ABSTRACT

Title of Document: MATHEMATICS TEACHERS' INTERPRETATIONS OF MESSAGES IN CURRICULAR RESOURCES AND THE RELATIONS OF THESE INTERPRETATIONS TO THEIR BELIEFS AND PRACTICES

Christy Danko Graybeal, Ph.D., 2008

Directed By: Dr. James T. Fey and
Dr. Anna O. Graeber,
Department of Curriculum and Instruction
University of Maryland, College Park

The curricular materials that teachers use, the assessments that teachers are required to administer, and the professional development activities in which teachers engage all contain messages about mathematics and mathematics teaching. The recent emphasis on both reform-oriented teaching and high-stakes testing in mathematics has increased the number and intensity of competing and conflicting messages. This qualitative study used survey, observation, and interview research methods to explore the messages that five experienced, elementary certified, middle school mathematics teachers interpreted from a variety of resources and the ways that those interpretations related to their beliefs and practices.

The teachers in this study interpreted messages in eleven themes. Four themes—*Concepts and Procedures*, *Question types*, *Source of solution methods*, and *Technology*—

created the most tension for the teachers. In general, when the teachers agreed with messages from professional resources about mathematics curriculum and teaching, they attempted to reflect those messages in their practice. However, the resources often lacked supports necessary for the teachers to follow through with the messages in their practice. When the teachers disagreed with particular messages they sometimes consciously decided to not reflect those messages in their practice. But usually the messages were so pervasive that the teachers were not able to ignore them. At times they felt obliged to reflect all of the messages in their practice, regardless of their personal beliefs. The amount of support that the resources provided for teachers was a strong indicator of the degree to which the teachers were successful in reflecting the messages in their practice. Frequently the resources only superficially presented messages to the teachers. This phenomenon was especially apparent when the messages were reform-oriented messages.

The study suggests that curriculum and policy writers need to consider the consistency of their messages, be more specific about their intentions, and provide more support to teachers as they try to translate recommendations into practice. Additionally, teacher educators and providers of professional development need to help teachers learn to critically examine curricular resources so that they can more consciously make decisions about to which messages they will attend.

MATHEMATICS TEACHERS' INTERPRETATIONS OF MESSAGES IN CURRICULAR RESOURCES AND THE RELATIONS OF THESE INTERPRETATIONS TO THEIR BELIEFS AND PRACTICES

By

Christy Danko Graybeal

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2008

Advisory Committee:
Professor James Fey, Chair
Professor Anna Graeber, Co-Chair
Professor Daniel Chazan
Professor Ann Edwards
Professor Steven Selden

Dedication

This work is dedicated to my husband, Ben, for his love, patience, and help.

Acknowledgements

Many people have contributed to this dissertation. Jim Fey and Anna Graeber have been outstanding mentors. They provided me with guidance over my years in the doctoral program and great help on this dissertation. I am also very grateful to Dan Chazan, Dara Sandow, and Eden Badertscher for their patience with me as I struggled to clarify my thoughts in our seminar meetings and their willingness to read draft chapters of this dissertation. Their questions and comments greatly shaped the direction of this research. I am indebted to the Mid-Atlantic Center for Mathematics Teaching and Learning for providing me the opportunity to study mathematics education on a fulltime basis. I am most thankful for the five teachers in this study. They allowed me into their classrooms and were generous with their time. I hope that this research gives voice to their perspectives and experiences and leads to improvements in the supports that they receive from policy makers, curriculum writers, teacher educators, and professional developers.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

AAAS	American Association for the Advancement of Science
AYP	Adequate Yearly Progress
HSA	(Maryland) High School Assessment
MSA	Maryland School Assessment
MSDE	Maryland State Department of Education
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
NSF	National Science Foundation
RTOP	Reformed Teaching Observation Protocol
VSC	(Maryland) Voluntary State Curriculum

Chapter 1: Introduction

Despite the many constraints placed upon them, teachers have significant influence over what and how they teach (Cohen & Hill, 2000; Cuban, 1995). Mathematics teachers who are using the same curricular materials can enact them in dramatically different ways and afford their students very different experiences (Chval, Grouws, Smith, & Ziebarth, 2006; Chávez-López, 2003; Kilpatrick, 2003; Remillard, 1996; Schwille et al., 1982). When making decisions about what and how to teach, among other things, teachers draw upon curricular materials and their own personal beliefs, experiences, and knowledge (Drake & Sherin, 2006; Remillard, 2005; Tarr, Chávez, Reys, & Reys, 2006). Remillard (1996) refers to “the range of tools (personal, collegial, published, structural) that teachers bring to, and draw on in their teaching” as *resources* (p. 90).

Resources such as the curricular materials that teachers use, the assessments that teachers are required to give to their students, and the professional development that teachers attend contain messages about mathematics and mathematics teaching. Frequently teachers interpret these messages to be competing and/or conflicting messages. It is unlikely that there has ever been or ever will be perfect alignment among the messages present in resources, however, it seems that the recent emphasis on both reform-oriented teaching (American Association for the Advancement of Science [AAAS], 1989, 1993; National Council of Teachers of Mathematics [NCTM], 1989, 1991, 1995, 2000, 2006; National Research Council [NRC], 2001) and high-stakes testing (No Child Left Behind Act, 2001) has increased the number and intensity of competing and conflicting messages. One possible reason for *why* it is that teachers can use the same resources in such a wide variety of ways is that each teacher is focusing on different messages and perhaps understanding the messages differently.

Since teachers play such a significant role in the mathematics education of students, the entire mathematics education community needs to support teachers in their efforts to reform mathematics education and ensure that all children succeed in learning mathematics. In order to better help teachers, curriculum and policy writers and professional developers need to first become more aware of how teachers are interpreting the messages they present in curricular materials, assessments, and professional development. All too often, messages are not interpreted by teachers as they are intended. Furthermore, competing and conflicting messages within resources are often interpreted by teachers and frequently it does not seem that writers and professional developers are aware of these incongruencies. Thus, curriculum and policy writers and professional developers need to learn how teachers are currently interpreting their resources and use this knowledge to modify these resources to better support teachers in their enactment of the intended messages.

Additionally, little is known about how teachers interpret messages from a variety of resources such as curricular materials, assessments, and professional development; almost all research on teachers' interpretations of resources has involved interpretation of only a single resource at a time (e.g., Berk, 2004). Just as there are competing and conflicting messages within resources, there are often competing and conflicting messages across resources. Curriculum and policy writers and professional developers need to become more aware of the messages that teachers interpret from various resources and consider how all of these messages fit together.

Furthermore, it has been hypothesized that in addition to influencing teachers' actions in the classroom (Aguirre & Speer, 2000; Thompson, 1992), teachers' beliefs influence what and how messages are interpreted (Spillane et al., 2002). But, especially when a variety of resources are being interpreted, it does not seem that there is a simple relation between beliefs and message interpretations. Thus, more research into the relation of messages, teachers' beliefs, and teachers' practices has been recommended.

In order to address these issues, this study explored the messages that five experienced, elementary certified, middle school mathematics teachers interpreted from curricular resources and the ways in which these messages related to their beliefs and practices. Learning more about what messages teachers interpret from resources and how these messages relate to teachers' beliefs and practices is an essential step toward supporting teachers in their efforts to improve their teaching.

Rationale

This study focused on the messages experienced, elementary certified, middle school mathematics teachers who were enrolled in a master's degree program focusing on mathematics education interpreted from the resources available to them. The relations between these messages and teachers' beliefs and practices were also examined. Below I give a rationale for these foci.

Rationale for focus on these teachers

This study focused on experienced, elementary certified, middle school mathematics teachers. This population was chosen because much of the research on teachers' beliefs and teaching has been conducted with preservice (Ball, 1990; Civil, 1990; Cooney, Shealy, & Arvold, 1998; Lerman, 1990; Raymond, 1997; Selden & Selden, 1997) elementary school (Ambrose, 2004; Ernest, 1988; Ng & Rao, 2003; Remillard & Bryans, 2004; Stipek, Givvin, Salmon, & MacGyvers, 2001) or high school (Andrews & Hatch, 1999; Cooney, 1985; Cooney et al., 1998) teachers.

Minimal research has focused on the beliefs and teaching of experienced, middle school, mathematics teachers. Experience plays an important role in shaping beliefs, and it has been found that preservice teachers hold different beliefs than inservice teachers (Weizman & Hoz, 2006). Moreover, because middle school teachers often teach only mathematics, their beliefs about the nature of mathematics and the ways in which they draw on resources are likely to be different than those of elementary school teachers who teach several content areas. Additionally, because middle school mathematics teachers are often certified in elementary education (grades 1-8) many have taken only the mathematics courses required to become certified in elementary education. Despite this lack of coursework, middle school mathematics teachers are often called upon to teach sections of high school Algebra I and high school Geometry. Thus their beliefs and interpretations of resources are likely to be different than those of high school teachers teaching the same courses.

Additionally, the teachers in this study were enrolled in a master's degree program focusing on mathematics education. As part of the master's degree program, the teachers studied the curricular materials provided to them by their school district and also other curricular options. As a consequence, some of the tensions they felt between the messages in different resources may have been exacerbated. This is especially important because, despite recommendations (Thompson, 1992), there have been very few studies of teachers with an informed philosophical perspective of mathematics.

Rationale for focus on messages represented in resources

Whether they are acknowledged or not, resources such as textbooks, curriculum guides, assessments, and professional development programs present messages about what is most important for students to learn and how students can best learn this (Goldenberg, 1999; Hill, 2001; Spillane et al., 2002). Messages are usually not explicit, but they can normally be rephrased as statements that begin with "Teachers should…"

Although there are often competing and/or conflicting messages both within and among resources, few studies have examined how teachers respond to these incongruencies. Additionally, messages often conflict with the teachers' own beliefs about mathematics and mathematics teaching (Smith, 1996). Bachkirova (2003) found that teachers whose personal values did not align with those of educational authorities (such as curriculum and assessment writers and professional development leaders) had higher levels of stress than those who had alignment. Similarly, Chávez-López (2003) found that congruence between a teacher's views of mathematics and mathematics teaching and the views presented in curricular resources resulted in a more positive attitude toward the curriculum. Neither study, however, focused on how teachers make sense of or respond to conflicting messages.

For these and other reasons, further study of teachers' interpretations of and responses to resources, especially with regard to teachers' beliefs, was recommended by Remillard (2005), Tarr et al. (2006), and Thompson (1992). Similarly, one of the NRC's (2002) four key questions to guide inquiry into the magnitude and direction of the influence of standards on the education system was "How are nationally developed standards being received and interpreted?" (p. 5).

Rationale for focus on beliefs held by teachers

A rationale for the study of mathematics teachers' beliefs and conceptions about the nature of mathematics was eloquently summarized by Hersh (1998): "One's conception of what mathematics *is* affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it…The issue, then, is not, What is the best way to teach? But, What is mathematics really all about?" (p. 13).

Furthermore, in her landmark piece, Thompson (1984) concluded that "teachers' beliefs, views, and preferences about mathematics and its teaching, regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle role, in the shaping of characteristic patterns of instructional behavior" (pp. 124-125). She added that "any attempt to improve the quality of mathematics teaching must begin with an understanding of the conceptions held by the teachers and how these are related to their instructional practice" (p. 106).

The importance of beliefs with regard to practice was also supported by Lerman (1983), Ng and Rao (2003), Schoenfeld (1992), Steiner (1987), Stipek et al. (2001), Thompson (1984), and Tymoczko (1998). Ernest (1989b) also supported the study of beliefs by pointing out that while knowledge is important, teachers with similar knowledge may teach in very different ways because of their different beliefs. Pajares (1992) goes as far as to state that "knowledge and beliefs are inextricably intertwined, but the potent affective, evaluative, and episodic nature of beliefs makes them a filter through which new phenomena are interpreted" (p. 325). Thus, while knowledge is indisputably important, this study focused on teachers' beliefs.

Additionally, since the relationship between teachers' beliefs and practices is more complex than expected (Andrews & Hatch, 1999; Weizman & Hoz, 2006), further research into this area has been recommended by many educational researchers (Lerman, 1983; Pajares, 1992; Richardson, 1996; Schoenfeld, 1992; Thompson, 1992; Weizman & Hoz, 2006). This study hopes to shed some light on this relationship by examining the relations between messages and beliefs and between messages and practices.

Research Questions

In order to address the above issues, this research study sought answers to the following questions:[1]

- What messages do elementary certified middle school mathematics teachers interpret[2] from curricular resources?

[1] I set out on this study with slightly different research questions. As is often the case in qualitative research, during data collection and data analysis, new questions emerged and I elected to shift my focus to these questions.

[2] Here *interpret* is used as defined in Webster's New World Dictionary: "to have or show one's own understanding of the meaning of" or "to bring out the meaning of; esp., to give one's own conception of (a work of art), as in performance or criticism" (Neufeldt & Guralnik, 1988, p. 706).

- How do these messages relate to the teachers' beliefs and observed classroom practices?[3]

Overview of Research Design

In order to address the questions listed above, a qualitative research study was conducted. Teachers were surveyed about their beliefs about mathematics and mathematics teaching using the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004 – see Appendix A for a copy of the inventory). To learn about their classroom practices, the teachers were observed teaching in their middle school classrooms. Observation notes were taken and Sawada et al.'s (2000) *Reformed Teaching Observation Protocol* (RTOP) was used to assess the degree to which each lesson embodied the recommendations and standards of reform-oriented teaching (see Appendix B for a copy of the RTOP). In order to learn more about how the teachers interpreted the messages in the curricular resources, the teachers were interviewed. They were asked about their teaching background and how they typically plan lessons. Additionally, for each of the curricular resources (their students' textbooks, school district's curriculum guides and assessments, state's assessments and curriculum framework, the master's degree program in which they were enrolled, and other resources which the teachers felt were significant influences on their teaching) the teachers were asked to talk about the messages they see in the resources and how these messages fit with their own beliefs and practices (see Appendix C for the complete interview protocol).

[3] Although the relation between beliefs and practices is important, it was not the focus of this study.

The observation notes and interview transcripts were analyzed using the qualitative procedures described by Bogdan and Biklen (2003). First, all of the observation notes and interview transcripts were read and examined for trends. Second, quotes from the teachers' interviews were sorted according to themes and the teachers' quotes about the messages in the different resources were paraphrased. Third, for each of the paraphrased messages for each of the teachers, I used the beliefs inventory, interview transcripts, observation notes, and RTOP data to determine the relation between the message and the teacher's beliefs and the relation between the message and the teacher's practices. Fourth, for each teacher I grouped the paraphrased messages by theme and for each theme examined the relation of the paraphrased messages to the teacher's beliefs. Because the teachers frequently interpreted a variety of messages within each theme, there were three possible relations between the interpreted messages and beliefs: agree with all of the messages in this theme, agree with some of the messages in this theme, and disagree with all of the messages in this theme. Similarly, I examined the relation of the paraphrased messages grouped by theme to each teacher's practices. Again, there were three possible relations between the messages and practices. The teacher could reflect all of these messages in her practices, reflect some of these messages in her practices, or reflect none of these messages in her practices. Since there are three possible relations between the messages grouped by theme and beliefs and also three possible relations between the messages grouped by theme and practices, there are theoretically nine possible ways in which the messages can relate to the teachers' beliefs and practices.

Chapter 2: Review of the Literature

This research study examined the messages about mathematics and mathematics teaching that five elementary certified middle school mathematics teachers interpreted from their students' textbooks, school district's curriculum guides and assessments, state's assessments and curriculum framework, a master's degree program in which they were enrolled, and other resources which the teachers felt were significant influences on their teaching. This study also examined how these messages related to the teachers' beliefs and classroom practices.

The focus and structure of this research study was significantly influenced by many other studies. In this chapter I review some of the most relevant literature. I begin with a brief description of *messages* and a summary of two of the most pertinent studies about mathematics teachers' interpretations of messages and a description of a framework for looking at teachers' interpretations of messages. A discussion of *message incongruence* describes some of the literature on what happens when the messages interpreted by teachers compete and/or conflict with each other or with the beliefs, goals, and values of teachers.

I posit that teachers' beliefs about mathematics and mathematics teaching play significant roles in the teachers' interpretations of messages. Thus, this chapter includes a review of the literature on *beliefs* in general and then goes on to review the existing scholarship and empirical research on beliefs about mathematics and beliefs about mathematics teaching. Because curricular materials greatly influence classroom practices, the following section focuses on some of the major perspectives and studies on *curriculum use*. Next, since so many of the messages revolve around the idea of *reform-oriented teaching*, a section on this topic expands upon the ideas embodied in this phrase. Both *test pressures* and *professional development* can have significant influences on teachers as well. Therefore a section is devoted to each of these influences.

Messages and Message Interpretation

Whether they are acknowledged or not, resources such as textbooks, curriculum guides, assessments, and professional development programs contain messages about what is most important for students to learn and how students can best learn this (Goldenberg, 1999; Hill, 2001; Spillane et al., 2002). Although they are usually not explicit, messages can be normally be rephrased as statements that begin with "Teachers should…" Goldenberg (1999) points out that frequently the authors of resources are not even aware of these messages; thus these messages are not given the thought, planning, and careful analysis that they deserve.

Furthermore, few of these messages directly impact what happens in the classroom; most messages are mediated by the teacher who must first interpret them (Spillane et al., 2002). Although it was recommended as a key area of needed research by the NRC (2002), few studies have examined how mathematics teachers interpret or make sense of policy messages. The two most relevant studies were conducted by Hill (2001) and Berk (2004, 2005).

Hill (2001) examined how a group of teachers on one school district's mathematics curriculum writing committee interpreted state standards. She found that language plays an important role in policy interpretation and that there is often a disconnect between how authors and readers use words such as *explore*, *construct*, and *understand*. Thus, authors' intentions are sometimes lost as readers make sense of the messages. To help clarify intentions, she recommended that policy writers use more than words to clarify their meanings. For example, she suggested that if the state had provided videos of teaching that depicted the intended messages, these messages would have been more accurately interpreted.

Berk (2004, 2005) followed a group of 14 middle school mathematics teachers as they read and discussed *Principles and Standards for School Mathematics* (NCTM, 2000). She found that the teachers came to view the document from multiple lenses: as a warrant for their current beliefs or practices, as a lever for effecting change, as a tool for their own learning, as a springboard for rich discussions with colleagues, and as a curriculum map. She also found that individual teachers often viewed the document from multiple lenses. The ways in which the teachers came to view the document were closely related to their local school contexts.

Spillane, Reiser, and Reimer (2002) have developed a cognitive framework to characterize how implementing agents[4] make sense of and implement messages. This framework focuses on the agents' existing cognitive structures (including knowledge, beliefs, and attitudes), their situation, and the policy signals. Their framework uses individual cognition theories, situated cognition theories, and theories about the role of representations to explain why different agents interpret the same messages differently and why agents can misunderstand new ideas as familiar and hinder change. It also explains why agents may focus on superficial features and miss deeper relationships and

[4] Because Spillane, Reiser, and Reimer (2002) use the term "implementing agents" I use it here. In other places in this dissertation I purposefully avoid the use of the word "implement" in order to emphasize the participatory nature of teachers with curricular materials.

why people are biased toward interpretations consistent with their prior beliefs and values. They argue that when policies are not implemented as intended it is not because implementing agents reject messages, but rather that it is because they understand them differently than policymakers intend. Although they mention that implementing agents often face contradicting messages, it seems that their model assumes that the messages present a consistent vision. It is unclear that their framework explains how teachers deal with competing or contradictory messages.

Message Incongruence

When making decisions about what and how to teach, teachers must consider the messages present in the available curricular materials, the assessments that they are required to give to their students, and the professional development that they attend. Often there is not consistency in messages among the different resources. Sometimes the messages contradict each other and frequently the messages compete for attention. Additionally, the beliefs, goals, and values of the teachers and the messages present in the resources are often not congruent.

Few studies have examined teachers' interpretations of incongruencies among resources. The two most relevant studies found that teachers tend not to notice incongruencies of messages among resources. Tomayko (2007) surveyed members of the Maryland Council of Teachers of Mathematics about the working conditions, challenges, and tensions they experience; 252 teachers completed the survey. Most were middle school or high school mathematics teachers. She found that more than 80% of the surveyed teachers agreed or strongly agreed with the statement "My school and my district have the same values regarding math content" (p. 79). Approximately 75% of the surveyed teachers agreed or strongly agreed with the statement "My school and my district have the same philosophy regarding math instruction" (p. 79). Similarly, although Hill (2001) and the writers of some of the resources saw significant differences in the messages in the different resources the teachers in her study were analyzing, the teachers did not appear to notice conflicts in messages. She attributed this to "humans proclivity to see order, not disorder, in their environments" (p. 313) and the teachers' blind acceptance of the textbook authors' claims of alignment with state standards.

Most studies of message incongruence have focused on the incongruencies between resources and teachers. Many studies have found that teachers are frequently not in agreement with the messages they interpret from their resources. For example, in a nationwide survey of over 4000 teachers' attitudes and opinions about state mandated testing programs Abrams, Pedulla, and Madaus (2003) found that "teachers are uncomfortable with the changes they feel they need to make to their instruction to conform to the demands of the state testing program" (pp. 23-24). Similarly, almost half of the teachers surveyed by Tomayko (2007) indicated that they agreed or strongly agreed with the statement "I am philosophically at odds with ways that I am expected to teach math" (p. 79).

Beliefs

Definitions of beliefs

There is not one universally agreed upon definition of *beliefs*, but most educational researchers agree that beliefs are "psychologically held understandings, premises, or propositions about the world that are felt to be true" (Richardson, 1996, p. 103). Beliefs can be held with varying degrees of conviction and beliefs are not consensual; that is, the believer is aware that others may hold different beliefs. These features make beliefs distinct from knowledge (Thompson, 1992). Beliefs develop over relatively long periods of time (McLeod, 1992) and are often resilient to efforts of change (Cooney et al., 1998; Pajares, 1992). Additionally, beliefs can be held in isolated disjoint clusters. This makes it possible for a person to hold what appear to be conflicting sets of beliefs (Thompson, 1992).

Conceptions are also frequently mentioned in conjunction with beliefs. Conceptions are a more general mental structure encompassing conscious and subconscious beliefs, concepts, rules, mental images, and preferences (Thompson, 1992). The distinction between beliefs and conceptions is not considered to be terribly important and it is "more natural at times to refer to a teachers' [*sic*] conception of mathematics as a discipline than to simply speak of the teachers' beliefs about mathematics" (Thompson, 1992, p. 130).

Professed beliefs are beliefs that teachers describe themselves as having. These may be articulated in an interview or in a written response to a beliefs inventory question. These beliefs may or may not be consistent with the teacher's observed behavior. *Attributed beliefs* are beliefs that an observer identifies to be consistent with a teacher's behavior (Aguirre & Speer, 2000).

Beliefs about the nature of mathematics

Beliefs about the nature of mathematics are “a general account of mathematics, a synoptic vision of the discipline that reveals its essential features and explains just how it is that human beings are able to do mathematics” (Tymoczko, 1998, p. xiii). These beliefs include “an individual’s understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior” (Schoenfeld, 1992, p. 358). Thus, beliefs about the nature of mathematics include a person’s beliefs about the source of mathematical ideas and what it means to *do* mathematics.

As early as the fourth century B.C., there have been discussions about the nature of mathematics. Plato and Aristotle were among the first major contributors to the dialogue (Dossey, 1992). Since then, mathematicians and philosophers have continued to discuss and disagree about the nature of mathematics. These discussions have resulted in a plethora of conceptions of mathematics and labels for these ideas. Although it may be a gross over simplification (Weizman & Hoz, 2006), beliefs about the nature of mathematics can be separated into two principal schools of thought: *External* conceptions of mathematics and *Internal* conceptions of mathematics (Dossey, 1992).

Dossey (1992) suggests that External conceptions of mathematics stem from Platonic views of mathematics. Central to this view is the idea that mathematical objects have an existence of their own outside of the mind. Thus, mathematics is a static discipline with a known set of concepts, principles, and skills (Dossey, 1992). Logicism (Dossey, 1992), Foundationalism (Tymoczko, 1998), Realism (Tymoczko, 1998), and Euclideanism (Lakatos, 1976; Lerman, 1983) are some of the philosophies of mathematics which would be considered External conceptions of mathematics.

Internal conceptions of mathematics stem from Aristotelian views of mathematics. Central to this view is the idea that mathematics is constructed through experimentation, observation, and abstraction (Dossey, 1992). Thus, mathematics is a dynamic, growing field of study (Dossey, 1992). Intuitionism (Dossey, 1992), Formalism (Dossey, 1992), Quasi-empiricism (Chazan, 1990; Lakatos, 1976), Constructivism (Tymoczko, 1998), and Problem-solving (Ernest, 1989a) are some of the philosophies of mathematics which would be considered Internal conceptions of mathematics.

It is rare for a person's beliefs about the nature of mathematics to fit neatly into one of these categorizations. Some feel that "most mathematicians live with two contradictory views on the nature and meaning of their work" (Hersh, 1998, p. 12). Mathematics teachers' beliefs about the nature of mathematics also tend to be a conglomeration of views (Weizman & Hoz, 2006), but most teachers tend to hold views of mathematics more closely aligned with External views of mathematics than Internal views of mathematics (Civil, 1990; Cooney et al., 1998; Ernest, 1989b; Raymond, 1997; Steele & Widman, 1997; Stipek et al., 2001; Thompson, 1992). Since teachers' views of mathematics seem to be related to their analysis of and decisions made in teaching situations (Lerman, 1990), these beliefs have important implications with regard to teachers' practices.

Beliefs about mathematics teaching

A teacher's beliefs and conceptions about mathematics teaching include "What a teacher considers to be desirable goals of the mathematics program, his or her own role in teaching, the students' role, appropriate classroom activities, desirable instructional approaches and emphases, legitimate mathematical procedures, and acceptable outcomes of instruction" (Thompson, 1992, p. 135). The beliefs that teachers have of mathematics teaching are shaped by their experiences as students and teachers of mathematics (Thompson, 1992). Some of the factors that influence teachers' beliefs about mathematics teaching include conceptions and mental models of mathematics, mathematical content knowledge, pedagogical content knowledge, pedagogical knowledge, educational policies, traditions, availability of resources, and student behaviors (Weizman & Hoz, 2006).

It seems logical to think that teachers' models of mathematics teaching are also closely related to their models of mathematics learning. But, "for most teachers it is unlikely that the two have been developed into a coherent theory of instruction. Rather, conceptions of teaching and learning tend to be eclectic collections of beliefs and views that appear to be more the result of their years of experience in the classroom than of any formal or informal study" (Thompson, 1992, p. 135).

There does, however, seem to be a relationship between a teacher's beliefs about mathematics and her beliefs about mathematics teaching (Lerman, 1983, 1990; Thompson, 1992). Lerman (1983) argues that "one's perspective on mathematics teaching is a logical consequence of one's epistemological commitment in relation to mathematical knowledge, and not merely one of expediency in response to societal pressures, or pedagogical convenience" (p. 59). He, like Lakatos (1976), goes on to suggest that there are two distinct pedagogical movements. The first, the Euclidean program, is knowledge centered while the second, the quasi-empirical program, is problem-solving centered.

These two categorizations are similar to Romberg's (1992). He separates beliefs about mathematics teaching into beliefs that focus on "knowing that" versus "knowing how." Like the Euclidean program, mathematics teaching that focuses on "knowing that" concentrates on helping students learn the massive record of knowledge of mathematics. Like the quasi-empirical program, mathematics teaching that focuses on "knowing how" concentrates on helping students learn how to *do* mathematical activity.

Of course, it is rare for teachers to fit perfectly into one of these categorizations. "As in the case of conceptions of mathematics, a given teacher's conception of mathematics teaching is more likely to include various aspects of several models than it is to fit perfectly into the description of a single model" (Thompson, 1992, p. 137). Similarly, Weizman and Hoz (2006) have concluded that categorizations of conceptions of mathematics teaching are more complex than this. Their results are based on a survey of 165 junior and senior high mathematics teachers in Southern Israel. Eleven of these teachers were also interviewed individually. The survey consisted of 23 items on the nature of mathematics and its importance and 46 items on the teaching of mathematics. One of the most surprising results of their research was that about half of the teachers they studied do not adhere to any official conception of mathematics or its teaching. Additionally, they found that the relationship between beliefs about mathematics and beliefs about mathematics teaching is much more complicated than is commonly thought.

Similarly, the relation between messages and beliefs is a complex one. Spillane et al. (2002) have found that beliefs play a significant role in determining how messages are interpreted. They found that “people are biased toward interpretations consistent with their prior beliefs and values” (p. 401) and teachers with different beliefs will interpret the same resources differently. In contrast, Berk (2004, 2005) found that messages can act as a lever for changes in beliefs.

Teaching Practice and Curriculum Use

Teaching practice can be summarized as what a teacher does or says within the classroom. One way of looking at the relation between messages in curricular resources and classroom practices is through the examination of *curriculum use*.[5] The term *curriculum* can have multiple meanings. It can refer to a course of study, overarching frameworks describing what should be taught, or the written resources and guides used by teachers.

Remillard (2005) describes the *formal curriculum* as the goals and activities outlined by school policies or designed in textbooks. The *intended curriculum* refers to teachers’ aims. The *enacted curriculum* is what actually takes place in the classroom. Similarly, Cuban (1995) defines the *official* curriculum as what the state and district officials set forth in curriculum frameworks and courses of study. The *taught* curriculum is what is taught while the *learned* curriculum is what students actually learn. The *tested* curriculum is what is on classroom, school, district, state, and national tests. These curricula may be similar, but are often quite different.

[5] Drawing on Remillard’s (2005) recommendations, I purposefully employ the word *use* rather than *implement* when referring to how teachers interact with, draw on, refer to, and are influenced by curricular materials when designing the enacted curriculum. This is meant to emphasize the participatory nature of teachers with curricular materials.

Through case study of two experienced elementary school teachers, Remillard (1999) identified three arenas for curriculum development activity that teachers engage in as they use curricular resources in teaching. The *design* arena involves the selection and design of mathematical tasks. The *construction* arena involves the enactment of the tasks in the classroom. The *mapping* arena involves the organization and selection of content over the school year.

When studying curriculum use, researchers have taken four theoretical perspectives: *following or subverting*, *drawing on*, *interpreting*, and *participating with* (Remillard, 2005, p. 217). Each of these perspectives has a different conception of curriculum materials and the teacher's role. These perspectives are summarized in Table 1 below. The *following or subverting* perspective conceives of curriculum materials as a fixed representation of the enacted curriculum while the teacher is the enactor of the formal curriculum. Here fidelity of implementation is a possible and desirable goal. The *drawing on* perspective conceives of curriculum materials as one of many available resources and the teacher is an active designer of the enacted curriculum. The teacher has agency over the curriculum. The *interpreting* perspective conceives of curriculum materials as representations of tasks and concepts and the teacher draws upon beliefs and experience to make meaning. Fidelity of implementation is not possible. The *participating with* perspective conceives of curriculum materials as artifacts or tools and the teacher designs the enacted curriculum through collaboration with curriculum materials. There is a participatory relationship between the teacher and curriculum which is influenced by both. These categorizations are not mutually exclusive, but in general, early studies on curriculum use employed *following or subverting* perspectives, while more recent studies use *interpreting* or *participating with* perspectives. In this study I use an *interpreting* perspective.

