that it is possible to design a realistic and simple bailout policy so that the welfare benefits of such an intervention outweigh its distortionary effects. In particular, we find that in the presence of a severe systemic financial crisis, it is optimal to engineer a transfer of funds from workers to firms.

This paper relates to different strands of literature. First, there is a growing literature that studies credit policy in quantitative DSGE models, building on the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). As in these papers, we study how government intervention can mitigate financial frictions that are activated during financial crises. For reasons of tractability, however, this literature mostly studies the optimal response to unanticipated crises or focuses on the log-linear dynamics around the deterministic steady state, thereby abstracting from

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risk considerations and the moral hazard effects of government intervention. Instead, a distinctive feature of this paper is the focus on how expectations of future bailouts affect ex-ante risk-taking within a non-linear DSGE model. This is crucial in order to assess the dynamic implications of credit intervention on financial stability and on welfare.²

This paper is also closely related to the theoretical literature that analyzes the incentive effects of bailouts on financial stability. Farhi and Tirole (2010) analyze how time-consistent systemic bailouts can generate strategic complementarities in private leverage choices, causing excessive financial fragility. Chari and Kehoe (2010) show that fire sale effects provide governments with stronger incentives to renegotiate contracts than private agents, making the time-inconsistency problem more severe for the government. Diamond and Rajan (2009) show that raising interest rates may be optimal to penalize excessive risk taking. There are two key differences between our paper and this literature: first, we develop a quantitative framework to assess the effects of bailouts on risk-taking; second, we emphasize the idea that systemic bailouts can be beneficial ex-ante as a way to provide insurance. In this aspect, this paper is related to Schneider and Tornell (2004) and Keister (2010) who also emphasize the insurance benefits of bailouts, but they focus on self-fulfilling crises.

This paper is also related to a growing literature that studies the normative implications of financial frictions and the role of macroprudential regulation. This

²Recently, Gertler, Kiyotaki, and Queralto (2010) develop a model where banks face a portfolio choice between short-term debt and equity and also consider moral hazard effects of credit policy in this portfolio choice. Gertler et al. consider a log-linear approximation around the risk-adjusted steady state in an equilibrium where financial constraints are always binding. An key structural difference in our model is that we disentangle the role of policy during normal times from the role of policy during times of systemic crisis by solving the model using global methods.
literature emphasizes the role of ex-ante prudential measures to correct pecuniary externalities due to financial accelerator effects.\textsuperscript{3} Our paper focuses instead on ex-post policy measures and their effects on ex-ante risk taking decisions.\textsuperscript{4}

3.2 Analytical Framework

Our model economy is a small open economy populated by firms and workers that are also the shareholders of the firms. This model shares with Jermann and Quadrini (2010) the consideration of dividend policy and with Mendoza (2010) the analysis of non-linear dynamics. We start by first describing the decisions of the different agents in the economy, and then we describe the general equilibrium.

3.2.1 Corporate entities

There is a measure one of identical firms with technology given by the production function $F(z_t, k_t, n_t)$ that combines capital denoted by $k_t$ and labor denoted by $h_t$ to produce a final good. TFP denoted by $z_t$ follows a first-order Markov process. Consistent with the typical timing convention, $k_t$ is chosen at time $t - 1$, and therefore, they are predetermined at time $t$. Instead, the input of labor $h_t$ can be flexibly changed in period $t$.

\textsuperscript{3}See for example Caballero and Krishnamurthy (2003), Lorenzoni (2008), Korinek (2009), Bianchi (2010), Bianchi and Mendoza (2010) and Jeanne and Korinek (2010).

\textsuperscript{4}Benigno, Chen, Otrok, Rebucci, and Young (2010) also consider ex-post policy measures in response to a pecuniary externality, but focus on policies that affect labor allocations as opposed to policies that affect the availability of credit.
Capital evolves according to:

\[ k_{t+1} = k_t(1 - \delta) + i_t \tag{3.1} \]

where \( i_t \) is the level of investment and \( \delta_t \) is the depreciation rate. Capital accumulation is subject to adjustment costs, given by \( \psi(\cdot) \).

Firms pay dividends, denoted by \( d_t \), and issue non-state contingent debt, denoted by \( b_{t+1} \). The flow of funds constraint for firms is then given by:

\[ b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq F(z_t, k_t, n_t) - w_t n_t + \frac{b_{t+1}}{R_t} + \Upsilon_t \tag{3.2} \]

where \( w_t \) is the wage rate, \( R_t \) is the gross interest rate determined in international markets, and \( \Upsilon_t \) is a transfer chosen by the government that will be specified below.

Firms face two types of constraints on their ability to finance investment. First, they are subject to a collateral constraint that limits the amount of borrowing to a fraction of the value of their assets such that:

\[ b_{t+1} \leq \kappa_t k_{t+1} \tag{3.3} \]

This constraint is similar to those used in existing literature (see Kiyotaki and Moore, 1997), and we interpret it as arising in an environment with limited enforcement between creditors and firms; \( \kappa_t \) is a financial shock and we interpret it, following Jermann and Quadrini (2010), as a shock originating in the financial system.
In addition, at each period firms are required to pay a minimum amount of dividends \( d \geq \bar{d} \). A value of \( \bar{d} \geq 0 \) implies that the issuance of new shares is not available. A special case is the restriction that dividends need to be non-negative. This constraint captures the notion that dividend payments are required in order to reduce agency frictions between shareholders and managers. In addition, it produces dynamics in dividend payouts which are in line with empirical evidence (see Jermann and Quadrini (2010)). We abstract, however, from an explicit microfoundation and instead focus on its implications. \(^5\)

Denoting by \( s \) the vector of aggregate states, i.e. \( s = \{ K, B, \kappa, z \} \), the optimization problem for firms can be written as:

\[
V(k, b, s) = \max_{d, h, k', b'} \left[ d + \mathbb{E}m'(s, s')V(k', b', s') \right] \\
\text{s.t.} \quad b + d + k' + \psi(k, k') \leq (1 - \delta)k + F(z, k, h) - wn + \frac{b'}{R} + \Upsilon \\
\quad b' \leq \kappa k' \\
\quad d \geq \bar{d}
\]

The function \( V(k, b, s) \) is the cum-dividend market value of the firm and \( m' \) is the stochastic discount factor, which will be equal in equilibrium to the ratio of

\(^5\)Endogeneizing such a constraint would require to model the divergence of interests between shareholder and corporate manager in an environment with asymmetric information (see e.g. Mayer 1986).
marginal utility of household consumption.

