ABSTRACT

Title of Dissertation: RISK-BASED BRIDGE MAINTENANCE STRATEGIES

Adel Abdel-Rahman Al-Wazeer, Doctor of Philosophy, 2007

Directed By: Professor Bilal M. Ayyub, Department of Civil and Environmental Engineering

The aging of bridges and traffic volume increase result in deterioration of bridge conditions and consequently increased risks to bridges and users. Highway agencies in the United States face challenges because of lack of sufficient resources to maintain bridges in good conditions. Current bridge management practices use cost as the main factor for determining maintenance strategies for bridges. Bridge management methods that identify bridges based on risk considerations and produce cost-effective maintenance strategies are needed to best utilize limited resources for reducing risks associated with bridge conditions.

This dissertation proposes and demonstrates a methodology for defining bridge maintenance strategies based on risks associated with conditions of bridge elements and costs needed to improve these conditions. The methodology is a systematic approach for assessing risks to bridge elements based on their failure probabilities and consequences and managing associated risks using cost-effective maintenance strategies. The proposed methodology defines maintenance scenarios, optimal policies that minimize cost of maintenance and risks to elements, and optimal timing for implementing or deferring maintenance policies. The element-level maintenance policies in the proposed methodology are integrated with bridge-level priority ranking to define practical maintenance strategies for an inventory of bridges. The bridges are prioritized for maintenance according to their risk values and risk-reduction effectiveness of their maintenance policies based on benefit-cost analysis. The proposed methodology builds on existing bridge management methods, and allows for the use of risks associated with bridge conditions to assist in making risk-informed decisions for allocating limited resources for cost-effective maintenance strategies of bridges most in need.

The study showed that risk is a viable tool for managing the maintenance of bridges in a cost-effective manner. The case study showed that the proposed methodology is feasible and can be implemented in currently used bridge management systems.

RISK-BASED BRIDGE MAINTENANCE STRATEGIES

By

Adel Abdel-Rahman Al-Wazeer

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2007

Advisory Committee:

Professor Bilal M. Ayyub, Chair
Research Professor Chung C. Fu, Co-Chair
Professor Amde M. Amde
Associate Professor Dimitrios G. Goulias
Professor Eyad H. Abed

Dedication

To My Late Mother and My Father

To My Wife Rola

To My Daughter Raghad, and My Sons Khaled and Abdelrahman

Acknowledgements

I would like to thank my advisor Professor Bilal M. Ayyub, Director of the Center for Technology and Systems Management, for his supervision, guidance, and support. His patience and encouragement is highly appreciated.

I would like to thank my co-advisor Research Professor Chung C. Fu, Director of Bridge Engineering Software & Technology (BEST) Center, for his support and guidance. I appreciate his support and encouragement

I would like to thank Professor Eyad H. Abed, Director of Institute for Systems Research, Professor Amde M. Amde, and Associate Professor Dimitrios G. Goulias for being part of my advisory committee.

Finally, I would like to thank my beloved wife for her patience during the time that I was busy doing this research.

Table of Contents

CHAPTER 1. INTRODUCTION 1

1.1 Background 1

1.2 Research Motivation 3

1.3 Research Objectives 5

1.4 Organization of the Dissertation 6

CHAPTER 2. CURRENT PRACTICES IN BRIDGE MANAGEMENT 7

2.1 History of Bridge Management Systems 7

2.2 Main Bridge Management Systems in the United States 10

2.3 Pontis Bridge Management System 11

2.4 Deterioration Prediction Model in Pontis BMS 13

2.5 Selection of Optimal Policies in Pontis Preservation Model 16

CHAPTER 3. METHODOLOGY FOR RISK-BASED BRIDGE MAINTENANCE STRATEGIES 22

3.1 Background 22

3.2 System Definition for Risk Analysis 25

3.3 System Breakdown 26

3.4 Element Level Modeling 28

3.5 Risk Assessment 38

3.5.1 Failure Mode Identification 39

3.5.2 Probability of Failure 44

3.5.3 Failure Consequences 55

3.5.4 Risk Estimation 58

3.6 Risk Management ... 72
3.6.1 Risk-Based Element Maintenance ... 72
3.6.2 Risk-Based Bridge Maintenance ... 99
3.6.3 Risk-Based Maintenance of Bridge Inventory ... 107
CHAPTER 4. CASE STUDY ... 114
4.1 Background ... 114
4.2 Tool Used ... 114
4.3 Analysis Objective ... 115
4.4 System Definition and Breakdown ... 116
4.5 Element-Level Modeling ... 117
4.6 Failure Probabilities in Condition States of Bridge Elements ... 130
4.6.1 Failure Probabilities for Steel Girder Element ... 130
4.6.2 Failure Probability for Concrete Deck Element ... 160
4.7 Estimation of Failure Consequences ... 181
4.8 Estimating Risk in Condition States of Bridge Elements ... 187
4.9 Risk Estimation at the Element Level ... 193
4.10 Risk Estimation at the Bridge Level ... 195
4.11 Element-Level Risk-Based Maintenance ... 196
4.12 Risk-Based Maintenance for Concrete Deck Element ... 196
4.12.1 Optimal Maintenance Actions for Concrete Deck Element ... 196
4.12.2 Maintenance Scenarios of Implementing/Deferring Optimal Maintenance Actions for Concrete Deck Element ... 222

4.12.3 Cost and Risk Associated with Maintenance Scenarios of Concrete Deck Element 225
4.12.4 Filtering Maintenance Scenarios for Concrete Deck Element 228
4.12.5 Ranking Maintenance Scenarios for Concrete Deck Element 230
4.12.6 Selecting the Optimal Maintenance Scenario for Concrete Deck Element 241
4.13 Risk-Based Maintenance for Steel Girder Element 244
4.13.1 Optimal Maintenance Actions for Steel Girder Element 244
4.13.2 Maintenance Scenarios of Implementing/Deferring the Optimal Maintenance Actions for Steel Girder Element 269
4.13.3 Cost and Risk Associated with Maintenance Scenarios of Steel Girder Element 271
4.13.4 Filtering the Maintenance Scenarios for Steel Girder Element 274
4.13.5 Ranking Maintenance Scenarios of Steel Girder Element 276
4.13.6 Selecting the Optimal Maintenance Scenario for Steel Girder Element 284
4.14 Bridge-Level Risk-Based Maintenance 289
4.15 Risk-Based Maintenance of the Bridge Inventory 294
CHAPTER 5. CONCLUSIONS AND RECOMMENDATIONS 297
5.1 Research Contributions 297
5.2 Main Conclusions 300
5.3 Recommendation for Future Research 301
APPENDIX A FAILURE CONSEQUENCES FOR BRIDGE ELEMENTS 303

List of Tables

Table 3-1. Weight Factors for Commonly Recognized Bridge Elements........... 28

Table 3-2. Materials Types of Commonly Recognized Bridge Elements........... 43

Table 3-3. Probabilities of Occurrence of N_f Failure Modes in N_s Possible Condition States of a Bridge Element... 53

Table 3-4. Risk Results Associated with a Bridge Element due to N_f Failure Modes in N_s Condition States of the Element.................................... 60

Table 3-5. Possible Scenarios for Analysis Periods of 1, 2, 3, and 4 in a Maintenance Planning Horizon... 81

Table 4-1. Elements of Bridge B_1 Used in the Case Study 121

Table 4-2. Quantity Distributions in Square Feet (SM) of the Concrete Deck Element Estimated by Five Bridge Inspectors................................. 121

Table 4-3. Quantity Distributions in Linear Feet (LF) of the Steel Girder Elements Estimated by Five Bridge Inspectors 122

Table 4-4. Deterioration Rates of Concrete Deck Element and Steel Girder Element of Bridge B_1 122

Table 4-5. Deterioration Transition Probabilities $P_{i,i}$ and $P_{i,i+1}$ for Concrete Deck Element and Steel Girder Element of Bridge B_1 123

Table 4-6. Feasible Maintenance Actions for Concrete Deck Element of Bridge B_1.. 124

Table 4-7. Feasible Maintenance Actions for Painted Steel Girder Element of Bridge B_1.. 125

Table 4-8. Repair Rates for Concrete Deck Element of Bridge B_1 126

Table 4-9. Repair Rates of Steel Girder Element of Bridge B_1 127

Table 4-10. Repair Transition Probabilities for Concrete Deck Element of Bridge B_1 128

Table 4-11. Repair Transition Probabilities for Steel Girder Element of Bridge B_1 129

Table 4-12. Section Properties for Steel Girder Element of Bridge B_1 140

Table 4-13. Reduced Cross-Sectional Dimensions in Condition States of Painted Steel Girder Element 147

Table 4-14. Section Properties in Condition States of Painted Steel Girder Element 147

Table 4-15. Calculation of Nominal Moment Resistance in Condition States of Steel Girder Element 148

Table 4-16. Calculation of Moment Load Effects for Steel Girder Element 148

Table 4-17. Calculation of Reliability Index Due to Flexural Failure Mode in Condition States of Steel Girder Element 149

Table 4-18. Calculation of Probability of Flexural Failure in Condition States of Steel Girder Element 149

Table 4-19. Calculation of Nominal Shear Resistance in Condition States of Steel Girder Element 150

Table 4-20. Calculation of Shear Load Effects for Steel Girder Element 150

Table 4-21. Reliability Index Calculation Due to Shear Failure Mode in Condition States of Steel Girder Element 151

Table 4-22. Calculation of Probabilities of Shear Failure in Condition States of Steel Girder Element 151

Table 4-23. Probabilities of Failure in Condition States of Steel Girder Element152

Table 4-24. Reliability Index in Condition States of Steel Girder Element due to Flexural Failure Mode for Five Years 159

Table 4-25. Reliability Index in Condition States of Steel Girder Element Due to Shear Failure Mode for Five Years.. 159

Table 4-26. Probabilities of Failure in Condition States of Steel Girder Element Due to Flexural and Shear Failure Modes for Five Years 160

Table 4-27. Cross Sectional Properties of Concrete Deck Element..................... 168

Table 4-28. Calculation of Nominal Moment Resistance in Condition States of Concrete Deck Element 169

Table 4-29. Calculation of Moment Load Effects for Deck Element 169

Table 4-30. Reliability Index Calculation Due to Flexural Failure Mode in Condition States of Deck Element... 170

Table 4-31. Calculation of Probabilities of Flexural Failure in Condition States of Concrete Deck Element 170

Table 4-32. Calculation of the Nominal Shear Resistance in Condition States of Concrete Deck Element 171

Table 4-33. Calculation of Shear Load Effects for Concrete Deck Element 171

Table 4-34. Reliability Index Calculation Due to Shear Failure Mode in Condition States of Concrete Deck Element 172

Table 4-35. Calculation of Probabilities of Shear Failure in Condition States of Concrete Deck Element 172

Table 4-36. Probabilities of Failure in the Condition States of Concrete Deck Element Due to Flexural and Shear Failure Modes 173

Table 4-37. Reliability Index in Condition States of Concrete Deck Element Due to Flexural Failure Mode for Five Years 180

Table 4-38. Reliability Index in Condition States of the Deck Element Due to Shear Failure Mode for Five Years.. 180

Table 4-39. Probabilities of Failure in Condition States of Concrete Deck Element Due to Flexural and Shear Failure Modes for Five Years 181

Table 4-40. Upper and Lower Limits for Consequences of Flexural and Shear Failure of Steel Girder Element 183

Table 4-41. Upper and Lower Limits for Consequences of Flexural and Shear Failure of Concrete Deck Element... 184

Table 4-42. Means and Standard Deviations for Consequences of Flexural and Shear Failure of Steel Girder Element 185

Table 4-43. Means and Standard Deviations for Consequences of Flexural and Shear Failure of Concrete Deck Element... 186

Table 4-44. Probabilities and Consequences of Flexural and Shear Failure Modes in Condition States of Steel Girder Element.................................... 188

Table 4-45. Consequences and Probabilities of Flexural and Shear Failure Modes in Condition States of Concrete Deck Element 188

Table 4-46. Risks to the Condition States of the Steel Girder Element 189

Table 4-47. Risks to the Condition States of Concrete Deck Element................ 190

Table 4-48. Risks Unit Costs in Condition States of Steel Girder Element for Five Years 191

Table 4-49. Risks Unit Costs in Condition States of Concrete Deck Element for Five Years 192

Table 4-50. Calculation of the Initial Risk Associated with Condition of Steel Girder Element.. 193

Table 4-51. Calculation of the Initial Risk Associated with Condition of the Concrete Deck Element 194

Table 4-52. Initial Risk Associated with Condition of Bridge B_1....................... 195

Table 4-53. Feasible Maintenance Actions for the Concrete Deck Element 204

Table 4-54. First-Year Optimal Maintenance Actions for the Concrete Deck Element 207

Table 4-55. Condition State Distribution of the Concrete Deck Element after Applying First-Year Optimal Maintenance Actions........................ 207

Table 4-56. Calculation of Maintenance Cost and Risk Associated with the First-Year Optimal Actions of the Concrete Deck Element..................... 208

Table 4-57. Second-Year Optimal Maintenance Actions for the Concrete Deck Element 211

Table 4-58. Third-Year Optimal Maintenance Actions for the Concrete Deck Element 214

Table 4-59. Fifth-Year Optimal Maintenance Actions for the Concrete Deck Element 219

Table 4-60. Risk-Based Optimal Maintenance Actions for the Concrete Deck Element in Each of the Five Years of Maintenance 222

Table 4-61. Five Years Maintenance Actions for the Concrete Deck Element for Different Cases of Delay and Application of Optimal Actions 224

Table 4-62. Maintenance Cost for the Different Maintenance Scenarios of Concrete Deck Element 226

Table 4-63. Risk Associated with the Concrete Deck Condition during the Five Years of Maintenance 227

Table 4-64. Deck Maintenance Scenarios Exceeding Maximum Risk Value Allowed for the Concrete Deck Condition 229

Table 4-65. Deck Maintenance Scenarios Exceeding the Annual Budget for the Concrete Deck Maintenance 230

Table 4-66. Risk Reduction Associated with Maintenance Scenarios of the Concrete Deck Element 237

Table 4-67. Benefit-Cost Ratio (BCR) and Net Present Value (NPV) for the Filtered Maintenance Scenarios of the Concrete Deck Element 238

Table 4-68. Means and Standard Deviations for Risk Reduction and Maintenance Cost Associated with Filtered Maintenance Scenarios for the Concrete Deck 239

Table 4-69. Benefit-Cost Reliability Index and Probability of Failure Associated with the Maintenance Scenarios for the Concrete Deck Element..... 240

Table 4-70. Comparison Between Maintenance Scenarios for the Concrete Deck in Terms of BCR, NPV, $\beta_{B/C}$, and $P_{f,B/C}$.. 242

Table 4-71. Ranking of Maintenance Scenarios for the Concrete Deck Based on Benefit-Cost Ratio (BCR), Net Present Value (NPV), and Benefit-Cost Reliability Index ($\beta_{B/C}$) and Probability of Failure ($P_{f,B/C}$)................ 242

Table 4-72. Feasible Maintenance Actions for the Steel Girder Element............ 248

Table 4-73. Third-Year Optimal Maintenance Actions for the Steel Girder 255

Table 4-74. Fifth-Year Optimal Maintenance Actions in the Condition States of the Steel Girder Element in the Different Cases of Delay of Previous-Years Optimal Actions.. 265

Table 4-75. Maintenance Actions for Different Scenarios of Delayed and Applied Optimal Actions for the Steel Girder Element................................. 270

Table 4-76. Maintenance Costs for the Different Maintenance Scenarios of Steel Girder Element.. 272

Table 4-77. Risk Associated with Maintenance Scenarios of the Steel Girder Element During Five Years of Maintenance 273

Table 4-78. Eliminated Scenarios Exceeding the Maximum Risk and/or the Annual Budget for the Steel Girder Maintenance 275

Table 4-79. Risk Reduction Associated with Maintenance Scenarios of the Steel Girder Element.. 279

Table 4-80. Benefit-Cost Ratio (BCR) and Net Present Value (NPV) for Filtered Maintenance Scenarios of Steel Girder Element 280

Table 4-81. Ranking of Filtered Maintenance Scenarios of Steel Girder Element based on Benefit-Cost Ratio (BCR) and Net Present Value (NPV) . 281

Table 4-82. Means and Standard Deviations for Risk Reductions and Maintenance Costs for Filtered Maintenance Scenarios for Steel Girder 282

Table 4-83. Benefit-Cost Reliability Index ($\beta_{B/C}$)and Probability of Failure ($P_{f,B/C}$) Associated with Filtered Maintenance Scenarios for Steel Girder Element 283

Table 4-84. Comparison Between the Maintenance Scenarios for the Steel Girder in Terms of BCR, NPV, $\beta_{B/C}$ and $P_{f,B/C}$ 286

Table 4-85. Ranking of the Maintenance Scenarios for the Steel Girder Element Based on BCR, NPV, $\beta_{B/C}$ and $P_{f,B/C}$ 287

Table 4-86. Maintenance Cost and Risk Reduction Associated with Bridge Maintenance 293

List of Figures

Figure 2-1. State Transition Diagram for a Three-State System 16

Figure 2-2. Calculation of Minimum Long-term Cost of Actions........................ 20

Figure 2-3. Methodology for Calculating the Minimum Failure Cost of Bridge Elements.. 21

Figure 3-1. Flowchart of Methodology for Risk-Based Bridge Maintenance Strategies (RBBMS) 24

Figure 3-2. Classification of Damage Categories for Commonly Recognized Bridge Elements.. 36

Figure 3-3. Transition between Condition States of a Bridge Element with Five Repairable Condition States and a Failure State................................ 37

Figure 3-4. Framework for Calculating Failure Probabilities of Bridge Elements in Possible Condition States.. 54

Figure 3-5. Procedure for Estimating Risk Associated with Bridge Elements in Different Condition States Due to One Failure Mode of the Element 61

Figure 3-6. Procedure for Estimating Risk due to All Failure Modes of a Bridge Element When the Element is in a Given Condition State 64

Figure 3-7. Procedure for Estimating Risks Associated with Bridge Elements.... 68

Figure 3-8. Procedure for Estimating Risks Associated with Conditions of All Elements of the Bridge.. 71

Figure 3-9. Priority Ranking for Maintenance of Bridges Based on Risk and Risk Reduction Effectiveness of Maintenance Scenarios for Bridges...... 111

Figure 3-10. Budget Allocation for Maintenance of Bridges Based on the Priority Ranking of Bridges 113
Figure 4-1. Break Down of Bridge B_1 in the Case Study into Elements 120
Figure 4-2. Cross Sectional Dimensions (inch) for Painted Steel Girder Element of Bridge B_1 140
Figure 4-3. Dimensions of Steel Girder Cross-Section with No Section Loss.... 141
Figure 4-4. Dimensions of Steel Girder Cross-Section in Condition State 1 142
Figure 4-5. Dimensions of Steel Girder Cross-Section in Condition State 2 143
Figure 4-6. Dimensions of Steel Girder Cross-Section in Condition State 3 144
Figure 4-7. Dimensions of Steel Girder Cross-Section in Condition State 4 145
Figure 4-8. Dimensions of Element Cross-Section in Condition State 5 146
Figure 4-9. Cumulative Distribution Functions for Maximum Moment Load Effects on Steel Girder Element 153
Figure 4-10. Cumulative Distribution Functions for Maximum Shear Load Effects on Steel Girder Element 154
Figure 4-11. Probability Density Functions for Maximum Moment Load Effects on Steel Girder Element 155
Figure 4-12. Probability Density Functions for Maximum Shear Load Effects on Steel Girder Element 156
Figure 4-13. Change of Maximum Moment Load Effects on Steel Girder Element for a Planning Horizon of Twenty Years 157
Figure 4-14. Change of Maximum Shear Load Effects on Steel Girder Element for a Planning Horizon of Twenty Years 158

