

ABSTRACT

Title of dissertation: **THREE CHAPTERS
ON HEDGE FUND RESERVE CAPITAL
AND SYSTEMIC RISK**

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Hedge fund industry has grown to be a key player in the financial markets. Just as large investment banks, the failure of this industry will greatly destroy the liquidity and stability of the whole system. However, contrast to regulated mutual funds, hedge funds are private and lightly regulated entities who are not obliged to disclose their activities to the general public. Hedge funds risk taking activity using ways such as short selling and excessive leverage and their increasingly correlated strategies pose substantial threats to the financial stability of the great economy.

In the First Chapter, we propose a simple framework which adopts the theory of acceptable risks and calculate capital requirements using the limited available data on hedge funds. We model the risky cash flow asset less liability (or Net Asset Value) directly using either a Gaussian process or a Variance Gamma process and apply the method to demeaned NAV data on 3622 hedge funds from January 2005 to April 2009. Funds are analyzed for their required capital and the value of the option to put losses back to the taxpayers.

The previous study has considered funds individually with no correlation between them. Focusing only on individual funds ignores the critical interactions between them and can cause the regulators to overlook important changes in the overall system. Because many hedge funds employ similar investment strategies they produce correlated returns. The failure of these correlated large funds will greatly affect the markets systematically either in a direct or an indirect way. In the Second Chapter, we propose a systemic approach with correlated largest market participants and we study the 30 largest funds as of April 2009 with total Asset Under Management over \$620 Bn. We demonstrate the systemic capital charges to be held by the broad economy, as well as the capital charges at the fund level accounting for the residual idiosyncratic risk component.

Hedge fund investment strategies often include the use of leverage in order for them to build up large positions. Extensive use of leverage has increased funds liabilities especially during market downturns and has posted a great systemic risk to the economy in large. In the Third Chapter, we recognize that with limited and incomplete information on hedge funds balance sheet positions, the public usually does not know how much leverage there is in a particular fund or how to distinguish its assets and liabilities from the observed returns. We estimate hedge fund leverage using a regression-based exercise on the individual fund level. The estimated leverage information is then combined with publicly known return and other fund information to separate from fund cash flows its asset side and liability side. The two sides of the cash flows are then modeled as exponentials of two correlated Lévy processes following [36]. Capital implications are then derived from the above setup.

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AND SYSTEMIC RISK

by

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Dedication

To Aaron, the love of my heart. And to the love of my life: Atti, Julius.

Acknowledgments

Do I really have to do this, again? Yes, I do! Aren't I supposed to be good at this now? Well, let's see.

A second Ph. D. is a painful experience, no matter from what angle you are coming in. It is beyond challenging. However, I would still categorize this as an achievement (a gain almost offset by the loss, but, still a gain). This achievement is a result of persistence, self discipline, and lots of help from people like family, advisors, friends and caregivers.

My husband, P. Aaron Lott has played an essential role in this experience. I wouldn't say I did this degree for him, but I did start this journey because of him. At the time of graduating from my first Ph. D., he was still in the AMSC program. Staying in Maryland and accepting the nice offer from the Finance department was at the time our best option to be together. Story continued, we got married, I got a job, and we had kids and even moved to California in the end. All of this didn't stop me, because I wouldn't give up, and because Aaron has always been there for me. I wouldn't say he is the silent author of my thesis, but without him, my thesis will probably look a lot more empty. Nothing more is enough to show my gratitude, I just want to say: I appreciate you and love you, let's walk, walk together, far and long... In the last three years, I also became a mother to my two wonderful boys Atticus X. Lott and Julius G. Lott. I have enjoyed the most precious moments with them and they have profoundly changed my being forever. I have to say they played a big role in this experience also, as they are the sunshine of my rainy days, and

the rain storm in sunny days. I will not forget those hard working days Julius and I spent in my study room/guest bedroom, with him either trying to “teach” me how to use my mouse sitting on my lap, or him looking at me through the mirror making sure I wasn’t slacking. As to my big boy Atticus, he’s really quite a teacher and I am being trained so well, in every different way. I love being with you and thank you both being there with me! Other family members I would like to say thank you are Aunt Sophie Qu, Aaron’s mother DeeAnn Smith, Aaron’s father Paul W. Lott, and especially my mother Yin Xiao who has helped me significantly.

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Life is like a current and carries me to places, islands, open waters. Some of them are wonderful and the others are treacherous, the rest are forgettable. They

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Chapter 1

Hedge Fund Reserve Capital and Taxpayer Put Option

1.1 Overview

The hedge fund industry has grown tremendously over the past years, fueled by the demand for higher returns in the face of stock market declines and mounting pension fund liabilities. A general estimate of the industry shows that it had managed around \$2.5 trillion at its peak in summer 2008 ([49]). Hedge funds contribute more than half of average trading volume in equity and corporate bond markets (back in 2005 [15]) and they are major liquidity providers in normal times. The credit crunch has caused assets under management (AUM) to fall sharply through a combination of trading losses and the withdrawal of assets from funds by investors. Nonetheless, recent estimates still find hedge funds with more than \$2 trillion in AUM and this industry is without a doubt a key player in the capital markets.

Despite its growing size and its importance in the market place, hedge fund industry has not been under the same scrutiny as banks or other investment entities. A hedge fund is typically set up as a limited partnership, a limited liability corporation in the United States, or as an offshore corporation. Most often, hedge funds that are based in the United States take the form of a limited partnership organized under section 3(c)(1) of the Investment Company Act (some organized under 3(c)(7)), and hence being exempt from most U.S. Securities and Exchange

Commission (SEC) regulations. These special investment entities can go long or short any number of securities, any type of securities, including derivatives and structured products, with varying degrees of leverage. Hedge fund managers enjoy enormous flexibility and discretion in pursuing fund performance and can change investment strategies at a moment's notice. The risks in hedge fund investment are easily underestimated and its transparency to the general public needs to be greatly improved. A number of empirical studies have highlighted the unique risk and reward profiles of hedge fund investments. For example, [2], [5], [6], [37], [40], [41], [43], [51], [53], [54], and [55] provide comprehensive empirical studies of historical hedge fund performance using various hedge fund databases. [4], [7], [16], [17], [18], [19], [20], [39], [42], and [57] present more detailed performance attribution and "style" analysis for hedge funds. Collectively, these studies show that the dynamics of hedge funds are quite different than those of more traditional investments, and the potential impact on systemic risk is apparent. The banking sector is exposed to hedge fund risks, especially smaller institutions. Even the largest banks are also exposed to hedge fund risks through proprietary trading activities, credit arrangements, structured products, and prime brokerage services. As a result, the risk exposures of the hedge fund industry may have a material impact on the banking sector, resulting in new sources of systemic risks ([27]). To include systemically important utilities into the realm of regulation and supervision has become the point of attention urged by recent crisis, and is exactly the motivation of our study here. As Dodd Frank Wall Street Reform and Consumer Protection Act ([1]) pass through the congress, Financial Stability Oversight Council (FSOC) has been set forth and

many other actions are underway. Questions remain however, how to identify who is systemically important and how should they be regulated once identified? Although a very important task, our objective is not to put ourselves in the position of such identifying process. Rather, we devote our effort here on demonstrating a theoretical framework using existent analytical theory, which provides regulatory bodies the guidance in the situation where these systemically important entities have been identified. Especially, we conduct our analysis using the data available for hedge funds and hope to provide some basic tool in the development of hedge fund regulation (for such approach applied on banks see [36], [59]).

Banks in the United States and many other countries must satisfy regulatory capital requirements that are intended to ensure that they can withstand reasonable losses. These are well know and commonly adopted as Basel capital requirements ([12]). These requirements are generally specified as a ratio of some measure of capital to some measure of assets, such as total assets or risk-adjusted assets. Maximum effort has been expanded on trying to assess the relative riskiness of assets that banks hold and determine the risk-based capital requirements (firms that hold riskier assets have higher capital requirements). [60] redefines the corporate balance sheet for relevance to two price markets related to Conic Finance introduced in [29], and argues however, that equity capital is a poor measure of financial health as it can be contaminated by the excessive value of the taxpayer put option ([60]). The taxpayer put option is first defined in [36], where they propose that the objective of a credit policy as an arm of regulation is to ensure that the value of this freely distributed put option is kept within limits and is not allowed to get excessively

valuable. Following the procedures outlined in [36] which lead to a new capital policy directed towards risk-based requisite levels of reserve capital, we propose an approach where the risky cash flow (asset less liability) is directly modeled. Based on the risks in total cash flow, we then construct the cash reserve capital as a buffer to be held by the firm. Here the term “capital” is in the sense of cash or cash equivalent reserves, distinguished from Basel equity capital. This cash buffer can also be viewed as a liquidity buffer as such preventative measure against liquidity shocks has been called for. As markets become more volatile and the the funds risk exposure increases significantly, this liquidity buffer prevents funds from liquidating large positions in short period of time which may lead to a wide-spread financial panic in the face of investors withdrawal and disruption of credit.

The classical Merton intuition ([64], [65], [66]) of a contingent claims analysis of equity has taught us how to see equity in a world with only random assets but no random liabilities. However, a lot has changed since the world has grown into a host of random unhedgeable liabilities with access to modern financial markets. Hedge funds are by nature good examples of holding balance sheets that contain both random cash flows as assets and another set of potentially unbounded random cash flows as liabilities. Among a variety of methods, hedge funds often use short-selling to increase rather than “hedge” their risk, with the expectation of increasing the return on their investment. The possibility that the liabilities may become limitless (unless the short position exactly hedges a corresponding long position) and dominate the assets determines that the fund cannot be permitted to exist as a limited liability entity if it is insufficiently capitalized. Once a fund is allowed to

exist as a limited liability entity, it accesses for free the option to put excessive losses back to the economy. This option of putting the excessive losses back to the economy is studied and termed the Taxpayer Put in [36]. Whether one calls it taxpayer put, or counter-party put, we have to realize that this put option is held as an asset by the limited liability entities and is distributed to them for free. If the value of this put is not being properly monitored and constrained, it will contaminate the equity ([60]) and it will give rise to incentives of manipulation and maximizing the put value, at least in the short term. Some did as we have clearly seen in the past crisis. Our argument is that no one can be permitted to exist and given a limited liability status if no funds are placed at stake with the capacity to absorb potential losses. Such level of funds placed at stake is the cash reserve capital studied in our research, as well as the put values. The research is from the perspective of generalizing risks acceptable to the general economy and to implement the theory of acceptable risks. The excessive leverage many hedge funds also employ only exacerbate the problem even further. This put option exists even with the presence of counter-party enforcement (such as raising collateral and margin requirement) and due diligence performed by the large institutional investors. The mechanisms and incentives in the market place enforced by market participants are rather segmented and discrete, and hence ineffective due to the complexity of relationships and limited market powers. In a system that faces substantial systemic risk, these mechanisms and incentives must be accompanied by new measures that are more systemic and transparent.

The need to regulate hedge funds arises from the presence of the implicit put

value. Once we recognize the existence of this put option born by the setup of limited liable entities holding also potentially unbounded liabilities (even without the presence of debt), the regulatory bodies must guarantee that sufficient capital is put at stake to ensure the risk of excess loss acceptable to taxpayers. Such precise link between capital reserve and acceptable risks ([31]) have been proposed by [59] and studied further in [36]. Following the earlier work of [10], [22], [50], the risks acceptable to the general economy have been given operational definition by focusing attention on the positive expectation under a sufficiently concave distortion of the probability distribution of the risks being undertaken. [30] give parameterized families of such distortions with parameter γ used to measure the level of stress being placed on the cash flow distribution to test for its acceptability at such level. In [59], such capital calculation is carried out with modeling assets and liabilities separately and paying attention to the correlation. For hedge funds however, this approach may not be easily applicable since it is unclear how to separate their balance sheets (for an attempt using estimated fund leverage to separate assets and liabilities and model them separately with correlation, see [72]). Contrast to regulated mutual funds, hedge funds are private and lightly regulated entities who are not obliged to disclose their activities to the general public. Data on hedge funds are reported as funds wish and usually incomplete and very limited. As a natural choice for an initial attempt, we model instead the net cash flow (on a per share basis) as a real-valued martingale and propose two models one with Gaussian components and one with Lévy jump components to account for skewness and kurtosis. The two models studied here are Bachelier model and Variance Gamma model ([61], [26]). We study 3622 funds who

have monthly NAV and return data for the period of Jan 2005 to April 2009. We fit the demeaned data with the models using maximum likelihood estimation and obtain model parameters that are then fed to simulation and derive risk determined capital requirement as well as put option values to be monitored. The distortion function is chosen to be MINMAXVAR and three increasing stress levels of γ , 0.25, 0.75, 1.25 are considered. The fatter-tailed distribution of VG normally generates higher capital requirement under otherwise same conditions. The results show that under VG model with stress 1.25 (the most stringent requirement) there were 127 funds insufficiently capitalized by April 30, 2009. We also show sensitivity of the calculated capital and the put value to the underlying risk parameters, i.e. volatility in the Gaussian model and volatility, skewness and kurtosis in the VG model. It is suggested in [59] that the level of γ may be calibrated by selecting the smallest value at which the preserved capital requirement mitigates the perverse risk incentives. We also show similar analysis on such mitigation of perverse risk incentives, and stress level 1.25 is shown to be a needed level for the sensitivity of capital to risk dominating the sensitivity of equity value to risk, given the presence of limited liability.

The outline of the rest of the chapter is as follows. Section 1.2 builds a model framework which allows correlated random assets and random liabilities both contributing to the final risky cash flows as an extension to the [65] model. Section 1.3 takes hedge fund perspective to model the total cash flow as a real-valued Martingale, and describes the calculation of required cash reserve and taxpayer put. In Section 1.4 we present two models for the total cash flows and their estimation. Sec-

tion 1.5 includes two numerical studies showing the effect of risk parameters on the capital charge and put value, as well as the corrected risk incentives in the presence of capital constraints. Section 1.6 presents results of analysis on 3622 funds. Finally, Section 3.4 concludes.

1.2 Extension to Merton (1974) - Managing Risks with Random Liabilities

[65] assumes that a company has a certain amount of zero-coupon debt that will become due at a future time T . The company defaults if the value of its assets is not equal to the promised debt repayment at time T . This model views balance sheet as of consisting assets that are random and liabilities that are stable with debt serving as the strike for equity viewed as an option at this strike. This world of no random unhedgeable liabilities has changed. Big investment banks nowadays write random unhedgeable liabilities on a daily basis. Firm volatilities (and skewness and kurtosis) consist of not only contribution from the assets but also the liabilities that are not perfectly hedged. In the case of a hedge fund, it is by nature that both assets and liabilities are random and correlated. The short-selling activity that hedge funds engage in on a daily basis is by nature a very risky one, since the losses incurred on a losing bet are theoretically limitless, unless the short position directly hedges a corresponding long position. Random liabilities and off balance sheet items are seen everywhere. How should the risks being managed with random assets and random liabilities? What is the equity of a hedge fund and what capital

needs to be posted to hold a long-short position? How should firms maximize their profit in this context and how should the government regulate them to be socially responsible?

We start by considering a long-short hedge fund that is long $A(t)$ dollars and short $L(t)$ dollars. For a balanced fund we have $A(0) = L(0)$. We extend the Merton model by assuming both random assets and liabilities with

$$A(t) = A(0)e^{rt + \sigma_A \sqrt{t} Z_A - \frac{\sigma_A^2}{2} t}$$

$$L(t) = L(0)e^{rt + \sigma_L \sqrt{t} Z_L - \frac{\sigma_L^2}{2} t}$$

$$\text{Corr}(Z_A, Z_L) = \rho$$

It follows that conditional on Z_L we have

$$Z_A = \rho Z_L + \sqrt{1 - \rho^2} Z \tag{1.1}$$

where Z is independent of Z_L .

This is a direct extension to Merton's model where assets and liabilities are now correlated log-normal processes. To calculate equity in this context, we first assume the fund has limited liability and post capital \mathcal{C} in future dollars. The equity in the fund at a future date t when the hedge fund is sold or liquidated is

$$(A(t) - L(t) + \mathcal{C})^+. \tag{1.2}$$

This is a call option with strike $-\mathcal{C}$, or it could also be viewed as an option written on $A(t)$ with random strike of $-\mathcal{C} + L(t)$ and we may value this call on the conditional law of $A(t)|L(t)$ and then integrating out the variable $L(t)$, or specifically,

$$E = E[E[e^{-rt}(A(t) - (L(t) - \mathcal{C}))^+ | L]] \quad (1.3)$$

Since we have that

$$f(A|L) = \frac{f(A, L)}{f(L)} = \frac{1}{\sqrt{2\pi}A(t)\sigma_A\sqrt{(1-\rho^2)t}} \exp\left[-\frac{\left(\log\left[\frac{A(t)}{A(0)}\left(\frac{L(0)}{L(t)}\right)^{\frac{\sigma_A\rho}{\sigma_L}}\right] - rt + qt + \frac{\sigma_A^2}{2}(1-\rho^2)t\right)^2}{2(1-\rho^2)\sigma_A^2t}\right],$$

where

$$q = \frac{\rho r \sigma_A}{\sigma_L} + \frac{\rho^2 \sigma_A^2}{2} - \frac{\sigma_A \sigma_L \rho}{2} \quad (1.4)$$

This is log-normal as in Merton's calculation with shifted spot, interest rate r , dividend yield q and volatility $\sigma_A\sqrt{1-\rho^2}$. The equity is call option on assets like in Merton with a strike of $-\mathcal{C} + L(t)$ and we may value this call using the Black Scholes formula

$$bsp(S, K, r, q, \sigma, t, call) \quad (1.5)$$

at the values

$$bsp\left(A(0) \left(\frac{L(t)}{L(0)}\right)^{\frac{\sigma_A\rho}{\sigma_L}}, L(t) - \mathcal{C}, r, \frac{\rho r \sigma_A}{\sigma_L} + \frac{\rho^2 \sigma_A^2}{2} - \frac{\sigma_A \sigma_L \rho}{2}, \sigma_A\sqrt{1-\rho^2}, t, call\right) \quad (1.6)$$

For the value of equity we integrate this Black Scholes call price with respect to the law of $L(t)$ which is

$$\frac{1}{\sqrt{2\pi}\sigma_L\sqrt{t}L(t)} \exp\left(-\frac{\left(\ln(L(t)/L) - rt + \frac{\sigma_L^2}{2}t\right)^2}{2\sigma_L^2t}\right). \quad (1.7)$$

Hence the extended model requires as inputs σ_A, σ_L, ρ and values equity as an integral of call options on assets perturbed by the conditional law of $A(t)$ given $L(t)$.

We next show models using Lévy processes as the underlying. Lévy processes are more flexible modeling agents than pure Gaussian ones in terms of admitting jumps and being capable of capturing skewness and kurtosis in addition to volatility. We start with the well-studied Variance Gamma family of distributions ([61], [26]), and show the dependence modeling in this context. One could extend this to the four parameter family of CGMY processes ([23]).

Recall that a VG process $VG(t; \theta, \sigma, \nu)$ can be considered a Brownian motion $\theta t + \sigma B(t)$ time-changed by a gamma process $G(t; 1, \nu)$. Here the gamma process $G(t; 1, \nu)$ with unit mean rate and variance rate ν has independent gamma increments. The characteristic function for $VG(t; \theta, \sigma, \nu)$ is given by

$$\Phi_{VG(t)}(u) = \left(\frac{1}{1 - i\theta\nu u + (\sigma^2\nu/2)u^2} \right)^{t/\nu} \quad (1.8)$$

We often model with centered or demeaned VG so that the mean is subtracted from the process as

$$H(t) = \theta(G(t) - t) + \sigma B(G(t)) \quad (1.9)$$

and the centered VG process has characteristic function

$$\Phi_{H(t)}(u) = \left(\frac{1}{1 - i\theta\nu u + (\sigma^2\nu/2)u^2} \right)^{t/\nu} \cdot e^{-iu\theta t} \quad (1.10)$$

We assume now the long and short sides are

$$\begin{aligned} A(t) &= A(0)e^{rt+X_1(t)-\omega_1 t} \\ L(t) &= L(0)e^{rt+X_2(t)-\omega_2 t} \end{aligned} \quad (1.11)$$

X_1 and X_2 are two centered VG processes and we wish to correlate them shortly. ω_1 and ω_2 are compensators for exponential VG processes on each marginal or specifically

$$\omega_i = \frac{1}{t} \log \Phi(-i) = -\frac{1}{\nu_i} \log\left(1 - \theta_i \nu_i - \frac{\sigma_i^2 \nu_i}{2}\right) - \theta_i \quad (1.12)$$

Unless it is specified otherwise, from now on we will always work with centered VG.

There are many ways to build dependence between the two VG variables, we illustrate the following two.

Building Dependence 1

We rewrite the parameters first by letting

$$\begin{aligned} C &= \frac{1}{\nu} \\ G &= \left(\sqrt{\frac{\theta^2 \nu^2}{4} + \frac{\sigma^2 \nu}{2}} - \frac{\theta \nu}{2} \right)^{-1} \\ M &= \left(\sqrt{\frac{\theta^2 \nu^2}{4} + \frac{\sigma^2 \nu}{2}} + \frac{\theta \nu}{2} \right)^{-1}, \end{aligned}$$

or change it back by

$$\nu = \frac{1}{C} \quad \theta = C \left(\frac{1}{M} - \frac{1}{G} \right) \quad \sigma^2 = \frac{2C}{GM}. \quad (1.13)$$

the Lévy measure associated with such $VG(t; C, G, M)$ process is

$$k_{VG}(x) = C[e^{-Mx} I_{(x>0)} + e^{Gx} I_{(x<0)}] / |x| \quad (1.14)$$

and the characteristic function is rewritten to be

$$\Phi_{VG(t)}(u) = \left[1 + i \left(\frac{1}{G} - \frac{1}{M} \right) u + \frac{u^2}{GM} \right]^{-Ct} \quad (1.15)$$

We would like to build dependence between two marginal VG variables by letting

$$\begin{aligned} X_1(t) &= Y_1(t) + Y(t) \\ X_2(t) &= Y_2(t) + Y(t) \end{aligned}$$

where Y_1 , Y_2 and Y are three independent VG with common parameters G, M and

$$\begin{aligned} C_1 &= C_2 = (1 - \alpha)C \\ C_Y &= \alpha C \end{aligned}$$

with $\alpha \in [0, 1]$. It is easy to derive the bivariate characteristic function between X_1 and X_2

$$\begin{aligned} &\Phi_{X_1(t), X_2(t)}(u_1, u_2) \\ &= E[e^{iu_1 X_1 + iu_2 X_2}] \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{iu_1 x_1 + iu_2 x_2} f_{x_1, x_2}(x_1, x_2) dx_1 dx_2 \tag{1.16} \\ &= \Phi_Y(u_1 + u_2) \Phi_{Y_1}(u_1) \Phi_{Y_2}(u_2) \\ &= \left[1 + i \left(\frac{1}{G} - \frac{1}{M} \right) (u_1 + u_2) + \frac{(u_1 + u_2)^2}{GM} \right]^{-\alpha C t} \cdot e^{i(u_1 + u_2) C \left(\frac{1}{G} - \frac{1}{M} \right) t} \\ &\quad \left[1 + i \left(\frac{1}{G} - \frac{1}{M} \right) u_1 + \frac{u_1^2}{GM} \right]^{-(1-\alpha) C t} \left[1 + i \left(\frac{1}{G} - \frac{1}{M} \right) u_2 + \frac{u_2^2}{GM} \right]^{-(1-\alpha) C t} \end{aligned}$$

Building Dependence 2

Following [52] we assume

$$A(t) = A(0)e^{X_A}$$

$$L(t) = L(0)e^{X_L}$$

where

$$X_A = rt - \omega_X t + \beta x(t) + y(t)$$

where $x(t)$ and $y(t)$ are two independent VG processes and $y(t)$ is associated with parameters σ_y , ν_y and θ_y , and

$$X_L = rt - \omega_x t + x(t)$$

where $x(t)$ is associated with parameters σ_x , ν_x and θ_x . Also we have that

$$\omega_x = -\frac{1}{\nu_x} \log\left(1 - \theta_x \nu_x - \frac{\sigma_x^2 \nu_x}{2}\right) \quad (1.17)$$

$$\omega_X = -\frac{1}{\nu_x} \log\left(1 - \beta \theta_x \nu_x - \frac{\sigma_x^2 \nu_x \beta^2}{2}\right) - \frac{1}{\nu_y} \log\left(1 - \theta_y \nu_y - \frac{\sigma_y^2 \nu_y}{2}\right) \quad (1.18)$$

The joint characteristic function of X_A and X_L can be easily derived

$$\begin{aligned} \Phi_X(u_1, u_2) &= \exp[iu_1(r - \omega_X)t + iu_2(r - \omega_x)t] \\ &= \left(\frac{1}{1 - i(u_1\beta + u_2)\theta_x \nu_x + \frac{1}{2}\sigma_x^2 \nu_x (u_1\beta + u_2)^2} \right)^{\frac{t}{\nu_x}} \left(\frac{1}{1 - iu_1\theta_y \nu_y + \frac{1}{2}\sigma_y^2 \nu_y u_1^2} \right)^{\frac{t}{\nu_y}} \end{aligned}$$

The calculation of the equity price in case of Lévy underliers can not be derived in closed form as in the Gaussian case, and it depends on the joint characteristic function. The evaluation of the equity contract is equivalent of a spread option and following [47] the computation uses a 2-D FFT method, which we describe in detail in the Appendix A. Before we end the section, let us illustrate a numerical example.

Numerical Example

In order to calculate the capital required from (3.32) we first need to simulate the cash flow X . VG density function is derived from Fourier transform the VG characteristic function (1.8) using FFT as described in [24], and the inversion method is then used to generate random independent VG variables.

We take $t = 1$, and let $\alpha = 0.4$ and $C = 1$, hence $C_1 = C_2 = 0.6$ and $C_Y = 0.4$. Also $G = 5$ $M = 8$, $r = 0.05$ and $A(0) = L(0) = 100$. We generate 3 sets (each 10000 draws) of independent VG variables

$$Y_1 \sim VG(C_1, G, M)$$

$$Y_2 \sim VG(C_2, G, M)$$

$$Y \sim VG(C_Y, G, M)$$

then building the two dependent VG

$$X_1 = Y_1 + Y \sim VG(C_1 + C_Y, G, M)$$

$$X_2 = Y_2 + Y \sim VG(C_2 + C_Y, G, M).$$

And then cash flows A and L are calculated with

$$\omega_i = -(C_i + C_Y) \log \left[1 + \left(\frac{1}{G} - \frac{1}{M} \right) - \frac{1}{GM} \right]$$

We show results with different stress levels being set at 0.25, 0.75 and 1.25. The equity calculation is also presented in the table (Table 1.1), while the detail numerical procedure to calculate the equity value is explained in the Appendix A.

Table 1.1: Numerical Example of Calculated Capital and Equity from Simulation.

stress	0.25	0.75	1.25
capital	9.6279	27.2734	42.6279
equity	15.8085	28.3582	41.5169

We see that the higher the stress level which are progressively higher levels of acceptability, the higher the capital needed, and higher capital corresponds to lower the negative strike of the call option and hence the higher the equity value as well.