The optimality condition for labor demand yields:

\[ F_h(z_t, k_t, h_t) = w_t \] (3.5)

There are also two Euler intertemporal conditions that relate the marginal benefit from distributing one unit of dividends today with the marginal benefit of investing in the available assets and distributing the resulting dividends in the next period. Denoting by \( \mu \) the multiplier associated with the borrowing constraint, \( \eta \) the multiplier associated with the dividend payout constraint, these Euler equations are given by:

\[ 1 + \eta_t = R_t \mathbb{E}_t m_{t+1} (1 + \eta_{t+1}) + \mu_t \] (3.6)

\[ (1 + \eta_t)(1 + \psi_2(t, t+1)) = \mathbb{E}_t m_{t+1} [1 - \delta + F_k(z_{t+1}, k_{t+1}, n_{t+1}) - \psi_1(t + 1, t + 2)] (1 + \eta_{t+1}) + \kappa_t \mu_t \] (3.7)

In the absence of the financial constraints on borrowing and dividend payments, the cost of raising equity (by reducing dividends), i.e. \( 1/\mathbb{E}_t m_{t+1} \), would be equal to the cost of debt \( R_t \), and firms would be indifferent at the margin between equity and debt financing. When the collateral constraint binds, there is a wedge between the marginal benefit of borrowing one more unit and distributing it as div-
idends in the current period and between the marginal cost of cutting dividends in
the next period to repay the debt increase. In addition, when the dividend payout
constraint binds, a positive wedge arises between the marginal benefit from invest-
ing one more unit in capital or bonds relative to the marginal cost of cutting one
more unit of dividends. Condition (3.6) suggests also that a binding collateral con-
straint is associated with a binding dividend payout constraint. Intuitively, both
constraints impose a limit on a firm’s funding ability, which implies that a tighter
constraint on borrowing imposes pressure on the need to finance with equity, which
given that households have concave utility functions, this increases the cost of equity.
Similarly, a binding dividend payout constraint forces higher levels of borrowing for
given investment choices.

3.2.2 Households

There is a continuum of identical households of measure one that maximize:

\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t - G(n_t)) \]  

(3.8)

where \( c_t \) is consumption, \( n_t \) is labor supply, \( \beta \) is the discount factor and \( G(\cdot) \) is a
twice-continuously differentiable, increasing and convex function. The utility func-
tion \( u(\cdot) \) has the constant-relative-risk-aversion (CRRA) form; the composite of the
utility function has the GHH form, eliminating wealth effects on the labor supply.
Households do not have access to bond markets and are the firms’ shareholders.
This yields the following budget constraint:

\[ w_t h_t + s_t (d_t + p_t) = s_{t+1} p_t + c_t + T_t \]  \hspace{1cm} (3.9)

where \( s_t \) represents the holdings of firm-shares, \( p_t \) represents the price of firm-shares, and \( T_t \) is a lump sum tax to finance the cost of the bailout policy to be specified below.

The first order conditions are given by:

\[ w_t = G'(n_t) \]  \hspace{1cm} (3.10)

\[ p_t u_c(t) = \beta E_t u_c(t + 1)(1 + p_{t+1}) \]  \hspace{1cm} (3.11)

The second condition determines the price of shares. Iterating forward on (3.11) yields that for the firm optimization problem to be consistent with the household optimization problem, it must be the case that the stochastic discount factor of firms is equal to \( m_{t+j} = (\beta u'(c_{t+j} - G'(n_{t+j}))) / (u'(c_{t+j-1} - G'(n_{t+j-1}))) \).

3.2.3 Government

The function of the government is to set a lump sum tax on workers and to transfer the proceeds to firms. A crucial element of the analysis is that transferring resources to firms entails an efficiency cost \( \phi \). Alternatively, this can be interpreted as a distortionary effect of taxation. We assume that the government follows a
balanced budget such that:

\[ T_t = \Upsilon_t (1 + \varphi) \] (3.12)

3.2.4 Competitive equilibrium

We consider a competitive equilibrium for a small open economy where firms borrow directly from abroad.

**Definition 5** A recursive competitive equilibrium is given by (i) firms’ policies \( \hat{d}(k, b, s), \hat{h}(k, b, s), \hat{k}(k, b, s) \) and \( \hat{b}(k, b, s) \); households’ policies \( \hat{s}(s, s), \hat{n}(s, s) \); firm’s value \( V(k, b, s) \); prices \( w(s), p(s), m(s, s') \); government policies \( \Upsilon(s), T(s) \); and a law of motion of aggregate variables \( s' = \Gamma(k, b, s) \) such that: (i) households solve their optimization problem characterized by (3.9)-(3.11), (ii) firms’ policies and firms’ value solve (3.4), (iii) prices clear labor market \( (h_t = n_t) \), equity market \( (s_t = 1) \), and goods market \( ((1 - \delta)k_t + F(z_t, k_t, n_t) + b_t = b_{t+1}/R + \psi(k_t, k_{t+1}) + k_{t+1} + c_t) \), (iv) the law of motion is \( \Gamma(\cdot) \) is consistent with individual policy functions and stochastic processes for \( \kappa \) and \( z \).

3.2.5 Some characterization

To illustrate the properties of the model and the effects of the bailout policy, it is useful to consider a few special cases.

**Proposition 3** If \( \beta R < 1 \), the collateral constraint is binding and the dividend payout constraint may or may not bind.
Proof: In a deterministic steady state, \( m_t = 1 \) and (3.6) is simplified to \( 1 = \beta R + \mu \). Since \( \beta R < 1 \), this implies that \( \mu > 0 \). Substituting the collateral constraint with equality in the firm’s flow of funds constraint yields that the dividend constraint binds if and only if \( \bar{d} > F(\bar{z}, \bar{k}, \bar{h}) - \bar{k}(\delta + \kappa(R - 1)/R) - G'(\bar{h})\bar{h} + \Upsilon \) where \( \bar{k} \) and \( \bar{h} \) are the steady state values of capital and labor given by (3.7), (3.10), and market clearing in labor markets. In general, in a stochastic steady state, these financial constraints may or may not bind depending primarily on the magnitude of the shocks, \( \beta R \), and the level of risk aversion.

**Proposition 4** Suppose (i) \( \bar{d} = -\infty \), and (ii) households have unrestricted access to international credit markets by borrowing and saving at the interest rate \( R \) (unlike the baseline model). Then the competitive equilibrium is unaffected by financial shocks.

Proof: From a household’s first order condition, it must be the case that \( RE'm' = 1 \). Using the firm’s first order condition \( RE'm' + \mu = 1 \) yields \( \mu = 0 \). Since the collateral constraint never binds, the model becomes a standard (RBC) model where the financial structure is independent on the real side.

**Remark 6** Suppose (i) \( \bar{d} = -\infty \), and (ii) households do not have access to international credit markets. Then the competitive equilibrium is affected by financial shocks.

This remark becomes obvious once we observe that if output is sufficiently low at a given period, households will demand positive dividends to smooth consumption,
which implies that firms would need to increase their debt position. Therefore, for low values of $\kappa$, the collateral constraint becomes binding.