Figure 4-15. Cross Section for Concrete Deck Element....................................... 168

Figure 4-16. Cumulative Distribution Functions for Maximum Moment Load Effects on Concrete Deck Element.. 174

Figure 4-17. Cumulative Distribution Functions for Maximum Moment Load Effects on Concrete Deck Element.. 175

Figure 4-18. Probability Density Functions for Maximum Moment Load Effects on Concrete Deck Element 176

Figure 4-19. Probability Density Functions for Maximum Shear Load Effects on Concrete Deck Element 177

Figure 4-20. Change of the Maximum Moment Load Effects on Concrete Deck Element for a Planning Horizon of Twenty Years............................ 178

Figure 4-21. Change of the Maximum Shear Load Effects on Steel Concrete Deck Element for a Planning Horizon of Twenty Years............................ 179

Figure 4-22. Scenarios of Actions with Lowest Summation of Maintenance Cost and Risk in the First Year of Maintenance for the Concrete Deck Element 205

Figure 4-23. Condition State Distribution for the Concrete Deck Element for the Five Scenarios with Lowest Summation of Maintenance Cost and Risk in the First Year of Maintenance 206

Figure 4-24. Scenarios of Optimal Actions with Lowest Summation of Maintenance Cost and Risk in the Second Year of Maintenance for the Concrete Deck Element... 209

Figure 4-25. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios with Lowest Summation of Maintenance Cost and Risk in the Second Year of Maintenance..... 210

Figure 4-26. Scenarios of Optimal Actions with Lowest Summation of Maintenance Cost and Risk in the Third Year of Maintenance for the Concrete Deck Element 212

Figure 4-27. Condition State Distribution for the Concrete Deck Element Associated with Optimal Scenarios of Lowest Summation of Maintenance Cost and Risk in the Third Year of Maintenance........ 213

Figure 4-28. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Concrete Deck Maintenance Where Third-Year Optimal Actions Were Applied 215

Figure 4-29. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Concrete Deck Maintenance Where Third-Year Optimal Actions Were Delayed.. 216

Figure 4-30. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of Maintenance with Applied Third-Year Optimal Actions 217

Figure 4-31. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of Maintenance with Delayed Third-Year Optimal Actions... 218

Figure 4-32. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios of Lowest Summation of Maintenance Cost and Risk in the Fifth Year of Maintenance with Applied Fourth-Year Optimal Actions 220

Figure 4-33. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios of Lowest Summation of Maintenance Cost and Risk in the Fifth Year of Maintenance with Delayed Fourth-Year Optimal Actions 221

Figure 4-34. Condition State Distribution Associated with Five Years Optimal Maintenance Strategy 243

Figure 4-35. Scenarios with Lowest Summation of Maintenance Cost and Risk in First Year of Maintenance for the Steel Girder Element 249

Figure 4-36. Condition State Distribution for the Steel Girder Element for the Five Scenarios with Lowest Summation of Maintenance Cost and Risk in the First Year of Maintenance 250

Figure 4-37. Scenarios of Optimal Actions with Lowest Summation of Maintenance Cost and Risk in the Second Year of Maintenance for the Steel Girder Element 251

Figure 4-38. Condition State Distribution for the Steel Girder Element Associated with the Optimal Scenarios with Lowest Summation of Maintenance Cost and Risk in the Second Year of Maintenance 252

Figure 4-39. Scenarios of Optimal Actions with Lowest Summation of Maintenance Cost and Risk in the Third Year of Maintenance for the Steel Girder Element 253
Figure 4-40. Condition State Distribution for the Steel Girder Element Associated with Optimal Scenarios of Lowest Summation of Maintenance Cost and Risk in the Third Year of Maintenance 254
Figure 4-41. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Steel Girder Maintenance with No Delay or Delay in First-Year Optimal Actions 256
Figure 4-42. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Steel Girder Maintenance with Delay in the Optimal Actions of the Second (and First) Year 257
Figure 4-43. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Steel Girder Maintenance with Delay in the Optimal Actions of the Third Year and One or Two Years Before .. 258
Figure 4-44. Condition State Distribution for the Steel Girder Element Associated with Fourth-Year Optimal Maintenance Scenarios for Cases with No Delay or Delay in First-Year Optimal Actions 259
Figure 4-45. Condition State Distribution for the Steel Girder Element Associated with Fourth-Year Optimal Maintenance Scenarios for Cases with Delay in the Optimal Actions of the Second (and First) Year 260
Figure 4-46. Condition State Distribution for the Steel Girder Element Associated with Fourth-Year Optimal Maintenance Scenarios for Cases with

Delay in the Optimal Actions of the Third Year and One or Two Years Before 261
Figure 4-47. Scenarios of Optimal Actions (A00100) in the Fifth Year of the Steel Girder Maintenance with No Delay and Delay of First and/or Second-Year Optimal Actions 262
Figure 4-48. Scenarios of Optimal Actions (A00120) in the Fifth Year of the Steel Girder Maintenance with Corresponding Cases of Delayed Optimal Actions 263
Figure 4-49. Scenarios of Optimal Actions (A00121) in Fifth Year of Maintenance with Corresponding Cases of Delayed Optimal Actions 264
Figure 4-50. Condition State Distribution for the Steel Girder Element for Cases That Use Scenario A00100 for Fifth-Year Optimal Maintenance Actions 266
Figure 4-51. Condition State Distribution for the Steel Girder Element for Cases That Use Scenario A00120 for Fifth-Year Optimal Maintenance Actions 267
Figure 4-52. Condition State Distribution for the Steel Girder Element for Cases That Use Scenario A00121 for Fifth-Year Optimal Maintenance Actions 268
Figure 4-53. Condition State Distribution Associated with the Five Years Risk-Based Optimal Maintenance Strategy for the Steel Girder Element 288

List of Abbreviations

AASHTO	American Association of State Highway and Transportation Officials
BCR	Benefit-Cost Ratio
BMS	Bridge Management System
CDF	Cumulative Distribution Function
CoRe	Commonly Recognized
CS	Condition State
EA	Each
FHWA	Federal Highway Administration
FM	Failure Mode
HBRRP	Highway Bridge Replacement and Rehabilitation Program
ISTEA	Intermodal Surface Transportation Efficiency Act
LF	Linear Foot
MR&R	Maintenance, Repair and Rehabilitation
NBI	National Bridge Inventory
NBIP	National Bridge Inspection Program
NBIS	National Bridge Inspection Standards
NCHRP	National Cooperative Highway Research Program
NPV	Net Present Value
PDF	Probability Density Function
RBBMS	Risk-Based Bridge Maintenance Strategies

SBRP	Special Bridge Replacement Program
SF	Square Foot
SM	Square Meter

List of Notations

α	Discount factor
$a(i,j)$	Action number i applied in condition state j
α_d	Density ratio of corrosion rust product to reinforcing steel in concrete sections
$a_{k,j}$	Maintenance action number k in condition state j
a_q	Multiplier for load component
A_w	Web area
b	Width of concrete section
β	Reliability index
$\beta_{B/C}$	Reliability index of the benefit exceeding the cost
BCR	Benefit-cost ratio
b_f	Flange width
β_T	Target reliability index
c	Concrete cover of reinforcement bars
$c(a_{k,j})$	Cost of maintenance action number k in condition state j
$C(i,a)$	Cost of maintenance action a in condition state i
C_a	Consequences of failure related to traffic accidents
C_b	Consequences of failure related to the bridge
C_e	Consequences of failure related to the element
C_{env}	Consequences related to the environment
C_F	Total consequences of failure

C_h	Consequences related to health and safety
$c_i(a_k,j)$	Unit cost of maintenance action a_k in condition state j estimated by expert i
C_{nb}	Consequences to nearby businesses
C_p	Consequences to the general public
C_p	Present value of total maintenance cost of a bridge element during the planning horizon
CR_t	Sum of maintenance cost at year t-1 and risk associated with the element at year t
C_t	Maintenance cost of a bridge element at year t
$C_{t,p}$	Present value of maintenance cost of a bridge element at year t
C_u	Consequences of failure related to the bridge users
d	Effective depth of concrete section
Δ	Section loss
δ	Reduction in thickness of the steel element surface
D	Reinforcing bar diameter in concrete section
ΔD	Reduction in reinforcement bar diameter
d_f	Flange depth
ΔR	Risk reduction
ΔR_p	Present value of total risk reduction for a maintenance scenario
$\Delta R_{t,p}$	Present value of risk reduction at year t
d_w	Web depth
Δw	Crack width in concrete section

$\Phi(\,)$	Cumulative distribution function of the standard normal
f'_c	Concrete compressive strength
$F_X(y)$	Cumulative distribution function of load effects
F_y	Steel yield stress
$F_{Yn}(y)$	Cumulative distribution function of extreme load effects in n years
γ	Load factor
h	Depth of concrete section
i_d	Real discount rate
λ	Bias factor
L	Load effect
l	Load variable
λ_{ij}	Deterioration rate from condition state i to condition state j
M	Bending moment
μ	Mean
μ_{ij}	Repair rate from condition state i to condition state j
M_n	Nominal moment resistance
$N_{a,j}$	Number of maintenance actions in condition state j
N_B	Number of bridges
N_e	Number of elements in the bridge
N_f	Number of failure modes of the element
N_h	Number of bridges in high risk group
n_{inp}	Number of bridge inspectors

n_L	Number of load components
N_l	Number of bridges in low risk group
N_m	Number of bridges in medium risk group
NPV	Net present value
Ns	Number of condition states
n_s	Number of maintenance scenarios for a given planning horizon
n_{sa}	Number of scenarios of actions for a bridge element
N_y	Number of years in a planning horizon for maintenance
P	Transition probability matrix
$P(\,)$	Probability of the bridge element being in condition state
P_f	Probability of failure
$P_{f,B/C}$	Reliability of the benefit failure to exceed the cost
P_{ij}	Transition probability from condition state i to condition state j
$P_{ij}(a)$	Transition probability from condition state i to condition state j when action a is applied
Q	Quantity of a bridge element
q_j	Quantity of a bridge element in condition state j
$R_{(i,j)}$	Risk associated with condition of the bridge element due to failure mode i when the element is in condition state j
R_B	Risk associated with a bridge
R_H	Risk threshold between the high risk group and the medium risk group

R_j	Risk associated with condition of the bridge element when the element is in condition state j
r_j	Risk associated with condition of unit measurement of the bridge element in condition state j
R_M	Risk threshold between the medium risk group and the low risk group
R_n	Nominal resistance
R_{n0}	Nominal resistance with no section loss
R_t	Mitigated risk at year t
$R_{t,\,B}$	Risk associated with a bridge at year t
$R_{t,0}$	Unmitigated risk at year t
σ	Standard deviation
$S(k)$	State of a system at time event k
t	Time in years
T	Year at which condition states of elements become steady state
t_f	Flange thickness
t_w	Web thickness
V	Coefficient of variation
$V(i)$	Minimum long-term cost in condition state i
$V(i,a)$	Minimum long-term cost in condition state i when an action a is applied
Vn	Nominal shear resistance
Z	Plastic section modulus

Chapter 1. Introduction

1.1 Background

State Highway Agencies face challenges because of aging of bridges, increasing traffic, using new materials, and lack of sufficient resources. According to the National Bridge Inventory (NBI) data of 2005 available in the Federal Highway Bridge Management Information Systems Laboratory, there are more than 608,000 bridge structures (including culverts and tunnels) in the United States. If culverts and tunnels are excluded, the total number of structures designated as bridges by the National Bridge Inspection Standards (NBIS) is around 473,000 bridges. More than two thirds of these bridges were constructed before 1980, including one third constructed before 1960. More than 30% of bridges are considered deficient. Steel, concrete, and prestressed concrete are the main material types used for most of bridges in the United States. Around 40% of steel bridges and 30% of concrete bridges are considered deficient (NBI 2005 in Bridge Management Information System Laboratory). To overcome the existing deficiencies of highway bridges, more than ten billion dollars per year over the next ten years are needed for the repair and replacement of these bridges (Estes, et al. 2003b).

Unavailability of adequate funds has been responsible for many of the decisions to defer maintenance (Golabi, et al. 1993). The limited resources of highway and bridge agencies are not enough to fix existing deficiencies in bridges and maintain all bridges in need in good conditions. Deterioration of bridges exposes them to more risks. Bridge management aims to identify bridges most in need of maintenance and apply efficient maintenance strategies to best utilize limited resources.

The conditions of bridges in the United States are rated using two methods. The first method uses NBI condition rating for bridge deck, superstructure and substructure, while the second method uses more detailed data to describe the conditions of bridge elements. NBI condition rating describes the conditions of bridge deck, superstructure and substructure using a scale of 0 to 9 (FHWA, 1995). Element-level condition rating uses condition states to describe the conditions of bridge elements (AASHTO, 1997). The prediction of NBI condition rating from element-level condition rating was investigated (Hearn, et al. 1997; Al-Wazeer, et al. 2007b). The main bridge management system in the United States, Pontis, uses the element-level condition rating method to describe the conditions of bridges. "Developing models for risk assessment" is the second item for recommended future research in Pontis Technical Manual (Golabi, et al. 1993).

The research in this dissertation provides a methodology for bridge maintenance strategies based on risks associated with conditions of bridge elements. The methodology is a systematic approach for assessing risks based on failure probabilities and failure consequences of bridge elements and managing associated risks using efficient strategies of optimal maintenance actions.

The proposed methodology allows for the selection of optimal maintenance actions in the condition states of bridge elements based on both maintenance costs and risks associated with elements conditions. If a bridge engineer decides to apply a particular scenario of maintenance for some years and no maintenance for other years in a planning horizon, the methodology allows him/her to select the optimal actions corresponding to that scenario. The proposed methodology provides the optimal timing for implementing and/or deferring optimal actions in a planning horizon based on a tradeoff between risk reductions and maintenance costs while considering acceptable risk levels and budget constraints.

The proposed methodology is a useful tool for making informed decisions to better utilize limited resources for managing existing bridges. The methodology integrates element-level maintenance policies with bridge-level priority ranking to define practical maintenance strategies for an inventory of bridges.

1.2 Research Motivation

Current bridge management practices use cost as the main factor for determining maintenance policies for bridges. Pontis is the predominant bridge management system in the United States. Pontis determines optimal policies for bridge elements based on the long-term cost of maintenance actions.

Risk is a viable tool that can be used for managing bridges and allocating resources efficiently. Conditions of bridge elements are associated with risks. Risks associated with conditions of bridge elements increase when elements deteriorate. Maintenance actions are applied to improve the elements conditions. Risks associated with elements conditions and costs of maintenance actions can be used to determine optimal policies for elements. Priority ranking of bridges for maintenance can be based on risk and benefit-cost analysis.

Risk is calculated based on the probability of failure and the consequences of failure. A probability of failure exists in any possible condition state of a bridge element. The calculation of the probability of failure in any condition state of an element can be based on reliability techniques. The cost associated with failure of a bridge element can be based on the consequences of failure due to the failure modes of the element. Probabilities and consequences of failure are used to estimate risks associated with the condition states of bridge elements.

The proposed methodology defines risk-based maintenance strategies that identify optimal maintenance actions in condition states of bridge elements, identify optimal timing for applying these actions, and prioritize bridges for maintenance based on risk and benefit-cost analysis.

1.3 Research Objectives

The main goal of this study is to develop and demonstrate a methodology for defining risk-based bridge maintenance strategies that best utilize available resources based on risks associated with conditions of bridge elements and costs needed to improve these conditions. In order to achieve this goal, the following tasks were carried out:

1. Estimation of failure probabilities in condition states of bridge elements,
2. Identification and estimation of failure consequences for bridge elements,
3. Assessment of risks associated with condition states of bridge elements based on probabilities and consequences of failure,
4. Selection of optimal maintenance actions in condition states of bridge elements based on maintenance costs and risks associated with elements conditions,
5. Selection of optimal timings for implementing and/or deferring optimal actions based on benefit-cost analysis of risk reduction, and
6. Priority ranking of bridges for maintenance and efficient allocation of resources for bridges most at risk.

The proposed methodology defines optimal maintenance strategies for bridge elements based on maintenance costs and risks associated with elements conditions. Optimal times for implementing and/or deferring optimal maintenance actions is determined in the methodology based on benefit–cost analysis of risk reduction effectiveness. The methodology allows for cost-effective bridge maintenance strategies by prioritizing bridges for maintenance based on risk and benefit-cost analysis. The proposed methodology builds on existing bridge management methods, and allows for the use of risk values associated with conditions of bridge elements to assist in making informed decisions for a system of bridges.

1.4 Organization of the Dissertation

This dissertation is organized into five chapters and one Appendix. Chapter 1 provides an introduction to the research subject. It includes background information, objective statement and structure of the dissertation. Chapter 2 presents an overview of the current practices for bridge management. Chapter 3 outlines the proposed methodology for defining risk-based maintenance strategies for bridges. Chapter 4 demonstrates the methodology using a case study. Chapter 5 provides conclusions and recommendations. Appendix A provides information about the classification of failure consequences for bridge elements. Finally, a bibliography is provided at the end of the dissertation. It includes all cited references and other sources that provide background information on bridge management practices and risk applications.

Chapter 2. Current Practices in Bridge Management

The goal of having a bridge management system (BMS) is to assist highway and bridge agencies in allocating available resources effectively among bridges by identifying bridge needs and establishing maintenance priorities.

2.1 History of Bridge Management Systems

The following milestones played an important role in the development of bridge management systems (BMS) in the United States:

1. Development of the National Bridge Inspection Standards (NBIS):

Following the collapse of the Silver Bridge in 1967, the U.S. Congress required the development and implementation of the National Bridge Inspection Standards (NBIS) which formed the basis for bridge inspection and bridge management (Small, et al. 1999). The National Bridge Inspection Program (NBIP), created by the Federal Highway Administration (FHWA) as a result of the Federal-Aid Highway Act of 1968, mandated biennial inspection of all bridges on the Federal aid system (FHWA, 1995). State agencies were required through this program to collect and maintain information on the condition of bridges on principal highways. The data collected as part of the NBIP are reported to the Federal Highway Administration (FHWA) where it is maintained in the National Bridge Inventory (NBI) database (FHWA, 1995). Initiated in 1972, the NBI database now contains detailed historical data on over

600,000 bridges (Chase, et al. 1999). The NBI information and inspection procedures were outlined by the Recording and Coding Guide for the Structure Inventory and Appraisal of the Nation's Bridges (FHWA, 1995).

2. Special Bridge Replacement Program (SBRP):

The Federal Highway Administration created the Special Bridge Replacement Program (SBRP) in 1970 to provide funding to States for the replacement of bridges located on the Federal-aid system based on the NBI information (Small, et al. 1999).

3. Highway Bridge Replacement and Rehabilitation Program (HBRRP):

The Surface Transportation Assistance Act of 1978 replaced the SBRP with the Highway Bridge Replacement and Rehabilitation Program (HBRRP) that allows for funding of bridge rehabilitation in addition to bridge replacement (Small, et al. 1999). Funding distributed through HBRRP program is based on a sufficiency rating formula for deficient bridges. The sufficiency rating includes NBI information related to structural adequacy and safety, serviceability and functional obsolescence, and essentiality for public use (FHWA, 1995). The sufficiency rating ranges between 0 and 100, where 100 represents a bridge in an excellent condition. Deficient bridges with sufficiency rating less than 50 are eligible for replacement funding, while deficient bridges with sufficiency rating between 50 and 80 are eligible for rehabilitation funding.