Table 1
Key assumptions and theoretical perspectives influencing conceptions of curriculum use

Conceptions of curriculum use	**Following or subverting**	**Drawing on**	**Interpreting**	**Participating with**
Conceptions of curriculum materials	Fixed representation of enacted curriculum	One of many available resources	Representation of tasks and concepts	Artifacts or tools; products of sociocultural evolution
Conceptions of the teachers' role	Enactor of planned curriculum	Active designer of the enacted curriculum	Meaning maker; draws on beliefs and experience to make meaning	Collaborator with curriculum materials to design enacted curriculum
View of teacher-curriculum relationship	Fidelity is possible and a desirable goal	Teacher has agency over curriculum	Fidelity is not possible	Participatory relationship influenced by both teacher and curriculum

Adapted from Remillard, 2005, p. 217

No matter the perspective used in the research, studies have concluded that teachers act with some autonomy with respect to what topics and skills they emphasize and ignore and with respect to what materials they use and how they use them (Chávez-López, 2003; Sosniak & Stodolsky, 1993; Tarr et al., 2006). Thus, even students within the same school taught by teachers using the same text are likely to experience different mathematics curricula. Teachers do, however, usually use the textbook to determine what content to teach and how the content will be sequenced, and as a source of activities and instructional ideas (Reys, Reys, & Chávez, 2004).

Several reasons for the variety in curriculum use have been proposed. All rely in some way on teachers' beliefs about mathematics and mathematics teaching. For example, Drake and Sherin (2006) have found that "teachers' narrative identities as learners and teachers of mathematics frame the ways in which they use and adapt a reform-oriented mathematics curriculum" (p. 154). Drake and Sherin's model of curriculum use was developed from the case study of two elementary school teachers as they used a reform-oriented mathematics curriculum for the first time. They focused on when and how the teachers made adaptations to the curriculum and found that each teacher had a distinctive pattern of adaptation.

Similarly, in his large-scale study of 53 middle school teachers and case study of three teachers, Chávez-López (2003) found that "teachers' views of the curriculum and the match, or lack of it, between their own views about mathematics and mathematics teaching and the philosophy of the textbook – whether it is explicit or not – were the primary factors that determined how the textbook was used" (p. 157). He, too, proposed a model of curriculum use that relies heavily on teachers' views of mathematics and mathematics teaching.

Remillard (1999) found that "the meanings the teachers made through reading the text grew out of interactions between their beliefs and elements of the textbook and were situated in the larger context of their teaching" (p. 319). Her model of teachers' construction of mathematics curriculum in the classroom is a result of case study of two elementary teachers as they used a publisher-generated reform-oriented curriculum for the first time. She considers teachers to be curriculum developers in the sense that they “develop curricular plans and ideals and translate them into classroom events” (Remillard, 1999, p. 318).

Although there is clearly a relation between beliefs and practices, it is a complex relation (Thompson, 1992). Raymond’s (1997) study of six beginning elementary school teachers’ beliefs and practices concluded that “factors, such as time constraints, scarcity of resources, concerns over standardized testing, and students’ behavior” can lead to inconsistencies between beliefs and practice (p. 567). Her results are based on data collected over 10 months through interviews, observations, document analysis, and a beliefs survey.

Leatham (2006), however, concludes that rather than inconsistencies, such discrepancies between stated beliefs and practice may be a result of context instead. Different contexts bring out different beliefs. For example, a teacher who believes that children benefit from working together, but has her students work independently may be making this decision based on beliefs about classroom management rather than beliefs about group work.

Thompson (1984) found that the extent to which experienced teachers’ conceptions are consistent with their practice depends greatly on their tendency to reflect. “It is through reflection that teachers develop coherent rationales for their views, assumptions, and actions, and become aware of viable alternatives” (Thompson, 1992, p. 139). Thus, teachers (such as the teachers in this study) who are involved in professional development that focuses on reflection are likely to have somewhat more consistent beliefs and practices.

Although we frequently talk about the ways in which beliefs impact practice, it is important to note that there is growing evidence that practice also has an impact on beliefs (Remillard, 2005; Remillard & Bryans, 2004; Richardson, 1996; Soloway, 1996; Thompson, 1992). For example, Guskey (as cited in Pajares, 1992) concluded that "change in beliefs follows, rather than precedes, change in behavior" (p. 321). Thus, the relation between beliefs and practice is likely to be cyclic in nature.

Reform-Oriented Teaching

The National Council of Teachers of Mathematics (NCTM) is the primary professional organization for teachers of mathematics in the United States. Its release of *An Agenda for Action* in 1980 marked the beginning of the most recent "reform" movement in mathematics education by outlining the shape that school mathematics programs should take. Its subsequent publication of *Curriculum and Evaluation Standards for School Mathematics* in 1989, *Professional Standards for Teaching Mathematics* in 1991, *Assessment Standards for School Mathematics* in 1995, *Principles and Standards for School Mathematics* in 2000, and *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* in 2006 further contributed to the movement. At around the same time, the American Association for the Advancement of Science (AAAS) released its own, similar, visions for mathematics and science reform with *Science for All Americans* (1989) and *Benchmarks for Science Literacy* (1993). Together these documents outline the essential components of a reform-oriented school mathematics program which is, in many ways, very different than conventional mathematics programs.

Reform-oriented instruction is often contrasted with *traditional* instruction. Traditional instruction "emphasizes factual knowledge, mastery of algorithms, and solving structured problems" (Stecher et al., 2006, p. 104). Historically mathematics in the United States has been taught through telling. Teachers tell students how to solve a certain type of problem by modeling a procedure, guide students through the process, and then provide problems on which students are to practice the procedure (Fey, 1979; Hiebert & Stigler, 2000).

Reform-oriented teaching recommendations challenge this model. However, traditional and reform-oriented teaching "should not be thought of as opposite ends of a single dimension but as separate dimensions; research shows that teachers use practices associated with both approaches" (Stecher et al., 2006, p. 104). In reform-oriented classrooms, the teacher is not the sole source of mathematical authority; students are expected to generate mathematical ideas of their own and to also learn by listening to each other. Tasks that can be approached in more than one way are encouraged. One of the goals set forth by the NCTM (2000) is for students to learn "to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments" (p. 17). Additionally, students are encouraged to come to see mathematics as the science of patterns and relationships (AAAS, 1993) and a dynamic body of knowledge (Romberg, 1992; Smith, 1996).

These reform-oriented messages have yet to make their way into all classrooms. Some teachers are reluctant to teach in ways that are different than the ways in which they learned mathematics or have always taught mathematics. For example, Smith (1996) found that teachers' sense of efficacy is often challenged by reform-oriented teaching methods. Teachers want to feel that they have a positive effect on student learning and a student-centered approach to learning makes the teacher's impact less visible.

Other teachers attempt to enact the reform ideals, but find it difficult to do so. Frequently teachers believe they are changing the ways they teach to be more reform-oriented, but it seems they are retaining the core of nonreform-oriented practice (D. K. Cohen, 1990; Hiebert & Stigler, 2000). For example, teachers may seat students in groups, yet maintain a teacher-centered classroom. Moreover, the messages presented in reform documents are often not easily enacted in classrooms. For example, despite extensive work in mathematics education, Cady (2006) and Chazan and Ball (1999) found it very challenging to lead classroom discourse in ways that they consider to be in line with the reform spirit.

Additionally, the materials to which teachers have access can support or hinder teachers' attempts to teach in a reformed way. In order to support teachers in reform-oriented teaching, the National Science Foundation (NSF) has supported the development of several textbook series. In their study of over 60 middle school teachers and 4000 sixth and seventh grade students, Reys, Reys, Tarr, and Chávez (2006) found that teachers who use the NSF funded materials are more likely to teach in ways consistent with the reform movement. "Specifically, 4%, 17% and 79% of the teachers using publisher generated mathematics textbooks were classified as high, medium, and low levels of [NCTM] standards-based practices, compared to 17%, 35% and 48% of the teachers using NSF supported mathematics curricula" (pp. 3-4).

It should be noted that use of publisher generated materials does not preclude teachers from teaching in reform-oriented ways, but Reys et al. (2006) also found that teachers who had access to a NSF-funded curricula are not only more likely to attempt to teach in reform-oriented ways, but they are also more likely to attain a reform-oriented classroom environment than teachers with access only to publisher-generated materials. Thus both the curricular materials to which teachers have access and the ways in which they use these materials are important.

Test Pressures

The No Child Left Behind Act (2001) has increased accountability for states, school districts, and schools. It requires that all public school students in grades 3-8 be tested annually in reading and mathematics and that states set annual statewide progress objectives to ensure that all groups of students reach proficiency within 12 years. Assessment results and progress objectives are broken out by poverty, race, ethnicity, disability, and limited English proficiency to ensure that no group of students is left behind. School districts and individual schools that fail to make adequate yearly progress (AYP) toward statewide proficiency goals are subject to corrective action. As a result of these high-stakes tests, many teachers feel great pressure.

Some of these pressures are evidenced by Tomayko (2007). Her most notable results regarding testing pressures include:

- Approximately 88% of the teachers indicated agreement or strong agreement with the statement "I feel pressure from my principal to raise scores on required math tests" (p. 100).

- Almost 98% of the teachers indicated agreement or strong agreement with the statement “The tests I am required to give significantly influence the content of my math course(s)” (p. 89).
- Approximately 78% of the teachers indicated agreement or strong agreement with the statement “The tests I am required to give significantly influence the methods of instruction used in my math course(s)” (p. 90).
- Approximately 90% of the teachers indicated agreement or strong agreement with the statement “I spend more than 30 hours per year preparing students specifically for the required math tests” (p. 89).

Thus, there is evidence that teachers feel pressures from high-stakes testing and these pressures influence what and how they teach.

Abrams et al. (2003) found that the pressures of high-stakes statewide testing negatively impact many teachers. While teachers generally have positive views toward their state’s curricular standards, the pressures of high-stakes tests lead them to “teach in ways that contradict their own notions of sound educational practice” (Abrams et al., 2003, p. 27). Furthermore, they reported that “high-stakes assessments increase stress and decrease morale among teachers” (Abrams et al., 2003, p. 20).

Teachers do not, however, always view the pressures they feel from testing as completely negative; some teachers accept that mechanisms such as standardized tests help ensure that students learn a minimum set of content and that their knowledge will be recognized by others. High school Advanced Placement Calculus teacher Marty Schnepp (Chazan & Schnepp, 2002) wrote about this when he described some of the tensions he experiences as he manages his (at times conflicting) commitments to his students and to the discipline of mathematics. In order to manage these tensions, he works with his students in different ways throughout the school year. One of the ways is strongly influenced by the fact that his students will be taking the Advanced Placement test in Calculus. He wants his students "to know standard terms and to have the skills and knowledge needed to convince others that they have learned Calculus, to enable them to be successful in collegiate mathematics, and to be successful on the AP [Advanced Placement] Calculus test" (p. 179).[6] In order to meet these goals, there are times that his teaching becomes more teacher directed than it is the rest of the year. While he "agonizes" over the decision of when to switch to this way of teaching, he sees the importance of the tests and does not wish that his students did not need to take this test.

Although Schnepp is certainly an exceptional teacher, the tensions he feels are not unique. Many other teachers change their ways of teaching because of test pressures. Through interview and observation of 63 fourth grade teachers in New Jersey, Schorr, Firestone, and Monfils (2003) found that "where teachers feel more pressure, they report increasing their 'didactic' instruction – that is, telling students exactly how to solve problems using algorithms and procedures with little or no attention to understanding" (p. 398). However, test pressures do not necessarily have to lead to more traditional instruction. Some tests, such as the ones in New Jersey and Maryland, are designed to act as an impetus for changes in teaching practices. By asking students to solve open-ended questions and explain their thinking, tests have the potential to push teachers to teach in more reform-oriented ways. Nevertheless, it seems that changing teaching practices to be more reform-oriented requires more than reform-oriented tests. Schorr et

[6] Schnepp also believes that the Advanced Placement test serves as an important motivational tool for his students (Chazan & Schnepp, 2002, p. 189).

al. (2003) found that while the teachers reported that they changed their practices to be more compatible with the reform-oriented state and national standards and the New Jersey test, their observed practices did not reflect these changes. They concluded that "in the absence of effective professional development, testing leads to minimal [reform-oriented] changes in teaching practice" (p. 373). Thus, professional development plays an important role in reform.

Role of Professional Development in Reform

Reform ultimately rests on teachers. The creation of new curriculum frameworks and assessments is pointless if teachers do not change their teaching practice. Usually it is not that teachers do not want to improve their practice, it is that they do not know how to improve their practice (Thompson & Zeuli, 1999). To support teachers in their efforts to reform their practice, high quality professional development is key (Campbell, Kramer, Bowden, & Yakimowski, 2005; Cohen & Hill, 2001; Thompson & Zeuli, 1999).

Research on effective professional development in mathematics education has resulted in the following principles:

- Professional development should be grounded in mathematical content.
- Professional development sessions should be part of a long term, cohesive plan.
- Professional development should encourage active and social learning by teachers.
- Professional development should be situated in classroom practice.

The teachers in this study were enrolled in a master's degree program aimed at helping them to reform their practices. Thus, the program[7] was developed with these principles in mind. In the following sections I describe and justify each of these principles.

Professional development should be grounded in mathematical content

Professional development should give teachers the opportunity to deepen their own subject matter knowledge and to think about how this content relates to their teaching. All too often professional development has focused on generic pedagogical strategies such as cooperative learning or the use of manipulatives (Cohen & Hill, 2001). While these strategies can be important to teaching, professional development that focuses only on them, without relating them to mathematical content, has not been found to be effective and has even been found to have negative effects on student achievement (Campbell et al., 2005). Conversely, professional development that is subject specific and focuses on mathematical knowledge has been found to have a positive effect on student achievement (Campbell et al., 2005; Wilson & Berne, 1999).

It is important for teachers to grapple with mathematical content and to think about issues such as mathematical reasoning and justification (Ball & Cohen, 1999; Garet, Porter, Desimone, Birman, & Yoon, 2001; Hill & Ball, 2004; Stein, Smith, & Silver, 1999). In her description of teachers working with *Developing Mathematical Ideas* professional development materials, S. Cohen (2004) illustrates how this type of professional development might look. In this and similar high quality professional development experiences, teachers develop the *human capital* (Spillane & Thompson,

[7] In Chapter 3 I describe this particular master's degree/professional development program in more detail.

1997), or in other words the knowledge, skills, and disposition to learn, that is necessary for teachers to make changes to their practice.

Professional development sessions should be part of a long term, cohesive plan

Making changes to teaching practice can be very difficult. Thompson and Zeuli (1999) describe these changes as *transformative learning* and define it as “thoroughgoing changes in deeply held beliefs, knowledge, and habits of practice” (p. 342). Unfortunately, often the professional development available to teachers is fragmented, superficial, and even contradictory; it does little to support teachers in their transformation (Cohen & Hill, 2001). In order to support teachers, professional development must be ongoing, intense, and cohesive (Cohen & Hill, 2001; Garet et al., 2001; Hill & Ball, 2004; Thompson & Zeuli, 1999).

Several studies have found that teachers benefit from ongoing professional development. Cohen and Hill (2001) found that teachers who had extended opportunities to study and learn, were more likely to change teaching practices. Similarly, Garet et al. (2001) concluded that time span and contact hours have a substantial positive influence on opportunities for active learning and coherence among teachers’ goals and experiences with standards. Although a long duration of professional development is often helpful, it is neither a necessary nor sufficient condition of effective professional development. Long, poorly designed professional development can be ineffective and Hill and Ball (2004) found that select well planned, but short, professional development programs seemed to be as beneficial as well planned long trainings.

Not only must professional development be ongoing, but it must also be intense. Garet et al. (2001) state that sustained and intensive professional development is more likely to have an impact on teaching (as reported by teachers) than shorter and less intense professional development.

Most importantly, professional development must be part of a larger, cohesive professional development plan. Cohen and Hill (2001) found that for many teachers, professional development typically consists of a few days of learning each year about discrete topics. As a result, teachers are often sent inconsistent and conflicting messages. If professional development is connected to other professional development experiences and is aligned with standards and assessments, it is more likely to enhance teachers' knowledge and skills and change their teaching practices (Garet et al., 2001).

Professional development should encourage active and social learning by teachers

Social constructivist theories on learning emphasize the need for active and social learning by teachers. Not only do these principles support learning, but they also support the creation of social support systems for teachers. Professional development is not a dissemination of knowledge activity. Wilson and Berne (1999) emphasize that teacher learning must be *activated* rather than *delivered*. Garet et al. (2001) found that active learning is related to the enhanced knowledge and skills of teachers. Learning can be hard and uncomfortable (Wilson & Berne, 1999). Many teachers do not expect to be active participants in professional development. Teachers expect to learn new theories or instructional strategies, but they do not expect to have their current theories or strategies questioned. Professional development must "create a sufficiently high level of cognitive dissonance to disturb in some fundamental way the equilibrium between teachers' existing beliefs and practices on the one hand and their experiences with subject matter, students' learning, and teaching on the other" (Thompson & Zeuli, 1999, p. 355).

During this active learning, social interactions are important. Learning can be conceptualized as "changes in participation in socially organized activities, and individuals' use of knowledge as an aspect of their participation in social practices" (Borko, 2004, p. 4). Spillane (2004) found that teachers who were most successful at teaching in ways closely resembling the standards described their efforts to make sense of the standards as a social endeavor. They also described a sense of obligation to their colleagues to improve practice.

Effective professional development helps teachers to develop trust and a sense of community with colleagues (Ball & Cohen, 1999; Garet et al., 2001; Stein et al., 1999; Wilson & Berne, 1999). These social interactions are also important in the building of social capital or professional networks which are instrumental in the realization of reform ideas (Spillane & Thompson, 1997). These networks need to include both teachers and outside experts (Ball & Cohen, 1999; Stein et al., 1999).

Professional development should be situated in classroom practice

Different situations give rise to different kinds of knowing and either support or limit the application of knowledge to different contexts (Greeno & Moore, 1993). Thus, it is often useful to ground teachers' learning in their own practice through the use of coteaching, coaching, assistance with planning, reflection on actual lessons, or guided group discussions about student work (Putnam & Borko, 2000; Stein et al., 1999).

In contrast, sometimes it is helpful for teachers to engage in learning away from their own classroom (Ball & Cohen, 1999; Thompson & Zeuli, 1999). "It can be hard to get enough distance on an example when it is from one's own practice" (Ball & Cohen, 1999, p. 22). Some of the constraints of such a setting can be reduced by using copies of student work, videotapes of lessons, curriculum materials, and other materials taken from real, but unknown, classrooms. A combination of approaches seems to be the most promising. Cohen and Hill (2001) found that only professional development which was grounded in practice and in which teachers had a chance to study and use student curriculum and assessments and see examples of student work (whether their own from their own students or others') had a constructive effect.

Summary

This chapter has discussed some of the existing literature on messages, message interpretation, message incongruence, beliefs, teaching practice and curriculum use, reform-oriented teaching, test pressures, and professional development. The recent proliferation of competing and/or conflicting messages suggests that further research is warranted. Thus, the current study examined the messages about mathematics and mathematics teaching that five elementary certified middle school mathematics teachers interpreted from various resources and how these messages related to the teachers' beliefs and classroom practices.

Chapter 3: Context and Methodology

This research study examined the messages about mathematics and mathematics teaching that five elementary certified middle school mathematics teachers interpreted from their students' textbooks, school district's curriculum guides and assessments, state's assessments and curriculum framework, a master's degree program in which they were enrolled, and other resources which the teachers felt were significant influences on their teaching. It also examined how these messages related to the teachers' beliefs and classroom practices. The data sources included the teachers' responses to the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004), classroom observation notes, scores on the *Reformed Teaching Observation Protocol* (Sawada et al., 2000), and transcripts from interviews and post-observation conversations.

This chapter begins with a description of the school district's and state's curriculum and assessments. This is followed by a description of the master's degree program in which the five teachers were enrolled and descriptions of the five teachers themselves. Next, the study's research questions are restated, the data sources are described, and lastly the data analysis methods are summarized.

School District

The teachers involved in this research study work in one of the largest school districts in the country. Just as in many other large school districts, in this district major decisions about the formal curriculum are made by the central office staff, not individual teachers or schools. Thus, all middle school teachers in the district are required to use the same textbooks, follow the same curriculum guides, and give their students the same formal assessments.

The school district offers six mathematics courses in its middle schools: a 6^{th} grade mathematics course, a 7^{th} grade mathematics course, an enrichment course in 7^{th} grade mathematics, an 8^{th} grade mathematics course, high school Algebra, and high school Geometry.[8] For each of these courses, the school district's central office staff has written a curriculum guide. These curriculum guides are quite detailed. For example, the 6^{th} grade mathematics course's guide is over 600 pages long. Each guide contains information about the scope and sequence of the curriculum as well as detailed expectations about each lesson. Some lessons have multi-page lesson plans. Additionally, pre-assessments and post-assessments for each unit of study are included. These assessments are required to be administered to the students at specific times of the year and the teachers must submit scores for each of the questions on the assessments for each student to the central office by pre-determined dates. These scores are then sent to the school district's superintendent's office.

The school district has adopted textbooks published by Glencoe/McGraw-Hill for all of its middle school mathematics courses. Students in the 6^{th} grade mathematics

[8] Although some of these courses are named by grade level, not all students enrolled in the 6^{th}, 7^{th}, or 8^{th} grade courses are in the given grade; younger students may take the courses as well. For example, it is not uncommon for 6^{th} graders to take the 7^{th} grade mathematics course or even the enrichment course in 7^{th} grade mathematics. In fact, students are encouraged to take more advanced coursework; by 2010 the school district aims to have more than 75% of its students successfully complete high school Algebra by the end of 8^{th} grade. In 2007, almost 60% of the school district's students successfully completed high school Algebra by the end of 8^{th} grade.

course receive[9] a copy of *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a). Students in the 7th grade mathematics course receive a copy of *Mathematics: Applications and Connections Course 3* (Collins et al., 2001b). Students in the enrichment course in 7th grade mathematics receive a copy of *Pre-Algebra* (Malloy, Price, Willard, & Sloan, 2008).[10] Students in the 8th grade mathematics course receive a copy of *Pre-Algebra* (Malloy, Price, Willard, & Sloan, 2003). Although the curriculum guides are loosely written around these textbooks, the guides do not follow the textbooks' sequence of lessons and the guides add lessons to and remove lessons from the textbooks. Additionally, the school district's curriculum guides frequently recommend the use of other instructional resources such as NCTM's *Navigations* series and Cuisenaire's *Super Source* series (ETA/Cuisenaire, 1996a and 1996b).

Some of the disparity between the school district's curriculum guides and textbooks may be because the school district's middle school mathematics textbook adoption process was quite contentious. The school district reported that they used work done by the AAAS to develop a set of evaluation criteria. A committee consisting of teachers, specialists, and the district's mathematics supervisors used those criteria to identify two textbook series for further consideration. The two series were *Connected Mathematics* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998) and the Glencoe/McGraw-Hill texts.[11] At around this time, several letters to the editors of local

---

[9] I use the word *receive* purposefully because the teachers in this study rarely assign work from the textbooks.

[10] As part of this study one of the teachers, Kathleen, was observed teaching the enrichment course in 7th grade mathematics. During the school year in which she was observed teaching, *Gateways to Algebra and Geometry: An Integrated Approach* (Benson et al., 1997) was the school district's textbook for this course and this is the book she talked about in her interview. The school district has since adopted *Pre-Algebra* (Malloy, Price, Willard, & Sloan, 2008) for this course.

[11] It is interesting to note that in its evaluation of middle grades mathematics textbooks, AAAS (2000) described *Connected Mathematics* as excellent and the Glencoe/McGraw-Hill texts as unsatisfactory.

newspapers were published. These letters, written by parents of school district students, other local citizens, and national figures in mathematics education, vigorously argued against the adoption of *Connected Mathematics*. After further consideration, the committee recommended the Glencoe/McGraw-Hill texts for adoption.

State of Maryland

The State of Maryland has developed a Voluntary State Curriculum (VSC). This curriculum defines what students should know and be able to do at each grade, preK through 8, in mathematics and eight other content areas. In mathematics, the VSC consists of indicators from seven standards: Knowledge of Algebra, Patterns, and Functions, Knowledge of Geometry, Knowledge of Measurement, Knowledge of Statistics, Knowledge of Probability, Knowledge of Number Relationships and Computation/Arithmetic, and Processes of Mathematics. In addition to listing the indicators in the VSC, the Maryland State Department of Education (MSDE) has developed a VSC Toolkit. This is meant to "provide Maryland educators with additional resources that will assist them in the instruction of content and skills contained in the Voluntary State Curriculum (VSC) . . . [and] enhance student learning in the classroom, as well as improve student success on all Maryland assessments" (MSDE, 2008). The toolkit consists of clarifications of indicators, sample lesson plans, lesson seeds, sample assessments, descriptions of prerequisite skills, examples of higher order thinking skills, technology suggestions, links to resources, and public release items from the Maryland School Assessment.

Although adoption of the VSC by school districts is *voluntary*, the VSC determines the content of the Maryland School Assessment (MSA). The MSA assesses the Maryland content standards in mathematics, reading, and science. The reading and mathematics tests are administered annually to students in grades 3 through 8. The science test is administered annually in grades 5 and 8. The tests include both selected response (multiple choice) and constructed response items.[12] The tests provide educators, parents, and the public information about student performance at the school, school system, and state levels and meet the requirements of the federal No Child Left Behind Act (MSDE, 2008).

In addition to the VSC, the State of Maryland has also developed a set of Core Learning Goals for high school Algebra/Data Analysis and Geometry. The High School Assessment (HSA) in Algebra/Data Analysis tests students' knowledge of the Core Learning Goals in this standard. Students take the Algebra/Data Analysis HSA after they have completed the Algebra/Data Analysis course. The tests contain multiple-choice questions and questions requiring written responses.[13] Starting with the graduating class of 2009, students must pass the HSA in order to graduate from high school.

Master's Degree Program

As a result of recent reform movements in mathematics education and increases in accountability through tests such as the MSA and HSA, middle school mathematics

[12] The constructed response items usually ask students to solve a word problem and then explain how they determined their answer. Occasionally they also ask students to explain how they know that their answer is correct. It is expected that students will spend approximately half of the testing time on selected response questions and the other half on constructed response questions.

[13] In order to expedite and ensure consistency in the grading of these tests, beginning in May 2009 the HSA will no longer include student constructed responses. They will be composed entirely of multiple choice questions.

teachers in this school district are being asked to teach more advanced mathematics content and are being asked to teach it differently than they have in the past. Although most middle school mathematics teachers in this school district teach only mathematics courses, many are certified in elementary education (grades 1-8) and have taken only the mathematics courses required to become certified in elementary education. As a result, the mathematics specialists in this school district's central office identified a need for professional development for their middle school mathematics teachers who are currently certified in elementary education, but are either currently teaching, or would like to teach, middle school mathematics. In 2003 they approached the University of Maryland to develop a master's degree program. The Mathematics Department at the University of Maryland, the Department of Curriculum and Instruction at the University of Maryland, and mathematics specialists in the school district worked together to design a new master's degree program. It was designed with research on effective professional development in mind; it is grounded in mathematical content, part of a long term, cohesive plan for reform, encourages active and social learning by teachers, and is situated in classroom practice.

The program consists of 10 courses: three mathematics education courses, three integrated mathematics and mathematics education courses, three mathematics courses, and one research course (see Appendix D for an overview of the program). The teachers take all 10 courses together as a cohort over three years. The primary goals of the program are that the courses will improve teachers' subject matter knowledge and pedagogy as well as encourage them to reflect on issues related to the philosophy of mathematics and mathematics teaching. Throughout the program, teachers are asked to think deeply about their own beliefs about mathematics and mathematics teaching.

The first cohort began the program in June 2005 and consisted of 14 teachers. A second cohort of 23 teachers began the program in 2006. A third cohort of 22 teachers from another local school district began the program in April 2008. As part of her dissertation research, Badertscher (2007) incorporated a mathematical inquiry strand into five of the first cohort's courses. In this strand, teachers engaged in full group, small group, and individual mathematical investigations.[14]

Teachers

In April 2007 all members of the first cohort of the master's degree program described above were invited to participate in this research study. Five teachers (Amelia, Beth, Emma, Kathleen, and Sarah)[15] responded that they were interested in participating. All five of these teachers are elementary certified, but teach middle school or high school mathematics courses in a middle school. Some of the professional experiences of these five teachers at the time of the study are described in Table 2 below.[16]

[14] See Badertscher (2007) for more detail about the inquiry strand.
[15] All teachers' names are pseudonyms.
[16] See Badertscher (2007) for more detail about Amelia's and Beth's mathematical experiences.

Table 2
Professional experiences of the teachers involved in this study

	Amelia	Beth	Emma	Kathleen	Sarah
Total years teaching	8 years	15 years	9 years	11 years	8 years
Years teaching middle school math	8 years	1 year	6 years	6 years	2 years
Middle school math courses taught	6^{th} grade 7^{th} grade	7^{th} grade 7^{th} grade enrichment 8^{th} grade Algebra	6^{th} grade 7^{th} grade 8^{th} grade Algebra	6^{th} grade 7^{th} grade 7^{th} grade enrichment 8^{th} grade Algebra	6^{th} grade 7^{th} grade enrichment Algebra
Was observed teaching	6^{th} grade math course with 6^{th} grade students	8^{th} grade math course with 7^{th} grade students	8^{th} grade math course with 8^{th} grade students	Enrichment course in 7^{th} grade math with 6^{th} and 7^{th} grade students	6^{th} grade math course with 6^{th} grade students
Noteworthy experiences		Is the chair of her school's math department Helped to write the school district's assessments Was a reviewer of the school district's curriculum guide		Holds a master's degree in curriculum design Worked on aligning the school district's objectives with the VSC Helped to write the school district's assessments	

Because the teachers were self-selected into the master's degree program and into this research study, they do not necessarily, and are not likely to, represent all middle school mathematics teachers. The teachers, however, are likely to be similar to teachers who typically enroll in such programs. Thus, their experiences in this program are likely to be indicative of the experiences of other teachers in similar programs. Additionally, these are skilled teachers who have put extensive thought into their beliefs about mathematics and mathematics teaching. They have studied the curricular materials provided to them by their school district and also other curricular options. As a consequence, they are especially interesting because, despite recommendations (Thompson, 1992), there have been very few studies of teachers with an informed philosophical perspective of mathematics.