3.3 Bailout Policy

3.3.1 A second best benchmark

To illustrate the role of bailouts, we start by considering a social planner that (i) directly chooses all allocations and (ii) is subject to the collateral constraint, but not the dividend payout constraint.

This problem can be written recursively as:

$$V(s) = \max_{d,n,k',b'} u(c - G(h)) + \beta \mathbb{E} V(s')$$

s.t. $$(1 - \delta)k + F(z,k,n) - wn + \frac{b'}{R} \geq b + d + k' + \psi(k,k')$$

$$b' \leq \kappa k'$$

We now highlight the main results regarding the implementability of this constrained social planner’s problem employing bailouts.

**Remark 7** If $\varphi = 0$, the second best allocations can be implemented by an appropriate state contingent bailout policy.

Intuitively, the planner can use cost-free transfers as a substitute for lower
dividend payments when the dividend payout constraint becomes binding.

**Proposition 5** If $\bar{d} = -\infty$ and $\varphi = 0$, the competitive equilibrium is unaffected by the bailout policy.

*Proof* The proof follows simply by noting that the transfers cancel out within the conditions characterizing the competitive equilibrium.

### 3.3.2 Policy Experiment

We now consider the general case where the government faces strictly positive efficiency costs from transferring resources from firms and households. Under these conditions, the second best allocations cannot be achieved: bailouts introduce a trade off between relaxing balance sheet constraints of firms and efficiency costs associated with the transfer.

The government function is limited to taxing workers and transferring those resources to firms in the same period.\(^6\) Specifically, we assume that the government can commit to following a bailout policy rule $\nu$ such that $\Upsilon = \nu(\cdot)$ where $\nu(\cdot)$ follows a parametric function that depends on the relevant state variables.

The objective of the government/planner is to choose $\nu(\cdot)$ to maximize

$$
\sum_{s_0} \left[ \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t^{bp} - G(n_t^{bp})) | s_0 \right] \Pi(s_0) \tag{3.14}
$$

subject to all allocations and prices being a competitive equilibrium for the specified

\(^6\)That is, we rule out the use of the government as a substitute for private credit. Allowing for government debt is likely to take the economy closer to a first best if there are no other frictions (e.g. sovereign default risk).
bailout policy, here \( \Pi \) represents the joint ergodic distribution of all aggregate states in the competitive equilibrium without bailout. That is, the policy rule is chosen to maximize the expected life-time utility from switching to the competitive equilibrium without intervention to the competitive equilibrium with a bailout policy. This welfare measure considers explicitly the effects associated with the transition from the stochastic steady state without bailout policy to the stochastic steady state with bailout policy.

This policy can be interpreted as a form of unconventional monetary policy (see e.g. Gertler and Karadi, 2010), as one can interpret the recent intervention of the Federal Reserve in credit markets as a way to facilitate credit to the corporate sector, given the strains in financial intermediaries.\(^7\) In the quantitative analysis we search for parametrization of these rules that are simple and realistic such and that the rules maximize expected life-time utility.

A key feature of this bailout policy is that bailouts are non-targeted, i.e. they apply to all market participants. That is, even if an individual agent is not under financial distress, it receives a bailout when the overall economy is under distress. This is akin to an interest rate policy (see Farhi and Tirole, 2010) in which even firms that are not under distress can refinance at a low interest rate when the

\(^7\)In practice, the Federal Reserve and the Treasury implemented a variety of policies with the aim of facilitating credit to the corporate sector including direct lending, debt guarantees, and equity injections in the banking sector. To simplify the analysis, we do not model financial intermediaries and capture this class of interventions in a crude way by modeling a direct transfer from workers to firms. What is crucial for our analysis is that this intervention relaxes balance sheets across the economy and mitigates the fall in credit and investment that occurs during crises. In our setup, absent of information asymmetries, these policies are likely to yield similar outcomes. See Philippon and Schnabl (2009) for an evaluation of optimal rescue packages in the context of a debt overhang problem using a mechanism design approach.
central bank conducts an expansionary monetary policy. This introduces strategic complementarities in firms’ financing decisions, as individual agents have more of an incentive to take a significant amount of risk when other firms in the economy are also taking a large amount of risk. On the other hand, conditioning bailouts on aggregate variables may also work to mitigate the amount of risk taking, as individual agents obtain a bailout only when the whole economy comes under financial distress.  

3.4 Quantitative Analysis

The numerical solution to the model involves several challenges. First, there are the well-known complications of non-linearities introduced by the absence of complete markets and in particular the occasionally binding collateral constraint. Moreover, the state variables in the model are not confined to a narrow region of the state space. We approximate the equilibrium functions over the entire state space and check that equilibrium conditions are satisfied at all grid points, allowing for the two financial constraints to bind only in some states of nature.  

The introduction of bailouts introduces an additional complication, which we handle using a nested fixed-point algorithm. First, for a given bailout policy, we compute the implied competitive equilibrium (inner loop). Second, we update the bailout policy accordingly (outer loop). These two procedures are followed until the two

---

8 Off equilibrium, while all firms receive the same amount of funds in case of a systemic crisis, the marginal value of those funds depends on how leveraged they are; i.e., those with a tighter financial constraint assign a higher value to the bailout.

9 An additional complication in our setup is that variations in consumption can lead to large changes in the value of the firm, which requires a slow adjustment in the update of the consumption function along the iterations. Further details are provided in an appendix upon request.
loops converge.

3.4.1 Calibration and Functional Forms

We calibrate the model to an annual frequency using data from the U.S. economy. The functional forms for preferences and technology are the following:

$$
 u(c - G(n)) = \frac{\left(c - \frac{n^{1+\omega}}{1+\omega}\right)^{1-\sigma} - 1}{1 - \sigma} \quad \omega > 0, \sigma > 1 
$$

$$
 F(z, k, h) = z k^\alpha h^{1-\alpha} 
$$

$$
 \psi(k_t, k_{t+1}) = \frac{\phi_k}{2} \left( \frac{k_{t+1} - k_t}{k_t} \right)^2 k_t 
$$

For a first group of parameters given by $\alpha, \delta, \sigma, \omega$, we choose values that we see as reasonably conventional in the literature. The capital share $\alpha$ is set to 0.32; the depreciation rate is set at 8 percent; the risk aversion $\sigma$ is set to 2; and $\omega$ is set so that the Frisch elasticity of labor supply is equal to 1. We normalize the labor disutility coefficient $\chi$ and the average value of the TFP shock so that employment and output are equal to one in the deterministic steady state.

