THE INSTITUTE FOR SYSTEMS RESEARCH

ISR TECHNICAL REPORT 2010-7

Distributed Topology Control for Stable Path Routing in Multi-hop Wireless Networks

Kiran K. Somasundaram, John S. Baras,
Kaustubh Jain, Vahid Tabatabaee

Distributed Topology Control for Stable Path Routing in Multi-hop Wireless Networks

Kiran K. Somasundaram, John S. Baras, Kaustubh Jain, Vahid Tabatabaee
Institute of Systems Research and Department of Electrical and Computer Engineering
University of Maryland, College Park
College Park, MD 20742
Email: kirans@umd.edu, baras@umd.edu, ksjain@umd.edu, vahidt@umd.edu

***Abstract*—In this paper, we introduce the topology control problem for stable path routing in mobile multi-hop networks. We formulate the topology control problem of selective link-state broadcast as a graph pruning problem with restricted local neighborhood information. We develop a multi-agent optimization framework where the decision policies of each agent are restricted to local policies on incident edges and independent of the policies of the other agents. We show that under a condition called the *positivity condition*, these independent local policies preserve the stable routing paths globally. We then provide an efficient algorithm to compute an optimal local policy that yields a minimal pruned graph, which we call the *Stable Path Topology Control* (SPTC) algorithm. Using simulations, we demonstrate that this algorithm, when used with the popular ETX metric, has lesser control overhead and the resulting pruned routing paths carry more upper layer traffic when compared with other topology control mechanisms commonly used for Mobile Ad Hoc Networks.**

***Index Terms*—stable paths; broadcast storm; graph-pruning; shortest-path problem; Bellman's optimality principle**

I. INTRODUCTION

Topology control in wireless multi-hop networks has been a topic of active research in recent years. A number of topology control mechanisms have been proposed for various purposes, including connectivity, energy-efficiency, throughput and robustness to mobility [1]. In particular, a number of topology control algorithms, both centralized and distributed, that are aimed to reduce the *broadcast storm problem* have been developed [2].

There are two popular approaches to reduce the broadcast storm problem [2]:

1) Graph pruning;
2) Controlled flooding.

Graph pruning approaches reduce the broadcast storm by limiting the link state information that is chosen to broadcast. Controlled flooding approaches reduce the broadcast storm by selecting a subset of the available stations to broadcast the chosen link states. Topology control mechanisms such as those in *Optimized Link State Routing* (OLSR) combine these approaches to reduce the broadcast storm problem. In this paper, we develop a framework to pose and solve the stable path topology control problem from a graph pruning perspective.

Several of the proposed pruning mechanism are distributed localized algorithms for static graphs [9], [11], [2], [14]. However, for mobile multi-hop wireless networks these static graph approaches are limited in solving the broadcast storm problem. Although there are a number of metrics that capture the link dynamics, very few algorithms use these link metrics for topology control in routing. Even those that do are only heuristic, which do not offer proof guarantees for the reduced topology and routing [3].

One important metric for routing in MANETs is *path longevity* or *path stability* [4]. In this paper, we refer to it as path stability. Although path stability has been studied for many reactive distance vector schemes [4], [5], there is little work that addresses topology control for stable paths in link state routing. We introduce a new topology control algorithm which guarantees stable path routing: *Stable path topology control* is a mechanism to prune the initial topology (to reduce the broadcast storm) and at the same time guarantees that the stable paths for routing (unicast) from every host to any target station are preserved in the pruned topology. Topology control for stable paths has a two-fold advantage: First, these long lived paths are *cheaper to maintain* (as they are less likely to change). Second, it offers the higher layer traffic long lived paths and consequently yields *improved traffic carrying performance*.

The main contributions of this paper are the following. We introduce the *stable path topology control* problem: we set up this problem as a *constrained multi-agent optimization problem*, where the agents include all the stations in the network, and these agents have access to only their local neighborhood information. We formulate the pruning problem as a policy on the incident edges for each of these agents. Then, we prove necessary conditions that these pruning policies must satisfy to preserve the stable routing paths. We introduce the notion of *loop freedom* for this problem and show that a certain *positivity* condition ensures that the necessary conditions become sufficient, even in the distributed setting. Finally, we develop a distributed pruning algorithm, which we call the *Stable Path Topology Control* (SPTC) algorithm, that solves the multi-agent optimization problem.

Our goal in this paper is not to engineer link stability metrics, but to develop a general framework for the stable path topology control problem that can make use of available link stability metrics. Several link stability metrics, which are commonly used in wireless networks, can be used with the SPTC algorithm. In this paper, we choose the ETX metric, a popular link stability metric, and apply it to the SPTC algorithm to demonstrate its pruning capabilities. We call this the SPTC-ETX algorithm. The SPTC algorithm can be implemented with minor modifications to OLSR's neighbor discovery [15] and topology selection mechanism [12]. Using OPNET [7] simulations, we compare the performance of the SPTC-ETX algorithm with that of an OLSR-ETX implementation [13]. For these simulations, we modified the default code of the OLSR model in OPNET to implement the SPTC algorithm. Simulation results for different scenarios suggest the SPTC-ETX outperforms the pruning mechanism of OLSR-ETX:

1) SPTC-ETX has lower topology control overhead compared to OLSR-ETX;
2) SPTC-ETX offers stable routing paths, which carry more upper-layer traffic, compared to OLSR-ETX.

This paper is organized as follows. In Section II, we summarize several link stability metrics proposed for wireless networks and the related topology control mechanisms. We also identify a fundamental limitation of the existing topology control algorithms. In Section III, we introduce the mathematical notation that is needed to formulate the stable path topology control problem. In Section IV, we develop the mathematical framework for the multi-agent pruning problem. We establish necessary and sufficient conditions for the pruning policies. Finally in Section V, we present the SPTC pruning algorithm. Using several simulation scenarios we demonstrate the performance of the SPTC-ETX algorithm.

II. Related Work

Routing protocols in mobile multi-hop networks are broadly classified as reactive, proactive and hybrid [8]. Reactive protocols request for route to a destination only when the source has a data to send (on-demand). On the other hand in proactive routing protocols, every source maintains at-least one route to every destination of interest, by periodic updates. Hybrid protocols adapt mechanisms from the both reactive and proactive protocols. Proactive protocols maintain routes by periodic broadcasts of link states [8], [2]. *Broadcasting* in a network is the process by which a packet sent from one station reaches all other stations in the network. However in the mobile multi-hop networks, link states are very dynamic, and consequently, a large number of packets, corresponding to every link state change, is broadcast in the network. This problem is referred to as the *broadcast storm problem* [2].

A class of proactive routing protocols called *controlled flooding* protocols reduce this control overhead (broadcast storm) by selecting a subset of stations to broadcast the link states to maintain routes; this includes [9], [10], [11], [12], [13]. Most of these controlled flooding algorithms, also use local neighborhood information to prune the link state locally [2], [14]. As an example, we will introduce OLSR's selective broadcasting in the next subsection.

A. Selective Broadcasting in OLSR

In Optimized Link State Routing (OLSR) protocol [11], [12], every host in the network discovers its local neighborhood by heartbeat periodic HELLO messages [15]. Since every host broadcasts to its neighbors the set of neighbors that it can hear, every host discovers its one-hop and two-hop neighbors. Figure 1 shows the neighborhood that host h discovers from the HELLO messages. The neighbor discovery protocol [15] is designed to discover only symmetric neighbors (that can hear each other), and consequently all the links discovered are undirected.

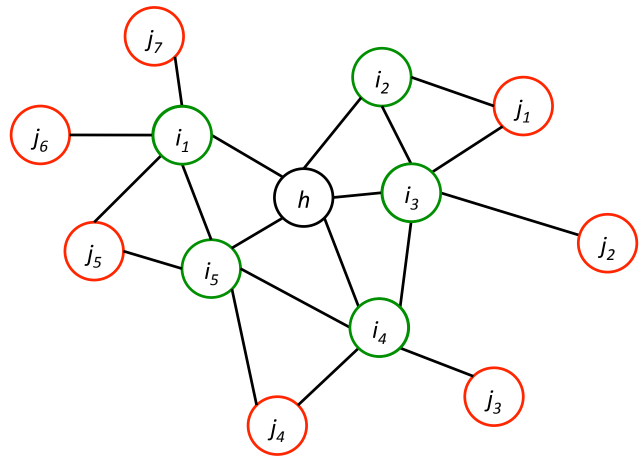


Fig. 1: Local View of Host h

The original version of OLSR [11], treats the topology pruning problem over a static graph. Every host solves a set-cover problem locally to find the minimum set of one-hop neighbors that cover all the two-hop neighbors. For instance, in the example graph shown in Figure 1, the host h selects from $\{i_1, i_2, \dots, i_5\}$ a minimal subset of neighbors that cover all two-hop neighbors $\{j_1, j_2, \dots, j_7\}$. This special set of one-hop neighbors is called Multi-Point Relays (MPRs) in OLSR. The pruning problem to compute the MPRs is shown to be NP-hard and a greedy heuristic was proposed in [16]. For this example of Figure 1, the host selects $\{i_1, i_3, i_4\}$ that covers all the two-hop neighbors. Then the host broadcasts links $\{(h, i_1), (h, i_3), (h, i_4)\}$ across the entire network. Every host in the network performs similar broadcast. It is was proved in [17] that this pruning mechanism preserves the shortest path, in *hop-count*, from every source to target host in the network.