4. NCHRP Project 12-28(2):

The objective of NCHRP Project 12-28 (Hudson, et al. 1987) was to develop a model bridge management system at the network level. The basic concepts of bridge management systems were analyzed and grouped into six computer modules that include database module; network level major maintenance, rehabilitation, and replacement (MR&R) selection module; maintenance module; historical data analysis module; project level interface module; and reporting module. The data base module contains the basic information needed for bridge management. The network MR&R selection module provides the analyses necessary for effective programming and budgeting decisions. The maintenance module assigns maintenance programs for preventive and responsive maintenance. The historical data analysis module tracks past and future actions and expenditures. The project level interface module uses the programmed activities at the network level in the selection of actions in the bridge level. The reporting module provides summaries of bridge conditions, budgeting, and bridge MR&R programs. The National Cooperative Highway Research Program (NCHRP) and the National Engineering Technology Corporation jointly developed a bridge management system, called BRIDGIT, as a result of the NCHRP Project 12-28 (Hudson, et al. 1987).

5. FHWA Demonstration Project 71:

The FHWA Demonstration Project 71 (O’Conner, et al. 1989) was initiated to develop sound bridge management practices. This demonstration project provided the foundation for the development of a comprehensive bridge management system that could be used by any State. The FHWA Project 71 continued with the development of the Pontis bridge management system (Small, et al. 1999).

6. Intermodal Surface Transportation Efficiency Act (ISTEA):

The U.S. Congress passed the Intermodal Surface Transportation Efficiency Act (ISTEA) in 1991. The ISTEA legislation mandated the development and implementation of bridge management systems for all State Departments of Transportation (Small, et al. 1999). The legislation was updated in 1993 to include the minimum standards for bridge management systems.

7. Pontis Bridge Management System:

In 1989, Pontis was developed by the FHWA in conjunction with six State Departments of Transportation and the joint venture of consultants Optima, Inc. and Cambridge Systematics. Pontis can be used to develop long-term maintenance and improvement programs (Golabi, et al. 1993).

2.2 Main Bridge Management Systems in the United States

The main bridge management systems in the United States are Pontis and BRIDGET. Pontis follows a top-down approach while BRIDGIT follows a bottom-up approach. In top-down approach, budgets and standards are used to develop optimal policies, which are then used to plan projects. In bottom-up approach, standards assist in the planning of projects and the planned projects are totaled to generate costs which are then compared to budgets (Small, et al. 1999).

Pontis is a network bridge management system and its models are designed to operate on inventories of bridges instead of individual structures. In 1994, the Association of American State Highway Officials (AASHTO) incorporated Pontis into the AASHTOWare program. The first Windows version was released as Pontis 3.0 in 1995 and the latest version of Pontis is version 4.4. Forty-six agencies in the United States and seven agencies outside the United States are using Pontis as their bridge management systems. (Robert, et al. 2003).

2.3 Pontis Bridge Management System

Pontis is the predominant bridge management system in the United States. Pontis is a network level bridge management system which incorporates probabilistic models, and a detailed bridge database to predict maintenance and improvement needs, recommend optimal policies, and schedule projects within budget and policy constraints (Golabi, et al. 1993). Pontis is a bridge management software tool that uses cost data and condition data of bridge elements to predict least cost long-term policies for a network of bridges.

Pontis consists of a number of models that include database, cost model, deterioration prediction model, maintenance, rehabilitation and replacement (MR&R) model, improvement model, and integration model. The data base model is used for storing the elements of each bridge, the condition states of each element, and the set of feasible actions associated with each condition state (Golabi, et al. 1993). The cost model includes costs for MR&R actions, improvement, replacement, and user costs. The prediction model estimates the deterioration rates between the different condition states of each element.

The MR&R or preservation model is used for finding the long-term minimum cost policy for each bridge element based on deterioration, condition states, feasible actions, and MR&R costs. The MR&R model is formulated using a Markov decision process. The MR&R model determines the optimal policy for each element and the steady state condition if optimal policy is followed. The optimal MR&R policy for an element is the set of optimal actions in the condition states. The optimal actions in the condition states are the actions with minimum long-term cost in each condition state.

The improvement model weighs the benefits of improvement against costs and prioritizes the bridges in need of improvement (Golabi, et al. 1993). Improvement activities, such as widening and raising, improve the level of service. The integration model combines MR&R and improvement activities into a bridge-level program.

The models of interest in this dissertation are the deterioration prediction model and the selection of the optimal policy in the MR&R model. These items are explained in the following sections.

2.4 Deterioration Prediction Model in Pontis BMS

Deterioration of bridges is modeled in Pontis using a Markov decision process. In a Markov process, the probability of transition from a given state to a future state is dependent only on the present state and not on the manner in which the current state was reached. A change of state occurs only at the end of the time period and nothing happens during the time period chosen. The probability of moving from any given state i to state j on the next time interval is called the transition probability P_{ij}. The transition probability matrix P, as shown in Equation 2-1, is non-negative and its rows sum to unity. It constitutes all transition probabilities between the set of possible states of the process.

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & .... & P_{1m} \\ P_{21} & P_{22} & P_{23} & .... & P_{2m} \\ .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... \\ P_{m1} & P_{m2} & P_{m3} & .... & P_{mm} \end{bmatrix} \qquad (2\text{-}1)$$

Assuming a unit time interval, each probability P_{ij} represents the probability of transition from state $S_i(t)$ to state $S_j(t+1)$ for any value of t. Transition probabilities are assumed to be stationary over the time period of interest (time invariant) and independent of how the current state was reached. A state transition diagram can be used to represent the transition probability matrix. Such a diagram shows the states and the probabilities represented by numbers on arrows between states. Figure 2-1 shows a state transition diagram for a three-state system.

The state of the system at any point in time can be estimated if the transition probability matrix and the initial state of the system are known. Given the initial state distribution of the system, $S(0) = [S_1, S_2, \ldots, S_m]$, and the transition probability matrix P, we can evaluate the state distribution of the system at the next point in time using Equation 2-2.

$$S(1) = S(0)P \tag{2-2}$$

Likewise, the state distribution of the system at any point in time k can be calculated as:

$$S(k) = S(0)P^k \tag{2-3}$$

where P^k is the k^{th} power of the transition probability matrix P.

For finite-state Markov chains, all condition states converge to a unique stationary (invariant) state distribution after a sufficient number of transitions. If P is the transition probability matrix of a Markov chain, and if P^k approaches a unique limiting matrix, then the process is said to have reached a steady state. Since the system reaches a steady state, the k^{th} and $(k+1)^{th}$ state distribution of the system will be the same as shown in Equation 2-4.

$$S(k+1) = S(0)P^{k+1} = S(0)P^{k} = S(k) \tag{2-4}$$

The steady state distribution $S(k)$ can be represented by Equation 2-5.

$$S(k) = (S_1^{(k)}, S_2^{(k)}, \ldots , S_m^{(k)}) \tag{2-5}$$

where

$$S_1^{(k)} + S_2^{(k)} + \cdots + S_m^{(k)} = 1 \tag{2-6}$$

When the system reaches a steady state, the state distribution of the system $S(k)$ remains unchanged and does not change with any number of steps. This distribution is called a steady state distribution as shown in Equation 2-7.

$$(S_1^{(k)}, S_2^{(k)}, \ldots , S_m^{(k)}) = (S_1^{(k)}, S_2^{(k)}, \ldots , S_m^{(k)})P \tag{2-7}$$

The steady state distribution represents the approximate probabilities of the system in each state at the end of a transition after a sufficiently large number of transitions. Thus, at equilibrium, the frequency of transitions into any given set of states must be equal to the frequency of transitions out of that set of states. Consequently, after a sufficiently long period of time, the distribution of states will be approximately invariant or stationary.

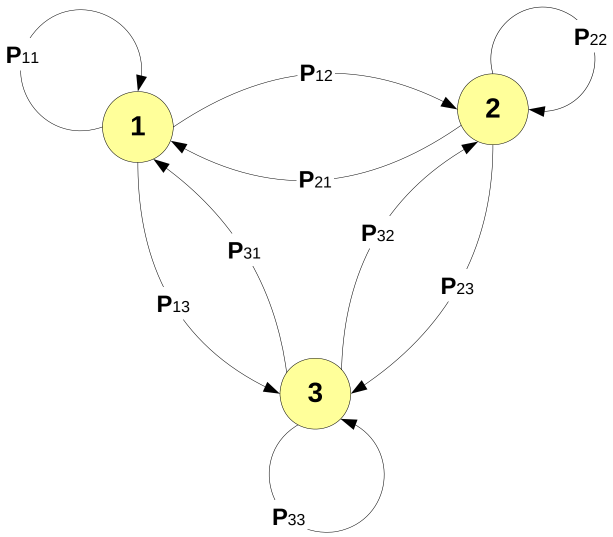


Figure 2-1. State Transition Diagram for a Three-State System

2.5 Selection of Optimal Policies in Pontis Preservation Model

Pontis preservation or MR&R model is used for selecting optimal actions in the condition states of bridge elements. Maintenance actions are selected at each time unit to improve The element performance. The behavior of the element at each time is determined by the state of the element, as well as the action chosen. The state and action fully determine the probability to move to any given state in the next time unit.

Optimal maintenance policies for bridge elements in the preservation model of Pontis bridge management system are selected based on the long-term cost of maintenance actions. The optimal policy in Pontis MR&R model is the policy that minimizes the long-term total expected cost and defined as shown in Equations 2-8 to 2-10.

$$V(i) = \underset{a}{Min}\ V(i,a) \tag{2-8}$$

$$V(i,a) = \underset{a}{Min}\left[C(i,a) + \alpha \sum_{j=1}^{N_s} P_{ij}(a) V(j) \right] \tag{2-9}$$

$$V(j) = \underset{a}{Min}\ V(j,a) \tag{2-10}$$

where,

N_s = the number of possible condition states of the element,

α = discount factor for future cost, $0<\alpha<1$,

i = current condition state of the element,

j = predicted condition state of the element,

$V(i,a)$ = expected long-term cost as a result of being in state i today when action a is applied,

$C(i,a)$ = initial cost of action a taken in state i,

$P_{ij}(a)$ = transition probability from condition state i to condition state j under action a, and

$V(j,a)$ = expected long-term cost if transition to condition state j occurs.

The long-term cost is optimal when $V(i) = Min\ V(i,a)$ for all i, with the requirement that a is the optimal action associated with state i. The calculation of the minimum long-term cost of actions can be illustrated graphically in an iterated procedure as shown in Figure 2-2.

The long-term cost of actions in the condition states of a bridge element depends on a failure cost and a probability of failure in the worst condition state of the element. Agencies that rely on using failure costs to trigger actions for an element in the worst condition state have focused on what failure cost is required to trigger preservation actions (Gurenich, 2002). A methodology for estimating a minimum failure cost was presented at the 2001 Pontis Task Force meeting in Denver, CO. (Al-Wazeer, 2001). The methodology is illustrated as shown in Figure 2-3. A method for calculating the minimum and maximum failure cost was also developed for Florida Department of Transportation (Thompson, et al. 2003). Gurenich (2002) developed a closed form method to calculate the minimum failure cost "inspired by the presentations on this topic at the 2001 Pontis User Training Meeting in Detroit made by Adel Al-Wazeer and Paul D. Thompson, as well as additional work performed by Mike Johnson and Richard Shepard of Caltrans" (Gurenich, 2002).

The optimal policies of bridge elements can be calculated based on risk. Risk can be defined as the potential of losses resulting from exposure to hazard (Ayyub, 2003). Hazard is an act or phenomenon posing potential harm or threat and could lead to failure (Ayyub, 2003). Traffic can be considered as a hazard because it has the potential to do harm to the bridge (Ryall, 2001).

Risk is commonly evaluated as the product of probability of failure and the consequences of failure. The probability of failure is calculated based on reliability theory. The consequences could be economic damage, environmental damage, injury or loss of human life, or other possible events (Ayyub, 2003). Risk may include other attributes such as the consequence significance and the population at risk (Ayyub, 2003).

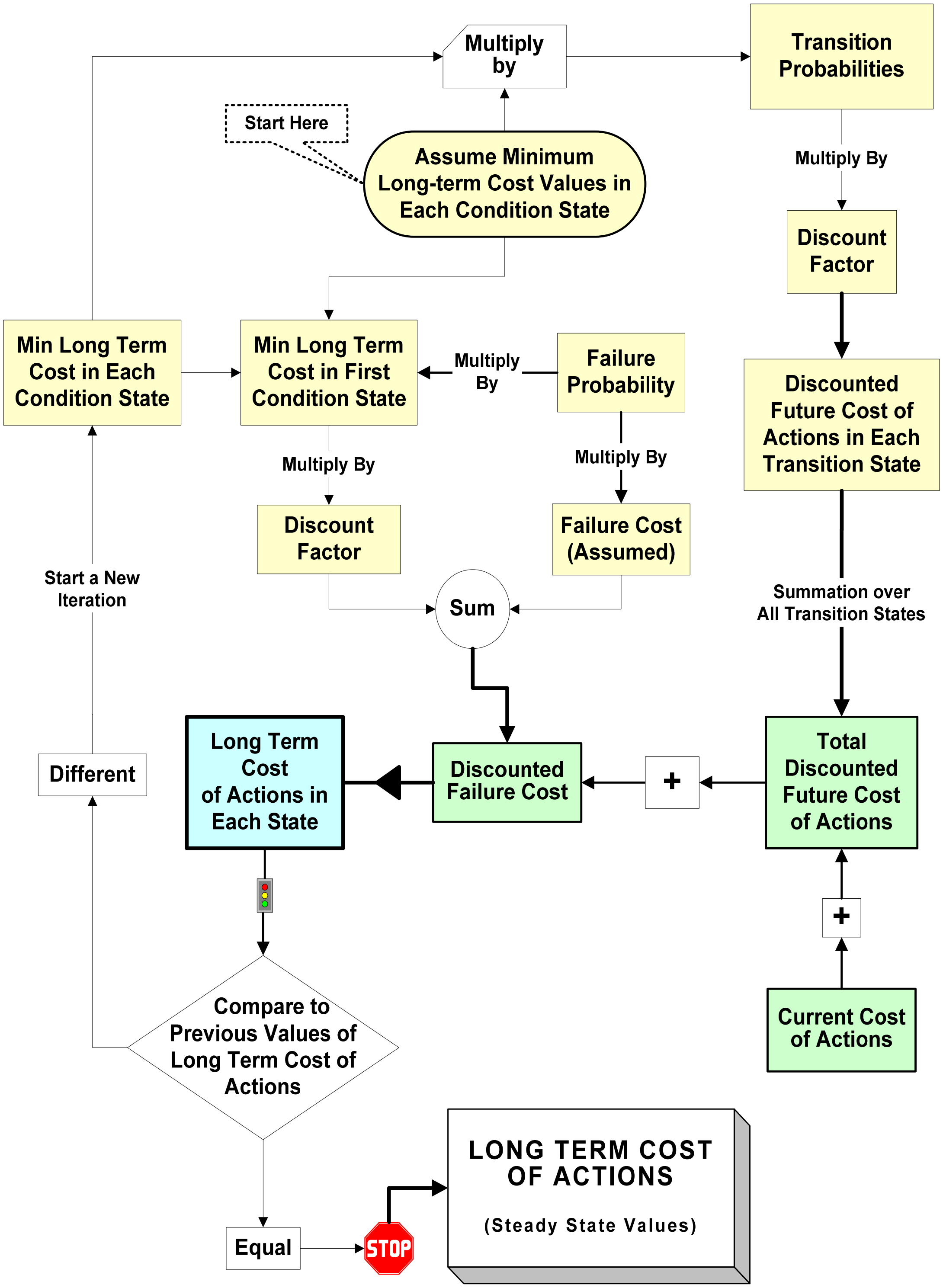


Figure 2-2. Calculation of Minimum Long-term Cost of Actions

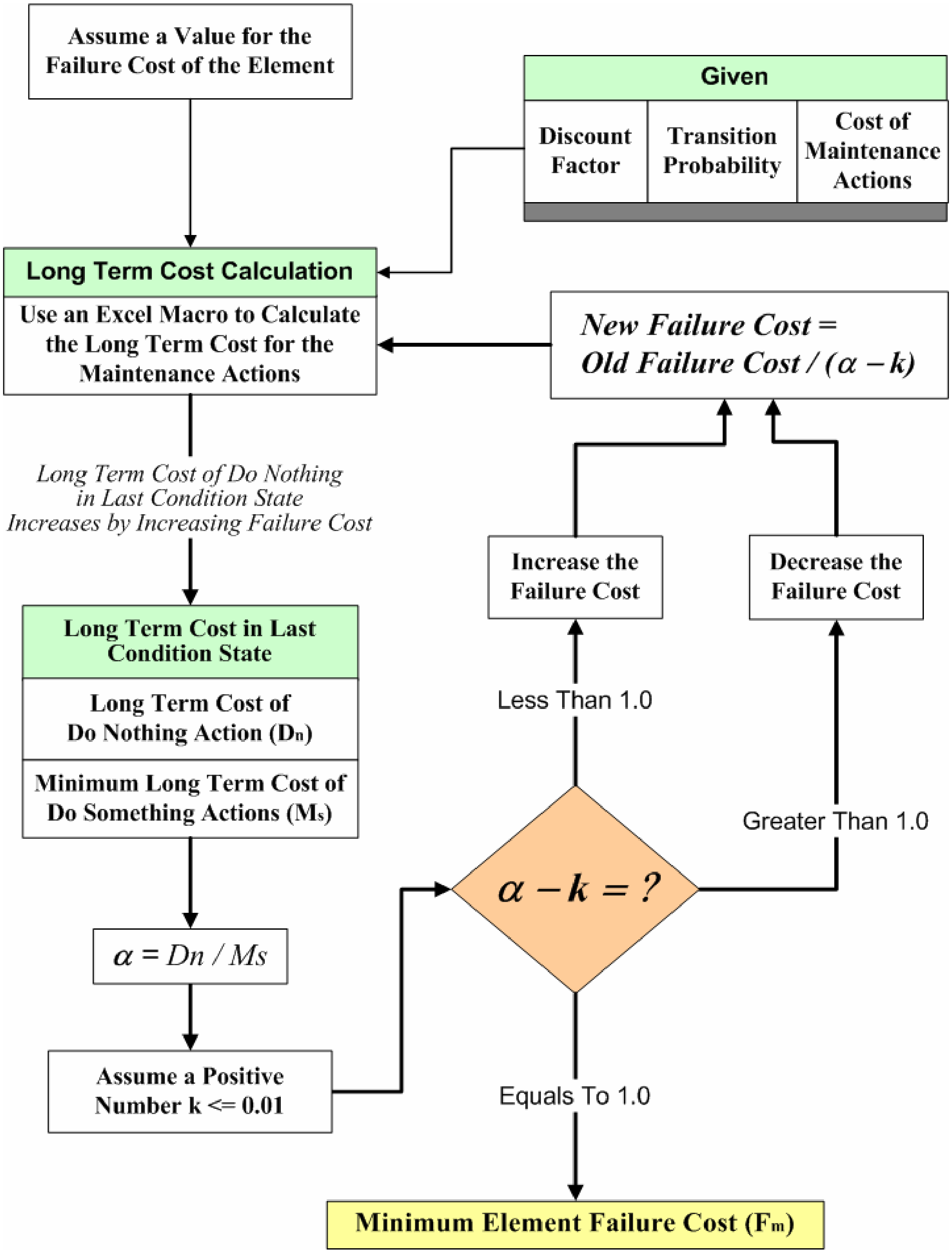


Figure 2-3. Methodology for Calculating the Minimum Failure Cost of Bridge Elements

Chapter 3. Methodology for Risk-Based Bridge Maintenance Strategies

3.1 Background

This chapter describes the methodology proposed in this research for identifying Risk-Based Bridge Maintenance Strategies (RBBMS). The methodology offers a systematic approach that combines reliability analysis algorithms, risk assessment methods, and decision analysis models.