Restatement of Research Questions

- What messages do elementary certified middle school mathematics teachers interpret from curricular resources?
- How do these messages relate to the teachers' beliefs and observed classroom practices?

Data Sources

This study explored the messages that elementary certified middle school mathematics teachers interpreted from curricular resources. It also examined how these messages related to the teachers' beliefs and classroom practices. A qualitative methodology was used. Teachers were surveyed about their beliefs about mathematics and mathematics teaching, were observed teaching, and were interviewed.

Beliefs inventory

All teachers enrolled in the master's degree program completed the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004 – see Appendix A for a copy of the inventory). This inventory is based on work done by Ross, McDougall, Hogaboam-Gray, and LeSage (2003). It focuses on the teachers' beliefs about mathematics and mathematics teaching and is meant to provide a measure of reform-oriented beliefs.[17] The inventory was administered to all of the teachers in the first cohort of the master's degree program in June 2005, May 2006, and May 2007. Teachers' responses to this beliefs inventory were used as one measure of the teachers' professed beliefs.

To summarize each teacher's professed beliefs, a score of 1 was associated with *Strongly disagree*, 2 with *Disagree,* and so on. Because some of the statements are worded to support nonreform-oriented models of instruction, before analysis, the scales for statements 2, 6, 8, 10, 14, 15, 16, 18, 19, 22, 25, 27, 29, and 30 were reversed. Next the professed beliefs for each teacher were summed. Totals can range from 30 (very nonreform-oriented) to 150 (very reform-oriented). Each teacher has three scores (from

[17] This inventory has been found to be a very reliable measure of teachers' beliefs. Campbell (personal communication, February 2007) administered the inventory to 996 elementary school teachers from 36 schools in 5 school districts in Virginia. She found that Cronbach's alpha coefficient of reliability for the entire inventory was 0.797. (Cronbach's alpha is a tool useful for assessing the reliability of scales. A coefficient of 0.797 is very good considering that a 0.70 is the cutoff value for being acceptable.) Her analysis also yielded two subscales in the inventory. Statements 1, 2, 3, 6, 9, 12, 13, 14, 15, 18, 19, 21, 22, 24, 26, 28, 29, and 30 can be grouped to assess the teachers' beliefs about mathematics, mathematics curriculum, and mathematics instructional practice. The Cronbach's alpha for this subgroup is 0.700. Statements 4, 5, 7, 8, 10, 11, 16, 17, 20, 23, 25, and 27 can be grouped to assess the teachers' beliefs about student needs and learning. The Cronbach's alpha for this subgroup is 0.609.

the three administrations of the inventory). These were averaged to provide an overall summary of each teacher's professed beliefs.

Middle school classroom observations

Each of the five teachers was observed teaching the same class section for three consecutive days in May 2007. Because single lessons are often not contained in a single class period, this ensured that at least one complete lesson was observed.[18] I focused on single lessons as the unit of analysis because "The individual lesson is a big enough unit of teaching to contain all the complex classroom interactions that influence the nature of learning opportunities for students. At the same time, the individual lesson is the smallest natural unit for teachers that retains such interactions. The benefit of defining small units is that they allow the detailed analyses of teaching/learning relationships that make up the core of a knowledge base for teaching" (Hiebert, Morris, & Glass, 2003, pp. 217-218). Obviously, observation of three class sessions did not provide me with a complete picture of each teacher's classroom practices.

When possible, short (5 minute) pre-observation conferences took place before the observations to allow for a description of the teacher's plans for the lesson. These pre-observation conferences took place in the teacher's middle school classroom and were not audio recorded.

Detailed observation notes were made of all middle school classroom observations. Additionally, Sawada et al.'s (2000) *Reformed Teaching Observation Protocol* (RTOP) was used to assess the degree to which each lesson embodied the

[18] Amelia's and Sarah's schools are on a block schedule, so each of their class sessions is twice as long as the other teachers'. Thus they were observed teaching for the equivalent of six class periods.

recommendations and standards of reform-oriented teaching (see Appendix B for a copy of the RTOP). This instrument is designed to reflect the recommendations and standards of the NCTM (1989, 1991, 1995), the NRC (1995), and the AAAS (1993) and it embodies a vision of reform-oriented teaching which is similar to the one depicted in Campbell's (2004) beliefs inventory. In order to become an approved user of this protocol, I completed an online training program that consisted of watching videos of teachers, rating them with the protocol, and comparing my ratings to those of expert raters.

When possible, middle school classroom observations were followed by a debriefing session. During these sessions, teachers were asked about the pedagogical choices made during the observed class session and what influenced these choices. These conversations were audio recorded and transcribed.

Interviews

In order to learn more about how the teachers interpreted the messages in the curricular resources, each of the observed teachers was interviewed individually. The interviews took place at a time and a location convenient for the teachers. Each interview lasted for approximately 60 minutes[19] and was audio recorded and transcribed. Curricular resources such as the school district's curriculum guides and textbooks were available during the interviews for teachers to reference.

In the interviews, the teachers were asked about their teaching backgrounds and how they typically plan lessons. Additionally, for each of the curricular resources (their

---

[19] Additionally, portions of interviews conducted by Badertscher (2007) with Amelia and Beth were analyzed.

students' textbooks,[20] school district's curriculum guides and assessments, state's assessments and curriculum framework, the master's degree program in which they were enrolled, and other resources[21] which the teachers felt were significant influences on their teaching) the teachers were asked to talk about the messages they see in the resources and how these messages fit with their own beliefs and practices. Interview questions included:

- How do you think the authors of [the resource] envision an ideal lesson's design and implementation? How does this fit with your own vision of an ideal mathematics lesson?
- What do you think the authors of [the resource] think is most important for students to learn about mathematics? How does this fit with your own priorities for your students?
- What kind of classroom culture do you imagine the authors of [the resource] would want? How do you think they would want students and the teacher to interact? How does this fit with your own thoughts about student and teacher interactions?

Although each teacher was asked each of these questions about each of the resources, sometimes the conversational nature of the interviews led to tangential topics and not every teacher answered every question (see Appendix C for the complete interview protocol).

---

[20] For Amelia and Sarah this was *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a). For Beth and Emma this was *Pre-Algebra* (Malloy, Price, Willard, & Sloan, 2003). For Kathleen this was *Gateways to Algebra and Geometry: An Integrated Approach* (Benson et al., 1997).

[21] Because Kathleen said that the NCTM is also a significant influence on her teaching, she was also asked about this organization. During the interview, Beth brought up messages that she sees in the *Super Source* (ETA/Cuisenaire, 1996a and 1996b) series which is listed in the school district's curriculum guide as an "additional mathematics support material."

Data Analysis

The observation notes and interview transcripts were used as the primary sources of data. These were analyzed using qualitative procedures described by Bogdan and Biklen (2003). First, all of the observation notes and interview transcripts were read and examined for trends. A preliminary set of codes was developed and the data were coded and re-examined for trends. This process was repeated several times. After several iterations of reading and coding the data, 11 themes emerged.[22] These themes provided a way of grouping all of the data (including the quantitative data from Campbell's (2004) beliefs inventory and the RTOP) by related topics. For example, all data relating to the use of technology by students or teachers was placed in the *Technology* theme.

Next, the teachers' quotes about their interpretations of messages in the different resources were paraphrased. Whenever possible, words from the teachers' vocabulary were used in the paraphrases, but quotations containing similar ideas were paraphrased in the same way in order to emphasize similarities in interpretations. Some quotes yielded messages in several themes. For example, when Beth talked about the textbook, she said:

> It seems to me their [the authors'] big goal is for them [the students] to learn procedures of how to do things. And the way they're laid out it's like procedures first, then problem solving. And so, yeah, that's all I think their goal is to have them memorize procedures and show them examples of different procedures. (personal communication, November 3, 2006)

This was paraphrased as "Teachers should have students memorize procedures" and "Teachers should provide students with methods and worked examples to follow" in the

[22] These 11 themes were: Concepts and procedures, Connections, Cooperative learning, Differentiation, Explanation, Manipulatives, Practice, Question types, Source of solution methods, Technology, and Timeline.

Source of solution methods theme and “Teachers should emphasize procedures” in the *Concepts and Procedures* theme.

For each of the paraphrased messages for each of the teachers, I determined the relation between the message and the teacher’s beliefs and the relation between the message and the teacher’s practices. I primarily used the teacher’s attributed beliefs as evidenced from the language used when talking about this message to decide if it seemed that the teacher agreed with or disagreed with this message. Also, when applicable, I used the teacher’s professed beliefs as indicated on the *Teachers’ Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004) in my decision of whether the teacher agreed or disagreed with the message she interpreted. Similarly, I used my observation notes and, when applicable, the RTOP to decide if the teacher’s practices were reflective of this message or not.

For example, I decided that Beth did not agree with the message “Teachers should have students memorize procedures.” This decision was based on Beth’s expressed disagreement with a similar statement on the beliefs inventory and her use of the phrase “that’s all I think their goal is” in the above quote. Similarly, she was not observed encouraging her students to memorize procedures, thus, I decided that her classroom practices were not reflective of this message. (A complete list of the paraphrased messages along with my analysis of the messages’ agreement with the teachers’ beliefs and practices is in Appendix E.)

Before observing or interviewing the teachers I looked through each of the resources in order to become familiar with them. After paraphrasing the messages that the teachers interpreted from the resources, I reexamined the resources while focusing upon the teachers' interpretations. I used my own professional judgment to decide if I thought the teachers' interpretations of the resources were reasonable and if I thought that the authors of the resources would agree with these interpretations. When I disagreed with the teachers' interpretations of a resource or I thought that the authors of the resource would disagree with the teachers' interpretations, I included some discussion in the following chapter.

For each teacher, I grouped the paraphrased messages by theme and determined whether the teacher agreed with all of the messages she interpreted within that theme, agreed with some of the messages she interpreted within that theme, or disagreed with all of the messages she interpreted within that theme. Similarly, I examined the relation of the paraphrased messages to the teacher's practices. Did her practices reflect all of the messages she interpreted in this theme? Did her practices reflect some of the messages she interpreted in this theme? Or did her practices reflect none of the messages she interpreted in this theme? For example, Beth interpreted seven messages in the *Source of solution methods* theme. She agreed with some of these and disagreed with others. Similarly, her practices were reflective of some, but not all, of these messages.

In addition to grouping the teachers' interpretations of messages by theme, I also grouped them by teacher and by resource. Each of these different ways of organizing the data informed my analysis, but it seemed that organizing the data by theme emphasized the saliency of some themes and the similarities and differences between teachers within the themes. Organizing the data by themes also seemed to best highlight some of the conflict in messages that the teachers interpret there to be and the tensions that these teachers feel as a result of this conflict. Thus, I elected to organize the bulk of the following chapter around themes.

Chapter 4: Results

This research study examined the messages about mathematics and mathematics teaching that five elementary certified middle school mathematics teachers interpreted from their students' textbooks, school district's curriculum guides and assessments, state's assessments and curriculum framework, a master's degree program in which they were enrolled, and other resources which the teachers felt were significant influences on their teaching. It also examined how these messages related to the teachers' beliefs and observed classroom practices. The data sources included the teachers' responses to the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004), classroom observation notes, scores on the *Reformed Teaching Observation Protocol* (Sawada et al., 2000), and transcripts from interviews and post-observation conversations.

The data revealed that the teachers interpreted messages in 11 themes. Within and among some of the 11 themes there were contradicting and/or competing messages. These contradicting and competing messages created tensions for the teachers. These tensions were most apparent in the three most salient themes (*Concepts and Procedures*, *Question types*, and *Source of solution methods*) as well as in *Technology*. Additionally, the teachers did not always agree with the messages they interpreted and their classroom practices were not always reflective of the messages they interpreted. Thus, this research study also examined how these messages related to the teachers' beliefs and observed classroom practices. In all, seven different types of relations between the messages and the teachers' beliefs and classroom practices were found.

In summary, when the teachers agreed with the messages they interpreted, they attempted to reflect these messages in their practices. However, the presence of these messages and the teachers' efforts to enact these messages were not always enough. The resources often lacked the supports necessary for the teachers to follow through with these messages in their practices. When the teachers disagreed with the messages they interpreted, they sometimes consciously made the decision to not reflect these messages in their practices. They had various degrees of success in this; sometimes they were successful in following their beliefs rather than the messages with which they disagreed, but usually the messages were so pervasive that the teachers were not able to overcome them. At other times, the teachers felt obliged to reflect all of the messages in their practices regardless of their own personal beliefs. The amount of support that the resources provided with regard to these messages was a strong indicator of the degree to which the teachers were successful in reflecting these messages in their practices. Frequently, the reform-oriented messages lacked support and thus the teachers found it difficult to reflect these messages in their practices.

These broad conclusions from the study were derived from synthesis and analysis of the data from several perspectives. Those perspectives and detailed findings are described in the following sections of this chapter. First, in order to answer *Research Question 1*, the messages which the teachers interpreted from the curricular resources are presented. To answer *Research Question 2*, the relations of these messages to the teachers' beliefs and practices are summarized for all 11 themes and detailed examples of the seven relations are given for the four most salient themes.

Research Question 1

What messages do teachers interpret from curricular resources?

The teachers interpreted a multitude of messages from the curricular resources. These messages can be grouped into 11 themes. A list of the paraphrased messages which the teachers interpreted from the curricular resources is presented below (see Appendix E for a complete list of the messages including references to specific resources).

Theme	**Paraphrase of message**
Concepts and Procedures	Teachers should emphasize both procedures and concepts.
	Teachers should emphasize concepts.
	Teachers should emphasize procedures.
	Teachers should emphasize skills.
	Teachers should focus more on concepts than procedures.
	Teachers should focus on "why" in addition to "how.”
	Teachers should make sure that students remember formulas.
	Teachers should value more than procedures.
Connections	Teachers should help students see how mathematical ideas connect.
Cooperative learning	Teachers should have students work alone.
	Teachers should have students work with others.
	Teachers should help students feel comfortable interacting with the teacher.
Differentiation	Teachers should differentiate instruction.
Explanation	Teachers should require students to explain their thinking.
Manipulatives	Teachers should incorporate manipulatives in lessons.
	Teachers should provide opportunities for "hands-on" learning.
Practice	Teachers should have students "practice.”
Question types	Teachers should ask students questions that have "relevance" to their lives.
	Teachers should ask students to solve straightforward questions.
	Teachers should focus lessons on "big problems.”
	Teachers should have students work on "authentic tasks.”
	Teachers should include "little problems" and "big problems" in lessons.
	Teachers should make sure that students are able to solve "application problems.”

	Teachers should make sure that students are able to solve "word problems."
Source of solution methods	Teachers should encourage students to develop their own solution methods.
	Teachers should expose students to "problem solving."
	Teachers should have students "investigate" mathematical ideas.
	Teachers should have students memorize procedures.
	Teachers should help students learn through "discovery."
	Teachers should provide students with methods and worked examples to follow.
	Teachers should try to "break away from that lecture style."
Technology	Teachers should allow students to use calculators.
	Teachers should incorporate technology in lessons.
	Teachers should require students to be able to calculate without a calculator.
	Teachers should show students how to use technology.
Timeline	Teachers should keep up with the timeline.

Because some themes are broader than others, it is perhaps unfair to compare frequencies across themes, but by looking at frequencies we can learn more about which themes were most salient to the teachers. Below Table 3 shows the number of messages[23] the teachers interpreted in each theme. This summary tells us that the teachers talked about messages related to *Concepts and Procedures* most frequently. This was followed by messages about *Source of solution methods,* and *Question types*.

[23] Each paraphrased message attributed to each teacher's discussion of a single resource was counted as one message in this table. For example, Emma and Sarah each interpreted a message in the *Explanation* theme for two different resources, so two messages were counted for each of these teachers. Each of the other three teachers talked about the *Explanation* theme for only one resource, thus only one message was counted for each of them. This resulted in a total count of seven messages in this theme.

Table 3
Number of messages the teachers interpreted in each theme

	Number of messages the teachers interpreted in this theme
Concepts and Procedures	22
Connections	4
Cooperative Learning	7
Differentiation	2
Explanation	7
Manipulatives	6
Practice	5
Question types	13
Source of solution methods	17
Technology	7
Timeline	2

When the messages were analyzed within themes,[24] it became apparent that the teachers interpreted some contradictory messages among the resources. For example, within the *Concepts and Procedures* theme, teachers interpreted that some of the resources send the message that they should focus on teaching concepts while other resources emphasize procedures, and others yet emphasize both concepts and procedures. These contradictory messages were troubling to the teachers. For example, in her interview, Kathleen talked about the fact that she does not see the different resources as aligned:

Kathleen: They're not - none of them [the resources] are aligned.

Christy: Okay. And so what does that do to your teaching? What does that do?

Kathleen: Well, it makes my life harder. Like, we're told they all go together and they honestly don't. . . . But I don't see them aligned at all. Like, I really don't, and I just feel like, you know, it creates a lot more - I mean, I don't mind doing it but it takes a lot more of my time for me to have a really good effective lesson I need to go out and find other resources and talk to other people. (Interview, June 7, 2007)

[24] During data analysis the messages were also sorted and analyzed by resource and by teacher. See Appendices F and G for summaries of these analyses.

Also, because teachers are limited in the amount of time that they can spend on any single activity or idea, even messages from different themes must compete for attention. For example, although they are not necessarily contradictory messages, the teachers find it difficult to both “help students learn through discovery” and “differentiate instruction.” Emma talked about this in her interview:

> We get all these awesome ideas [from the master’s degree program]. They’re great, but in the real world you can’t do that every day. Sometimes you just have to stand up there and teach . . . I think almost to the extreme where they [the instructors in the master’s degree program] want everything to be student-centered, discovery learning. . . . The big thing is I think the time. . . . but it’s like in the real world you can’t use all these ideas. . . . You can’t do everything, even though everything sounds so great. It’s great to take anecdotal notes on every kid and have a portfolio for everybody and monitor how they’re doing and, “Okay. Johnny didn’t get this so let’s go back and work with Johnny.” But you’re between the curriculum guide with deadlines. They’re more strict on the deadlines now than they probably were when you were [a teacher in the district]. I mean, you have like a two-week opening where you have to get those scores in. And then that’s it. So between those deadlines, making sure they’re covering all the objectives for the state assessments before their state assessment is given . . . You can’t do that for every single lesson. It’s just not feasible. (Interview, October 8, 2007)

All of the teachers expressed similar frustrations about the conflicting and/or competing messages they interpreted. They said it is impossible to teach everything that they are supposed to teach in the ways that they are supposed to teach. Additionally, their beliefs about what and how to teach are often at odds with what they feel they are being told to do. As will be shown in the following sections, these tensions among the messages, beliefs, and practices can play out in many different ways.

Research Question 2

How do these messages relate to the teachers' beliefs and observed classroom practices?

One may imagine that teachers interpret messages from the resources which support their personal beliefs, thus using the resources as a *warrant* for their beliefs (Berk, 2004). Conversely, one may imagine that when the teachers are asked to talk about the messages that they see in the resources, they might focus upon messages with which they disagree because these are a source of consternation for them. Similarly, they might interpret messages from the resources which support their classroom practices, thus using the resources as a warrant for their practices (Berk, 2004). Or, they might focus upon messages which are not reflected in their practices because these messages are also disconcerting for them.

Analysis of the relations of the messages that the teachers interpreted from the resources to the teachers' beliefs yielded three primary interactions. These were (a) interpreting there to be messages with which the teacher agrees, (b) interpreting there to be a variety of messages (within a theme) with some of which the teacher agrees and with some of which the teacher disagrees, and (c) interpreting there to be messages with which the teacher disagrees.

Similarly, analysis of the relations of the messages that the teachers interpreted from the resources to the teachers' classroom practices yielded three primary interactions. These were (a) interpreting there to be messages which are reflected in the teacher's classroom practices, (b) interpreting there to be a variety of messages (within a theme) some of which are reflected in the teacher's classroom practices and some of which are not reflected in the teacher's classroom practices, and (c) interpreting there to be messages which are not reflected in the teacher's classroom practices.

When the relation between the teachers' interpretations of messages and beliefs is analyzed along with the relation between the teachers' interpretations of messages and classroom practices, theoretically nine possible relations result. These are summarized in Table 4 below.

Table 4
Possible relations among messages, beliefs, and practices

	(a) Teacher interprets there to be messages which are reflected in her practices	**(b) Teacher interprets there to be messages some of which are reflected in her practices**	**(c) Teacher interprets there to be messages which are not reflected in her practices**
(a) Teacher interprets there to be messages with which she agrees	(aa) Teacher interprets there to be messages with which she agrees and her practices reflect all of these messages	(ab) Teacher interprets there to be messages with which she agrees, but her practices reflect only some of these messages	(ac) Teacher interprets there to be messages with which she agrees, but her practices do not reflect any of these messages
(b) Teacher interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees)	(ba) Teacher interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees), but her practices reflect all of these messages	(bb) Teacher interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees) and her practices reflect some of these messages	(bc) Teacher interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees), but her practices do not reflect any of these messages
(c) Teacher interprets there to be messages with which she disagrees	(ca) Teacher interprets there to be messages with which she disagrees, but her practices reflect all of these messages	(cb) Teacher interprets there to be messages with which she disagrees, but her practices reflect some of these messages	(cc) Teacher interprets there to be messages with which she disagrees and her practices do not reflect any of these messages

In order to analyze the relation of the teachers' interpreted messages to their beliefs and classroom practices, one must first know what the teachers' professed beliefs are. One measure of the teachers' beliefs is the *Teachers' Beliefs about Mathematics and*

Mathematics Teaching inventory (Campbell, 2004 – see Appendix A for a copy of the inventory).   Table 5 summarizes the results of three administrations of this inventory.

Table 5
Results of beliefs inventory

	June 2005	May 2006	May 2007	Average
Amelia	73	70	Did not participate	71.5
Beth	83	83	87	84.3
Emma	88	81	79	82.7
Kathleen	82	86	89	85.7
Sarah	94	89	89	90.7

Results of this beliefs inventory can range from a 30 (very nonreform-oriented) to 150 (very reform-oriented). Thus, Sarah professes to hold the most reform-oriented beliefs of the group followed by Kathleen, Beth, Emma, and Amelia respectively.

In addition to responding to the beliefs inventory, the teachers' talked about their beliefs about mathematics and mathematics teaching in the post-observation conferences and interviews. In particular, the teachers were asked how they envision an ideal mathematics lesson, what they think is most important for their students to learn about mathematics, and how they think students and the teacher should interact. The results of the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory and post-observation conference and interview transcripts were used as evidence of professed beliefs.

In order to analyze the relation of the teachers' interpreted messages to their beliefs and classroom practices, we must also know what their classroom practices are. One measure of the teachers' classroom practices is the *Reformed Teaching Observation*

Protocol (Sawada et al., 2000). Table 6 summarizes the scores on this protocol for three observations of each teacher.

Table 6
Results of observation protocol

	1st Observation	2nd Observation	3rd Observation	Average
Amelia	21	20	17	19.3
Beth	65	84	92	80.3
Emma	28	41	26	31.7
Kathleen	55	37	30	40.7
Sarah	50	33	34	39

Results on this observation protocol can range from 0 (very nonreform-oriented) to 100 (very reform-oriented). Overall, Beth was observed to be most reform-oriented, followed by Kathleen, Sarah, Emma, and Amelia.

In addition to completing the *Reformed Teaching Observation Protocol* (Sawada et al., 2000) for each classroom observation, I took extensive notes of the teachers' language and actions. Also, in post-observation conferences and interviews the teachers talked about their classroom practices and their reasons for these practices. The *Reformed Teaching Observation Protocol* scores, observation notes, and post-observation conference and interview transcripts were used as evidence of classroom practices.

The orderings of teachers from most reform-oriented to least by professed beliefs and observed classroom behaviors are similar in some ways. In both orderings Amelia is the most nonreform-oriented and Emma is the second most nonreform-oriented. However, Sarah professed to having the most reform-oriented beliefs, but Beth was observed to be teaching in the most reform-oriented ways.

In order to gain an overall view of how the teachers' interpretations of messages in the resources were related to their beliefs and practices, I compared the messages that each teacher talked about to the teacher's professed and attributed beliefs and observed classroom practices. I then grouped the messages by theme.[25] For example, in her interview Amelia brought up messages from eight themes. For the messages in four of these eight themes it seemed that she believed in the messages that she interpreted and her classroom practices were reflective of these messages. Thus, for four themes the relation of the messages that Amelia interpreted to her beliefs and practices was (aa). However, for one theme, Amelia interpreted messages with which she agreed, but not all of these messages were reflected in her practices (ab). The other three themes which she talked about fell into (ca), and (cc). A summary for all of the teachers is presented in Table 7 below.

[25] In addition to analyzing the messages in groups by theme, I also analyzed them individually. The results follow: 52 of the 92 interpreted messages seemed to be aligned with the teachers' beliefs and practices (aa); 8 of the messages seemed to be aligned with the teachers' beliefs, but not their practices (ac); 19 of the messages seemed to be aligned with the teachers' practices, but not their beliefs (ca); 13 of the messages seemed to not be aligned with the teachers' practices or beliefs (cc). In order to better describe the tensions that the teachers feel within themes, in this chapter I elected to present the data by theme.

Table 7
Relations among messages, beliefs, and practices by teacher

	(a) Teacher interprets there to be messages which are reflected in her practices	**(b) Teacher interprets there to be messages some of which are reflected in her practices**	**(c) Teacher interprets there to be messages which are not reflected in her practices**
(a) Teacher interprets there to be messages with which she agrees	(aa) Amelia 4 themes Beth 5 themes Emma 4 themes Kathleen 4 themes Sarah 3 themes	(ab) Amelia 1 theme	(ac)
(b) Teacher interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees)	(ba) Beth 1 theme Emma 1 theme Kathleen 3 themes	(bb) Beth 2 themes Emma 1 theme Sarah 1 theme	(bc) Emma 1 theme Kathleen 1 theme
(c) Teacher interprets there to be messages with which she disagrees	(ca) Amelia 2 themes Beth 2 themes Sarah 2 themes	(cb)	(cc) Amelia 1 theme

Looking at the how the interpreted messages relate to the teachers' beliefs and practices in this way tells us several things about these five teachers. Twenty of the 39 message themes[26] are in (aa). This means that for more than half of the time, the teachers focused upon groups of messages with which they agreed and which were reflective of their practices. This may be expected because teachers (and people in general) are often biased toward interpretations consistent with their beliefs and practices (Berk, 2004; Spillane et al., 2004).

[26] All five of the teachers did not talk about each of the 11 message themes. Thus, there are fewer than the 55 possible message themes.

What may be more surprising is that for 18 of the 39 message themes, the teachers either focused entirely upon messages with which they disagreed (row c) or on a combination of messages with some of which they agreed and with some of which they disagreed (row b). This may be because these messages were a source of consternation for the teachers. In contrast, only eight of the 39 message themes were ones of which the teachers' practices are not entirely reflective (columns b and c). Thus it seemed that these teachers focused more often upon messages with which they disagreed (rows b and c) than upon messages from which their practices differed (columns b and c). This may mean that the teachers find messages that go against their beliefs more salient than they find messages that are not reflected in their practices. On the other hand, it may indicate that the teachers often feel obligated to enact messages with which they disagree.

Similarly, it is interesting to look at how the message themes are distributed among the different types of interactions. For example, all five of the teachers talked about *Concepts and Procedures*, but for only one of the teachers, it seemed that her beliefs and practices were aligned with the messages she interpreted about this theme. Thus, the first cell (aa) in Table 8 has the number "1" after *Concepts and Procedures*. Similarly, two of the teachers talked about some messages about *Concepts and Procedures* with which they agreed and also about some messages with which they disagreed. But, their classroom practices were reflective of all of these messages. Thus, the cell (ba) has the number "2" after *Concepts and Procedures*. A summary for all of the themes is presented in Table 8 below.

Table 8
Relations among messages, beliefs, and practices by theme

	(a) Teacher interprets there to be messages which are reflected in her practices	**(b) Teacher interprets there to be messages some of which are reflected in her practices**	**(c) Teacher interprets there to be messages which are not reflected in her practices**
(a) Teacher interprets there to be messages with which she agrees	(aa) Concepts and Procedures 1 Connections 2 Cooperative learning 1 Differentiation 2 Explanation 5 Manipulatives 2 Practice 2 Question types 1 Source of solution methods 1 Technology 3	(ab) Concepts and Procedures 1	(ac)
(b) Teacher interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees)	(ba) Concepts and Procedures 2 Cooperative learning 2 Question types 1	(bb) Concepts and Procedures 1 Question types 2 Source of solution methods 1	(bc) Source of solution methods 2
(c) Teacher interprets there to be messages with which she disagrees	(ca) Manipulatives 1 Practice 1 Question types 1 Source of solution methods 1 Timeline 2	(cb)	(cc) Technology 1

Looking at how the interpreted messages relate to the teachers' beliefs and practices in this way helps us to see similarities in these relations among themes. The teachers' beliefs and practices related in a wide variety of ways to the messages in the *Concepts and Procedures*, *Question types,* and *Source of solution methods* themes. But, all of the teachers who talked about *Connections*, *Differentiation*, and *Explanation* agreed with the messages they saw in these themes and their practices were reflective of these messages (aa). Although some of the teachers agreed and others disagreed with the messages about *Cooperative learning*, *Manipulatives*, *Practice*, and *Timeline*, the practices of all of the teachers who talked about these themes were reflective of the messages they interpreted (column a). Three of the teachers who talked about *Technology* agreed with the messages they interpreted and their practices were reflective of these messages (aa), but one teacher disagreed with the messages she heard about technology and her practices did not reflect these messages (cc).

Although there are nine possible interactions between the messages that the teachers interpreted from the resources and the teachers' beliefs and classroom practices, only seven of these interactions were observed in this study. Examples of each of these seven are discussed in the following sections. The examples come from four message themes: *Concepts and Procedures, Question types*, *Source of solution methods,* and *Technology*. These four themes were chosen because (a) at least four of the five teachers in this study interpreted messages from each of these themes in the resources, (b) more than one message was interpreted for each of these themes, and (c) together these four themes provide examples of the seven types of interactions found in this study. Closer examination of these four themes will help us to understand the relations of the messages the teachers interpreted from the resources to the teachers' beliefs and classroom practices. When the teachers have interpreted messages in the resources that I do not see or that I do not think the authors of the resources would see, I have included some commentary.

Concepts and Procedures

Over the past century, the pendulum of emphasis on concepts and procedures in mathematics instruction has swung back and forth repeatedly. For the first half of the twentieth century, mathematics instruction primarily focused on mathematical procedures. In the 1950s and 1960s the “new math movement” attempted to redirect instruction to mathematical concepts. This was followed by the “back to basics” movement which again focused on procedures. In the 1980s and 1990s the “reform” movement again focused on concepts. In response to this, there have been more recent “back to basics” movements (NRC, 2001, p. 115).