B. Stability Metrics for Multi-Hop Wireless Networks

The majority of the routing protocols proposed for wireless multi-hop networks, both reactive and proactive, are mechanisms that use *hop count* as the metric for routing [8]. However, the wireless links in a multi-hop network are vulnerable to frequent breakage due to mobility and channel erasures [18], [19], [20]. Hence, schemes based merely on hop count, which are inherently insensitive to the dynamic stability of the paths, have shown poor performance [21]. This limitation has inspired a number of protocols that use *link stability* as a metric for routing. In particular, several link stability metrics for Mobile Ad Hoc Networks (MANETs) and Wireless Mesh Networks (WMNs) have been proposed.

Perhaps, the earliest MANET protocol to use link stability metric for routing is the *Associativity Based Routing* (ABR) scheme [22], which uses an *associativity* threshold used to predict the stability of a neighboring station. It assumes that neighbors that remain associated beyond this threshold are less likely to move away and hence form stable neighbors or links. *Signal Stability based Adaptive routing* (SSA) [23] is another link stability based routing protocol that uses signal strength and location information from the neighboring stations to estimate stability of the links. *Route-lifetime Assessment Based Routing* (RABR) [24] is an extension to SSA that uses thresholding of link ages to choose routes. Mobility prediction was suggested in [18] to improve unicast and multicast routing protocols for MANETs. This scheme uses GPS location information to estimate the residual lifetime for links.

In [19], the authors present a simulation study of the empirical distribution of link lifetimes for various mobility models [25]. From these empirical distributions, they also derive a method to compute the residual life time distribution for these models. The study reveals that there are strict thresholds beyond which the residual lifetimes exhibit a positive correlation with the link age. Another statistical characterization of link lifetimes is presented in [4]. Their simulation results show that the longer lifetime paths tend to have longer length (in hop count), and hence, there is a clear tradeoff between path stability and path delay. The *Stability and Hopcount based Algorithm for Route Computing* (SHARC) [6] identifies this tradeoff and combines the link stability metric and the hop count to find short paths (in terms of hop count) that also have good stability.

Another simulation study, presented in [26], shows that the path life is inversely related to the maximum velocity and the hop count and is directly related to the transmission range. The authors observe that under high mobility patterns, the path durations can be approximated using exponential distributions. In [27], *Han* et. al. use Palm calculus to show that under certain conditions, the path durations converge to an exponential distribution as the number of hop count increases.

The wireless mesh networking community has also been actively developing several stability metrics for routing. Since these backbone routers of a WMN are stationary, routing using link stability metrics, rather than mere hop-count, is more feasible compared with the MANET case, where the network topology is more dynamic [28]. The first metric proposed, for multi-hop wireless networks, is the Expected Transmission Count (ETX) metric in [29], [30]. The ETX link metric computes the expected number of transmissions, including retransmissions, for a packet to reach the other station of the link. The authors of [29] design the ETX metric for 802.11 MAC with acknowledgements. Thus the ETX metric accounts for the link stability both in the forward and reverse direction of the link. The ETX of a path, i.e., to deliver the packet to the destination, is the sum of the link ETX metrics along the path. Although, it is not in the RFC [11], the ETX metric has been incorporated in popular OLSR implementations [31], [13]. In [32], the authors argue that the ETX metric, being additive, suffers from route oscillations. Instead, they propose, a multiplicative metric, Minimum Loss (ML) metric that computes the loss probability of a path. Another shortcoming of the ETX metric computation is that the network data and control packets are typically larger than the probe packets used to compute the metric. This problem is identified and a new metric called Expected Transmission Time (ETT), which computes the expected transmission time instead of the count, was proposed in [33]. ETT adapts the ETX for different PHY transmission rates and packet sizes. They also proposed the Weighted Cumulative ETT (WCETT) that changes ETT to also consider intra-flow interference. This metric is a sum of end-to-end delay and channel diversity. A tunable parameter is used to combine both components. Unlike ETX and ETT, WCETT is an end-to-end metric. However, in [34], the authors argue that WCETT guarantees neither shortest paths nor inter-flow interference. They develop an alternative metric, Metric of Interference and Channel-switching (MIC), which addresses these shortcomings.

In wireless networks, the link stability is usually highly dynamic, and consequently, several of the metrics proposed, if used crudely, can cause significant control overhead or route oscillations [28]. In [28], the authors propose two metrics: modified ETX (mETX) and Effective Number of Transmissions (ENT) that consider the variance of the link-stability while computing the stability metric. Another metric that considers link-quality variation is iAWARE [35]. This metric uses the signal to noise ratio and signal to interference and noise ratio to continuously reproduce neighboring interference variations onto routing metrics. A number of other link stability metrics have also been proposed [22], [5], [36], [4], [37], [18], [6], [20], [23], [24] that capture the stability of the links in a MANET.

C. Limitations of Existing Topology Control Mechanisms for Controlled Flooding

The algorithms that make use of the link stability metrics, in most cases, are modifications of reactive distance vector protocols such as Dynamic Source Routing (DSR) [38] and *Ad Hoc On-demand Distance Vector* (AODV) [39]; these include Link Quality Source Routing (LQSR) [33], Multi-Radio LQSR (ML-LQSR) [33], SrcRR [30] and others. There are a few proactive routing protocol that incorporate these link stability metrics for topology control for controlled flooding. Most of these are variants of OLSR's [11] pruning methods (Subsection II-A). In [40], [3], [41], the authors modify OLSR's MPR selection algorithm using a weighted set-cover algorithm [42]. In [31], [13], the ETX metric is used as link-stability weights. Consider the example local view of Figure 1. Let $\mathit{ETX}(u, v)$ denote the ETX metric of the link (u, v). The ETX of an non-existent link is ∞. In these implementations, the host h computes the ETX metric, the best two-hop path to reach a two-hop neigbor j, by

$$\min_{l} \mathit{ETX}(h, i_l) + \mathit{ETX}(i_l, j),$$

where i_l's are the one-hop neighbors of the host h. The host then selects a minimal set of its one-hop neighbors (MPRs) that are in these paths for all the two-hop neigbors j. In essence, this is another set-cover problem where all two-hop neighbors are covered using one-hop neighbors using modified weights.

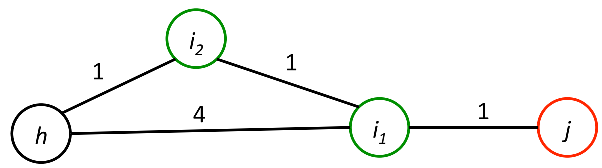


Fig. 2: Example Local View with ETX metric for each link indicated

However, these set-cover methods offer no proof guarantees for the stability of the pruned paths, i.e., the stable paths for routing need not be preserved by these pruning methods. Consider the example local view shown in Figure 2. The symmetric ETX metrics for the different links are also indicated. The host h has two one-hop neighbors i_1 and i_2 and one two-hop neighbor j. The long-distance link (h, i_1) is unstable (*ETX* $= 4$), while all other links are stable. For this example, the set cover method of the implementations in [13], [31] has only one feasible (two-hop) path (h, i_1, j) to reach j, which has an ETX cost 5. However, if we relax the artificial constraint of two-hop feasible paths, there exists an alternative better path (h, i_2, i_1, j) of ETX cost 3. Clearly, the set-cover pruning methods (of OLSR and its variants) will not preserve this stable path. Note that this example is not a mere pedagogical example. In wireless radio networks, the unstable long-distance one-hop neighbors, typically, cover more two-hop neighbors than shorter (more stable) one-hop neighbors [17].

In the forthcoming sections, we will formulate and solve a distributed pruning (topology control) problem that can provably preserve all the stable paths between every source-destination pair in the pruned topology. Our pruning method is not specific to any particular stability metric and can be applied for all the stability metrics discussed in this section.

III. Mathematical Notations and Definitions

A. Graphs and Neighborhoods

Let $G(V, E)$ denote the communication graph, where V is the vertex set of stations and E is the undirected edge set (communication adjacency between the vertices). For $(u, v) \in E$, there is an associated symmetric link stability metric $a(u, v) = a(v, u) \geq 0$ (all the metrics introduced in Subsection II-B satisfy this condition). Thus, G is an undirected edge-weighted graph.

A subgraph of G, denoted by $G' \subseteq G$, is a graph $G'(V', E')$ such that $V' \subseteq V$ and $E' \subseteq E$ (restricted to $V' \times V'$). For any vertex $i \in V'$, the set of edges incident to i in any subgraph G' is denoted by $\Omega_{G'}(i)$. The set of paths in any subgraph G' between a pair of vertices $i, j \in G'$ is denoted by $P_{ij}^{G'}$. For any path $p \in P_{ij}^{G'}$, the *successor vertex* for a vertex i in p is denoted by η_p^i.

We introduce the notion of hop-based neighborhoods. The hop count hc of a path is the number of edges in the path. Then the minimal hop count distance between a pair of vertices (i, j) in G is defined as

$$d_{hc}(i, j) = \min_{p \in P_{ij}^G} hc(p).$$

We define the *k-hop neighborhood* for a host $h \in V$ by

$$N_h^k = \{j \in V : d_{hc}(h, j) \leq k\}.$$

Here, k is called the *size of the neighborhood*. The boundary set for the neighborhood N_h^k is given by

$$\partial N_h^k = N_h^k \backslash N_h^{k-1},$$

where $N_h^0 = \{h\}$, and $N_h^k = \emptyset, k < 0$. Let $N_h^{k^-} = N_h^k \backslash \{h\}$ denote the *exclusive* neighborhood, which is the neighborhood excluding h.