In the risk-based approach for managing the maintenance of bridges, risks associated with conditions of bridge elements are estimated by assessing failure probabilities and failure consequences of bridge elements in different condition states using a quantitative approach. Risk is managed using decision analysis techniques for optimizing the expenditure of limited resources allocated for managing the maintenance, repair, rehabilitation or replacement of bridge elements. Optimal policies for bridge elements are selected as sets of maintenance actions in condition states with minimum sums of risks and maintenance costs for each year of maintenance. Optimal times for applying or deferring maintenance polices for bridge elements are selected based on tradeoffs between risk reductions and maintenance costs . Optimal maintenance policies for bridge elements are used to set up maintenance strategies for bridges. Risks to bridges and risk reduction effectiveness of maintenance strategies are used to set priorities for bridges identified for maintenance in an inventory. Available resources are allocated for maintenance based on the priority ranking of bridges.

The methodology is essentially a five-step process that provides a systematic framework for optimum utilization of available resources. This framework combines models for condition rating, deterioration, reliability analysis, cost estimation, and risk assessment; and directs them towards the identification, prioritization, and overall management of bridge maintenance problems.

The basic steps followed for defining the proposed methodology for risk-based bridge maintenance strategies are shown in Figure 3-1. The five steps of the methodology are:

1. System definition,
2. System breakdown,
3. Element-level modeling,
4. Risk assessment, and
5. Risk management.

Each of the essential steps outlined in the flowchart of Figure 3-1 is discussed in the subsequent sections.

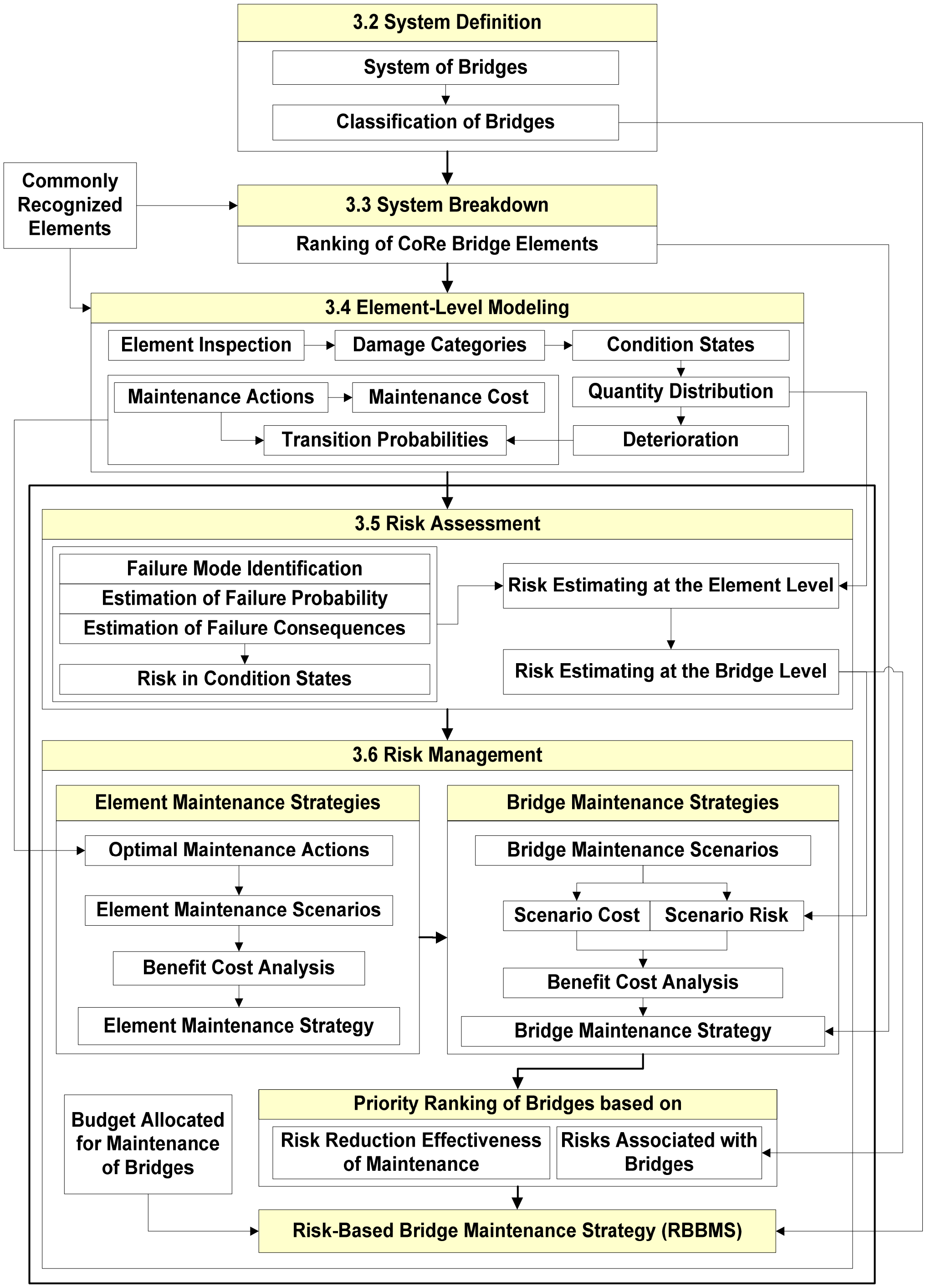


Figure 3-1. Flowchart of Methodology for Risk-Based Bridge Maintenance Strategies (RBBMS)

3.2 System Definition for Risk Analysis

The first step in the risk-based methodology is the definition of the system. Definition of the system is based on achieving a set of objectives related to risk-based optimal maintenance and efficient utilization of available resources. Defining the system provides the risk-based methodology with the information it needs to achieve the analysis objectives. The system includes inventory of bridges in a highway agency that are identified for maintenance purposes. Each bridge is defined as the physical model of an assembly of structural elements with their individual quantities.

Although the methodology proposed herein is general, and can be applied to all aspects of bridge management, emphasis in this research is placed on risk assessment and management based on visual inspection of bridge conditions. The system boundaries have the following limitations:

1. Bridge elements are limited to the commonly recognized (CoRe) elements defined by AASHTO (AASHTO, 1997),
2. Similar elements in different bridges in the inventory are assumed to have similar deterioration and cost models,
3. The characteristics and environments of bridge elements are limited to maintenance purposes, and
4. Risk assessment and management of bridge elements are based on their conditions.

3.3 System Breakdown

The system breakdown is a top-down division of each bridge identified for maintenance purposes in the inventory into its subsystems and components. The subsystems of each bridge include decks, superstructures and substructures. The subsystems are divided into structural elements. Similar elements of different bridges are classified under one element type. Elements are defined according to the AASHTO Guide for Commonly Recognized (CoRe) Structural Elements (AASHTO, 1997). Some State highway agencies define non-CoRe elements to identify some of their bridge elements (Al-Wazeer, et al. 2007a). The non-CoRe elements are not considered as part of the system breakdown in this research.

The presence of defects in bridge elements creates risks that affect the maintenance policies for bridge elements and the prioritization of bridge maintenance decisions for efficient utilization of available resources. In order to prioritize maintenance policies for bridge elements, elements are ranked according to their importance to the bridge. The ranking scheme for the elements is based on the relative importance of the element function to the bridge and how the element's failure affects the bridge's structural integrity and services or both. The criteria used to determine the importance of the bridge elements are based on weight factors of the AASHTO CoRe elements used in the Pontis bridge management system, and the AASHTO guidelines for fracture critical members.

The AASHTO CoRe elements are classified based on element weight factors available in Pontis bridge management system. The weight factors of elements are based on a scale of 1 to 20, where 20 represents the elements with the highest relative importance to the bridge, while 1 represents the elements with the lowest relative importance to the bridge. An element with a weight factor of 20 is given an importance of 20 times an element with a weight factor of 1. The element weights for the different types of CoRe elements are summarized in Table 3-1.

The critical elements of the bridge whose failure is expected to cause the collapse of the bridge are designated as failure-critical elements. Elements whose failure is not expected to cause collapse of the bridge are designated non-failure-critical (AASHTO, 2004). The fracture critical members or member components, as defined in the AASHTO Manual for Condition Evaluation of Bridges (AASHTO, 2000), are *"tension members or tension components of members whose failure would be expected to result in the collapse of a bridge*." Elements that can be classified as fracture-critical members are assigned a weight values of 20 regardless of their weight value available in the Pontis bridge management system.

Table 3-1. Weight Factors for Commonly Recognized Bridge Elements

Element Weight	Type of Commonly Recognized Element			
20	Pin / Pin and Hanger Assembly			
15	Column or Pile Extension			
14	Truss		Arch	
12	Pier Cap			
10	Girder	Stringer	Cable	Beam
9	Slab			
8	Pier Wall		Abutment	
6	Deck			
5	Bearing		Culvert	
4	Submerged Pile/Footing		Approach Slab	
3	Joint	Seal	Railing	
1	Smart Flag			

3.4 Element Level Modeling

Element-level modeling includes the steps needed to provide the necessary information for developing optimal risk-based strategies for maintenance, rehabilitation and replacement of bridge elements. The methodology depends on the condition rating results based on visual inspection of bridge elements. An element inspection is used to collect data that describe the condition of the element with a distribution of the damage in the element condition. The damage in the condition is assessed using damage categories. Each damage category describes bridge elements that have similar deterioration in the condition. The damage categories that are used for the commonly recognized bridge elements are shown in Figure 3-2.

Condition assessment of bridge elements is measured using condition state distribution as defined by the AASHTO Guide for Commonly Recognized (CoRe) Structural Elements (AASHTO, 1997). The least number of possible states that can model elements of repairable systems is two: a normal state or a failed state. In general, models for elements of repairable systems usually have more than two states. Damage extent and distribution in the conditions of bridge elements are represented by condition state distributions using a number of condition states N_s. Each condition state represents a damage level in the element condition. Condition state 1 represents the least damage while condition state Ns represents the highest damage. In the existing definition of commonly recognized elements, three to five condition states are defined for each bridge element. Elements of the same damage category have a similar number of condition states.

The condition state distribution of a bridge element is represented by element quantities in the different condition states. Each CoRe element has a unit of measurement and a quantity Q of that measurement. For example, a concrete deck element of a bridge is measured in unit area in terms of square foot (SF) or square meter (SM). The quantity Q of the deck element is the area of the deck element in SF or SM. The element quantity Q is distributed into quantities q_j among the possible condition states j of the element. For example, if a 1000-SF concrete deck has five possible condition states with 100 SF in condition state 1, 800 SF in condition state 2, 50 SF in condition state 3, 40 SF in condition state 4, and 10 SF in condition state 5 , then $Q = 1000$, $q_1 = 100$, $q_2 = 800$, $q_3 = 50$, $q_4 = 40$, and $q_5 = 10$.

For any bridge element with N_s possible condition states, the probability of the element being in a particular condition state j at year t is calculated as the ratio of the element quantity q_j in condition state j at year t to the total quantity of the element Q as shown in Equation 3-1.

$$P_j(t) = \frac{q_j(t)}{\sum_{j=1}^{N_s} q_j} = \frac{q_j(t)}{Q} \tag{3-1}$$

where

$P_j(t)$ = Probability of the bridge element being in Condition State j at year t,

$q_j(t)$ = Quantity of the element in Condition State j at year t, and

Q = Total quantity of the element in all possible condition states.

Guidelines for assessing the condition of bridge elements are used to assure uniformity in the inspections performed by different inspectors. However, inspection results conducted by different inspectors may differ based on the experience and judgment of each inspector; therefore, uncertainty in the inspection results should be quantified. The means and standard deviations for the element quantities in the condition states can be evaluated based on inspection results estimated by a number of bridge inspectors, n_{insp}. Assuming equal weights for different inspectors, the means and standard deviations of element quantities in the condition states of a bridge element can be calculated as shown in Equations 3-2 and 3-3.

$$\mu_{q_j} = \frac{\sum_{i=1}^{i=n_{insp}} q_{j,i}}{n_{insp}} \tag{3-2}$$

$$\sigma_{q_j} = \sqrt{\frac{\sum_{i=1}^{i=n_{insp}} (q_{j,i} - \mu_{q_j})^2}{n_{insp} - 1}} \tag{3-3}$$

where

$q_{j,i}$ = Element quantity in condition states j as inspected by inspector i,

μ_{q_j} = Mean of element quantity in condition state j, and

σ_{q_j} = Standard deviation of element quantity in condition state j.

Bridge elements deteriorate with time due to environmental conditions and traffic volume increase. Maintenance actions are needed to improve the conditions of bridge elements. Transition between the possible condition states of a bridge element due to deterioration or maintenance actions is modeled using Markov process (Golabi, et al. 1993). The element condition in the current Pontis bridge management system is allowed to deteriorate by, at most, one condition state when no repair actions are applied. The transition probability from condition state i to condition state j of a bridge element can be defined as shown in Equation 3-4.

$$P_{ij} = \begin{cases} \lambda_{ij} & i < j \\ 1 - \sum_{j=1}^{i-1} \mu_{ij} - \sum_{j=i+1}^{Ns} \lambda_{ij} & j = i \\ \mu_{ij} & i > j \end{cases} \tag{3-4}$$

where

P_{ij}: Transition probability of the element from Condition State *i* to Condition State *j*

λ_{ij}: Deterioration rate from Condition State *i* to Condition State *j*

μ_{ij}: Repair rate from Condition State *i* to Condition State *j*

N_s: Number of possible condition states for the bridge element

The transition between the condition states of a bridge element with five condition states can be illustrated as shown in Figure 3-3. An important idea illustrated in Figure 3-3 is that a probability of failure exists for each condition state of a bridge element.

Means and standard deviations for the transition probabilities can be calculated based on the means and standard deviation of deterioration and repair rates as shown in Equations 3-5 and 3-6.

$$\mu_{P_{ij}} = \begin{cases} \mu_{\lambda_{ij}} & i < j \\ 1 - \sum_{j=1}^{i-1} \mu_{\mu_{ij}} - \sum_{j=i+1}^{Ns} \mu_{\lambda_{ij}} & j = i \\ \mu_{\mu_{ij}} & i > j \end{cases} \tag{3-5}$$

$$\sigma_{P_{ij}} = \begin{cases} \sigma_{\lambda_{ij}} & i < j \\ \sqrt{\sum_{j=1}^{i-1} \sigma_{\mu_{ij}}{}^2 + \sum_{j=i+1}^{Ns} \sigma_{\lambda_{ij}}{}^2} & j = i \\ \sigma_{\mu_{ij}} & i > j \end{cases} \tag{3-6}$$

where

$\mu_{P_{ij}}$, $\mu_{\lambda_{ij}}$, and $\mu_{\mu_{ij}}$ are the means of transition probabilities, deterioration rates, and repair rates, respectively, from condition states *i* to condition state *j*, and

$\sigma_{P_{ij}}$, $\sigma_{\lambda_{ij}}$, and $\sigma_{\mu_{ij}}$ are the standard deviations of transition probabilities, deterioration rates, and repair rates, respectively, from condition states *i* to condition state *j*.

The probability of the element being in a particular condition state *i* at year *t*+1 after a deterioration/repair cycle can be calculated from the initial probability at time *t* as shown in Equation 3-7.

$$P_i(t+1) = \sum_{j=1}^{N_s} P_{ji} P_j(t) \tag{3-7}$$

where

$P_i(t)$ = Probability of the element being in condition state *i* at year *t*

$P_j(t)$ = Probability of the element being in condition state *j* at year *t*

$P_i(t+1)$ = Probability of the element being in condition state *i* at year *t*+1

P_{ji} = Transition probability from condition state *j* to condition state *i*

N_s = Number of possible condition states for the bridge element

The element quantity $q_i(t+1)$ in condition state *i* at year *t*+1 can be calculated from the element quantity at time *t* as shown in Equation 3-8.

$$q_i(t+1) = \sum_{j=1}^{N_s} P_{ji} q_j(t) \tag{3-8}$$

The means and standard deviations for the element quantity in condition state *i* at year *t*+1 can be calculated as shown in Equations 3-9 and 3-10.

$$\mu_{q_i(t+1)} = \sum_{j=1}^{N_s} \mu_{P_{ji}} \mu_{q_j(t)} \tag{3-9}$$

$$\sigma_{q_i(t+1)} = \sqrt{\sum_{j=1}^{N_s} (\mu_{P_{ji}}{}^2 \sigma_{q_j(t)}{}^2 + \sigma_{P_{ji}}{}^2 \mu_{q_j(t)}{}^2)} \tag{3-10}$$

where

$\mu_{P_{ji}}$, $\mu_{q_{i(t)}}$, and $\mu_{q_{i(t+1)}}$ are the means of the transition probabilities and element quantities in condition states *t* and *t*+1, and

$\sigma_{P_{ji}}$, $\sigma_{q_{i(t)}}$, and $\sigma_{q_{i(t+1)}}$ are the standard deviations of the transition probabilities and element quantities in condition states *t* and *t*+1.

Maintenance actions are needed to improve the condition of the element. Maintenance actions are defined for the different condition states of bridge elements. The definition of the maintenance actions is in accordance with the AASHTO Guide for Commonly Recognized (CoRe) Structural Elements (AASHTO, 1997). Each maintenance action applied incurs a unit cost. The unit costs of maintenance actions are estimated using expert elicitation from a number of bridge experts. The unit costs estimated by different experts may differ based on the experience and judgment of each expert; therefore, uncertainty in the unit costs should be quantified. The means and standard deviations for the unit costs of maintenance in the condition states of bridge elements can be evaluated based on unit costs estimated by a number of experts n_{exp}. Assuming equal weights for different experts, the means and standard deviations of the unit costs of maintenance actions in the condition states of a bridge element can be calculated as shown in Equations 3-11 and 3-12.

$$\mu_{c(a_k,j)} = \frac{\sum_{i=1}^{i=n_{\exp}} c_i(a_k, j)}{n_{\exp}} \tag{3-11}$$

$$\sigma_{c(a_k,j)} = \sqrt{\frac{\sum_{j=1}^{j=n_{\exp}} (c_i(a_k, j) - \mu_{c(a_k,j)})^2}{n_{\exp} - 1}} \tag{3-12}$$

where

$c_i(a_k j)$ = Unit cost of maintenance action a_k in condition state j estimated by expert i

$\mu_{c(a_k j)}$ = Mean for unit cost of maintenance action a_k in condition state j

$\sigma_{c(a_k j)}$ = Standard deviation for unit cost of maintenance action a_k in condition state j

The uncertainties in the quantity distributions in condition states, the transition probabilities between condition states, and unit costs of maintenance actions in the condition states of bridge elements are used for estimating uncertainties in risks and maintenance costs associated with the selection of optimal maintenance scenarios in the proposed risk-based methodology.