Kilpatrick, Swafford, and Findell (NRC, 2001) define *conceptual understanding* as “comprehension of mathematical concepts, operations, and relations” and *procedural fluency* as “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (p. 5). They also argue that both are important and necessary parts of mathematical proficiency.

Although it is clear that conceptual understanding and procedural fluency are not mutually exclusive instructional goals, many teachers feel that they are and that they must choose between the two. Despite recent efforts to reform mathematics instruction, most students in the United States of America continue to receive instruction which “continues to emphasize the execution of paper-and-pencil skills in arithmetic through demonstrations of procedures followed by repeated practice” (NRC, 2001, p. 4).

All five of the teachers in this study specifically brought up messages about concepts and procedures that they interpreted from the resources. In summary, they said that the various resources send the messages that:

- Teachers should emphasize both procedures and concepts.
- Teachers should emphasize concepts.
- Teachers should emphasize procedures.
- Teachers should emphasize skills.
- Teachers should focus more on concepts than procedures.
- Teachers should focus on "why" in addition to "how."
- Teachers should make sure that students remember formulas.
- Teachers should value more than procedures.

Some of these interpreted messages are similar to the beliefs of the teachers, but some are quite different. In their interviews Amelia, Emma, and Sarah indicated that it is important to find a balance between procedures and concepts, while Beth and Kathleen indicated that concepts are more important than procedures. None of the statements on the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004) specifically focuses on both concepts and procedures. Table 9 contains a summary of how the teachers responded to a statement about procedures.

Table 9
Teachers' responses to statement #27 on beliefs inventory: Learning mathematics requires a good memory because you must remember how to carry out procedures and, when solving an application problem, you have to remember which procedure to use.

	June 2005	May 2006	May 2007	Average
Amelia	4 Agree	3	Did not participate	3.5
Beth	4 Agree	4	2	3.3
Emma	2 Disagree	2	2	2
Kathleen	3 Not sure	2	2	2.3
Sarah	3 Not sure	2	2	2.3

Note. This statement is phrased to support nonreform-oriented models of instruction so a lower score reflects a more reform-oriented belief.[27]

The ordering of the teachers' responses to this statement is somewhat different than the ordering resulting from the comparison of total scores on the inventory. This may be because the statement talks about both procedures (which are considered to be important by both reform and nonreform supporters) and memorization (which reform supporters generally discourage or urge to be delayed until after procedures are understood). For this statement Emma responded with the most reform-oriented response, followed by Kathleen and Sarah, Beth, and Amelia. For the overall inventory scores, Sarah was most reform-oriented, followed by Kathleen, Beth, Emma, and Amelia.

In their observed lessons, Emma, Kathleen, and Sarah emphasized both concepts and procedures, while Amelia emphasized primarily procedures and Beth emphasized primarily concepts. None of the items on the *Reformed Teaching Observation Protocol* (Sawada et al., 2000) focuses on the balance between concepts and procedures, but in

[27] The scoring scale for this and other statements which are phrased to support nonreform-oriented models of instruction were reversed in the calculation of the overall beliefs inventory score.

Table 10 below is a summary of how the teachers scored[28] on one item on the observation protocol about conceptual understanding.

Table 10
Teachers' scores on item #7 on observation protocol: The lesson promoted strongly coherent conceptual understanding.

	1st Observation	2nd Observation	3rd Observation	Average
Amelia	2	1	2	1.7
Beth	2	3	3	2.7
Emma	1	1	1	1
Kathleen	2	3	2	2.3
Sarah	1	1	3	1.7

The teachers' scores on this item indicate that for this aspect of teaching, Beth was observed to be the most reform-oriented followed by Kathleen, Amelia and Sarah, and Emma. For the overall RTOP scores, Beth was observed to be teaching in the most reform-oriented ways, followed by Kathleen, Sarah, Emma, and Amelia. Note that a teacher's *emphasis* on concepts does not necessarily mean that the lesson promoted strongly coherent conceptual understanding. For example, although Emma emphasized both concepts and procedures, it did not seem that this was enough for her students to gain coherent conceptual understanding. In contrast, although Amelia focused primarily on procedures, it did seem that her instruction promoted conceptual understanding.

The relation of the interpreted messages in this theme and the teachers' beliefs and practices was different for each teacher. Sarah interpreted there to be messages with which she agreed and her practices were reflective of these messages (aa). Amelia also

[28] Each item was rated on a scale ranging from 0 to 4. A "0" was chosen if the characteristic never occurred in the lesson. A score of "4" indicates that the item is very descriptive of the lesson. Intermediate ratings do not reflect the number of times an item occurred, but rather the degree to which that item was characteristic of the lesson observed (Sawada et al., 2000).

interpreted there to be messages with which she agreed, but her practices did not reflect all of these messages (ab). Emma and Kathleen interpreted there to be a variety of messages (with some of which they agreed and with some of which they disagreed), but their practices were reflective of all of these messages (ba). Beth also interpreted there to be a variety of messages. She agreed with some of these messages and disagreed with others and her practices reflected some of these messages and not others (bb). In the following sections, more detail about the messages about *concepts and procedures* which Sarah, Amelia, Kathleen, and Beth interpreted from the resources and the relation of these messages to their beliefs and classroom practices is presented.

Sarah – Interprets there to be messages with which she agrees and her practices reflect all of these messages (aa)

Sarah interpreted there to be messages about concepts and procedures in three of the resources that we discussed. With regard to the master's degree program, she specifically talked about the first course in the program. This course focused on trends in mathematics education and in this course, the teachers read parts of *Adding It Up* (NRC, 2001). Reading this book helped to change Sarah's thinking about the balance between procedures and concepts:

> *Christy*: Okay. And then what about the program? What do you think the professors – or you could choose just one professor if you feel like they've had different views, but what do you think the program wants students to learn?

> *Sarah*: I think the program's focus is to kind of like give the teachers insight, to then be able to give the students – and change their way of teaching for the students. . . . but also *Adding It Up* – that book and resource was presented in that course.

Christy: Did you read that book?

Sarah: She [the professor] pulled Chapter 4 of that book, talking about the strands, conceptual, procedural, and that was kind of like the first time like I had an explanation attached. Like I could understand it but I couldn't articulate the difference between procedural and conceptual knowledge and then the strategic competence and those five strands, and that basically like – I think in my mind, that's what I've adapted as – and I've since got the book and use it on every research and just refer to it, because it's like an organizational method, just to help me organize and see like *how balanced it actually has to be* and just what all it encompasses, and it's not your mother's math course or whatever. It definitely gave me the bigger perspective I think, and then I was able to run with it. (personal communication, November 4, 2007)

This was paraphrased as "Teachers should emphasize both procedures and concepts."

Similarly, in talking about the school district's curricular materials, Sarah interpreted there to be an emphasis on both procedures and concepts. However, she thinks that she is one of the few teachers to see this in the materials. She thinks that most other teachers see the materials as emphasizing procedures above concepts:

Christy: So starting with the [school district] materials, what do you think the authors of the [school district] materials most want kids to learn?

Sarah: Well, I think it's kind of inferred that they want the kids to be proficient in not just procedural understanding but conceptual understanding. But it's also – I think through my other experiences that I'm able to pull that out. . .

Christy: How do you think they want teachers to be teaching?

Sarah: Well, I think it's kind of like mixed messages, because they'll give like essential problems from the book, but if teachers just see 14 to – I forget how they do it in the book – where they'll say, in the guide, they'll say like Selected problems on Page Such-and-Such, from numbers 14 - 37, and teachers will just say, okay, do 14 - 37. Like there is kind of just that step missing that they can't necessarily jump out and say from the guide that you kind of have to infer, and if you don't, *it's more procedural based, but if you actually read it and make sense of it, that's not what they want*, but how do they get that message across?

Christy: So you said that you think you're getting the message, but everyone –

Sarah: Right. I think I'm getting the message, but I feel like I'd probably be in the minority in that because of the program and the undergrad and the other work I've done. (personal communication, November 4, 2007)

This was also paraphrased as "Teachers should emphasize procedures and concepts."

When asked about the goals of the authors of *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) Sarah said, "I think they're kind of still in the same tract of the procedural. . . " (personal communication, November 4, 2007). This was paraphrased as "Teachers should emphasize procedures."

Although the messages that teachers should "emphasize both procedures and concepts" and "emphasize procedures" may appear to be conflicting messages, for Sarah they did not seem to be. Concepts and procedures may compete for time and attention, but Sarah believes that it is important to find a balance between the two. This was especially apparent in her talk about *Adding It Up*. It is likely, however, that she would disagree with a message statement such as "Teachers should emphasize procedures over concepts."

Sarah's classroom practices also reflected a balance between concepts and procedures. For example, in one of the lessons she was observed teaching, her students were reviewing how to solve equations such as y + 5 = 8. Although her students were proficient at solving this by subtracting 5 from both sides of the equation, Sarah pushed her students to think about more than the procedure by asking questions such as "Why subtraction?" (observation notes, May 17, 2007).

Amelia – Interprets there to be messages with which she agrees, but her practices reflect only some of these messages (ab)

Amelia talked about messages about concepts and procedures in three resources. When asked how she imagines the Maryland State Superintendent of Schools might want her to teach, she said, that in order to answer the questions on the state assessments, you have to “focus more on the conceptual side than the procedurals” (personal communication, November 1, 2006). This was paraphrased as “Teachers should focus more on concepts than procedures.” When asked how she perceives the writers of *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) would describe what and how students should learn, Amelia said, “they're definitely procedure” (personal communication, November 1, 2006). This was paraphrased as “Teachers should emphasize procedures.” When talking about the master’s degree program, she said that this program has taught her to value more than students’ abilities to apply procedures. This was paraphrased as “Teachers should value more than procedures.”

Although some of these messages may seem to be conflicting, Amelia indicated that she believes in all of them. She feels that both concepts and procedures need to be emphasized. She even noted that she believes procedures are important even though this is not a widely accepted position. She said:

> I feel like people think it's a crime to say that you know, knowing how to do the procedure is important. So and I think that that [number sense] is important and I think you get that with the conceptual piece, but I also don't think that procedure is like a bad thing. Like people would say procedural almost like it's a bad thing. . . . I kind of don't think it's a – you know, like in conjunction with conceptual understanding. (personal communication, November 1, 2006)

Although Amelia claimed to believe in the importance of both procedures and concepts, her classroom practices reflected more of a focus on procedures. For example, she showed her students how to solve division by decimals questions by moving the decimal point in the divisor and dividend and then had her students practice the algorithm for homework. The next day she modeled decimal division with base-ten blocks on the overhead projector and then had the students work in small groups to practice solving similar problems and make sure they got the same answer as they would with the paper-and-pencil algorithm.

It is interesting to note that for the topic of decimal division she reversed the order of lessons that the school district and textbook recommend. Both the school district's curriculum guide and the *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) textbook have students use base-ten blocks to divide decimals (Lesson 4-6A) before learning the traditional algorithm of moving the decimal point in the divisor and dividend (Lesson 4-6).

When asked why she changed the order of the lessons, she said:

> Sometimes I feel like – I never really know what's better, like what comes first the egg or the chicken (Laughter) whatever the chicken and the egg. Like sometimes, I feel like with our lower level kids that sometimes procedure first actually is more helpful to them when it comes to trying to understand the concept, the concept behind it and then I think some things lend themselves to the other way around. I don't know. I opted to go procedural with the division. They could relate to it with whole numbers. . . . (personal communication, November 1, 2006)

Although Amelia was attempting to help her students understand the concepts by first gaining fluency in the procedures, it did not seem that the students learned anything more than procedures from this activity.

This focus on procedures may be because when Amelia was learning mathematics as a student, she feels that she focused only on procedures. But, Amelia is now making an effort to focus on both procedures and concepts. She said:

> I feel that I learned procedure. I never learned meaning behind anything, but I always thought that I was very good at remembering my directions. . . . but you know, I'm changing, like things are making more sense to me. I tell the kids all the time, like I understand fractions and decimals and all these other things so much more now because I am teaching them, you know, which is why I try and have them talk to each other more and explain themselves more, and maybe teach each other – because I feel that's how you kind of learn it. You know? So that to me has changed a lot, like I am trying to go away from procedure now. And now I feel like I am asking why more often.… (interview with Badertscher, September 2006)

Amelia is finding it difficult, however, to incorporate more of a focus on concepts into her teaching. She said, "I feel like I am confident in the fact that I am trying to do the right thing. But I am really lacking how to teach it conceptually in a lot of ways" (interview with Badertscher, June 2007).

The authors of *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) might disagree with Amelia's interpretation that they focus on procedures, but others agree with Amelia. In their evaluation of the textbook, AAAS said, "In some lessons, there are prompts in the margins that attempt to address prerequisites but these focus only on procedures. . . . There are few strategies to build conceptual thinking and no suggestions for correcting procedural errors" (2000, p. 119). Thus, this resource provides little support to Amelia as she tries to learn "to teach it conceptually."

Kathleen – Interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees), but her practices reflect all of these messages (ba)

Kathleen interpreted there to be messages about concepts and procedures in all four of the resources about which every teacher was asked and also in NCTM documents. When asked what she thinks the authors of *Gateways to Algebra and Geometry: An*

Integrated Approach (Benson et al., 1997) think is most important for students to learn about mathematics she said, “This is very procedural. So I’m going to say they just want to memorize processes. Because even when they do examples, they don’t tell you why they do stuff, they just tell you what to do” (personal communication, June 7, 2007). This was paraphrased as “Teachers should emphasize procedures.” Kathleen indicated that she disagrees with this when she was asked how this fits with her own priorities for her lessons. She said, “It doesn’t because I like to teach it conceptually” (personal communication, June 7, 2007).

Similarly, Kathleen indicated disagreement with some of the messages she sees in the school district’s materials. When asked what she thinks the authors of the school district’s materials think is most important for students to learn about mathematics, Kathleen said:

> Yeah, *procedures*. I mean, these are nice that they do give really good definitions and they have all the bold printed things, bold printed. But, again, it’s like, you know, they are the methods, these are the ways. And even with the equations it’s, “These are the steps.” So, I think it’s the same idea. I mean, but I think they are trying to get away from - *I think this was a start to get away from just learning the straight procedures, and then now we’ve kept going so now we’re going to get more, I guess more conceptual.* But I think this was, like, their start to try to teach - because I do know that of all the curriculums when I came into [the school district] in 2000, of all the curriculums, and this was written in ’98, this was the most abstract, top one. That, you know, like now, it’s [the other curriculum guides] because they’ve rewritten them or have more, like, thought provoking and conceptual stuff in it and more of the, you know, how’s and why’s and explain. . . . But I don’t see them aligned at all. Like, I really don’t, and I just feel like, you know, it creates a lot more - I mean, I don’t mind doing it but it takes a lot more of my time for me to have a really good effective lesson I need to go out and find other resources and talk to other people. And, you know, if I just relied on this stuff I would be teaching all procedures. (personal communication, June 7, 2007)

Two messages were paraphrased from this passage: “Teachers should emphasize procedures” and “Teachers should emphasize both procedures and concepts.” Unlike

Sarah, it seemed that Kathleen saw a conflict between these two messages. Her tone when talking about procedures indicated that she disagreed with an emphasis on procedures. When talking about the other resources it became apparent that she thinks the emphasis should be on concepts.

Kathleen was also asked about what she thinks the master's degree program wants students to learn about mathematics. She said, "I think it wants them to know the big picture, like, how everything connects and works together and not just how but why." This was paraphrased as "Teachers should focus on 'why' in addition to 'how.'" Kathleen indicated agreement with this message by adding "that's where I'm going with my teaching" (personal communication, June 7, 2007).

Similarly, Kathleen agreed with the focus on concepts that she saw in the State of Maryland's assessments and the NCTM documents. She said that for students to be able to answer the questions on the state assessments, "the kids really have to conceptually understand all the foundation stuff to be able to answer them because most of them are not direct computation. So they have to have some concepts, some understandings of some concepts. . . . So I think it's, you know, the word problems are looking for more of a conceptually based understanding" (personal communication, June 7, 2007). Kathleen also said that the authors of NCTM documents "definitely want students talking and feel like they want a conceptual, you know, they want it to be hands-on, they want conceptual learning to be, you know, to take place. . ." (personal communication, June 7, 2007). These messages from the State of Maryland and the NCTM were paraphrased as "Teachers should emphasize concepts."

Although Kathleen seemed to have mixed feelings about the messages she saw about concepts and procedures in the resources, all of these messages were reflected in her teaching. As indicated on the RTOP, her lessons promoted coherent conceptual understanding. But, her lesson warm-ups and homework assignments typically focused on building procedural fluency. Thus, she promoted both procedures and concepts in her teaching. It seemed that she assigned work that focused on procedures not because she valued procedural fluency, but rather because these types of questions were readily available in the textbook and school district's curricular materials.

It is interesting that Kathleen saw the NCTM as emphasizing concepts over procedures. Throughout their documents, they talk about both concepts and procedures. For example, in the introduction of *Principles and Standards for School Mathematics* (NCTM, 2000), the NCTM describes an ideal classroom. It states "The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding" (p. 3). Later, it stresses the importance of factual knowledge, procedural proficiency, and conceptual understanding (NCTM, 2000, p. 20). Kathleen is not alone, however, in seeing the NCTM as focusing on concepts significantly more than on procedures. This misrepresentation has been frequently made by critics of the NCTM.

Beth – Interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees) and her practices reflect some of these messages (bb)

Beth interpreted there to be a variety of messages about concepts and procedures. She agreed with some of these messages, but disagreed with others.

When asked about *Pre-Algebra* (Malloy et al., 2003), Beth said:

> It seems to me their big goal is for them to learn procedures of how to do things. And the way they're laid out it's like procedures first, then problem solving. And so, yeah, that's all I think their goal is to have them memorize procedures and show them examples of different procedures. . . . (personal communication, November 3, 2006)

This was paraphrased as "Teachers should emphasize procedures." Beth's tone and word choices such as "that's all I think their goal is" indicated that she disagreed with this message.

Similarly, Beth disagreed with the messages about concepts and procedures that she saw in the State of Maryland documents. When asked what she thinks the writers of these documents would say is most important for kids to learn, she said:

> *Beth*: Basic skills. That's what it seems like.
>
> *Christy*: And do you get a feeling for how you would – what a classroom would look like, what they would think a good classroom would look like?
>
> *Beth*: I really don't 'cause it seems extremely disjointed to me the way it's written. It is like a collection of skills so I don't know how they would envision teaching a collection of skills. I don't know. They – and on the MSDE [Maryland State Department of Education] website they have all this great stuff about cognitive levels of demand and extending thinking and all this, but it's so narrow, the scope of the VSC [Voluntary State Curriculum]. And the way it's written, it's just all these separate skills. It's not written where there's any connections or big ideas to me, in my opinion. (personal communication, October 10, 2007)

This was paraphrased as "Teachers should emphasize skills."

Beth also saw this message in the textbook and some of the school district's materials. She described *Pre-Algebra* (Malloy et al., 2003) as "what's the skill, let's practice the skill, and then they throw in a couple of application problems" (personal communication, May 16, 2007). She also described the school district's curriculum guides as “all skill, skill, skill” (personal communication, May 16, 2007). Beth’s description of these messages made clear that she disagreed with an emphasis on skills and procedures.

On the other hand, Beth did see some messages with which she agreed. When asked about the school district’s curriculum guides, she also said,

> *Beth*: The curriculum guide says they want you to start with concept development and have them you know, have exposure to problem solving, but the curriculum guide is written around the textbook. So the guide will say, "Do this sequence of lessons in the textbook – but make sure it's problem solving."
>
> *Christy*: . . . What do you think they would say is the most important or what do you see in the curriculum guide as the most important thing for kids to learn? What do you think they think is the most important thing for kids to learn about math?
>
> *Beth*: I think they think the big ideas, *the general concepts,* and the *conceptual understanding* are the most important because that’s the way they start everything out and then it gets drilled down into the specific indicators and I think that’s where it gets lost and the conceptual knowledge becomes procedural. Central office starts out with, “Kids should know these grand ideas and these connections and everything,” but then when it gets into the classroom, some teachers are focusing on each specific indicator and not making the connections themselves. But yeah, that’s what I think. (personal communication, October 10, 2007)

This was paraphrased as “Teachers should emphasize concepts.” Note that Beth saw this message, but thought that the message was not obvious to other teachers. All of the instances in which the teachers talked about how they were able to “correctly” interpret the resources while others were not, were in discussion of the school district’s materials. This may be because one of the primary writers of the school district’s materials was a member of the master’s degree program in which they were enrolled. Because of their familiarity with this writer, the teachers may feel that they know what her intentions are. They do not have this familiarity with the writers of any of the other resources.

Beth also thought that the master's degree program wants the teachers "to teach conceptually. They want us to teach the concepts and teach for – I keep calling it 'teach for understanding.' Not teach for process learning but really how does it all work. . . ." (personal communication, October 10, 2007). This focus on concepts was clearly in line with Beth's beliefs. She said, "I used to think you needed a skill before you could do the abstract stuff. Now, I know in my heart that that isn't true" (personal communication, May 16, 2007).

Beth's classroom practices were reflective of the messages with which she agreed and not reflective of the messages with which she disagreed. One reason she may have felt a certain degree of freedom in choosing which messages to attend to may have been because she is chair of the mathematics department in her school.

She stated that one of her goals of her students is for them to leave her class at the end of the year knowing "that math isn't just this set of rules that you learn. It's a way of thinking and a way of looking at things and you can apply it" (personal communication, November 3, 2006). This goal was apparent in her teaching. Her instruction focused primarily on concepts rather than skills and procedures. For example in a conversation with Eden Badertscher, Beth shared how she feels about memorization of basic facts:

> … they [her students] don't all know all their facts, but we don't care, 'cause we use a calculator. Using a calculator has helped some of them to learn, like umm…positives and negatives, how to multiply, divide, add, subtract integers, like…they get it now. Like, after seeing it…cause I kept saying this…I don't care if you don't know your facts…I saw a kid in there who's 13, who still goes like "this" with his fingers. And I say, just use the calculator… "No, no no, I gotta figure it out myself." I'm like, "Why? Why memorize something you don't have to if it's going to slow you down?" (interview with Badertscher, April 2007)

It seemed that Beth felt that students should focus on concepts rather than skills and procedures by taking full advantage of the help that technology can provide.

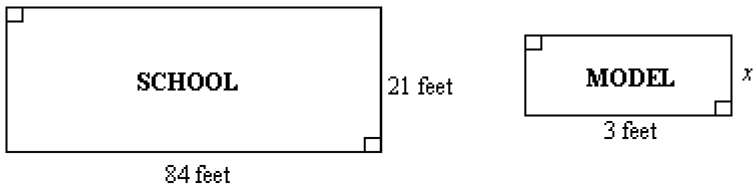
This acceptance of procedural aids may stem from Beth's own experiences as a student. When Eden Badertscher asked about mathematical experiences that have challenged how she saw herself as a learner, Beth shared:

> The only ones in high school that challenged me were the one algebra 2/trig class that I took, and that was the class where the teacher recognized that every time I had to do anything with a unit circle, I would flip things and transpose them, and she saw that I understood the concept, but I couldn't get all the particulars straight. And she let me keep a sheet on my desk with a unit circle with all the cosines and sines and all the relationships, and just being allowed to have that reference made me think I was smart, and that I could do it - even though I couldn't remember everything. Like, all previously I thought "I am so stupid" because I couldn't remember, I mean, you know "beat yourself up," and if I was smart, I would remember this stuff and I would be able to do it, and that changed the way I thought of myself as a learner. And like, the girl behind me couldn't do computation. She could do everything else, but she couldn't multiply, so she got to use a calculator. I thought that was the coolest thing. Like, I needed something and the girl behind me needed something else….and now I mean, going back to being a teacher…but she showed me that everybody learns in a different way. So that just because I didn't do it the same way someone else did, it was still valuable, my way of learning was still valuable. (personal communication, September 2006)

Beth saw this experience as pivotal in shaping her thoughts about herself as a student (Badertscher, 2007, p. 91) and by extension her thoughts about herself as a teacher. This story also sheds light on Beth's beliefs about mathematics; to Beth mathematics is much more than computation and this is reflected in her teaching.

It is surprising that Beth interpreted the Maryland School Assessments to emphasize only skills. It seems to me that some of the questions on the Maryland School Assessment require both conceptual understanding and procedural knowledge. This is especially evident on the Extended Constructed Response questions. For example, one of the sample Extended Constructed Response questions for 8th grade mathematics is shown in Figure 1 below.

Cedric is making a scale model of his school. The dimensions of his school and of his model are shown below.



Note: The figures are not drawn to scale.

Step A

What is the length, in feet, of side *x* of the model?

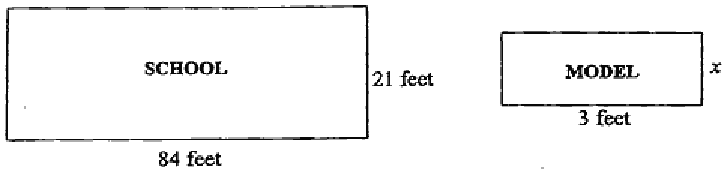
Step B

- Use what you know about similar polygons to justify why your value of side *x* is correct. Use words, numbers, and/or symbols in your justification.
- If Cedric changes the length of his model from 3 feet to 6 feet, explain how this change will affect the value of side *x*. Use words, numbers, and/or symbols in your explanation.

Figure 1. Sample Extended Constructed Response question from the 8th grade Maryland School Assessment (MSDE, 2003).

Obviously procedural knowledge is necessary to perform the calculations to answer Step A. But, in order to receive full credit on Step B, students must also demonstrate conceptual understanding of similarity and scale. Figure 2 below, contains a sample student response to this question.

Cedric is making a scale model of his school. The dimensions of his school and of his model are shown below.



Note: The figures are not drawn to scale.

Step A

What is the length, in feet, of side *x* of the model?

x: .75 feet

Step B

- Use what you know about similar polygons to justify why your value of side *x* is correct. Use words, numbers, and/or symbols in your justification.
- If Cedric changes the length of his model from 3 feet to 6 feet, explain how this change will affect the value of side *x*. Use words, numbers, and/or symbols in your explanation.

Side x=.75. I know this because 84÷3=28, and 21÷28 rounds to .75. If Cedric changes the length from 3 to 6 feet, side x= 1.5 feet, because 84÷6=14 and 21÷14=1.5.

Figure 2. Sample student response to an Extended Constructed Response question from the 8th grade Maryland School Assessment (MSDE, 2003).

This student received full credit on Step A, but because this student did not include connections to these concepts, he received only two of a possible three points on Step B. The MSDE describes the rationale for this score below:

> This response demonstrates a general understanding and analysis of the problem. A reasonable strategy of using proportional reasoning to determine the value for side x is indicated. The justification, presented as a numerical explanation, for why this value is correct, is only partially developed: "84/3=28. . . 21/28 rounds to .75." The explanation for how the change will affect the value of side x is clear:

> "84/6=14. . . 21/14=1.5." Appropriate supportive numbers are provided. However, connections to the *concepts* of similarity and scale are missing [emphasis added]. (MSDE, 2003)

Thus, it seems that the Maryland School Assessment requires both conceptual understanding and procedural knowledge.

Question types

One of the most prominent features of any curricular resource is the types of questions that it asks. Do the questions tell what operations to use or is that left to the readers to determine? Are the questions contextualized? If so, are the contexts realistic and/or meaningful to the students?

Traditionally textbooks in the United States of America have emphasized computational questions devoid of contexts. As a result, our students have had great difficulty in applying what they learn in a particular lesson to other situations both inside and outside of school. In order to help students succeed in applying their knowledge to new situations, some have argued that students should learn mathematics in contexts similar to the situations in which the knowledge will be needed (Barab & Plucker, 2002; Greeno & Moore, 1993). Others have argued that mathematics questions need to not only be contextual, but that these contexts need to be relevant to the students' lives (Gutstein, 2003; Secada, 1992; Tate, 1994). On the other hand, Goldenberg (1999) argues that "real life application" problems are not necessarily more interesting to or motivating for students than are "good puzzles."

All five of the teachers in this study specifically brought up messages they interpreted from the resources about the types of questions they are supposed to ask their students. They talked about *relevant*, *straightforward*, *big*, *authentic*, *little*, *application*, and *word* problems. In summary, they said that the various resources send the messages that:

- Teachers should ask students questions that have "relevance" to their lives.
- Teachers should ask students to solve straightforward questions.
- Teachers should focus lessons on "big problems."
- Teachers should have students work on "authentic tasks."
- Teachers should include "little problems" and "big problems" in lessons.
- Teachers should make sure that students are able to solve "application problems."
- Teachers should make sure that students are able to solve "word problems."

Some of these interpreted messages were similar to the beliefs of the teachers, but some were quite different. Most of the teachers thought that all types of questions are important at times, but that they should receive different levels of emphasis. None of the statements on the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004) specifically addresses all of the issues present in the messages the teachers interpreted about question types, but Table 11 contains a summary of how the teachers responded to a statement about "real-life problems."

Table 11
Teachers' responses to statement #5 on beliefs inventory: I regularly have my students work though real-life math problems that are of interest to them.

	June 2005	May 2006	May 2007	Average
Amelia	4 Agree	3	Did not participate	3.5
Beth	4 Agree	4	4	4
Emma	4 Agree	4	4	4
Kathleen	4 Agree	5	4	4.3
Sarah	5 Strongly Agree	5	5	5

It should be noted that the teachers are required to administer school district-wide assessments and state-wide assessments. Thus, the teachers' responses to this statement may not be a reflection of their beliefs about the types of questions that they feel they should assign to their students. Rather, it may reflect their thoughts about how realistic and interesting they think these required assessments and the types of questions they assign to their students to prepare them for these assessments are.

Similarly, none of the items on the *Reformed Teaching Observation Protocol* (Sawada et al., 2000) specifically addresses the issues present in the teachers' statements about question types. Table 12 contains a summary of how the teachers scored[29] on one item about "real world phenomena."

[29] Each item was rated on a scale ranging from 0 to 4. A "0" was chosen if the characteristic never occurred in the lesson. A score of "4" indicates that the item is very descriptive of the lesson. Intermediate ratings do not reflect the number of times an item occurred, but rather the degree to which that item was characteristic of the lesson observed (Sawada et al., 2000).

Table 12
Teachers' scores on item #10 on observation protocol: Connections with other content disciplines and/or real world phenomena were explored and valued.

	1st Observation	2nd Observation	3rd Observation	Average
Amelia	2	1	1	1.3
Beth	1	1	3	1.7
Emma	0	0	0	0
Kathleen	2	1	0	1
Sarah	1	2	3	2

The teachers' scores on this item indicate that for this aspect of teaching, Sarah was observed to be the most reform-oriented followed by Beth, Amelia, Kathleen, and Emma. For the overall RTOP scores, Beth was observed to be teaching in the most reform-oriented ways, followed by Kathleen, Sarah, Emma, and Amelia.