The TFP shock and the financial shock are modeled as independent stochastic processes. Each of these shocks is discretized using a two-state Markov chain. The TFP shock is constructed using a symmetric simple persistence rule. The realization of the shock and the persistence of the TFP shock are chosen so that in the model, fluctuations in GDP are roughly consistent with those in the data. This yields TFP realizations of plus/minus 1 percent and a probability of remaining in the same state
of 78 percent. The calibration of the financial shock is meant to be suggestive. This shock follows an asymmetric process such that during "bad times", the pledgability of capital falls by 20 percent; this event occurs 3 percent of the time and lasts on average for two years.

The capital adjustment cost parameter \( \phi_k \) is set so that the standard deviation of investment in the competitive equilibrium without bailouts roughly matches the standard deviation of investment in the data. This yields \( \phi_k = 7.8 \). The remaining parameters are the discount factor \( \beta \), the mean value of \( \kappa \), and the minimum dividend payment \( \bar{d} \). These parameters govern the values of leverage as well as how frequent the constraints on borrowing and dividend bind. For now, we set \( \beta = 0.92 \), the mean value of \( \kappa = 0.32 \) and \( \bar{d} = 0.04 \). With these values, the mean value of leverage-defined as the ratio of debt to market value of the firm-equals 0.29, which is in the upper range of corporate leverage documented by Masulis (1988); the crisis dynamics are also roughly in line with US data.

### 3.4.2 Results

Figures 1 and 2 show the laws of motion for capital and debt in the competitive equilibrium with and without bailout policy. The x-axis in the two figures is given by the current level of debt. (The level of capital is approximately the average value of capital, and the shocks are given by adverse TFP and adverse financial shocks). Let us first describe the behavior of the competitive equilibrium without bailouts.

---

10To facilitate convergence, for the time being, we use an endogenous discount factor without internalization (see Schmitt-Grohe and Uribe (2003)), such that on average \( \beta = ln(1 + c)\zeta = 0.96 \), yielding \( \zeta = 0.10 \).
Figure 3.1: Law of Motion for Capital

before analyzing the effects of credit intervention.

As Figure 1 shows, an increase in debt holdings reduces the demand for capital, since capital is risky and the valuation of future dividends becomes more sensitive to adverse shocks. Since borrowing ability depends on the next period capital, this produces a non-monotonic law of motion for debt, as shown in Figure 2. For low values of current debt, the collateral constraint is not binding. As the value of current debt increases, the demand for debt increases; but as investment is reduced, the ability to borrow shrinks. When the current level of debt is about 0.84, the collateral constraint becomes binding. This is indicated in the first vertical in Figures 1 and 2.
For $b > 0.84$, the next period debt holdings decrease in current debt holdings as the level of capital and debt are reduced in an endogenous feedback loop. The second vertical line indicates when the dividend payout constraint becomes binding. Notice that both investment and the level of future debt are reduced more sharply once the dividend payout constraint binds. This is intuitive, since a binding dividend payout constraint limits even more the access to financing.

For now, we consider bailouts that are strictly positive if and only if the dividend payout constraint is binding and it is increasing in the shadow interest rate, i.e. the interest rate that will make the collateral constraint hold with equality.
but not bind at the margin, given prices and allocations.\textsuperscript{11} This makes the bailout increase in the level of debt and decrease in the level of capital.

To understand how bailouts affect the competitive equilibrium, it is useful to analyze first its effects during periods in which the dividend payout constraint becomes binding. As Figure 2 shows, bailouts allow firms to borrow more during these periods. This occurs because as firms receive the bailout transfer, they can allocate these funds to invest more in capital, which boosts the firms’ capacity to borrow. In the region where the dividend constraint \textit{is not} binding, firms also borrow more in the competitive equilibrium with bailouts. This occurs because given that crises become less severe as a result of bailouts, there is a lower incentive to accumulate precautionary savings during normal times. This leads in turn to a higher probability of the economy becoming financially constrained in the future.\textsuperscript{12}

\section*{3.4.3 Discussion}

In our benchmark model, a bailout from workers to firms does not involve any wealth redistribution since workers are the owners of firms. This allows us to focus on the bailout’s consequences for efficiency. We can also extend our analysis to allow bailouts to have wealth redistribution effects. In particular, consider a model where firms are owned by entrepreneurs and workers do not hold any shares of the firms. In this context, while workers face a negative wealth effect as a result of carrying the burden of the bailout, there are labor-market spillovers that might

\footnotesize\textsuperscript{11}Formally, the shadow interest rate is defined as $R^{ef} = (1 + \eta_t) / E_t m_{t+1}(1 + \eta_{t+1})$

\footnotesize\textsuperscript{12}The specified bailout policy increases welfare by about 0.1 percentage point of permanent consumption (please check future updates of the paper for a complete welfare analysis).
raise their welfare. In particular, as firms do not reduce investment as much during a financial crisis, this leads to higher wages in the recovery of the crisis. For plausible calibrations, however, the welfare of workers is reduced by bailouts. Intuitively, this results from capital not increasing enough to make the increase in wages compensate for the direct cost of financing the bailout.

In our setup, there is also scope for ex-ante prudential measures. The reason is a form of overborrowing externality. While committing to a bailout in some states of nature is desirable, these interventions also impose costs that are paid by all workers. Not internalizing that becoming financially constrained during a systemic financial crisis triggers a costly intervention by the planner, firms borrow too much. Notice that while firms benefit from other firms being constrained due to the systemic nature of the bailout, this constitutes only a private gain, as the bailout imposes costs at the social level. A similar point is also made by Chari and Kehoe (2009) and Farhi and Tirole (2010). In their setup, regulation is designed to reduce the temptation to conduct bailouts. In our setup, however, regulation also improves the commitment solution.

We should point out that the possibility to improve welfare in our setup depends on the ability of the government to redirect funds from workers to financially constrained firms. Workers do not have the incentive to transfer these funds to firms because this entails only costs at the individual level. Ex ante, the rationale for committing to such interventions is to provide a form of systemic insurance against financial crises. This result also points towards the desirability of enhancing the development of private insurance markets. Clearly, however, there are reasons
why the availability of private insurance against systemic episodes is limited by a host of credit market imperfections (e.g., insurers may go bankrupt in crises). Under these conditions, our analysis suggests that the government should have a direct role in providing insurance against systemic financial crises.

Another point that we should emphasize is that we have assumed that the planner can commit to a specific bailout policy. This is motivated by debates about how to specify a legal framework to put limits on the ability of central banks to bail out the financial sector (see e.g. the Dodd-Frank Act). Under some states of nature, however, it might still be feasible for central banks to evade the legal framework. It would be interesting in our framework to study how the belief that central banks would deviate from previous commitments could generate an incentive for private agents to take even more risk as compared to an environment where the planner can commit to future policies.