Consider a special induced subgraph G_h^{local}, which is a subgraph of G, that contains only the vertices in N_h^k and all the edges between them, expect those between any two vertices of the boundary set, i.e, the vertex set is N_h^k and the edge set is $\{(u, v) \in E : \ u, v \in N_h^k \text{ and } \{u, v\} \nsubseteq \partial N_h^k\}$. We will, later, call this edge-weighted subgraph the *local view* of h (Subsection IV-A). For brevity of notation, we define for this special sub-graph paths rooted at h: For $j \in N_h^k$, $P_j^{h-local} = P_{hj}^{G_h^{local}}$.

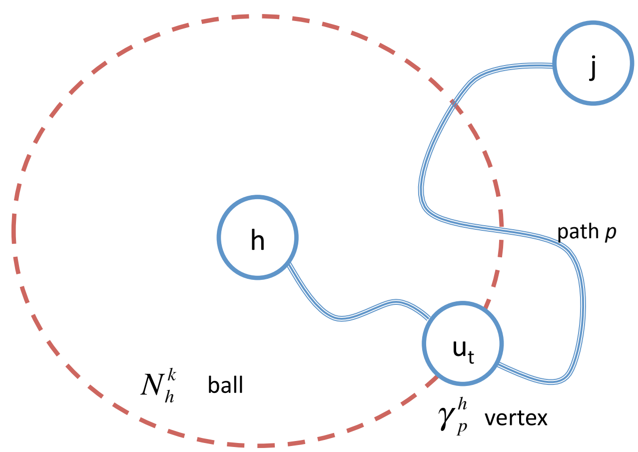


Fig. 3: Gateways for path p in local view

Finally, we introduce the notion of gateways for paths restricted to the special sub-graph G_h^{local} (Figure 3). For any path $p = (h = u_1, u_2, \ldots, u_n = j) \in P_{hj}^G$, the *gateway* of p in G_h^{local}, denoted by γ_p^h, is the first vertex of p that is in the boundary set ∂N_h^k. i.e., $\gamma_p^h = u_t$ if and only if $u_t \in \partial N_h^k$ and $u_s \notin \partial N_h^k, 1 \leq s < t$. If the path p never intersects ∂N_h^k, i.e., $u_s \notin \partial N_h^k$, $1 \leq s \leq n$, then γ_p^h is not defined.

B. Path Stability

For the stability metrics discussed in Subsection II-B, the stability of path p, denoted by $w(p)$, is computed by composing the link stability metrics $a(u, v)$, $(u, v) \in p$. Most of the metrics from Subsection II-B follow either additive or multiplicative compositions. Since a multiplicative composition can be transformed to an additive composition (i.e., using logarithms), we only consider additive compositions for path stability:

$$w(p) = \sum_{(u,v)\in p} a(u,v).$$

The optimal value of the path stability metric between a vertex pair (i,j) in G' is

$$\begin{aligned} w^*_{G'}(i,j) &= \min_{p\in P_{ij}^{G'}} w(p) \\ &= \min_{p\in P_{ij}^{G'}} \sum_{(u,v)\in p} a(u,v), \end{aligned}$$

and the corresponding optimal path set, which is the set of paths that achieve the optimal value of the optimal path stability metric from i to j in the subgraph G', is

$$P_{ij}^{G'^*} = \{p \in P_{ij}^{G'} : w(p) = w^*_{G'}(i,j)\}.$$

In essence, computing the optimal paths corresponds to computing the shortest paths in the restricted graph G'. From hereon, we will call these shortest paths, optimal paths.

We call the optimal paths in G_h^{local} from i to j, *h-locally optimal*. The set of h-locally optimal paths from i to $j \in N_h^k$ is

$$P_{ij}^{h-local*} = \{p \in P_{ij}^{G_h^{local}} : w(p) = w^*_{G_h^{local}}(i,j)\},$$

and their corresponding optimal weights are denoted by $w^*_{h-local}(i,j)$. Again for brevity of notation, we will denote the h-locally optimal paths from h to $j \in N_h^k$ by $P_j^{h-local*}$.

Finally, note that this additive path stability metric follows the *Bellman's optimality principle*:

Lemma 3.1: For a stable path $p = (i = u_1, u_2, \ldots, u_n = j) \in P_{ij}^{G'^*}$, any sub-path $(u_k, u_{k+1}, \ldots, u_l) \in P_{kl}^{G'^*}$ for $1 \leq k < l \leq n$.

The above lemma is proved in [42].

IV. Theory of Local Pruning

Topology control by local pruning [2] in multi-hop wireless networks is an interesting graph optimization problem. These pruning algorithms make use of the local neighborhood information that is provided by neighbor discovery protocols [15]. From this local neighborhood information they select a subset of the topology information that is broadcast to the network. This subset is chosen so that the resulting pruned graph preserves some properties of the original graph. The non-triviality in these problems is establishing a relation between the local and pruned-global graph. Though there have been many local pruning algorithms [3], [12], [13], [2], to the best of our knowledge, there is no good optimization formulation of the local pruning problem that relates to the global properties of the pruned graph, except for [44], [14].

In [45], we extended the notion of local and global views introduced in [14] to encompass edge-weighted dynamic graphs. We summarize these extensions in the forthcoming subsections.

A. Heartbeat Discovery and Local View

We assume that every host has a neighbor discovery module [11], [12], [13]. It discovers its local neighborhood information using periodic *HELLO* messages. The *HELLO* message from each host contains both the communication adjacency and the link stability information for all of its $(k-1)$-hop neighbors $(k \geq 2)$. Consequently, every host $h \in V$ has access to the dynamic edge-weighted graph G_h^{local}, where the edge-weights correspond to the symmetric link stability metrics $a(u,v),\ (u,v) \in G_h^{local}$. In OLSR, $k = 2$ because every host exchanges its one-hop link state information. This notion is formally abstracted as the *local view*:

Definition: At every host station $h \in V$, the *local view* consists of the edge-weighted subgraph G_h^{local}, with a neighborhood size k, that is exposed by the neighbor discovery mechanism at h. The edge weights $a(u,v),\ (u,v) \in G_h^{local}$ are the symmetric link stability metrics.

B. Pruning with Local Policies

In local pruning algorithms, the host $h \in V$, which has discovered the edge-weighted graph G_h^{local}, chooses a subset of its incident edges, which we call the *pruned edge set* of h. The set of such *pruning policies* at host h is the set of functions

$$F_h^{prune} = \{f : G_h^{local} \to 2^{\Omega_{G_h^{local}}(h)}\},$$

where $2^{\Omega_{G_h^{local}}(h)}$ is the power-set of $\Omega_{G_h^{local}}(h)$ (set of of all subsets of $\Omega_{G_h^{local}}(h)$).

For a given pruning policy $f_h \in F_h^{prune}$ at $h \in V$, the pruned edge set is $\Omega_h = f_h(G_h^{local})$. From henceforth, f_h and Ω_h will represent the pruning policy and pruned edge set at host h respectively. Ω_h is, then, broadcast network-wide. If the subset Ω_h is small compared to $\Omega_{G_h^{local}}(h)$, then the broadcast information rate is significantly reduced. In essence, this controlled flooding (of pruned link states) reduces the broadcast storm.

In [14], the authors show several local pruning methods, in essence, try to construct a *Connected Dominating Set* (CDS) for the dynamic graph G by local pruning. For every construction discussed in [14], there is a different objective function for the CDS construction. For instance in [44], *Lee* et. al. present a CDS construction that is efficient w.r.t. energy and graph resiliency. In OLSR [11], [12], the *Multi-Point Relay* (MPR) set, which is constructed by local pruning, yields a global CDS that preserves the shortest hop-count paths. Each of these local pruning policies can be associated with some $f_h \in F_h^{prune},\ h \in V$.

However, the CDS constructions [14], in general, do not offer guarantees on the quality of the routing paths in the pruned CDS (see Subsection II-C). To the best of our knowledge, there has been no class of local pruning algorithms that offer path quality guarantees on the pruned global graph.

C. Global View

As explained in Subsection IV-B, the pruning policy f_h selects a pruned edge set Ω_h and the host h broadcasts this

Ω_h and their corresponding edge-weights (in most protocols [12], [13], [46], the link stability information $a(u, v)$ is also broadcast). The corresponding broadcast edge set is given by $E^{broadcast} = \cup_{h \in V} \Omega_h$, and this induces an edge-weighted subgraph $G^{broadcast}$, which we call the *broadcast view*.

Definition: At every host station $h \in V$, the *global view* G_h^{global} is the edge-weighted graph union $G_h^{local} \cup G^{broadcast}$, where G_h^{global} and $G^{broadcast}$ are exposed by some neighbor discovery and link state broadcast mechanisms respectively.