The relation of the interpreted messages in this theme and the teachers' beliefs and practices was somewhat different for each teacher. Beth interpreted there to be messages with which she agreed and her practices were reflective of these messages (aa). Kathleen interpreted there to be a variety of messages (with some of which she agreed and with some of which she disagreed), but her practices reflected all of these messages (ba). Emma and Sarah also interpreted there to be a variety of messages. They agreed with some of these messages and disagreed with others and their practices reflected some of these messages and not others (bb). Amelia interpreted there to be messages with which she disagreed, but her practices were reflective of these messages (ca). In the following sections, more detail about the messages about *question types* which Beth, Kathleen, Emma, and Amelia interpreted from the resources and the relation of these messages to their beliefs and classroom practices is presented.

Beth – Interprets there to be messages with which she agrees and her practices reflect all of these messages (aa)

In her interview, Beth interpreted there to be messages about the types of questions she should ask her students in two of the resources that we discussed. The first instance was when we discussed the *Pre-Algebra* textbook (Malloy et al., 2003). Beth said:

> …it's ok for practice. It's built the same way the [school district's curriculum] guide is – they built the guide around the book. It's what's the skill, let's practice the skill, and then they throw in a couple of application problems. So what I do is take the application problems out and use them to teach. And then, what are the skills that we need. That's what I finally started doing with the book. It was better for them [her students]. To have an actual problem to solve. And then if they needed a skill to solve it, I could teach it to them and then it was useful – working backwards. (personal communication, May 16, 2007)

This was paraphrased as "Teachers should make sure that students are able to solve 'application problems.'" The above quote indicated that Beth did not agree with the suggested order of instruction she interpreted from the textbook (focusing on skills before applications), but her use of the application problems indicated that she believes that they are important.

Later, when talking about the Super Source series (ETA/Cuisenaire, 1996a and 1996b), Beth compared the types of questions in this series to the types of questions in the textbook. She said:

> It's all stuff like designing kites and marking out steepness of stairways and just actual real world stuff where they have to solve a problem, but there's no math – there's no traditional math involved. It's not a word problem. It's like an authentic task where math has to be used but they don't realize they're using it until after. (personal communication, October 10, 2007)

This was paraphrased as "Teachers should have students work on 'authentic tasks.'" Beth's decision to use this supplementary resource and her positive tone indicated that she believed that it is important to have her students work on authentic tasks. It is also significant that Beth noted that she saw a difference between *word problems* and *authentic tasks*.

These messages about application problems and authentic tasks are closely related to Beth's goals for her students. She wants her students to see the applicability of mathematics. When asked what she most wants her students to learn about mathematics she said:

> It wouldn't be like a specific content. I – honestly I want them to come away knowing that they can think about what they're seeing mathematically. . . . that math isn't just this set of rules that you learn. It's a way of thinking and a way of looking at things and you can apply it. (personal communication, November 3, 2006)

Beth's classroom practices were reflective of these messages as well. For example, on the days on which she was observed, Beth's students were learning about modular arithmetic and transformational geometry with the goal of using these to produce geometric artwork. Thus, Beth helped her students see an application of mathematics to another discipline.

Kathleen – Interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees), but her practices reflect all of these messages (ba)

In her interview, Kathleen interpreted there to be messages about the types of questions she should ask her students in two of the resources that we discussed. She disagreed with some of these messages and agreed with others. She described the questions in the school district's curriculum guide as "just very straightforward questions." This was paraphrased as "Teachers should ask students to solve straightforward questions." She indicated her disagreement with these questions by saying that they are not very thought provoking (personal communication, June 7, 2007).

When asked about the state assessments Kathleen said, "there's a lot of word problems in there where they have to explain their thinking. And the nice thing with the word problems that they have in there is there's not one right way to solve the problem" (personal communication, June 7, 2007). This was paraphrased as "Teachers should make sure that students are able to solve 'word problems.'" Kathleen indicated her agreement with this by saying "the nice thing with the word problems. . ."

Despite her mixed feelings about these messages, Kathleen's classroom practices reflected both of these messages. Most of the questions she assigned to her students to work on during class were straightforward computations, but most of the questions she assigned for homework were word problems. Most of these questions came from workbooks written by the school district.

Emma – Interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees) and her practices reflect some of these messages (bb)

In her interview, Emma interpreted messages about the types of questions she should ask her students from both the school district's curriculum guide and her principal. When talking about the school district's curriculum guide she talked at length about the types of application problems the assessments ask. This was paraphrased as "Teachers should make sure that students are able to solve 'application problems.'" She expressed her disagreement with these problems by calling them “obscure” (personal communication, October 8, 2007).

In contrast, she spoke positively about her principal’s emphasis on “relevance” (personal communication, October, 8, 2007). This was paraphrased as “Teachers should ask students questions that have ‘relevance’ to their lives.” One of her primary goals of teaching is for her to help her students learn how mathematics is relevant to their lives.

Interestingly, Emma was observed enacting the message with which she disagreed, but not observed enacting the message with which she agreed. Although she disliked the application problems that the school district’s and state’s assessments contain, she asked her students to solve similar problems for homework on one of the days she was observed teaching. It seemed that she did this because she feels obligated to prepare her students for the school district’s and state’s assessments. Conversely, despite her agreement with the message that it is important to ask students questions which are relevant to their lives, Emma has difficulty reflecting this in her teaching. She said:

> *Emma:* I have trouble with that [relevance] in math. For me, that’s very difficult. . . . Things like integers, it’s very easy, fractions, but when you get to some of those obscure things like box-and-whisker plots, it’s very hard to make that connection. I have a big problem now, “Why do we need to know what a quadratic is?” Well, if you don’t go into any higher-level mathematical thing, or something involving the sciences or something, you don’t use it.
>
> *Christy:* So what do you say to your students if they would ask that question?
>
> *Emma:* I haven’t come across that one lately, but usually I would say, “Well, it gives you the opportunity to advance. You need to know this if you do want to go further, so it opens you up so you have all these opportunities.” So it’s kind of the “you’ve got to learn it” answer. (personal communication, October 8, 2007)

In her observed lessons, almost none of the questions she asked were contextualized. The few questions which might be considered to be “application problems” did not seem to be very relevant to her students’ lives. For example, one question was:

> A cone-shaped icicle on a gingerbread house will be dipped in frosting. The icicle is 1 centimeter in diameter and the slant height is 7 centimeters. What is its total surface area? (Malloy et al., 2003, p. 581)

Although Emma’s students may bake, it is hard to imagine a situation in which one would need to calculate the surface area of an icicle on a gingerbread house.

It is not surprising, however, that Emma finds it difficult to help her students see the relevancy of mathematics to their lives. Although the school district’s curriculum guide says that one of the primary goals is for students to be able to “use mathematics to solve problems in authentic contexts” (school district curriculum guide, 2003, p. 3) and many of the questions in the guide are contextual, very few are what I would consider to be authentic contexts or contexts which are relevant to adolescents’ lives.

Amelia – Interprets there to be messages with which she disagrees, but her practices reflect all of these messages (ca)

Amelia interpreted there to be messages about the types of questions she is supposed to ask her students in two of the resources that we discussed. When asked what she thinks the authors of the *Mathematics: Applications and Connections Course 1* textbook (Collins et al., 2001a) would describe mathematics is about and how the authors think students should learn math, Amelia said:

> Everyone's trying to get very PC [politically correct] these days. So it's not like you know, like they're definitely procedure, but then there's like – like I think our book has a lot of "labs" for each section or, or every few sections. So they're trying to do a hands on thing. There's a lot of practice and it's a lot of

> straightforward practice. Although I do think that there are more like applications. I think there's a lot more like *application type problems* than there used to be you know. (personal communication, November, 1, 2006)

This was paraphrased as "Teachers should make sure that students are able to solve 'application problems.'"

When talking about the school district's curriculum guide, she said:

> They're [the authors are] trying to do all the different pieces like – although I think that the curriculum guide especially on the assessments, *they don't do enough straightforward problems*. How do I – like you spend so much time on learning how to multiply and divide decimals – And it's truly what the heart of it is and you want to know if they're able to do that. And yet there's like two problems that are actually here's a multiplication problem and this is a division problem on the test. And to me that doesn't seem right. You know, like – and *there are a lot of word problems*. There are a lot of very difficult word problems in terms of the wording. (personal communication, November 1, 2006)

This was paraphrased as "Teachers should make sure that students are able to solve 'word problems.'"

It appears that Amelia has negative feelings about the messages she interpreted about the types of questions she is supposed to ask her students. By saying that the inclusion of application problems in the textbook is because the authors are trying to be politically correct and that there are not enough straightforward questions on the school district's assessments Amelia indicated that she did not believe that application problems and word problems are as important as straightforward questions.

Although Amelia did not seem to be enthusiastic about application problems or word problems, Amelia's classroom practices included these. For example, every lesson began with a warm-up consisting of three questions similar to those found on the school district's and state's assessments. Each day at least one of these questions was a word problem that required students to apply their mathematical knowledge to a situation in which a mathematical solution strategy is not obvious. Additionally, several of the questions she assigned to her students for homework were word problems requiring students to apply their knowledge to a context. Her inclusion of these types of questions may have been because Amelia feels it is important to prepare her students to do well on the school district's and state's assessments.

Source of solution methods

One of the most contentious issues in modern mathematics education involves the question of how students should become acquainted with solution methods. Some argue that "students learn by creating mathematics through their own investigations of problematic situations, and that teachers should set up situations and then step aside so that students can learn" (NRC, 2001, p. xiv). Others claim that "students learn by absorbing clearly presented ideas and remembering them, and that teachers should offer careful explanations followed by organized opportunities for students to connect, rehearse, and review what they have learned" (NRC, 2001, p. xiv). Of course, others argue for a teaching approach that is between these two extremes. Yet others contend that these views should not be thought of as opposite ends of a single dimension, but rather as separate dimensions of teaching (Stecher et al., 2006). Nevertheless, these two ways of acquainting students with solution methods have historically been seen as opposites.

Different terms have been used in conjunction with each of these ways of acquainting students with solution methods and each of these terms has been defined somewhat differently and is often used differently by different people. Some of the terms used for the first approach described above include *discovery learning*, *inquiry*, *student centered teaching*, *reform teaching*, and *problem-based teaching*. The terms *direct instruction*, *teaching by telling*, *guided instruction*, *teacher centered teaching*, and *traditional teaching* are frequently used to describe the second approach described above.

The question of *how* to teach mathematics is closely related to the question of *what* to teach about mathematics. Thus, research alone cannot answer the question: How should students become aware of solution strategies and methods? There are philosophical issues and additional questions to consider: What is it that we want students to learn from mathematics instruction? What do we want students to learn about the discipline of mathematics? How do we weight the value of the ability to recall mathematical facts and efficiently apply mathematical algorithms to the value of learning what it means to problem solve and *do* mathematics? What learning should we attempt to measure and how can we measure it? The answers to these and other questions can influence the selection of teaching approach (Kirschner, Sweller, & Clark, 2006).

All five of the teachers in this study specifically talked about how they think the authors of the resources want their students to become acquainted with solution methods. In summary, they said that the various resources send the messages that:

- Teachers should encourage students to develop their own solution methods.
- Teachers should expose students to "problem solving."
- Teachers should have students "investigate" mathematical ideas.
- Teachers should have students memorize procedures.
- Teachers should help students learn through "discovery."

- Teachers should provide students with methods and worked examples to follow.
- Teachers should try to "break away from that lecture style."

Some of these interpreted messages were similar to the beliefs of the teachers, but some were quite different. Table 13 summarizes how the teachers responded to a statement about source of solution methods on the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004).

Table 13
Teachers' responses to statement #10 on beliefs inventory: Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing.

	June 2005	May 2006	May 2007	Average
Amelia	3 Not sure	4	Did not participate	3.5
Beth	2 Disagree	2	1	1.7
Emma	4 Agree	4	3	3.7
Kathleen	3 Not sure	2	1	2
Sarah	1 Strongly Disagree	1	1	1

Note. This statement is phrased to support nonreform-oriented models of instruction so a lower score reflects a more reform-oriented belief.[30]

The ordering of the teachers' responses to this statement is similar to the ordering resulting from the comparison of total scores on the inventory. For this statement Sarah responded with the most reform-oriented response, followed by Beth, Kathleen, Amelia, and Emma. For the overall inventory scores, Sarah was most reform-oriented, followed by Kathleen, Beth, Emma, and Amelia.

[30] The scoring scale for this and other statements which are phrased to support nonreform-oriented models of instruction were reversed in the calculation of the overall beliefs inventory score.

Below, Table 14 summarizes how the teachers scored[31] on one item on the *Reformed Teaching Observation Protocol* (Sawada et al., 2000).

Table 14
Teachers' scores on item #3 on observation protocol: In this lesson, student exploration preceded formal presentation.

	1st Observation	2nd Observation	3rd Observation	Average
Amelia	0	0	0	0
Beth	2	3	3	2.7
Emma	0	3	2	1.7
Kathleen	1	0	0	0.3
Sarah	2	1	1	1.3

The teachers' scores on this item indicate that for this aspect of teaching, Beth was observed to be the most reform-oriented followed by Emma, Sarah, Kathleen, and Amelia. For the overall RTOP scores, Beth was observed to be teaching in the most reform-oriented ways, followed by Kathleen, Sarah, Emma, and Amelia.

The relation of the interpreted messages in this theme and the teachers' beliefs and practices was different for each teacher. Amelia interpreted there to be messages with which she agreed and her practices were reflective of these messages (aa). Beth interpreted there to be a variety of messages. She agreed with some of these messages and disagreed with others and her practices reflected some of these messages and not others (bb). Emma and Kathleen also interpreted there to be a variety of messages (with some of which they agreed and with some of which they disagreed), but their practices were not reflective of these messages (bc). Sarah only talked about messages with which

[31] Each item was rated on a scale ranging from 0 to 4. A "0" was chosen if the characteristic never occurred in the lesson. A score of "4" indicates that the item is very descriptive of the lesson. Intermediate ratings do not reflect the number of times an item occurred, but rather the degree to which that item was characteristic of the lesson observed (Sawada et al., 2000).

she disagreed, but her practices were reflective of these messages (ca). In the following sections, more detail about the messages about *source of solution methods* which Amelia, Beth, Emma, and Sarah interpreted from the resources and the relation of these messages to their beliefs and classroom practices is presented.

Amelia – Interprets there to be messages with which she agrees and her practices reflect all of these messages (aa)

In her interview, Amelia interpreted there to be a message about the *source of solution methods* in the school district's materials. When asked how she imagines the authors of the school district's materials would want her to teach, she said:

> But I do think to that, the curriculum guide tries to be creative. So if you're able to do some of the activities and things that it suggests then you may have that opportunity to do things. I just – I don't know. I don't know if somebody were to come in from the [central office of the school district] and see something I don't think that they would disagree, 'cause I think you're always like focused toward – you know, if you're dividing decimals and *you have a different way of doing it* then I don't think anyone's going to argue that, you know what I mean? . . . I don't know, cause I think the – you can actually – I think it would be *encouraged to show something in more than one way* as long as it is age appropriate and grade level appropriate. (personal communication, November 1, 2006)

Although at first glance it seems that Amelia thinks that the school district is open to a variety of ways of teaching, when this quote is looked at in the context of the rest of the interview it seems that she means something slightly different. She means that the school district is open to having the teacher present a variety of procedures or solution methods. She did not interpret the school district to be promoting student development of a variety of solution methods. Amelia's statements here were paraphrased as "Teachers should provide students with methods and worked examples to follow."

This message seemed to be aligned with Amelia’s beliefs. On the beliefs survey she had an average score of 3.5 (between *not sure* and *agree*) on Statement #10: Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing. Furthermore, Amelia does not indicate that she disagrees with this message. Additionally, she believes that she herself learns best in this way. In an interview, Eden Badertscher asked her about the most effective way for her to learn mathematics. As part of her answer, Amelia said, “I just watch and try and soak it in. . . more of the traditional way, I guess, is my comfort zone” (interview with Badertscher, July 2006).

Amelia’s classroom practices also reflected this message. When she talked about her teaching, she frequently used the words “show,” “explain” and “present” to describe what she does with her students. In the lessons I observed, every new mathematical idea and solution method was presented by Amelia rather than developed by her students. For example, she showed her students how to solve division by decimals questions by moving the decimal point in the divisor and dividend and then had her students practice the algorithm for homework. The next day she modeled decimal division with base-ten blocks on the overhead projector and then had the students work in small groups to practice solving similar problems and make sure they got the same answer as they would with the paper-and-pencil algorithm.

It is interesting to note that for the topic of decimal division she reversed the order of lessons that the school district and textbook recommend. Both the school district’s curriculum guide and the *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) textbook have students use base-ten blocks to divide decimals (Lesson 4-

6A) before learning the traditional algorithm of moving the decimal point in the divisor and dividend (Lesson 4-6).

It is likely that the authors of the school district's curricular materials would disagree with Amelia's interpretation of their materials. For the observed lesson on decimal division, the curriculum guide states:

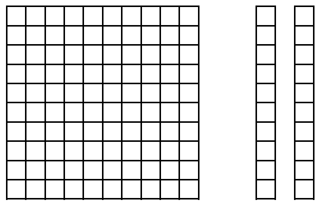
> The purpose of these lessons is to use modeling and estimation to explore dividing decimals. Students use base-ten blocks to model decimal division in the 4-6A Dividing by Decimals Hands-On Lab. Then, students explore decimal division and use estimation skills to decide if their answer is reasonable in 4-6 Dividing by Decimals and 4-7 Zeros in the Quotient. . . . (District-wide Curriculum Guide, 6th Grade Unit 2A, p. 15)

The use of the word *explore* indicates that the students should have a hand in developing methods to solve decimal division problems. However, the idea of having the students explore and develop their own solution methods is not as apparent in the textbook.

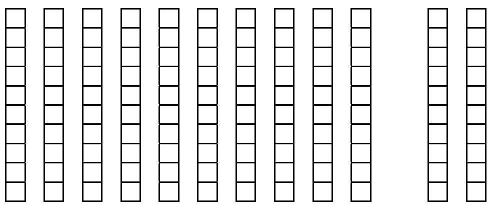
For example, in 4-6A Hands-On Lab on Dividing by Decimals (Collins et al., 2001a, p. 156) students are supposed to use base-ten blocks to model dividing a decimal by a decimal. The Hands-On Lab directions (shown in Figure 3 below) tell students to use a 10-by-10 block to represent 1 and to follow the steps on the page. After students work through this example and a similar example also modeled in the text, they are to use base-ten blocks to find the answer to four division questions and then answer one division question without using models.

To model 1.2 ÷ 0.3, follow these steps.

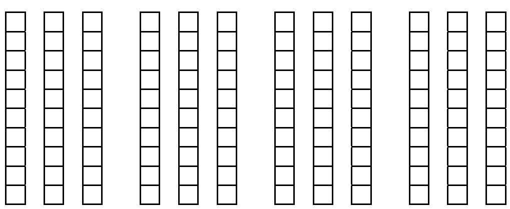
Place one and two tenths in front of you.



Trade the ones block for tenths.



Separate the tenths into groups of three tenths.



There are four groups of three tenths. Therefore, 1.2 ÷ 0.3 = 4.

Figure 3. Hands-On Lab on Dividing by Decimals (Collins et al., 2001a, p. 156).

Immediately following the Hands-On Lab on Dividing by Decimals is a lesson on the same topic (4-6). In it students are told “When dividing decimals by decimals, change the divisor to a whole number. To do this, multiply both the divisor and dividend by the same power of 10. Then divide as with whole numbers" (p. 157). After this statement several worked examples are given. In the margin of the teacher's edition is a Suggested Reteaching Activity. Teachers are told to:

> *Illustrate* 2.5 ÷ 0.5 on the overhead projector using decimal models. Then multiply the divisor and the dividend by ten. *Show* 25 ÷ 5 with models. *Guide* students to conclude that multiplying the divisor and the dividend by a power of ten does not change the quotient [italics added]. (p. 158)

Words like *illustrate*, *show*, and *guide* provide evidence that Amelia's interpretation of the school district's curricular materials is reasonable. Although the school district seems to be attempting to at least occasionally have students be the source of solution methods, the school district selected a textbook which undermines this by presenting methods and worked examples to follow.

Beth – Interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees) and her practices reflect some of these messages (bb)

Beth talked about messages related to the *sources of solution methods* when she talked about the textbook, the school district's curricular materials, the *Super Source* series (ETA/Cuisenaire, 1996a and 1996b), and the master's degree program. She sees a wide variety of messages in these resources.

When talking about the *Pre-Algebra* textbook (Malloy et al., 2003), Beth said:

> *Beth*: . . . but it's still written the same way every other textbook is written. *They give you a couple of examples, definitions, practice*, then a couple of word problems at the end, next topic. I don't know. It's – next topic. It's the same old thing.
>
> *Christy*: So what do you think that they would want students to learn? What would – when you look at that, what do you think their goals are?
>
> *Beth*: For them to learn the procedures.

> *Christy*: And how do you imagine that they would want the class set up? What kind of interactions would there be?
>
> *Beth*: I don't think they envision a lot of it just by the way it's written. You can read it like a textbook and *have the kids actually sit down and read it by themselves*, notice the text features. . . . It seems to me their big goal is for them to learn procedures of how to do things. And the way they're laid out it's like procedures first, then problem solving. And so, yeah, that's all I think their goal is to have them *memorize procedures* and *show them examples* of different procedures. (personal communication, November 3, 2006)

The messages in this exchange were paraphrased as "Teachers should have students memorize procedures" and "Teachers should provide students with methods and worked examples to follow."

Beth's word choices and tone in the above conversation indicated that she disagreed with these messages. This was also indicated by her professed disagreement to Statement #10: "Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing" on the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004).

She, however, agreed with the messages related to this theme that she saw in the school district's curricular materials, the *Super Source* series (ETA/Cuisenaire, 1996a and 1996b), and the master's degree program. With regard to the school district's curricular materials, Beth said:

> The curriculum guide says they want you to start with concept development and have them you know, have exposure to problem solving, but the curriculum guide is written around the textbook. So the guide will say, "Do this sequence of lessons in the textbook – but make sure it's problem solving." (personal communication, November 3, 2006)

This was paraphrased as "Teachers should expose students to 'problem solving.'" Beth's meaning of "problem solving" is not clear in this exchange, but as part of the master's degree program she has read NCTM's *Principles and Standards for School Mathematics* (2000) which defines problem solving as "engaging in a task for which the solution method is not known in advance" (p. 52). In her interview, she frequently talked about the importance of having students work on problems that they do not already know how to solve, so it is likely that she is referring to this meaning.

Beth also talked about how she imagined the authors of the school district's materials would want teachers to teach:

Christy: Well, what do you their intent is for instruction in [the school district]?

Beth: . . . I can see that their intent is really to show teachers how to teach the curriculum in – I don't want to say a more creative way, but just with good practice. 'Cause what I found is when we get to middle school, all those good elementary school methods go out the window and I don't know if it's because of the time constraint or the way the kids are or if it's teacher training, but it seems to be a lot more lecture style.
And I think the county's intent is to try and *get teachers to break away from that lecture style* and get kids in groups, so that guide, which was just written, really addresses those things concretely and specifically more so than the other guides just 'cause I think they're older. The [7th grade] guide is from 2003, which is not that old, but a teacher in elementary school teaching it does it differently than somebody in middle school.

Christy: How so?

Beth: I think the middle school people focus on the textbook more than elementary school. Elementary school teachers will do more manipulatives, more group work, more projects, and base that on the tests. Like, "This is what my kids need to know. This is how I can get them to do it," and I'll use the textbook every now and then if I need just some drill and kill or anything to pull assessment items for it, but this does not – the textbook does not guide me. I've seen teachers in middle school where that's their Bible.
You do pages and pages and then when we're through with the pages

> we have a test and, "Oh, the kids didn't get it? Well, let's give them more pages to do." It's just a different mindset. (personal communication, October 10, 2007)

This was paraphrased as "Teachers should try to 'break away from that lecture style.'" In this exchange, it seemed that Beth agreed with what she saw as the intentions of the authors of the school district's materials.

Similarly, it seemed that Beth agreed with the messages that she saw in the *Super Source* series (ETA/Cuisenaire, 1996a and 1996b), which is one of the resources that the school district recommends in the curriculum guides. In an interview, she said that she thinks these resources emphasize "the big ideas" and having "kids discovering them" (personal communication, November 3, 2006). This was paraphrased as "Teachers should help students learn through 'discovery.'"

Beth also spoke positively about the messages that she saw in the master's degree program. She said that the professors in the master's degree program want to show the teachers "… how the inquiry – what do you call investigations, how that can work to develop content understanding" (personal communication, November 3, 2006). This was paraphrased as "Teachers should have students 'investigate' mathematical ideas."

The master's degree program[32] also seemed to be influencing her instruction. When asked about this program, she said that the lesson she had taught just before the interview was reflective of the messages she hears from the program. In this lesson she had her students work in small groups to develop different methods to multiply two-digit numbers by two-digit numbers. She said:

> It's like – well, for an example today I'm trying to teach them the distributive property and I finally understand, because of the program, the big idea of distributive property and I'm kind of able to guide my kids into seeing that too.

[32] For more about Beth's experiences with inquiry in the master's degree program, see Badertscher, 2007.

> And today *one of my students came up with* a connection between using distributive property and the long algorithm for multiplying 2-digit numbers. And so now that I understand that it's number sense – my number sense is so much better since being in the program than it ever was. Since I can figure out all this stuff now I can show them. They're intuitively – they have a lot of intuition and the program has helped me kind of recognized the kids' own intuition about stuff, so it's just like – I don't know. It seems more – *it's more about them and them finding the best way for themselves* and they keep saying to me, "But we're getting the same answers but I did it a different way." There must've been 6 different ways the kids did this 15 times 12 and it all worked for them and they get – I think 'cause my attitude's different. They get that it's okay to *do it their way* that makes sense to them 'cause I don't care. It all makes sense to me as soon as they explain it… (personal communication, October 10, 2007)

This was paraphrased as "Teachers should encourage students to develop their own solution methods." Beth spoke positively of the program and her classroom practices were reflective of the messages she interpreted from the program.

Similarly, her classroom practices were reflective of the other messages with which she agreed. In her observed lessons, her students worked over several days to develop computational techniques for modular arithmetic. On the first day, Beth introduced the topic simply by writing "2 + 3 = 1" on board. She did not say anything after writing this. Students called out things like "That's wrong!," "It should be 5!," and "Change the addition sign to a minus sign. No, wait, that would be -1." After having the students share ideas as a whole class for several minutes, Beth said, "I want you to talk to a partner for about 1 minute. Talk about what you think is going on." During this time, several students brought their partners to the board to demonstrate their thinking as they wrote on the board. After a few minutes, Beth said, "I had a bunch of people come up and show what is going on. Could someone come up and demonstrate how 2 + 3 = 1?" Two students shared their ideas about modular addition and subtraction with the whole class. Other students also posed questions about multiplication and division. Beth then posed a few more questions and gave students time to work in pairs or small groups to attempt to answer these questions. While the students were working, Beth wandered among the groups asking questions like, “What patterns do you notice?,” “What do you call something like that?,” “How would you define subtraction?” and saying things like “I’m not going to tell you if you’re right or wrong because I want you to think about it more tonight.” and “We need to reconcile this – even if you do it different ways, you should get the same answers” (observation notes, May 16, 2007). In this lesson it was apparent that Beth was exposing her students to problem solving, having her students develop their own solution methods, having students investigate mathematical ideas, and helping her students learn through discovery. She has also certainly broken “away from that lecture style.”

The authors of the *Pre-Algebra* textbook (Malloy et al., 2003) might disagree with the messages that Beth has interpreted from this text,[33] but there is evidence that supports most of her interpretations. The authors seem to be making a superficial effort to have students investigate, explore, and discover mathematical ideas. There are 27 Algebra Activities and three Geometry Activities in the text. Some of the stated objectives for these activities say that the activities provide opportunities for students to “investigate” (pp. 39, 158, 180, 512, 562, 583), “explore” (pp. 368, 386, 392, 640), and “discover” (p. 476) mathematical content. Verbs such as these are usually used when students are to develop rather than be presented with mathematical ideas (Van de Walle, 2004, p. 13). But, as can be seen in the Geometry Activity on Similar Solids (p. 583 - described in the following section on Emma), these activities are typically very structured and provide

[33] The messages that Beth interpreted from the textbook were paraphrased as “Teachers should have students memorize procedures” and “Teachers should provide students with methods and worked examples to follow.”

little room for investigation, exploration, or discovery. Similarly, the frequent inclusion of worked examples makes it seem that the authors believe that these should be provided to students. However, it is not clear why Beth interpreted that the authors want students to *memorize* procedures.

It is interesting that Beth only interpreted the school district to be emphasizing problem solving and encouraging teachers to break away from teaching through lecture. These messages seem to be present, but there also seem to be messages that are counter to these ideas. For example, in each of the district-wide mathematics curriculum guides teachers are told that each "Mathematics Instructional Block" should contain:

5 minutes – Warm-Up
- Connection to prior learning
- Essential question

20 minutes – Focus Problem/Lesson
- Exploration
- Direct instruction
- Guided practice

15 minutes – Independent Practice/Evaluation
- Differentiation

5 minutes – Closure

Note the specific inclusion of "direct instruction" and "guided practice" in the model. Additionally teachers are told to "Keep students actively engaged in instruction. Independent practice is only appropriate after *the teacher has taught* [emphasis added] the concepts and the student has demonstrated the capacity to work independently" (District-wide curriculum guide, 2003, p. 13).

Emma – Interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees), but her practices do not reflect any of these messages (bc)

In her interview, Emma interpreted there to be messages about *sources of solution methods* in the school district's curricular documents, the textbook, and the master's degree program. She agreed with some of these messages, but her practices did not reflect any of these messages.

When asked how she thinks the writers of the school district's curriculum guides envision an ideal lesson, Emma said, "I think fairly structured. . . . what I kind of call the 'old school' approach, where you do the activator, you model the activity, you do the guided practice, you do the independent practice, and then you summarize. . . . I think they expect the lessons to be very structured and teacher-centered" (personal communication, October 8, 2007). Emma also assumed that the school district's curriculum writers want teachers to teach from the textbook. She saw the Glencoe/McGraw-Hill textbooks to be "totally teacher-centered." She thought that the authors of these texts imagine an ideal lesson as when the teacher is standing in front of the room saying something like, "Let's read Page 101. Okay. Everybody understand? Let's do these. This is how you do it. Let me model it for you" (personal communication, October 8, 2007). These messages were paraphrased as "Teachers should provide students with methods and worked examples to follow."