3.5 Conclusion

The quantitative analysis shows that it is possible to design a realistic and simple bailout policy so that the welfare benefits of such an intervention outweigh its distortionary effects. A key feature of this intervention is that it is reserved for extreme episodes of financial distress.

Our results are relevant for ongoing debates about the appropriate role of central banks during financial crises. Our analysis points towards giving a specific mandate of intervention that supports credit flows, albeit in a strictly limited way.
Refining our analysis would require us to consider explicitly the temptation of central banks to renege on policy commitments. Within our framework, it would also be interesting to study how certain policies like capital requirements can help to offset these problems of credibility.
Appendix A

Proofs (Chapter 1)

A.1. Proof of Proposition 1 (Constrained Inefficiency)

This is a proof by contradiction. Suppose the decentralized equilibrium yields the same allocations as the constrained-efficient allocations. Then, we can combine (1.4) and (1.12), yielding:

\[ \lambda_t^{de} = \lambda_t^{sp} + \mu_t^{sp} \Psi_t \tag{A.1} \]

where we denote with superscript ‘sp’ the Lagrange multipliers of the social planner and with ‘de’ those of the decentralized equilibrium. Updating this equation one period forward and taking conditional expectations at time \( t \):

\[ \mathbb{E}_t \lambda_{t+1}^{de} = \mathbb{E}_t \lambda_{t+1}^{sp} + \mathbb{E}_t \mu_{t+1}^{sp} \Psi_{t+1} \tag{A.2} \]

Suppose that at time \( \tilde{t} \), \( b_{\tilde{t}+1} > -\left( \kappa^N p_{\tilde{t}}^N y_{\tilde{t}}^N + \kappa^T y_{\tilde{t}}^T \right) \). Combining (1.6),(1.7),(1.13), and (1.14) we obtain:

\[ \mathbb{E}_t \lambda_{\tilde{t}+1}^{de} = \mathbb{E}_t \lambda_{\tilde{t}+1}^{sp} \tag{A.3} \]

If at time \( \tilde{t}+1 \) the credit constraint binds with positive probability, comparing (A.3) and (A.2) yields a contradiction. \[\blacksquare\]
A.2. Proof of Proposition 2 (Optimal tax on debt)

This a proof by construction. Combining the optimality conditions for the social planner (1.12) and (1.13) yields:

\[ u_T(t) = \beta(1 + r)\mathbb{E}_t(u_T(t + 1) + \mu_{t+1}^{sp}\Psi_{t+1}) + \mu_t^{sp}(1 - \Psi_t) \quad (A.4) \]

First, notice that the constrained-efficient allocations are characterized by stochastic sequences \( \{c_T, c_N^N, b_{t+1}, p_{t+1}^N, \mu_t^{sp}\} \) such that the following conditions hold: (1.5),(1.8),(1.9),(1.14), (A.4) and \( \mu_t^{sp} \geq 0 \).

Second, the decentralized equilibrium allocations with taxes on debt are characterized by stochastic sequences \( \{c_T^T, c_N^N, b_{t+1}, p_{t+1}^N, \mu_t, \tau_t, T_t\} \) such that the following conditions hold: (1.5),(1.7),(1.8),(1.9),(1.17), \( T_t = b_t(1 + r)\tau_{t-1} \) and \( \mu_t \geq 0 \).

Defining the tax as \( \tau_t^* = (\mathbb{E}_t\mu_{t+1}^{sp}\Psi_{t+1}) / (\mathbb{E}_t u_T(t + 1)) - (\mu_t^{sp}\Psi_t) / (\beta(1 + r)\mathbb{E}_t u_T(t + 1)) \) and redistributing the proceeds lump sum yields that the conditions characterizing the decentralized equilibrium with the specified tax on debt are identical to those characterizing the constrained-efficient allocations.

Appendix B

An Equivalence Result (Chapter 1)

We show in this appendix that the constrained-efficient allocations can be decentralized with regulatory measures directed to the banking sector. Consider the following simple model. Banks make loans to households at rate \( r^L \) and imposing the constraint (1.2) to guarantee repayment. Banks finance these loans by accepting
deposits from the rest of the world at rate $r$ and issuing equity in the domestic markets. We assume that the required return on equity $r^e$ is higher than the rate on deposits, i.e., $r^e > r$. This could be the outcome of moral hazard or tax disadvantages on equity, but we abstract from explicitly modeling this relationship. Financial intermediation is costless. Banks last for one period, and every period new banks are set up with free entry into banking.

Without any regulation or any other frictions, banks would finance loans only with deposits, and the resulting equilibrium would be equivalent to the decentralized equilibrium. We introduce two regulatory measures. First, the planner imposes capital requirements: banks are required to finance a fraction $\gamma$ of their assets with equity. Second, the planner imposes reserve requirements: banks are required to hold a fraction $\phi$ of deposits in the form of unremunerated reserve. Thus the banks’ balance sheets become:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tbody>
<tr>
<td>$b$ Loans</td>
<td>$d$ Deposits</td>
</tr>
<tr>
<td>$f$ Reserve requirements</td>
<td>$e$ Equity</td>
</tr>
</tbody>
</table>

The objective of the bank is static and consists of maximizing shareholder value, net of the initial equity investment:

$$\max_{b,f,e,d} b(1 + r^L) + f - d(1 + r) - e(1 + r^e)$$
subject to
\[ f \leq d + e \]
\[ f \geq \phi d \]
\[ e \geq \gamma (b + f) \]

Given that holding reserves and capital is privately costly, banks do not hold excess reserves or excess capital. In equilibrium, the return from assets must be equal to the return on liabilities, i.e., \( r_L^d(1 - \phi(1 - \gamma)) = \gamma r^e + (1 - \gamma) r \). Therefore, by setting \((\phi_t, \gamma_t)\) such that \((1 + r_t^L) = (1 + r)(1 + \tau_t^*)\), the social planner can raise the cost of borrowing and induce agents to hold the socially optimal amount of debt. Assuming only capital requirements are used yields: \( \gamma_t^* = (\tau_t^*(1 + r))/ (r^e - r) \). When only reserve requirements are used, this yields: \( \phi_t^* = (\tau_t^*(1 + r))/ (r(1 + \tau_t^*) + \tau_t^*) \).

Appendix C

Sensitivity Analysis (Chapter 1)

We continue here the sensitivity analysis presented in the body of the paper. We discuss separately the effects of varying each of the parameter values of the model, using the analysis of the externality term and the elasticity decomposition studied in the body of the paper. The main quantitative results of all experiments are shown in Table 4. The table shows for each experiment the average welfare loss, the average implied tax on debt, the relative volatility of consumption, the probability of a financial crisis for the decentralized equilibrium and constrained-efficient allocations, and the effects of a median crisis in consumption, the real
exchange rate and the current account for the two equilibria.