D. Expressing Global Constraints

The fundamental pruning problem for each host $h \in V$, is to construct a minimal pruned edge set Ω_h^* such that G_h^{global} preserves some desired properties of G. This is an interesting multi-agent optimization problem where the objective function (finding a minimal pruned edge set) for each agent (host) depends only on local neighborhood information (local view). However, the agents (hosts) together must satisfy a global constraint (the global view must preserve some desired properties of G). Before we consider the optimization problem (of finding the minimal pruned edge set), we will mathematically express the global constraint for stable path topology control. This is non-trivial because the global constraint involves the global view, while the hosts have access to strictly their local view. We will introduce more notation for this purpose.

Consider the pruning methods summarized in [2], [14]. All these methods construct a minimal pruned set such that the vertices of G_h^{global} form a CDS of G. We will represent this by a *CDS-of* property, denoted by π^{CDS}. $\pi^{CDS}(G)$ is said to hold for a subgraph G' if the vertices of G' constitute a CDS of G. The set of subgraphs of G for which the property π^{CDS} holds is given by

$$\Pi^{CDS}(G) = \{G' \subseteq G : \ \pi^{CDS}(G) \text{ holds for } G'\}.$$

Then, the global constraint (of preserving a CDS) for each agent (host) h can be expressed as $G_h^{global} \in \Pi^{CDS}(G)$.

For a generic property π, let

$$\Pi(G) = \{G' \subseteq G : \ \pi(G) \text{ holds for } G'\}$$

denote the set of subgraphs for which the property $\pi(G)$ holds. Several global constraints, including those in [2], [14], can be expressed in the form $G_h^{global} \in \Pi(G)$ by choosing a suitable property π.

Although several of the global constraints for pruning problems can be expressed via the π^{CDS} property, the stable path topology control problem cannot be expressed via this property. For stable path routing, we want G_h^{global} to preserve all stable routing paths (of G) from h to every other vertex $j \in V$. This can be expressed as a shortest path property:

$$\pi_h^S : \exists p \in {P_{(h,j)}^G}^*, \ j \in V. \tag{1}$$

Then, $\Pi_h^S(G)$ corresponds to all the subgraphs of G that contain the optimal path tree rooted at h. Then the stable path preserving global constraint can be expressed as $G_h^{local} \subseteq \Pi_h^S(G)$.

Although we have mathematically expressed the global constraint for stable path pruning, this constraint cannot be directly imposed on the local view. In other words, we need local constraints that will guarantee that the global constraints are satisfied. In the forthcoming subsections, we will develop such local constraints.

E. Necessary Conditions and Lack of Loop Freedom

We first establish a necessary (local) condition for any pruning policy to preserve all stable paths globally (Equation (1)). Since the local pruning policies of interest at host $h \in V$ are given by the functions $f_h \in F_h^{prune}$, it is natural to establish the necessary conditions that these functions must satisfy.

Consider a local property $\pi_h^{\Omega-S}$ at $h \in V$. Given the local view G_h^{local}, the property $\pi_h^{\Omega-S}(G_h^{local})$ is said to hold for a pruning function $f_h \in F_h^{prune}$, if for all $j \in \partial N_h^k$ there exists a path $p \in P_j^{h-local*}$ such that $(h, \eta_p^h) \in f_h(G_h^{local})$. Let $\Pi_h^{\Omega-S}(G_h^{local})$ denote the subset of functions (of F_h^{prune}) for which $\pi_h^{\Omega-S}(G_h^{local})$ holds. Clearly, the property $\pi_h^{\Omega-S}$ is local because it depends on only G_h^{local}. The following theorem establishes that this local constraint is necessary for the stable path pruning.

Theorem 4.1: If, for an arbitrary edge-weighted graph G, $G_h^{global} \in \Pi_h^S(G)$, then at every $h \in V$ the pruning policy $f_h \in \Pi_h^{\Omega-S}(G_h^{local})$.

Proof: We will prove the contrapositive. If we show that for some graph G, not satisfying the pruning condition $f_h \notin \Pi_h^{\Omega-S}(G_h^{local})$ yields $G_h^{global} \notin \Pi_h^S(G)$, the proof is complete. Consider a line graph shown in Figure 4, where all edge-weights are 1. Let the size of the neighborhood be $k = 2$. We will show that if the pruning condition is not satisfied, then edge (h_2, h_3) is not chosen for broadcast. Clearly, in this example, only the hosts h_2 and h_3 are responsible of selecting (h_2, h_3) (by the virtue of the local pruning policy in Subsection IV-B). Let us consider the pruning policy at h_2. Here, $\partial N_{h_2}^k = \{h_4\}$. (h_2, h_3, h_4) is the only path from h_2 to h_4. And, $\partial N_{h_3}^k = \{h_1, h_5\}$. (h_3, h_2, h_1) is the only path from h_3 to h_1. If $(h_2, h_3) \notin \Omega^*(h_2)$ and $(h_3, h_2) \notin \Omega^*(h_3)$, then $(h_2, h_3) \notin G^{broadcast}$. Since $(h_2, h_3) \notin G_{h_5}^{local}$, $(h_2, h_3) \notin G_{h_5}^{global}$. Consequently, $G_{h_5}^{global}$ does not contain the globally optimalss paths to h_1 and h_2. ∎

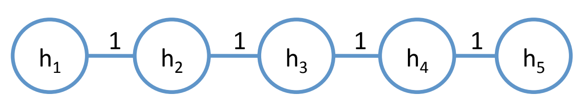


Fig. 4: Example line graph illustrating the necessary condition

However, the necessary condition of the above theorem are not sufficient to ensure $G_h^{global} \in \Pi_h^S(G)$, since it does not guarantee *loop-freedom*. This is a well-known problem for distributed routing protocols [8]: Loops typically occur in distributed graph algorithms when tie-breaking mechanisms are not employed. Using an example, we illustrate that a similar problem is likely to occur in stable path distributed pruning without tie-breaking.

Consider the example of an edge-weighted graph shown in Figure 5, where the edge-weights correspond to some

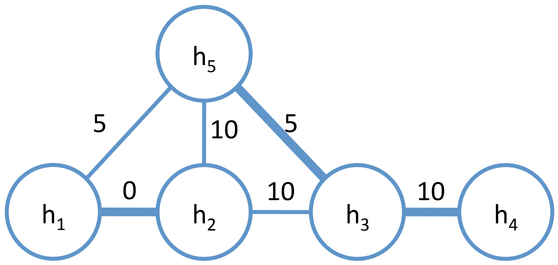


Fig. 5: An example to illustrate loops in pruning. The edges of the $G^{broadcast}$ are indicated with thick lines.

link stability metric (Section II). Consider any stable path pruning policy at stations $h_1, h_2, \ldots, h_5$. Let the size of the neighborhood exposed be $k = 2$. The neighborhood boundary sets are $\partial N^2_{h_1} = \{h_3\}$, $\partial N^2_{h_2} = \{h_4\}$, $\partial N^2_{h_3} = \{h_1\}$, $\partial N^2_{h_4} = \{h_2, h_5\}$ and $\partial N^2_{h_5} = \{h_4\}$. If the pruning mechanisms at these stations satisfy the necessary conditions, then station h_4 choses (h_4, h_3) and station h_5 choses (h_5, h_3). However, the pruning mechanisms at stations h_1, h_2 and h_3 have multiple optimal paths to choose from. For station h_1 to reach h_3, there are two optimal paths, (h_1, h_2, h_3) and (h_1, h_5, h_3). For station h_2 to reach h_4, there are two optimal paths, (h_2, h_3, h_4) and $(h_2, h_1, h_5, h_3, h_4)$. For station h_3 to reach h_1, there are two optimal paths, (h_3, h_2, h_1) and (h_3, h_5, h_1). The Figure 5 illustrates one pruning policy that satisfies the necessary conditions: h_1 chooses (h_1, h_2) for path (h_1, h_2, h_3), h_2 chooses (h_2, h_1) for path $(h_2, h_1, h_5, h_3, h_4)$, and h_3 chooses (h_3, h_5) for path (h_3, h_5, h_1). The pruned graph $G^{broadcast}$, shown in the Figure 5, is then disconnected! Clearly, the distributed pruning does not preserve the stable optimal paths in the different global views.

F. Positivity Assumption, Sufficiency and Local Reducibility

The example of Figure 5 suggests a sufficient condition, which we call the *positivity condition*: all the edge weights are strictly positive, $a(u, v) > 0,\ (u, v) \in E$. We will show that under the positivity assumptions, the necessary conditions become sufficient.

Lemma 4.2: For any $p = (h = u_1, u_2, \ldots, u_n = j) \in P^{G\,*}_{hj}$, let $p_1 = (u_1, u_2, \ldots, u_l)$ be the sub-path from h to first occurrence γ^h_p, i.e., $l = \min\{1 \le s \le n :\ u_s = \gamma^k_p\}$, and let $p_2 = (u_l, u_{l+1}, \ldots, u_n)$ be the remnant of path p. Then for any $p'_1 \in P^{h-local\,*}_{\gamma^p_h}$, the concatenated path $(p'_1, p_2) \in P^{G\,*}_{hj}$.

Proof: We will assume that $(p'_1, p_2) \notin P^{G\,*}_{u_1 u_n}$ and derive a contraction. Since p_1 and p_2 are sub-paths of p, $p_1 \in P^{G\,*}_{u_1 u_l}$ and $p_2 \in P^{G\,*}_{u_l u_n}$ (Lemma 3.1). This implies that $p'_1 \notin P^{G\,*}_{u_1 u_l}$. Since p_1 is completely contained in G^{local}_h, $w^*_G(u_1, u_l) = w(p_1) = w^*_{h_1-local}(u_1, u_l) = w(p'_1)$. This is a contradiction. ■

Theorem 4.3: Under the positivity assumption, if $h \in V$, $f_h \in \Pi^{\Omega-S}_h(G^{local}_h)$, then $G^{global}_h \in \Pi^S_h(G)$.