We have shown in this section how to manage risks with random assets as well as random liabilities by modeling the asset and the liability as correlated positive random variables. Alternatively we may model net assets \widetilde{X} as a real valued process. In the case of hedge funds, in fact, we do not have readily available data to enable the separation of assets and liabilities and perform any empirical study. One approach is to estimate the fund leverage and separate balance sheet information from know data on NAV and AUM, etc. This is studied in a future paper [72]. However, to start with our initial interest on demonstrating capital regulation on hedge funds, we take what is available and model only the total cash flow without separating the positive and the negative sides. The next section begins our theoretical framework.

The rest of the chapter takes this simplified framework as model and demonstrate empirical studies done both numerically and with actual hedge fund data.

1.3 Hedge Fund Required Cash Reserve and Taxpayer Put

Any financial entity, be it an investment bank, or an insurance company, or a hedge fund, faces the same question of capital reserving to cope with potential losses. Hedge funds that take for example balanced long-short positions, have a balance sheet with random assets and random liabilities supported by cash equivalents constituting their equity.

We take a hedge fund which operates on random asset value A and random liabilities L . The Net Asset Value(NAV) of this fund is then $X = A - L$. Due to the random liability that contributes to cash flow X , one has to hold this position with some initial non-random cash or cash equivalent amount \mathcal{C} , so that such capitalization generates cash flow $Y = \mathcal{C}e^{rt} + X(t)$ at some time t that is acceptable to the external economy. We want to emphasize that the shareholder equity commonly held in Tier 1 capital is not part of the reserve capital that concerns us here as we focus on the events when all equity is destroyed and the firm is being put back to the general economy as an exercise of its limited liability. [46] compares such a magnitude with a margin requirement, or leverage being permitted. Following [59] and [35] we employ the theory of acceptable risks and take stress function $MINMAXVAR$ at level γ , and require

$$\int_{-\infty}^{\infty} y d\Psi^{\gamma}(F_Y(y)) \geq 0 \tag{1.19}$$

where the stress function *MINMAXVAR* is

$$\Psi^\gamma(u) = 1 - (1 - u^{\frac{1}{1+\gamma}})^{1+\gamma}. \quad (1.20)$$

Since

$$F_Y(y) = F_X(y - \mathcal{C}e^{rt}), \quad (1.21)$$

so

$$\int_{-\infty}^{\infty} y d\Psi^\gamma(F_X(y - \mathcal{C}e^{rt})) \geq 0. \quad (1.22)$$

We change variable $x = y - \mathcal{C}e^{rt}$ then

$$\int_{-\infty}^{\infty} (x + \mathcal{C}e^{rt}) d\Psi^\gamma(F_X(x)) \geq 0, \quad (1.23)$$

or

$$\mathcal{C}e^{rt} = - \int_{-\infty}^{\infty} x d\Psi^\gamma(F_X(x)). \quad (1.24)$$

We now have an analytical function of capital reserve requirement in terms of risk parameters. The effects of different distributions of the cash flow X on the capital is studied in later sections. To compute this expression, one may directly integrate if distribution function is known since

$$\mathcal{C}e^{rt} = - \int_{-\infty}^{\infty} x d\Psi^\gamma(F_X(x)) = - \int_0^1 F^{-1}(u) d\Psi^\gamma(u). \quad (1.25)$$

If the distribution of X is not analytically known one may follow the procedure outlined in [30]. From a simulation of outcomes from the distribution of underlying variables one can sort the outcomes in increasing order as $x_1 \leq x_2 \leq \dots \leq x_n$ and

$$\mathcal{C}e^{rt} \approx - \sum_{i=1}^n x_i \left(\Psi^\gamma\left(\frac{i}{n}\right) - \Psi^\gamma\left(\frac{i-1}{n}\right) \right). \quad (1.26)$$

For a hedge fund set up with limited liability with random real-valued cash flow $X(t) = A(t) - L(t)$, the value of the fund or the equity at any time t is

$$(X(t) + \mathcal{C}e^{rt})^+, \quad (1.27)$$

which is a call option on $X(t)$ that struck at $-\mathcal{C}e^{rt}$. The limited liability concept ensures that

$$(X(t) + \mathcal{C}e^{rt})^+ \geq X(t) + \mathcal{C}e^{rt}, \quad (1.28)$$

with the excess being

$$(-\mathcal{C}e^{rt} - X(t))^+, \quad (1.29)$$

which is a put option on $X(t)$ with strike $-\mathcal{C}e^{rt}$. This is what is being termed as “taxpayer put” ([36]). The limited liability status of the corporation gives rise to this implicit put option. Especially, with the access to modern financial markets, firms nowadays have taken on random unheadable liabilities that are potentially unbounded, and it is precisely when this liability goes over the value of asset, the put option comes into money. It is essentially the option a limited liability entity holds to put losses back on society. In terms of time zero values we have

$$e^{-rt}\mathbf{E}_0^Q[(X(t) + \mathcal{C}e^{rt})^+] = \mathcal{C} + X(0) + e^{-rt}\mathbf{E}_0^Q[(-\mathcal{C}e^{rt} - X(t))^+], \quad (1.30)$$

which is also the put-call parity. If we let $E = e^{-rt}\mathbf{E}_0^Q[(X(t) + \mathcal{C}e^{rt})^+]$ and $P = e^{-rt}\mathbf{E}_0^Q[(-\mathcal{C}e^{rt} - X(t))^+]$, then

$$E = \mathcal{C} + X(0) + P. \quad (1.31)$$

We need to again distinguish our “capital” \mathcal{C} , which is the cash reserve, from the commonly adopted Basel capital requirement ([12]). The equity capital (E) concerned in Basel is the same as the cash reserve if it is a long-short balanced fund (A and L always offset each other and taxpayer put has no value in this situation). In general, they are not the same, and equity is larger when taxpayer put has value. [60] redefined the corporate balance sheet for relevance to two price markets related to Conic Finance introduced in [29], showing that equity capital (in Basel sense) is a poor measure of financial health as it can be contaminated by the excessive value of the taxpayer put option.

From the construct of the put option, we see that this option comes into value exactly when the potentially unbounded liability exceeds the asset. When this is the case, all equity is destroyed and the excess (negative of X) is being put back on to the economy being that the firm has only limited liability. One can easily place some debt in the formula and observe that the value of this put option is enjoyed by both the equity holders and the debt holders (unlike the classical limited liability of equity) and it distorts the debt holders incentive to monitor risk (see detail in [36]). We place the put as an asset if the hedge fund is too large to unwind and has to be put to the government. For a small fund the government may not get involved but then the liability holders receive L minus the put or someone else in the system, like the exchanges take the loss embedded in the put. Whoever is burdened with this put, the important point here is that the firm owns the value of this put and the need to regulate the hedge fund arises from the presence of this implicit contract.

Some argue that creditor counter-parties with the liability L will force the

firm to hold an acceptable level of funds for them to be willing counter-parties, although this put option has mathematical value, in reality, there are incentives and mechanisms (such as raising collateral and margin requirement) in the market place to enforce that this option does not come to existence even when the firm has limited liability. We argue that this put option exists even with the presence of counter-party enforcement and due diligence performed by the large institutional investors. When managers of funds are paid in line with the funds performance (or CEO's paid in line with the stock price), they have an incentive, especially in the short term, to build up the value of this put, and some did. The market forces did not curtail them as there is not and probably will not be sufficient transparency. Furthermore, there are many players with limited renegotiation powers. When one brings in as liabilities the retirement funds of employees for example, these parties can be small and diverse with insufficient market power to enforce capital levels on the firm. Recent FSA studies have also shown the new trend of "retailization" of hedge funds. It is unclear that these parties can demand collateral, or reprice the deal especially when the risk taking takes place ex-post all contracts in place. This is especially true in hedge fund domain as these entities are private and exempt from most SEC regulations. Upon Lehman's collapse, investors realized that no prime broker was too big to fail and spread their counter-party risk across several prime brokerages. The multi-prime brokerage relation adds some extra complexity to the due diligence even for the sophisticated institutional investors, as it becomes very complicated to perform proper assets reconciliation between the fund's administrator and its counter-parties. This is in addition to the fact that hedge fund assets are

hard to value due to their exposure to structured products and illiquid products. Hedge funds often utilize substantial leverage in their trading and this exacerbates the impact of a pricing mis-mark even further. Consequently, the mechanisms and incentives in the market place enforced by market participants are rather segmented and discrete, and hence ineffective due to the complexity of relationships and limited market powers. In the system that faces substantial systemic risk these mechanisms and incentives must be accompanied by new measures that are more systemic and transparent. The implicit put option is an opportunity created and will be abused if we simply choose to ignore it and place it in the hands of “market mechanisms”. The world is just a more complicated place and markets are not perfect.

As a result, we argue that it is necessary to set up the externally determined cash or cash equivalent amount \mathcal{C} so that the risky cash flow coming from the balance sheet of a limited liability entity with access to unbounded random liabilities is acceptable to the outside market. In other words, this cash buffer is to ensure that the taxpayer put value is not too large and the destruction of the balance sheet will not spill over outside of the box jeopardizing the health of the general economy. The firm cannot be the one to set the level of funds \mathcal{C} supporting the business as agency arguments designed to align CEO compensation with shareholders end up distorting financial health by maximizing the value of the embedded put option (by choosing the maximum strike of zero). It is necessary that a regulatory agency representing the government sets up externally the cash reserve and monitor the taxpayer put value, since it is the government who granted the limited liability status to the corporations. The economy is not burdened with extra risk, if \mathcal{C} calculated at

certain high enough stress level γ is big enough to account for the possibility of unbounded liability. As \mathcal{C} sets up such risk control for the external economy, the put value is also limited so that what is given to the firm for free does not have some excessive value. We show (in Section 1.5.2) similar analysis as in [59] that the stresses can be set at high enough level so that the incentive of perverse risk taking of firms will be corrected and put values are limited.

1.4 Models

As the intuition and the need for capital requirement and taxpayer put monitoring are articulated, we now move on to the next issue of modeling the underlying risks. As we have discussed, the random cash flow of the fund is

$$X = A - L \tag{1.32}$$

which we now model as real-valued Martingale after discounting.

Model One - Bachelier Model

$$X(t) = X(0)e^{rt} + \int_0^t e^{r(t-u)} \sigma dW(u) \tag{1.33}$$

where W is a Wiener process.

$$dX(t) = rX(t)dt + \sigma dW(t) \tag{1.34}$$

$X(t)$ is in fact normally distributed as

$$N(X(0)e^{rt}, \sigma^2 \frac{e^{2rt} - 1}{2r}), \tag{1.35}$$

and

$$F_X(x) = \Phi\left(\frac{x - X(0)e^{rt}}{\sigma\sqrt{\frac{e^{2rt}-1}{2r}}}\right). \quad (1.36)$$

We always demean the data and always calculate capital on demeaned NAV since any risk is certainly credited for its mean. The demeaned time series are assumed to follow

$$X(t+h) - X(t) = \sigma W(h). \quad (1.37)$$

Using monthly observed data on X we can easily estimate the maximum likelihood estimate for σ on demeaned time series data.

Model Two - VG

The VG process as a specific example of Lévy process can be expressed as Brownian motion with drift, time changed by a subordinator ([26], [34]). We can also model the real-valued X as a centered variance gamma (VG) process so that

$$X(t) = X(0)e^{rt} + \int_0^t e^{r(t-u)} dH(u) \quad (1.38)$$

where $H(t)$ is the centered VG

$$H(t) = \theta(G(t) - t) + \sigma W(G(t)) \quad (1.39)$$

here the time change $G(t)$ is a Gamma process with parameter ν , whose increments $G(t+h) - G(t) = g$ have Gamma density with mean h and variance νh and density function

$$f_h(g) = \frac{g^{h/\nu-1} \exp(-g/\nu)}{\nu^{h/\nu} \Gamma(h/\nu)} \quad (1.40)$$

The characteristic function of $H(t)$ is

$$\Phi_{H(t)}(u) = \left(\frac{1}{1 - i\theta\nu u + (\sigma^2\nu/2)u^2} \right)^{t/\nu} \cdot e^{-iu\theta t} \quad (1.41)$$

and we know ([26]) also the distribution function

$$f_{H(t)}(x) = \frac{2e^{\frac{\theta x}{\sigma^2}}}{\nu^{\frac{t}{\nu}} \sqrt{2\pi}\sigma \Gamma(\frac{t}{\nu})} \left(\frac{x^2}{\frac{2\sigma^2}{\nu} + \theta^2} \right)^{\frac{t}{2\nu} - \frac{1}{4}} K_{\frac{t}{\nu} - \frac{1}{2}} \left(\frac{\sqrt{x^2(\frac{2\sigma^2}{\nu} + \theta^2)}}{\sigma^2} \right) \quad (1.42)$$

where K is the modified Bessel function of the second kind.

We again work with demeaned increments

$$X(t+h) - X(t) = \theta(g-h) + \sigma\sqrt{g}Z \quad (1.43)$$

which can also be estimated using maximum likelihood to obtain parameter estimates for σ , ν and θ . Following the methods of [23], for each fund, we standardized the monthly changes (of NAV) to zero mean and unit variance and then binned the data into 100 evenly spaced bins in the interval $+/- 5$ standard deviations and maximized the log likelihood of the binned data.

1.5 Numerical Analysis

Before we jump into the empirical analysis on actual hedge fund data, we would like to take another closer look at the models using some numerical experiments. In the first subsection, we demonstrate the sensitivity analysis on model outputs, to show for example how would the change of VG model parameters affect the

calculation of capital and the put values. The second subsection shows numerically the effect of requiring cash reserves on risk incentives.

1.5.1 The Effect of Risk Parameters

Since VG model has richer risk dimensions including volatility, skewness and kurtosis, we study this model to show the effect of these risks on the value of required cash reserve and the associated taxpayer put. We impose a long-short balanced position so that $X(0) = 0$. We simulate model process (3.35) with some pre-defined parameter values out to one year ($t = 1$) assuming interest rate $r = 0.05$. γ is set at 0.25, 0.75 and 1.25 as three increasing stress levels or levels of acceptability. We then calculate cash reserve required $e^{rt}\mathcal{C}^*$ using (3.32) and the associated put value as $P = e^{-rt}E_0^Q[(-\mathcal{C}^*e^{rt} - X(t))^+]$ (one may also use distribution function directly such as normal in (1.36) and VG in (1.42)). The equity value is hence derived from the put-call parity (1.31). We first start with a base case where there is zero skewness $\theta = 0$ and no kurtosis $\nu = 0.001$. The model is now actually a Gaussian model. Since the hedge fund data analysis generally show very high volatility for the models, we set $\sigma = 13$. The cash required and the value of the taxpayer put as well as the fund equity value are shown in the following Table 1.2.

Table 1.2: Numerical Analysis with zero skewness no kurtosis and $\sigma = 13$.

stress	0.25	0.75	1.25
\mathcal{C}^*	5.2960	14.0251	21.0979
Put	2.8689	0.8766	0.2661
Equity	8.1648	14.9018	21.3640

We see that the higher the stress level, the higher the required cash level, and lower the negative strike of the options which render higher call value (the equity) and lower put value (the taxpayer burden).

We then increase the volatility to $\sigma = 130$ and keeping $\nu = 0.001$ $\theta = 0$ to show the effect of volatility, in Table 1.3

Table 1.3: Numerical Analysis with zero skewness no kurtosis and $\sigma = 130$.

stress	0.25	0.75	1.25
\mathcal{C}^*	52.9517	139.4298	209.0974
Put	28.7936	8.6840	2.5495
Equity	81.7453	148.1138	211.6470

The response to volatility is quite substantial at all stress levels.

To see the effect of kurtosis we now increase $\nu = 0.8$ keeping $\sigma = 13$ and $\theta = 0$, in Table 1.4

Table 1.4: Numerical Analysis with $\nu = 0.8$ and keeping $\sigma = 13$ and $\theta = 0$.

stress	0.25	0.75	1.25
\mathcal{C}^*	5.3821	15.1969	24.8151
Put	2.5497	0.8182	0.2565
Equity	7.9317	16.0151	25.0716

We observe that there are moderate responses to kurtosis at level 1.25 and only slight or no responses at lower stress levels.

Next we would like to study the effect of skewness, both positive and negative, on the calculated values. We show cases with $\theta = -1$ and $\theta = 1$ in Table 1.5 ($\sigma = 13$ $\nu = 0.8$) and again we see no response for low stress level with moderate response

at level 1.25.

Table 1.5: Effect of skewness in VG model, $\sigma = 13$ $\nu = 0.8$.

$\theta = -1$	stress	0.25	0.75	1.25
	\mathcal{C}^*	5.4037	15.6063	25.8747
	Put	2.5746	0.8328	0.2641
	Equity	7.9813	16.4391	26.1388
$\theta = 1$	stress	0.25	0.75	1.25
	\mathcal{C}^*	5.2917	14.9276	24.4824
	Put	2.4459	0.7733	0.2464
	Equity	7.7375	15.7010	24.7288

1.5.2 Risk Incentives in The Presence of Capital Constraints

To see the effect of requiring cash reserves on risk incentives for individual funds, we evaluate numerically (center differences are used) the derivatives of the fund profit with respect to risk parameters. The fund profit is equity less the cash reserved $E - \mathcal{C}$. Again we set $X(0) = 0$, $r = 0.05$ and $t = 1$, the results are shown in Table 1.6

Table 1.6: Risk Incentives Corrected by Capital Presence as Stress Levels Increase.

VG	$\sigma = 0.3$	$\nu = 0.5$	$\theta = 0.1$
stress	0.25	0.75	1.25
$\frac{\partial(E-\mathcal{C})}{\partial\sigma}$	0.1864	0.0073	-0.0751
$\frac{\partial(E-\mathcal{C})}{\partial\nu}$	-0.0268	-0.0173	-0.0266
$\frac{\partial(E-\mathcal{C})}{\partial\theta}$	0.0772	-0.0880	-0.1694
Gaussian	$\sigma = 0.3$		
stress	0.25	0.75	1.25
$\frac{\partial(E-\mathcal{C})}{\partial\sigma}$	0.1974	0.0195	-0.0548

We see that as stress level reach the highest 1.25, all the risk incentives are corrected with the appropriate amount of cash reserve adjustment. In actual calculation with real market data, the appropriate stress level (ideally fund dependent stress level) can be calibrated so that the risk incentives are all pointing at the right direction ([59]). Ideally, this calibration should also be updated from time to time to reflect recent changes. Alternatively, one could have a set level of stress as what we have shown in the numerical examples. In the next section, we begin our hedge fund analysis using actual fund data to implement our model calculations.

1.6 Hedge Fund Analysis

Data on hedge funds are reported as funds wish and it is usually incomplete and very limited. For example in CISDM hedge fund database, the reported NAV is on a per share basis without much knowledge of actual number of shares outstanding; AUM has frequent missing data and leverage information is mostly stale if even reported. Little is known on hedge fund balance sheet to separate the assets and liabilities.

We obtain hedge fund monthly NAV data from CISDM for Jan 2005 to April 2009. We only take funds that have continuous monthly data for at least 3 years in this period to be studied. There are a total of 3622 funds in our sample. The most data available for a particular fund is 52 monthly data (for the entire inquired time period). For each fund, we standardize the monthly changes (of NAV) to zero mean and unit variance and use maximum likelihood estimation to estimate

the parameters of each model. The estimation of Gaussian model is trivial. The estimation of VG model, follows the methods explained in [23], working with the data binned into 100 evenly spaced bins in the interval $+/- 5$ standard deviations and maximize the log likelihood of the binned data.

We then simulate each model process (3.34) and (3.35) out to one year ($t = 1$) assuming interest rate is $r = 0.05$ which is the one-month T-bill rate on May 1, 2009. γ is set at 0.25, 0.75 and 1.25 as three increasing stress levels. We then calculate cash reserve required as in $e^{rt}C^*$ using (3.32) (one may also use distribution function F_X as normal in model one and (1.42) in model two directly) and the associated put value as in $P = e^{-rt}E_0^Q[(-C e^{rt} - X(t))^+]$.

We show in Table 1.7 the results of calculated cash capital required and the corresponding taxpayer put values. We only demonstrate results for the first 40 (sorted by fund ID number) funds since it is impossible and unnecessary to show all 3622 of them. The first column of the table is fund ID number. The second to fourth columns are capital required using Gaussian model with stress levels being 0.25, 0.75, and 1.25 respectively. The fifth to seventh columns are capital required using VG model also with three increasing stress levels. We see that as the stress level increases, the capital required as cash reserve increases, as the acceptable condition is more and more stringent. Comparing the results from two different models with fixed stress levels, we also notice that VG model usually requires more capital than the Gaussian model. This is due to the fact that VG model is a fatter-tailed model compared to the Gaussian model, and there are more dimensions of risks (skewness and kurtosis) being incorporated in the VG model. We also show in

the table the values calculated as taxpayer put. The eighth to tenth columns are put values calculated in the Gaussian model corresponding to three stress levels and the eleventh to thirteenth columns are put values for the VG model with respect to the same stress levels. We observe that the put values in either model decrease as the stress level increases which is as expected. The put values can not be too high since they are “given for free” to the firms, and the higher value of this free put option should correspond to higher capital requirement and more stringent regulation.

Table 1.7: Results for the Capital Reserve and Taxpayer Put Values of the First 40 funds.

ID	$C^*(\text{Gaussian})(0,25)$	(0.75)	(1.25)	$C^*(\text{VG})(0,25)$	(0.75)	(1.25)	put(Gaussian)(0,25)	(0.75)	(1.25)	Put(VG)(0,25)	(0.75)	(1.25)
9	54.0190	145.1844	221.2350	57.4459	155.1050	242.6252	29.2367	8.8245	2.5864	29.6628	8.8572	2.5584
10	70.4309	188.4225	284.9862	71.4805	197.3027	308.4215	38.9440	11.6966	3.4931	38.0060	11.8079	3.4670
11	7.5928	20.2222	30.6061	6.4333	17.7011	27.4723	4.1179	1.2451	0.3695	3.4204	1.0924	0.3489
12	196.3013	294.5223	60.2266	6.02266	176.2622	284.6676	39.0334	11.6864	3.3692	32.9555	10.6028	3.2901
14	6.5068	17.2590	26.0835	6.7111	17.4837	26.7704	3.5072	1.0753	0.3159	3.3897	0.9748	0.2671
16	26.7622	70.6266	105.9344	37.1175	106.5201	169.6831	14.4759	4.4862	1.3494	20.3930	6.6783	2.0398
17	82.0308	217.7099	329.7025	110.7806	334.4556	553.7432	43.8313	13.3644	3.8631	61.2454	19.9975	6.0478
18	35.4388	92.6722	137.8915	37.9955	103.1492	158.3365	19.0998	5.9437	1.8445	20.6924	6.1586	1.7234
19	192.7504	515.0249	776.1639	201.3363	562.3234	886.1575	105.4532	32.4713	9.8618	108.0268	33.5610	9.7543
20	99.5994	261.5733	393.9767	109.1171	301.7073	467.3446	53.1270	16.3352	4.8268	59.9920	18.9458	5.6249
21	18.0884	47.6668	71.5248	19.1377	51.2843	79.1054	9.8229	2.9925	0.8712	9.8865	3.1482	0.9209
22	13.6683	37.6716	58.1279	14.7533	38.6250	58.2674	7.6664	2.2843	0.6415	7.7625	2.3500	0.7043
23	29.6821	77.6343	116.9713	33.1111	96.3418	154.3760	15.6063	4.7084	1.3719	18.3626	5.7614	1.8007
27	17.2424	45.8417	69.6671	17.3756	45.3861	68.3045	9.3406	2.7746	0.7759	9.1206	2.8090	0.8464
29	43.8586	116.5876	176.8019	44.1127	119.2670	183.2841	23.6998	7.1923	2.0619	23.4493	7.2801	2.1900
32	100.2921	269.2327	410.3142	88.5254	234.4484	354.9530	54.7930	16.5648	4.7157	47.1190	14.1130	4.1344
34	10.4089	28.2248	43.0881	8.1481	23.3323	37.0404	5.7773	1.7474	0.4928	4.4870	1.4307	0.4431
39	388.3245	1013.5807	1533.6629	368.6456	1004.5339	1536.7734	203.1369	60.2982	16.9463	202.9050	62.4305	18.8746
41	14.2814	38.8332	58.6785	15.2801	41.7869	65.2631	8.2052	2.4802	0.6979	8.0733	2.4636	0.7077
43	8.3236	21.8344	33.1774	8.9233	25.1659	39.8012	4.3825	1.3139	0.3578	4.8811	1.4833	0.4266
45	152.3887	406.5514	614.3462	154.8991	432.5858	679.8977	82.8653	25.1867	7.4359	84.4334	26.2410	7.4832
48	7.7093	20.3735	30.8546	7.9703	20.2412	30.2208	4.1609	1.2534	0.3486	4.0498	1.1930	0.3438
51	15.1523	39.8311	60.7729	13.2578	37.5507	59.8454	7.9397	2.4023	0.6462	7.0267	2.2750	0.7049
52	233.4610	618.4067	932.5740	218.5304	557.3683	834.4202	127.6117	38.4165	10.9452	110.4230	31.8428	9.3427
61	5.5907	15.5191	23.7198	4.6046	12.9785	20.6058	3.2332	0.9766	0.2879	2.4876	0.7951	0.2344
63	0.6933	1.8266	2.7362	0.7206	1.9557	3.0401	0.3768	0.1137	0.0334	0.3797	0.1157	0.0342
65	0.5105	1.3765	2.1041	0.5148	1.4209	2.2103	0.2802	0.0843	0.0240	0.2780	0.0872	0.0259
67	23.0193	61.2214	93.0128	24.6522	67.5964	106.4698	12.3850	3.7405	1.0493	12.8866	3.9595	1.1560
70	12.6584	33.0726	49.7967	11.8208	32.9975	51.6471	6.6986	2.0251	0.5826	6.4576	1.9810	0.5898
73	1.5035	3.9477	5.9216	1.4083	3.9420	6.2347	0.8069	0.2457	0.0724	0.7633	0.2393	0.0662
78	243.8213	657.5218	999.3176	259.3883	712.2139	1096.7520	135.1457	40.4450	11.5916	143.4458	44.6915	12.9929
79	20.3864	54.8327	83.7019	24.6519	71.1856	114.1209	11.1993	3.3109	0.8980	13.5722	4.3819	1.3299
83	5.1763	13.4297	20.1840	5.9519	16.6214	26.1553	2.7243	0.8199	0.2298	3.1665	1.0154	0.3255
85	3.2604	8.7629	13.2794	4.9510	13.5075	21.0319	1.7740	0.5372	0.1638	2.5848	0.8000	0.2457
86	57.3063	155.0517	234.7061	37.6164	101.4815	157.7379	31.8194	9.5794	2.8780	19.2709	5.7826	1.7005
89	401.1407	1059.9925	1592.8535	365.8641	963.0910	1474.0036	217.5209	65.0558	19.5413	184.1791	59.0425	17.8893
92	391.9490	1015.1648	1525.7334	376.0361	1060.0086	1685.6916	202.3997	60.5381	18.2397	201.4200	62.0797	17.5022
96	2.74798	7.1675	10.7527	2.6931	7.4301	11.5128	1.4649	0.4405	0.1281	1.4735	0.4509	0.1324
97	403.3785	1087.9808	1653.8336	400.2266	1090.2164	1681.9934	222.0202	67.3186	20.0278	219.2158	65.1376	18.3269
98	0.4810	1.2624	1.9084	0.4830	1.2904	1.9823	0.2529	0.0763	0.0223	0.2551	0.0773	0.0224

In the simulation, we must specify the starting value $X(0)$. We entertain with the idea of assuming a long-short balanced fund, and hence $X(0) = 0$. For each fund we simulate out to one year as if we start from the end of its available data and this tells us the required cash reserve \mathcal{C}^* . We also take the NAV of the last observed for each fund which in general may be composed of non-random asset, random asset and random liability. By assuming starting NAV $X(0) = 0$ we effectively assumed that the observed NAV was in fact the value of the non-random cash asset being held, or \mathcal{C} , and this can be compared to the calculated \mathcal{C}^* and determine which funds are under-capitalized or over-capitalized. If \mathcal{C}^* is greater than \mathcal{C} , these funds are considered inadequate since they are not holding enough cash and too much risk has been put on the taxpayers.