Emma's interpretation that authors of *Pre-Algebra* (Malloy et al., 2003) want to provide students with methods and worked examples to follow is reasonable. In the introductory pages of the *Pre-Algebra* text, it is touted that "**Completely worked-out**

examples [emphasis in original] with clear explanations are paralleled by the **Guided Practice** and **Practice and Apply** [emphasis in original] exercises that follow" (p. T2). It also states,

> **Key Concept** [emphasis in original] boxes use words, symbols, models, and examples to illustrate new rules, properties, and definitions so students can build their reading skills as they build their math skills. **Concept Summary** [emphasis in original] boxes provide a concise overview of key topics. (p. T2)

Thus, it seems that the writers believe it is important for students to receive clear explanations of concepts.

In contrast, Emma said that her principal and the professors of the master's degree program have other goals related to the source of solution methods. First she talked about her principal:

> *Emma*: I've been working really hard in a lot of this, just because of my principal right now, trying to make sure it's student-centered, that the students are doing the learning. I'm not just giving them the information and saying, "Here you go, go practice." I try and *let them figure it out on their own* and make the connections on their own and things like that.
>
> *Christy*: So what has your principal been doing?
>
> *Emma*: That's a whole other story. Her thing is rigor, relevance, and relationships. That's her famous quote. . . And the rigor aspect is really the part of teaching, you know that. I think we all try and do it, but never heard the word used that way. Again, to get the students to develop the – you guide them, but obviously they're making the connections and *doing the learning themselves*, so therefore they get the conceptual understanding versus just the procedural. (personal communication, October 8, 2007)

This was paraphrased as "Teachers should encourage students to develop their own solution methods." Later, she talked about the master's degree program:

> *Christy*: What would you say that the professors in the master's degree program would think? If I asked you what's most important that you want your students to know about mathematics. How do you think

that they would answer that question? What would they say is the most important for kids to know about math?. . .

Emma: I think there – I don't know, probably the same thing as the conceptual understanding, and almost probably a little bit more to the extreme that *it's almost totally just discovery learning*. It's kind of like – and that's our biggest complaint. It's like we complained about this the other day. We get all these awesome ideas. They're great, but in the real world you can't do that every day. Sometimes you just have to stand up there and teach, whether it's __ or whatever it is, so I think almost to the extreme where *they want everything to be student-centered, discovery learning*. (personal communication, October 8, 2007)

This was paraphrased as "Teachers should help students learn through 'discovery.'"

It appeared that Emma agreed with some of these contrasting messages about the source of solution methods, but disagreed with others. It also seemed that she holds conflicting beliefs. On the *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004) she primarily agreed with Statement #10: Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing. But, when asked to describe her ideal lesson, she said, it would be "student-centered" and involve "discovery learning."

Her classroom practices reflected a mix of approaches, but none of her practices was clearly inline with the messages she interpreted from the resources. Rather, her practices seemed to be midway between the messages. In the lessons in which she was observed, it seemed that Emma was attempting to have her students discover mathematical ideas, but she provided much guidance and instruction. She, however, did not provide worked examples for the students to follow.

For example, the objective of one lesson was for students to “Compare the volume and surface area of 3-D figures” (observation, May 23, 2007). Emma distributed snap cubes and had students work in groups of three to make a 1x1x1, 2x2x2, and 3x3x3 cube and find the volume and surface area of each. As a whole class, they filled in the table:

Dimensions	Volume (units^3)	Surface Area (units^2)
1	1	6
2	8	24
3	27	54

For homework she told students to “Do 4x4x4 and 5x5x5 without cubes and look for a pattern. What’s going to happen to volume and surface area as dimensions double, triple, … (all based on 1x1x1). What happens to the volume and surface area?” The next day in class she asked students to describe the patterns that they found.

The content of this lesson is similar to that of the Geometry Activity on Similar Solids in the *Pre-Algebra* textbook (Malloy et al., 2003, p. 583). But, the textbook’s instructions are much more explicit. In the textbook, students are to work through the two activities and 15 questions to “Investigate similar solids using sugar cubes or centimeter cubes” (Malloy et al., 2003, p. 583). Activity 1 is shown below:

Collect the Data

- If each square of a sugar cube is 1 unit long, then each face is 1 square unit and the volume of the cube is 1 cubic unit.
- Make a cube that has sides twice as long as the original cube.

Analyze the Data

1. How many small cubes did you use?
2. What is the area of one face of the original cube?
3. What is the area of one face of the cube that you built?
4. What is the volume of the original cube?

5. What is the volume of the cube that you built? (Malloy et al., 2003, p. 583).

Note that each of these questions has one correct answer and all can be answered with a single number or short phrase. Fourteen of the 15 questions in the Geometry Activity are similar. The 15th question asks students to "Research the scale factor of a model car. Use the scale factor to estimate the surface area and volume of the actual car" (p. 583). Although Emma did not write as many questions as the textbook has, she orally asked similar questions and most of her questions could be answered with a single word or phrase. However, some of her questions such as "What do we look for in a sequence?" have more than one correct answer. This was unlike the questions asked in the textbook.

Emma realizes that she has not yet reached her goal to "make sure everything is student-centered, make sure the kids are discovery learning, [and] make sure they have conceptual understanding of every single thing. . ." But, she feels that she would be closer to this goal if other teachers in her school had similar goals. She said:

> And I know I'm not the world's best teacher, but then I see people who think other people are the world's best teacher, and all they do is stand up there and, "Here's how you do this. Practice this." And so when I try and do something different from that, it's, "Well, that's too much time," or, "I don't understand why you're doing that." And I get that battle, so I get frustrated with that. . . (personal communication, October 8, 2007)

She is not frustrated because she feels that she is not allowed to teach through discovery; rather, she is frustrated that she does not have the support she needs to do so.

Sarah – Interprets there to be messages with which she disagrees, but her practices reflect all of these messages (ca)

In her interview, Sarah said that although her students are provided *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) she rarely uses the text.

This is “because a lot of times the book will be just procedure, do this, do this, and it’s not application, but that’s all the assessments are, and it’s like how can they jump without this happening? So I have to find other resources to make sure that they’re getting what they need, rather than just here’s *a procedure, go try it*” (personal communication, May 15, 2007). Sarah’s statement that the textbook presents students with procedures to try was paraphrased as “Teachers should provide students with methods and worked examples to follow.”

It seemed that Sarah did not believe that students should receive solution methods and worked examples from the teacher. On the *Teachers’ Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004), Sarah consistently disagreed with the Statement #10: Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing.

Sarah also attempted to have students “discover” mathematical ideas and to her this meant that *the teacher should not tell*. For example, at the beginning of one of her observed lessons, she displayed the following table:

Expression	Equation
OK	I am feeling fine.
2x	m + 10 = 24
ab	14 = 2y
Awesome!!	You are the best.
m + 17	m + 17 = 53

Below the table were the directions, “Using the above examples, determine your own definition for expression and equation” (observation notes, May 15, 2007). Sarah encouraged students to work with other students on this and when a student stated, “I don’t get this” rather than instructing her how to proceed, Sarah responded “Did you talk to the people at your table?”

Although Sarah did not want to be the one to *tell* students information, she always had a specific goal toward which she was guiding them. Later in the same class period, Sarah was attempting to get the students to say that “expressions have an operation” but the students were not saying this. Rather than explicitly making this statement herself, she had the students play the game Hangman to come up with the word “operation.” After the observation, Sarah talked about this part of the lesson:

> I’m not going to just tell them this is the way it is, but having them, like with “operation” with the Hangman, like they just love that. They were all engaged and just so excited, but yet I didn’t tell them the word, and the one kid even commented on it. Like, “Why didn’t you just tell us that? We spent 20 minutes.”. . . I’m not going to just say, here, here it is, like I was taught. (personal communication, May 15, 2007)

The above quote indicated that Sarah’s decision not to tell students information was largely based on her desire to engage her students’ attention. Similarly, when describing her attempts at different teaching approaches, Sarah said, “I’ve just really noticed that they [my students] don’t respond to direct instruction for very long. Their attention span is very short. . . . but, it’s hard to have them discover everything” (personal communication, May 21, 2007).

Although Sarah was attempting to teach without telling, she did not follow through with this. In fact, during each of the observed lessons she provided her students with worksheets which consisted of worked examples and similar questions on which to practice. Many of these worksheets came from the supplementary resources to the textbook.

Sarah's interpretation of the *Mathematics: Applications and Connections* textbooks (Collins et al., 2001a) seems to be reasonable, but the authors may disagree. In the introductory pages, teaching without telling is seemingly promoted. For example, an introductory page in *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) has the following heading: "Hands-On Labs and Mini-Labs help students discover concepts on their own." Directly below this heading it says:

> Glencoe's ***Mathematics: Applications and Connections*** encourages students to **do** mathematics. **Hands-On Labs** give students hands-on experiences, with a partner or group, in discovering mathematical concepts for themselves. The **Hands-On Lab Masters** provide students with a way to record what they observe and discover in the Hands-On Labs. (Collins et al., 2001a, p. T5, emphasis in original)

But, despite these promises, learning through discovery is not always clearly promoted in the text of the books. Though the *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) textbook contains 30 Hands-On Labs, there are few chances for students to truly discover mathematical concepts for themselves. Hands-On Lab 4-6A (described in the discussion of Amelia's lesson) on decimal division is an example of one of these labs. Additionally, the lessons in this textbook consist primarily of worked examples followed by similar questions to practice upon.

In summary, Sarah felt that the authors of the textbook expected students to learn by following worked examples. This was contrary to her belief that *the teacher should not tell*, but in her classroom practices, she and the materials she provided to her students, rather than her students, were frequently the sources of solution methods. It seemed that she used materials that did not support her beliefs because these materials were readily available to her; in order to follow her beliefs she would have had to have created or found other resources to use.

Technology

One of the most controversial topics in mathematics education concerns the role of technology[34] in mathematics classrooms. Most often, at the middle school level, the technology in question is the handheld calculator. Since the early 1980s inexpensive handheld calculators have been widely available for use in classrooms, yet almost 30 years later there are still frequent debates about if and how calculators should be used in classrooms. At the extremes are those who feel that students should have unlimited access to calculators and those who oppose the use of calculators in grade school mathematics classrooms entirely. Many in the middle feel that students should only use calculators after they have demonstrated mastery of the procedure for which they are using the calculator.

Those opposed to unlimited access to calculators fear that extensive use of calculators interferes with students' mastery of computational skills. However, "a large number of empirical studies of calculator use, including long term studies, have generally shown that the use of calculators does not threaten the development of basic skills and that it can enhance conceptual understanding, strategic competence, and disposition toward mathematics" (NRC, 2001, p. 354).

Although there is research based support for the use of calculators there are also philosophical issues to consider. The use of technology can change what mathematics is taught. Technology can amplify, that is, extend the existing curriculum by increasing

---

[34] Here *technology* refers to electronic technologies which aid in computation or facilitate the representation of mathematical concepts. It does not refer to the use of computers as tutors or the use of generic instructional tools such as projection systems.

“the number and range of examples with which students can come in contact” (Heid, 1997, p. 7) or it can reorganize the curriculum by causing one to question what knowledge of mathematics is necessary. It can also change how it is taught by allowing the exploration of messy data sets and allowing students to focus on reasoning and conceptualization rather than computation. But, it can also obscure mathematical ideas. “Tools may reveal or hide the mathematics underlying them, and they may make it easier or harder for students to portray their individual mathematics conceptualizations” (Heid, 1997, p. 7). Thus there is not a clear, universally accepted answer to the question, “How should calculators and other technology be used by students?”

Four of the teachers in this study (Amelia, Beth, Emma, and Kathleen) specifically brought up messages about technology that they interpreted from the resources. In summary, they said that the various resources send the messages that:

- Teachers should allow students to use calculators.
- Teachers should incorporate technology in lessons.
- Teachers should require students to be able to calculate without a calculator.
- Teachers should show students how to use technology.

Some of these interpreted messages were similar to the beliefs of the teachers, but some were quite different. Below, Table 15 summarizes how the teachers responded to a statement about technology use on the *Teachers’ Beliefs about Mathematics and Mathematics Teaching* inventory (Campbell, 2004). The ordering of the teachers’ responses to this statement is closely aligned with the ordering resulting from the comparison of total scores on the inventory. For this statement Sarah and Beth responded with the most reform-oriented responses, followed by Kathleen, Emma, and Amelia. For the overall inventory scores, Sarah was most reform-oriented, followed by Kathleen, Beth, Emma, and Amelia.

Table 15
Teachers' responses to statement #22 on beliefs inventory: If students use calculators they won't master the basic math skills they need to know.

	June 2005	May 2006	May 2007	Average
Amelia	3 Not sure	3	Did not participate	3
Beth	2 Disagree	2	1	1.7
Sarah	1 Strongly Disagree	2	2	1.7
Kathleen	2 Disagree	2	2	2
Emma	2 Disagree	2	3	2.3

Note. This statement is phrased to support nonreform-oriented models of instruction so a lower score reflects a more reform-oriented belief.[35]

The *Reformed Teaching Observation Protocol* (Sawada et al., 2000) does not have a measure of technology use, but all five teachers' students were observed using calculators in some way. Beth, Emma, Kathleen, and Sarah frequently allow or encourage their students to use technology, while Amelia limits its use to very specific conditions.

Of the four teachers who talked about messages about technology use, three of them (Beth, Emma, and Kathleen) primarily interpreted there to be messages about technology use with which they agreed and their classroom practices were reflective of these messages (aa). The fourth teacher (Amelia) primarily interpreted there to be messages about technology use with which she disagreed and her practices were generally not reflective of these messages (cc). In the following sections, more detail about the messages about technology which Kathleen and Amelia interpreted from the

[35] The scoring scale for this and other statements which are phrased to support nonreform-oriented models of instruction were reversed in the calculation of the overall beliefs inventory score.

resources and the relation of these messages to their beliefs and classroom practices is presented.

Kathleen – Interprets there to be messages with which she agrees and her practices reflect these messages (aa)

In her interview, Kathleen interpreted there to be messages about technology use in two of the resources that we discussed. The first instance was when we discussed the *Gateways to Algebra and Geometry: An Integrated Approach* (Benson et al., 1997) textbook:

Christy: What do you think the authors of this textbook - how would they envision an ideal lesson, an ideal mathematics lesson? What do you think they would –

Kathleen: From this? I'm gonna say they - it's seatwork. I really am. I mean, I could see the Think and Discuss being like guided practice but to me it's all seatwork. There's nothing in here for cooperative learning. There's nothing in here, you know, even like the Investigations, they're - it's something you would just look at the book and do it. I just don't - like, in their spreadsheet things why don't they have you actually go on the computer and use Excel? This is how they teach spreadsheets to the kids.

Christy: Okay. So that would, then I guess that was page 43?

Kathleen: Forty-three, yeah. Like, they just, "This is what it looks like. Let's put some numbers in." Why don't they just go on Excel and put formulas in and actually learn how to use it. You know?

Christy: Yeah.

Kathleen: Yeah, because this is here on page 30. That's when they introduce a spreadsheet, that's what they do.

Christy: Okay.

Kathleen: That's how they teach it to them. Like, I just would think it would make more sense to actually go, you know, then they tell you here to add these things together. Well, why can't we just go on the computer and do that? (personal communication, June 7, 2007)

Kathleen's statements here were paraphrased as "Teachers should show students how to use technology."

The second instance in which Kathleen brought up a message about technology use was in our discussion of the National Council of Teachers of Mathematics (NCTM):

Christy: And what kinds of classroom interactions do you think NCTM, for example, would –

Kathleen: Oh, I would definitely say cooperative learning. They definitely want students talking and feel like they want a conceptual, you know, they want it to be hands-on, they want conceptual learning to be, you know, to take place… [In the first course of the master's degree program] we had to pick an issue and I actually did mine on calculator use and I actually found out so much from that because in *NCTM always says that you should let kids use calculators*. Well, prior to that paper I was anti-calculator. I was so anti-calculator. And then after doing the paper I was like, you know, "Wow," I was like, you know, "for my kids that are like Special Ed[ucation] or my 504's, if I just let them use a calculator, they don't [mess up] on the computations. They can focus on the concepts."

Christy: Um-hum, and how about the rest of your students?

Kathleen: I let them, no, any time I teach something new they all have calculators out. I let them all use them because I'm like, you know what? Right now it's not the computations that are important, like, even with today's lesson with the graphing calculator with the lines I know they can go plug numbers into equations and solve them, but I wanted them to see how the coefficient effected the way the lines were. So why waste time filling in tables with completing math in our head? Let's just put them on the graphing calculator and start discussing them. So I feel like allowing the calculator in the classroom actually gives you more time for the student discourse because it gets through that rut of the computation. So it doesn't take as much time and then you can - you have that thirty minutes in class then to have this really rich lesson with conversation. Whereas before, you know, fifteen, twenty minutes would have been taken up with them just doing math in their head. (personal communication, June 7, 2007)

Kathleen's statements here were paraphrased as, "Teachers should allow students to use calculators."

It seemed that the messages that Kathleen interpreted from the resources about technology use were aligned with her beliefs. Although it did not seem that Kathleen agreed with *how* she thinks the authors of the textbook want students to learn how to use technology, it was clear that she saw that they want students to learn how to use spreadsheets and that she, too, was committed to this goal. Similarly, she talked positively about her interpretation of the ways in which the NCTM recommends calculators are to be used by students. In fact, she attributed recent changes in her beliefs about calculator use to her reading of NCTM documents.

Additionally, Kathleen's classroom practices were reflective of the messages that she interpreted about technology use. For example, she has had her students use graphing calculators to perform operations on matrices. Also, in the above interview excerpt she talked about how her students used technology when they were learning how the coefficient of x in an equation such as y = 3x influences the graph of the equation. She knew that her students "can do their math" so she had her students use graphing calculators in order to minimize the amount of time spent on computation and maximize the amount of time spent on analysis (personal communication, June 7, 2007).

It is interesting that Kathleen interpreted that the authors of *Gateways to Algebra and Geometry: An Integrated Approach* (Benson et al., 1997) want students to learn how to use technology solely by reading about it. On the pages to which she referred (pages 30 and 43) the authors present sample spreadsheets and ask questions about these spreadsheets. There is no indication, however, that the authors think that the textbook should be the students' *only* interaction with spreadsheets. It seems to me that the authors present these examples because they are unsure that students have access to spreadsheet software.

It is also interesting that Kathleen interpreted that the National Council of Teachers of Mathematics "always says that you should let kids use calculators." But, she is not alone in this interpretation. In NCTM's *Principles and Standards for School Mathematics* (2000), the Technology Principle is one of six principles addressed. The Technology Principle states that, "Technology is essential in teaching and learning mathematics; it influences the mathematics that it taught and enhances students' learning" (NCTM, 2000, p. 11). It goes on to say that "In the mathematics classrooms envisioned in *Principles and Standards*, every student has access to technology to facilitate his or her mathematics learning under the guidance of a skillful teacher" (NCTM, 2000, p. 25).

Although NCTM (2000) states that "technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions. In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics" (p. 25), some have interpreted NCTM's Technology Principle to mean that the council does not oblige students to learn how to compute without a calculator. In response to this interpretation, NCTM published a statement in May 2005 to clarify its position on technology. In it NCTM states that "all students should develop proficiency in performing efficient and accurate pencil-and-paper [mathematical] procedures. At the same time, students no longer have the same need to perform these procedures with large numbers or lengthy expressions that they might have had in the past without ready access to technology." Furthermore, "the teacher should help students learn when to use a calculator and when not to, when to use pencil and paper, and when to do something in their heads. Students should become fluent in making decisions about which approach to use for different situations and proficient in using their chosen method to solve a wide range of problems" (NCTM, May 2005). In 2006 NCTM further clarified its expectations related to calculators by stating that students in grades 6-8 are expected to "select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods" (p. 35).

Despite these clarifications, Kathleen and many others interpreted NCTM to be advocating unrestricted use of calculators. It seems that Kathleen may be focusing only on aspects of these messages about technology which support her beliefs and practices.

Amelia - Interprets there to be messages with which she disagrees and her practices do not reflect these messages (cc)

In her interview, Amelia brought up interpretations of messages about technology use twice. The first time was in discussion of *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a):

Christy: …talk about how you perceive the writers of the text book would describe what math is all about, what and how students should learn math.

Amelia: Everyone's trying to get very PC [politically correct] these days…. now everyone's trying to like incorporate you know, technology in some way shape or form or manipulatives in some way shape or form

> you know…. So it's like a little overboard sometimes…. (personal communication, November 1, 2006)

This was paraphrased as “Teachers should incorporate technology in lessons.”

The second time that Amelia brought up a message about technology use was in discussion of the school district’s documents. She said that the textbook and school district’s curriculum guide are very similar and that it also tries “to incorporate technology” (personal communication, November 1, 2006). This was also paraphrased as “Teachers should incorporate technology in lessons.”

It seemed that Amelia did not agree with the messages she heard about technology usage. In her interview, Amelia indicated that the incorporation of technology was a result of “political correctness” gone astray rather than a well-reasoned pedagogical move. Also, Amelia repeatedly said that she is “not a big fan of the calculator.” This may be a result of three factors: She herself is uncomfortable with technology, she feels that she learns better when she is not using a calculator, and she wants her students to learn procedures and is afraid that if her students use calculators they will not learn how to compute without a calculator. Each of these factors is discussed below.

Amelia is uncomfortable with technology. In an interview with Eden Badertscher, Eden asked Amelia to describe her ideal classroom and Amelia stated:

> I don’t feel like I have much knowledge when it comes to technology, like computer programs, graphing calculator, any of that. And so I would like…I don’t want them in the room ‘cause I am not going to know how to use them. Like a couple of years ago they were like, “Do you want more computers?” and I was like, “Not yet, don’t waste them here.” (interview with Badertscher, June 2007)

Amelia also feels that she learns better when she is not using a calculator. In an interview, Eden Badertscher asked, "Is there a way that you think you like to learn math that is the most effective way for you to learn?" and Amelia responded:

> I don't know if it's the best way for me, but it's what I'm used to, which seems to be my comfort zone. So start in small groups, and try and practice, and understand where it comes from. But I really need a lot of practice before it gets harder. I like learning the computation of it, I don't like putting stuff into the calculator, and I think that's 'cause I don't want my kids to do that at their age if it's not necessary. So trying to do it by hand. I get really lost when I have to just punch something into a calculator, 'cause it doesn't have any meaning for me. Last night I was doing a problem, and I was IMing [instant messaging] another student, and I was like, "You are already done?" "Yeah, I did them all on the calculator." And I was like, "Oh, I'm doing them all by hand." But I really need the computation to have it make sense. That helps me. (interview with Badertscher, July 2006)

As Heid (1997) pointed out, technology can obscure mathematical ideas. It seems that Amelia felt that this is often the case and is therefore reluctant to use technology when she is learning.

Most of all, Amelia limits her students' access to technology because she is afraid that if her students have unlimited access to calculators, they will not learn how to compute by hand and she believes that it is important that they can do so. Additionally, she feels that all of her students should learn how to compute without a calculator even if they are allowed to use a calculator at all times because of a documented special need:

> *Amelia*: I'm not a big fan of the calculator. That's always gonna be a battle, but I feel like I've found a happy medium between when it's appropriate and when it's not and I'm kind of the, the decider of that in my classroom and as a department in our school we've kind of come to an agreement on how we handle that.
>
> *Christy*: Are you in agreement with the other –
>
> *Amelia*: Yeah, we're pretty – as a math department we're pretty much in agreement. The biggest struggle used to – the conflict used to come between like special ed[ucation] and, and regular ed[ucation]. And since I'm an inclusion teacher that was always my biggest problem. So

> I've tried to like meet them halfway and now like a lot of my kids will use the calculator, but they still have to learn how to do the algorithm. So – because again you know, they at some point whether it's on the MSA [Maryland School Assessment] or where, whatever all the tests. They have to explain that. Like even if they say, "I put this into my calculator," which is not what I would count on them to say, but they have to be able to explain that and my special ed[ucation] department knows that. So they were willing to meet me halfway and now there's like a happy medium between the two. That's, you know, that's one example. (personal communication, November 1, 2006)

Note that Amelia feels that she and the mathematics department in her school are able to decide what is considered to be appropriate use of calculators.

There are, however, times that Amelia felt that calculators use is appropriate. One of her criticisms of the textbook is that it asks students to perform computations that she feels would best be done with a calculator. For example, on page 158 of *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) students are instructed to "Find each quotient" and some problems include numbers with many digits such as 40.99524 ÷ 14.3 and 5,885.9514 ÷ 703.22.

> *Amelia*: [I] am very critical of using the book for a lot of things, especially with decimals.
>
> *Christy*: Okay.
>
> *Amelia*: Division with decimals – they have problems in there that would basically give them a 3 and 4 digit divisor and I think that that's ridiculous. And actually Dr. [professor of a course in the master's degree program] just said tonight, she's like, "You know, what are you – why teach them really past the 2 [digit] by 2 [digit] multiplying or the 3 [digit] by 2 [digit]." You know, like –
>
> *Christy*: As an adult (Laughter) –
>
> *Amelia*: Yeah, we just do it on a calculator. I just – and I agree. Like I'm not a huge fan of the calculator, but I think that there's a certain point where I'm not gonna do that. You know, you can have number sense and hopefully go with that. So we make our own [worksheets] a lot of times for those things. (personal communication, November 1, 2006)

Note that rather than have her students use a calculator to perform these computations, Amelia created her own worksheets, which consisted of computations that are less tedious to perform by hand.

In determining her practices, Amelia seemed to be following her beliefs about technology usage rather than following the messages that she heard. Amelia provided multiplication tables to all of her students, but only reluctantly gave calculators to students when she was required to do so either because the students had a documented special need which granted them access to calculators at all times or because an assessment stated that calculators were allowed.

It is interesting that Amelia interpreted the authors of *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) to be promoting the use of technology. In the introductory pages of this textbook, the use of technology is seemingly promoted. For example, in the introductory letter to students, teachers, and parents, it states that students will "have opportunities to use technology tools such as the internet, CD-ROM, graphing calculators, and computer applications like spreadsheets" (Collins et al., 2001a, p.iii). Additionally, on the inside cover of that text there is a list of nine "Features and Benefits" of the textbook. *Technology* is listed as one of these nine and it states that the "Technology strand prepares students to function in a technological society through a variety of instruction and activities, including Technology Labs." However, despite these promises, the role of technology is not always clearly promoted in the text of the book.

The Course 1 textbook contains six Technology Labs. Four of these labs focus on the use of spreadsheets and the remaining two focus on the use of graphing calculators. The use of calculators or computers in most of the other 137 lessons in the textbook is neither explicitly encouraged nor discouraged. Even when a question mentions technology it is not clear if students are to use technology to answer the question. For example, one of the exercises in the section on dividing decimals by whole numbers states: “The spreadsheet shows the unit price for a jar of jelly. To find the unit price, divide the cost of the item by its size. Find the unit price for the next three items. Round to the nearest cent” (Collins et al., 2001a, Question 31, p. 154). Next to this exercise is a table in the format of a spreadsheet with the following data:

	A	B	C	D
1	Item	Cost	Size	Unit Price
2	Jelly	$1.59	12 oz.	0.1325
3	Cereal	$3.35	18 oz.	
4	Bread	$1.19	16 oz.	
5	Ketchup	$0.89	14 oz.	

Even if students are encouraged or allowed to use a spreadsheet to answer this question, considering the amount of time it would take to set up a spreadsheet, this method of solution may not be the most appropriate.

Similarly, it is not clear to me that the authors of the school district’s materials promote the use of technology as strongly as Amelia interpreted. In the mathematics curriculum guides, one of the four “Overarching Enduring Understandings” that the school district has chosen is that “Technology influences the mathematics that is taught and essential for our world” (School district’s Mathematics Instructional Guide, 2003, p. 3). By including this, the district has placed technology in a potentially prominent position.

But, most of the district-wide unit assessments require students to complete at least some, and usually about half of the questions, without a calculator. The questions for which calculators are not allowed to be used are primarily purely computational questions although there are a few word problems. Most of the problems for which calculators are allowed to be used are word problems. For example, on the 6th grade district-wide assessment on decimals and fractions, students are not allowed to use a calculator to find 21.63 ÷ 0.3, but are allowed to use a calculator to answer the question: "Hia walks 0.75 miles per day. How many days will it take her to walk 4.5 miles? Use mathematics to explain why your answer is correct. Use words, symbols, and/or numbers in your explanation." Calculators are also allowed to be used on problems for which they will not be helpful such as, "Which of the following is the best measure for the capacity of a large fish tank? mL, L, kg, g."

Although the school district's curriculum guides rely heavily on the Glencoe/McGraw-Hill textbooks, the school district has changed the sequence of lessons and has added some lessons to and removed some lessons from the textbooks. The removal of lessons is striking when the use of technology is focused upon. For example, in the curriculum guide for the 6th grade mathematics course the school district only requires that teachers teach three of the six "Technology Labs" in the *Mathematics: Applications and Connections Course 1* (Collins et al., 2001a) textbook. The other three Technology Labs are considered to be for enrichment or acceleration.

There are, however, times that the school district's curriculum guide encourages the use of technology beyond that which the textbooks present. For example, when 6th grade students are learning how to multiply decimals, the school district's curriculum guide provides a detailed lesson plan for teachers which has students use estimation, examination of patterns, base-ten models, and a calculator to develop an algorithm to determine the placement of the decimal point in the product of two numbers containing decimals. By doing so, the school district shows that it values the use of calculators and it seems that it considers them to be a source of mathematical authority. In contrast, the textbook uses base ten models and estimation to place the decimal point. After a few examples demonstrating how these methods can be used, the textbook simply states: "Another way to place the decimal point in the product is by counting the decimal places in each factor. The product will have the same number of decimal places as the sum of the number of decimal places in the factors" (Collins et al., 2001a, p. 141). There is no mention of using a calculator and no further justification for this mathematical rule is given.

Although the school district clearly allows the use of calculators in mathematics classrooms, it is primarily left to individual teachers to determine when and how they will be used. For example, in the 6^{th} grade mathematics course there is a 13 week unit on decimals, fractions, and percents. 172 pages of the school district's curriculum guide are devoted to this unit. Besides the use of calculators in the lesson on multiplication of decimals described above, there are no further mentions of the use of calculators by students in this unit's instructional pages. The only other times that calculators are mentioned in this unit are on the district-wide assessments where it is specified whether a calculator may or may not be used to answer particular questions.

When mathematics teachers in this school district are observed teaching, administrators consult a list of instructional practices which are considered to be consistent with the district-wide curriculum guide. Administrators are supposed to note if students “use calculators to develop and enhance conceptual understanding and as a tool in problem solving” and if “appropriate mathematical tools and models [are] accessible to students.” It is left to teachers and administrators to determine the meaning of *appropriate*.