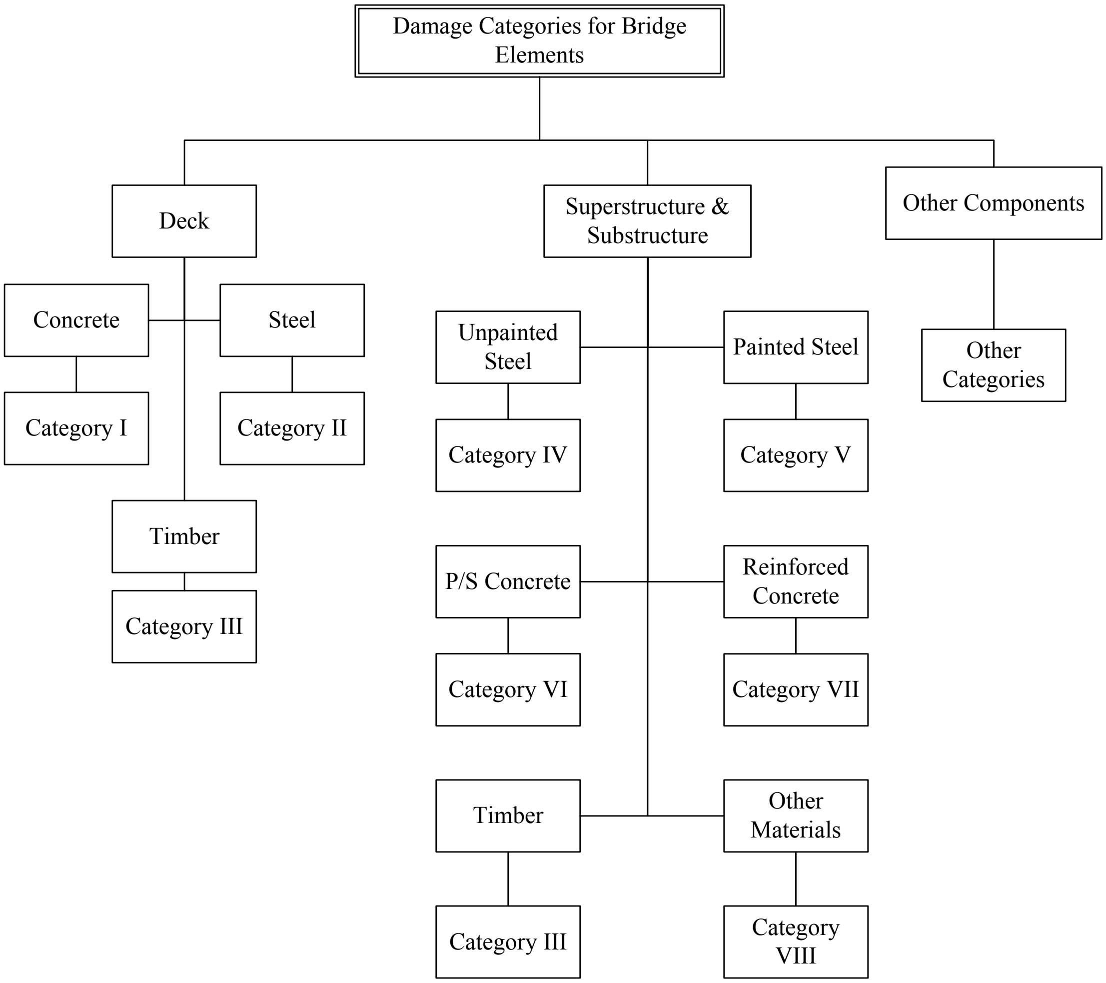


Figure 3-2. Classification of Damage Categories for Commonly Recognized Bridge Elements

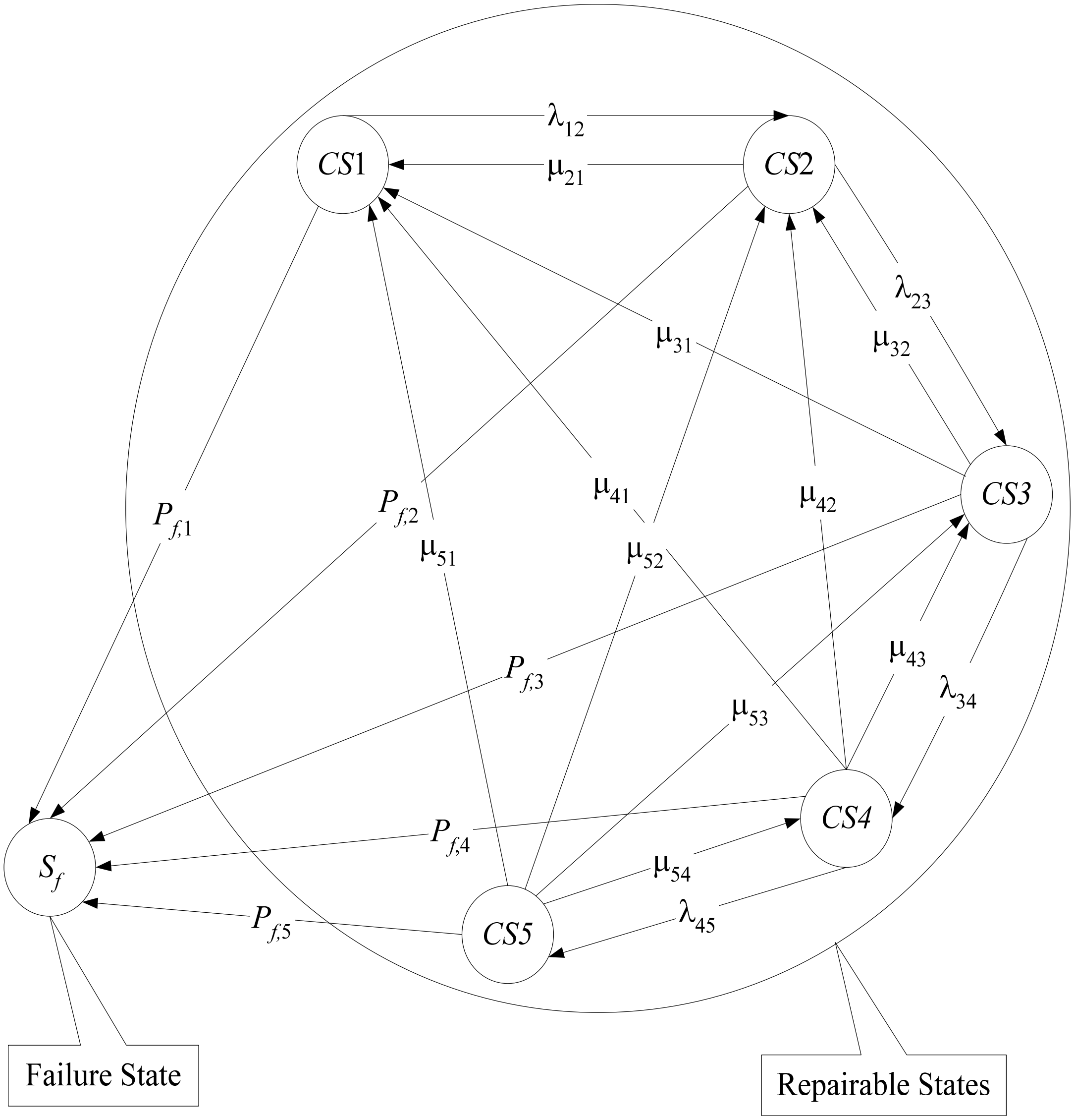


Figure 3-3. Transition between Condition States of a Bridge Element with Five Repairable Condition States and a Failure State

3.5 Risk Assessment

Risk assessment attempts to answer three questions: What can go wrong? What is the likelihood that it will go wrong? And what are the consequences if it does go wrong? Answers to these questions will help identify, measure, quantify, and evaluate potential risks and their consequences. They also provide tools for subjective risk assessment and evaluation (Ayyub, 2003).

Risk assessment can be used to identify failure modes for bridge elements, and estimate probabilities of failure and consequences of failure. Sources of failure probabilities and consequences include in-house rehabilitation and replacement records, failure and damage records from in-house database, data from similar studies, published results based on literature review, probabilistic analysis, and expert elicitation.

Risks associated with conditions of bridge elements can be estimated for different levels of damage in the conditions of bridge elements. The probability of failure and the consequences of failure for each failure mode of an element can be estimated in each condition state of the element. Risk associated with a condition state of the element is calculated as the product of the probability of failure and consequences of failure when the element is in that condition state.

The outcome of risk assessment is a risk profile for the element in the different condition states. The risks associated with the element in the condition states and the quantity distribution of the element in the different condition states are used for estimating risk at the element level. The estimated risks at the element level for the different bridge elements are used for estimating risk at the bridge level. In order to estimate risks for bridge elements, data are needed for the different failure modes of each element and the probabilities and consequences of each failure mode in the different condition states.

3.5.1 Failure Mode Identification

Bridge elements can have one or more modes of failure. Identification of failure modes for bridge elements is needed in order to estimate risks associated with elements conditions based upon the probabilities and consequences of failure for the different failure modes. Each mode of failure is defined using performance criteria expressed in terms of limit states based on AASHTO specifications, which implies that each limit state of a failure mode is considered separately.

A limit state function describes the performance of a structural element and defines the failure surface that separates the failure and survival regions. A limit state equation is expressed in terms of capacity minus demand. A positive result indicates survival and a negative result represents failure of the element. A result equal to zero indicates a point on the failure surface. In general, a limit state equation for the performance function of each failure mode can be expressed in terms of basic random variables such as load components, resistance parameters, material properties, and dimensions.

Failure modes for a bridge element may include ultimate strength failure modes, serviceability failure modes, and extreme event failure modes. Ultimate strength failure modes lead to structural damage. Ultimate strength limit states ensure strength and stability for resisting the load combinations on the bridge and maintain the overall structural integrity (AASHTO, 2004). Examples of ultimate strength limit states include flexure, shear, stability, buckling, fatigue, fracture and crushing.

Structural elements of a bridge respond to load effects in brittle or ductile behavior. Some bridge elements fail in a brittle manner, others are more ductile. Brittle behavior results in a sudden loss of load carrying capacity immediately when elastic limit is exceeded. On the other hand, ductile behavior provides warning of structural failure by significant inelastic deformations before any loss of load carrying capacity occurs. Shear failure is an example of brittle failure while flexural failure is an example of ductile failure. The consequences due to damages resulting from a flexural failure at the ultimate limit state are less likely than from a shear failure since flexural failure is usually preceded by extensive cracking or deformation. Maintenance priority should be given to bridge elements that are likely to have brittle failures that result in sudden collapse without warning. When considering mitigation actions, more attention should be given to bridge elements that are weak in brittle failures than bridge elements that are weak in ductile failures.

Serviceability failure modes result in decreasing the functional performance and serviceability of bridge elements. Serviceability failure modes are defined by limit states related to comfort of bridge users rather than strength and stability of the bridge. Examples of serviceability limit states include cracking, deformation, settlement, vibration, and deflection.

Extreme event failure modes are defined by limit states that ensure the structural survival of the bridge during a major hazard event with a return period more than the bridge life (AASHTO, 2004). An extreme hazard event could be a major earthquake, flood, scour or collision. Damages due to earthquake for bridges located in areas of high seismic risk depend on the characteristics of the bridge and its immediate surroundings, the regional likelihood of an earthquake of a particular severity, the bridge’s design and construction characteristics, and on the local geology and soils conditions that may influence ground motion and accelerations (Hawk, 2003). Flood hazard may cause a structural damage due to lateral forces imposed by high-water flows and impact of flood-borne debris on a bridge’s superstructure and supports (Hawk, 2003). Extreme flooding may also cause erosion of bridge approaches (Hawk, 2003). Footing scour in elements adjacent to water, such as the pier scour, results from soil erosion under bridges crossing water (Hawk, 2003). Scour depends on the water depth and angle of flow, the element shape and width, the soil characteristics, and other factors (Hawk, 2003). Potential damage to a bridge could occur due to collisions with the bridge or any of its elements such as the railing.

Collision could be caused by oversized or out-of-control vehicles, trains on bridges crossing rail lines, barges and ships on navigable waterways (Hawk, 2003).

Failure modes may change from one commonly recognized bridge element to another depending on many factors including the element type, failure type, and material type. Identification of the failure modes for commonly recognized bridge elements is needed for evaluating the probabilities and consequences of failure. Elements made of the same material have common damage behavior and share a number of common failure modes. For demonstration purposes, it is assumed in this research that the material of the element is the main factor in classifying the failure modes for bridge elements. The possible materials used in commonly recognized bridge elements are unpainted steel, painted steel, prestressed concrete, reinforced concrete, timber, and other material. Table 3-2 summarizes the possible types of material used for the different commonly recognized bridge elements.

The element types in Table 3-2 are ordered based on their importance to the bridge and are in accordance with the rating scheme shown in Table 3-1. Table 3-2 can be used, among other factors, to help identify the potential failure modes for the commonly recognized elements based on the element type and material. Each element type made of a certain material has a number of failure modes. Other elements such as bearings and joints could vary in their materials and have different failure modes based on their materials.

Table 3-2. Materials Types of Commonly Recognized Bridge Elements

Element Type	Unpainted Steel	Painted Steel	Prestressed Concrete	Reinforced Concrete	Timber	Other Material
Pin and/or Pin and Hanger Assembly	X	X				
Column or Pile Extension	X	X	X	X	X	
Truss	X	X			X	
Arch	X	X	X	X	X	X
Cap	X	X	X	X	X	
Girder/Beam	X	X	X	X	X	
Stringer	X	X	X	X	X	
Floor Beam	X	X	X	X	X	
Cable	X	X				
Slab				X	X	
Pier Wall				X		X
Abutment				X	X	X
Deck	X			X	X	
Culvert	X			X	X	X
Submerged Pile/Footing	X		X	X	X	
Approach Slab			X	X		
Railing	Uncoated Metal	Coated Metal		X	X	X

3.5.2 Probability of Failure

The likelihood of a failure mode occurrence for a bridge element is assessed using a probability of failure due to that failure mode. According to the AASHTO Guide for Commonly Recognized (CoRe) Structural Elements, CoRe elements can have three to five possible condition states. A failure probability exists for any of the possible condition states of the element as shown previously in Figure 3-3. The failure probabilities in the different condition states of the element can be estimated quantitatively based on the available information on failure rates.

The probability of an element failure for a particular failure mode can be estimated based on reliability analysis. The failure probability is defined as one minus the element reliability. Reliability can be estimated based on a performance function; the supply minus the demand for a given criteria. The performance function for ultimate strength limit states is defined as the element capacity or resistance R_n minus the load effects L. The element resistance and the load effects could have different distributions including normal and lognormal distributions. For demonstration purposes, the element resistance is assumed to have a lognormal distribution and the load effects are assumed to have normal distributions.

For a given failure mode of a bridge element, the probability of failure can be calculated when the element is in each possible condition state. The following steps can be used to calculate the probabilities of failure due to ultimate strength failure modes when the element is in different condition states:

1. Identify the failure modes for the element,
2. Find the performance function that defines each failure mode,
3. Define the random variables used in defining the resistance and the load effect in each limit state function,
4. For each failure mode, use the material properties of the element and the properties of the element section (with no section loss) to calculate the nominal value of the resistance R_{n0},
5. Calculate the mean μ_{R0} and the standard deviation σ_{R0} of the resistance using the bias factor λ_R (ratio of mean to nominal) and the coefficient of variation V_R of the resistance, respectively, as shown in Equations 3-13 and 3-14 (Nowak, 1999),

$$\mu_{R_0} = \lambda_R R_{n_0} \tag{3-13}$$

$$\sigma_{R_0} = V_R \lambda_R R_{n_0} \tag{3-14}$$

6. Define the load effect L using the load combination for the different load components L_q and their load factors γ_q and define each load factor γ_q as a function of its bias factor λ_q and coefficient of variation V_{Lq} for each load component, as shown in Equation 3-15 and 3-16 (Nowak, 1999),

$$L = \sum_{q=1}^{n_L} \gamma_q L_q \tag{3-15}$$

where

$$\gamma_q = \lambda_q(1+2V_{Lq}) \tag{3-16}$$

7. Define the mean μ_L and the standard deviation σ_L of the load effect L using the load factors γ_q, the bias factors λ_{Lq} and coefficients of variation V_{Lq} for the load components, as shown in Equations 3-17 to 3-20,

$$\mu_L = \sum_{q=1}^{n_L} \gamma_q \mu_{Lq} \tag{3-17}$$

$$\sigma_L = \sqrt{\sum_{q=1}^{n_L} {\gamma_q}^2 {\sigma_{Lq}}^2} \tag{3-18}$$

where

$$\mu_{Lq} = \lambda_q L_q \tag{3-19}$$

$$\sigma_{Lq} = \lambda_q V_q L_q \tag{3-20}$$

8. For each failure mode, use the means and the standard deviations of the resistance and the load effect to calculate the reliability index β_0 corresponding to a cross section of the element with no section loss, as shown in Equation 3-21, assuming a normal distribution for the load and a lognormal distribution for the resistance (Nowak, 1999),

$$\beta_0 = \frac{\mu_{R0}(1-2V_R)[1-\ln(1-2V_R)]-\mu_L}{\sqrt{[\sigma_{R0}(1-2V_R)]^2+{\sigma_L}^2}} \tag{3-21}$$

9. If the components of the load effect on the bridge element are known, steps 5 to 8 can be used to calculate the reliability index β_0 corresponding to the full cross section of the element,
10. If the components of the load effect on the bridge element are not known,

a. Assume the AASHTO design value for target reliability index ($\beta_T = 3.5$) as the reliability index corresponding to the full section with no section loss, i.e. $\beta_0 = \beta_T$,

b. Define the different load components in terms of a load variable l and factors a_q that can be assumed to represent the proportions between the different load components as shown in Equation 3-22,

$$L_q = a_q l \tag{3-22}$$

c. Substitute for the unknown load components L_q in Equation 3-22 into Equations 3-19 and 3-20, then substitute Equations 3-19 and 3-20 into Equations 3-17 and 3-18 to find the mean and the standard deviation for the load effect in terms of the variable l as shown in Equation 3-23 and 3-24,

$$\mu_L = \sum_{q=1}^{n_L} \gamma_q \lambda_{L_q} a_q l \tag{3-23}$$

$$\sigma_L = \sqrt{\sum_{q=1}^{n_L} {\gamma_q}^2 {\lambda_q}^2 {V_q}^2 {a_q}^2 l^2} \tag{3-24}$$

d. Substitute for the mean μ_L and the standard deviation σ_L of the load effect in Equation 3-21 and use the known values for the target reliability index β_T, the mean μ_{R0} and the standard deviation σ_{R0} of the resistance to calculate the variable l using Equation 3-25,

$$\beta_T = \frac{\mu_{R0}(1-2V_R)[1-\ln(1-2V_R)] - \sum_{q=1}^{n_L} \gamma_q \lambda_{L_q} a_q l}{\sqrt{{\sigma_{R0}}^2 (1-2V_R)^2 + \sum_{q=1}^{n_L} {\gamma_q}^2 {\lambda_q}^2 {V_q}^2 {a_q}^2 l^2}} \tag{3-25}$$

e. Substitute for the load components L_q using the value of l calculated from Equation 3-25,

11. Define a section loss corresponding to each condition state of the bridge element,
12. Calculate the modified section properties of the element based on the defined section loss of the element when the element is in each condition state,
13. Calculate the nominal resistance R_{nj} corresponding to each condition state CS_j, where j is from 1 to N_s, using the material and section properties,
14. Calculate the mean μ_{Rj} and the standard deviation σ_{Rj} of the resistance using the calculated value for the nominal resistance R_{nj}, the bias factor λ_R, and the coefficient of variation V_R for the resistance as shown in Equations 3-26 and 3-27,

$$\mu_{R_j} = \lambda_R R_{n_j} \tag{3-26}$$

$$\sigma_{R_j} = V_R \lambda_R R_{n_j} \tag{3-27}$$

15. Use the load components L_q and their bias factors λ_q and coefficients of variation V_q for a given failure mode to calculate the mean μ_L and the standard deviation σ_L of the load effect using Equations 3-17 to 3-20,
16. For each failure mode, use the means and standard deviations for the resistance and the load effect to calculate the reliability index β_j of the bridge element when the element is in condition state CS_j corresponding to a given section loss. The reliability index β_j can be calculated as shown in Equation 3-28, assuming the same load effects in the different condition states,

$$\beta_j = \frac{\mu_{Rj}(1-2V_R)[1-\ln(1-2V_R)] - \mu_L}{\sqrt{\sigma_{Rj}{}^2(1-2V_R)^2 + \sigma_L{}^2}} \tag{3-28}$$

17. For each failure mode FM_i,
    a. Calculate the reliability index $\beta_{(j,i)}$ of the element in each condition state CS_j from $j = 1$ (best condition state of the element) to $j = N_s$ (worst condition state of the element) using Equation 3-29,

$$\beta_{(j,i)} = \frac{\mu_{R(j,i)}(1-2V_{R,i})[1-\ln(1-2V_{R,i})]-\mu_{L,i}}{\sqrt{\sigma_{R(j,i)}{}^{2}(1-2V_{R,i})^{2}+\sigma_{L,i}{}^{2}}} \tag{3-29}$$

b. Calculate the probability of the element failure $P_{f(j,i)}$ in each condition state CS_j from j =1 to N_s as a function of the reliability indexes $\beta_{(j,i)}$ using Equation 3-30,

$$P_{f(j,i)} = 1- \Phi(\beta_{(j,i)}) \tag{3-30}$$

18. For each condition state of the element, calculate the probabilities of failure resulting from the different modes of failure. For a bridge element with N_s possible condition states and N_f potential failure modes, the probabilities of failure $P_{f(j,i)}$, for j from 1 to N_s and i from 1 to N_f, can be calculated as shown in Table 3-3,
19. The element probability of failure in a given condition state can be assumed as the one with maximum value among the different modes of failure as shown in Equation 3-31, and

$$P_{f,j} = \text{ Max } P_{f(i,j)} \tag{3-31}$$

20. Use the final probabilities of failure of the element in the different condition states in the transition probability matrix of the bridge element (see Figure 3-3).