Proof: We need to show that G^{global}_h has at-least one optimal path to any vertex $j \in V$. We will prove by construction that one optimal path is preserved under pruning.

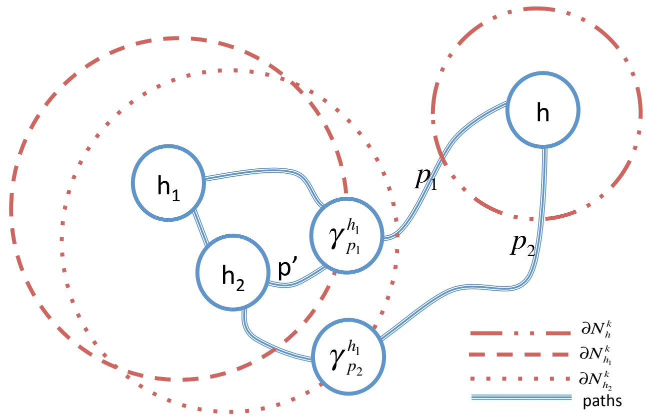


Fig. 6: Path construction

Suppose an optimal path to j is contained in G^{local}_h, then the proof is trivial. Consider the other case (Figure 6): an optimal path to j is not contained in G^{local}_h. Let $j = h_1$. Let $p_1 \in P^{G\,*}_{h_1 h}$ be a globally optimal path from h_1 to h. $\gamma^{h_1}_{p_1}$ is the corresponding gateway vertex in $G^{local}_{h_1}$. Let p^1_1 be the sub-path of p_1 from h to $\gamma^{h_1}_{p_1}$ and p^2_1 is the remnant path. Since f_{h_1} satisfies the pruning condition ($f_{h_1} \in \Pi^{\Omega-S}_{h_1}(G^{local}_{h_1})$), there exists $p' \in P^{h_1-local\,*}_{\gamma^{h_1}_{p_1}}$ (h-locally optimal path from h_1 to $\gamma^{h_1}_{p_1}$). By Lemma 4.2, p' is a globally optimal path from h_1 to the gateway. Thus the concatenated path $(p', p^2_1) \in P^{G\,*}_{h_1 h}$. Suppose $p = (h_1, h_2, \ldots, \gamma^{h_1}_{p_1})$, then $w^*_G(h_1, h) = a(h_1, h_2) + w^*_G(h_2, h)$. Since $a(h_1, h_2) > 0$ (positivity assumption), $w^*_G(h_1, h) > w^*_G(h_2, h)$. Consequently, the pruning policy at h_2 to reach h can never choose h_1 ($\because w^*_G(h_1, h) > w^*_G(h_2, h)$). In essence, this relation ensures local loop-freedom, which translates to global loop-freedom: By recursively applying this argument, we obtain a sequence of vertices $h_1, h_2, \ldots, h_n$ that are distinct from each other. We can terminate at h_n, where any optimal path from h to h_n is completely contained in G^{global}_h. Since $|V|$ is finite, the recursive procedure converges constructing an optimal path from j to h. ■

Theorem 4.4: Under the positivity assumption, $G^{global}_h \in \Pi^S_h(G)$, $h \in V$ if and only if $f_h \in \Pi^{\Omega-S}_h(G^{local}_h)$, $h \in V$.

Proof: Theorem 4.1 establishes the forward implication:

$$G^{global}_h \in \Pi^S_h(G), h \in V \Rightarrow f_h \in \Pi^{\Omega-S}_h(G^{local}_h).$$

Under the positivity assumption $a(u, v) > 0,\ (u, v) \in E$, the reverse implication holds (Theorem 4.3):

$$f_h \in \Pi^{\Omega-S}_h(G^{local}_h), h \in V \Rightarrow G^{global}_h \in \Pi^S_h(G), h \in V.$$

■

In other words, the above theorem establishes that under the positivity assumption, the global constraint reduces to local constraints. Consequently, we call such instances of the stable path pruning problem as locally reducible.

G. Optimal Pruning as a Local Set-Cover Problem

With the notation introduced in the previous sections, the stable path pruning problem can be expressed mathematically as follows. Every host $h \in V$, given only its *local view*, solves for a minimal pruned edge set:

$$\begin{aligned} \min_{f_h \in F_h^{prune}} \quad & |\Omega_h| \\ \text{subject to} \quad & G_h^{global} \in \Pi(G). \end{aligned} \tag{2}$$

The above version of the pruning problem has the global constraint $G_h^{global} \in \Pi_h^S(G)$. If the edge-weights are all strictly positive (positivity condition), this global version (Equation (2)) reduces to a local problem:

$$\min_{f_h \in \Pi_h^{\Omega - S}(G_h^{local})} |\Omega_h| \tag{3}$$

Attempting to list out all feasible pruning policies $f_h \in \Pi_h^{\Omega-S}(G_h^{local})$, in general, is computationally intractable. We will show that this problem can be reduced to a set-cover problem. To formulate this set-cover problem, we introduce further notation. Let $\zeta_h : \partial N_h^1 \to 2^{\partial N_h^k}$ denote the covering function: for $i \in \partial N_h^1$ and

$$\zeta_h(i) = \{j \in \partial N_h^k : \exists p \in P_j^{h-local^*} \text{ such that } i = \nu_p^h\}.$$

The corresponding inverse function $\zeta_h^{-1} : \partial N_h^k \to 2^{\partial N_h^1}$ is: for $j \in \partial N_h^k$

$$\zeta_h^{-1}(j) = \{i \in \partial N_h^1 : j \in \zeta_h(i)\}.$$

This function ζ_h can be computed efficiently using any shortest path procedures [42] (see Section V). Then the set-cover problem is

$$\begin{aligned} \min_{\Delta \in 2^{\partial N_h^1}} \quad & |\Delta| \\ \text{subject to} \quad & \cup_{i \in \Delta} \zeta_h(i) = \partial N_h^k. \end{aligned} \tag{4}$$

Theorem 4.5: For any minimizer Δ^* of the problem in Equation (4), $\{(h,i) : i \in \Delta^*\}$ solves the minimal pruning problem of Equation (3).

Proof: Since $\cup_{i \in \Delta} \zeta_h(i) = \partial N_h^k$, $f_h(G_h^{local}) = \{(h,i) : i \in \Delta^*\} \in \Pi_h^{\Omega-S}(G_h^{local})$. ∎

V. Stable Path Topology Control Algorithm

In this section, we present the Stable Path Topology Control (SPTC) algorithm that solves the set-cover problem (to an approximation) in Equation (4) introduced in Subsection IV-G. Finally, we demonstrate the performance of the SPTC algorithm by using the ETX metric (see Subsection II-B).

A. Computing ζ_h

Algorithm 1 computes the covering function ζ_h used in the set-cover formulation (Equation (4)). The local view G_h^{local} is input to the algorithm and it outputs ζ_h. Given G_h^{local}, the function `computeAllPairSPFloydWarshall`, used in Algorithm 1, computes the all pair shortest paths (h-locally optimal paths) in the exclusive neighborhood $N_h^{k^-}$, using the well-know *Floyd-Warshall* algorithm [42]. It returns a function $SP_{N_h^{k-}}$ that yields the shortest (h-locally optimal) path metrics: for $i, j \in N_h^{k^-}$

$$SP_{N_h^{k-}}(i,j) = w^*_{h-local}(i,j)$$

Algorithm 1 Compute covering function ζ_h at $h \in V$

INPUT: G_h^{local}

//Compute all-pair-shortest paths in exclusive neighborhood
$SP_{N_h^{k-}} \leftarrow$ `computeAllPairSPFloydWarshall`(G_h^{local});

//Vertex expansion
for all $j \in \partial N_h^k$ **do**
  $\zeta_h^{-1}(j) \leftarrow arg \min_{i \in \partial N_h^1} a(h,i) + SP_{N_h^{k-}}(i,j)$;
end for
OUTPUT: ζ_h

B. Greedy Approximation Algorithm to Solve Set-Cover Problem

Given ζ_h, Algorithm 2 is a greedy algorithm that approximately solves Equation (4).

Algorithm 2 Greedy Set-Cover Algorithm at h

INPUT: $\zeta_h, G_h, \partial N_h^1, \partial N_h^k$
Init: $R_{h-greedy} \leftarrow \emptyset$, $U \leftarrow \partial N_h^k$;

// Find and append essential cover elements
for all $\{j \in \partial N_h^k : |\zeta_h^{-1}(j)| = 1 \}$ **do**
  $R_{h-greedy} \leftarrow R_{h-greedy} \cup \zeta_h^{-1}(j)$;
  $U \leftarrow U \backslash \{j\}$;
end for

// Greedy selection
while $U \neq \emptyset$ **do**
  $i^* \leftarrow \arg\max_{i \in \partial N_h^1} |\{j \in U : j \in \zeta_h(i)\}|$
  $R_{h-greedy} \leftarrow R_{h-greedy} \cup \{i^*\}$
  $U \leftarrow U \backslash \{j \in U : i^* \in \zeta_h(j)\}$
end while
Output: $R_{h-greedy}$

Let $d_h^* = \max_{i \in \partial N_h^1} |\zeta_h(j)|$. Then the following Lemma gives the approximation bounds for the greedy solution $R_{h-greedy}$:

Lemma 5.1: Let the optimal solution to Equation (4) be Δ_h^* and $R_{h-greedy}$ be the output of Algorithm 2 at host h, then $|R_{h-greedy}| \leq H(d^*)|\Delta_h^*|$, where $H(N) = \sum_{n=1}^{N} \frac{1}{n}$.