Amelia has interpreted the ambiguity in the messages about technology use as promotion of the widespread use of calculators. She, however, does not believe that students should use them very often and her practices were more reflective of her beliefs than the messages she interpreted.

Summary of Results and Discussion

This research study examined the messages about mathematics and mathematics teaching that five elementary certified middle school mathematics teachers interpreted from their students’ textbooks, school district’s curriculum guides and assessments, state’s assessments and curriculum framework, a master’s degree program in which they were enrolled, and other resources which the teachers felt were significant influences on their teaching. The teachers in this study interpreted messages in 11 themes from the curricular resources. Three of these themes (*Concepts and Procedures*, *Question types*, and *Source of solution methods*) were the most salient to the teachers.

Within and among some of the 11 themes there were contradicting and/or competing messages. These contradicting and competing messages created tensions for the teachers. These tensions were most apparent in the three most salient themes (*Concepts and Procedures*, *Question types*, and *Source of solution methods*) as well as in

Technology. Additionally, the teachers did not always agree with the messages they interpreted and their classroom practices were not always reflective of the messages they interpreted. Thus, this research study also examined how these messages related to the teachers' beliefs and observed classroom practices.

In all, seven different types of relations between the messages and the teachers' beliefs and classroom practices were found:

1. Roughly half of the time the teachers interpreted there to be messages with which they agreed and their practices were reflective of these messages (aa). This was to be expected because people are often biased toward interpretations consistent with their beliefs and practices (Berk, 2004; Spillane et al., 2004). For the most part, the teachers were fair in their interpretations of the resources, but occasionally it seemed that they focused only upon aspects of the resources which were consistent with their beliefs and practices.
2. One teacher (Amelia on *Concepts and Procedures*) interpreted there to be messages with which she agreed, but her practices reflected only some of these messages (ab). It seemed that she was trying to reflect all of these messages in her practices, but her past experiences and a lack of support in the curricular materials limited her.
3. When the teachers interpreted there to be a variety of messages (with some of which they agreed and with some of which they disagreed), but their practices reflected all of these messages (ba), it seemed that the messages were more influential on their practices than their beliefs were. Even when the teachers

attempted to follow their beliefs rather than the messages, they were often unsuccessful in doing so.

4. When the teachers interpreted there to be a variety of messages (with some of which they agreed and with some of which they disagreed) and their practices reflected some of these messages (bb), two scenarios were observed. For one of the teachers (Beth on *Concepts and Procedures* and *Source of solution methods*), there was alignment between beliefs and practices; her practices reflected the messages with which she agreed and did not reflect the ones with which she disagreed. She may have felt a certain degree of freedom in choosing which messages to attend to because she is chair of the mathematics department in her school. In contrast, for two of the teachers (Emma and Sarah on *Question types*), there was not alignment between beliefs and practices; their practices reflected the messages with which they disagreed and did not reflect the ones with which they agreed. This may have been because these two teachers felt obliged to follow all of the messages (even the ones with which they did not agree), but the resources did not provide enough support to do so.
5. Two of the teachers (Emma and Kathleen on *Source of solution methods*) interpreted there to be a variety of messages (with some of which they agreed and with some of which they disagreed), but their practices did not reflect any of these messages (bc). They were successful in not enacting the messages with which they disagreed, but despite their conscious efforts to reflect the messages with which they agreed, they were unable to do so. Again, it seemed that this was

because there was not enough support in these resources for the teachers to follow through with these messages in their practices.

6. When the teachers interpreted there to be messages with which they disagreed, but their practices reflected all of these messages (ca), most of the time it seemed that this was because the teachers felt obliged to follow the messages. For example, although Amelia did not like the types of questions that the school district's assessments asked, she felt that she needed to prepare her students for these assessments and she therefore asked similar questions of her students. However, for one teacher (Sarah on *Source of solution methods*) it seemed that despite her conscious efforts to not reflect the messages in her classroom practices, she was unable to do so because the messages were so pervasive in the resources.
7. One teacher (Amelia on *Technology*) interpreted there to be messages with which she disagreed and her practices did not reflect these messages (cc). It seemed that she felt the freedom to follow her beliefs with regard to this aspect of her practice.

In summary, when the teachers agreed with the messages they interpreted, they attempted to reflect these messages in their practices. However, the presence of these messages and the teachers' efforts to enact these messages were not always enough. The resources often lacked the supports necessary for the teachers to follow through with these messages in their practices. When the teachers disagreed with the messages they interpreted, they sometimes consciously made the decision to not reflect these messages in their practices. They had various degrees of success in this; sometimes they were successful in following their beliefs rather than the messages with which they disagreed, but frequently the messages were so pervasive that the teachers were not able to overcome them. At other times, the teachers felt obliged to reflect all of the messages in their practices regardless of their own personal beliefs. The amount of support that the resources provided with regard to these messages was a strong indicator of the degree to which the teachers were successful in reflecting these messages in their practices. Frequently, the reform-oriented messages lacked support and thus the teachers found it difficult to reflect these messages in their practices.

Chapter 5: Conclusions

This research study examined the messages about mathematics and mathematics teaching that five experienced, elementary certified, middle school mathematics teachers interpreted from their students' textbooks, school district's curriculum guides and assessments, state's assessments and curriculum framework, a master's degree program in which they were enrolled, and other resources which the teachers felt were significant influences on their teaching. This study also examined how these messages related to the teachers' beliefs and classroom practices.

In this chapter, findings of the study are summarized and compared to the findings of previous research. Implications of these findings are discussed and several recommendations for future research are made.

Summary of Findings

The teachers in this study interpreted numerous messages about what and how to teach from the curricular resources. These messages can be sorted into 11 themes. Within and among some of the 11 themes there were contradicting and/or competing messages. These contradicting and competing messages created tensions for the teachers. These tensions were most apparent in the following themes: *Concepts and Procedures*, *Question types*, *Source of solution methods*, and *Technology*. Additionally, the teachers did not always agree with the messages they interpreted and their classroom practices were not always reflective of the messages they interpreted.

When the messages are sorted by themes, there are nine possible relations between the messages and the teachers' beliefs and classroom practices. Seven of these nine were observed in this study. The distribution of these relations with regard to the 11 themes, originally presented in Table 8, is reprinted below as Table 16.

Table 16
Relations among messages, beliefs, and practices by theme

	(a) Teacher interprets there to be messages which are reflected in her practices	**(b) Teacher interprets there to be messages some of which are reflected in her practices**	**(c) Teacher interprets there to be messages which are not reflected in her practices**
(a) Teacher interprets there to be messages with which she agrees	(aa) Concepts and Procedures 1 Connections 2 Cooperative learning 1 Differentiation 2 Explanation 5 Manipulatives 2 Practice 2 Question types 1 Source of solution methods 1 Technology 3	(ab) Concepts and Procedures 1	(ac)
(b) Teacher interprets there to be a variety of messages (with some of which she agrees and with some of which she disagrees)	(ba) Concepts and Procedures 2 Cooperative learning 2 Question types 1	(bb) Concepts and Procedures 1 Question types 2 Source of solution methods 1	(bc) Source of solution methods 2
(c) Teacher interprets there to be messages with which she disagrees	(ca) Manipulatives 1 Practice 1 Question types 1 Source of solution methods 1 Timeline 2	(cb)	(cc) Technology 1

In general, when the teachers agreed with the messages they interpreted, they attempted to reflect these messages in their practices. However, the presence of the messages and the teachers' stated desire to enact these messages did not always result in their teaching evidencing the messages. The resources from which the teachers interpreted these messages often lacked the supports necessary for the teachers to follow through with these messages in their practices.

When the teachers disagreed with the messages they interpreted, they sometimes consciously made the decision to not reflect these messages in their practices. They had various degrees of success in this; sometimes they were successful in following their beliefs rather than the messages with which they disagreed, but usually the messages were so pervasive that the teachers were not able to ignore them. At other times, the teachers felt obliged to reflect all of the messages in their practices, regardless of their own personal beliefs. For example, because the teachers felt obligated to prepare their students for the school district's and state's assessments they asked their students questions which were similar to the questions found on these assessments.

The amount of support that the resources provided to the teachers with regard to these messages was a strong indicator of the degree to which the teachers were successful in reflecting them in their practices. Frequently the resources superficially presented messages to the teachers without providing support for the teachers to follow through with the messages in their classroom practices. This phenomenon was especially apparent when the messages were reform-oriented messages.

Comparison of Findings with Previous Research

The majority of studies of educators' interpretations of advisory messages have found that teachers tend to focus primarily on messages with which they agree and which are already reflected in their classroom practices (e.g., Berk, 2004; Hill, 2001; Spillane & Callahan, 2000). In this study, the teachers interpreted messages with which they agreed and which were reflected in their practices only about half of the time. Similarly, previous studies have found that teachers tend not to see conflicts between messages (Hill, 2001; Spillane & Callahan, 2000), but the teachers in this study saw many competing and conflicting messages both within and among the resources.

The fact that these teachers were enrolled in a master's degree program which encouraged teachers to critically examine curricular resources may have contributed to the variety of interpreted messages. In fact, several of the teachers stated that their experiences in the master's degree program changed how they view the resources. Alternatively, this difference may be because this study asked the teachers to interpret messages from a variety of resources while most other studies focused on only a single resource. The comparison of resources may have stimulated the teachers to talk about competing and contrasting messages.

Additionally, many studies have found that teachers often "traditionalize" reform-oriented curricular materials (e.g., D. K. Cohen, 1990; Hiebert & Stigler, 2000; Tarr et al., 2006). In contrast, the teachers in this study seemed to be attempting to "reform-ize" somewhat traditional materials. The teachers held primarily reform-oriented beliefs, but they interpreted many nonreform-oriented messages in the materials. Four of the five teachers seemed to be attempting to teach in primarily reform-oriented ways and the fifth teacher seemed to be attempting to integrate certain aspects of reform into her teaching.

Spillane et al. (2002) attributed the “limited” implementation of policy messages to teachers’ misunderstandings of the messages. They argued that the primary problem in the implementation of policy messages is that implementing agents understand them differently than policymakers intend. The teachers in this study, however, usually seemed to have reasonable interpretations of the messages.

Others have attributed the limited implementation of reform-oriented messages to teachers’ rejection of these messages. Most of the teachers in this study did not reject the reform-oriented messages. In fact, they embraced them. But, despite faithful efforts, they were not always able to follow through with these messages in their practices. It seemed that the degree to which the teachers were successful in reflecting reform-oriented messages in their practices was closely related to the degree to which the materials provided actual (as opposed to superficial) support for these messages. In contrast, the teachers did seem to be rejecting most of the nonreform-oriented messages. Their practices, however, frequently reflected these messages because there was so much support for these messages in the materials. This was especially apparent in the *Question types* and *Source of solution methods* themes.

Surprisingly, the teachers in this study did not talk much about the effects of standardized testing on their teaching. I, however, suspect that, like the teachers in Tomayko’s (2007) study, the teachers in this study would say that the pressures they feel from high-stakes testing influence what and how they teach. Specifically, although the teachers were not always enthusiastic about it, the teachers were observed assigning questions similar in content and format to those found on the school district's and state's assessments. Like Schnepp (Chazan & Schnepp, 2002), the teachers seem to have accepted that high-stakes tests are a part of today's schools and thus have changed their practices to reflect these assessments. This is also in line with Raymond's (1997) observations that factors such as time constraints and standardized testing can influence teachers' practices.

Implications

In some ways the tensions that the teachers feel as a result of the conflicting and/or competing messages seem to be helpful; they help the teachers clarify their own beliefs and commitments. Having a wide variety of messages from which to choose allows them to feel that no matter what they choose to do, they have at least nominal backing. But, because it is impossible to follow all of these messages and the teachers often do not agree with the messages, the wide variety of messages also causes the teachers stress and frustration.

To help teachers, the entire mathematics education community (including curriculum writers, policy writers, school district officials, teacher educators, and professional developers) needs to become more conscious of the messages put forth in curricular resources. First, we need to consider the consistency of our messages. We should ask ourselves, "Are the messages within this resource consistent and how do these messages fit with the messages in other resources?" This is especially important for school district officials. Although it is unlikely that there will ever be complete curricular coherence (especially across all aspects of mathematics education) and it is at times politically challenging to achieve consensus, school district officials have an obligation to send coherent messages through the resources they select.

Second, we need to be more specific about our intentions. Phrases such as "emphasize concepts *and* procedures," "have students solve *application* problems," "have students *discover*," and "use technology *appropriately*" can be interpreted in a wide variety of ways. Curriculum writers, policy writers, and school district officials especially need to clarify what they mean by these and other word choices.

Third, and perhaps most importantly, we need to provide support to teachers. This is especially important when reform-oriented messages are being put forth. For example, in addition to telling teachers to teach through problem solving, materials need to illustrate for teachers what this might look like and provide true problems (rather than only computational practice questions) for teachers to assign to students.

Teacher educators and professional developers have an important role in this as well. They must help teachers learn to critically examine curricular resources in order to become more aware of the messages contained in them. Both pre-service and in-service teachers should be asked to compare and contrast the messages they see both within resources and among resources and to compare these messages to their own beliefs and to what is known about effective mathematics education. By doing so, teacher educators and professional developers can help teachers become more conscious of their decisions regarding which messages they will attend to and which they will try to ignore.

Recommendations for Future Research

This study was an exploratory study of the messages that five teachers interpreted from curricular resources and the relations of these messages to the teachers' beliefs and practices. Because of its exploratory nature and the small number of teachers studied, all findings are quite tentative. In order to strengthen my claims, further research would be helpful.

One of the main findings of this study was the list of 11 themes into which the messages the teachers interpreted from the resources fell. These 11 themes became apparent after all data were collected. Similarly, *Concepts and Procedures, Question types*, *Source of solution methods*, and *Technology* did not emerge as the most salient and/or tension creating themes until the data were analyzed. As such, the observations did not focus upon these themes. It would be helpful to re-observe the teachers while focusing upon these themes.

Additionally, because the teachers were broadly asked to interpret messages from the resources, the messages that the teachers talked about were the most salient messages, but not necessarily the only messages that the teachers interpreted. Thus, the teachers may interpret more messages than they talked about in their interviews. For example, the teachers were asked how they imagined the authors of the textbooks would envision an ideal lesson. This question is likely to have resulted in a different response than a question such as "How do you imagine the authors of the textbook envision students should become aware of solution methods?" would have. In order to more thoroughly capture the messages that the teachers interpret from the resources, it would be helpful to re-interview each teacher while focusing on these themes.

This research focused on the relations between the messages teachers interpreted from resources and their beliefs and the relations between these messages and their practices. It did not focus directly on the relations between beliefs and practices. In order to better understand *why* teachers are using curricular resources as they are, more research on teachers' decision making would be useful. In this study, perhaps the most interesting instances were when the teachers' beliefs and practices were not aligned. When teachers followed messages in which they did not believe, why did they do so? When and why did the teachers feel obligated to follow certain messages? When their practices were not reflective of messages in which they believed, why was this? Additional conversation with the teachers might help to clarify these issues.

Also, the previously developed instruments that I used to assess the teachers' professed beliefs and classroom practices were not entirely consonant with the themes that the teachers found to be most salient. For instance, Campbell's (2004) *Teachers' Beliefs about Mathematics and Mathematics Teaching* inventory did not have any statements about the balance of concepts and procedures. It also did not always provide evidence of teachers' beliefs regarding the specific themes they discussed. For example, the statement most closely related to *Question types* only gauged how often teachers ask certain types of questions. It did not help me to understand if they were using these questions because they believe they are a valuable type of question to ask of students, because they are readily available, or because they feel obligated to use such questions. Similarly, Sawada et al.'s (2000) *Reformed Teaching Observation Protocol* did not have any items that focused on the balance between concepts and procedures and none of the items talked about the use of technology. Thus, it would be useful to create instruments that more thoroughly capture the teachers' beliefs and practices with regard to the themes found in this study.

In order to determine if the results of this study are widely applicable, further research with other teachers would be helpful. In particular, it would be interesting to compare the results of this study to how teachers who were not in the master's degree program, who teach other grades, and/or who teach in other school districts interpret curricular resources and how the messages they interpret relate to their beliefs and practices.

For a larger-scale study, a questionnaire could be developed. This questionnaire would focus on the four most salient and tension creating themes. The teachers would be asked to indicate their level of agreement to statements about these themes with regard to each of the resources, their personal beliefs, and their classroom practices. Some of the statements on this questionnaire might include:

- It seems that the authors of my students' textbook believe it is important to provide students with solution methods and worked examples to follow.
- I believe it is important to provide my students with solution methods and worked examples to follow.
- I provide my students with solution methods and worked examples to follow.

Space for comments about these statements would also be provided. Such a questionnaire would provide insight into the messages the teachers are interpreting from resources and how these messages relate to their beliefs and practices. Some of the teachers would also be observed in order to provide a measure of the teachers' observed classroom practices in addition to their professed practices.

In another related line of research, it could be helpful to ask policy and curriculum writers and professional developers about their intentions. Although it seemed that the teachers in this study made reasonable interpretations of the resources, I often thought that these interpretations were not what the writers or professional developers intended. In interviews, policy and curriculum writers and professional developers could be asked questions such as “How do you intend for students to become aware of solution methods?” or “How and when do you imagine that calculators should be used by students?” Afterwards, I can imagine sharing teachers’ interpretations of the resources with the respective writers and professional developers and discussing areas in which the writers’ and professional developers’ intentions were and were not well matched with the teachers’ interpretations. Such comparison could be helpful to writers and professional developers in the future development of resources.

Further research could be helpful in understanding more about why teachers use curricular resources in the ways that they do and how the mathematics education community can better support teachers in their efforts to reform their teaching.

Appendices

Appendix A: Teachers' Beliefs about Mathematics and Mathematics Teaching

The following inventory is reprinted with permission of the author.

Teachers' Beliefs about Mathematics and Mathematics Teaching

Name: ____

Date: ____

This scale presents a listing of sentences. You are to indicate the degree to which you agree or disagree with the opinion or belief expressed in each of the sentences.

> If you strongly disagree with the opinion or belief expressed in a sentence, circle the letters SD to the right of that sentence.
>
> If you disagree with the opinion or belief expressed in a sentence, but not so strongly, circle the letter D to the right of that sentence.
>
> If you are not sure how you feel about the opinion or belief expressed in a sentence, that is you cannot decide or you do not really have an opinion, circle the letter N to the right of that sentence.
>
> If you agree with the opinion or belief expressed in a sentence, circle the letter A to the right of that sentence.
>
> If you strongly agree with the opinion or belief expressed in a sentence, circle the letters SA to the right of that sentence.

There are no "right" or "wrong" answers. The only correct responses are those that reflect what you believe to be true. Be sure to respond to each item in a way that reflects your personal beliefs.

Do not spend too much time pondering each sentence. Work briskly, but carefully.

Be sure to respond to every statement.

	Strongly Disagree	Disagree	Not Sure No Opinion	Agree	Strongly Agree
1. I like to use math problems that can be solved in many different ways.	SD	D	N	A	SA
2. Using computers to solve math problems distracts students from learning basic skills.	SD	D	N	A	SA
3. When two students solve the same math problem correctly using two different strategies I have them share the steps they went through with each other.	SD	D	N	A	SA
4. Every child in my room should feel that mathematics is something he/she can do.	SD	D	N	A	SA
5. I regularly have my students work through real-life math problems that are of interest to them.	SD	D	N	A	SA
6. It is not very productive for students to work together during math time.	SD	D	N	A	SA
7. I often learn from my students during math time because my students come up with ingenious ways of solving problems that I have never thought of.	SD	D	N	A	SA
8. No child should associate mathematics with frustration so, during mathematics class, a teacher should limit the questions he or she asks of a child to those questions that the teacher is reasonably confident that the child can answer correctly.	SD	D	N	A	SA
9. I integrate math assessment into most math activities.	SD	D	N	A	SA
10. Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing.	SD	D	N	A	SA
11. In mathematics class, each student's solution process should be valued.	SD	D	N	A	SA
12. I encourage students to use manipulatives to explain their mathematical ideas to other students.	SD	D	N	A	SA
13. I tend to integrate multiple strands of mathematics within a single unit.	SD	D	N	A	SA

	Strongly Disagree	Disagree	Not Sure No Opinion	Agree	Strongly Agree
14. A lot of things in math must simply be accepted as true and remembered.	SD	D	N	A	SA
15. When students are working on math problems, I put more emphasis on getting the correct answer than on the process followed.	SD	D	N	A	SA
16. When students are grouped for instruction on the basis of their past mathematical performance, each student may then receive the level of mathematics instruction that is most appropriate for that student.	SD	D	N	A	SA
17. In my class, students learn math best when they can work together to discover mathematical ideas.	SD	D	N	A	SA
18. I like my students to master basic mathematical operations before they tackle complex problems.	SD	D	N	A	SA
19. No matter whether I am teaching mathematics to the whole class or to one group of students at a time, I know that I am most comfortable when I first model the activity, then provide some practice and immediate feedback, and, finally, clarify what the assignment is and how I expect it to be completed.	SD	D	N	A	SA
20. Teachers should incorporate students' diverse ideas and personal experiences into mathematics instruction.	SD	D	N	A	SA
21. In my class it is just as important for students to learn data management and probability as it is to learn multiplication facts.	SD	D	N	A	SA
22. If students use calculators they won't master the basic math skills they need to know.	SD	D	N	A	SA
23. Students can figure out how to solve many mathematics problems without being told what to do.	SD	D	N	A	SA
24. I teach students how to explain their mathematical ideas.	SD	D	N	A	SA

	Strongly Disagree	Disagree	Not Sure No Opinion	Agree	Strongly Agree
25. Prior achievement in mathematics determines a student’s potential for learning mathematics in the future.	SD	D	N	A	SA
26. Creating rubrics for math is a worthwhile assessment strategy.	SD	D	N	A	SA
27. Learning mathematics requires a good memory because you must remember how to carry out procedures and, when solving an application problem, you have to remember which procedure to use.	SD	D	N	A	SA
28. I don’t necessarily answer students’ math questions but rather let them puzzle things out for themselves.	SD	D	N	A	SA
29. The best way to teach students to solve mathematics problems is to model how to solve one kind of problem at a time until the students achieve mastery and then to foster frequent practice.	SD	D	N	A	SA
30. You have to study math for a long time before you see how useful it is.	SD	D	N	A	SA

The items in this instrument are either modifications of items drawn from the following sources or items drawn directly from the following sources. The permission of the authors of these sources has been granted to Patricia F. Campbell for use of their items within this instrument.

Campbell, P. F. (1990). *Project IMPACT Mathematics Beliefs Scales.* College Park, MD: University of Maryland, Center for Mathematics Education.

Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction, 6*, 1-40.

Ross, J. A., McDougall, D., Hogaboam-Gray, A., & LeSage, A. (2003). A survey measuring elementary teachers' implementation of standards-based mathematics teaching. *Journal for Research in Mathematics Education*, *34*, 344-363.

June 2004

 Developed with the support of the National Science Foundation, Grant No. ESI-0353360. Decisions regarding content selection reflect the opinions of the compiler and Management Team and not necessarily those of the Foundation.

Appendix B: Reformed Teaching Observation Protocol

The following observation protocol is reprinted with permission of the authors.

Reformed Teaching Observation Protocol (RTOP)

Daiyo Sawada — External Evaluator
Michael Piburn — Internal Evaluator

and

Kathleen Falconer, Jeff Turley, Russell Benford and Irene Bloom
Evaluation Facilitation Group (EFG)

Technical Report No. IN00-1
Arizona Collaborative for Excellence in the Preparation of Teachers
Arizona State University

I. BACKGROUND INFORMATION

Name of teacher ____________________ Announced Observation? ____________________ (yes, no, or explain)

Location of class ____ (district, school, room)

Years of Teaching ____________________ Teaching Certification ____________________ (K-8 or 7-12)

Subject observed ____________________ Grade level ____________________

Observer ____________________ Date of observation ____________________

Start time ____________________ End time ____________________

II. CONTEXTUAL BACKGROUND AND ACTIVITIES

In the space provided below please give a brief description of the lesson observed, the classroom setting in which the lesson took place (space, seating arrangements, etc.), and any relevant details about the students (number, gender, ethnicity) and teacher that you think are important. Use diagrams if they seem appropriate.

Record here events which may help in documenting the ratings.

Time	Description of Events

2000 Revision

III. LESSON DESIGN AND IMPLEMENTATION

		Never Occurred				Very Descriptive
1)	The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.	0	1	2	3	4
2)	The lesson was designed to engage students as members of a learning community.	0	1	2	3	4
3)	In this lesson, student exploration preceded formal presentation.	0	1	2	3	4
4)	This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.	0	1	2	3	4
5)	The focus and direction of the lesson was often determined by ideas originating with students.	0	1	2	3	4

IV. CONTENT

Propositional knowledge

6)	The lesson involved fundamental concepts of the subject.	0	1	2	3	4
7)	The lesson promoted strongly coherent conceptual understanding.	0	1	2	3	4
8)	The teacher had a solid grasp of the subject matter content inherent in the lesson.	0	1	2	3	4
9)	Elements of abstraction (i.e., symbolic representations, theory building) were encouraged when it was important to do so.	0	1	2	3	4
10)	Connections with other content disciplines and/or real world phenomena were explored and valued.	0	1	2	3	4

Procedural Knowledge

11)	Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.	0	1	2	3	4
12)	Students made predictions, estimations and/or hypotheses and devised means for testing them.	0	1	2	3	4
13)	Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.	0	1	2	3	4
14)	Students were reflective about their learning.	0	1	2	3	4
15)	Intellectual rigor, constructive criticism, and the challenging of ideas were valued.	0	1	2	3	4

2000 Revision

Continue recording salient events here.

Time	Description of Events

V. CLASSROOM CULTURE

Communicative Interactions		Never Occurred				Very Descriptive
16)	Students were involved in the communication of their ideas to others using a variety of means and media.	0	1	2	3	4
17)	The teacher's questions triggered divergent modes of thinking.	0	1	2	3	4
18)	There was a high proportion of student talk and a significant amount of it occurred between and among students.	0	1	2	3	4
19)	Student questions and comments often determined the focus and direction of classroom discourse.	0	1	2	3	4
20)	There was a climate of respect for what others had to say.	0	1	2	3	4
Student/Teacher Relationships						
21)	Active participation of students was encouraged and valued.	0	1	2	3	4
22)	Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.	0	1	2	3	4
23)	In general the teacher was patient with students.	0	1	2	3	4
24)	The teacher acted as a resource person, working to support and enhance student investigations.	0	1	2	3	4
25)	The metaphor "teacher as listener" was very characteristic of this classroom.	0	1	2	3	4

Additional comments you may wish to make about this lesson.

Appendix C: Interview Protocol

Materials:

- Have available a copy of the textbook, curriculum guide (including district-wide assessments), and sample state assessment questions.
- Audio recorder

Thank you again for agreeing to be a part of this study. The purpose of this study is to learn more about how teachers perceive of and respond to different messages about mathematics and mathematics teaching. First, I'd like you to talk a little about your teaching background. Second, I'd like to hear about how you typically plan your lessons. Third, I'd like to hear your thoughts about the different resources that are available to you.

Remember, that I will be using a pseudonym when I write about your thoughts and practices.

Let's start by having you tell me a little about your teaching background.

- How many years have you been teaching? What grades have you taught? Where have you taught? What mathematics courses have you taught? How many years have you taught each of the middle school mathematics courses?
- Have you been involved in the writing of the curriculum guides and assessments or the selection of the textbooks? If so, please tell me about that.

Next, I'd like to hear about how you plan your lessons and what most influences your planning decisions.

- How do you typically plan a lesson? What resources do you use?
- How would you rank the amount of influence that each of these resources (the textbook, curriculum guide (and local assessments), statewide assessments, master's degree program) has on your teaching? Is there anything else that significantly influences your teaching?

Next, I'd like to hear your thoughts about your textbook, curriculum guide (including district-wide assessments), statewide assessments, the University of Maryland master's degree program, (and any other significant influences that the teacher mentioned). For each of these different resources, I'd like you to talk about the messages about mathematics and the messages about mathematics teaching that you perceive are evident, how these fit with your own beliefs, and how you respond to these messages in your teaching.

Let's start by talking about the textbook and accompanying supplemental resources.

- How do you think the authors of the textbook materials envision an ideal lesson's design and implementation? How does this fit with your own vision of an ideal mathematics lesson?

- What do you think the authors of the textbook materials think is most important for students to learn about mathematics? How does this fit with your own priorities for your students?
- What kind of classroom culture do you imagine the authors of the textbook materials would want? How do you think they would want students and the teacher to interact? How does this fit with your own thoughts about student and teacher interactions?

Next, let's talk about the curriculum guide (and district-wide assessments).

- How do you think the authors of the curriculum guide envision an ideal lesson's design and implementation? How does this fit with your own vision of an ideal mathematics lesson?
- What do you think the authors of the curriculum guide think is most important for students to learn about mathematics? How does this fit with your own priorities for your students?
- What kind of classroom culture do you imagine the authors of the curriculum guide would want? How do you think they would want students and the teacher to interact? How does this fit with your own thoughts about student and teacher interactions?

Now, let's talk about the statewide assessments.

- How do you think the authors of the statewide assessments envision an ideal lesson's design and implementation? How does this fit with your own vision of an ideal mathematics lesson?
- What do you think the authors of the statewide assessments think is most important for students to learn about mathematics? How does this fit with your own priorities for your students?
- What kind of classroom culture do you imagine the authors of the statewide assessments would want? How do you think they would want students and the teacher to interact? How does this fit with your own thoughts about student and teacher interactions?

Finally, let's talk about the master's degree program.

- How do you think the instructors of the master's degree program courses envision an ideal lesson's design and implementation? How does this fit with your own vision of an ideal mathematics lesson?
- What do you think the instructors of the master's degree program courses think is most important for students to learn about mathematics? How does this fit with your own priorities for your students?
- What kind of classroom culture do you imagine the instructors of the master's degree program courses would want? How do you think they would want students and the teacher to interact? How does this fit with your own thoughts about student and teacher interactions?

If there are any other resources that the teacher draws upon in planning lessons, ask:

- How do you think the authors of the ___ materials envision an ideal lesson's design and implementation? How does this fit with your own vision of an ideal mathematics lesson?
- What do you think the authors of the ___ materials think is most important for students to learn about mathematics? How does this fit with your own priorities for your students?
- What kind of classroom culture do you imagine the authors of the ___ materials would want? How do you think they would want students and the teacher to interact? How does this fit with your own thoughts about student and teacher interactions?

If it seems that the teacher sees conflicting messages in the different resources:

- It sounds like you see some conflict in message among the different resources. Is there a particular conflict that is most difficult for you?
- How do you decide what to do with regard to this conflict?

Is there anything you'd like to add?

Thank you very much for your time!