The following graphs 1.1 and 1.2 show the results for all 3622 funds. First figure plots the required \mathcal{C}^* for both models each at three stress levels, together with \mathcal{C} (which is the last observed NAV). Since the values are usually tens of thousands, we plot the logarithm of the values instead to be more clear on the graphs. The cash reserve required is generally higher for the VG model than the Gaussian model at same stress level. The second figure plots the put values for different stress levels and different models. For clarity, we plot the sorted logarithm of the values.

Among the 3622 funds, there are 127 inadequate funds whose holding $\mathcal{C} < \mathcal{C}^*$ where \mathcal{C}^* is taken to be highest values of all cases which is VG at $\gamma = 1.25$. We also plot similar graphs just for these 127 funds in 1.3 and 1.4. The last figure 1.5 clearly shows the inadequacy of the cash holding by plotting only the maximum of all \mathcal{C}^* (VG case with $\gamma = 1.25$) and the actual holding \mathcal{C} .

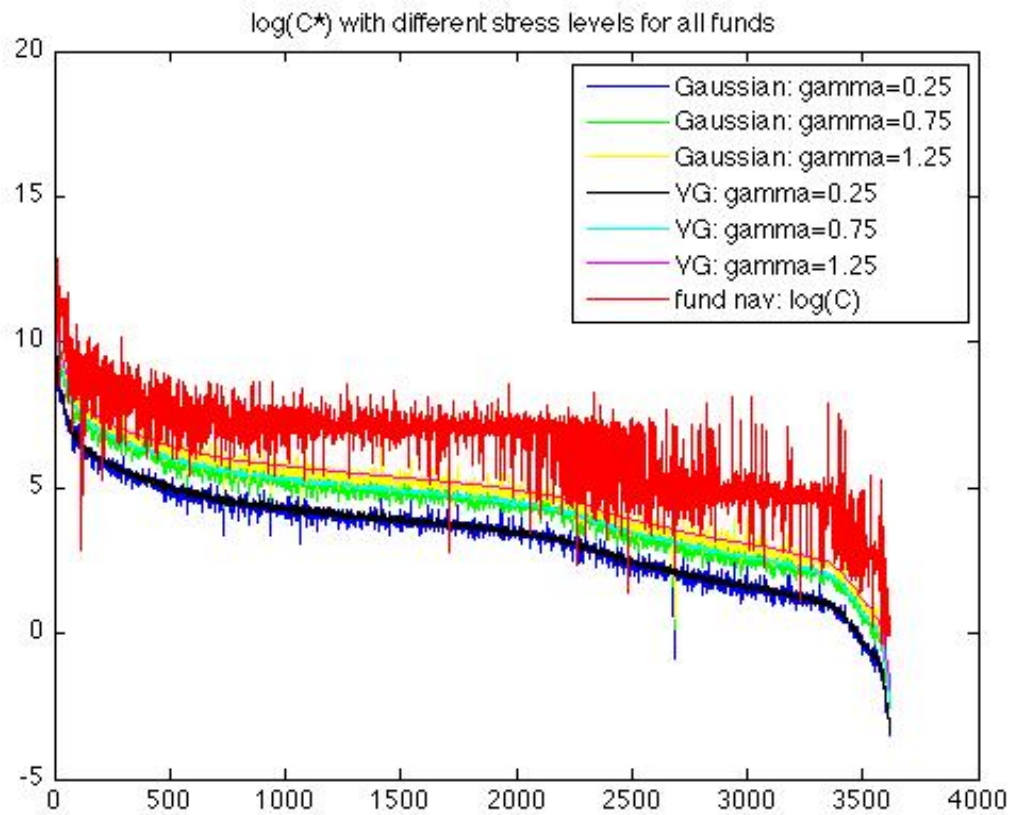


Figure 1.1: Plot of the Cash Reserve Required: C^* in Both Models, Together with Observed NAV: C , for all 3622 Funds.

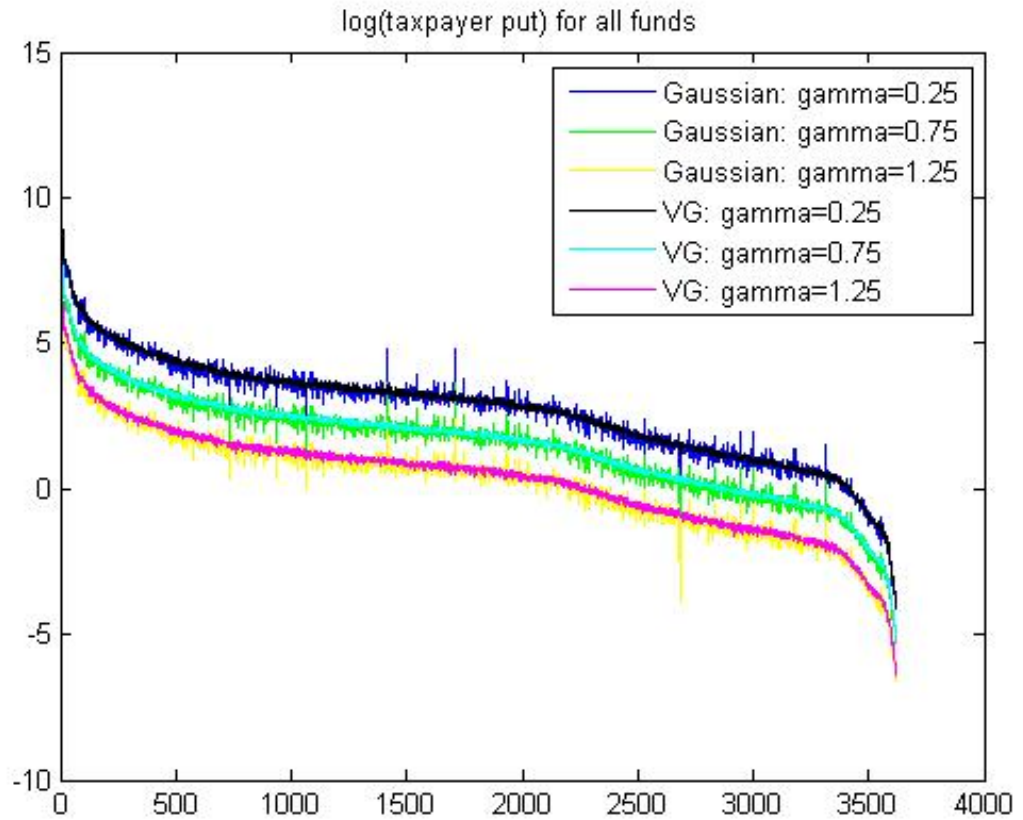


Figure 1.2: Plot of the Taxpayer Put Values in Both Models, for all 3622 Funds.

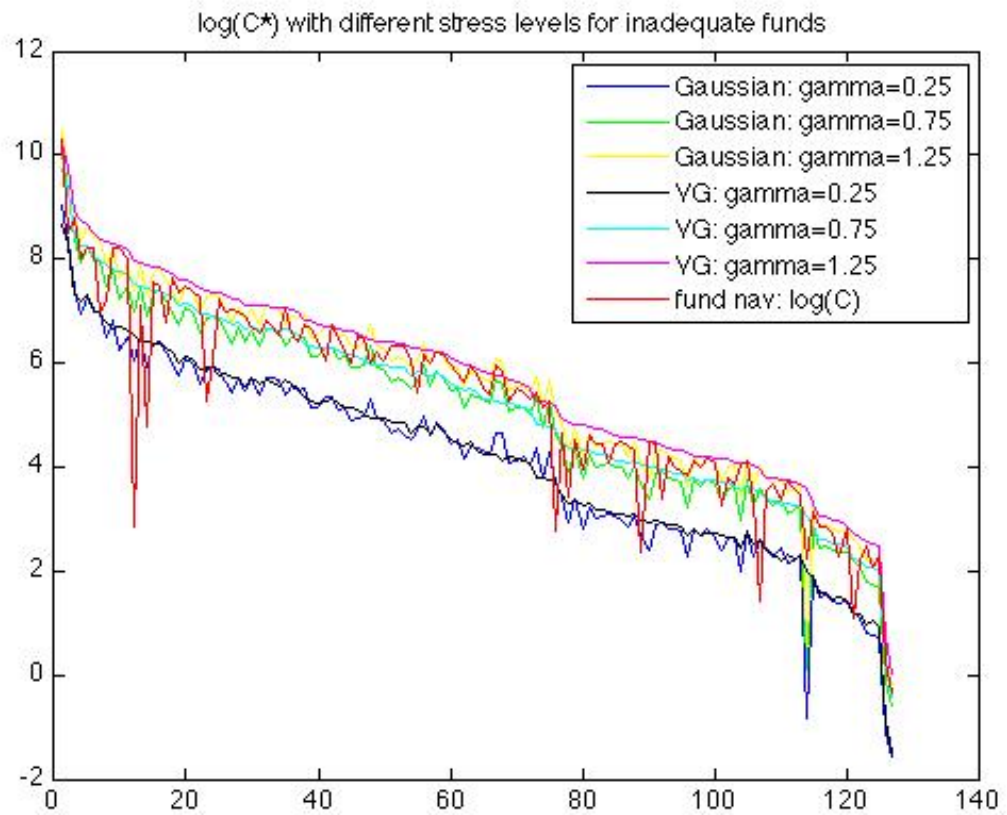


Figure 1.3: Plot of the Cash Reserve Required: C^* in Both Models, Together with Observed NAV: C , for 127 Inadequate Funds.

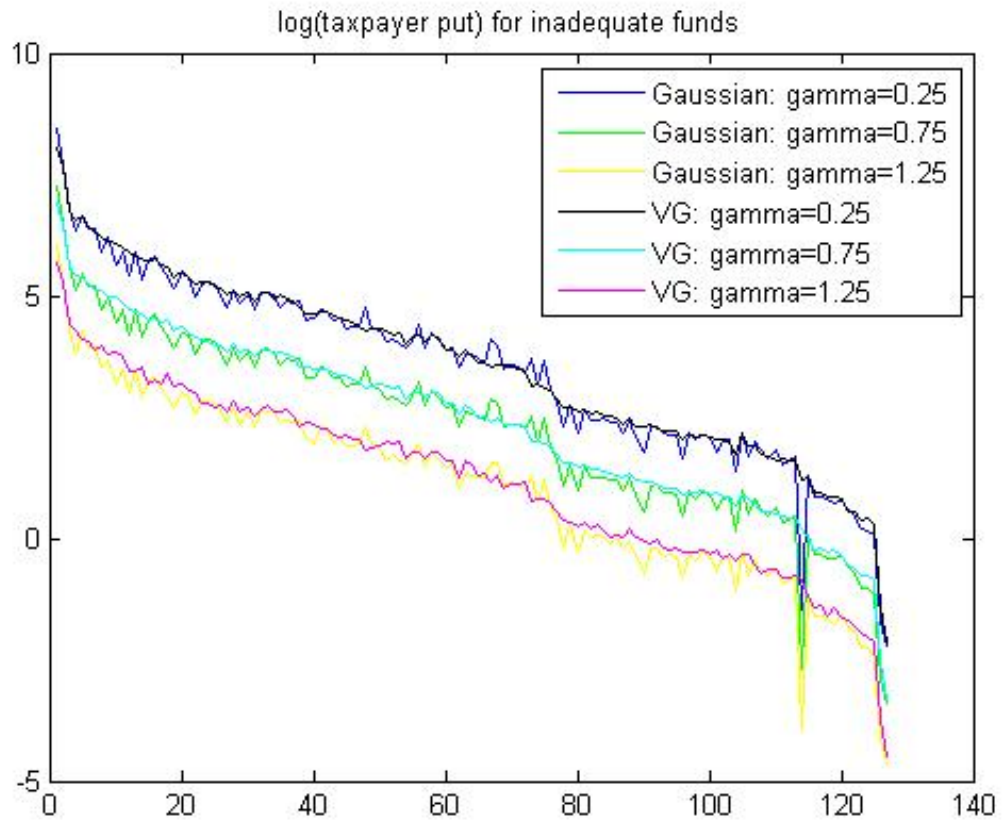


Figure 1.4: Plot of Taxpayer Put Values in Both Models, for 127 Inadequate Funds.

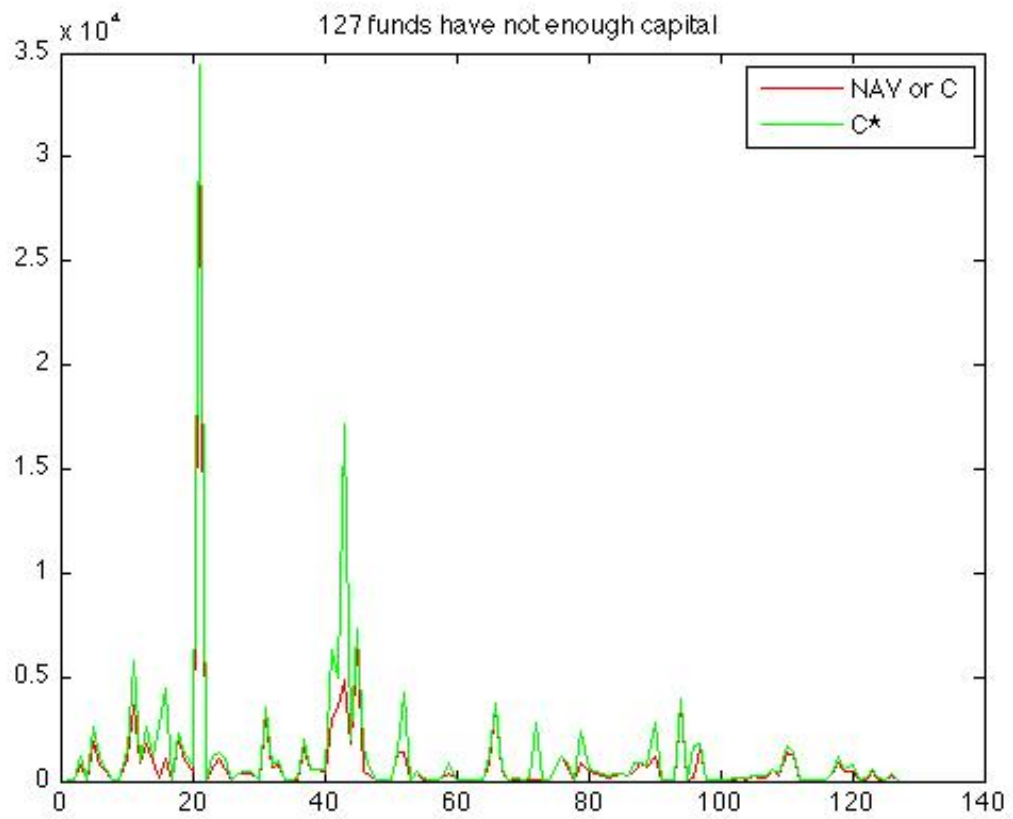


Figure 1.5: Plot of the Maximum C^* (VG case with $\gamma = 1.25$) and the actual holding C for 127 Inadequate Funds.

1.7 Conclusion

Hedge funds are systemically important financial entities. Despite the fact that hedge fund industry has grown to be a key player in the markets, its regulation has lagged compared to other institutions such as investment banks or mutual funds and its transparency to the general public needs to be greatly improved. Hedge funds are by nature good examples of holding balance sheets that contain both random cash flows as assets and another set of potentially unbounded random cash flows as liabilities. The possibility that the liabilities may become limitless (unless the short position exactly hedges a corresponding long position) and dominate the assets determines that the fund cannot be permitted to exist as a limited liability entity if it is insufficiently capitalized. We propose an approach that builds a framework in which the risks of the total cash flow of assets less the liabilities are directly modeled. Based on the risks in total cash flow we construct the capital reserve (cash or cash equivalent) as buffer to be held by the firm so that the remaining risk is acceptable to the general economy. We conduct our analysis using the data available for hedge funds and hope to provide some basic tool in the development of hedge fund regulation.

We model the net cash flow (on a per share basis) as a real-valued martingale and propose two models one with Gaussian components and one with Lévy jump (Variance Gamma) components to account for skewness and kurtosis. Hedge fund monthly NAV data from CISDM is obtained from Jan 2005 to April 2009. We only take funds that have continuous monthly data for at least 3 years in this period

to be studied. There are 3622 funds in total. We fit the demeaned data with the models using maximum likelihood estimation and obtain model parameters that are then fed to simulation and derive risk determined capital requirement as well as put option values to be monitored. The risks acceptable to the general economy have been given operational definition by focusing attention on the positive expectation under a sufficiently concave distortion of the probability distribution of the risks being undertaken. The distortion function is chosen to be MINMAXVAR and three increasing stress levels of γ , 0.25, 0.75, 1.25 are considered. The fatter-tailed distribution of VG normally generates higher capital requirement under otherwise same conditions. The results show that under VG model with stress 1.25 (the most stringent requirement) there were 127 funds insufficiently capitalized on April 30, 2009. We also show sensitivity of the calculated capital and the put value to underlying risk parameters, i.e. volatility in the Gaussian model and volatility, skewness and kurtosis in the VG model. It is suggested in [59] that the level of γ may be calibrated by selecting the smallest value at which the preserved capital requirement mitigates the perverse risk incentives. We also show similar analysis on such mitigation of perverse risk incentives and stress level 1.25 is shown to be a needed level for the sensitivity of capital to risk dominating the sensitivity of equity value to risk, given the presence of limited liability.

Chapter 2

Hedge Fund Reserve Capital and Systemic Risk

2.1 Overview

Just like large investment banks, large hedge funds play a significant role in facilitating the efficient allocation of the risks of investments. With the status of limited liability, large hedge funds also pose great threats to the health of the general economy when things are not so great. Large institutions such as these big hedge funds must satisfy capital requirements intended to ensure that they can sustain reasonable losses. The capital requirements introduced in our previous paper ([73]), are designed so that the total risky cash flow as the assets less the liabilities is acceptable to the general economy, as specified by the theory of acceptability ([10], [50], [31]). The capital requirements however, are designed as if each fund is an isolated entity. Focusing only on individual funds ignores the critical interactions between them and can cause the regulators to overlook important changes in the overall system. Because many hedge funds employ similar investment strategies they produce correlated returns. The failure of these correlated large funds will greatly affect the markets systematically either in a direct or an indirect way. The high leverage employed by these funds also have the potential to exacerbate instability in the market as a whole. In this paper we take into considerations the systemic effects when setting the capital requirements.

As we see from the past crisis, the systemic problems have been far more severe and far more global this time around. Unregulated or less regulated financial firms have played a much more significant role than they did 20 years ago. The hedge fund industry is huge in size and with the tremendous amount of leverage it has employed, it is without a doubt a big player in the capital markets. A key determinant of hedge fund risk is the degree of similarity between the trading strategies of different funds. Many hedge funds by nature employ similar investment strategies and chase correlated returns. Similar trading strategies can heighten risks when funds have to close out comparable positions in response to a common shock. The outcome may be direct losses inflicted on creditors and trading counter-parties as well as an indirect impact on other market participants through price changes resulting from the disappearance of investors willing to bear higher risks. The episode around LTCM ([71]) and the 2007/2008 credit crisis are cases in point. Extensive rules and tremendous supervisory resources are focused on banks, with far less of both devoted to other types of financial firms. As these other types of firms became much more significant in the delivery of financial services through the growth of securitization, structured products, and derivatives, our attention is again focused on hedge fund industry as its size and proper regulation has not been proportionally developed. The damage it could cause as a trillion dollar industry is potentially enormous ([44] analyzes hedge funds and their implication for financial stability and see also [27] for a thorough, 109-page discussion on the topic).

Capital requirements are typically designed as if each firm is an isolated entity, with little concern for the effect of losses on one firm can have on other in-

stitutions. The fragmented regulation structure is apparently not enough to tackle issues when the system faces substantial systemic risk and shall be accompanied by new measures which take into account of the correlations in a systemic way. Capital requirements for regulated financial institutions should depend on the systemic risk they pose. Large entities holding illiquid assets and relying heavily on short-term debt or security lending financing should be required to hold proportionally more capital than smaller entities with more liquid assets and more stable financing arrangements. The recent crisis has seen the systemic crash caused by a financial system that has outgrown the existing set of rules. Bail-out plans despite its unfairness, the moral hazard it created, and the severe political problems it generated, did resolve problems to some extent post crisis. However, what we hope to build is a system that tries to avoid the clear unfairness of this provision by adopting a new orderly resolution mechanism ex ante that can achieve the same effects without bailing out uninsured stakeholders. The Squam Lake Report ([70], [69]) has pointed out that the solution to this narrow institutional focus is to urge a central regulatory organization to oversee the health and stability of the overall financial system: “The role of the systemic regulator should include gathering, analyzing, and reporting information about significant interactions between and risks among financial institutions; designing and implementing systemically sensitive regulations, including capital requirements; ...”. In the United States, as Dodd Frank Wall Street Reform and Consumer Protection Act ([1]) pass through the congress, Financial Stability Oversight Council (FSOC) has been set forth and many other actions are underway. We focus here on proposing a mathematical framework for

such regulation in a systemic context. In this section we conduct our analysis again in the hedge fund industry and argue that capital requirements should also vary with other characteristics that are linked to the systemic problems a hedge fund might create. Our hope here is to provide guidance on how hedge funds should be charged cash (or cash equivalent) capitals in order to account for both risks they pose to the economy and as an individual. Regulators will then have much stronger tools to address systemic financial crises, including a new resolution mechanism for addressing the failures of systemically significant firms. Once we have these tools and implement them correctly, it is what we hope to expressively design the tools to be used to maintain financial stability and prevent from the event of a potential failure of a systemically significant firm.

To take a more systemic approach, we propose to model the funds risky cash flows jointly with correlations specified by the underlying laws of motions. The capital charges on the biggest funds in the industry must account for both their contribution to systemic risk and their own idiosyncratic risk. The charges on each fund towards the systemic contribution will be collected by external hedge fund authority and these charges must sum to account for the aggregate risky outcome that is imposed on the systemic level. In addition, each fund has to also reserve cash capital according to the residual risk induced by idiosyncratic component. The residual charge is then held at the fund level. The biggest funds that have AUM larger than \$2Bn as of April 30, 2009 and have all data from April 2006 to April 2009 are studied. There are total 30 of these funds. The total AUM of these 30 funds is about 620 billion dollars and is 64% of the total AUM of the 3622 funds

(there are 3622 funds studied previously, these are funds that have all data for the studied period and they are considered the universe of the funds here). We again assume marginal distributions of fund cash flows following a Gaussian process or a Variance Gamma (VG) process ([61], [26]). For the Gaussian case, the joint laws of the changes in NAV are multivariate normal. The VG process as a specific example of Lévy process can be expressed as a Brownian motion with drift, time changed by a subordinator and we introduce the dependence by merely correlating the Brownian motions that are being time changed ([26], [34]). It is unreasonable to believe, however, that these hedge funds generate co-monotonic cash flows. Hence following the theory of acceptability, we know that this always leads to sum of the individual charge being not equal to the charge on the sum (e.g. [28]). This does not give us the additivity that we wish to obtain by charging individual funds and collect the sum. We need additivity of the charges so that the sum accounts for the total risk of the total cash flow and the charges should not be too conservative so that they deter growth and activities. We show in two Theorems that additivity can be achieved by separating the cash flow into two components: the systemic component and the residual or idiosyncratic component. The systemic component is constructed to be a conditional expectation of the underlying cash flow given the industry total return. Such defined conditional function as risky random flow is postulated to be an increasing function of the total return. In other words, these are the funds where we will have higher (lower) expectation of their returns whenever the total return in the industry is high (low). These conditional expectations are then co-monotonic functions and enjoy additivity (see Theorem 2.1). Theorem 2.2 then shows that the

charges calculated on these random risky flows are able to sum to account for the aggregate systemic risk. The residual charge is then held at the fund level to account for the idiosyncratic risk component. We present this framework and work out the capital charges for these largest funds. As far as our knowledge the calculation of such systematic approach has not been done before.

The outline of the rest of the chapter is as follows. Section 2.2 describes the theoretical framework allowing us to take into account systemic risk in the context of correlated funds. In Section 2.3 and Section 2.4 we present two detail models one with Gaussian underliers and one with Variance Gamma processes as underliers. Section 2.5 describes the data and analysis done in the empirical study for the 30 largest funds. Charges accounting for systemic component and idiosyncratic component and taxpayer put values are calculated for each of 30 funds. Section 3.4 concludes.

2.2 Systemic Risk with Correlated Funds

In the paper preceding the current one ([73]), we propose and study a new capital regulatory approach based on acceptable risk control theory ([22], [30],[31]) in the hedge fund domain (for an application in bank domain see [59], [36]). The previous paper has considered funds individually with no correlation between them. Unfortunately, an important source of risk imposed on the general economy by hedge funds is the systemic risk. Focusing only on individual funds ignores the critical interactions between them and can cause the regulators to overlook important changes

in the overall system.

Capital requirements are typically designed as if each firm is an isolated entity, with little concern for the effect losses on one firm can have on other institutions. An important source of risk imposed on the general economy by hedge funds is the systemic risk. The failure of a large national bank, for example, is almost surely to have a bigger impact on the banking system and the wider economy than the failure of several small regional banks. Following the same vein, large funds and funds that have correlated returns will certainly affect the whole system even when individually run into problems. The fragmented regulation structure is apparently not enough to tackle issues when the system faces substantial systemic risk and shall be accompanied by new measures which take into account of the correlations in a systemic way. Capital requirements for regulated financial institutions should depend on the systemic risk they pose. Large entities holding illiquid assets and relying heavily on short-term debt or security lending financing should be required to hold proportionally more capital than smaller entities with more liquid assets and more stable financing arrangements. The recent crisis has seen the systemic crash caused by a financial system that has outgrown the existing set of rules. Bail-out plans despite its unfairness, the moral hazard it created, and the severe political problems it generated, did resolve problems to some extent post crisis. However, what we hope to build is a system that tries to avoid the clear unfairness of this provision by adopting a new orderly resolution mechanism *ex ante* that can achieve the same effects without bailing out uninsured stakeholders. The Squam Lake Report ([70], [69]) has pointed out that the solution to this narrow institutional focus is to urge

a central regulatory organization to oversee the health and stability of the overall financial system: "The role of the systemic regulator should include gathering, analyzing, and reporting information about significant interactions between and risks among financial institutions; designing and implementing systemically sensitive regulations, including capital requirements; . . .". In the United States, as Dodd Frank Wall Street Reform and Consumer Protection Act ([1]) pass through the congress, Financial Stability Oversight Council (FSOC) has been set forth and many other actions are underway. We focus here on proposing a mathematical framework for such regulation in a systemic context. In this section we conduct our analysis again in the hedge fund industry and argue that capital requirements should also vary with other characteristics that are linked to the systemic problems a hedge fund might create. Our hope here is to provide guidance on how hedge funds should be charged cash (or cash equivalent) capitals in order to account for both risks they pose to the economy and as an individual. Regulators will then have much stronger tools to address systemic financial crises, including a new resolution mechanism for addressing the failures of systemically significant firms. Once we have these tools and implement them correctly, it is what we hope to expressively design the tools to be used to maintain financial stability and prevent from the event of a potential failure of a systemically significant firm.