Group	Parameter	Value
MAC and PHY	Protocol	802.11b
	Transmission Rate	11 Mbps
	Transmit Power	5 mW
	Receiver Sensitivity	-95 dBm
	Error Correction Capabilities	None
Routing and TC	Protocol	OLSR-ETX or SPTC-ETX
	HELLO message interval	2 s
	Neighbor hold time	32 s
	TC message interval	5 s
ETX Computation	ETX Memory Length	32 s
	ETX Memory Interval	2 s
	ETX Hello Timeout Expiry	2.5 s
Traffic	Type	UDP CBR
	Packet length	1024 bits

TABLE I: Parameters for simulation

This lemma is proved in Chapter 11 of [42]. We call the Algorithms 1 and 2 together as the SPTC algorithm.

C. Simulation Setup

All simulations were carried out in OPNET Modeler 14.5 [7]. For the simulations, the mobile node model *manet station advanced* was chosen. The parameters given in Table I were used in the simulations.

We modified the default code for the OLSR model, which is an OLSR version 1 [11] implementation. We made suitable modifications to the neighbor discovery mechanism, as per [13], to compute the ETX metric online. We also modified the MPR selection algorithms to implement the SPTC algorithm. For the simulations, we used $k = 2$, size of the neighborhood.

To study the performance of the SPTC algorithm, we compare it with the OLSR implementation in [13], which uses the ETX metric to select MPRs using set-cover methods (illustrated in Subsection II-C). We implemented both the OLSR and the SPTC algorithm to use the ETX metric. We call these two implementations, OLSR-ETX and SPTC-ETX respectively.

In the simulations, we compared both the data traffic carrying and Topology Control (TC) overhead performance of SPTC-ETX and OLSR-ETX. For the data traffic carrying performance, we studied the *carried load* for various *offered loads*. We set up a *UDP* traffic generator that sends a *Constant Bit Rate* (CBR) traffic between pairs of stations. We then swept across this CBR rate to study the traffic performance with OLSR-ETX and SPTC-ETX.

In link state mechanisms such as OLSR, the TC broadcast mechanism is proactive, and consequently not all TC messages broadcast correspond to topology changes. To study the overhead due to topology changes, we measured the rate of reactive TC messages and the total number of actual topology changes. Reactive TC messages are those that are generated due to changes in the selected topology. This is a good estimate of the actual topology overhead for the pruned network. We will compare this topology control overhead for SPTC-ETX against that of OLSR-ETX.

D. Scenario Illustrating CDS Limitations

Before we present the results for complicated topologies, we will study the performance of SPTC-ETX and OLSR-ETX for a simple topology shown in Figure 7. This scenario corresponds to the example topology in Subsection II-C that illustrates the fundamental limitation of CDS constructions. The topology is set up such that there is a long-distance unstable wireless link between *manet_0* and *manet_1*. All other links are short and hence more stable compared to *(manet_0,manet_1)*.

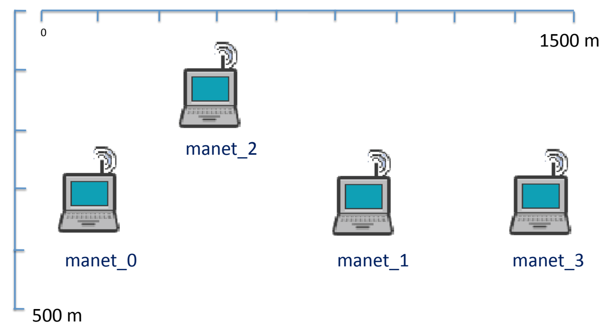


Fig. 7: 4 node topology to illustrate the limitation of CDS constructions

Consider OLSR-ETX's MPR selection process at node *manet_0*. Since the link *(manet_0,manet_1)* is unstable, it goes ON and OFF frequently. Whenever this link is ON, *manet_3* $\in \partial N^2_{manet\_0}$. The OLSR-ETX's set-cover construction, selects the unstable link *(manet_0,manet_1)* as Ω_{manet_0} because it the only edge in the two-hop path to reach *manet_3*. When the link *(manet_0,manet_1)* is OFF, *manet_1* $\in \partial N^2_{manet\_0}$, and consequently, OLSR-ETX chooses *(manet_0,manet_2)* as Ω_{manet_0}. Thus as the unstable link *(manet_0,manet_1)* goes ON and OFF, Ω_{manet_0} oscillates between *(manet_0,manet_1)* and *(manet_0,manet_2)*. This is illustrated in Figure 8, which shows a realization of the topology selection process at *manet_0* obtained by OPNET simulation.

Fundamental limitation of the set-cover OLSR construction is that it is not designed to exploit the local path diversity. This limitation is overcome by the SPTC-ETX that provides a more stable Ω_{manet_0}. From simulations we observed that SPTC-ETX almost always chooses $\Omega_{manet\_0} = \{$*manet_2*$\}$ and $\Omega_{manet\_2} = \{$*manet_1*$\}$, thus preserving the stable path *(manet_0,manet_2,manet_1)*. For a simulation period of 1 *hour*, we observed 96 topology changes for OLSR-ETX and 6 for SPTC-ETX.

E. Static Grid Topology

The next topology that we consider is a 100-node static grid topology shown in Figure 9. The network consists of many stable and unstable links. This topology suffers from the same

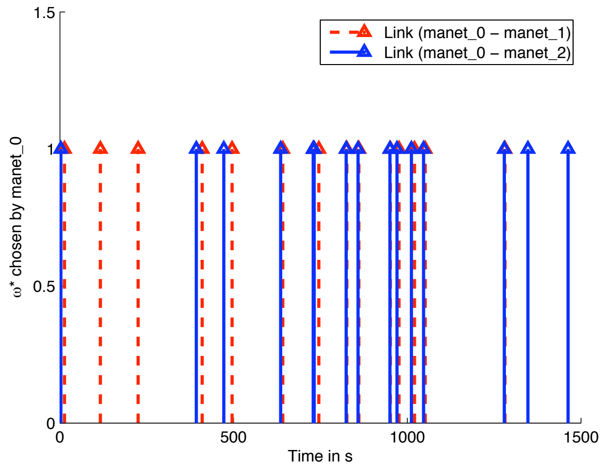


Fig. 8: Topology selection process at manet_0 for OLSR-ETX

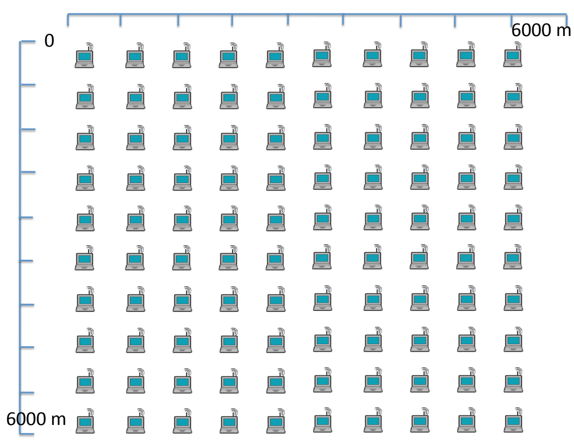


Fig. 9: 100-node grid network

problem explained in Subsection V-D. The one-hop neighbors that are far off (in physical distance) typically cover more two-hop neighbors. However, by the nature of radio propagation, these links are unstable.

The UDP CBR traffic is sent between 5 different random source-destination pairs. The comparison of the traffic-carrying performance of SPTC-ETX and OLSR-ETX is shown in Fig. 10. The simulation results indicate that SPTC-ETX has a saturation capacity of $86kbps$, while that of OLSR-ETX is $75kbps$.

The average number of total topology changes (for many runs of the simulation) was 11254 and 8280 for OLSR-ETX and SPTC-ETX respectively. The corresponding rate of reactive TC messages was $930bps$ and $681bps$ respectively. This implies that the pruned subnetwork of SPTC-ETX is stable/long lived compared to that of OLSR-ETX.

F. Random Waypoint Mobility Scenario

Random waypoint mobility pattern is a commonly used to study protocol performances in a mobile environment [25].