*Discount Factor* ($\beta$).— An increase in the discount factor leads to a shift of the distribution of bond holdings towards a lower amount of debt, leading to less frequent binding constraints and causing the distribution of the externality term to concentrate higher probability in a region where its value is zero. This effect leads to smaller effects from the externality. There is an opposite effect from an increase in the discount factor. Recall that the maximum welfare gains from the externality arise in relatively tranquil times because of the reduction in future vulnerability to financial crises. Hence, a higher discount factor makes the economy value relatively more the benefits from a reduction in future variability, which should lead to higher welfare effects from correcting the externality. Quantitatively, we find that the first effect is more important. Increasing the discount factor by 0.02 reduces the average implied tax on debt to 3.3 percent, although large differences remain in the probability of financial crises: the probability of a crisis is 0.2 percent for the social planner and 4.1 percent for the decentralized equilibrium.

*Interest Rate* ($r$).— An increase in the interest rate has effects similar to an increase in the discount factor since both reduce the willingness to borrow and shifts the economy away from binding constraints. For a given amount of debt, however, a higher interest rate implies an increase in the debt service, which causes a larger depreciation of the real exchange rate. Quantitatively, we find that increasing the interest rate 100bps reduces the implied tax on debt from 5.2 percent to 4.4 percent, but the effects on the incidence and severity of financial crises remain very similar.
Risk Aversion (σ).— An increase in the risk aversion implies a higher disutility from consumption variability. This implies that a large drop in consumption generates a higher shadow value from relaxing the credit constraint at a given state where the constraint binds; therefore, this yields a higher externality term. At the same time, an increase in risk aversion makes both the social planner and private agents accumulate more precautionary savings making the constraint less likely to bind and shifting away the distribution of the externality term towards zero. Quantitatively, as shown in Table 4, we find that the effects of the externality decreases (increases) modestly when we consider $\sigma = 5$ ($\sigma = 1$).

Independent shocks.— We model tradable and nontradable endowment shocks as independent AR(1) processes and analyze the effects over the externality. When shocks are correlated, both tradable and nontradable shocks typically fall during financial crises. The fact that nontradables fall, however, mitigates the fall in the price of nontradables and the tightening of financial constraints. This channel suggests that making the two shocks independent should reduce the effects of the externality. There is another channel, however, by which making the shocks independent causes the externality to have higher effects. For the baseline calibration, the risk aversion and the elasticity of substitution between tradables and nontradables are such that tradable and nontradable goods are Edgeworth substitutes. As a result, a fall in the endowment of nontradables when the credit constraint binds, increases the marginal utility from tradable consumption, which increases the desire to borrow and increases the shadow value from relaxing the credit constraint. Quantitatively,
we find that the effects over the shadow value from relaxing the credit constraint are stronger than those affecting the price effects, so that the differences in severity of financial crises become even stronger.

**Volatility and Persistence** \((\text{Cov}(\varepsilon))\).— An increase in the volatility of endowment shocks increases the severity of financial crises, in terms of the amplification effects and the disutility cost from a binding constraint. This effect increases the externality term. At the same time, private agents have an incentive to increase relatively more precautionary savings in response to the increase in volatility. This occurs because the concavity of the utility function implies that a given increase in variability is more costly in the decentralized equilibrium compared to the constrained-efficient allocations. In fact, when we vary simultaneously the volatility of the shocks to the endowment processes by 15 percent, we find that the externality decreases modestly with a higher volatility.

An increase in persistence leads to a higher probability of financial crises for a given level of precautionary savings although it does not alter the size of the shocks and the severity of financial crises. When we vary the autocorrelation of the endowment shocks by 15 percent, we find that a higher autocorrelation is associated with larger effects from the externality. In fact, the experiment with higher autocorrelation yields larger differences in the incidence and severity of financial crises, and this leads to larger welfare effects.

**Elasticity of Substitution** \((1/(1+\eta))\).— As explained in the paper, the elasticity of substitution between tradables and nontradables determines the debt service elas-
ticity of the real exchange rate, which is in turn a key component of the externality term. Moreover, the elasticity of substitution also affects the incentive to accumulate precautionary savings: the lower the elasticity of substitution the higher the disutility from drops in consumption during financial crises. This second channel is similar to the increase in the risk aversion, but we find that the channel affecting directly the price effects are quantitatively more important.

*Share of tradables* ($\omega$).— As explained in the paper, the weight of tradables in the utility function determines the borrowing limit elasticity of the real exchange rate and is key for the effects on the externality. There is another effect of this parameter. A higher share of tradables in the utility function implies that large drops in tradables consumption during financial crises are more costly, causing an increase in precautionary savings. As explained before, this second channel becomes qualitatively ambiguous, but we find that the price effects, which unambiguously increase the externality, are more significant.

*Credit Coefficient* ($\kappa$).— We set $\kappa^T = \kappa^N = \kappa$. An increase in $\kappa$ has two effects. First, it increases directly the externality term, because for a given drop in the price of nontradables the effects over the borrowing ability are directly proportional to $\kappa$. Second, it makes the constraint less likely to bind, hence reducing the effects of the externality. On one hand, when $\kappa$ is 0, there is no borrowing; therefore an increase in $\kappa$ raises the effects of the externality. On the other hand, for a very large $\kappa$, the credit constraint never binds and there are no effects from the externality in the long run. Quantitatively, we find that increasing $\kappa$ from 0.32 to 0.36 increases the welfare
effects of the externality to 0.22 percentage points of permanent consumption. In addition, consumption during a median crisis drops almost three times as much in the decentralized equilibrium compared to the constrained-efficient equilibrium. Reducing $\kappa$ to 0.28 reduces also slightly the effects of the externality but crises in the decentralized equilibrium remain ten times more likely than in the constrained-efficient equilibrium.
Table C.1: Sensitivity Analysis