A key determinant of hedge fund risk is the degree of similarity between the trading strategies of different funds. Many hedge funds by nature employ similar investment strategies and chase correlated returns. Similar trading strategies can heighten risk when funds have to close out comparable positions in response to a

common shock. For example, many funds had to close out positions during the LTCM crisis to meet margin calls and satisfy risk management constraints. Talented hedge fund managers are able to exploit market inefficiencies that cannot be exploited by conventional asset managers and/or design innovative investment strategies that may yield excellent returns. However, the existence of pricing inefficiencies is very limited. Moreover, any successful strategy will quickly be imitated. As a result, many hedge funds employ similar investment strategies and produce correlated returns. Such highly correlated behavior poses great threat to the financial stability of the economy as a whole. There are many ways to assess the similarity of hedge fund strategies. The approach taken in general is to examine how closely together the funds returns move. If the returns of many funds are either high or low at the same time, the funds could record losses simultaneously, with possible adverse consequences for market liquidity and stability (see e.g. [3] among others for some study on rising correlations in hedge fund returns during crisis and as a recent trend and their effects and differences, using a measure of cross-sectional dispersion of returns). Furthermore, when leveraged investors are overwhelmed by market or liquidity shocks, the risks they have assumed will be discharged back into the market. Thus, highly leveraged investors have the potential to exacerbate instability in the market as a whole. The outcome may be direct losses inflicted on creditors and trading counter-parties as well as an indirect impact on other market participants through price changes resulting from the disappearance of investors willing to bear higher risks. The indirect impact is potentially the more serious effect. Volatility and sharp declines in asset prices can heighten uncertainty about credit risk and dis-

rupt the intermediation of credit. These secondary effects, if not contained, could cause a contraction of credit and liquidity and, ultimately, heighten the risk of a contraction in real economic activity. A conclusive assessment of the systemic risks posed by hedge funds however, requires certain data that is currently unavailable, and is unlikely to become available in the near future, i.e., counter-party credit exposures, the net degree of leverage of hedge fund managers and investors, the gross amount of structured products involving hedge funds, etc. Since return correlation is a key risk in hedge fund industry and key contributor to systemic risk, and it is relatively easy to incorporate, we propose to impose capital requirements with correlation in the context.

Let us now be interested in the hedge funds that are most influential to the systemic risk on the whole economy. We would like to correlate these funds who have AUM (Asset Under Management) larger than a certain threshold. Each with random real-valued cash flow X_i , $i = 1, 2, \dots, N$. The aggregate risk is now $\sum_{i=1}^N X_i(t)$ and we require that $\sum_{i=1}^N X_i(t) + \mathcal{C}$ be acceptable and again the smallest such capital is ([30], [36])

$$\mathcal{C}(\sum_{i=1}^N X_i(t))e^{rt} = - \inf_{Q \in \mathcal{D}} E^Q[\sum_{i=1}^N X_i(t)], \quad (2.1)$$

and the specific form of acceptability employed is positive expectation under a concave distortion of the cash flow distribution ([30]), so that for $Y = \sum_{i=1}^N X_i$

$$\mathcal{C}(Y)e^{rt} = - \int_{-\infty}^{\infty} y d\Psi^\gamma(F_Y(y)), \quad (2.2)$$

where the stress function $MINMAXVAR$ is

$$\Psi^\gamma(u) = 1 - (1 - u^{\frac{1}{1+\gamma}})^{1+\gamma}. \quad (2.3)$$

This amount of capital \mathcal{C} is deposited to the hedge fund regulation authority (HFRA) in order to cover the total risk or systemic risk of these largest funds combined. In order to charge each individual for its contribution to the systemic risk, we may also work out the capital required in isolation as

$$\mathcal{C}(X_i)e^{rt} = - \inf_{Q \in \mathcal{D}} E^Q[X_i(t)] \quad (2.4)$$

The charge to each fund i on account for the aggregate risk must sum to \mathcal{C} in order to ensure the acceptability of the total cash flow. However, we have no reason to believe that cash flows X_i are all co-monotonic and following the theory of acceptability, we know that this always leads to sum of the individual charge being not equal to the charge on the sum (e.g. [28]). This does not give us the additivity that we wish to obtain by charging individual funds $\mathcal{C}(X_i)$ and collect the sum. What we can do is shown below by separating the cash flow X_i into two components: the systemic components and the residual or idiosyncratic component. The systemic component is constructed to be a conditional expectation of the underlying cash flow given the industry total return. These conditional expectations are co-monotonic functions and hence they will be added up and charged to account for the systemic risk. The residual charge is then held at the fund level to account for the idiosyncratic risk component. Let us consider the return as

$$R_i(t) = \frac{X_i(t) - X_i(t-1)}{X_i(t-1)} \quad (2.5)$$

and the total return

$$R(t) = \sum_{i=1}^N R_i(t). \quad (2.6)$$

Given the total return, we are interested in the risky cash flows $X_i(t)$ that have conditional expectations that are positively related to the total return. Such risks on average move together with the system and are exactly the subject of suspect for contributing to systemic risks, and the fact that these risks are also correlated (due to the increasingly similar positioning of individual hedge funds within broad hedge fund investment strategies), the threats they post to the general economy in a systemic way is even exacerbated. Formally, let us define for $i = 1, 2, \dots, N$

$$V_i(t) = E[X_i(t)|R(t)] \quad (2.7)$$

So we have now separated cash flow X_i

$$X_i = V_i + e_i \quad (2.8)$$

where e_i is the residual component. What we are going to charge on each fund is the capital required on the systemic component $\mathcal{C}(V_i)$,

$$\mathcal{C}(V_i)e^{rt} = - \inf_{Q \in \mathcal{D}} E^Q[V_i(t)] \quad (2.9)$$

which is collected and stored in the hedge fund authority and we hope to see the sum of $\mathcal{C}(V_i)$ equal to the charge on the sum $\mathcal{C}(\sum_i V_i)$. Additionally, we also require fund to put aside cash reserve in the amount of $\mathcal{C}(X_i) - \mathcal{C}(V_i)$ on the fund level to account for the idiosyncratic risk.

Such defined risk $V_i(t)$ is a function of the total return $R(t)$. Given $R(t)$ we are interested in funds whose L is an increasing function of the total return. In other words, these are the funds where we will have higher (lower) expectation of their return whenever the total return in the industry is high (low). Even if this function may be decreasing in R_t , in this case this is a fund who bets against the market and takes opposite positions than others, we may then charge negative capital on those particular funds as they have opposite effect on contributing to systemic risk (see detail in the proof of Theorem 2.2). The hence selected group of increasing functions guarantee that V_i 's are pairwise monotonic. (Recall that two random variables X and $Y \in L^1(P)$ (the collection of real-valued integrable random variables on a probability space (Ω, \mathcal{F}, P)) are co-monotonic if $(X(\omega_2) - X(\omega_1))(Y(\omega_2) - Y(\omega_1)) \geq 0$ for $P \times P$ -a.e. ω_1, ω_2 , or in other words, they move together.) The following theorem guarantees the additivity of the capital as long as the V_i s are pairwise monotonic.

Theorem 2.1 *For $V_i(t), V_j(t)$ defined as above, we have*

$$-\inf_{Q \in \mathcal{D}} E^Q[V_i(t) + V_j(t)] = -\inf_{Q \in \mathcal{D}} E^Q[V_i(t)] - \inf_{Q \in \mathcal{D}} E^Q[V_j(t)] \quad (2.10)$$

if and only if $V_i(t)$ and $V_j(t)$ are co-monotone.

Proof. Realizing that $-\inf_{Q \in \mathcal{D}} E^Q[\cdot]$ is a coherent utility function, the results above is a direct deduction from Theorem 5.1 in [28]. ■

So once the V_i function is an increasing function of the R , we have that all V_i 's are pairwise co-monotone. And in fact we further realize the fact that if a series of random variables are pairwise co-monotonic, then any combination of the summation will also be co-monotonic (eg. V_1 and $V_2 + V_3$). Now we are ready to prove the following theorem where additivity of the total capital is achieved.

Theorem 2.2 *If $V_i(t)$ $i = 1, 2, \dots, N$ are pairwise co-monotone, then we have the capital charge on the total cash flow $\sum_{i=1}^N V_i$ equal to the sum of all the individual charges on V_i , specifically,*

$$-\inf_{Q \in \mathcal{D}} E^Q \left[\sum_{i=1}^N V_i(t) \right] = \sum_{i=1}^N \left(-\inf_{Q \in \mathcal{D}} E^Q [V_i(t)] \right) \quad (2.11)$$

Proof.

$$\begin{aligned} RHS &= -\inf_{Q \in \mathcal{D}} E^Q \left[\sum_{i=1}^2 V_i(t) \right] + \sum_{i=3}^N \left(-\inf_{Q \in \mathcal{D}} E^Q [V_i(t)] \right) \\ &= -\inf_{Q \in \mathcal{D}} E^Q \left[\sum_{i=1}^3 V_i(t) \right] + \sum_{i=4}^N \left(-\inf_{Q \in \mathcal{D}} E^Q [V_i(t)] \right) \\ &\quad \dots \\ &= -\inf_{Q \in \mathcal{D}} E^Q \left[\sum_{i=1}^N V_i(t) \right] = LHS \end{aligned}$$

■

Now let us evaluate a bit further on our assumption that V_i should all be increasing function in R_i . In reality, many funds strategically bet against the market and in fact make profit using such strategy, in which case their expected returns will only be decreasing functions of the total return. In such case, we will charge the negative of the calculated (say $-\mathcal{C}(V_i)$ for illustration) to be included in the sum,

and we have

$$\begin{aligned}
&= - \inf_{Q \in \mathcal{D}} E^Q[V_1(t)] - \inf_{Q \in \mathcal{D}} E^Q[V_2(t)] + \dots - \inf_{Q \in \mathcal{D}} E^Q[-V_i(t)] + \dots \\
&\quad - \inf_{Q \in \mathcal{D}} E^Q[V_N(t)]
\end{aligned}$$

where the functions inside expectations are still co-monotonic, so that

$$\begin{aligned}
&= - \inf_{Q \in \mathcal{D}} E^Q[V_1(t)] - \inf_{Q \in \mathcal{D}} E^Q[V_2(t)] + \dots - \inf_{Q \in \mathcal{D}} E^Q[-V_i(t)] + \dots \\
&\quad - \inf_{Q \in \mathcal{D}} E^Q[V_N(t)] \\
&= - \inf_{Q \in \mathcal{D}} E^Q[V_1 + V_2 + \dots - V_i \dots + V_N]
\end{aligned}$$

which is still the correct capital charge on total capital only with the adjustment that the negative of the cash flow is added in the sum if the fund contribute to the systemic risk in an opposite way.

2.3 Correlated Gaussian Model

The correlation structure could be still either the Gaussian case or the VG case. In the Gaussian case, let us assume

$$X_i(t) = X_i(0)e^{rt} + \int_0^t e^{r(t-u)} \sigma dW_i(u) \quad (2.12)$$

where W_i are correlated Brownian motions. In other words, for $i = 1, 2, \dots, N$

$$\Delta X_i = X_i(t+h) - X_i(t) = \sigma_i \sqrt{h} Z_i \quad (2.13)$$

and if we let

$$\rho_{i,j} = \text{corr}(Z_i, Z_j) \quad (2.14)$$

then we have variance covariance matrix for the vector $\Delta \mathbf{X} = [\Delta X_1, \Delta X_2, \dots, \Delta X_N]$

$$\Sigma = \begin{bmatrix} \sigma_1^2 h & \sigma_1 \sigma_2 h \rho_{1,2} & \cdots \\ \sigma_1 \sigma_2 h \rho_{1,2} & \sigma_2^2 h & \\ \cdots & & \sigma_N^2 h \end{bmatrix}. \quad (2.15)$$

This matrix can be estimated using MLE. We have all the $X_i(t)$ are also multivariate Normal with

$$\begin{aligned} \mu_{X_i} &= X_i(0)e^{rt} \\ \sigma_{X_i}^2 &= \sigma_i^2 \frac{e^{rt} - 1}{2r} \\ \text{cov}(X_i, X_j) &= \sigma_i \sigma_j \rho_{i,j} \frac{e^{2rt} + 1 - 2e^{rt}}{r^2 \delta} \end{aligned}$$

For the total cash flow $Y = \sum_i X_i$, it is also a Normal variable with

$$\begin{aligned} \mu_Y &= \sum_{i=1}^N \mu_{X_i} \\ \sigma_Y^2 &= \sum \sigma_{X_i}^2 + 2 \sum \text{cov}(X_i, X_j) \end{aligned}$$

The variable $X_i|Y = y$ is distributed as

$$N\left(\mu_{X_i} + \frac{\sigma_{X_i}}{\sigma_Y} \rho_{X_i, Y} (y - \mu_Y), (1 - \rho_{X_i, Y}^2) \sigma_{X_i}^2\right) \quad (2.16)$$

where

$$\rho_{X_i, Y} = \frac{\sum_{j \neq i} \text{cov}(X_i, X_j) + \sigma_{X_i}^2}{\sigma_{X_i} \sigma_Y} \quad (2.17)$$

To implement the model, we would first obtain monthly changes of cash flows from monthly NAV and then demean each univariate data. The estimated parameters (variances and correlations) of the correlated Gaussian model will then be used in a simulation to generate 10000 readings from this joint law by generating correlated Gaussian random variables to form a reading on an N vector in line with equation (2.13). The result is an N by 10000 matrix of draws from the correlated Gaussian law. This matrix of simulated draws from the estimated model will be used subsequently in our calculation of cash capital.

2.4 Correlated VG Model

Each fund has cash flow

$$X_i(t) = X_i(0)e^{rt} + \int_0^t e^{r(t-u)} dH_i(t) \quad (2.18)$$

where

$$H_i(t) = \theta_i(G_i(t) - t) + \sigma_i W_i(G_i(t)) \quad (2.19)$$

We introduce dependence between them by merely correlating the Brownian motions and keeping the time changing subordinator independent ([26], [34]). The demeaned changes are

$$\Delta X_i(h) = X_i(t+h) - X_i(t) = \theta_i(g_i - h) + \sigma_i \sqrt{g_i} Z_i \quad (2.20)$$

where variables $Z_i, i = 1, 2, \dots, N$ are now standard normal variates with correlations ρ_{ij} for $i \neq j$. The joint probability density and characteristic functions are not available in closed form as one has to integrate out a large number of independent

gamma densities but they appear as products of square roots that do not separate out in either the density or the characteristic function. The joint law, however, is easily simulated from a multivariate normal simulation coupled with drawings from gamma densities.

We see that there is now dependence between the unit changes as the covariances

$$E[\Delta X_i(h), \Delta X_j(h)] = \sigma_i \sigma_j E[\sqrt{g_i}] E[\sqrt{g_j}] \rho_{i,j} \quad (2.21)$$

are not zero.

We see from this equation that once we have estimated the marginal laws and have the specification of the time change and the parameters σ_i , ν_i , θ_i , and the parameters for the subordinator we may estimate the correlation between the Brownian motions implied by the time change model by

$$\rho_{i,j} = \frac{E[\Delta X_i(h), \Delta X_j(h)]}{\sigma_i \sigma_j E[\sqrt{g_i}] E[\sqrt{g_j}]} \quad (2.22)$$

as the numerator is estimated by evaluating a sample covariance and we need to compute the expectation of the square root of the subordinator. In the case of VG model the density of the time change at unit time has a single parameter ν_i and

$$\begin{aligned} E[\sqrt{g_i}] &= \int_0^\infty \frac{1}{\nu_i^{\frac{1}{\nu_i}} \Gamma(\frac{1}{\nu_i})} \sqrt{x} x^{\frac{1}{\nu_i}-1} e^{-\frac{x}{\nu_i}} dx \\ &= \frac{\sqrt{\nu_i} \Gamma(\frac{1}{\nu_i} + \frac{1}{2})}{\Gamma(\frac{1}{\nu_i})} \end{aligned}$$

Once we have estimated the marginal distribution parameters σ_i , ν_i , θ_i by maximum

likelihood on time series monthly data, we estimate the correlation implied between the Brownian motions. Specifically, we center the data to a zero sample mean and estimate the marginal distribution functions on the univariate data. This gives us a matrix of marginal VG parameters

$$\sigma_i, \nu_i, \theta_i, \quad i = 1, \dots, N \quad (2.23)$$

We then infer the correlations between the Gaussian variates from the observed matrix of covariances between observed returns. This procedure inflates Gaussian component correlations relative to observed correlations by a factor of de-correlation induced by the independent gamma time changes that depends on just the marginal laws (see [34]). We follow [34] and construct the closest correlation to our symmetric matrix using the procedures of [67]. We then generate 10000 readings from this joint law by generating correlated Gaussian random variables and independent gamma variates to form a reading on an N vector in line with equation (2.20). The result is an N by 10000 matrix of draws from the correlated VG law. This matrix of simulated draws from the estimated model will be used subsequently in our calculation of cash capital.

2.5 Data and Analysis

In the previous study we have collected data for 3622 funds that have monthly NAV and return data for the period of Jan 2005 to April 2009. Using this pool of funds as the universe of funds, we choose from them the biggest funds: funds that have AUM larger than \$2Bn as of end of April 30, 2009. We also require that the

funds have all data from April 2006 to April 2009 available, therefore 3 years of monthly data. There are 30 of these funds in total. The total AUM of these 30 funds is about 620 billion and is 64% of the total AUM of 3622 funds. Because our need to estimate the laws jointly, we must have same length of data for each fund, and this is the reason we choose the 30 largest funds who all have 3 years of data in the same period. Even though some funds have more data, we must drop them. This is also the reason that some large funds such as JPMorgan are not included in our analysis due to lack of relevant data (JPMorgan for example stopped reporting in CISDM after 2005).

We show first in Table 2.1 a categorization of these 30 funds in terms of fund types and strategies as defined in CISDM.

We also show in Table 2.2 and Table 2.3 some summary statistics of the funds performance and their characteristics. These tables show the mean returns, standard deviations (SD), medians, skewness (Skew), Min-Max skewness (MM Skew), kurtosis, minimum and maximum realizations and Sharpe Ratios (SR) for the individual Hedge Funds during April 2006 to April 2009. We calculate the Sharpe Ratio considering a risk-free rate of 0.0027. Min-Max skewness is computed as

$$(\textit{Maximum} + \textit{Minimum} - (2 * \textit{Mean})) / (\textit{Maximum} - \textit{Minimum}). \quad (2.24)$$

NAV is dollar per share, and AUM is in unit of billion dollars.

Next we begin our empirical study and first the change in NAV data is organized and demeaned and fed into optimizer to estimate the joint laws in both the Correlated Gaussian model and the correlated VG model. The eigenvalues and

Table 2.1: Fund Types and Strategies Categorization for 30 Largest Funds on April 30, 2009.

FUND TYPE	STRATEGY	N
Hedge Fund	Equity Long/Short	5
	Equity Long Only	8
	Equity Market Neutral	1
	Global Macro	1
	Multi Strategy	1
	Emerging Markets	1
	Event Driven Multi Strategy	1
Fund of Funds	Multi Strategy	2
	Market Neutral	1
	Unspecified	1
		4
Commodity Trading Advisor	Systematic	5
		5
Commodity Pool Operator	Single Strategy	2
	Multi Strategy	1
		3

eigenvectors of the correlation matrices are reported in Table 2.4 and Table 2.5. We only report the largest eigenvalues since these indicate the significance of the underlying factors. We see from the results that for example in the VG case there are only five components that have significant impact on the correlated returns. The dimensionality of the returns are reduced from 30 funds to five underlying factors, although the question remain what constitute these factors.

We then simulate jointly the cash flows $X_i(t)$ out to one year (and also one month before for the return calculation). We sum together the one-month returns for individual funds and calculate the function $V_i(t)$ as in (2.7). This conditional

expectation is calculated in our simulated sample numerically. Once the cash flows are generated, the capital charges on X_i and V_i are calculated as in (2.4) and (2.9). This capital calculation is done using the specific form of acceptability employed as positive expectation under a concave distortion of the cash flow distribution with the concave distortion function again chosen to be the MINMAXVAR function as described in detail in [36]. We charge each fund $\mathcal{C}(V_i)$ to be held at an aggregate level to account for its systemic contribution and also $\mathcal{C}(X_i) - \mathcal{C}(V_i)$ to be held at the fund level. We also show results for taxpayer put values in the setup of correlated funds, here the put value for firm i is determined by the underlying firm cash flow and the capital charge associated with it.

$$TP_i = e^{-rt} \mathbb{E}_0^Q [(-\mathcal{C}(X_i)e^{rt} - X_i(t))^+] \quad (2.25)$$

Since in capital charge calculation, any risk is always credited for its mean, we focus our attention on the demeaned risks and start the simulation always from initial value zero. Tables 2.6 and 2.7 report the capital charges for each model with acceptability level set to 1.25. NAV and AUM on April 30 2009 are also shown for comparison and numbers of shares are estimated by simply dividing AUM by NAV. We also show in the tables the charges as percentages of AUM for each fund. All numbers shown are in dollars per share basis except for AUM.

Table 2.2: Summary Statistics of the 30 Largest Funds as of April 30, 2009. NAV is dollar per share, and AUM is in unit of billion dollars.

394	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0118	0.0520	-0.0050	-0.1340	0.1120	-0.3026	0.0064	3.1793	-0.2784
NAV	28634.07	5565.38	31684.28	18396.88	34066.86				
AUM	7.3292	3.1942	7.8000	2.5000	14.1000				
446	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0050	0.0297	0.0053	-0.0709	0.0500	-0.6894	-0.2548	3.2683	0.0759
NAV	4610.82	353.45	4541.72	4156.03	5228.34				
AUM	5.8454	0.2980	5.8080	5.3440	6.6660				
2239	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0051	0.0257	-0.0008	-0.0704	0.0346	-0.5407	-0.2429	2.6561	-0.3057
NAV	4373.05	457.39	4460.27	3368.63	5000.32				
AUM	8.0272	1.6528	8.3043	4.5158	10.2300				
2526	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0001	0.0351	0.0109	-0.1289	0.0482	-1.8017	-0.4541	6.8776	-0.0809
NAV	395.24	52.00	392.96	322.11	472.19				
AUM	2.8035	0.4758	2.6982	2.0050	3.6017				
2995	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0069	0.0394	0.0180	-0.1380	0.0614	-1.3741	-0.4536	6.0667	0.1072
NAV	2812.69	392.15	2890.50	2151.32	3522.27				
AUM	2.0783	1.0128	2.0720	0.7220	3.8620				
3257	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0035	0.0184	0.0030	-0.0330	0.0450	0.2187	0.0644	2.8906	0.0427
NAV	67.60	3.04	68.04	62.15	73.85				
AUM	4.1586	0.6921	4.1830	2.6910	5.2170				
3831	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0009	0.0222	0.0047	-0.0747	0.0313	-1.4757	-0.3924	5.4093	-0.1627
NAV	1817.96	135.27	1827.45	1582.47	2019.86				
AUM	1.9581	0.5811	2.1671	0.8347	2.7798				
3837	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0005	0.0237	0.0064	-0.0859	0.0281	-1.7554	-0.4989	6.4701	-0.1336
NAV	2359.11	192.02	2369.64	2055.98	2631.81				
AUM	3.5696	0.5512	3.4410	2.5260	4.4260				
5077	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0130	0.0322	0.0137	-0.0777	0.0766	-0.3935	-0.1755	3.2335	0.3193
NAV	2922.37	455.24	2854.77	2315.24	3745.87				
AUM	1.8541	1.2229	1.7856	0.2277	3.6451				
5144	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0142	0.0372	0.0114	-0.0446	0.0845	0.1663	0.0894	2.0181	0.3089
NAV	1.528.66	291.26	1555.22	1093.03	2008.72				
AUM	19.0035	7.2780	16.8260	10.9440	42.2880				
5292	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0122	0.0342	0.0148	-0.0593	0.0795	-0.0190	-0.0296	2.2628	0.2762
NAV	595.78	95.55	590.47	464.45	740.84				
AUM	4.4006	1.6800	4.1569	2.1603	7.2850				
5353	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0046	0.0374	-0.0041	-0.1081	0.0781	-0.2055	-0.1114	3.5999	-0.1959
NAV	1067.76	62.82	1059.08	954.63	1233.46				
AUM	7.7087	2.6878	8.2424	3.4263	10.9705				
5598	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0154	0.0362	0.0133	-0.0407	0.0902	0.1794	0.1430	2.0352	0.3501
NAV	1642.64	337.72	1668.26	1154.13	2197.48				
AUM	2.7075	1.2612	2.2250	1.4380	4.8940				
6016	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0139	0.0205	0.0141	-0.0381	0.0753	0.3089	0.0823	4.3212	0.5472
NAV	3712.37	459.51	3600.84	3114.33	4982.62				
AUM	55.1748	16.3687	57.9190	23.6340	80.4590				
6102	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	0.0119	0.0510	0.0100	-0.0754	0.1545	0.4721	0.2405	3.0206	0.1805
NAV	81.35	14.06	80.65	62.99	107.45				
AUM	3.0987	0.6340	2.8902	2.3661	4.3084				

Table 2.3: Summary Statistics of the 30 Largest Funds as of April 30, 2009, part II