A framework for calculating the failure probability of bridge elements in their possible condition states is shown in Figure 3-4. The calculated value for the failure probability in any given condition state of the element for a particular failure mode represents the quantitative probability that the element will fail due to that failure mode when it is in that condition state.

The probability of failure due to a given failure mode is expected to increase by worsening the condition state of the element. For example, $P_{f1,1}$, $P_{f2,1}$, $P_{f3,1}$, $P_{f4,1}$, $P_{f5,1}$ can be used to represent the probabilities of failure due to Failure Mode 1 ($FM1$) for an element with five condition states ($N_s = 5$) when the element is in Condition State 1, 2, 3, 4, and 5 respectively. The probability of failure $P_{f5,1}$ due to Failure Mode $FM1$ when the element is in Condition State 5 is expected to be larger than the probability of failure $P_{f4,1}$ when the element is in Condition State 4. Similarly, the probability of failure $P_{f4,1}$ when the element is in Condition State 4 is expected to be larger than the probability of failure $P_{f3,1}$ when the element is in Condition State 3. Likewise $P_{f3,1}$ is expected to be larger than $P_{f2,1}$ and $P_{f2,1}$ is expected to be larger than $P_{f1,1}$.

The probabilities of failure for bridge elements are calculated for future years based on predicted element conditions and loading. This involves the prediction of the load effects on the elements during the planning horizon. The cumulative distribution function $F_{Y_n}(y)$ of the maximum load effects Y_n during a number of years n in the future can be derived from the cumulative distribution function of the initial load effects $F_X(y)$ as shown in Equation 3-32 (Ang and Tang, 1984).

$$F_{Y_n}(y) = [F_X(y)]^n \tag{3-32}$$

Since the probability $F_X(y)$ for the initial load effects is less than one, the value $[F_{Y_n}(y)]^n$ decreases with increasing the number of years n; therefore, the cumulative distribution function (CDF) for the maximum load effects in the future will shift to the right with increasing values of n (Ang and Tang, 1984). The CDFs of the initial load effects are generated using the means and standard deviations of the initial load effects derived from the reliability index equation. The corresponding probability density function $f_{Y_n}(y)$ for the maximum load effects Y_n is calculated as shown in Equation 3-33 (Ang and Tang, 1984).

$$f_{Y_n}(y) = \frac{\partial F_{Y_n}(y)}{\partial y} \tag{3-33}$$

The PDFs of the maximum load effects are generated from the CDFs of the maximum load effects using numerical differentiation. The first moments of the PDFs around the origin are used to calculate the means of the maximum load effects. The second moments of the PDFs around the mean are used to calculate the standard deviations of the maximum load effects. The increased load effects over the years result in reducing the reliability in the condition states of bridge elements, and therefore, increase the probabilities of failure.

Steel and concrete are the main materials in most of bridge elements in the United States. Flexural and shear failure modes are among the most common failure modes for bridge elements. Calculations of the probabilities of failure due to flexural and shear failure modes for steel and concrete elements in different condition states are illustrated in Chapter 4 following the steps shown in Figure 3-4.

Table 3-3. Probabilities of Occurrence of N_f Failure Modes in N_s Possible Condition States of a Bridge Element

Failure Mode	Condition State of Bridge Element					
	$CS1$	$CS2$	…	CSj	…	CS_{Ns}
$FM1$	$P_{f1,1}$	$P_{f2,1}$	…	$P_{f(j,1)}$	…	$P_{f(Ns,1)}$
$FM2$	$P_{f1,2}$	$P_{f2,2}$	…	$P_{f(j,2)}$	…	$P_{f(Ns,2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
FMi	$P_{f(1,i)}$	$P_{f(2,i)}$	…	$P_{f(j,i)}$	…	$P_{f(Ns,i)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
FM_{Nf}	$P_{f(1, Nf)}$	$P_{f(2, Nf)}$	…	$P_{f(j, Nf)}$	…	$P_{f(Ns,Nf)}$

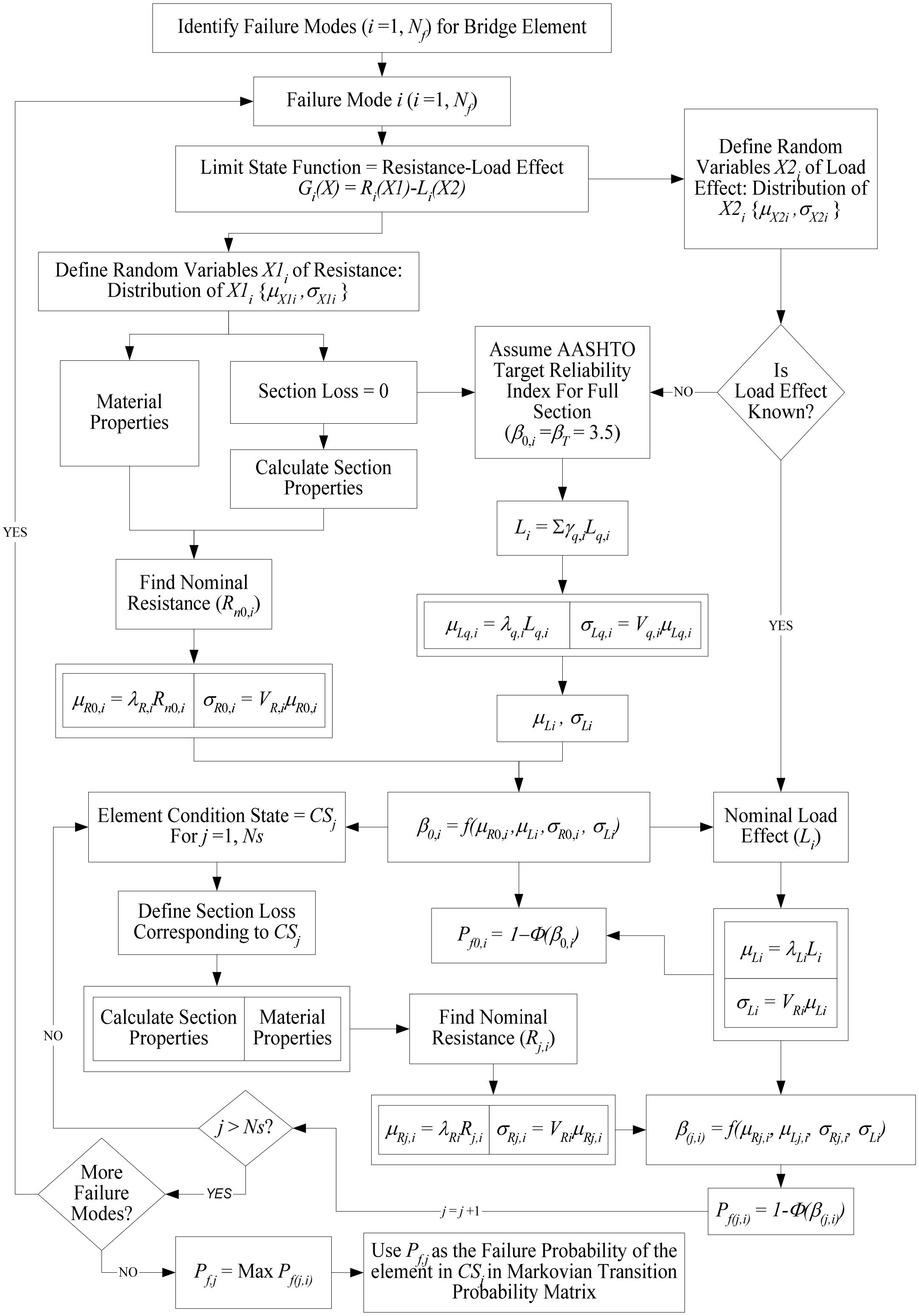


Figure 3-4. Framework for Calculating Failure Probabilities of Bridge Elements in Possible Condition States

3.5.3 Failure Consequences

Failure consequences of bridge elements may include economic consequences, human injuries and/or loss of lives, traffic delays, and environmental impacts. It is assumed that consequences of failure of a bridge element include consequences on the highway agency related to the element and the bridge, consequences on the bridge users, consequences on health and safety, consequences that increase the traffic accident rates, consequences on the environment, consequences to the nearby businesses, and consequences on the public.

The extent or amount of potential consequences of bridge elements failure can be measured by consequence severities that can be used to quantify the consequences. Consequence and severity estimations can be based on historical data, expert elicitation, or analytical prediction. The severity of consequences can be measured using agency cost of repair or replacement, user cost, accident cost, expected value of injury or loss of life, cost of environmental damage, loss of revenue to affected businesses, or impact on the general public.

It is assumed that one or more of the following consequences (costs) may take place at different levels for a particular failure mode of a bridge element:

1. Agency consequences related to the element (C_e),
2. Agency consequences related to the bridge (C_b),
3. Consequences related to the bridge users (C_u) due to time delay, traffic diversion and/or bridge closure,

4. Consequences related to traffic accident (C_a),
5. Consequences related to health and safety (C_h) in terms of injuries and/or deaths,
6. Consequences related to the environment (C_{env}),
7. Consequences to nearby businesses (C_{nb}), and
8. Consequences to the general public (C_p).

The different consequences of the element failure are discussed in detail in Appendix A.1. The values for the different types of consequences are added and used along with the probabilities of failure of the element to estimate the risk. It is clear from the consequences listed above that some of them are hard to evaluate. There is a need for future research to find approaches that can quantify the identified consequences accurately.

The total consequence of failure $C_{F,i}$ for a bridge element due to a particular failure mode i can be estimated as shown in Equation 3-34.

$$C_{F,i} = C_{e,i} + C_{b,i} + C_{u,i} + C_{a,i} + C_{h,i} + C_{env,i} + C_{nb,i} + C_{p,i} \qquad (3\text{-}34)$$

The value for the consequence of failure $C_{F,i}$ represents the effect of the element failure on the bridge system due to a particular failure mode i of the element. For example, if shear failure mode is represented by a failure mode 1, then $C_{F,1}$ is the consequence of the element failure on the system due to shear failure mode.

It is assumed that the probabilities of occurrence of the failure consequences in the different condition states due to a particular failure mode are the same as the failure probability of the element due to that failure mode. The appropriate cost in each type of consequence can be estimated and combined to find the total consequence of each type. The total values for the different types of consequences are substituted in Equation 3-34 to find the total consequences of the element failure due to a particular failure mode.

Since the estimation of the different types of consequences for bridge elements is subjective, uncertainty in the failure consequences should be considered when estimating risk associated with conditions of bridge elements. Probability distributions with means and standard deviations can be used for each type of consequences. The mean $\mu_{C_{F,i}}$ and standard deviation $\sigma_{C_{F,i}}$ for the total consequences on the element due to a particular failure mode *i* can be calculated using the means and standard deviations of the different types of consequences due to that failure mode as shown in Equations 3-35 and 3-36, respectively.

$$\mu_{C_{F,i}} = \mu_{C_{e,i}} + \mu_{C_{b,i}} + \mu_{C_{u,i}} + \mu_{C_{a,i}} + \mu_{C_{h,i}} + \mu_{C_{env,i}} + \mu_{C_{nb,i}} + \mu_{C_{p,i}} \qquad (3\text{-}35)$$

$$\sigma_{C_{F,i}} = \sqrt{\sigma_{C_{e,i}}{}^2 + \sigma_{C_{b,i}}{}^2 + \sigma_{C_{u,i}}{}^2 + \sigma_{C_{a,i}}{}^2 + \sigma_{C_{h,i}}{}^2 + \sigma_{C_{env,i}}{}^2 + \sigma_{C_{nb,i}}{}^2 + \sigma_{C_{p,i}}{}^2} \qquad (3\text{-}36)$$

3.5.4 Risk Estimation

Risk associated with condition of a bridge element is considered to have potential loss resulting from the element failure. Risk can be estimated based on estimates of the probability of failure and the failure consequences. Risk can be calculated as follows:

$$\text{\textit{Risk}} = \text{\textit{Failure Probability}} \text{ x } \text{\textit{Failure Consequences}} \qquad (3\text{-}37)$$

Risk estimation for bridge elements can be performed at the following levels:

1. Estimating risk in the condition states of the element due to one failure mode,
2. Estimating risk in the condition states of the element due to all failure modes,
3. Estimating risk associated with conditions of each element in the bridge, and
4. Estimating risk associated with conditions of the bridge based on the risks of the elements.

The abovementioned levels for estimating risk are explained in the following subsections.

3.5.4.1 Risk in Condition States of Bridge Elements Due to One Failure Mode

For any bridge element in a particular condition state, there is a risk associated with the element in that condition state. Each failure mode of the element results in risk values associated with the condition states of the element. The risk associated with the element due to a particular failure mode when the element is in a given condition state can be estimated as the product of the probability of the element failure and the consequences due to that failure mode when the element is in that condition state. Risk due to failure mode i when the element is in a given condition state j can be expressed as:

$$R_{(i,j)} = P_{f(i,j)}C_{F,i} \tag{3-38}$$

where

$R_{(i,j)}$ = Risk associated with a bridge element due to a particular failure mode i when the element is in a given condition state j.

$P_{f(i,j)}$ = Probability of failure due to failure mode i when the element is in condition state j.

$C_{F,i}$ = Consequences of the element failure due to failure mode i.

For a bridge element with N_s condition states and N_f potential failure modes, the risks $R(i,j)$ associated with the element in the different condition states $j = 1$ to N_s due to each failure mode $i = 1$ to N_f are shown in Table 3-4. The steps for estimating risk $R(i,j)$ are shown in Figure 3-5.

To represent the probability distribution of the risk associated with condition of a particular element due to a particular failure mode i, the probability of failure $P_{f(i,j)}$ in the different condition states is assumed as a point estimate and the failure consequence distribution is represented by its mean $\mu_{C_{F,i}}$ and standard deviation $\sigma_{C_{F,i}}$.

The mean and standard deviation of the risk associated with the element in a given condition state *j* due to a particular failure mode *i* can be calculated as shown in Equations 3-39 and 3-40, respectively.

$$\mu_{R_{(i,j)}} = P_{f_{(i,j)}} \mu_{C_{F,i}} \tag{3-39}$$

$$\sigma_{R_{(i,j)}} = P_{f_{(i,j)}} \sigma_{C_{F,i}} \tag{3-40}$$

Table 3-4. Risk Results Associated with a Bridge Element due to N_f Failure Modes in N_s Condition States of the Element

Failure Mode	Condition State of the Bridge Element					
	CS1	CS2	…	CS$_j$	…	CS$_{Ns}$
FM1	$R_{(1,1)}$	$R_{(1,2)}$	…	$R_{(1,j)}$	…	$R_{(1,Ns)}$
FM2	$R_{(2,1)}$	$R_{(2,2)}$	…	$R_{(2,j)}$	…	$R_{(2,Ns)}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
FM$_i$	$R_{(i,1)}$	$R_{(i,2)}$	…	$R_{(i,j)}$	…	$R_{(i,Ns)}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
FM$_{Nf}$	$R_{(Nf,1)}$	$R_{(Nf,2)}$	…	$R_{(Nf,j)}$	…	$R_{(Nf, Ns)}$

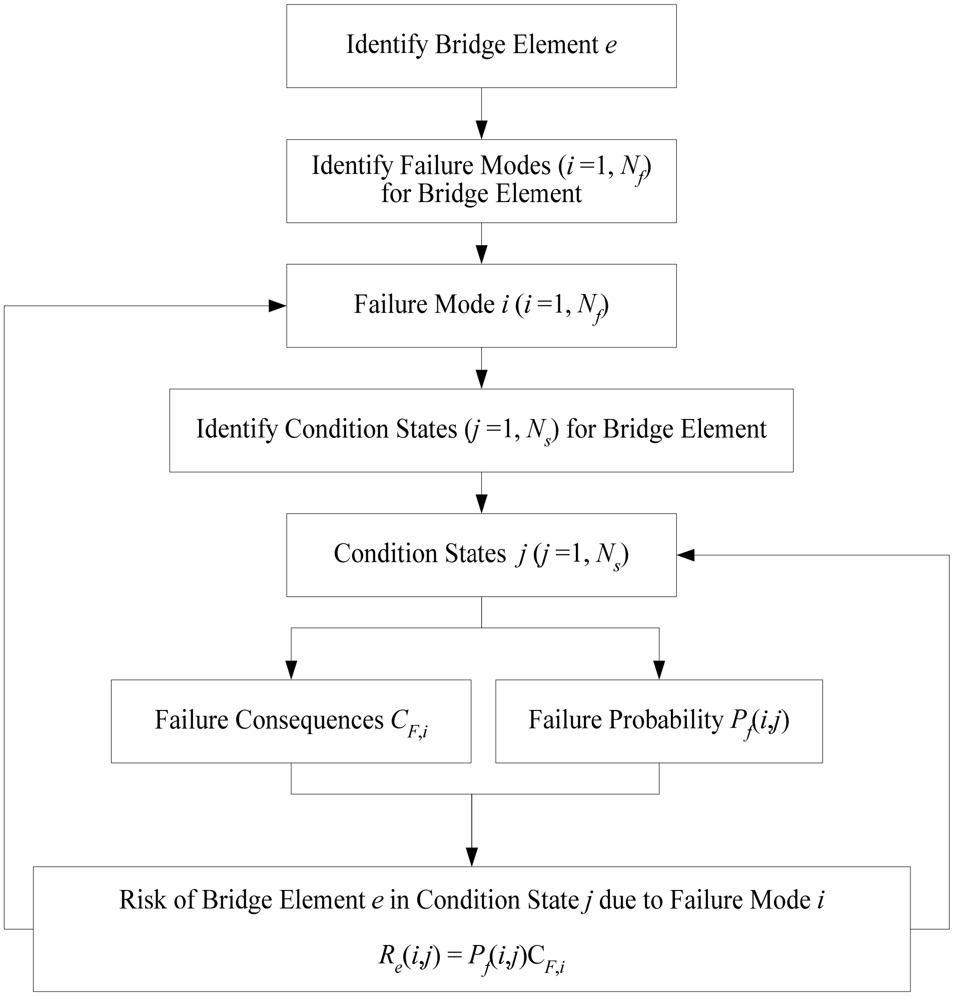


Figure 3-5. Procedure for Estimating Risk Associated with Bridge Elements in Different Condition States Due to One Failure Mode of the Element

3.5.4.2 Risk in Condition States of Bridge Elements due to Multiple Failure Modes

All types of failure modes should be considered in calculating the risks associated with conditions of bridge elements. Different failure modes have different probabilities of occurrence and they result in different consequences. The criticality of each failure modes is represented in the severity of its consequences. For example, ductile failure modes like flexure have warning signs in terms of deflection, while non-ductile failure modes like shear occurs suddenly without warning; therefore, the consequences due to damages resulting from an element failure in shear are more critical than in flexure.