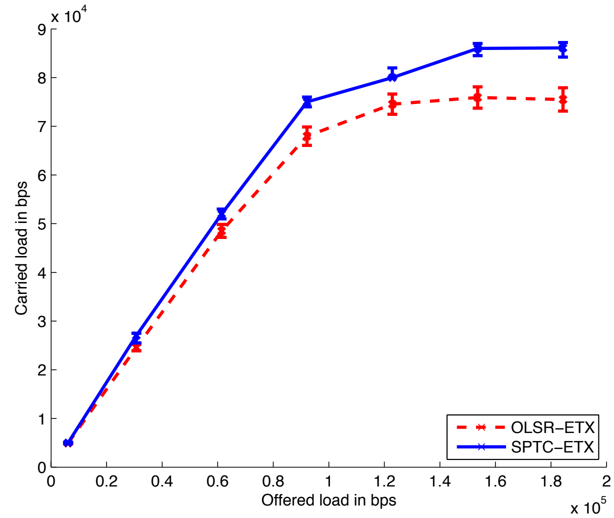


Fig. 10: Carried load vs. Offered load for 100-node grid shown with 95% confidence intervals

Parameter	Value
No of stations	25
Simulation Area	3000m $\times$ 3000m
Speed	(5,20] m/s
Pause time	0 s

TABLE II: Random waypoint mobility parameters

The mobility parameters that we used for the random waypoint mobility pattern are shown in Table II. All statistics were collected once the simulations reached stochastic stationarity.

Again, UDP CBR traffic is sent between two different random source-destination pairs. The sample mean of the carried load as a function of the offered load is shown in Fig. 11. We observe that SPTC-ETX is capable of carrying 13% more load than OLSR-ETX. This is because in OLSR, we observed that significantly more traffic is routed through unstable links.

The average number of topology changes was 80124 and 60874 in one hour of simulation time for OLSR-ETX and SPTC-ETX respectively. The corresponding rate of reactive TC messages was $8.3kpbs$ and $6kpbs$ respectively.

G. Battlefield Scenario

Finally, we consider a battlefield scenario, introduced in [47], with an initial topology as shown in Sub-figure 12a. It comprises of 3 platoons of stations: Platoon A consists of nodes 0 to 9, Platoon B consists of nodes 10 to 19, and Platoon C consists of nodes 20 to 29. The three platoons move in the trajectories shown in Sub-figure 12b: Platoon B moves forward along the x direction, and Platoons A and C move forward and away from platoon B at speed of 1.5 m/s in the y direction. Then the platoons move together back to the initial formation. To ensure better connectivity among the platoons, two supporting nodes 30 (to support connections between Platoon A and B) and 31 (to support connections between Platoon B and C) move alongside the platoons (in

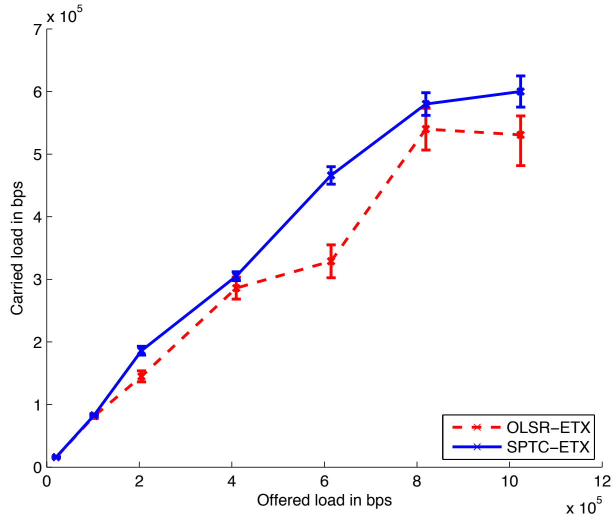


Fig. 11: Carried load vs. Offered load of Random Waypoint shown with 95 % confidence intervals

Type	Source-Destination	Offered Load (*kbps*)
Intra-Platoon	(1,3),(2,9),(4,6),(7,5),(20,29), (14,17),(16,11),(17,18),(19,12), (21,22),(23,27),(23,28)	12
Inter-Platoon	(1,18) (20,11),(20,0) (10,1),(21,10)	2.4 6 12

TABLE III: Traffic connections for Battlefield scenario

the x direction). The simulations were carried out with the parameters shown in Table I. This yields a radio range of approximately 900m. Hence within each platoon, all the nodes are at most two hops from each other. When the platoons are close together, the inter-platoon communication is stable without using the supporting nodes 30 and 31. However, when the platoons move away from each other, the direct inter-platoon connections become unstable and the supporting stations become necessary for delivering high traffic. Again for SPTC, we chose the *current age* as link stability metric (this is only a heuristic).

UDP traffic was sent between 17 source-destinations pairs. Table III shows the base traffic for the scenario. For the traffic analysis, we focus on the connection $(20, 0)$ (from Platoon C to A) because this is a long connection and would be potentially sensitive to path stability. We scale the base traffic (offered load) of all connections (in Table III) by the same factor and obtain the carried load vs. offered load performance for connection $(20, 0)$ shown in Fig. 13. Again, we observe that SPTC carries significantly more load than OLSR for this connection. This is because when the platoons are maximally apart, we observe that for connection $(20, 0)$, SPTC-ETX routes significantly more traffic (about 1.5 times more) through the supporting nodes 30 and 31 when compared to OLSR-ETX's routing mechanism. We observe that the carried load for the other connections is also higher. Thus the overall network throughput is improved. For example, when the offered load to the network (all connections) was $2Mbps$, SPTC-ETX was able to carry $923kbps$, while OLSR-ETX is able to carry only $890kpbs$. Figure 13 compares the traffic carrying performance for the long connection $(20, 0$ for SPTC-ETX and OLSR-ETX.

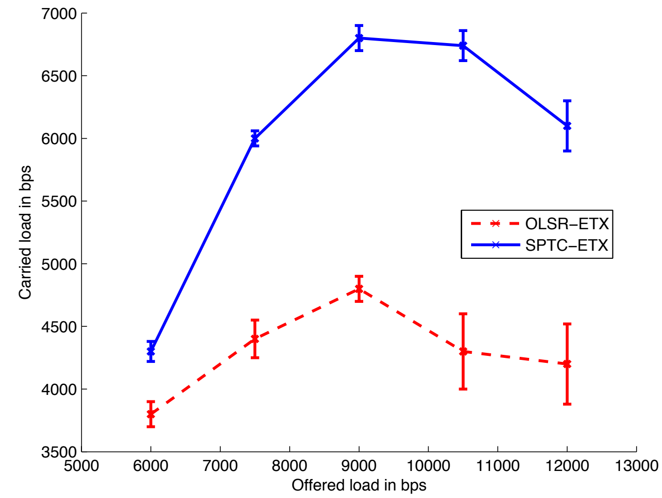


Fig. 13: Carried load vs. Offered load for the longest connection of Battlefield Scenario shown with 95 % confidence intervals

In the TC study, we observed that the average number of topology changes was 12266 and 5360 changes in one hour of simulation time for OLSR-ETX and SPTC-ETX respectively. The corresponding rate of reactive TC messages was $884bps$ and $338bps$ respectively.

VI. Conclusion

In this paper, we introduced a new topology control problem for preserving stable routing paths. We formulated the problem as a constrained multi-agent optimization problem with only local neighborhood information. We established necessary and sufficient conditions that reduce the global pruning constraint to a local constraint on the pruning policies. We presented the SPTC algorithm that approximately solves the stable path topology control problem. Finally, we quantified the two-fold advantage of SPTC with different simulation scenarios. By using the popular ETX metric, we showed that the topology formed by SPTC-ETX is stable and is able to carry significantly higher traffic compared to OLSR-ETX.

References

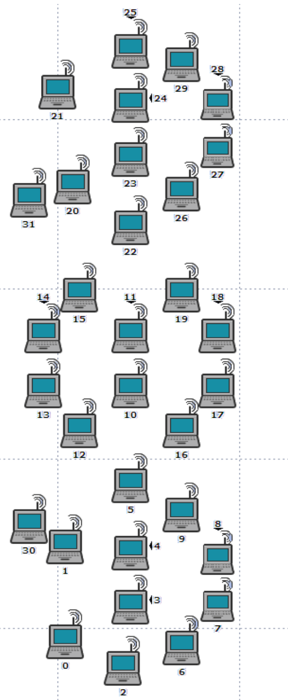
[1] R. Rajaraman, "Topology control and routing in ad hoc networks: a survey," *ACM SIGACT News*, vol. 33, no. 2, pp. 60–73, 2002.

[2] B. Williams and T. Camp, "Comparison of broadcasting techniques for mobile ad-hoc networks," in *Proceedings of the ACM International Symposium on Mobile Ad-Hoc Networking and Computing (MOBIHOC)*, 2002, pp. 194–205.

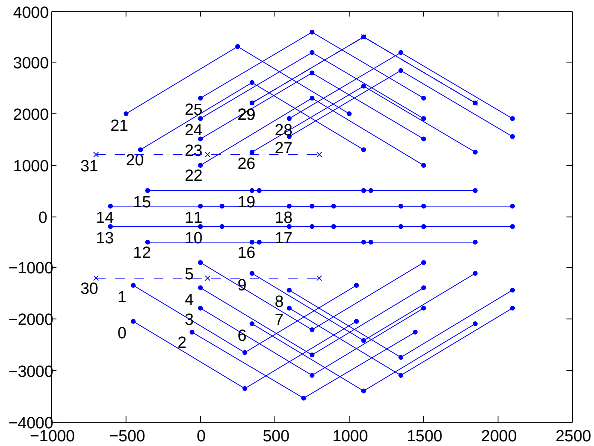
[3] S. Obilisetty, A. Jasti, and R. Pendse, "Link stability based enhancements to olsr (ls-olsr)," in *IEEE 62nd Vehicular Technology Conference*, 2005, pp. 306– 310.