6988	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0058	0.0376	-0.0051	-0.1092	0.0776	-0.2000	-0.1067	3.5799	-0.2272
NAV	2881.54	195.34	2831.08	2517.77	3348.77				
AUM	4.1425	1.3012	4.3952	2.1091	5.8331				
7306	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0056	0.0250	0.0042	-0.0912	0.0211	-1.8384	-0.5252	6.2479	-0.3317
NAV	109385.52	10145.02	113627.68	87934.08	119135.39				
AUM	115.4168	14.7817	121.0102	85.9310	132.5293				
7307	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0053	0.0249	0.0047	-0.0908	0.0212	-1.8477	-0.5256	6.2764	-0.3230
NAV	107980.39	9847.52	111695.56	87223.43	117626.99				
AUM									
8312	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0131	0.0626	-0.0140	-0.2040	0.1240	-0.4331	-0.1640	4.4162	-0.2525
NAV	1131.50	249.84	1244.82	643.63	1415.67				
AUM	10.0879	2.5133	9.3000	6.4110	15.2000				
8725	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0043	0.0268	0	-0.1300	0.0300	-2.8482	-0.5716	14.1993	-0.2597
NAV	1038.47	94.62	1000.40	914.70	1187.59				
AUM	10.8760	2.5862	11.5000	6.0000	16.0000				
9510	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0133	0.0602	-0.0110	-0.2000	0.1290	-0.2211	-0.1347	4.7846	-0.2665
NAV	2719.90	515.27	2910.98	1636.03	3352.00				
AUM	279.7297	36.2236	304.0000	193.0000	306.0000				
9512	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0028	0.0606	-0.0020	-0.1440	0.1630	-0.0973	0.0804	3.8640	-0.0914
NAV	107.18	18.42	111.15	66.03	131.90				
AUM	9.7283	1.6172	10.5480	5.9000	11.3700				
9676	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0245	0.0622	-0.0260	-0.2150	0.0600	-0.8111	-0.3856	3.5519	-0.4369
NAV	157.71	48.18	172.42	76.49	227.60				
AUM	12.6999	2.6133	14.1000	7.6380	14.9350				
10964	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0119	0.0651	-0.0140	-0.2030	0.1450	-0.2710	-0.0983	4.2546	-0.2241
NAV	8869.53	1920.38	9552.15	5100.38	11145.82				
AUM									
10966	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0237	0.0616	-0.0260	-0.2150	0.0600	-0.8379	-0.3911	3.6900	-0.4290
NAV	14743.34	4455.72	16101.48	7186.55	20796.02				
AUM									
10967	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0239	0.0616	-0.0260	-0.2150	0.0600	-0.8382	-0.3897	3.6769	-0.4318
NAV	14707.14	4478.57	16080.12	7139.77	20791.95				
AUM									
10969	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0242	0.0622	-0.0260	-0.2140	0.0610	-0.8012	-0.3807	3.5266	-0.4316
NAV	158.12	47.85	172.62	77.29	227.60				
AUM									
11882	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0051	0.0269	-0.0077	-0.0736	0.0501	-0.2639	-0.1074	3.1876	-0.2901
NAV	13670.47	1231.01	13636.12	11098.21	15953.09				
AUM	4.8787	1.3200	5.0918	2.4112	7.7930				
11883	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0060	0.0268	-0.0087	-0.0745	0.0491	-0.2924	-0.1093	3.2055	-0.3240
NAV	12109.32	1125.64	12176.00	9699.73	14114.53				
AUM	4.8787	1.3200	5.0918	2.4112	7.7930				
12644	mean	std	median	min	max	skew	MM skew	kurtosis	SR
Ret	-0.0008	0.0144	0.0005	-0.0451	0.0276	-0.6984	-0.2176	4.1285	-0.2455
NAV	1170.58	18.94	1169.66	1138.48	1217.58				
AUM	4.0080	0.6431	4.2617	2.9639	4.8491				

Table 2.4: The largest five Eigenvalues and associated Eigenvectors for estimated correlation matrix in Gaussian case

Eigenvalues	1.4504	1.9823	2.8259	5.3521	12.8918
Eigenvectors	-0.1296	-0.0444	-0.1128	-0.0397	0.2343
	0.1239	-0.1182	0.3390	0.2503	-0.0168
	0.0959	0.0002	0.2359	0.0715	0.2142
	0.0762	-0.0859	0.2335	-0.0424	0.2270
	0.0882	-0.2037	0.3045	0.2229	0.0340
	0.6326	-0.2152	-0.0067	-0.1363	0.0788
	-0.1055	-0.0994	0.2889	0.0005	0.2274
	-0.0301	-0.1395	0.2415	-0.0345	0.2371
	-0.3731	-0.3494	0.0873	-0.1062	0.0140
	-0.0815	-0.1518	-0.1564	0.3750	0.0498
	0.0196	-0.1527	0.0424	0.3525	0.0311
	-0.1384	0.1589	-0.0380	0.3166	0.1015
	-0.0580	-0.1769	-0.1520	0.3788	0.0385
	-0.2394	0.0494	0.0184	-0.1816	0.1084
	0.2361	-0.1720	-0.1006	0.3443	-0.0241
	-0.1335	0.1563	-0.0490	0.3165	0.1012
	-0.1123	-0.0263	0.2216	-0.0635	0.2410
	-0.1147	-0.0232	0.2250	-0.0619	0.2400
	-0.0339	0.0377	-0.1109	-0.0759	0.2628
	0.2452	0.4329	0.0762	0.1124	0.1037
	0.1036	-0.1291	-0.2341	-0.1498	0.2070
	0.2940	-0.2392	-0.0236	-0.1456	0.1996
	-0.0469	-0.0260	-0.1813	0.0365	0.2542
	-0.0094	0.0247	-0.1113	-0.0768	0.2615
	-0.0526	-0.0218	-0.1713	0.0317	0.2561
	-0.0523	-0.0188	-0.1725	0.0326	0.2558
	-0.0497	-0.0293	-0.1800	0.0359	0.2544
	0.1197	0.3824	0.0494	0.0588	0.1988
	0.1047	0.3798	0.0502	0.0625	0.2003
	0.1161	-0.1438	-0.3760	-0.0034	0.0868

Table 2.5: The largest five Eigenvalues and associated Eigenvectors for estimated correlation matrix in VG case

Eigenvalues	0.6309	0.9432	1.0858	5.5765	21.7636
Eigenvectors	-0.0036	0.0085	-0.0105	0.0399	0.2134
	-0.0425	-0.1928	-0.1624	-0.4033	-0.0359
	-0.0090	-0.0108	-0.0138	0.0009	0.2143
	-0.0070	-0.0121	-0.0143	0.0193	0.2141
	-0.0435	-0.3296	-0.3491	-0.3263	0.0885
	0.6858	-0.5420	-0.0426	0.1273	0.1239
	-0.0168	-0.0162	-0.0298	0.0163	0.2140
	-0.0097	-0.0105	-0.0256	0.0223	0.2140
	-0.0240	0.3066	-0.8538	0.1287	0.0353
	0.1159	0.2706	0.0311	-0.3319	0.1188
	0.1052	-0.1083	-0.1187	-0.3842	0.0812
	-0.1264	0.0522	0.1024	-0.1861	0.1897
	0.1703	0.2980	0.0140	-0.3515	0.0979
	-0.1194	-0.0294	-0.0659	0.2044	0.1859
	0.1905	0.0420	0.0782	-0.4121	-0.0317
	-0.2597	0.0913	0.1816	-0.1995	0.1783
	-0.0119	-0.0081	-0.0158	0.0276	0.2139
	-0.0121	-0.0081	-0.0158	0.0274	0.2139
	-0.0055	0.0048	-0.0017	0.0475	0.2130
	-0.3432	-0.2691	0.1402	-0.0345	0.1952
	0.0133	0.0152	0.0010	0.0764	0.2108
	0.0177	-0.0115	-0.0183	0.0666	0.2116
	-0.0034	0.0062	-0.0024	0.0238	0.2140
	-0.0049	0.0046	-0.0013	0.0464	0.2131
	-0.0041	0.0055	-0.0033	0.0244	0.2140
	-0.0042	0.0056	-0.0031	0.0244	0.2140
	-0.0019	0.0039	-0.0015	0.0166	0.2142
	-0.0527	-0.0308	0.0178	0.0243	0.2137
	-0.0336	-0.0190	0.0095	0.0174	0.2141
	0.4636	0.4516	0.1699	0.0881	0.1657

Table 2.6: Cash Charges on Systemic Level and Fund Level, As Well As Taxpayer Put Values for 30 Largest Funds on April 30, 2009, Using Correlated Gaussian Model ($\gamma = 1.25$)

Fund id	NAV	AUM	shares ($\frac{AUM}{NAV}$)	$C(X_i)$	$C(V_i)$	$C(X_i) - C(V_i)$	$C(V_i)$ *shares	%AUM	$(C(X_i) - C(V_i))$ *shares	%AUM	TP	TP *shares	%AUM
394	20164.5995	3200000000	158693.9527	8270.7504	7568.4053	702.3451	1201060156	0.3753	111457920.1	0.0348	126.3510	20051139.56	0.0063
446	4984.3726	6122000000	1228238.836	810.0697	755.2078	54.8619	927575579.1	0.1515	67383511.62	0.0110	11.4452	14057445.54	0.0023
2239	3421.5173	4515828078	1319832.017	631.8143	586.3068	45.5076	773826446.8	0.1714	60062344.55	0.0133	8.9481	11810044.52	0.0026
2526	331.9306	2485060000	7486685.47	79.9426	73.4960	6.4466	550241270.9	0.2214	48263568.02	0.0194	0.9729	7284003.786	0.0029
2995	2794.0749	2517000000	900834.8345	703.2369	643.6899	59.5470	579858273.9	0.2304	53642037.73	0.0213	9.5591	8611173.023	0.0034
3257	70.91	2888000000	40727682.98	6.9488	6.1793	0.7694	251671602.8	0.0871	31337753.06	0.0109	0.1126	4586231.308	0.0016
3831	1630.3629	2167071990	1329196.089	234.8577	220.4343	14.4234	293000358.1	0.1352	19171572.62	0.0088	3.0609	4068488.808	0.0019
3837	2121.8184	2587000000	1219237.235	322.1585	301.8797	20.2788	368063055.7	0.1423	24724609.29	0.0096	4.5695	5571364.011	0.0022
5077	3745.87	2725000000	727467.8513	539.5878	484.0834	55.5044	352155086.1	0.1292	40377658.33	0.0148	7.412	5392214.283	0.0020
5144	1939.9676	4228800000	21798302.2	322.3363	295.6236	26.7127	6444093268	0.1524	582291892.1	0.0138	3.8005	82843470.45	0.0020
5292	705.2728	3300000000	4679040.507	113.5055	104.4036	9.1018	488508828.6	0.1480	42587848.66	0.0129	1.7212	8058417.071	0.0024
5353	954.6326	3426300000	3589129.472	246.2073	222.0088	24.1985	790818334.9	0.2326	86851420.76	0.0253	3.1089	11373426.69	0.0033
5598	2127.9107	4468000000	2099712.173	340.1081	311.8289	28.2792	654751012.1	0.1465	59378087.08	0.0133	4.0640	8533146.366	0.0019
6016	4982.6236	6234900000	12513287.18	484.2376	436.7769	47.4607	5465514958	0.0877	593889101.4	0.0095	6.4815	81105213.53	0.0013
6102	94.8593	32185051.41	33929252.49	23.7710	21.4957	2.2754	729331475.9	0.2266	77202272.99	0.0240	0.3106	10536980.07	0.0033
6988	2517.7683	2109057467	837669.4023	671.4089	605.9285	65.4804	507567802.5	0.2407	54850921.83	0.0260	8.5905	7195969.334	0.0034
7306	90311.62	85930980000	951494.171	15221.9741	14193.7579	1028.2162	13505277929	0.1572	978341691.7	0.0114	199.0755	189419196.2	0.0022
7307	89375.19	85930980000	961463.4665	15042.2594	14044.0208	998.2387	13502812872	0.1571	959770015.6	0.0111	221.4303	212897172.5	0.0025
8312	766.1211	7640000000	9972313.777	366.2463	334.3930	31.8533	3334671476	0.4365	317651060.4	0.0416	4.7375	47243362.18	0.0062
8725	922.0193	9764000000	10589800.02	166.6520	149.2099	17.4421	1580103203	0.1618	184708534.5	0.0189	2.4277	25708442.88	0.0026
9510	1938.5292	2.26E+11	116583232.3	821.5415	734.4662	87.0753	85626439720	0.3789	10151520273	0.0449	10.9062	1271479138	0.0056
9512	84.0144	7300000000	86889866.5	31.4166	28.5301	2.8866	2478973410	0.3396	250812683.1	0.0344	0.4692	40771123.66	0.0056
9676	84.812	8472000000	99891524.78	52.9009	48.3320	4.5659	4827965299	0.5699	465388837	0.0549	0.6316	63089307.18	0.0074
10964	6082.2079	6206000000	1020353.152	2998.3721	2732.6241	265.7480	2788241621	0.4493	271156829.3	0.0437	39.9013	40713416.94	0.0066
10966	7975.9353	8539000000	1070595.445	4789.7150	4381.0048	408.7101	4690283831	0.5493	437563211.3	0.0512	59.7679	63987282.75	0.0075
10967	7916.5309	7909000000	999048.7121	4782.0751	4372.34006	409.7350	4368180706	0.5523	409345230.8	0.0518	59.6567	59599947.51	0.0075
10969	85.8656	8988000000	104675213.4	53.1679	48.5129	4.6550	5078097258	0.5650	487264506.1	0.0542	0.6348	66452344.62	0.0074
11882	11098.2059	2411197743	217260.1378	2165.0767	1965.2674	199.8094	426974255.9	0.1771	43410612.64	0.0180	32.0173	6956088.708	0.0029
11883	9699.7339	2411197743	248583.9063	1908.3911	1730.7020	177.6891	430224665.5	0.1784	44170661.05	0.0183	24.4477	6077301.14	0.0025
12644	1189.6427	2996000000	2518403.215	98.0796	89.1759	8.9037	224580990.5	0.0750	22423088.22	0.0075	1.2412	3125894.213	0.0010

Table 2.7: Cash Charges on Systemic Level and Fund Level, As Well As Taxpayer Put Values for 30 Largest Funds on April 30, 2009, Correlated VG Model ($\gamma = 1.25$)

Fund id	NAV	AUM	shares ($\frac{AUM}{NAV}$)	$C(X_i)$	$C(V_i)$	$C(X_i) - C(V_i)$	$C(V_i)$ *shares	%AUM	$(C(X_i) - C(V_i))$ *shares	%AUM	TP	TP *shares	%AUM
394	20164.5995	3200000000	158693.9527	8752.3562	8113.8318	638.5244	1287616036	0.4024	101329957.9	0.0317	89.5739	14214842.53	0.0044
446	4984.3726	6122000000	1228238.836	854.0421	765.8024	88.2397	940588205.8	0.1536	108379478.2	0.0178	10.9769	13482249.51	0.0022
2239	3421.5173	4515828078	1319832.017	666.1752	573.6309	92.5443	757096451.6	0.1677	122142954.6	0.0270	7.7600	10241889.89	0.0023
2526	331.9306	2485060000	7486685.47	93.1997	83.3694	9.8302	624161119.5	0.2512	73595946.18	0.0296	1.0852	8124230.483	0.0033
2995	2794.0749	2517000000	900834.8345	832.8967	755.6601	77.2367	680724932	0.2705	69577473.64	0.0276	9.1391	8232842.733	0.0033
3257	70.91	2888000000	40727682.98	8.7142	7.6054	1.1088	309751460.2	0.1073	45157519.35	0.0156	0.0903	3676036.539	0.0013
3831	1630.3629	2167071990	1329196.089	283.6147	247.5828	36.0319	329086139.1	0.1519	47893430.19	0.0221	2.6382	3506710.035	0.0016
3837	2121.8184	2587000000	1219237.235	407.3146	368.5642	38.7504	449367230	0.1737	4245937.34	0.0183	4.8685	5935822.417	0.0023
5077	3745.87	2725000000	727467.8513	563.4704	516.5028	46.9675	375739196.7	0.1379	34167373.61	0.0125	6.4998	4728413.418	0.0017
5144	1939.9676	4228800000	21798302.2	301.0143	265.6227	35.3917	5790122809	0.1369	771478594.4	0.0182	3.2964	71855673.1	0.0017
5292	705.2728	3300000000	4679040.507	116.8162	104.4703	12.3458	488820987.4	0.1481	57766698.31	0.0175	1.2979	6072792.457	0.0018
5353	954.6326	3426300000	3589129.472	249.4119	224.2888	25.1231	805001674.3	0.2349	90169947.95	0.0263	2.8146	10101906.54	0.0029
5598	2127.9107	4468000000	2099712.173	331.2086	296.5677	34.6409	622706731.6	0.1394	72735953.87	0.0163	3.7752	7926911.259	0.0018
6016	4982.6236	6234900000	12513287.18	484.1751	434.6155	49.5597	5438468390	0.0872	620154269.4	0.0099	5.5118	68971281.14	0.0011
6102	94.8593	32185051.41	33929252.49	21.6453	19.4097	2.2356	65857250.6	0.2046	75852652.97	0.0236	0.2637	8948723.314	0.0028
6988	2517.7683	2109057467	837669.4023	797.5214	719.9214	77.6000	603056109.5	0.2859	65003124.69	0.0308	8.3898	7027838.022	0.0033
7306	90311.62	8593080000	951494.171	18319.7943	15609.9718	2709.8226	14852797211	0.1728	2578380360	0.0300	199.7745	190084286.2	0.0022
7307	89375.19	8593080000	961463.4665	18277.7136	16693.2702	1584.4433	1604969462	0.1868	1523384379	0.0177	195.8946	188345514.7	0.0022
8312	766.1211	7640000000	9972313.777	379.2682	332.9389	46.3293	3220171377	0.4346	462010401.8	0.0605	3.7955	37849978.59	0.0050
8725	922.0193	9764000000	10589800.02	244.9961	215.5623	29.4338	282761553	0.2338	311698481.1	0.0319	2.7245	28851617.06	0.0030
9510	1938.5292	2.26E+11	116583232.3	850.8799	754.5991	96.2808	87973603772	0.3893	11224727210	0.0497	9.9169	1156140695	0.0051
9512	84.0144	7300000000	86889866.5	35.0929	31.1733	3.9196	2708644334	0.3710	340574428.5	0.0467	0.3883	33741943.26	0.0046
9676	84.812	8472000000	99891524.78	52.2070	45.6194	6.5876	4556993162	0.5370	658041139.7	0.0777	0.5114	51085891.8	0.0060
10964	6082.2079	6206000000	1020353.152	3319.5136	2915.2057	404.3079	2974539300	0.4793	412536842.6	0.0665	35.2444	35961717.37	0.0056
10966	7975.9353	8539000000	1070595.445	4656.1532	4083.3461	572.8071	4371611686	0.5120	113244718.5	0.0718	46.7592	50060168.87	0.0059
10967	7916.5309	7909000000	999048.7121	4683.3965	4097.8501	585.5464	4093951896	0.5176	584989363.4	0.0740	45.9526	45908878.33	0.0058
10969	85.8656	8988000000	104675213.4	59.1708	52.8830	6.2878	5535539513	0.6159	658171664.5	0.0732	6.6795	71127272.77	0.0079
11882	11098.2059	2411197743	217260.1378	2783.9539	2518.6108	265.3431	547193725.9	0.2269	57648473.89	0.0239	27.7958	6038917.848	0.0025
11883	9699.7339	2411197743	248583.9063	1987.7663	1795.9872	192.4791	446279501.4	0.1851	47847211.54	0.0198	21.3543	5308341.053	0.0022
12644	1189.6427	2996000000	2518403.215	127.3544	116.4828	10.8716	293350760.2	0.0979	27379002.68	0.0091	1.5018	3782098.351	0.0013

2.6 Conclusion

Just like large investment banks, large hedge funds play a significant role in facilitating the efficient allocation of the risks of investments. With the status of limited liability, large hedge funds however, also pose great threat to the health of the general economy when things are not so great. Large institutions such as these big hedge funds must satisfy capital requirements intended to ensure that they can sustain reasonable losses. The capital requirements introduced in our previous paper are designed so that the total risky cash flow as the assets less the liabilities is acceptable to the general economy as specified by the theory of acceptability ([10], [50], [31]). The capital requirements however are designed as if each fund is an isolated entity. Because many hedge funds employ similar investment strategies they produce correlated returns. As a huge trillion dollar industry the failure of these correlated large firms will greatly affect the markets systematically either in a direct or indirect way. The high leverage employed by these funds also have the potential to exacerbate instability in the market as a whole. In this paper we take into considerations the systemic effects when setting the capital requirements.

The previous study has considered funds individually with no correlation between them. Unfortunately, focusing only on individual funds ignores the critical interactions between them and can cause the regulators to overlook important changes in the overall system. We do realize that a conclusive assessment of the systemic risks posed by hedge funds requires certain data that is currently unavailable such as counter-party credit exposures, the net degree of leverage of hedge fund man-

agers and investors, the gross amount of structured products involving hedge funds, etc. Nonetheless, we build upon our current modeling framework and propose a systemic approach with largest market participants and their return correlations incorporated. We study the 30 largest funds as of April 2009 with total Asset Under Management over \$620 Bn. The capital charges on these biggest funds must account for both their contribution to systemic risk and their own idiosyncratic risk. We separate the cash flows of the funds which are marginally Normal or marginally VG, into two components: the systemic component and the idiosyncratic component. The systemic component is constructed to be a conditional expectation of the underlying cash flow given the industry total return. These conditional expectations are postulated to be co-monotonic functions and enjoy additivity shown in Theorem 2.1. Theorem 2.2 then shows that the charges calculated on these random risky flows are able to sum to account for the aggregate systemic risk. The residual charge is then held at the fund level to account for the idiosyncratic risk component. In our empirical study, we select from a pool of 3622 funds the 30 largest funds who have AUM larger than \$2Bn as of April 30, 2009. These funds account for 64% of the total AUM and are the most influential in the industry. We use available data on the returns and NAVs to estimate the model parameters and infer the correlations among these firms. Many of them show significant correlations as expected. Once the models are estimated, we simulate the cash flows out to one year and calculate the capital charges that would guarantee to cover potential losses in a year. The charges on each fund towards the systemic contribution will be collected by external hedge funds authority. These charges will sum to account for the aggregate risky

outcome as long as the funds expected returns are increasing functions of the total aggregate return. Even if expected returns are decreasing functions of the total return as some firms bet against the market, we will charge the negative of the calculated capital to be included in the sum to account for the opposite effect they have on the systemic risk contribution. Our results are shown in Section 2.5 with both types of capital charges compared against actual AUM level. We also show taxpayer put values in this correlated setup. Each firm's taxpayer put value is determined by the underlying firm cash flow and the capital charge associated with it. Our hope here is to provide some guidance on how hedge funds should be charged cash (or cash equivalent) capitals in order to account for risks they pose to the economy systemically and as an individual. We hope by proposing such methods, regulators will have much stronger tools to address systemic financial crises, not only in the hedge fund industry, but also in the banking industry, including a new resolution mechanism for addressing the failures of systemically significant firms.

Chapter 3

Hedge Fund Leverage Estimation and Hedge Fund Reserve Capital

3.1 Overview

In our previous studies on hedge fund capital reserving ([73], [74]), hedge fund information is limited to only the total return without a separation of its plus side and its minus side. It is very important however, pointed out in [59] and studied further in bank domain ([36]) and in hedge fund domain ([73]), to consider the risks from liabilities together with the ones from assets in a correlated way. The taxpayer put option ([36]) born by the setup of limited liable entities holding also potentially unbounded liabilities (even without the presence of debt) must be monitored. The regulatory agencies must require reserved capital charges in order to ensure that the value of this put option is not too large. It is necessary to set up such externally determined cash or cash equivalent amount so that the risky cash flow, coming from the balance sheet of a limited liability entity with access to unbounded random liabilities, is acceptable to the outside market. This is linked to the context of acceptable risks and such precise link between capital reserve and acceptable risks has been proposed by [59] and studied further in [36], [73] and [74]. In order to extract balance sheet information from publicly available data on hedge funds, we propose to start with an estimation of fund leverages. This is by itself an interesting and challenging task since fund leverage, in spite of its importance, is generally not

reported and unknown to the public. Assisted with these estimated leverage values, we are then able to obtain the separated cash flows and the implied joint laws to infer capital implication.

Many hedge funds use leverage to enhance their returns and, consequently, their risks. Hedge funds often leverage the funds to the extent only restricted by their creditors. The net asset value of a hedge fund can run into many billions of dollars, and the gross assets of the fund will usually be even larger since hedge funds typically also borrow money or trade on margin in addition to the money invested by the investors. This is usually labeled as funding leverage. An alternative way of achieving leverage is through the choice of investment instruments, such as derivatives and structured notes. This type of leverage is labeled as instrument leverage. Either way, leverage is quite often being employed in such a way, so that the investment in the long position is a multiple of the hedge fund equity, in the hope of making enlarged returns on the original equity. A big leverage, unfortunately, also gives rise to the possibility that an adverse shock to the fund returns might lead to negative net worth. If for example a hedge fund has borrowed \$9 for every \$1 received from investors, a loss of only 10% of the value of the investments will wipe out 100% of the value of the investors' stake in the fund, once the creditors have called in their loans. In September 1998, shortly before its collapse, Long Term Capital Management (LTCM) had \$125 billion of assets on a base of \$4 billion of investors' money, a leverage of over 30 times. It also had off-balance sheet positions with a notional value of approximately \$1 trillion. The excessive leverage that is used by hedge funds to achieve their return is outlined as one of the main factors

of the hedge funds' contribution to systemic risk. In the absence of more detailed information on hedge fund investments, the estimation of leverage can serve as a tool for the surveillance of stability of the financial system.

The interesting challenge of estimating hedge fund leverage has been studied in [63] as a way to measure any resulting systemic risks to the financial system. In their paper, the estimation of leverage is done with an extension of “regression-based style analysis” that has also been employed by many other papers (to cite a few such as [43] [7] and [32]) to study hedge fund performance. The leverage in concern is only the funding leverage. However, at the end of the day, if a fund rises twice as much as the market on “up” days and falls twice as much on “down” days, then the source of leverage is less relevant. In fact, deriving leverage based on historical returns will also capture the leverage implicit in the balance sheets or business models of individual funds. We will follow the same footsteps here with slightly different implementation as to the choosing of right-hand-side regression factors including the nonlinear option-like factors. We also add in the regression the lagged returns which result in substantially increased leverage estimates. The most distinguishing character of our study compared to previous studies, is that we perform the estimation on an individual fund level. In [63], leverage indicators are obtained on a fund-family level and there are a total of nine families based on different investment strategies. In order for our capital implication to be meaningful, we would require fund level estimates of fund leverages. This sort of estimation requires a relatively long time series of fund returns and better identification of the right-hand-side regressors. We take CISDM hedge fund monthly data and only

retain funds who have continuous data for returns, from February 1996 to September 2008. This results in 177 funds each with 152 monthly data points. This obviously creates a survivorship bias in our sample. Nonetheless, since our main objective is to estimate leverage on individual funds and not to make inference about overall performance, our filter may not be as problematic. We do however, expand our sample so that funds only need shorter time period of survival from January 2002 to September 2008. This results in 1797 funds and the results are in agreement with the ones from longer period. The representative market factors are chosen to be a set of indices which come from equity, bond, and commodity markets. Since hedge fund returns exhibit nonlinear option-like exposures to standard asset classes ([42], [43]), we follow [7], [32], and [63] to estimate leverage including some synthetic option factors to take into account such nonlinearity. These options are termed power-tail options and are written on S&P500 as their underlying. We mainly focus on equity markets since it has been shown (e.g. [63]) that returns across most fund families seem to be heavily influenced by equity market factors. In addition, [63] found in their analysis that the returns on the broad equity market index (S&P500) and the associated synthetic option factors are almost always important drivers of performance. We perform a Principal Component Analysis on the option returns and retain the first two principal components as they explain about 99% of the variance. This procedure eliminates the multicollinearity problem and these two principal factors will enter into the regression equation instead of the 18 option factors. In addition to broad market indices and option-like factors, we have also added lagged returns in the regression model. After adding the lagged returns, the

model is in fact an ARX model (Autoregressive model with exogenous inputs) and allows for dependence of current returns on returns from previous periods. Although theoretically, return serial correlation implies market inefficiency and presence of predictability in returns which contradicts the common belief that hedge funds are operated under highly capable fund managers with optimal investment strategies, in practice, there might be other reasons for the returns to show serial correlation. Most importantly, the impact on our leverage estimation is of true interest here. Finally, we adopt the approach implemented in [63] so that the leverage estimator is simply a sum of the absolute values of the estimated coefficients of each fund. We estimate time-varying parameters in a rolling regression with 36 months rolling window. Within each window, a two-stage stepwise regression is performed.