Risk associated with a bridge element in a particular condition state due to a number of failure modes of the element can be defined as the sum of the risks associated with the failure modes in that condition state. The risk R_j associated with a bridge element in Condition State *j* due to a number N_f of potential failure modes of the element can be defined as:

$$R_j = \sum_{i=1}^{N_f} P_{f(i,j)} C_{F,i} \qquad (3\text{-}41)$$

where

R_j = Risk associated with a bridge element in condition state *j*

$P_{f(i,j)}$ = Probability of the element failure due to failure mode *i* when the element is in condition state *j*

$C_{F,i}$ = Consequences of the element failure due to failure mode i

N_f = Number of failure modes of the element

The procedure for calculating the risk R_j is shown in Figure 3-6. The mean and standard deviation for the risk R_j in condition state j are calculated as shown in Equations 3-42 and 3-43, respectively.

$$\mu_{R_j} = \sum_{i=1}^{N_f} \mu_{R_{(i,j)}} \tag{3-42}$$

$$\sigma_{R_j} = \sqrt{\sum_{i=1}^{N_f} \sigma_{R_{(i,j)}}{}^2} \tag{3-43}$$

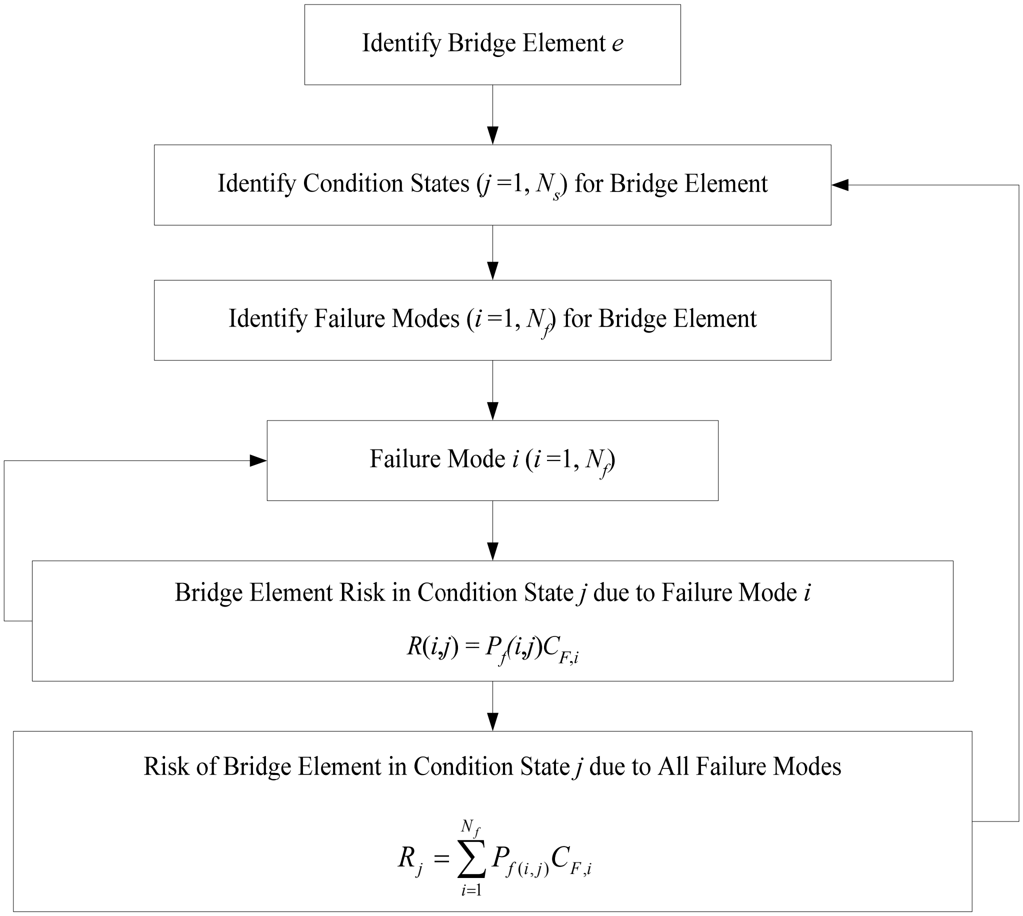


Figure 3-6. Procedure for Estimating Risk due to All Failure Modes of a Bridge Element When the Element is in a Given Condition State

3.5.4.3 Risk Estimation at the Element Level

Risk associated with condition of a bridge element is calculated using the risks associated with the condition states of the element and the probability of the element being in the different condition states. The risk associated with a bridge element condition distributed in a number of condition states is calculated by multiplying the risk values R_j in each condition state j by the probability $P(j)$ of the element being in condition state *j* and summing the results for all possible condition states of the element. The risk *R* associated with a bridge element with N_s condition states can be defined as shown in Equation 3-44.

$$R = \sum_{j=1}^{N_s} \frac{q_j}{Q} R_j \tag{3-44}$$

where

$R =$ Risk associated with condition of the bridge element

$q_j =$ Quantity of the element in Condition State *j*

$Q =$ Total quantity of the bridge element

$R_j =$ Risk associated with the bridge element when the element is in Condition State *j*

$N_s =$ Number of possible condition states of the element

Assuming the risk per unit measurement of the element, r_j, as the risk R_j in condition state *j* divided by the element quantity *Q* as shown in Equation 3-45.

$$r_j = \frac{R_j}{Q} \tag{3-45}$$

The risk *R* associated with a bridge element with N_s condition states can be defined in terms of r_j as shown in Equation 3-46.

$$R = \sum_{j=1}^{N_s} q_j r_j \tag{3-46}$$

The procedure for estimating the risk at the element level is shown in Figure 3-7. The initial risk R_0 associated with condition of a bridge element can be calculated using the initial condition state distribution $q_{(t = 0)}$ of the element in the different condition states as shown in Equation 3-47.

$$R_0 = \sum_{j=1}^{N_s} r_j q_j (t = 0) \qquad (3\text{-}47)$$

The condition state distributions of bridge elements change from time to time due to deterioration and application of maintenance actions. The condition state distribution of the element $q_{(t)}$ after any number of years t can be calculated using the transition probabilities between the condition states of the element and the initial condition state distribution. The risk R_t associated with the element condition after any number of years t can be calculated using the element condition state distribution $q_{j(t)}$ and the condition state risk per unit of the element r_j(t)at that time as shown in Equation 3-48.

$$R_t = \sum_{j=1}^{N_s} r_j (t) q_j (t) \qquad (3\text{-}48)$$

where

$q_{j(t)}$ = Quantity of the element in condition state j at time t, where t is the number of years from the present, and

$r_{j(t)}$ = Risk per unit of the element in condition state j.

The mean μ_{R_t} and standard deviation σ_{R_t} for the risk associated with the element condition at year *t* are calculated as shown in Equations 3-49 and 3-50, respectively.

$$\mu_{R_t} = \sum_{j=1}^{N_s} \mu_{q_j(t)} \mu_{r_j(t)} \tag{3-49}$$

$$\sigma_{R_t} = \sqrt{\sum_{j=1}^{N_s} (\sigma_{q_j(t)}{}^2 \mu_{r_j(t)}{}^2 + \mu_{q_j(t)}{}^2 \sigma_{r_j(t)}{}^2)} \tag{3-50}$$

where,

μ_{R_t}, σ_{R_t} = Mean and standard deviation for the risk associated with the element condition at year *t*.

$\mu_{r_j(t)}$, $\sigma_{r_j(t)}$ = Mean and standard deviation for the risk per unit of the element in condition state *j* at year *t*, and

$\mu_{q_j(t)}$, $\sigma_{q_j(t)}$ = Mean and standard deviation for the element quantity in condition state *j* at year *t*.

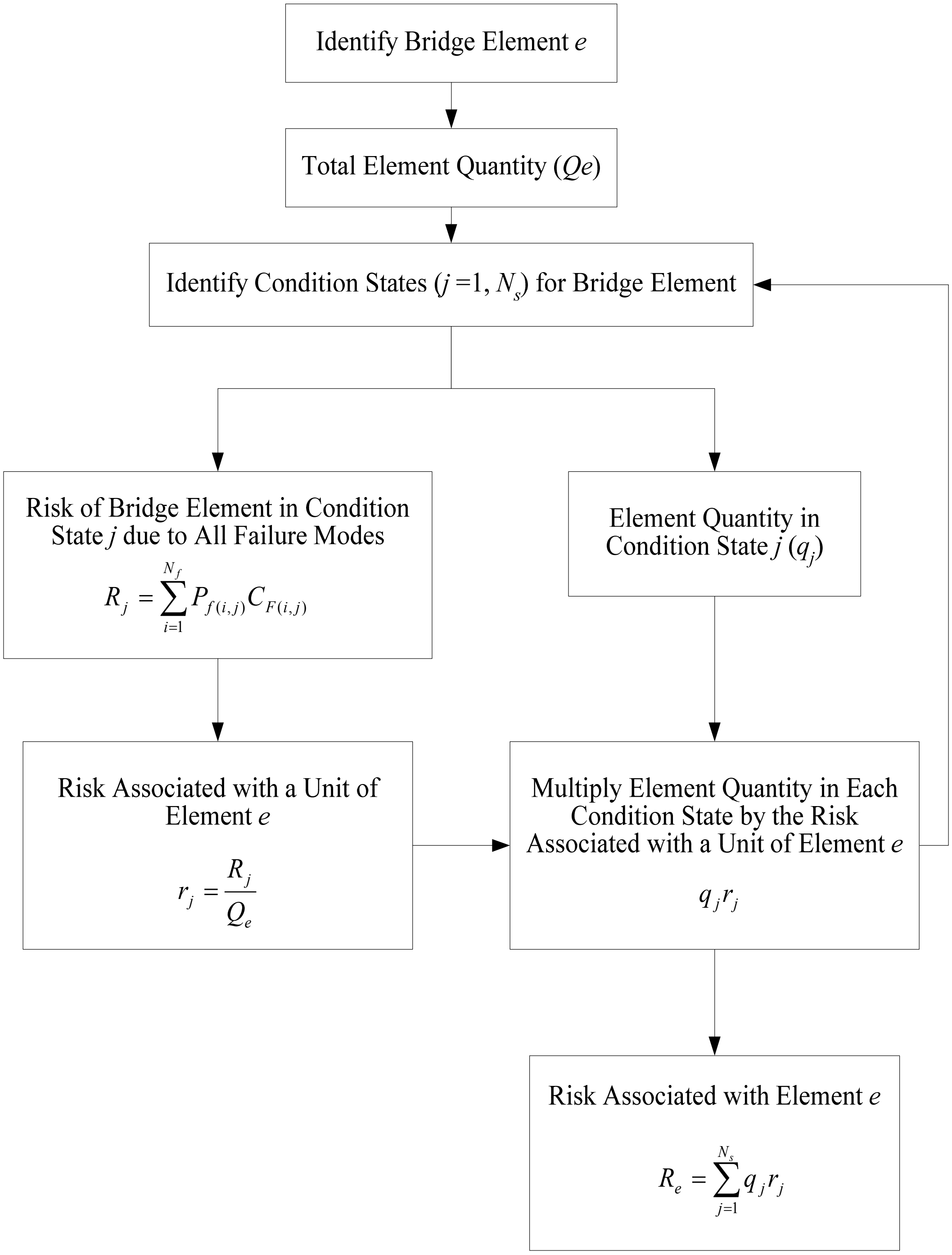


Figure 3-7. Procedure for Estimating Risks Associated with Bridge Elements

3.5.4.4 Risk Estimation at the Bridge Level

In this study, a bridge is assumed as a set of structural elements, and risk values associated with conditions of each element are used to estimate total risk associated with bridge conditions. Risk R_B associated with the condition of a bridge composed of a number of elements N_e can be calculated based on the risks R_e associated with the conditions of elements of the bridge as shown in Equation 3-51.

$$R_B = \sum_{e=1}^{N_e} R_e = \sum_{e=1}^{N_e} \sum_{j=1}^{N_s} r_j q_j \qquad (3\text{-}51)$$

where

R_e = Risk associated with condition of Element e

r_j = Condition state risk per unit of the element in condition state j

q_j = Quantity of the element in condition state j

N_s = Number of condition states of the element

N_e = Number of elements in the bridge

The risk $R_{t,B}$ associated with the bridge condition after a number of years t is calculated using condition state distributions $q_{j(t)}$ and condition state risks per unit of the element r_j(t) at that time for all bridge elements as shown in Equation 3-52.

$$R_{t,B} = \sum_{e=1}^{N_e} R_{t,e}(t) = \sum_{e=1}^{N_e} \sum_{j=1}^{N_s} r_j(t) q_j(t) \qquad (3\text{-}52)$$

The procedure for estimating the risk associated with the bridge condition is shown in Figure 3-8. The mean $\mu_{R_{t,B}}$ and standard deviation $\sigma_{R_{t,B}}$ associated with bridge maintenance at any year t can be calculated as shown in Equations 3-53 and 3-54, respectively.

$$\mu_{R_{t,B}} = \sum_{e=1}^{N_e} \mu_{R_{t,e}} \tag{3-53}$$

$$\sigma_{R_{t,B}} = \sqrt{\sum_{e=1}^{N_e} \sigma_{R_{t,e}}{}^2} \tag{3-54}$$

where $\mu_{R_{t,e}}$ and $\sigma_{R_{t,e}}$ are the means and standard deviations of risks associated with bridge elements and N_e is the number of elements in the bridge.

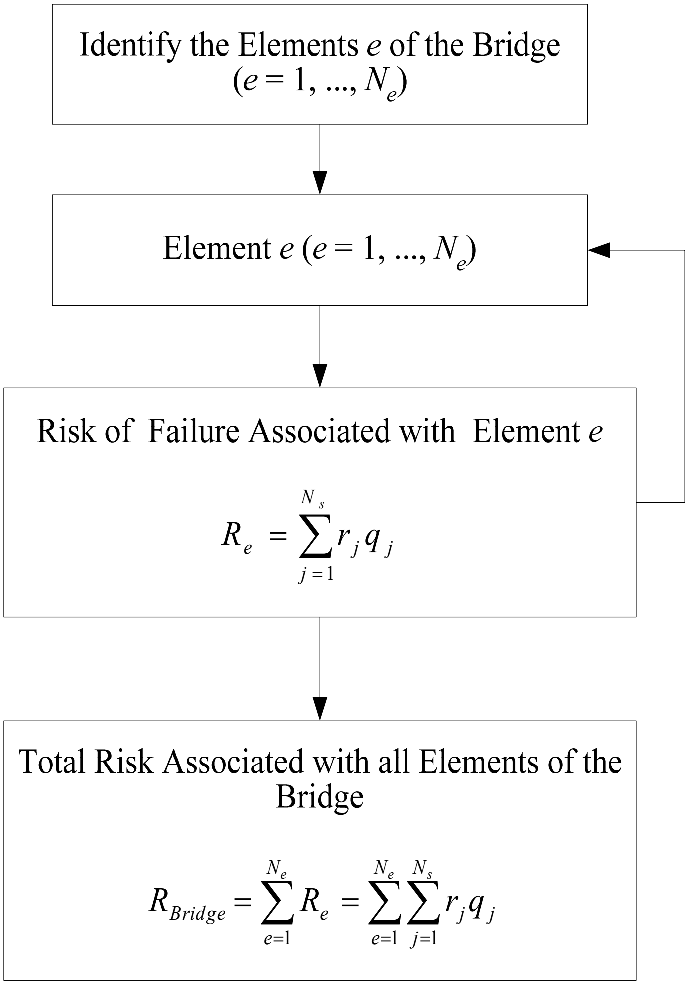


Figure 3-8. Procedure for Estimating Risks Associated with Conditions of All Elements of the Bridge

3.6 Risk Management

Risk management is used to control risk by improving the conditions of bridge elements through the optimal application of maintenance actions based on risk reduction effectiveness. Risk management for bridge maintenance includes three levels. The first level is focused on the element maintenance. The second level is focused on the bridge maintenance. Finally, the third level is focused on the maintenance priority of bridges identified for maintenance in the bridge inventory of a highway agency. The three maintenance levels are explained in the following subsections.

3.6.1 Risk-Based Element Maintenance

Bridge management decisions include alternatives that have costs and risks associated with conditions of bridge elements. A number of feasible maintenance actions are used as maintenance alternatives in the condition states of bridge elements. Each feasible action incurs a cost and poses a risk. The maintenance alternatives include doing nothing and maintenance actions that can improve the element condition and control potential risk. Optimal actions in the condition states of a bridge element are selected based on both cost of maintenance actions and risks associated with element conditions.

Different scenarios are created for the maintenance of bridge elements in a given planning horizon. An element maintenance scenario is the sequence of maintenance or no-maintenance options during the planning horizon. In each year of a maintenance scenario, the maintenance options are to apply the optimal actions in the different condition states of the element or to apply doing nothing in all condition states of the element. The goal is to select optimal maintenance actions in the condition states of bridge elements and select the optimal scenarios of applying and/or deferring the optimal actions during the planning horizon. To achieve this goal, the following steps are used in developing risk-based maintenance strategies for bridge elements:

1. Identify the optimal maintenance actions in the different condition states of the element,
2. Create maintenance scenarios for the element,
3. Calculate the risk associated with the maintenance scenarios,
4. Calculate the risk reduction due to the application of the maintenance scenarios,
5. Calculate the cost of actions applied in the maintenance scenarios,
6. Use the benefit-cost analysis to study the effectiveness of the maintenance scenarios,
7. Compare the different maintenance scenarios based on benefit-cost analysis, and
8. Select the optimal maintenance strategy for the bridge element

The abovementioned steps are explained in the following subsections.

3.6.1.1 Optimal Maintenance Actions in Condition States of the Element

The proposed methodology assumes that a probability of failure exists in any possible condition state of the element. The probability and consequences of failure in a particular condition state of the element result in a risk value associated with the element in that condition state. The objective of this subsection is to identify the set of optimal actions in the condition states of a bridge element based on the cost and risk associated with the different condition states of the element. This objective can be achieved by applying the following procedure:

1. Creating scenarios for the different actions in the possible condition states of the element,
2. Calculating the maintenance cost for each scenario of actions in the condition states of the element,
3. Calculating the risk associated with each scenario of actions in the condition states of the element, and
4. Selecting the optimal set of actions in the condition states of the element.

The proposed methodology uses the consequences of failure and the probabilities of failure calculated in the risk assessment section to calculate the risk associated with the possible scenarios of actions in the condition states of the element. The optimal set of maintenance actions in the different condition states of the element are selected based on minimizing the costs and the risks associated with the element condition for the different scenarios of maintenance actions.