[4] Z. Cheng and W. B. Heinzelman, "Exploring long lifetime routing (llr) in ad hoc networks," in *International workshop on Modeling Analysis and Simulation of Wireless and Mobile Systems*, 2004, pp. 203–210.

[5] N. Sarma and S. Nandi, "Route stability based qos routing in mobile ad hoc networks," *Wireless Personal Communications*, 2009.



(a) Initial topology



(b) Node trajectories

Fig. 12: Battlefield Scenario

[6] K. Sridhar and M. Chan, "Stabilty and hop-count based approach for route computation in manet," in *International Conference on Computer Communications and Networks*, 2005, pp. 25–31.

[7] OPNETTechnologies, "Opnet modeler," URL. [Online]. Available: http://www.opnet.com/solutions/network_rd/modeler.html

[8] C. E. Perkins, *Ad hoc networking*. Addison Wesley, 2001.

[9] R. Ogier, F. Templin, and M. Lewis, "Topology dissemination based on reverse-paht forwarding," RFC, February 2004. [Online]. Available: http://www.ietf.org/rfc/rfc3684.txt

[10] S. Nelakuditi, S. Lee, Y. Yu, J. Wang, Z. Zhong, G. H. Lu, and Z. L. Zhang, "Blacklist-aided forwarding in static multihop wireless networks," in *IEEE Conference on Sensor and Ad Hoc Communications and Networks*, 2005, pp. 252–262.

[11] T. Clausen and P. Jacquet, "Optimized link state routing protocol (olsr)," RFC, Oct 2003. [Online]. Available: http://www.ietf.org/rfc/rfc3626.txt

[12] T. Clausen, C. Dearlove, and P. Jacquet, "The optimized link state routing protocol version 2," Draft-IETF, September 2009. [Online]. Available: http://www.ietf.org/id/draft-ietf-manet-olsrv2-10.txt

[13] H. Rogge, E. Baccelli, and A. Kaplan, "Packet sequence number based etx metric for mobile ad hoc networks," IETF Draft, December 2009. [Online]. Available: http://www.ietf.org/id/draft-funkfeuer-manet-olsrv2-etx-00.txt

[14] J. Wu and F. Dai, "A generic distirbuted broadcast scheme in ad hoc wireless networks," *IEEE Transactions of Computers*, vol. 53, no. 10, pp. 1343–1354, 2004.

[15] T. Clausen, C. Dearlove, and J. Dean, "Mobile ad hoc network (manet) neighborhood discovery protocol (nhdp)," Draft-IETF, October 2009. [Online]. Available: http://www.ietf.org/id/draft-ietf-manet-nhdp-11.txt

[16] A. Qayyum, L. Viennot, and A. Laouiti, "Multipoint relaying for flooding broadcast messages in mobile wireless networks," in *35th Annual Hawaii International Conference on System Sciences (HICSS'2001)*, 2001, pp. 3866–3875.

[17] P. Jacquet, A. Laouiti, P. Minet, and L. Viennot, "Performance analysis of olsr multipoint relay flooding in two ad hoc wireless network models," INRIA, Tech. Rep., September 2001.

[18] W. Su, S.-J. Lee, and M. Gerla, "Mobility prediction in wireless networks," in *MILCOM*, vol. 1, 2000, pp. 491–495.

[19] M. Gerharz, C. d. Waal, M. Frank, and P. Martini, "Link stability in mobile wireless ad hoc networks," in *Proceedings of the 27th Annual IEEE Conference on Local Computer Networks*, 2002, pp. 30–42.

[20] O. A. Al-Amoudi, M. S. El-Azhari, M. Woodward, and I. Awan, "Performance analysis of angle routing in manets," in *Lecture Notes in Computer Science*, 2008, pp. 21–29.

[21] D. S. J. D. Couto, D. Aguayo, B. A. Chambers, and R. Morris, "Performance of multihop wireless networks: Shortest path is not enough," *ACM SIGCOMM Computer Communication Review*, vol. 33, pp. 83–88, 2003.

[22] C.-K. Toh, "Associativity-based routing for ad hoc networks," *Wireless Personal Communications*, vol. 4, pp. 103–139, 1997.

[23] R. Dube, C. D. Rais, K.-Y. Wang, and S. K. Tripathi, "Signal stability-based adaptive routing (ssa) for ad hoc mobile networks," *IEEE Personal Communication*, vol. 4, no. 1, pp. 36–45, 1997.

[24] S. Agarwal, A. Ahuja, J. P. Singh, and R. Shorey, "Route-lifetime assessment based routing (rabr) protocol for mobile ad-hoc networks," in *IEEE International Conference on Communications*, vol. 3, 2000, pp. 1697–1701.

[25] T. Camp, J. Boleng, and V. Davies, "A survey of mobility models for ad hoc network research," *Wireless Communications and Mobile Computing: Special Issues on Mobile Ad Hoc Networking: Research, Trends and Applications*, vol. 2, no. 5, pp. 483–502, 2002.

[26] N. Sadagopan, F. Bai, B. Krishnamachari, and A. Helmy, "Paths: analysis of path duration statistics and their impact on reactive manet routing protocols," in *ACM Internation Symposium on Mobile Ad Hoc Networking and Computing*, 2003, pp. 245–256.

[27] Y. Han, R. J. La, A. M. Makowski, and S. Lee, "Distribution of path durations in mobile ad hoc networks: Palm's theorem to the rescue," *The International Journal of Computer and Telecommunications Networking:*

Special Issue on Network Modeling and Simulation, vol. 50, pp. 1887–1900, 2006.
[28] C. E. Koksal and H. Balakrishnan, "Quality-aware routing metrics for time-varying wireless mesh networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 11, pp. 1984–1994, 2006.
[29] D. S. J. D. Couto, D. Aguayo, J. Bicket, and R. Morris, "A high-throughput path metric for multi-hop wireless routing," in *9th Internation Conference on Mobile Computing and Networking*, 2003, pp. 134–146.
[30] D. S. J. D. Couto, "High-throughput routing for mullti-hop wireless networks," Ph.D. dissertation, MIT, 2004.
[31] www.olsr.org, "http://www.olsr.org/docs/readme-link-quality.html."
[32] D. Passos, D. V. Teixeira, D. C. Muchaluat-Saade, L. C. S. Magalhaes, and C. V. N. de Albuquerque, "Mesh netwok performance measurements," in *International Information and Telecommunications Technologies Symposium*, 2006.
[33] R. Draves, J. Padhye, and B. Zill, "Routing in multi-radio, multi-radio wireless mesh networks," in *ACM International Conference on Mobile Computing and Network*, 2004, pp. 114–128.
[34] Y. Yang, J. Wang, and R. Kravets, "Designing routing metrics for mesh networks," in *IEEE Workshop on Wireless Mesh Networks (WiMesh)*, 2005.
[35] A. P. Subramanian, M. M. Buddhikot, and S. C. Miller, "Interference aware routing in multi-radio wireless mesh networks," in *IEEE Workshop on Wireless Mesh Networks (WiMesh)*, 2006, pp. 55–63.
[36] G. Lim, K. Shin, S. Lee, H. Yoon, and J. S. Ma, "Link stability and route lifetime in ad-hoc wireless networks," in *Proceedings of the International Conference on Parallel Processing Workshops*, 2002, p. 116.
[37] P. Yang and B. Huang, "Qos routing based on link stability with dynamic delay prediction in manet," in *IEEE Pacific-Asia Workshop on Computational Intelligence and Industrial Application*, vol. 1, 2008, pp. 515–518.
[38] D. Johnson and Y. Hu, "The dynamic source routing protocol (dsr) for mobile ad hoc networks for ipv4," RFC, Feb 2007. [Online]. Available: http://www.ietf.org/rfc/rfc4728.txt
[39] P. C., "Ad hoc on-demand distance vector (aodv) routing," RFC 3561, July 2003. [Online]. Available: http://www.ietf.org/rfc/rfc3561.txt
[40] T. Aure and F. Y. Li, "Load-balancing in multi-homed olsr networks using airtime metric and radio-aware path selection: implementation and testing," White paper, 2008.
[41] K. Kowalik, B. Keegan, and M. Davis, "Making olsr aware of resources," in *International Conference on Wireless Communications, Networking and Mobile Computing*, 2007, pp. 1488–1493.
[42] J. Kleinberg and É. Tardos, *Algorithm Design*. Addison Wesley, 2005.
[43] M. Ehrgott, *Multicriteria Optimization*. Springer, 2005.
[44] S. Lee, B. Bhattacharjee, A. Srinivasan, and S. Khuller, "Efficient and resilient backbones for multihop wireless networks," *IEEE Transactions on Mobile Computing*, vol. 7, no. 11, pp. 1349–1362, 2008.
[45] K. K. Somasundaram and J. S. Baras, "Semiring pruning for information dissemination in mobile ad hoc networks," in *Workshop on Applications of Graph Theory in Wireless Ad hoc Networks and Sensor Networks*, 2009.
[46] J. Moy, "Ospf version 2," RFC 2328. [Online]. Available: http://www.ietf.org/rfc/rfc2328.txt
[47] J. S. Baras, S. Perumal, V. Tabatabaee, K. Somasundaram, and P. Purkayastha, "Loss network models and multiple metric performance sensitivity analysis for mobile wireless multi-hop networks," in *4th Wireless Internet Conference*, 2008.