Contrast to regulated mutual funds, hedge funds are private and lightly regulated entities who are not obliged to disclose their activities to the general public. Data on hedge funds are reported as funds wish and usually incomplete and very limited. For example in CISDM hedge funds database, the reported NAV is on a per share basis without much knowledge of actual number of shares outstanding, AUM has frequent missing data and leverage information is mostly stale if even reported. Little is known on hedge funds balance sheet to separate the assets and liabilities. Once we estimate the leverage on a fund level, we build a more comprehensive framework in which the segregated balance sheet information can be extracted and capital implication can be obtained. Following [36], we model the logs of assets and liabilities as linear mixture of some unknown latent variables. These latent variables are assumed to follow Variance Gamma (VG) distribution and such joint

laws have been considered in the time series context by [62], [58] and [52]. In the time series applications the required mixing matrix is estimated using Independent Components Analysis (ICA) ([48]). The latent variables are assumed non-Gaussian and mutually independent, and they are also found by the ICA. We also demonstrate a benchmark model by modeling assets and liabilities as marginal lognormal with correlated Gaussian components. We show in comparison the modeling of total cash flow (asset less liability) as a real-valued Martingale (see [73] for detail) and include also two models one with Gaussian components one with VG components. Maximum likelihood estimation can be performed to obtain model parameters. Using the theory of acceptable risks one would require that the simulated cash flows be acceptable. We report the required capital for these four different models and compare the requirement to the last observed AUM which is viewed as the “cash” on hand held at the fund.

The outline of the rest of the chapter is as follows. Section 3.2 describes the estimation of leverage detailed as methodology, data description, building of non-linear factors, using of lagged returns and discussion of results. Section 3.3 demonstrates how to use the leverage estimates in separating assets and liabilities as well as modeling framework used in calculating capital reserve results. Finally, Section 3.4 concludes.

3.2 Estimating Hedge Fund Leverage

3.2.1 Methodology

A number of empirical studies have highlighted the unique risk and reward profiles of hedge fund investments. For example, [2], [40],[41], [43], [53], [54], [55], [6], [5], [37], [51], and [9] provide comprehensive empirical studies of historical hedge fund performance using various hedge fund databases. [18], [19], [20], [42], [39], [17], [7], [4], [16], and [57] present more detailed performance attribution and “style” analysis for hedge funds. Collectively, these studies show that the dynamics of hedge funds are quite different from those of more traditional investments, and the potential impact on systemic risk is apparent (for a comprehensive review on this literature see [27]).

We follow [63] and use “regression-based style analysis” to conduct our estimation of hedge fund leverage. The linear regression involved here attributes portfolio returns to a set of risk factors which represent different asset classes that the portfolio is considered to be exposed to. The estimated coefficients on these risk factors measure the sensitivity of the portfolio returns to changes in the returns on the underlying factors. We also adopt the notion that leverage employed by the funds acts as an amplifier to the estimated sensitivity and hence is a re-interpretation of these estimated coefficients. We will elaborate on this notion after we introduce the definition of hedge fund leverage used in our study.

We only focus on funding leverage which basically can be viewed as debt (or funds raised by short-selling) borrowed by the fund to increase initial fund assets

on top of investor money (equivalent to AUM - Asset Under Management). If we denote A as all assets raised outside of AUM, funding leverage is then defined as

$$\rho = \frac{A + AUM}{AUM}. \quad (3.1)$$

We now illustrate in a simple example how leverage acts as amplifier to the exposures to underlying risk factors. Assume AUM is 10 and A is 90. We have now ρ with a value of 10 which we also term as a “10 to 1 leverage”. If end-of-period return on portfolio is R , assuming interest rate in this period is r , the actual return on investment is then

$$\tilde{R} = \frac{100R - 90r}{10}, \quad (3.2)$$

or

$$\tilde{R} = R + 9(R - r). \quad (3.3)$$

If we know the allocation of the portfolio and portfolio is fully invested, then portfolio return R can be written as the weighted average of returns on the individual assets,

$$R = \sum_{i=1}^k w_i R_i \quad (3.4)$$

where the weights sum to one $\sum_{i=1}^k w_i = 1$. Rewriting (3.3), we have

$$\tilde{R} = \sum_{i=1}^k w_i R_i + 9 \sum_{i=1}^k w_i R_i - 9r, \quad (3.5)$$

or

$$\tilde{R} - r = 10 \sum_{i=1}^k w_i (R_i - r), \quad (3.6)$$

or in a general case

$$\tilde{R} - r = \rho \sum_{i=1}^k w_i (R_i - r). \quad (3.7)$$

We see that weighted average of the excess returns on individual non-cash assets in the portfolio is scaled up by our leverage parameter ρ .

Equation 3.4 is usually the case where information on exact allocation of the portfolio is known. However, most of the time there is no such information and an investment style analysis typically involve a regression analysis which uses (as explanatory variables) returns on broad market indices which proxy the asset classes included in the portfolio

$$\tilde{R} - r = \alpha + \sum_j \beta_j (R_j - r) + \epsilon. \quad (3.8)$$

The constant α is in the sense of Jensen's alpha and the estimated coefficients $\hat{\beta}_j$ indicate the sensitivity or exposure of the portfolio excess return to the underlying factors excess returns. ϵ gives the error since the underlying factors are only proxies and (3.8) is at best an approximation. Given the estimates from a regression of equation (3.8) for a particular time period, we have

$$\sum_j \hat{\beta}_j = \hat{\rho} \sum_{i=1}^k w_i = \hat{\rho}. \quad (3.9)$$

This means that after each estimation, the estimated coefficients can be simply summed up and the sum is an estimator for the leverage. Since short positions

would appear as negative estimated coefficients, we shall sum up the absolute values of these coefficients.

We intend to estimate leverage on individual fund level which is quite different from [63] where they have estimated leverage at a fund family level. Nine fund families are distinguished in their paper based on different investment strategies. [43] and [7] also study fund performance on a broad fund family level. [32] also evaluate hedge fund performance only for some groups of hedge funds sharing similar strategies that are identified using Principal Component Analysis (PCA). However, since we would like to eventually study capital implication by separating fund cash flow into assets and liabilities using leverage information, we wish to see leverage information obtained on an individual fund level. This fund-level leverage estimator also sheds more granular insight on individual fund behavior than only obtaining one leverage number for the hundreds and thousands of different funds within the same broad investment strategy. This goal however, poses challenges to the estimation itself, since an estimation of this sort requires a relatively long time series of fund returns and better identification of the right-hand-side regressors. We will spend the next couple of sections going through the detail on data choices, right-hand-side factor selections and the strategy of the actual estimation in (3.8).

3.2.2 Data

In order to estimate (3.8), we first need to have time-series data for the LHS which is hedge fund return data. Due to the nature of voluntary reporting of hedge

fund data, incomplete data and fund disappearing is often observed in any hedge fund database. In order to obtain long time series for the individual fund level estimation and also make certain that later on AUM and NAV data are also available (for separating assets and liabilities in later use of calculating capital required), we take CISDM hedge fund monthly data and only retain funds who have continuous data for returns, from February 1996 to September 2008. This results in 177 funds each with 152 monthly data points. This obviously creates survivorship bias in our sample, but since our main objective is to estimate leverage on individual funds and not to make inferences about overall performance, our filter may not be as problematic. Nonetheless, we do expand our sample later so that funds only need shorter time period of survival from January 2002 to September 2008. This then results in 1797 funds in our sample and the results are also reported.

The representative market factors on the right-hand-side of equation (3.8) are chosen to be a set of indices which come from equity, bond, and commodity markets. We mainly focus on equity markets since it is shown (e.g. [63]) that returns across most fund families seem to be heavily influenced by equity market factors. We include three equity indices: S&P500 index (from WRDS), the MSCI World Excluding US index (MSDUWxUS) and the MSCI Emerging Market index (MSCIEM) (from Bloomberg). Together these indices represent the major equity markets. Similarly, we include three bond indices: the Salomon Brothers World Government Bond Index US (SBWGU) and CSFB High Yield Index (CSHY) and Salomon Brothers Corp Bond Index (from Bloomberg). Further, we include a commodity index: Moody's Commodity Index (from Datastream) reflect positions in

commodities. Finally, we include gold prices from Bloomberg and also the famous SMB, HML and UMD indices suggested by [38], which measure a Size factor and a Value-Growth factor, and Momentum respectively. All returns are calculated from price series between January 1996 and September 2008. We also take one-month T-bill rates as the risk-free interest rates.

We summarize the statistics of these factors (using excess returns) in Table 3.1. In calculating Sharpe Ratio (SR) we use average interest rate of the period: 0.003.

Table 3.1: Summary Statistics of the right-hand-side Factors During Jan. 1996 to Sep.2008

Index	Mean	SD	Median	Min	Max	Skew	MM Skew	Kurtosis	SR
Equity									
S&P500	0.0035	0.0429	0.0081	-0.1474	0.0938	-0.5045	-0.2518	3.4305	0.0128
MSCI Wrld Excl US(MSDUWxUS)	0.0003	0.0435	0.0035	-0.1483	0.0995	-0.6810	-0.1998	3.7376	-0.0610
MSCI Emer Market(MSCIEM)	0.0025	0.0690	0.0063	-0.2972	0.1301	-0.8687	-0.4029	4.6074	-0.0066
FF SMB factor	0.0026	0.0401	-0.0001	-0.1685	0.2199	0.7871	0.1188	9.8375	-0.0091
FF HML factor	0.0045	0.0362	0.0037	-0.1237	0.1387	0.0834	0.0230	5.3959	0.0411
Momentum factor	0.0084	0.0540	0.0085	-0.2504	0.1835	-0.5489	-0.1928	7.0545	0.0998
Bond									
SB Wrld Gov Bond(SBWGU)	0.0016	0.0192	-0.0000	-0.0436	0.0559	0.3249	0.0902	2.8935	-0.0712
CSFB High Yield(CSHY)	0.0019	0.0191	0.0041	-0.0722	0.0545	-0.8632	-0.1686	6.3584	-0.0603
SB Corp Bond(SBCRP)	0.0033	0.0535	0.0061	-0.1704	0.1556	-0.5551	-0.0655	4.3022	0.0054
Commodity									
Moody's Commodity Index	0.0036	0.0323	0.0011	-0.1125	0.1403	0.3321	0.0817	5.3375	0.0179
Other									
Gold Prices	0.0029	0.0424	-0.0038	-0.0939	0.1646	0.6893	0.2513	3.8431	-0.0033

3.2.3 Building Non-linear Factors

Another alternative to using large leverage position is to use derivatives to hedge market risk. For example, a manager could buy some stock considered to be underpriced with a simultaneous short position in stock options or other derivatives. Since hedge fund returns exhibit non-linear option-like exposures to standard asset classes ([42] and [43]), following [8], [32], [63] estimate leverage including synthetic option factors to take into account such non-linearity. [63] found in their analysis that the returns on the broad equity market index (S&P500) and the associated synthetic option factors are almost always important drivers of performance. Hence, in addition to the factors chosen and described in the last section, we also build some option-like payoffs which proxy for the non-linearity of hedge fund return exposures. In [8], [32], [63], historical options price series are calculated using Black Scholes formula ([13]) with historical volatilities as inputs. This however, is quite problematic since it is well documented in the literature that realized volatilities are not implied volatilities. Historical option prices must be available to calculate actual historical implied volatilities.

In this paper, we choose options that are termed power-tail options ([25]) and are written on S&P500 as their underlying. We are here only focused in the equity markets since equity factors and their options have been shown to be the most significant explanatory factors ([63]) among the others. We price these options base on calibration of historical vanilla prices. In order to price these structured payoffs historical prices, one would like to incorporate stochastic models capable of

synthesizing the surface of historical vanilla option prices. These prices are typically represented by the matrix of implied volatilities indexed by the strike and maturity dimensions. Recently, [21] showed that a wide class of additive processes (with independent but inhomogeneous increments) can synthesize the surface of option prices, remarkably with as few as four parameters. These processes are associated with the law at unit time of a subclass of Lévy processes defined by the condition that the law at unit time be self decomposable or a limit law. These additive processes have been studied in [68] and are termed SATO processes in [33]. The SATO model is calibrated to vanilla option surfaces of each time point and the specific payoffs are then calculated using a Fast Fourier Transform method.

We begin by describing the payoffs of these power-tail options. We only consider OTM options due to their liquid nature and the payoff for OTM power-tail call is

$$E[S(T)^n, I(S > K)], \quad K = 1.1, 1.2, 1.3 \quad (3.10)$$

and the payoff for OTM power-tail put is

$$E[S(T)^n, I(S < K)], \quad K = 0.7, 0.8, 0.9 \quad (3.11)$$

These products emphasize the payouts on the tails. The underlying index S is assumed to follow an exponential SATO process introduced in e.g. [21] and [33] (and note that we will always start from $S(0) = 1$)

$$S(T) = S(0) \frac{e^{rT+X(T)}}{E[e^{X(T)}]} \quad (3.12)$$

where $X(t)$ is a SATO process and we assume $E[e^{X(t)}]$ is finite and is the normalizing factor so that S is an exponential Martingale. The SATO process $X(t)$ is constructed from the law at unit time of a subclass of pure jump Lévy processes such as Variance Gamma (VG) process ($X = VG(1)$), and has characteristic function (if employing VG)

$$\begin{aligned} \phi_{X(t)}(u) &= E[e^{iuX(t)}] \\ &= \phi_X(ut^\gamma) \\ &= \left(\frac{1}{1 - iu\theta\nu t^\gamma + \frac{\sigma^2\nu}{2}u^2t^{2\gamma}} \right)^{\frac{1}{\gamma}} \end{aligned}$$

It is then trivial to obtain the characteristic function for $\ln S(T)$

$$\begin{aligned} \phi_T(u) &= E[e^{iu \ln S(T)}] \\ &= \exp[iu(\ln S(0) + rT - \ln(\phi_{X(T)}(-i)))]\phi_{X(T)}(u) \end{aligned}$$

We employ the Fast Fourier Transform method detailed in [24] to value options and obtain SATO parameters $\sigma \nu \theta$ and γ for every month end from January 1996 to September 2008 by calibrating the SATO model to market prices on index options.

Once these calibrated parameters are obtained for each time, we may then price the options in (3.10) and (3.11) using same approach of FFT described in [25]. We price these options for both 2 month and 3 month maturities with power

$n = 2, 3, 4$ and build the return time series (monthly returns) month by month from February 1996 to September 2008. So for each month end, we have returns on 18 option contracts, 9 OTM calls and 9 OTM puts. In case of calls for example, the Fourier transform for modified prices in (3.10) is described following [25]

$$\begin{aligned}\delta(u) &= e^{-rt} \int_{-\infty}^{\infty} e^{(\alpha+iv)k} \int_k^{\infty} e^{ns} ds dk \\ &= e^{-rt} \frac{\phi_T(v - i(\alpha + n))}{\alpha + iv}\end{aligned}$$

where $k = \ln K$ and $S = e^s$, α is set to be 1.1. Interest rate r is always set to 0, so that we do not need to subtract to get excess returns. The mean rate of return on these options are shown in Table 3.2. As expected all are negative and more negative as one goes further out of money reflecting a U-shaped pricing kernel ([11], the increasing region of a U-shaped pricing kernel causes the expected rate of return of a payout on the upside to be negative for strikes that are beyond a threshold).

Table 3.2: The mean rate of return on calculated option prices

	$K = 1.1$	$K = 1.2$	$K = 1.3$
$n = 2$	-0.6845	-1.3955	-2.1222
$n = 3$	-0.6935	-1.4038	-2.1114
$n = 4$	-0.7032	-1.4127	-2.1042

Finally, we have built returns for these synthetic options that are out of the money and are written on the equity index S&P500. These 18 options (9 calls 9 puts) will be combined with the index factors described in last section to capture the non-linear risk exposures of hedge fund portfolios, following the similar approach which have been taken by [63], [32] and [8]. We are here only focused in the equity markets since equity factors and their options have been shown to be the most

significant explanatory factors ([63]) among the others.

Before we move on to the regression, we must perform one more transformation to these synthetic factors, since it is recognized that the option returns are very highly correlated. If we use these highly correlated returns directly in the regression process, the regression will suffer from problems caused by such multi-collinearity of the right-hand-side variables. Although multi-collinearity will not bias the estimated coefficients, it will unreasonably inflate the estimates due to high standard errors.

We perform a Principal Component Analysis on the option returns and retain the first two principal components as they explain about 99% of the variance. This procedure eliminates the multi-collinearity problem and these two principal factors will enter into the regression equation instead of the 18 option factors.

3.2.4 Adding Lagged Returns in Regression Model

In the search of representative right-hand-side factors, we also consider adding lagged returns in the regression model. Adding lagged returns on the right-hand-side basically allows dependence on current returns on returns from previous periods (how many lags are appropriate still need to be determined). Although theoretically, return serial correlation implies market inefficiency and presence of predictability in returns which contradicts the common belief that hedge funds are operated under highly capable fund managers with optimal investment strategies, in practice, there might be other reasons for the returns to show serial correlation. It is shown in empirical research that the returns to hedge funds are often highly serially correlated

due to the illiquidity exposure and return smoothing (see e.g. [45], [14]). [45] and [14] argue that serial correlation is the outcome of illiquidity exposure, and intentional return smoothing reporting by the fund managers in order to produce misleading performance statistics such as volatility, Sharpe ratio, correlation, and market-beta estimates to attract investors. Given the nature of hedge-fund compensation contracts and performance statistics, managers have an incentive to “smooth” their returns by marking their portfolios to less than their actual value in months with large positive returns so as to create a “cushion” for those months with lower returns. Such return-smoothing behavior yields a more consistent set of returns over time, with lower volatility and, therefore, a higher Sharpe ratio, but it also produces serial correlation as a side effect. If the securities in the managers portfolio are actively traded, the manager has little discretion in marking the portfolio. It is “marked to market”. The more illiquid the portfolio, the more discretion the manager has in marking its value and smoothing returns, creating serial correlation in the process. The impact of smoothed returns and serial correlation is considered in more detail in [56] and [45],

Regardless of the particular mechanism by which hedge fund returns are smoothed and serial correlation is induced, the economic impact of serial correlation can be quite real and the impact on our leverage estimation is of true interest here. We build on original model in 3.8 and adding lagged returns

$$\tilde{R}_t = \alpha + \sum_{i=1}^k \eta_i \tilde{R}_{t-i} + \sum_j \beta_j R_t^j + \epsilon \quad (3.13)$$

here these returns are still excess returns after subtracting risk free rate. This model is in fact an ARX model(Autoregressive model with exogenous inputs). To ensure stationarity of the model and the convergence to equilibrium, the eigenvalues associated with the characteristic AR polynomial must lie in unit circle. For $k = 1$ with just one-period lag, this is equivalent to auto-correlation coefficient $|\eta_1| < 1$, and our leverage estimate is then

$$\hat{\rho} = \frac{\sum_j \hat{\beta}_j}{1 - \eta_1} \quad (3.14)$$

3.2.5 Estimation of Leverage

In this section we explain in more detail how the estimation of leverage is performed. As shown in (3.9) the leverage estimator is simply a sum of the absolute values of the estimated coefficients β_j in equation (3.8). To obtain the estimation of these estimators, however, is not so straight-forward due to two reasons: we have no knowledge about what these right-hand-side factors actually are; and because we wish to perform estimation on individual fund level. Although we have put together a reasonably representative set of factors proxying risks across major markets and we will include the non-linear variables as proxied by the synthetic options (or their principal components to be exact), which factors should enter into which fund's equation at which period of time is still a difficult question to answer. Each fund has its own unique trading strategy and likely to put more emphasis on certain markets over others. In different time periods, funds are also likely to shift between

different strategies. In addressing these issues, we adopt the approach used in [63]. To point out the difference however, our estimation is done on individual fund level and obtains more meaningful fund-level leverages.

For each fund, we estimate time-varying parameters $\hat{\beta}_j$ in a rolling regression with 36 months as the rolling window. Within each window, a two-stage stepwise regression is performed. First stage selects which right-hand-side factors best explain the portfolio returns within the specific window, and the second stage yields the actual estimates of the betas that are used to calculate leverage. The identification of those risk factors most relevant for the specific fund (with particular investment style and strategy) and in the specific time window is performed by means of stepwise regression of funds' excess monthly returns on the full set of risk factors. The stepwise procedure combines forward-selection (variables are added one by one to the model until no remaining variable produces a significant statistic) and backward-elimination (testing variables one by one for statistical significance and deleting any that are not significant) steps and only retain variables if their statistical significance exceeds a certain threshold. The tolerances for inclusion and exclusion of right-hand-side variables in the stepwise procedure are set at p-values of 0.10 and 0.11. The second stage engages a fixed-effects regression of fund returns on the set of factors already identified in the first stage. These estimates from the second stage regression are used then in calculating leverage estimator for the particular fund and regression window. Since short positions will appear as a negative estimated coefficient, the leverage estimator is the sum of all absolute values of the estimated betas.

When adding lagged returns on the right hand side. We did separate regres-

sions for adding one lag versus adding two lags and the results show that adding one lag significantly improves leverage estimation, but adding two lags do not necessarily improve further. This is also consistent with our initial analysis using Durbin-Watson test and plotting ACF and PACF graphs within regressions, which show that generally the return autocorrelations are only up to the second lag and mostly focused on the first lag. We focus all of the following analysis on only having one lagged returns and results are reported accordingly.

The results of estimation show consistency with results in other similar research. Firstly, we notice that usually equity market factors (especially S&P500 and SMB) are often included in the factor selections and show significant influence on majority of the funds (in our sample 177 funds). Secondly, in general 3 to 5 factors are selected in the first stage stepwise regression. Thirdly, the inclusion of factors proxying the non-linear returns as explanatory variables can improve the estimation results. We also notice that in general, the regression model with lagged returns give higher leverages on average. We select three funds with ID numbers 9038, 12924, and 12939 to show the different leverages estimators obtained from different models. First, the estimators are obtained in three different models using just indices as factors and then adding options and further adding lagged returns. The average leverage estimators are compared with CISDM reported historical average gross leverages and they are shown in Table 3.3. Then we show time series estimators in graphs shown in Figures 3.1, 3.2 and 3.3 for fund 9038, 12924, and 12939 respectively. Each figure shows estimated leverages for all rolling periods starting from January 1999 to September 2008. Each figure shows three estimators

for models with only indices, and indices plus equity options, and indices plus equity options plus one-period lagged returns, in green, red and blue respectively. We see from the graphs that including options in the regression formula improves leverage estimation as documented in [63], [32] and [8]. We also see that including lagged returns can improve the leverage estimated for majority of the time. However, lagging two periods do not necessarily improve results further at all. Our estimation results shown only included one-lagged returns in model.

Table 3.3: Average Leverage Comparison for Funds 9038, 12924 and 12939

ID	cisdm avg-gross-lev	just indices	add options	add lagged returns
9038	1.5	1.6567	2.3447	2.6348
12924	2	2.6201	2.9289	3.1690
12939	2	2.5076	2.8368	3.1315

Finally, we would like to see if our estimated leverages indeed reflect real leverage positions. This however, is rather a naive wish since as mentioned before, we do not have reliable source of leverage information on hedge funds at all and this is the exact reason we wish to estimate them. In CISDM database, there is a variable indicating leverage use (1 for yes, 0 for no) and also a variable reporting average-gross-leverage. The data however, is rather uninformative and sometimes meaningless. The average-gross-leverage is one stale number no matter how long the fund has existed which makes little sense. Besides, there are cases where funds have average-gross-leverage numbers reported however indicating no leverage and vice versa. We nonetheless would like to still utilize such information provided in the database and see if our leverage estimates show any comparison to the reported

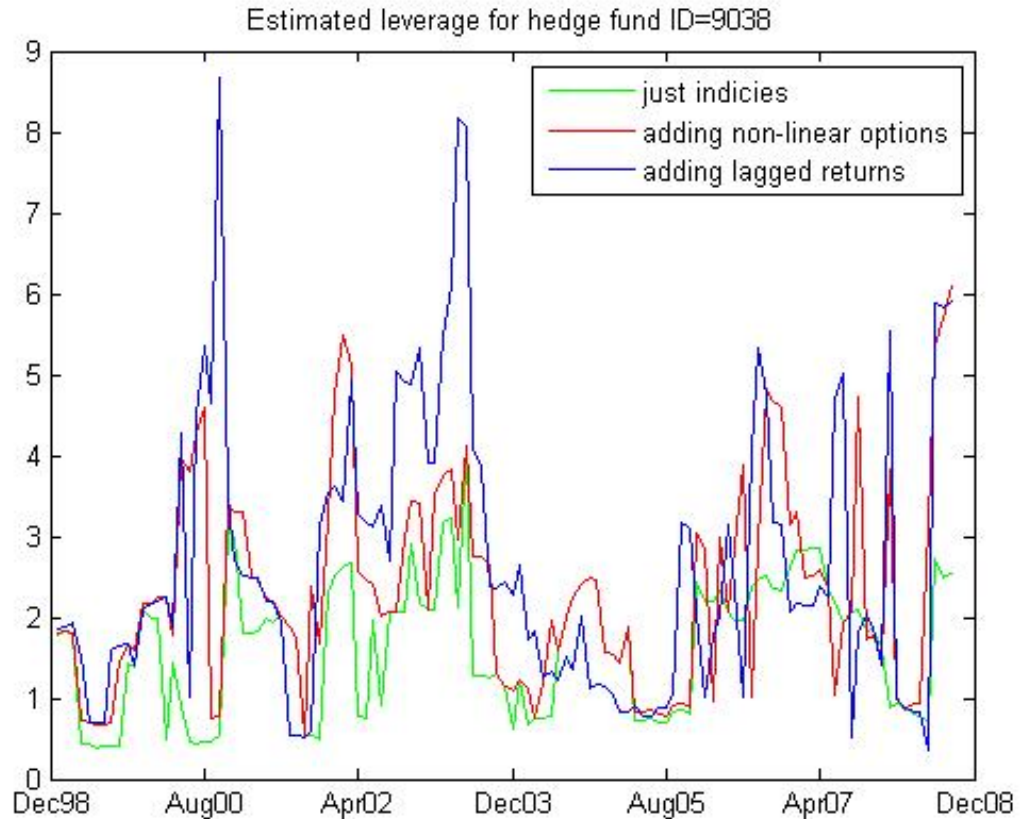


Figure 3.1: Estimated Leverages for Fund 9038.

leverage. In the 177 funds we sampled, there are 62 of them indicating using leverage ($\text{avg-gross-lev} > 1$) and 115 of them with no information on whether or not leverage is used or leverage numbers are 1 or less. We compare the average-gross-leverage numbers for these 62 funds with the average (across all rolling windows) leverages from our estimation (indices plus options plus one lagged returns) and plot them together in Figure 3.4.