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Welfare</th>
<th>Tax on Debt</th>
<th>$\sigma_{\text{cde}} / \sigma_{\text{csp}}$</th>
<th>Probab. Crisis</th>
<th>Consumption</th>
<th>RER</th>
<th>Current Account</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>DE</td>
<td>SP</td>
<td>DE</td>
<td>SP</td>
</tr>
<tr>
<td>baseline</td>
<td>0.13</td>
<td>5.2</td>
<td>1.13</td>
<td>5.5</td>
<td>0.4</td>
<td>-16.7</td>
<td>-10.0</td>
</tr>
<tr>
<td>$\beta = 0.93$</td>
<td>0.08</td>
<td>3.3</td>
<td>1.08</td>
<td>4.1</td>
<td>0.2</td>
<td>-13.0</td>
<td>-9.2</td>
</tr>
<tr>
<td>$\beta = 0.89$</td>
<td>0.19</td>
<td>7.3</td>
<td>1.18</td>
<td>6.4</td>
<td>0.4</td>
<td>-17.9</td>
<td>-10.0</td>
</tr>
<tr>
<td>$\tau = 0.05$</td>
<td>0.11</td>
<td>4.4</td>
<td>1.11</td>
<td>4.7</td>
<td>0.3</td>
<td>-16.1</td>
<td>-10.2</td>
</tr>
<tr>
<td>$\tau = 0.03$</td>
<td>0.16</td>
<td>6.0</td>
<td>1.16</td>
<td>6.0</td>
<td>0.4</td>
<td>-17.3</td>
<td>-10.1</td>
</tr>
<tr>
<td>$\sigma = 5$</td>
<td>0.11</td>
<td>4.7</td>
<td>1.06</td>
<td>2.4</td>
<td>0.2</td>
<td>-12.8</td>
<td>-7.9</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>0.15</td>
<td>5.3</td>
<td>1.21</td>
<td>6.4</td>
<td>0.4</td>
<td>-19.4</td>
<td>-10.1</td>
</tr>
<tr>
<td>Independent shocks</td>
<td>0.15</td>
<td>5.0</td>
<td>1.2</td>
<td>4.9</td>
<td>0.3</td>
<td>-12.4</td>
<td>-2.2</td>
</tr>
<tr>
<td>Volatility $\varepsilon$ (15% less)</td>
<td>0.17</td>
<td>5.3</td>
<td>1.21</td>
<td>4.9</td>
<td>0.0</td>
<td>-18.9</td>
<td>-10.1</td>
</tr>
<tr>
<td>Volatility $\varepsilon$ (15% more)</td>
<td>0.13</td>
<td>5.0</td>
<td>1.10</td>
<td>5.5</td>
<td>0.4</td>
<td>-10.5</td>
<td>-6.5</td>
</tr>
<tr>
<td>Autocorrelation $\varepsilon$ (15% less)</td>
<td>0.12</td>
<td>5.1</td>
<td>1.12</td>
<td>6.0</td>
<td>0.4</td>
<td>-15.4</td>
<td>-9.6</td>
</tr>
<tr>
<td>Autocorrelation $\varepsilon$ (15% more)</td>
<td>0.16</td>
<td>5.2</td>
<td>1.17</td>
<td>4.9</td>
<td>0.1</td>
<td>-18.6</td>
<td>-10.5</td>
</tr>
<tr>
<td>$\kappa = 0.36$</td>
<td>0.21</td>
<td>5.9</td>
<td>1.15</td>
<td>4.3</td>
<td>0.4</td>
<td>-14.5</td>
<td>-5.6</td>
</tr>
<tr>
<td>$\kappa = 0.28$</td>
<td>0.09</td>
<td>4.7</td>
<td>1.10</td>
<td>6.4</td>
<td>0.6</td>
<td>-13.5</td>
<td>-8.2</td>
</tr>
<tr>
<td>$\omega = 0.34$</td>
<td>0.08</td>
<td>4.4</td>
<td>1.11</td>
<td>6.5</td>
<td>0.9</td>
<td>-13.8</td>
<td>-9.9</td>
</tr>
<tr>
<td>$\omega = 0.29$</td>
<td>0.22</td>
<td>5.8</td>
<td>1.14</td>
<td>4.1</td>
<td>0.0</td>
<td>-22.3</td>
<td>-10.2</td>
</tr>
<tr>
<td>$1/(\eta + 1) = 1.0$</td>
<td>0.07</td>
<td>4.3</td>
<td>1.10</td>
<td>7.7</td>
<td>1.3</td>
<td>-9.8</td>
<td>-6.3</td>
</tr>
<tr>
<td>$1/(\eta + 1) = 0.7$</td>
<td>0.19</td>
<td>5.8</td>
<td>1.15</td>
<td>4.5</td>
<td>0.5</td>
<td>-17.0</td>
<td>-8.0</td>
</tr>
<tr>
<td>Intermediate inputs</td>
<td>0.14</td>
<td>5.4</td>
<td>1.10</td>
<td>5.5</td>
<td>0.3</td>
<td>-15.4</td>
<td>-9.0</td>
</tr>
<tr>
<td>$\kappa^N / \kappa^T = 0.5$</td>
<td>0.04</td>
<td>3.8</td>
<td>1.08</td>
<td>7.0</td>
<td>2.0</td>
<td>-17.1</td>
<td>-13.8</td>
</tr>
</tbody>
</table>

Note: 'DE' represents the decentralized equilibrium, 'SP' represents the social planner. Tax on debt is the unconditional average of the optimal tax on debt. Welfare represent the unconditional average of the welfare gains from correcting the externality. $\sigma_{\text{cde}} / \sigma_{\text{csp}}$ represents the relative volatility in consumption. The tax on debt, welfare and the probability of a crisis are expressed in percentage. Consumption and the depreciation of the real exchange rate (RER) are expressed as percentage deviations from long run values during a median crisis (see footnote 12 in the body of the paper). Current account is expressed as a percentage of GDP.
Appendix D

Numerical Solution Method (chapter 1)

The computation of the competitive equilibrium requires solving for functions $B(b,y), P^N(b,y), C^T(b,y)$ such that:

\begin{equation}
P^N(b,y) = \left(\frac{1 - \omega}{\omega}\right) \left(\frac{C^T(b,y)}{y^N}\right)^{\eta+1}
\end{equation}

\begin{equation}
u_T(C^T(b,y), y^N) \geq \beta(1 + r)E_{y'\mid y}u_T(C^T(B(b,y), y'), y^{N'}) \end{equation}

\begin{equation}B(b,y) \geq - \left(\kappa N P^N(b,y) y^N + \kappa T y^T\right) \quad \text{with } = \text{ if (25) holds with strict inequality}
\end{equation}

\begin{equation}B(b,y) + C^T(b,y) = b(1 + r) + y^T \end{equation}

where $u_T(C^T(b,y), y^N) = u_C(C(b,y))C^T(b,y), C(b,y) = \left[\omega (C^T(b,y))^{-\eta} + (1 - \omega) (y^N)^{-\eta}\right]^{-\frac{1}{\eta}}$

and $y = (y^T, y^N)$.

The algorithm employed to solve for the competitive equilibrium is based on the time iteration algorithm modified to address the occasionally binding endogenous constraint. The algorithm follows these steps:

1. Generate a discrete grid for the economy’s bond position $G_b = \{b_1, b_2, \ldots b_M\}$ and the shock state space $G_Y = \{y_1, y_2, \ldots y_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 800

\footnote{For the social planner’s allocations, we use a standard value function iteration algorithm.}
points in the grid for bonds and interpolate the functions using a piecewise linear approximation.

2. Conjecture \( \mathcal{P}^N_K(b, y), \mathcal{B}_K(b, y), \mathcal{C}^T_K(b, y) \) at time \( K \) \( \forall b \in G_b \) and \( \forall y \in G_Y \).