Appendix D: Overview of Master's Degree Program

The Middle School Mathematics Partnership Program consists of 10 courses in mathematics and mathematics education. Participants in Cohort 1 took one course at a time, as a cohort, between June 2005 and May 2008. The schedule was as follows:

Summer 2005	EDCI 650	Trends in Mathematics Education
	EDCI 655	Teaching and Learning Algebra in the Middle School
Fall 2005	MATH 480	Algebra for Middle School Teachers
Spring 2006	EDCI 656	Teaching and Learning Statistics and Data Analysis in the Middle School
Summer 2006	MATH 481	Statistics and Data Analysis for Middle School Teachers
Fall 2006	EDCI 657	Understanding and Engaging Students' Conceptions of Mathematics
Spring 2007	EDCI 688c	Teaching and Learning Geometry in the Middle School
Summer 2007	MATH 482	Geometry for Middle School Teachers
Fall 2007	EDCI 654	Assessing Mathematical Understanding
Spring 2008	EDMS 696	Conducting Research on Teaching

Appendix E: Messages Sorted by Theme

Theme	Paraphrase of message	Resource	Teacher	Beliefs agree?	Reflected in practice?
Concepts and Procedures	Teachers should value more than procedures.	Master's program	Amelia	yes	yes
Concepts and Procedures	Teachers should focus more on concepts than procedures.	State	Amelia	yes	no
Concepts and Procedures	Teachers should emphasize procedures.	Textbook	Amelia	yes	yes
Concepts and Procedures	Teachers should emphasize concepts.	Master's program	Beth	yes	yes
Concepts and Procedures	Teachers should emphasize concepts.	School district	Beth	yes	yes
Concepts and Procedures	Teachers should emphasize skills.	School district	Beth	no	No
Concepts and Procedures	Teachers should emphasize skills.	State	Beth	no	no
Concepts and Procedures	Teachers should emphasize procedures.	Textbook	Beth	no	no
Concepts and Procedures	Teachers should emphasize skills.	Textbook	Beth	no	no
Concepts and Procedures	Teachers should emphasize concepts.	Master's program	Emma	yes	yes
Concepts and Procedures	Teachers should emphasize both procedures and concepts.	School district	Emma	yes	yes
Concepts and Procedures	Teachers should make sure that students remember formulas.	State	Emma	no	yes
Concepts and Procedures	Teachers should emphasize procedures.	Textbook	Emma	yes	yes
Concepts and Procedures	Teachers should focus on "why" in addition to "how."	Master's program	Kathleen	yes	yes
Concepts and Procedures	Teachers should emphasize concepts.	NCTM	Kathleen	yes	yes
Concepts and Procedures	Teachers should emphasize both procedures and concepts.	School district	Kathleen	yes	yes
Concepts and Procedures	Teachers should emphasize procedures.	School district	Kathleen	no	yes
Concepts and Procedures	Teachers should emphasize concepts.	State	Kathleen	yes	yes
Concepts and Procedures	Teachers should emphasize procedures.	Textbook	Kathleen	no	yes
Concepts and Procedures	Teachers should emphasize both procedures and concepts.	Master's program	Sarah	yes	yes

Theme	Paraphrase of message	Resource	Teacher	Beliefs agree?	Reflected in practice?
Concepts and Procedures	Teachers should emphasize both procedures and concepts.	School district	Sarah	yes	yes
Concepts and Procedures	Teachers should emphasize procedures.	Textbook	Sarah	yes	yes
Connections	Teachers should help students see how mathematical ideas connect.	Master's program	Beth	yes	yes
Connections	Teachers should help students see how mathematical ideas connect.	School district	Beth	yes	yes
Connections	Teachers should help students see how mathematical ideas connect.	Super Source	Beth	yes	yes
Connections	Teachers should help students see how mathematical ideas connect.	Master's program	Kathleen	yes	yes
Cooperative learning	Teachers should have students work with others.	School district	Beth	yes	yes
Cooperative learning	Teachers should have students work with others.	Super Source	Beth	yes	yes
Cooperative learning	Teachers should have students work alone.	Textbook	Beth	no	yes
Cooperative learning	Teachers should help students feel comfortable interacting with the teacher.	Master's program	Emma	yes	yes
Cooperative learning	Teachers should have students work with others.	NCTM	Kathleen	yes	yes
Cooperative learning	Teachers should have students work alone.	School district	Kathleen	no	yes
Cooperative learning	Teachers should have students work alone.	Textbook	Kathleen	no	yes
Differentiation	Teachers should differentiate instruction.	School district	Amelia	yes	yes
Differentiation	Teachers should differentiate instruction.	School district	Sarah	yes	yes
Explanation	Teachers should require students to explain their thinking.	State	Amelia	yes	yes
Explanation	Teachers should require students to explain their thinking.	Super Source	Beth	yes	yes

Theme	Paraphrase of message	Resource	Teacher	Beliefs agree?	Reflected in practice?
Explanation	Teachers should require students to explain their thinking.	School district	Emma	yes	yes
Explanation	Teachers should require students to explain their thinking.	State	Emma	yes	yes
Explanation	Teachers should require students to explain their thinking.	State	Kathleen	yes	yes
Explanation	Teachers should require students to explain their thinking.	School district	Sarah	yes	yes
Explanation	Teachers should require students to explain their thinking.	State	Sarah	yes	yes
Manipulatives	Teachers should incorporate manipulatives in lessons.	School district	Amelia	no	yes
Manipulatives	Teachers should incorporate manipulatives in lessons.	Textbook	Amelia	no	yes
Manipulatives	Teachers should provide opportunities for "hands-on" learning.	Textbook	Amelia	no	yes
Manipulatives	Teachers should incorporate manipulatives in lessons.	Super Source	Beth	yes	yes
Manipulatives	Teachers should incorporate manipulatives in lessons.	NCTM	Kathleen	yes	yes
Manipulatives	Teachers should provide opportunities for "hands-on" learning.	NCTM	Kathleen	yes	yes
Practice	Teachers should have students "practice."	School district	Amelia	yes	yes
Practice	Teachers should have students "practice."	Textbook	Amelia	yes	yes
Practice	Teachers should have students "practice."	Textbook	Beth	no	yes
Practice	Teachers should have students "practice."	Textbook	Beth	no	yes
Practice	Teachers should have students "practice."	School district	Emma	yes	yes
Question types	Teachers should make sure that students are able to solve "word problems."	School district	Amelia	no	yes
Question types	Teachers should make sure that students are able to solve "application problems."	Textbook	Amelia	no	yes
Question types	Teachers should have students work on "authentic tasks."	Super Source	Beth	yes	yes

Theme	Paraphrase of message	Resource	Teacher	Beliefs agree?	Reflected in practice?
Question types	Teachers should make sure that students are able to solve "application problems."	Textbook	Beth	yes	yes
Question types	Teachers should ask students questions that have "relevance" to their lives.	School district	Emma	yes	no
Question types	Teachers should make sure that students are able to solve "application problems."	School district	Emma	no	yes
Question types	Teachers should ask students to solve straightforward questions.	School district	Kathleen	no	yes
Question types	Teachers should make sure that students are able to solve "word problems."	State	Kathleen	yes	yes
Question types	Teachers should focus lessons on "big problems."	School district	Sarah	yes	no
Question types	Teachers should make sure that students are able to solve "application problems."	State	Sarah	yes	no
Question types	Teachers should ask students to solve straightforward questions.	Textbook	Sarah	no	yes
Question types	Teachers should include "little problems" and "big problems" in lessons.	Textbook	Sarah	yes	no
Question types	Teachers should make sure that students are able to solve "word problems."	Textbook	Sarah	yes	yes
Source of solution methods	Teachers should provide students with methods and worked examples to follow.	School district	Amelia	yes	yes
Source of solution methods	Teachers should encourage students to develop their own solution methods.	Master's program	Beth	yes	yes
Source of solution methods	Teachers should have students "investigate" mathematical ideas.	Master's program	Beth	yes	yes
Source of solution methods	Teachers should expose students to "problem solving."	School district	Beth	yes	yes

Theme	Paraphrase of message	Resource	Teacher	Beliefs agree?	Reflected in practice?
Source of solution methods	Teachers should try to "break away from that lecture style."	School district	Beth	yes	yes
Source of solution methods	Teachers should help students learn through "discovery."	Super Source	Beth	yes	yes
Source of solution methods	Teachers should have students memorize procedures.	Textbook	Beth	no	no
Source of solution methods	Teachers should provide students with methods and worked examples to follow.	Textbook	Beth	no	no
Source of solution methods	Teachers should help students learn through "discovery."	Master's program	Emma	yes	no
Source of solution methods	Teachers should encourage students to develop their own solution methods.	School district	Emma	yes	no
Source of solution methods	Teachers should provide students with methods and worked examples to follow.	School district	Emma	no	no
Source of solution methods	Teachers should provide students with methods and worked examples to follow.	Textbook	Emma	no	no
Source of solution methods	Teachers should have students "investigate" mathematical ideas.	Master's program	Kathleen	yes	no
Source of solution methods	Teachers should provide students with methods and worked examples to follow.	School district	Kathleen	no	no
Source of solution methods	Teachers should have students memorize procedures.	Textbook	Kathleen	no	no
Source of solution methods	Teachers should provide students with methods and worked examples to follow.	Textbook	Kathleen	no	no
Source of solution methods	Teachers should provide students with methods and worked examples to follow.	Textbook	Sarah	no	yes
Technology	Teachers should incorporate technology in lessons.	School district	Amelia	no	no
Technology	Teachers should incorporate technology in lessons.	Textbook	Amelia	no	no

Theme	Paraphrase of message	Resource	Teacher	Beliefs agree?	Reflected in practice?
Technology	Teachers should allow students to use calculators.	School district	Beth	yes	yes
Technology	Teachers should incorporate technology in lessons.	School district	Emma	yes	yes
Technology	Teachers should require students to be able to calculate without a calculator.	State	Emma	yes	yes
Technology	Teachers should allow students to use calculators.	NCTM	Kathleen	yes	yes
Technology	Teachers should show students how to use technology.	Textbook	Kathleen	yes	yes
Timeline	Teachers should keep up with the timeline.	School district	Beth	no	yes
Timeline	Teachers should keep up with the timeline.	School district	Sarah	no	yes

Appendix F: Messages Summarized by Teacher

Each of the teachers in this study tended to frequently bring up messages in the same themes. Below is a table showing the number of messages in each theme for each teacher.

	Amelia	Beth	Emma	Kathleen	Sarah
Concepts and Procedures	3	6	4	6	3
Connections	0	3	0	1	0
Cooperative Learning	0	3	1	3	0
Differentiation	1	0	0	0	1
Explanation	1	1	2	1	2
Manipulatives	3	1	0	2	0
Practice	2	2	1	0	0
Question type	2	2	2	2	5
Source of solution methods	1	7	4	4	1
Technology	2	1	2	2	0
Timeline	0	1	0	0	1

Amelia spoke about *Concepts and Procedures* and *Manipulatives* slightly more than any of the other themes. Beth and Emma spoke about both *Concepts and Procedures* and *Source of solution methods* twice as often as they talked about messages in any other theme. Kathleen spoke most often about messages in *Concepts and Procedures* and *Source of solution methods*. Sarah spoke about messages in *Question type* and *Concepts and Procedures* more than any other theme. All five teachers brought up messages related to *Concepts and Procedures* either most often or second most often.

Beth, Emma, and Kathleen brought up messages related to *Source of solution methods* either most often or second most often.

Although each of the teachers tended to focus on a few themes of messages, the messages they interpreted within those themes were often quite different for the different resources. For example, within the *Concepts and Procedures* theme Beth interpreted the master's degree program and the school district to be saying that "Teachers should emphasize concepts" while she interpreted the textbook to be saying that "Teachers should emphasize procedures." This indicates that the teachers do not only bring up messages with which their beliefs agree or disagree. Similarly, in her teaching, Beth focuses on concepts rather than procedures. Thus, she does not only bring up messages which are reflective of her practices.

Appendix G: Messages Sorted by Resource

A list of the messages which the teachers interpreted from the curricular resources is presented below. Here the messages are sorted by resource.

Resource	Paraphrase of message
NCTM	Teachers should emphasize concepts.
	Teachers should have students work with others.
	Teachers should incorporate manipulatives in lessons.
	Teachers should provide opportunities for "hands-on" learning.
	Teachers should allow students to use calculators.
Master's degree program	Teachers should emphasize both procedures and concepts.
	Teachers should emphasize concepts.
	Teachers should focus on "why" in addition to "how."
	Teachers should value more than procedures.
	Teachers should help students see how mathematical ideas connect.
	Teachers should help students feel comfortable interacting with the teacher.
	Teachers should encourage students to develop their own solution methods.
	Teachers should have students "investigate" mathematical ideas.
	Teachers should help students learn through "discovery."
School district	Teachers should emphasize both procedures and concepts.
	Teachers should emphasize concepts.
	Teachers should emphasize procedures.
	Teachers should emphasize skills.
	Teachers should help students see how mathematical ideas connect.
	Teachers should have students work alone.
	Teachers should have students work with others.
	Teachers should differentiate instruction.
	Teachers should require students to explain their thinking.
	Teachers should incorporate manipulatives in lessons.
	Teachers should have students "practice."
	Teachers should ask students questions that have "relevance" to their lives.
	Teachers should ask students to solve straightforward questions.
	Teachers should focus lessons on "big problems."
	Teachers should make sure that students are able to solve "application problems."
	Teachers should make sure that students are able to solve "word problems."

	Teachers should encourage students to develop their own solution methods.
	Teachers should expose students to "problem solving.”
	Teachers should provide students with methods and worked examples to follow.
	Teachers should try to "break away from that lecture style.”
	Teachers should allow students to use calculators.
	Teachers should incorporate technology in lessons.
	Teachers should keep up with the timeline.
State of Maryland	Teachers should emphasize concepts.
	Teachers should emphasize skills.
	Teachers should focus more on concepts than procedures.
	Teachers should make sure that students remember formulas.
	Teachers should require students to explain their thinking.
	Teachers should make sure that students are able to solve "application problems.”
	Teachers should make sure that students are able to solve "word problems.”
	Teachers should require students to be able to calculate without a calculator.
Super Source	Teachers should help students see how mathematical ideas connect.
	Teachers should have students work with others.
	Teachers should require students to explain their thinking.
	Teachers should incorporate manipulatives in lessons.
	Teachers should have students work on "authentic tasks.”
	Teachers should help students learn through "discovery.”
Textbook	Teachers should emphasize procedures.
	Teachers should emphasize skills.
	Teachers should have students work alone.
	Teachers should incorporate manipulatives in lessons.
	Teachers should provide opportunities for "hands-on" learning.
	Teachers should have students "practice.”
	Teachers should ask students to solve straightforward questions.
	Teachers should include "little problems" and "big problems" in lessons.
	Teachers should make sure that students are able to solve "application problems.”
	Teachers should make sure that students are able to solve "word problems.”
	Teachers should have students memorize procedures.
	Teachers should provide students with methods and worked examples to follow.
	Teachers should incorporate technology in lessons.
	Teachers should show students how to use technology.

Analysis of the messages sorted by resource indicates that the teachers interpreted some of the resources similarly and other resources quite differently. For example, all of the teachers' interpretations of messages about *Source of solution methods* in the master's degree program indicate that teachers should have students "develop their own solution methods," "investigate mathematical ideas," and "learn through discovery." The teachers saw these messages to be quite similar. Conversely, the teachers' interpretations of messages about *Source of solution methods* in the school district documents range from "Teachers should encourage students to develop their own solution methods" to "Teachers should provide students with methods and worked examples to follow." The teachers saw these messages as quite disparate.

References

Abrams, L. M., Pedulla, J. J., & Madaus, G. F. (2003). Views from the classroom: Teachers' opinions of statewide testing programs. *Theory into Practice, 42*(1), 18-29.

Aguirre, J., & Speer, N. M. (2000). Examining the relationship between beliefs and goals in teacher practice. *Journal of Mathematical Behavior, 18*(3), 327-356.

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education, 7,* 91-119.

American Association for the Advancement of Science. (1989). *Science for all Americans.* New York: Oxford University Press.

American Association for the Advancement of Science. (1993). *Benchmarks for science literacy*. New York: Oxford University Press.

American Association for the Advancement of Science. (2000). *Middle grades mathematics textbooks: A benchmarks-based evaluation.* Washington, DC: Author.

Andrews, P., & Hatch, G. (1999). A new look at secondary teachers' conceptions of mathematics and its teaching. *British Educational Research Journal, 25*(2), 203-223.

Bachkirova, T. (2003). *Role of values in teacher stress.* Retrieved June 7, 2006 from http://www.brookes.ac.uk/schools/eduation/rescon/Stress.pdf

Badertscher, E. M. (2007). *An inquiry into relationships with mathematics: How identities and personal ways of knowing mediate and respond to mathematics content experiences.* Unpublished doctoral dissertation, University of Maryland, College Park.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-467.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). San Francisco: Jossey-Bass.

Barab, S. A., & Plucker, J. A. (2002). Smart people or smart contexts? Cognition, ability, and talent development in an age of situated approaches to knowing and learning. *Educational Psychologist, 37*(3), 165-182.

Benson, J., Dodge, S., Dodge, W., Hamberg, C., Milauskas, G., & Rukin, R. (1997). *Gateways to algebra and geometry: An integrated approach.* Evanston, IL: McDougal Littell.

Berk, D. M. (2004). *The interplay between instructional policy and its constituents: How mathematics teachers interact with and understand Principles and Standards for School Mathematics.* (Doctoral dissertation, UMI No. 3158671)

Berk, D. M. (2005). Educational policy as a vehicle for teacher learning. In G. M. Lloyd, M. Wilson, J. L. M. Wilkins, & S. L. Behm (Eds.), *Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.*

Bogdan, R. C., & Biklen, S. K. (2003). *Qualitative research for education: An introduction to theories and methods* (4th ed.) Boston: Allyn and Bacon.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33*(8), 3-15.

Cady, J. (2006). Implementing reform practices in a middle school classroom. *Mathematics Teaching in the Middle School, 11*(9), 460-466.

Campbell, P. F. (2004). *Teachers' beliefs about mathematics and mathematics teaching beliefs scales*. College Park, MD: University of Maryland, Center for Mathematics Education.

Campbell, P. F., Kramer, S., Bowden, A. R., & Yakimowski, M. E. (2005). *District-wide reform in elementary mathematics: The value-added impact of professional development on student achievement.* Manuscript in preparation.

Chávez-López, O. (2003). *From the textbook to the enacted curriculum: Textbook use in the middle school mathematics classroom.* (Doctoral dissertation, UMI No. 3099613)

Chazan, D. (1990). Quasi-empirical views of mathematics and mathematics teaching. *Interchange, 21,* 14-23.

Chazan, D., & Ball, D. (1999). Beyond being told not to tell. *For the Learning of Mathematics, 19*(2), 2-10.

Chazan, D., & Schnepp, M. (2002). Methods, goals, beliefs, commitments, and manner in teaching: Dialogue against a calculus backdrop. *Social Constructivist Teaching, 9*, 171-195.

Chval, K. B., Grouws, D. A., Smith, M., & Ziebarth, S. (2006, April). *A cross-site study of local factors affecting mathematics curriculum enactment.* Paper presented at the meeting of the American Educational Research Association, San Francisco, CA.

Civil, M. (1990). "You only do math in math": A look at four prospective teachers' views about mathematics. *For the Learning of Mathematics, 10*(1), 7-9.

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis, 12*(3), 311-329.

Cohen, D. K., & Hill, H. C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record, 102*(2), 294-343.

Cohen, D. K., & Hill, H. C. (2001). *Learning policy: When state education reform works*. New Haven, CT: Yale University Press.

Cohen, S. (2004). *Teachers' professional development and the elementary mathematics classroom*. Mauwah, NJ: Erlbaum.

Collins, W., Dritsas, L., Frey, P., Howard, A. C., McClain, K., Molina, D., et al. (2001a). *Mathematics: Applications and connections, Course 1*. Columbus, OH: Glencoe/McGraw Hill.

Collins, W., Dritsas, L., Frey, P., Howard, A. C., McClain, K., Molina, D., et al. (2001b). *Mathematics: Applications and connections, Course 3*. Columbus, OH: Glencoe/McGraw Hill.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education, 16*, 324-336.

Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29*, 306-333.

Cuban, L. (1995). How organizations influence teacher responses to secondary science curriculum reform. *Theory into Practice, 34*(1), 4-11.

Dossey, J. A. (1992). The nature of mathematics: Its role and its influence. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 39-48). New York: Macmillan.

Drake, C., & Sherin, M. G. (2006). Practicing change: Curriculum adaptation and teacher narrative in the context of mathematics education reform. *Curriculum Inquiry, 36*(2), 153-187.

Ernest, P. (1988). The attitudes and practices of student teachers of primary school mathematics. In A. Borbas (Ed.), *Proceedings of 12th International Conference on the Psychology of Mathematics Education* (pp. 288-295). Hungary: Veszprem.

Ernest, P. (1989a). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249-254). New York: Falmer.

Ernest, P. (1989b). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15,* 13-33.

ETA/Cuisenaire. (1996a). *The super source grades K-6 resource library*. Vernon Hills, IL: Author.

ETA/Cuisenaire. (1996b). *The super source grades 7-8 resource library*. Vernon Hills, IL: Author.

Fey, J. T. (1979). Mathematics teaching today: Perspectives from three national surveys. *Arithmetic Teacher, 27*(2), 10-14.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38*, 915-945.

Goldenberg, E. P. (1999). Principles, art, and craft in curriculum design: The case of Connected Geometry. *International Journal of Computers for Mathematical Learning, 4,* 191-224.

Greeno, J. G., & Moore, J. L. (1993). Situativity and symbols: Response to Vera and Simon. *Cognitive Science, 17*(1), 49-59.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education*, *34*(1), 37-73.

Heid, M. K. (1997). The technological revolution and the reform of school mathematics. *American Journal of Education, 106*, 5-61.

Hersh, R. (1998). Some proposals for revising the philosophy of mathematics. In T. Tymoczko (Ed.), *New directions in the philosophy of mathematics* (pp. 9-28). Princeton, NJ: Princeton.

Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An "experiment" model for teaching and teacher preparation in mathematics. *Journal of Mathematics Teacher Education, 6,* 201-222.

Hiebert, J., & Stigler, J. W. (2000). A proposal for improving classroom teaching: Lessons from the TIMSS Video Study. *The Elementary School Journal, 101*(1), 3-20.

Hill, H. C. (2001). Policy is not enough: Language and the interpretation of state standards. *American Educational Research Journal, 38*(2), 289-318.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*, *35*, 330-351.

Kilpatrick, J. (2003). What works? In S. S. Senk & D. R. Thompson (Eds.), *Standards-based school mathematics curricula: What are they? What do students learn?* (pp. 471-488). Mahwah, NJ: Erlbaum.

Kirschner, P. A., Sweller, J., and Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*(2), 75-86.

Lakatos, I. (1976). A renaissance of empiricism in the recent philosophy of mathematics. *The British Journal for the Philosophy of Science, 27*(3), 201-223.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (1998). *Connected Mathematics series.* Palo Alto, CA: Dale-Seymour.

Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education*, *9*, 91-102.

Lerman, S. (1983). Problem-solving or knowledge-centred: the influence of philosophy on mathematics teaching. *International Journal of Mathematical Education in Science and Technology, 14*(1), 59-66.

Lerman, S. (1990). Alternative perspectives on the nature of mathematics and their influence on the teaching of mathematics. *British Educational Research Journal, 16*(1), 53-60. Retrieved February 7, 2006 from http://weblinks1.epnet.com/

Malloy, C., Price, J., Willard, T., & Sloan, L. L. (2003). *Pre-Algebra.* New York: Glencoe/McGraw-Hill.

Maryland State Department of Education. (2003). *Sample item: Sample Extended Constructed Response item for grade 8.* Retrieved March 13, 2008 from http://mdk12.org/assessments/k_8/index.html

Maryland State Department of Education (2008). *Voluntary State Curriculum Toolkit.* Retrieved January 24, 2008 from http://mdk12.org/instruction/curriculum/mathematics/vsc_toolkit.html

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: Macmillan.

National Council of Teachers of Mathematics. (1980). *An agenda for action.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2005, May). *Computation, calculators, and common sense.* Retrieved June 18, 2008, from http://www.nctm.org/positionstatements

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence.* Reston, VA: Author.

National Research Council. (1995). *National science education standards.* Washington, DC: National Academy Press.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Research Council. (2002). Executive summary of *Investigating the influence of standards: A framework for research in mathematics, science, and technology education.* I. R. Weiss, M. S. Knapp, K. S. Hollweg, & G. Burrill (Eds.).

Committee on Understanding the Influence of Standards in K-12 Science, Mathematics, and Technology Education. Retrieved April 13, 2008 from http://www.nap.edu/catalog/10023.html

Neufeldt, V., & Guralnik, D. B. (Eds.). (1988). *Webster's new world dictionary of American English* (3rd ed.). New York: Simon & Schuster.

Ng, S. S. N., & Rao, F. L. R (2003). Early mathematics teaching: The relationship between teachers' beliefs and classroom practices. *International Group for the Psychology of Mathematics Education.* Retrieved February 21, 2005, from http://onlinedb.terc.edu/PME2003/PDF/RR-López-real.pdf

No Child Left Behind Act of 2001 (executive summary). (2001). [On-line]. Retrieved January 7, 2004, from www.ed.gov/nclb/overview/intro/execsumm.html

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332.

Putnam, R. T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4-15.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28,* 550-576.

Remillard, J. T. (1996). *Changing texts, teachers, and teaching: The role of curriculum materials in mathematics education reform.* (Doctoral dissertation, UMI No. 9734181)

Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry, 29*(3), 315-342.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246.

Remillard, J. T., & Bryans, M. B. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education, 35*(5), 352-388.

Reys, B. J., Reys, R. E., & Chávez, O. (2004). Why mathematics textbooks matter. *Educational Leadership, 61*(5), 61-66.

Reys, R., Reys, B., Tarr, J., & Chávez, O. (2006). *Assessing the impact of standards-based middle school mathematics curricula on student achievement and the* *classroom learning environment* (Final Report No. R305T010735). Columbia, MO: University of Missouri.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula, T. Buttery, & E. Guyton (Eds.), *Handbook of research on teacher education: A project of the Association of Teacher Educators* (pp. 102-119). New York, NY: Macmillan.

Romberg, T. A. (1992). Problematic features of the school mathematics curriculum. In P. W. Jackson (Ed.), *Handbook of research on curriculum* (pp. 749-788). New York: Macmillan.

Ross, J. A., McDougall, D., Hogaboam-Gray, A., & LeSage, A. (2003). A survey measuring elementary teachers' implementation of standards-based mathematics teaching. *Journal for Research in Mathematics Education*, *34*(4), 344-363.

Sawada, D., Piburn, M., Falconer, K., Turley, J., Benford, R., & Bloom, I. (2000). *Reformed teaching observation protocol* (ACEPT Tech. Rep. No. IN00-1). Arizona State University.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Schorr, R. Y., Firestone, W. A., Monfils, L. (2003). State testing and mathematics teaching in New Jersey: The effects of a test without other supports. *Journal for Research in Mathematics Education*, *34*(5), 373-405.

Schwille, J., Porter, A., Belli, G., Floden, R., Freeman, D., Knappen, L., et al. (1982). *Teachers as policy brokers in the content of elementary school mathematics* (Report No. ED 225 854 SE 040 213). East Lansing, MI: Institute for Research on Teaching.

Secada, W.G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623-660). Reston, VA: NCTM.

Selden, A., & Selden, J. (1997). Preservice teachers' conceptions of mathematics and how to teach it. *Mathematical Association of America Online.* Retrieved November 11, 2004, from http://www.maa.org/t_and_l/samplers/rs_3.html

Smith, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*(4), 387-402.

Soloway, E. (1996). Teachers are the key. *Communications of the ACM, 39*(6), 11-14.

Sosniak, L. A., & Stodolsky, S. S. (1993). Teachers and textbooks: Materials use in four fourth-grade classrooms. *The Elementary School Journal*, *93*(3), 249-275.

Spillane, J. P. (2004). *Standards deviation: How schools misunderstand education policy*. Cambridge, MA: Harvard University Press.

Spillane, J. P., & Callahan, K. A. Implementing state standards for science education: What district policy makers make of the hoopla. *Journal of Research in Science Teaching, 37*(5), 401-425.

Spillane, J. P., Reiser, B. J., & Reimer, T. (2002). Policy implementation and cognition: Reframing and refocusing implementation research. *Review of Educational Research, 72*(3), 387-431.

Spillane, J. P., & Thompson, C. L. (1997). Reconstructing conceptions of local capacity: The local education agency's capacity for ambitious instructional reform. *Educational Evaluation and Policy Analysis*, *19*, 185-203.

Steele, D. F., & Widman, T. F. (1997). Practitioner's research: A study in changing preservice teacher's conceptions about mathematics and mathematics teaching and learning. *School Science and Mathematics, 97,* 184-191.

Stein, M. K., Smith, M. S., & Silver, E. A. (1999). The development of professional developers: Learning to assist teachers in new settings in new ways. *Harvard Educational Review, 69*, 237-269.

Steiner, H. G. (1987). Philosophical and epistemological aspects of mathematics and their interaction with theory and practice in mathematics education. *For the Learning of Mathematics, 7,* 7-13.

Stecher, B., Le, V., Hamilton, L., Ryan, G., Robyn, A., & Lockwood, J. R. (2006). Using structured classroom vignettes to measure instructional practices in mathematics. *Educational Evaluation and Policy Analysis, 28*(2), 101-130.

Stipek, D. L, Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17,* 213-226.

Tarr, J. E., Chávez, O., Reys, R. E., & Reys, B. J. (2006). *From the written to enacted curricula: The intermediary role of middle school mathematics in shaping students' opportunity to learn.* Manuscript submitted for publication.

Tate, W.F. (1994). Race, retrenchment, and the reform of school mathematics [Electronic version]. *Phi Delta Kappan, 75*(6), 477-484.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, *15*, 105-127.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Thompson, C. L., & Zeuli, J. S. (1999). The frame and the tapestry: Standards-based reform and professional development. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 341-375). San Francisco: Jossey-Bass.

Tomayko, M. C. (2007). *An examination of the working conditions, challenges, and tensions experienced by mathematics teachers.* Unpublished doctoral dissertation, University of Maryland, College Park.

Tymoczko, T. (Ed.). (1998). *New directions in the philosophy of mathematics* (Rev. ed.). Princeton, NJ: Princeton.

Van De Walle, J. A. (2004). *Elementary and middle school mathematics: Teaching developmentally* (5th ed.). New York: Longman.

Weizman, G., & Hoz, R. (2006). *A revised theorization of the relationship between teachers' conceptions of mathematics and its teaching.* Manuscript submitted for publication.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of research in education* (Vol. 24, pp. 173-209). Washington, D.C.: American Educational Research Association.