Again this is at best a naive comparison since we do not know how reliable these reported leverages are to start with. However the comparison does show our

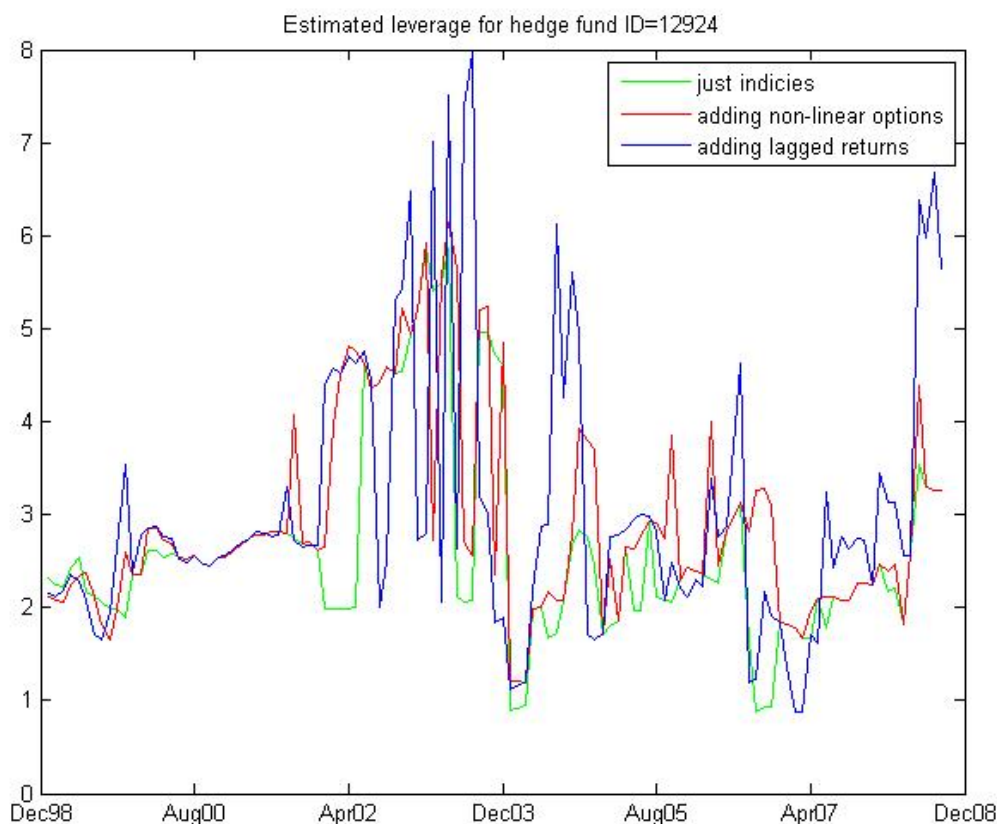


Figure 3.2: Estimated Leverages for Fund 12924.

leverage estimators on average are comparable to the reported ones. Although we see much improvement on higher and more plausible leverage numbers after options and lagged returns are included in the right-hand-side factor selection, the leverage estimated is still not as high as what is possibly engaged in practice. For the whole sample, 177 funds, the estimated leverages average across the whole period of February 1996 to September 2008 can be as high as 22, but also as low as 0.07 which does not have any practical meaning as leverage being one indicates no leverage. For the 177 funds studied, there are only 12 reported historical averages greater than

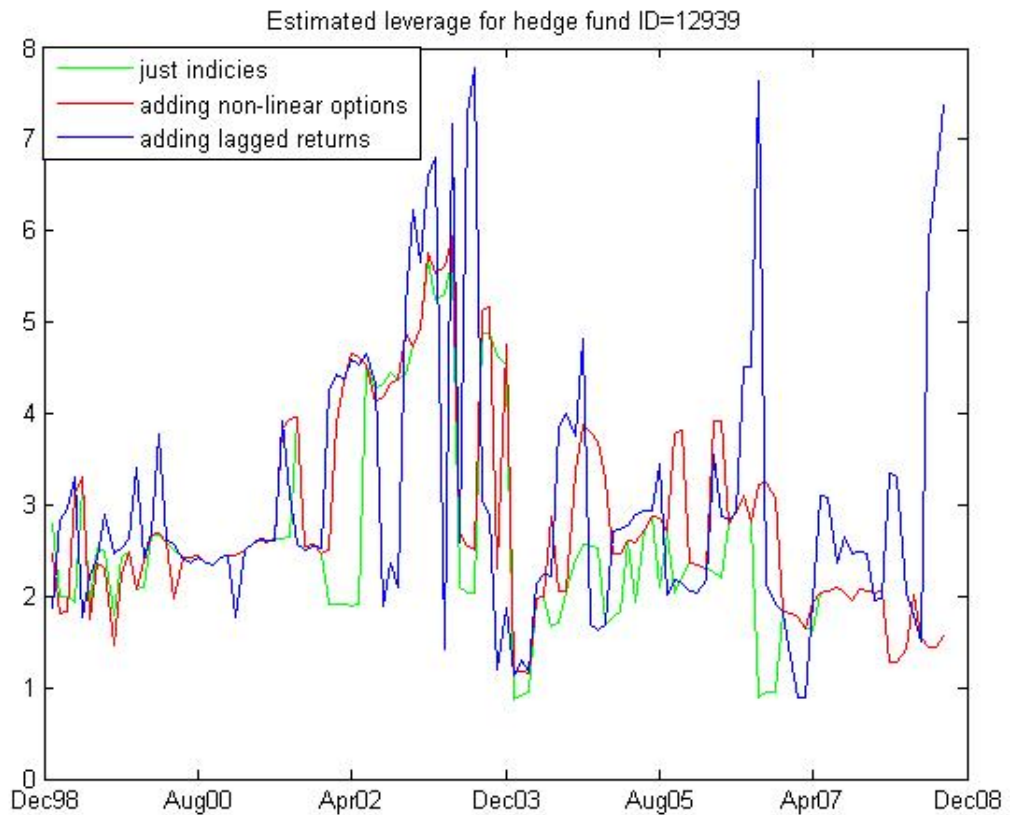


Figure 3.3: Estimated Leverages for Fund 12939.

2, but in the estimation we have 35 of them with average leverage greater than 2. Even though some average leverage can be above 2 for example, when looking at the whole rolling time periods, most of funds have leverages estimated below 1 for at least one period. For the purpose of separating hedge fund balance sheet in the next few sections, these leverage numbers that are less than one should not be considered. Among 177 funds being studied here, there are only 2 funds with all 117 rolling time periods with estimated leverage higher than one.

The leverage estimators although improved with added non-linear factors and

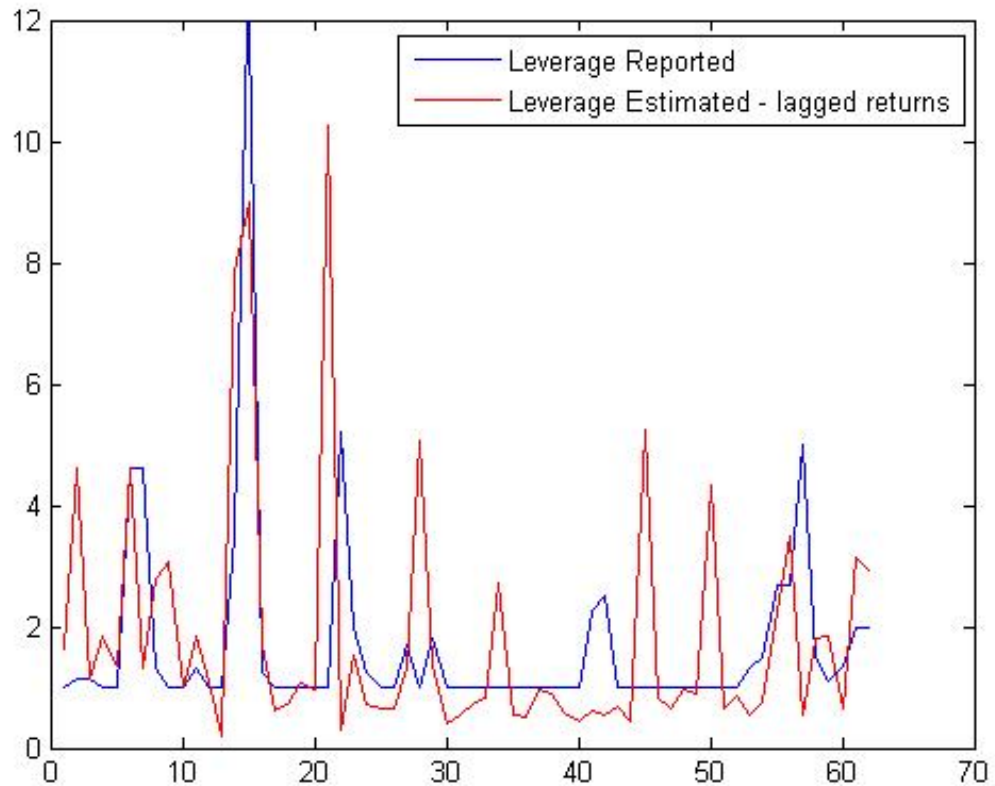


Figure 3.4: Leverage Reported Compared to Leverage Estimated for 62 Hedge Funds.

added lagged return factors are still lower than what is possibly engaged in practice. Our selection of time period of February 1996 to September 2008 is quite a long time and there might be survival bias introduced in such selection (the funds that could survive such long period are the funds that employ lower leverages). To address this concern, we next choose a shorter period of survival for the funds to be included in the sample and obtain results for those chosen funds.

3.2.6 Estimation Using Shorter Time Period 2002-2008

Due to the survival bias we may have introduced to the estimation by requiring funds to have data from 1996 to 2008, we also shorten our period to 2002 to 2008 in order to include more funds in our sample. This results in 1797 funds who have all monthly return data from January 2002 to September 2008 (81 data points). Our results show similar characteristics about our approach here as: 1) leverage estimates usually are improved after options are added and even further improved after one-period lagged returns are included on the right-hand-side; 2) the estimated leverage averages are in general comparable to the leverage reported, shown in Figure 3.5 where the leverages estimated and leverages reported are plotted together; 3) in the data, out of 1797 funds, there are only 815 funds have average leverage numbers and in these, 407 of them have numbers greater than 1 and 92 of them have numbers greater than 2, versus in the estimation, we have 1069 funds have estimated average greater than 1 and 397 funds with estimated average greater than 2.

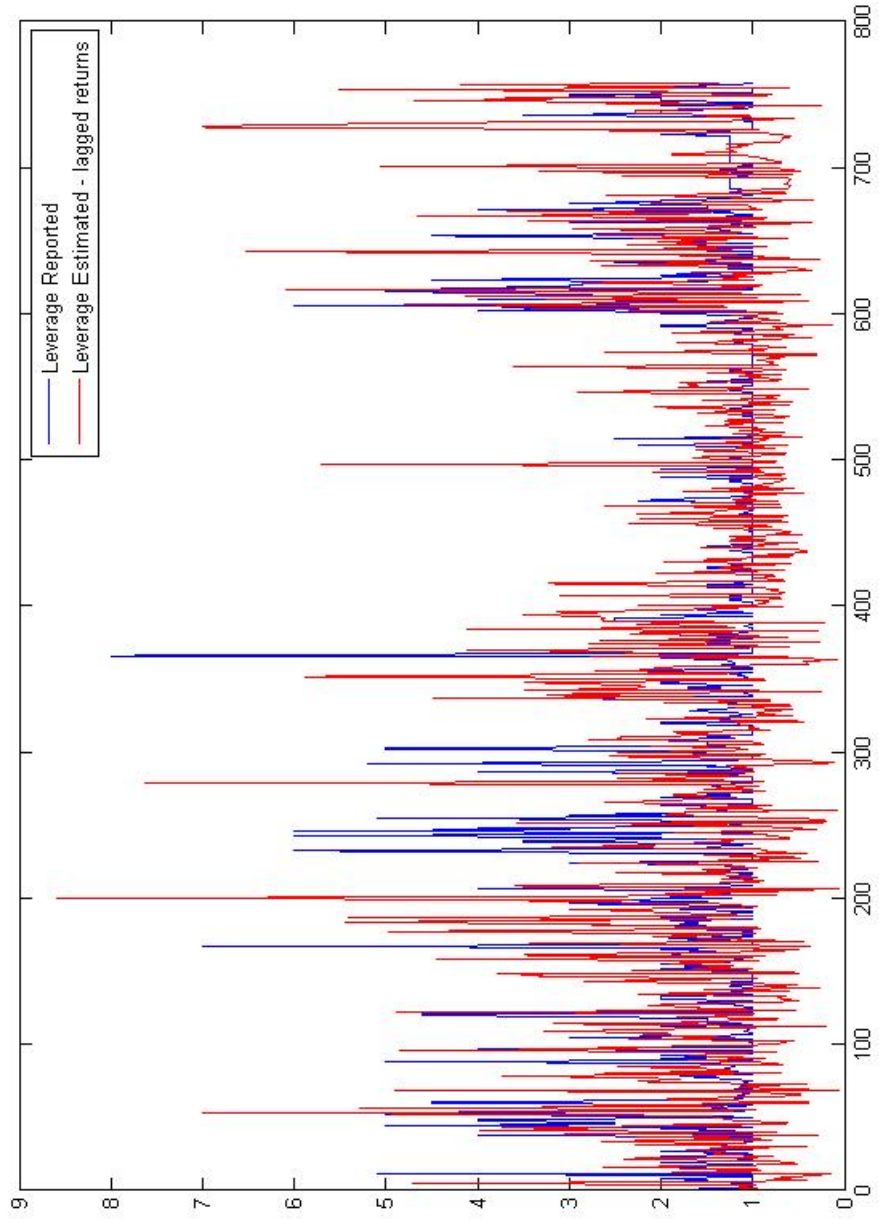
We summarize the statistics of these factors (excess returns) in Table 3.4. We use average interest rate of the period 0.0022 in the calculation for Sharp Ratio.

In the sample of 1797 funds, there are 181 funds with leverage estimators greater than one at all 46 time points. These will be the object of study in later section when we calculate required capital after separating fund cash flows.

Table 3.4: Summary Statistics of the right-hand-side Factors During Jan. 2002 to Sep. 2008

Index	Mean	SD	Median	Min	Max	Skew	MM Skew	Kurtosis	SR
Equity									
S&P500	0.0004	0.0365	0.0085	-0.1104	0.0872	-0.5646	-0.1220	3.8019	-0.0698
MSCI Wrld Excl US(MSDUWxUS)	0.0029	0.0427	0.0084	-0.1483	0.0910	-0.9199	-0.2640	4.4041	-0.0019
MSCI Emer Market(MSCIEM)	0.0109	0.0601	0.0140	-0.1786	0.1070	-0.7503	-0.3275	3.2379	0.1322
FF SMB factor	0.0038	0.0243	0.0005	-0.0518	0.058	0.1270	-0.0122	2.5198	0.0316
FF HML factor	0.0054	0.0201	0.004	-0.065	0.0449	-0.3236	-0.2804	3.6941	0.1176
Momentum factor	0.0060	0.0435	0.0064	-0.1628	0.1256	-0.6699	-0.1705	5.708	0.068693
Bond									
SB Wrld Gov Bond(SBWGU)	0.0046	0.0203	0.0040	-0.0436	0.0559	0.2057	0.0315	2.7507	0.0771
CSFB High Yield(CSHY)	0.0037	0.0185	0.0066	-0.0719	0.0517	-0.7703	-0.2220	5.9206	0.0352
SB Corp Bond(SBCRP)	0.0033	0.0546	0.0054	-0.1704	0.1556	-0.6557	-0.0655	4.9529	0.0050
Commodity									
Moody's Commodity Index	0.0124	0.0371	0.0099	-0.1125	0.1403	-0.0165	0.0117	4.9722	0.2762
Other									
Gold Prices	0.0130	0.0449	0.0177	-0.0939	0.1177	0.0564	-0.0104	2.6912	0.2403

Figure 3.5: Leverage Reported Compared to Leverage Estimated for Funds in 2002-2008.



3.2.7 More Discussion on Estimated Leverages

As we improve our leverage estimators by ways of incorporating option-like returns and lagged periods, we still feel puzzled that the leverage numbers are not as high as expected. Especially when leverage numbers are not even greater than one. We discuss in the following some possible explanations that could have caused the observed results from our study.

The specific mechanisms by which a hedge fund determines its leverage can be quite complex and often depend on a number of factors including market volatility, credit risk, and various constraints imposed by investors, regulatory bodies, banks, brokers, and other counter-parties. But the basic motivation for typical leverage dynamics is the well-known trade-off between risk and expected return. By increasing its leverage ratio, a hedge fund boosts its expected returns proportionally, but also increases its return volatility and, eventually, its credit risk or risk of default. Therefore, counter-parties providing credit facilities for hedge funds will impose some ceiling on the degree of leverage they are willing to provide. More importantly, as market prices move against a hedge funds portfolio, thereby reducing the value of the funds collateral and increasing its leverage ratio, or as markets become more volatile and the funds risk exposure increases significantly, creditors (and, in some cases, securities regulations) will require the fund to either post additional collateral or liquidate a portion of its portfolio to bring the leverage ratio back down to an acceptable level.

On the other hand, low leverage estimates could result in mis-specification

of the regression model. Although we try to cover major market indices and the stepwise regression tries to fit the best model to the returns, the actual fund strategy and hence their choosing of actual markets and products are still very complicated and beyond what we could at best approximate here. Hedge funds generally have very complex and discretionary and time varying investment strategies. The funds claim of superior returns are often achieved with low risk and low correlation with conventional investments. To this extent, our estimation model constructed using a selective set of factors can be very well overly subjective and well “missing the dots”. This is also another reason that some of the leverage numbers are low and basically the funds are out of the “market” we have constructed. For the periods that the fund returns are “out of the model”, the leverage estimates which are the sum of the coefficients will tend to be low.

Another explanation for the estimated leverage numbers being low and even below one is that returns reported are over longer period (monthly) than actual holding period of most portfolios, hence the amplification effect by the leverage on returns are deflated over the longer period. The theory behind our regression style analysis to estimate leverage is that leverage can amplify portfolio returns. However, in reality, most strategies require active trading and hedging and the holding period for a particular leveraged portfolio can be as short as days or hours and much more volatile. The reported returns on monthly basis could be much less dramatic and much more smoothed out over this relatively long observation period and hence dampens the original amplification effect from taking on big leverage.

In reality, it is also true that some funds heavily use leverage and some do

not. In fact, recent industry reports (from example UBSs Alexander Ineichen and Merrill Lynch and Morgan Stanley) have documented that hedge funds are indeed “de-leveraging” as the credit markets deteriorated throughout 2008. Britains Financial Services Authority (FSA) recently found that hedge fund leverage was nearly extinct. The European Central Bank (ECB) reported gross leverage (longs plus absolute value of shorts) from Hennessee Group and found leverage levels around $1.5\times$. This seems also consistent with what managers were telling Merrill Lynch in a survey cited by the ECB - that a majority of managers were actually using no leverage at all. In [7], their leverage estimates, although on fund family level and not directly comparable to our fund-level ones, also show similar conclusion such as bared any leverage used by equity hedge strategies. After all, what we hope to demonstrate here is a general approach which builds a bridge between observed market data and some unobserved characteristics in interest such as hedge fund leverage. Furthermore, the estimation of leverage, once rationalized, can be used to study hedge fund balance sheet and draw implication of hedge fund capital requirement. This is what we are going to demonstrate in the next section.

3.3 Separating Hedge Fund Balance Sheet and Cash Reserve Requirement

We have data for AUM and NAV and by our definition of funding leverage in (3.1), it is easy to estimate random assets (other than AUM) which we term A from

$$A = \rho * AUM - AUM \quad (3.15)$$

These assets are the ones raised outside of asset under management invested by the investors and are borrowed assets or securities directly contribute to the leverage the fund is taking. These assets have random cash flow $A(t)$ and we see that ρ must be great than one (at all times) to ensure positive cash flows. For the random liability side L , we assume that at any time

$$NAV = A - L + AUM \quad (3.16)$$

In the data however, the NAV is on a per share basis and there is no information on number of shares sold by the fund. Hence we actually have

$$NAV = \frac{A - L + AUM}{N} \quad (3.17)$$

where N denotes the number of shares and we would like to estimate N first. This is done by taking the average of previous 36 months AUM/NAV from data and we impose that on average the total cash flow $X = A - L$ have zero mean (in our analysis we would always demean the cash flows). And the \bar{N} is used to estimate L

$$L = A + AUM - \bar{N} * NAV \quad (3.18)$$

Hence, from February 1996 to September 2008, every 36 month, we estimate the share number from the last 36 observations of AUM/NAV by taking average, then obtain the separated assets and liabilities A and L for each month from January 1999 to September 2008, using leverage estimates and NAV AUM data. There are 117 time series point for both A and L .

3.3.1 Modeling Random Assets and Liabilities

We model assets and liabilities as correlated processes. Following [36], we model the log of A and L as linear mixture of VG processes (LM). We also show a bench-mark model by modeling A and L as marginal lognormal with correlated Gaussian components (LNC).

Model One - LNC

$$A(t) = A(0)e^{X_A(t)}$$

$$L(t) = L(0)e^{X_L(t)}$$

where

$$X_A(t) = \mu_A t - \frac{1}{2}\sigma_A^2 t + \sigma_A \sqrt{t} Z_A$$

$$X_L(t) = \mu_L t - \frac{1}{2}\sigma_L^2 t + \sigma_L \sqrt{t} Z_L$$

$$Z_A = \rho Z_L + \sqrt{1 - \rho^2} Z \tag{3.19}$$

Z and Z_L are independent standard normals and hence Z_A and Z_L are correlated normals with correlation being ρ . If we let

$$X_1(t) = \sigma_A \sqrt{t} Z_A$$

$$X_2(t) = \sigma_L \sqrt{t} Z_L$$

and observing $h = 1/12$, we have actually demeaned log daily return,

$$X_1(t+1) - X_1(t)$$

$$X_2(t+1) - X_2(t)$$

and

$$X_1(h) = X_1(t+1) - X_1(t) = \sigma_A \sqrt{h} Z_A$$

$$X_2(h) = X_2(t+1) - X_2(t) = \sigma_L \sqrt{h} Z_L$$

Maximum likelihood estimation can be performed on the data by recognizing

$$LH = \sum_{i=1}^{n-1} \log f(x_1, x_2) \quad (3.20)$$

where

$$f(x_1, x_2) = \frac{1}{2\pi\sigma_A\sigma_L h \sqrt{1-\rho^2}} \exp\left[-\frac{z}{2(1-\rho^2)}\right] \quad (3.21)$$

and

$$z = \frac{x_1^2}{\sigma_A^2 h} + \frac{x_2^2}{\sigma_L^2 h} - \frac{2\rho x_1 x_2}{\sigma_A \sigma_L h} \quad (3.22)$$

The parameters estimated using $h = 1/12$ are already annualized. We then from this model generate correlated X_1 and X_2 10000 each and build cash flows $A(t)$ and $L(t)$ with A_0 and L_0 being the last data and $t = 1$.

Model Two - LM

In this model, the data variables are assumed to be linear mixtures of some unknown latent variables. Such joint laws have been considered in the time series context by [62], [58] and [52]). The latent variables are assumed non-Gaussian

and mutually independent, and they are called the independent components of the observed data. These independent components, also called sources or factors, can be found by ICA (Independent Component Analysis). In the time series applications the required mixing matrix is also estimated using ICA ([48]). We begin with

$$A(t) = A(0)e^{X_A(t)}$$

$$L(t) = L(0)e^{X_L(t)}$$

where

$$X_A(t) = \mu_A t - \omega_1 t + X_1(t)$$

$$X_L(t) = \mu_L t - \omega_2 t + X_2(t)$$

X_1 and X_2 are constructed from linear mixture of independent VG processes

$$X_1 = a_{11}Y_1 + a_{12}Y_2$$

$$X_2 = a_{21}Y_1 + a_{22}Y_2$$

where Y_i is centered VG process with parameters σ_i , ν_i and θ_i and original mean h and variance νh .

$$Y_i = \theta_i(g_i(h) - h) + \sigma_i W_i(g_i(h)) \quad (3.23)$$

and ω_1 and ω_2 are compensators for exponential VG processes on each marginal or specifically

$$\omega_i = \frac{1}{t} \log \Phi(-i) = -\frac{1}{\nu_i} \log(1 - \theta_i \nu_i - \frac{\sigma_i^2 \nu_i}{2}) - \theta_i. \quad (3.24)$$

We use fast ICA algorithm to estimate mixing matrix \mathcal{A} and the components Y_i and then estimate VG parameters on the univariate data on Y_i . In ICA estimation, it is assumed that Y_i 's have zero mean and unit variance, hence the variance is all taken care of in mixing matrix $\mathcal{A} = [a11 \quad a12; a21 \quad a22]$. We estimate VG parameters on the components at unit time and then annualized by multiplying \mathcal{A} by $1/\sqrt{h}$.

The position $X = A - L$ can not be held without imposed capital requirement and the capital is cash like or cash equivalent, denoted by \mathcal{C} . Hence, the whole economy admits the risky cash flow

$$Y = A - L + \mathcal{C} \quad (3.25)$$

Using the new theory of acceptable risks ([36], [59]) one would require that this cash flow be acceptable. If we take stress function *MINMAXVAR* at level γ ([35]) we require

$$\int_{-\infty}^{\infty} y d\Psi^\gamma(F_Y(y)) \geq 0 \quad (3.26)$$

where the stress function *MINMAXVAR* is

$$\Psi^\gamma(u) = 1 - (1 - u^{\frac{1}{1+\gamma}})^{1+\gamma} \quad (3.27)$$

Since

$$F_Y(y) = F_X(y - \mathcal{C}) \quad (3.28)$$

so

$$\int_{-\infty}^{\infty} y d\Psi^\gamma(F_X(y - \mathcal{C})) \geq 0 \quad (3.29)$$

We change variable $x = y - \mathcal{C}$ then

$$\int_{-\infty}^{\infty} (x + \mathcal{C}) d\Psi^\gamma(F_X(x)) \geq 0 \quad (3.30)$$

or

$$\mathcal{C} = - \int_{-\infty}^{\infty} x d\Psi^\gamma(F_X(x)) \quad (3.31)$$

We now have an analytical function of capital requirement in terms of risk parameters. To compute this expression, one may follow the procedure outlined in [30]. From a simulation of outcomes from the distribution of underlying variables one can sort the outcomes in increasing order as $x_i, i = 1, \dots, n$ and evaluate the required capital as

$$\mathcal{C} \approx - \sum_{i=1}^n x_i \left(\Psi^\gamma\left(\frac{i}{n}\right) - \Psi^\gamma\left(\frac{i-1}{n}\right) \right) \quad (3.32)$$

3.3.2 Modeling Total Real-Valued Cash Flow

Follow [73], we also model the random cash flow of the fund

$$X = A - L \quad (3.33)$$

directly as real-valued Martingale. We use again two comparing models:

Model Three - Bachelier Model (Gaussian)

$$X(t) = X(0)e^{rt} + \int_0^t e^{r(t-u)} \sigma dW(u) \quad (3.34)$$

Model Four - VG

We can also model the real-valued X as a centered variance gamma (VG) process so that

$$X(t) = X(0)e^{rt} + \int_0^t e^{r(t-u)} dH(u) \quad (3.35)$$

where $H(t)$ is the centered VG

$$H(t) = \theta(G(t) - t) + \sigma W(G(t)) \quad (3.36)$$

here the time change $G(t)$ is a Gamma process with parameter ν , whose increments $G(t+h) - G(t) = g$ have Gamma density with mean h and variance νh .