3. Set \( j = 1 \)

4. Solve for the values of \( \mathcal{P}^N_{K-j}(b, y), \mathcal{B}_{K-j}(b, y), \mathcal{C}^T_{K-j}(b, y) \) at time \( K - j \) using (D.1),(D.2),(D.3),(D.4) and \( \mathcal{B}_{K-j+1}(b, y), \mathcal{P}^N_{K-j+1}(b, y), \mathcal{C}^T_{K-j+1}(b, y) \), \( \forall b \in G_b \) and \( \forall y \in G_Y \):

   (a) Set \( \mathcal{B}_{K-j}(b, y) = - (\kappa^N \mathcal{P}^N_{K-j+1}(b, y)y^N + \kappa^T y^T) \) and compute \( \mathcal{C}^T_{K-j}(b, y) \) from (D.4)

   (b) Compute

   \[
   U = u_T(\mathcal{C}^T_{K-j}(b, y), y^N) - \beta(1 + r)E_{y'/y}u_T(\mathcal{C}^T_{K-j}(\mathcal{B}_{K-j}(b, y), y'), y'^N)
   \]

   (c) If \( U > 0 \), the credit constraint binds; move to (e).

   (d) Solve for \( \mathcal{B}_{K-j}(b, y), \mathcal{C}^T_{K-j}(b, y) \) using (D.2) and (D.4) with a root finding algorithm.

   (e) Set \( \mathcal{P}^N_{K-j}(b, y) = \left( \frac{1 - \omega}{\omega} \right) (\frac{\mathcal{C}^T_{K-j}(b, y)}{y^N})^{\eta+1} \)

5. Evaluate convergence. If \( \sup_{b \in G_b, y \in G_Y} \| x_{K-j}(b, y) - x_{K-j+1}(b, y) \| < \varepsilon \) for \( x = \mathcal{B}, \mathcal{C}^T, \mathcal{P}^N \) we have found the competitive equilibrium. Otherwise, set \( x_{K-j}(b, y) = \alpha x_{K-j}(b, y) + (1 - \alpha)x_{K-j+1}(b, y) \) and \( j \sim j + 1 \) and go to step 4. We use values of \( \alpha \) close to 1.
Appendix E

Numerical Solution Method (chapter 2)

The computation of the competitive equilibrium requires solving for functions $B(b, \varepsilon), q(b, \varepsilon), C(b, \varepsilon), N(b, \varepsilon), \mu(b, \varepsilon)$ such that:

\[
C(b, \varepsilon) + \frac{B(b, \varepsilon)}{R} = \varepsilon F(K, N(b, \varepsilon)) + b \tag{E.1}
\]

\[
-\frac{B(b, \varepsilon)}{R} + \theta G'(N(b, \varepsilon))N(b, \varepsilon) \leq \kappa q(b, \varepsilon)K \tag{E.2}
\]

\[
u'(t) = \beta RE_{\delta'/\varepsilon} \left[ u'(C(B(b, \varepsilon), \varepsilon')) \right] + \mu(b, \varepsilon) \tag{E.3}
\]

\[
\varepsilon F_n(K, N(b, \varepsilon)) = G'(N(b, \varepsilon))N(b, \varepsilon)(1 + \theta \mu(b, \varepsilon)/u'(C(b, \varepsilon))) \tag{E.4}
\]

\[
q(b, \varepsilon) = \frac{\beta E_{\delta'/\varepsilon} \left[ u'(c(B(b, \varepsilon), \varepsilon')) \varepsilon' F_k(K, N(B(b, \varepsilon), \varepsilon')) + q(B(b, \varepsilon), \varepsilon') \right]}{(u'(C(b, \varepsilon)) - \mu(b, \varepsilon)\kappa)} \tag{E.5}
\]

We solve the model using a time iteration algorithm developed by Coleman (1990) modified to address the occasionally binding endogenous constraint. The algorithm follows these steps:\footnote{For the social planner’s allocations, we use the same algorithm operating on the planner’s optimality conditions.}

1. Generate a discrete grid for the economy’s bond position $G_b = \{b_1, b_2, ... b_M\}$ and the shock state space $G_\varepsilon = \{\varepsilon_1, \varepsilon_2, ... \varepsilon_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 300
points in the grid for bonds and interpolate the functions using a piecewise linear approximation.

2. Conjecture $B_K(b, \varepsilon), q_K(b, \varepsilon), C_K(b, \varepsilon), N_K(b, \varepsilon), \mu_K(b, \varepsilon)$ at time $K$ for all $b \in G_b$ and all $\varepsilon \in G_\varepsilon$.

3. Set $j = 1$

4. Solve for the values of $B_{K-j}(b, \varepsilon), q_{K-j}(b, \varepsilon), C_{K-j}(b, \varepsilon), N_{K-j}(b, \varepsilon), \mu_{K-j}(b, \varepsilon)$ at time $K-j$ using (E.1), (E.2), (E.3), (E.4), (E.5) and $B_{K-j+1}(b, \varepsilon), q_{K-j+1}(b, \varepsilon), C_{K-j+1}(b, \varepsilon), N_{K-j+1}(b, \varepsilon), \mu_{K-j+1}(b, \varepsilon)$ for all $b \in G_b$ and all $Y \in G_Y$:

   (a) Assume collateral constraint (E.2) is not binding. Set $\mu_{K-j}(b, \varepsilon) = 0$ and solve for $N_{K-j}(b, \varepsilon)$ using (E.4). Solve for $B_{K-j}(b, \varepsilon)$ and $C_{K-j}(b, \varepsilon)$ using (E.1) and (E.3) and a root finding algorithm.

   (b) Check whether $-\frac{B_{K-j}(b, \varepsilon)}{R} + \theta G'(N(b, \varepsilon)) N_{K-j}(b, \varepsilon) \leq \kappa q_{K-j+1}(b, \varepsilon) \overline{K}$ holds.

   (c) If constraint is satisfied, move to next grid point.

   (d) Otherwise, solve for $\mu(b, \varepsilon), N_{K-j}(b, \varepsilon), B_{K-j}(b, \varepsilon)$ using (E.2), (E.3) and (E.4) with equality.

   (e) Solve for $q_{K-j}^N(b, \varepsilon)$ using (E.5)

5. Evaluate convergence. If $sup_{B, \varepsilon}\| x_{K-j}(B, \varepsilon) - x_{K-j+1}(B, \varepsilon) \| < \epsilon$ for $x = B, C, q, \mu, N$ we have found the competitive equilibrium. Otherwise, set $x_{K-j}(B, \varepsilon) = x_{K-j+1}(B, \varepsilon)$ and $j \sim j + 1$ and go to step 4.
Bibliography


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