3.3.3 Results

There are 181 funds with all 46 leverage estimators greater than one (for the shorter period of January 2002 to September 2008 estimation). We obtain AUM and NAV data from WRDS for these 181 funds still from January 2002 to September 2008. After manually deleting the ones with incomplete data, there are 81 funds left with all 46 time series points for AUM and NAV available. We first summarize the categorization of these 81 funds in terms of fund type and strategy used in Table 3.5.

With the data on AUM and NAV together with our estimators for leverages, we then separate assets and liabilities and obtain time series estimation on A and L following (3.15) and (3.18). After estimating the model parameters using data series A and L or $X = AUM + A - L$ in the four different models, we simulate 10000 paths and assume A_0 L_0 to be the last observed values. We report the capital required for

Table 3.5: Fund Types and Strategies Categorization for 81 Funds

FUND TYPE	STRATEGY	<i>N</i>
Hedge Fund	Equity Long/Short	16
	Equity Long Only	8
	Equity Market Neutral	2
	Global Macro	3
	Emerging Markets	6
	Distressed Securities	1
	Convertible Arbitrage	3
	Sector	9
	Short Bias	5
Fund of Funds	Multi Strategy	5
	Market Neutral	2
	Single Strategy	3
		10
Commodity Trading Advisor	Systematic	8
		8
Commodity Pool Operator	Single Strategy	4
	Multi Strategy	4
	Unspecified	2
		10

the different model framework and different models and compare the requirement to the last observed AUM which is viewed as the “cash” on hand held at the fund. We finally calculate capital requirement for each fund for four consecutive quarters: December 2007, March 2008, June 2008 and September 2008. Each calculation uses the previous 36 months of data for fitting the models. It is not necessary and impossible to show results for all 81 funds and we pick one from each fund type to show the calculation results with setting the stress level at $\gamma = 0.25$. Table 3.6 and 3.7 show the results in million-dollar unit.

The results indicate that generally, modeling with correlating assets and liabilities gives higher capital requirement than directly modeling final cash flows, since the correlation between assets and liabilities is an added source of uncertainty. This is then reflected as a higher requirement for reserve capital. We also see that once we take the correlation of underlying assets and liabilities into account and model them jointly, the calculated cash reserves are very sensitive to the choices of models, either fat-tailed or not. In the joint models, both A and L are positive random variables. It is useful to work in these terms and assume the fate of the business is determined by the joint probability law of these variables, as opposed to just the probability law of the difference, as one can relate matters better to classical corporate balance sheets by keeping both entities in mind. The four estimates for the four quarters also generally increase as it approaches September 2008, which is consistent with empirical evidence that cash balances as percentage of total assets have steadily increased over the course of the past crisis.

Table 3.6: Capital Requirement Sample Results

ID=39	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	7.1275	3.3941	3.7334	2.3015	0.96934	2.6488	0.43179	0.52377
Mar08	13.418	10.303	3.115	2.6071	2.9722	5.0538	0.43721	0.51859
Jun08	10.526	6.8876	3.6382	3.4933	2.0525	6.5876	0.44391	0.52651
Sep08	19.201	16.638	2.5639	3.0042	5.032	13.62	0.43715	0.42577
ID=131	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	160.51	44.69	115.82	123.4	11.431	81.833	3.6143	3.7542
Mar08	166.04	63.547	102.5	107	8.9671	88.632	4.1745	4.4344
Jun08	166.57	67.134	99.437	106.5	8.2527	77.18	5.1422	5.4771
Sep08	599.46	509.79	89.671	96.437	86.158	124.03	5.6509	6.4067
ID=348	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	465.8	368.63	97.167	125	40.013	79.456	3.3105	3.3442
Mar08	146.42	50.909	95.509	125	16.153	84.2	4.001	3.727
Jun08	140.68	39.167	101.52	125	17.624	82.168	4.5514	5.2028
Sep08	136.31	53.656	82.658	100	12.4	73.752	6.9615	7.8457
ID=2617	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	676.65	472.03	204.62	179.7	9.0445	13.656	12.124	12.621
Mar08	990.22	764.98	225.24	202.7	18.081	14.9228	13.486	14.298
Jun08	1094.3	836.34	257.99	232.4	19.1	25.201	14.169	12.417
Sep08	290.87	90.599	200.28	183	19.505	28.19	16.938	18.941
ID=3078	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	1627.3	1246	381.28	380.6	234.61	642.08	23.007	24.023
Mar08	1472.1	1135.4	336.68	358.5	210.02	343.88	28.148	28.765
Jun08	1494.4	1134.9	359.54	371.4	156.01	208.21	30.754	31.607
Sep08	266.46	34.394	232.07	204.9	21.362	160.91	36.909	37.143
ID=2757	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	78.892	67.641	11.25	25.984	4.6258	7.311	0.95165	1.0698
Mar08	141.02	129.34	11.679	24.677	9.1813	8.222	1.06	1.1567
Jun08	63.539	52.016	11.523	21.276	2.8841	9.5153	1.4177	1.4207
Sep08	55.459	47.46	7.999	12.736	2.9543	5.2733	3.6513	4.8421
ID=3134	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	304.91	125.18	179.73	172.57	23.652	119.25	6.8277	6.6667
Mar08	439.69	276.74	162.96	154.9	17.621	128.11	7.9	9.3809
Jun08	532.08	350.07	182.01	174.26	28.915	139.55	8.1354	8.175
Sep08	409.11	271.6	137.51	131.74	22.739	106.45	10.738	12.718
ID=3620	A_0	L_0	X_0	AUM_0	LNC	LM	Gauss	VG
Dec07	153.77	116.98	36.791	56	1.8308	1.4849	1.4153	1.2911
Mar08	353.78	306.49	47.295	86	2.9429	31.487	1.7979	1.7292
Jun08	374.03	321.15	52.874	95	2.899	28.445	2.4113	2.1312
Sep08	414.5	351.4	63.105	103	2.8764	11.411	2.764	2.5962

Table 3.7: Capital Requirement Sample Results - Continued

ID=4377	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	446.74	316.96	129.77	159	8.772	114.0	4.6983	4.8252
Mar08	439.93	319.7	120.23	144	10.341	111.81	5.5134	5.7084
Jun08	376.11	243.67	132.44	153	6.0396	67.166	6.1491	6.2101
Sep08	344.71	234.5	110.21	123.7	6.3794	372.78	7.3047	8.5555
ID=6844	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	38.298	7.4434	30.854	30.696	2.4894	17.445	1.7987	1.7354
Mar08	44.962	8.8317	36.13	35.621	2.9372	19.77	2.3216	2.2619
Jun08	47.861	9.1995	38.661	38.187	3.158	20.417	2.3255	2.2015
Sep08	40.272	5.7326	34.539	33.83	3.2007	23.241	2.4837	2.4645
ID=3476	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	121.7	43.763	77.936	71.2	20.779	100.16	3.8694	4.2534
Mar08	184.27	118.8	65.472	59.2	35.772	48.135	4.2059	4.6939
Jun08	107.36	50.208	57.152	59.2	22.989	71.159	4.5862	5.274
Sep08	160.13	109.61	50.517	59.2	41.592	99.562	4.882	5.2144
ID=5333	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	665.47	602.2	63.27	69.813	33.797	43.545	5.3805	5.3835
Mar08	674.68	587.23	87.452	92.863	22.611	76.072	8.079	7.5238
Jun08	2119.9	1984.7	135.23	282.11	100.46	116.87	11.206	11.222
Sep08	468.54	398.8	69.732	116.6	15.952	43.686	15.363	15.183
ID=3622	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	212.81	137.21	75.602	84	4.3888	36.857	4.201	4.7166
Mar08	415.85	329.49	86.358	143	15.573	68.009	4.1547	4.0631
Jun08	616.08	526.25	89.833	151	24.523	88.576	4.9749	4.9766
Sep08	601.74	496.94	104.8	155	21.672	74.3	5.5558	5.3044
ID=3714	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	29.107	18.772	10.335	9.44	1.2678	8.4383	0.2314	0.24914
Mar08	23.582	13.888	9.694	9.13	0.93782	5.2927	0.27903	0.3076
Jun08	15.975	5.9213	10.054	9.7	1.6494	7.5797	0.27298	0.36633
Sep08	23.564	14.975	8.5888	8.8	3.9488	5.6648	0.37248	0.45016
ID=6998	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	16.894	12.756	4.1371	3.2921	3.9944	3.636	0.55841	0.54062
Mar08	5.8768	1.6354	4.2413	3.4119	0.80091	3.9202	0.52911	0.53293
Jun08	5.9712	1.8011	4.1701	3.2402	0.71404	3.1561	0.51741	0.53633
Sep08	4.4436	0.83533	3.6083	2.9416	0.65873	3.199	0.51464	0.48122
ID=12030	A_0	L_0	X_0	AUM_0	LNC	LM	Gaussian	VG
Dec07	101.25	54.526	46.721	50.113	11.184	24.624	2.0983	2.3871
Mar08	98.536	56.277	42.259	50.113	11.373	38.031	2.5571	2.8474
Jun08	89.251	46.263	42.988	50.113	9.1163	28.853	2.6042	2.8102
Sep08	92.278	59.427	32.852	50.113	2.3304	45.34	3.0759	3.5772

3.4 Conclusion

In this paper, we estimate hedge fund leverage from available public information. The estimated values are then used to separate hedge fund balance sheet so that a more comprehensive framework of modeling and calculating capital implication is developed. The estimation is done with regression-based style analysis following [63] using time series data and we perform the estimation on an individual fund level. The return cash flow is then separated into assets and liabilities and these are modeled using joint laws built from either Brownian processes or Variance Gamma processes. We follow [36], [73] and [74] to carry out the cash capital reserve calculation.

We adopt the theory that funding leverage acts as an amplifier of the return exposures to the underlying risks. Following the approach implemented in [63] so that the leverage estimator is simply a sum of the absolute values of the estimated coefficients of each fund. The most distinguishing character of our study compared to previous studies, is that we perform the estimation on an individual fund level. In [63], leverage indicators are obtained on a fund-family level and there are a total of nine families based on different investment strategies. In order for our capital implication to be meaningful, we would require fund level estimates of fun leverages. This sort of estimation requires a relatively long time series of fund returns and better identification of the right-hand-side regressors. We take CISDM hedge fund monthly data and only retain funds who have continuous data for returns, from February 1996 to September 2008. This results in 177 funds each with 152 monthly

data points. The results of estimation are in agreement with results from other similar research. Firstly, we notice that usually equity market factors (especially S&P500 and SMB) are often included in the factor selections and show significant influence on majority of the funds. Secondly, in general 3 to 5 factors are selected in the first stage stepwise regression. Thirdly, the inclusion of factors proxying the non-linear option returns as explanatory variables can improve the estimation results as also documented in [63]. We also notice that in general, the regression model with lagged returns give higher leverages on average. For the 177 funds studied, there are only 12 reported historical average greater than 2, but in the estimation we have 35 of them with average leverage greater than 2. Although our leverage estimators on average are comparable to the reported ones, and we see much improvement on higher and more plausible leverage numbers after options and lagged returns are being included on the right hand side, the estimated leverages are still not as high as what is perceived. Nonetheless, our leverage estimation is in line with the level of leverage actually engaged in practice as counter-parties may impose restrictions on acceptable leverage levels. Our estimation is also consistent with the recent phenomenon of de-leveraging observed in the industry especially as the credit markets deteriorated throughout 2008. Moreover, we argue that misspecification of models is not likely to be the key source of lower values of leverage estimation, since the implemented regression has taken the step of model fitting and our proxy factors are generally comprehensive as commonly practiced. Other explanations such as dampened amplifying effect by using monthly observed data rather than more frequent activities could also be crucial in interpreting the results.

We also report results for a similar study using shorter period of January 2002 to September 2008 with 1797 funds in the sample. The results and conclusions are similar to the longer period.

The separation of assets and liabilities from estimated leverage values enables a framework similar as in [36] and is an extension to our previous studies in [73] and [74]. Even though some average leverage can be above 2 for example, when looking at the whole rolling time periods, most of funds have leverages estimated below 1 for at least one period. For the purpose of separating hedge fund balance sheet these leverage numbers that are less than one should not be considered. For the period January 2002 to September 2008, there are 81 funds left in our sample after requiring all leverage values greater than one and the funds have both data on AUM and NAV. We finally calculate capital requirement for each fund for four consecutive quarters, December 2007, March 2008, June 2008 and September 2008, each using the previous 36 months of data for model fitting. We report calculated capital requirement for these four quarters using four comparing models. The results show that generally, modeling with correlating assets and liabilities gives higher capital requirement than directly modeling final cash flows, since the correlation between assets and liabilities is an added source of uncertainty. The capital requirement for the four quarters in general also increase as it approaches September 2008.

Appendix A

Equity Valuation – Using 2D-FFT on Spread Option

As we have shown the equity of a hedge fund of long-short positions posting capital \mathcal{C} and with limited liability is

$$E = e^{-rt} E[(A(t) - L(t) + \mathcal{C})^+] \quad (\text{A.1})$$

this is a spread option with a negative strike. However, we also know by put-call parity that

$$E = A(0) - L(0) + \mathcal{C}e^{-rt} + w \quad (\text{A.2})$$

where w is a put option

$$w = e^{-rt} E[(-\mathcal{C} - A(t) + L(t))^+] \quad (\text{A.3})$$

or it could be viewed again as a spread (call) option

$$w = e^{-rt} E[(L(t) - A(t) - \mathcal{C})^+]. \quad (\text{A.4})$$

We wish to price this option and then use (A.2) to obtain the interested equity value.

Since we have assumed (1.11) and if we define

$$X_L = rt + X_2 - \omega_2 t$$

$$X_A = rt + X_1 - \omega_1 t$$

(A.5)

we have the joint characteristic function with $X = [X_L, X_A]$

$$\begin{aligned}\Phi_X(u_1, u_2) &= E[e^{iu_1 X_L + iu_2 X_A}] \\ &= \exp[iu_1(rt - \log \Phi_{X_1, X_2}(-i, 0))] \\ &\quad \exp[iu_2(rt - \log \Phi_{X_1, X_2}(0, -i))] \Phi_{X_1, X_2}(u_1, u_2)\end{aligned}$$

Following [47] we write the price as

$$w = e^{-rt+c} \int \int dx_L dx_A (e^{a_L - c + x_L} - e^{a_A - c + x_A} - 1)^+ f_X(x_L, x_A) \quad (\text{A.6})$$

where $c = \log \mathcal{C}$, $a_L = \log L(0)$ and $a_A = \log A(0)$.

Define $y_L = -x_L$, $y_A = -x_A$, and notice

$$E[\exp(iu_1 Y_L + iu_2 Y_A)] = \Phi_X(-u_1, -u_2) = \Phi_Y(u_1, u_2) \quad (\text{A.7})$$

we may also write

$$\begin{aligned}w &= e^{-rt+c} \int \int dy_L dy_A (e^{a_L - c - y_L} - e^{a_A - c - y_A} - 1)^+ f_Y(y_L, y_A) \\ &= e^{-rt+c} h(a_L - c, a_A - c)\end{aligned}$$

where

$$\begin{aligned}h(a, b) & \quad (\text{A.8}) \\ &= \int \int dy_L dy_A (e^{a - y_L} - e^{b - y_A} - 1)^+ f_Y(y_L, y_A) \\ &= \int \int dy_L dy_A (e^{a - y_L} - e^{b - y_A} - 1)^+ e^{\lambda_1(a - y_L) + \lambda_2(b - y_A)} e^{-\lambda_1(a - y_L) - \lambda_2(b - y_A)} f_Y(y_L, y_A) \\ &= e^{-\lambda_1 a - \lambda_2 b} \int \int dy_L dy_A (e^{a - y_L} - e^{b - y_A} - 1)^+ e^{\lambda_1(a - y_L) + \lambda_2(b - y_A)} e^{\lambda_1 y_L + \lambda_2 y_A} f_Y(y_L, y_A)\end{aligned}$$

for appropriate choices of λ_1 and λ_2 . After transforming the right hand side treating the two functions separately, we obtain (detail as in Appendix in [36], or [47])

$$\begin{aligned}
& h(a, b) \\
&= \frac{e^{-\lambda_1 a - \lambda_2 b}}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-iu_1 a - iu_2 b} \\
&\quad \frac{\Gamma(-\lambda_1 - \lambda_2 - 1 - iu_1 - iu_2)\Gamma(\lambda_2 + iu_2)}{\Gamma(1 - \lambda_1 - iu_1)} \Phi_X(i\lambda_1 - u_1, i\lambda_2 - u_2) du_1 du_2 \\
&= \frac{e^{-\lambda_1 a - \lambda_2 b}}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{iu_1 a + iu_2 b} \\
&\quad \frac{\Gamma(-\lambda_1 - \lambda_2 - 1 + iu_1 + iu_2)\Gamma(\lambda_2 - iu_2)}{\Gamma(1 - \lambda_1 + iu_1)} \Phi_X(i\lambda_1 + u_1, i\lambda_2 + u_2) du_1 du_2
\end{aligned}$$

the second equality comes after a simple change of variables.

To compute $h(a, b)$, we approximate the double integral by a double sum over the lattice

$$\mathbb{T} = \{u(k) = (u(k_1), u(k_2)) | k = (k_1, k_2) \in \{0, \dots, N-1\}^2\}, \quad u(k) = -\bar{u} + k\delta \quad (\text{A.9})$$

for appropriate choices of N , δ , $\bar{u} := N\delta/2$. For the FFT it is convenient to take N to be a power of 2 and lattice spacing δ such that truncation of the u -integrals to $[-\bar{u}, \bar{u}]$ and discretization leads to an acceptable error. Finally we choose initial values $X(0) = (\log L(0), \log A(0))$ to lie on the reciprocal lattice with spacing $\eta = \frac{2\pi}{N\delta} = \frac{\pi}{\bar{u}}$,

$$\begin{aligned}
\mathbb{T}^* &= \{x(l) = (x(l_1), x(l_2)) | l = (l_1, l_2) \in \{0, \dots, N-1\}^2\}, \\
& \quad x(l) = -\bar{x} + l\eta, \bar{x} = N\eta/2
\end{aligned}$$

We then have the approximation with $X(0) = x(l) \in \mathbb{T}^*$

$$h(x_1, x_2) \approx \frac{\delta^2}{(2\pi)^2} \sum_{k_1, k_2=0}^{N-1} e^{[iu_1(k_1) - \lambda_1]x_1 + [iu_2(k_2) - \lambda_2]x_2}$$

$$\frac{\Gamma(-\lambda_1 - \lambda_2 - 1 + iu_1(k_1) + iu_2(k_2))\Gamma(\lambda_2 - iu_2(k_2))}{\Gamma(1 - \lambda_1 + iu_1(k_1))} \Phi_X(u_1(k_1) + i\lambda_1, u_2(k_2) + i\lambda_2)$$

As usual for the discrete FFT, as long as N is even

$$iu(k)x(l)' = i\pi(k_1 + k_2 + l_1 + l_2) + 2\pi ikl'/N \pmod{2\pi i} \quad (\text{A.10})$$

This leads to the double inverse discrete Fourier transform

$$h(x_1, x_2) \approx (-1)^{l_1+l_2} \left(\frac{\delta N}{2\pi}\right)^2 e^{-\lambda_1 x_1(l_1) - \lambda_2 x_2(l_2)} \left[\frac{1}{N^2} \sum_{k_1, k_2=0}^{N-1} e^{2\pi ikl'/N} H(k) \right]$$

$$= (-1)^{l_1+l_2} \left(\frac{\delta N}{2\pi}\right)^2 e^{-\lambda_1 x_1(l_1) - \lambda_2 x_2(l_2)} [ifft2(H)](l)$$

where

$$H(k) =$$

$$(-1)^{k_1+k_2} \frac{\Gamma(-\lambda_1 - \lambda_2 - 1 + iu_1(k_1) + iu_2(k_2))\Gamma(\lambda_2 - iu_2(k_2))}{\Gamma(1 - \lambda_1 + iu_1(k_1))} \Phi_X(u_1(k_1) + i\lambda_1, u_2(k_2) + i\lambda_2)$$

For our numerical implementation, we choose parameters same as in [47]:

$$N = 2^9 \quad \lambda_1 = -3, \quad \lambda_2 = 1 \quad \text{and} \quad \delta = 2^{-3}.$$

Appendix B

Fund Type and Strategy Categorization

In CISDM, funds report a certain fund type as well as some description on the general strategy they employ. Fund types indicate whether the fund is a Hedge Fund, a Fund of Funds, a Commodity Trading Advisor or a Commodity Pool Operator. The Strategy is more of a free form description of the general investment strategy of the fund. Hedge funds employ many different trading strategies, which are classified in many different ways, with no standard system used. A hedge fund will typically commit itself to a particular strategy, particular investment types and leverage limits via statements in its offering documentation, thereby giving investors some indication of the nature of the particular fund. We categorize hedge funds here generally following the CISDM description and each strategy can be said to be built from a number of different elements.

Hedge Fund: Basic type of funds. The following is a group of strategies hedge funds may employ.

1. Relative value: (Arbitrage, Market neutral) - Exploit pricing inefficiencies between related assets that are mis-priced.

a. Fixed income arbitrage - exploit pricing inefficiencies between related fixed income securities.

b. Equity market neutral (Equity arbitrage) - being market neutral by maintaining a close balance between long and short positions.

c. Convertible arbitrage - exploit pricing inefficiencies between convertible securities and the corresponding stocks.

d. Fixed income corporate - fixed income arbitrage strategy using corporate fixed income instruments.

e. Asset-backed securities (Fixed-Income asset-backed) - fixed income arbitrage strategy using asset-backed securities.

f. Credit long / short - as long / short equity but in credit markets instead of equity markets.

g. Statistical arbitrage - equity market neutral strategy using statistical models.

h. Volatility arbitrage - exploit the change in implied volatility instead of the change in price.

i. Yield alternatives - non-fixed income arbitrage strategies based on the yield instead of the price.

j. Multi-strategy - fund uses a combination of strategies or diversification through different styles to reduce risk.

k. Regulatory arbitrage - the practice of taking advantage of regulatory differences between two or more markets.

l. Capital-structure arbitrage - seeks opportunities created by differential pricing of various instruments issued by one corporation. Consider, for example, traditional bonds and convertible bonds. The latter are bonds that are, under con-

tracted conditions, convertible into shares of equity. The stock-option component of a convertible bond has a calculable value in itself. The value of the whole instrument should be the value of the traditional bonds plus the extra value of the option feature. If the spread, the difference between the convertible and the non-convertible bonds grows excessively, then the capital-structure arbitrageur will bet that it will converge.

2. Global Macro: (Macro, Trading) - Global Macro funds attempt to anticipate global macroeconomic events, generally using all markets instruments to generate a return.

a. Discretionary macro - trading is carried out by investment managers selecting investments, instead of being generated by software.

b. Systematic macro - trading is carried out using mathematical models, executed by software without any human intervention other than the initial programming of the software.

3. Sector: emerging market, technology, health care etc.

Fund of Funds: a hedge fund with a diversified portfolio of numerous underlying hedge funds and a fund invested in other funds of hedge funds. The fund of funds may be also grouped into:

1. Conservative: - a low volatility, absolute-return fund of funds emphasizing consistent returns and capital preservation.

2. Opportunistic: - designed to target high absolute and high risk adjusted returns. The fund will capitalize on Primores' fund expertise and allow for a broad

mandate in pursuing investment opportunities. Investments can include offshore seeding of onshore funds, allocation to emerging managers, investments into less liquid strategies offering access to non mainstream strategies as well as special situation investments.

Commodity Trading Advisor (CTA Managed futures, Trading): The funds originally operated predominantly in commodities markets, but today they invest in any liquid futures (or options) market. The two major types of advisors are technical traders and fundamental traders. Technical traders may use computer software programs to follow price trends and perform quantitative analysis. Fundamental traders forecast prices by doing the analysis of supply and demand factors and other market information.

Commodity Pool Operator: an enterprise in which funds (or securities, property, either directly or through capital contributions) contributed by a number of persons are combined for the purpose of trading futures contracts, options on futures, or retail off-exchange FOREX contracts, or to invest in another commodity pool.

For fund strategies, since unlike the traditional investment arena, there does not exist a universally accepted norm to classify hedge funds' different strategies, we present one popular and commonly used categorization segregated mainly as "Non-Directional" and "Directional" strategies. Hedge fund strategies with low ex-

posures to standard asset markets (ones following Relative Value, Long-Short, or Risk Arbitrage type strategies) are classified as non-directional, while those having high correlation with the market are classified as directional.

Non-directional Strategies: These strategies have less correlation with any specific market. They are commonly referred to as “market neutral” strategies. These strategies aim to exploit short term pricing discrepancies and market inefficiencies between related securities while keeping the market exposure to a minimum. As most of the times, liquidity is limited in such strategies, they frequently run smaller pools of capital than their counterparts following directional strategies. Included in this group are the following strategies:

1. Event Arbitrage - A strategy of purchasing securities of a company being acquired, and shorting that of the acquiring company. The risk associated with such strategies is more of a “deal” risk rather than market risk.

2. Event Driven - A strategy which hopes to benefit from mis-pricing arising in different events such as merger arbitrage, restructurings etc. Manager takes a position in an undervalued security that is anticipated to rise in value because of events such as mergers, reorganizations, or takeovers. The main risk in such strategies is non-realization of the event.

a. Distressed securities (Distressed debt) - specialized in companies trading at discounts to their value because of (potential) bankruptcy.

3. Equity Hedge A strategy of investing in equity or equity-like instruments where the net exposure (gross long minus gross short) is generally low. Also referred

to as Long-Short strategy. The manager may invest globally, or have a more defined geographic, industry or capitalization focus. The risk primarily pertains to the specific risk of the long and short positions.

4. Restructuring - A strategy of buying and occasionally shorting securities of companies under Chapter 11 and/or ones which are undergoing some form of reorganization. The securities range from senior secured debt to common stock. The liquidation of financially distressed company is the main source of risk in these strategies.

5. Fixed Income Arbitrage - A strategy having long and short bond positions via cash or derivatives markets in government, corporate and/or asset-backed securities. The risk of these strategies varies depending on duration, credit exposure and the degree of leverage employed.

6. Capital Structure Arbitrage - A strategy of buying and selling different securities of the same issuer (e.g. convertibles/common stock) seeking to obtain low volatility returns by arbitraging the relative mis-pricing of these securities.

Directional Strategies: These strategies hope to benefit from broad market movements. Some popular directional strategies are:

1. Macro - A strategy that seeks to capitalize on country, regional and/or economic change affecting securities, commodities, interest rates and currency rates. Asset allocation can be aggressive, and leverage and derivatives may be utilized. The method and degree of hedging can vary significantly.

2. Long - A strategy which employs a “growth” or “value” approach to

investing in equities with no shorting or hedging to minimize inherent market risk. These funds mainly invest in the emerging markets where there may be restrictions on short sales.

3. Hedge (Long Bias) - A strategy similar to equity hedge with significant net long exposure.

4. Short - A strategy that focuses on selling short over-valued securities, with the hope of repurchasing them in the future at a lower price. Short bias - take advantage of declining equity markets using short positions.

5. Long/short equity (Equity hedge) - long equity positions hedged with short sales of stocks or stock market index options.

6. Emerging markets - specialized in emerging markets, such as China, India etc.

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