For any bridge element with N_s condition states where each condition state j has $N_{a,j}$ feasible maintenance actions, the number of scenarios n_{sa} of maintenance actions in the condition states of the element are calculated as shown in Equation 3-55.

$$n_{sa} = \prod_{j=1}^{N_s} N_{a,j} \tag{3-55}$$

The risk associated with each scenario is calculated based on the condition state distribution of the element due to the application of the scenario. The initial risk before applying the scenarios is calculated based on the initial condition state distribution of the element $q_{(0)}$ and risk R_j associated with the condition states of the element as shown in Equation 3-56.

$$R_0 = \sum_{j=1}^{N_s} R_j \frac{q_j(0)}{Q} \tag{3-56}$$

The risk associated with the element condition at year t can be calculated as shown in Equations 3-57.

$$R_t = \sum_{j=1}^{N_s} \frac{R_{j,t}}{Q} q_j(t) \tag{3-57}$$

where

R_t = Risk associated with all condition states of an element at year t

$R_{j,t}$ = Risk associated with condition state j of an element at year t

$q_j(t)$ = Element quantity in condition state j at year t

Q = Total quantity of the element

Ns = Number of possible condition states of the element

The cost of applying a set of maintenance actions in the condition states of an element at year t can be calculated as shown in Equation 3-58.

$$C_t = \sum_{j=1}^{N_s} c(a_{k,j})q_j(t) \tag{3-58}$$

where

$c(a_{k,j})$ = Unit cost of maintenance action $a_{k,j}$ applied in condition state j at year t

When a set of maintenance actions are applied in the condition states of the element at year t-1, the condition state distribution of the element at year t (after one year) improves in accordance with the transition probabilities associated with applying these actions. Risk associated with element condition is estimated at year t after applying the actions. The sum of the maintenance cost of the set of actions applied in the condition states of the element at year t-1 and the risk associated with the element condition at year t can be expressed as:

$$CR_t = (1 + i_d)C_{t-1} + R_t \tag{3-59}$$

where

CR_t = Sum of maintenance cost at year t-1 and risk associated with element condition at year t

C_{t-1} = Maintenance cost of the set of actions applied in the condition states of the element at year t-1

R_t = Risk associated with element condition at year t after one year of applying the set of maintenance actions in the condition states of the element

i_d = Real discount rate

If we substitute Equations 3-57 and 3-58 in Equation 3-59, the sum CR_t of maintenance cost of actions and risks associated with element conditions can be expressed as:

$$CR_t = (1+i_d)\sum_{j=1}^{N_s} c(a_{k,j})q_j(t-1) + \sum_{j=1}^{N_s} R_{j,t}\frac{q_j(t)}{Q} \tag{3-60}$$

The selection of the optimal set of actions in the condition states of a bridge element depends on the objective. For any year t of the maintenance scenario, the objective is to minimize the sum of the risk associated with element condition at year t and the maintenance cost of the set of actions applied in the condition states of the element at year (t-1). The optimization problem can be defined as:

Minimize the objective function

$$CR_t = (1+i_d)\sum_{j=1}^{N_s} c(a_{k,j})q_j(t-1) + \sum_{j=1}^{N_s} R_{j,t}\frac{q_j(t)}{Q} \tag{3-61}$$

Subject to the conditions

$$\begin{aligned} &q_i(t-1) \geq 0 \\ &q_j(t) \geq 0 \\ &\sum_{j=1}^{N_s} q_j(t-1) = Q \\ &\sum_{j=1}^{N_s} q_j(t) = Q \end{aligned} \tag{3-62}$$

The scenario of actions that satisfies both the conditions of the maintenance process and the objective is the optimal scenario of maintenance actions in the condition states of the element. Therefore, the optimal actions in the condition states of the element at any year *t* in the future are assumed as the set of actions in the scenario of condition states that result in the least sum of maintenance cost at year *t*-1 and risk associated with the element condition at year *t* among all scenarios. The minimum value for the summation $CR_{t,min}$ can be calculated as shown in Equation 3-63

$$CR_{t,\min} = \underset{a_{k,i}}{Min}\left[(1+i_d)\sum_{j=1}^{N_s} c(a_{k,j})q_j(t-1) + \sum_{j=1}^{N_s} R_{j,t}\frac{q_j(t)}{Q}\right] \tag{3-63}$$

In the long run, the element condition reaches a steady state distribution at time t>T, where T is the year after which the condition state distribution of the element remains unchanged over the years. The long-term optimal actions in the condition states of the element are the set of actions that result in the least sum of maintenance costs (at year *T*-1) and risks associated with the element conditions (at year *T*) among all condition state scenarios in the long run. The minimum value for the summation $CR_{T,min}$ of the cost and risk at time *T* can be calculated as shown in Equation 3-64.

$$CR_{T,\min} = \underset{a_{k,j}}{Min}\left[\sum_{j=1}^{N_s}\left((1+i_d)c_j(a_{k,j}) + \frac{R_{j,T}}{Q}\right)q_j(T)\right] \tag{3-64}$$

The outcome of this subsection is a set of optimal maintenance actions in the condition states of a bridge element at any given year *t*. The scenario of these optimal actions in the condition states of the element results in the least sum of maintenance cost and risk associated with element condition for any year in the planning horizon. The optimal actions are used for creating element maintenance scenarios throughout the planning horizon as explained in the following section.

3.6.1.2 Element Maintenance Scenarios

The set of optimal maintenance actions in the condition states of a bridge element are used for creating maintenance scenarios for the element during the planning horizon. For any planning horizon with a number of years N_y, different scenarios can be created for the maintenance of a bridge element. For each year in a maintenance scenario, there are two options of maintenance or no maintenance of the element. Maintenance of an element refers to applying the optimal maintenance actions in the condition states of the element. No maintenance refers to applying do-nothing options to all condition states of the element, and therefore leaving the element without maintenance.

Each scenario is called based on whether or not to do maintenance for the element at a certain year. The scenario name will be composed of a chain of ones and zeros from left to right, representing the years in the analysis period, where one stands for maintenance and zero stands for no maintenance of the element at a given year. For example, the scenario '1011' in an analysis period of four years means element maintenance in years 1, 3 and 4 and no maintenance for the element in year 2 from the base year. The number of maintenance scenarios n_s for an analysis period of N_y years from the base year can be calculated using Equation 3-65.

$$n_s = 2^{N_y} \tag{3-65}$$

Table 3-10 shows the possible scenarios for analysis periods of 1, 2, 3 and 4 years. A one-year analysis period has only two scenarios for the element maintenance; either to do maintenance ‘1’ by applying the optimal actions in the condition states of the element or to doing nothing ‘0’ in all condition states of the elements. A two-year analysis period has four scenarios; to do nothing in both years ‘00’, to do nothing in the first year and maintenance in the second year ’01’, to do maintenance in the first year and nothing in the second year ‘10’, or to do maintenance in both years ‘11’. Likewise, analysis periods of three and four years will have nine and sixteen scenarios, respectively, as shown in Table 3-5.

Table 3-5. Possible Scenarios for Analysis Periods of 1, 2, 3, and 4 in a Maintenance Planning Horizon

Scenario No.	Year after Base Year			
	1	2	3	4
1	0	00	000	0000
2	1	01	001	0001
3		10	010	0010
4		11	011	0011
5			100	0100
6			101	0101
7			110	0110
8			111	0111
9				1000
10				1001
11				1010
12				1011
13				1100
14				1101
15				1110
16				1111

3.6.1.3 Risk Calculations for the Maintenance Scenarios of the Element

The objective of this subsection is to calculate the risk associated with each maintenance scenario of the element in each year of the planning horizon. This objective can be achieved by applying the following procedure:

1. Calculating the probability of failure in the condition states of the element for each year of the maintenance scenario,
2. Estimating the consequences of failure for each year of the maintenance scenario,
3. Calculating the risk in the condition states of the element for each year of the maintenance scenario,
4. Calculating the quantity distribution of the element in the condition states for each year of the maintenance scenario based on the transition in the element quantity that results from the maintenance or no-maintenance options, and
5. Calculating the risk associated with the element conditions for each year of the maintenance scenario based on the risk values in the condition states and the condition state distribution of the element.

The failure probabilities, failure consequences, and risks associated with the condition states of the element are calculated as discussed in sections 3.5.2, 3.5.3, and 3.5.4, respectively. The quantity distribution of the element $q(t)$ at year t, following the application of the maintenance actions in the condition states of an element, is calculated based on distribution of the element quantity $q(t\text{-}1)$ at year t-1 before applying the actions and the transition probabilities associated with the maintenance actions. The quantity of the element in any condition state j at a particular year t is calculated as shown in Equation 3-66.

$$q_j(t) = \sum_{i=1}^{N_s} P_{ij}(a_{k,i}) q_i(t-1) \tag{3-66}$$

where

$q_j(t)$ = Element quantity in condition state j at year t

$P_{ij}(a_{k,i})$ = Transition probability from condition state i to condition state j when action $a_{k,i}$ is applied in condition state i

$q_i(t\text{-}1)$ = Element quantity in condition state i at year t-1

N_s = Number of possible condition states of the element

Mitigated and unmitigated risk values are estimated at each year of the maintenance scenario. Mitigated risk in a particular year of a maintenance scenario is associated with the set of optimal actions for that year, while unmitigated risk is associated with doing nothing in the condition states of the element for that year of the scenario. The mitigated risk R_t and unmitigated risk $R_{t,0}$ associated with the element condition at any year t can be expressed as shown in Equations 3-67 and 3-68, respectively:

$$R_t = \sum_{j=1}^{N_s} \left(\frac{R_{j,t}}{Q} \sum_{i=1}^{N_s} P_{ij}(a_k, i) q_i(t-1) \right) \tag{3-67}$$

$$R_{t,0} = \sum_{j=1}^{N_s} \left(\frac{R_{j,t}}{Q} \sum_{i=1}^{N_s} P_{ij}(a_{0,i}) q_i (t-1) \right) \tag{3-68}$$

where

R_t = Mitigated risk associated with applying optimal maintenance actions a_k in the condition states of the element,

$R_{t,0}$ = Unmitigated risk associated with applying do nothing action a_0 in the condition states of the element,

$R_{j,t}$ = Risk associated with condition state j of the element at year t,

$P_{ij}(a_{k,}i)$ = Transition probability from condition state i to condition state j when action a_k is applied in condition state i,

$P_{ij}(a_{k,}i)$ = Transition probability from condition state i to condition state j when do-nothing action a_0 is applied in condition state i,

$q_i(t\text{-}1)$ = Element quantity in condition state i at year t-1,

N_s = Number of possible condition states of the element

Risk R_t for each year of the planning horizon is compared to a target risk value associated with a threshold for the acceptable damage level in the element condition. Risk R_t should not exceed the target risk at any year of the planning horizon in order to be considered.

The mean and standard deviation for the mitigated and unmitigated risks are calculated as shown in section 3.5.4.3. Mitigated and unmitigated risks at any year of the planning horizon are used to calculate risk reduction associated with the optimal maintenance actions for that year as discussed in the following section.

3.6.1.4 Risk Reduction Associated with Maintenance Scenarios of Bridge Elements

Risk reduction for a given year of the maintenance scenario is calculated by comparing the mitigated risk with the unmitigated risk. The mitigated risk for a given year results from applying the set of optimal actions in the condition states of the element in that year. The unmitigated risk for a given year results from delaying the optimal actions for that year and applying the do-nothing option by leaving the element without maintenance in that year.

The risk reduction ΔR_t for any year t in the maintenance scenario can be calculated as the unmitigated risk $R_{t,0}$ minus the mitigated risk R_t as shown in Equation 3-69.

$$\Delta R_t = R_{t,0} - R_t \tag{3-69}$$

where

ΔR_t = Risk reduction associated with element condition at year t of the maintenance scenario

$R_{t,0}$ = Unmitigated risk associated with element condition at year t of the maintenance scenario

R_t = Mitigated risk associated with element condition at year t of the maintenance scenario

Risk reduction ΔR_t measures the benefit of reducing the risk associated with element condition at year t of the maintenance scenario as a result of applying the optimal maintenance actions for that year. The mean and standard deviation for the risk reduction $\sigma_{\Delta R_t}$ at any year t of the maintenance scenario is calculated based on the means and standard deviations for the mitigated and unmitigated risks associated with element conditions in year t. The mean of risk reduction is equal to the mean of unmitigated risk minus the mean of mitigated risk. The standard deviation of risk reduction is calculated as shown in Equation 3-70.

$$\sigma_{\Delta R_t} = \sqrt{\sigma_{R_t}{}^2 + \sigma_{R_{t,0}}{}^2} \tag{3-70}$$

where,

$\sigma_{\Delta R_t}$ = Standard deviation for the risk reduction at year t of the maintenance scenario

σ_{R_t} = Standard deviation for the mitigated risk at year t of the maintenance scenario

$\sigma_{R_{t,0}}$ = Standard deviation for the unmitigated risk at year t of the maintenance scenario

The present value of risk reduction $\Delta R_{t,p}$ for any year t in the analysis period of a maintenance scenario can be calculated as shown in Equation 3-71.

$$\Delta R_{t,p} = \frac{\Delta R_t}{(1+i_d)^t} \tag{3-71}$$

where i_d is the real discount rate.

The present value of the risk reduction ΔR_p associated with the maintenance scenario is calculated as the summation of the present values for risk reductions associated with all years of the scenario (t = 1 to N_y) as shown in Equation 3-72.

$$\Delta R_p = \sum_{t=1}^{N_y} \frac{\Delta R_t}{(1+i_d)^t} = \sum_{t=1}^{N_y} \frac{R_{t,0}}{(1+i_d)^t} - \frac{R_t}{(1+i_d)^t} \tag{3-72}$$

The present value of the total risk reduction ΔR_p measures the benefit of reducing the risk associated with element condition during the years of the maintenance scenario as a result of applying the different maintenance options in each year of the maintenance scenario. The mean and standard deviation of the total risk reduction for a maintenance scenario is calculated based on the means and standard deviations for the risk reductions in all years of planning horizon. The mean of total risk reduction in present value is equal to the summation of the means in of risk reductions present value over the years of the planning horizon. The standard deviation of risk reduction in present value $\sigma_{\Delta R_p}$ is calculated using the standard deviations of risk reductions $\sigma_{\Delta R_{t,p}}$ for all years t in the planning horizon as shown in Equation 3-73.

$$\sigma_{\Delta R_p} = \sqrt{\sum_{t=1}^{N_y} \sigma_{\Delta R_{t,p}}{}^2} \tag{3-73}$$

Different maintenance scenarios for a bridge element result in different risk reductions associated with the element maintenance. The risk reductions alone are not enough to measure the efficiency of a maintenance scenario. The cost of applying the maintenance actions during a scenario also should be considered. The cost associated with the maintenance scenarios is discussed in the following section.

3.6.1.5 Cost of Element Maintenance Scenarios

The cost of applying a maintenance scenario for a bridge element depends on the maintenance actions applied during the planning horizon of the scenario. For each year of a maintenance scenario, there is an option of applying the optimal actions in the condition states of the element or doing nothing. The cost of the maintenance scenario in a particular year is the sum of the costs of the maintenance actions applied in the different condition states of the element in that year. The cost $C(a_k, j)$ of applying a maintenance action a_k in a particular condition state j of the element at year t is calculated by multiplying the element quantity $q_j(t)$ in Condition State j by the unit cost $c(a_k, j)$ of the maintenance action a_k as shown in Equation 3-74.

$$C(a_k, j) = q_j(t)c(a_k, j) \tag{3-74}$$

The cost C_t of applying a set of maintenance actions in N_s possible condition states of the element at year t can be calculated as shown in Equation 3-75.

$$C_t = \sum_{j=1}^{N_s} C(a_k, j) = \sum_{j=1}^{N_s} q_j(t) c(a_k, j) \tag{3-75}$$

The mean μ_{C_t} and standard deviation σ_{C_t} for the maintenance cost associated with the element at year *t* is calculated as shown in Equations 3-76 and 3-77, respectively.

$$\mu_{C_t} = \sum_{j=1}^{N_s} \mu_{q_j(t)} \mu_{c(a_k, j)} \tag{3-76}$$

$$\sigma_{C_t} = \sqrt{\sum_{j=1}^{N_s} (\sigma_{c(a_k,j)}{}^2 \mu_{q_j(t)}{}^2 + \mu_{c(a_k,j)}{}^2 \sigma_{q_j(t)}{}^2)} \tag{3-77}$$

where,

μ_{C_t}, σ_{C_t} = Mean and standard deviation for maintenance cost at year *t* of the maintenance scenario

$\mu_{c(a_k,j)}$, $\sigma_{c(a_k,j)}$ = Mean and standard deviation for the unit cost of optimal maintenance action in condition state *j*.

$\mu_{q_j(t)}$, $\sigma_{q_j(t)}$ = Mean and standard deviation for the element quantity in condition state *j* at year *t*.

The present value $C_{t,p}$ of a maintenance cost C_t at year *t* can be calculated as shown in Equation 3-78.

$$C_{t,p} = \frac{C_t}{(1 + i_d)^t} \tag{3-78}$$

Where i_d is the real discount rate.

The present value $C_{t,p}$ for any cost C_t at any year t in the maintenance scenario can be calculated as shown in Equation 3-79.

$$C_{t,p} = \sum_{j=1}^{N_s} q_j(t) \frac{c(a_k, j)}{(1+i_d)^t} \tag{3-79}$$

The present value of total maintenance costs of actions applied during a planning horizon of N_y years for a maintenance scenario can be calculated as the sum of present values of maintenance costs $C_{t,p}$ from year t =0 to t =N_y-1 as shown in Equation 3-80.

$$C_p = \sum_{t=0}^{N_y - 1} C_{t,p} = \sum_{t=0}^{N_y - 1} \frac{C_t}{(1+i_d)^t} \tag{3-80}$$

The present value C_p of total maintenance cost for any maintenance scenario with an analysis period of N_y years can be calculated as shown in Equation 3-81.

$$C_p = \sum_{t=0}^{N_y - 1} \sum_{j=1}^{N_s} q_j(t) \frac{c(a_k, j)}{(1+i_d)^t} \tag{3-81}$$

where

C_p = Present value of total maintenance cost for a maintenance scenario with an analysis period of N_y years

q_j(t) = Element quantity in Condition State j at year t from the base year of the maintenance scenario

$c(a_k, j)$ = Unit cost of applying optimal maintenance action a_k in Condition State j at year t of the maintenance scenario

i_d = Real discount rate.

The present value of the total maintenance cost C_p measures the cost of actions during the years of the maintenance scenario as a result of applying the different maintenance options in each year of the scenario. The mean and standard deviation of the total cost for a maintenance scenario is calculated based on the means and standard deviations for the costs in all years of planning horizon. The mean of total cost in present value is equal to the summation of the means in of maintenance costs in present value over the years of the planning horizon. The standard deviation of maintenance cost in present value σ_{C_p} is calculated using the standard deviations of costs $\sigma_{C_{t,p}}$ for all years t in the planning horizon as shown in Equation 3-82.

$$\sigma_{C_p} = \sqrt{\sum_{t=0}^{N_y - 1} {\sigma_{C_{t,p}}}^2} \tag{3-82}$$

The present value of total maintenance cost for a scenario is needed to find how much money in present value is needed to apply the maintenance options defined by the scenario. A tradeoff between maintenance costs and risk reductions associated with the maintenance scenarios can be used to measure the cost effectiveness of these scenarios. On the other hand, the maintenance cost for a scenario should not exceed the maintenance budget of the highway agency in any year of the planning horizon. If the maintenance cost exceeds the budget, the scenario could be eliminated.

3.6.1.6 Filtering Maintenance Scenarios of Bridge Elements

Maintenance costs and risks associated with the maintenance scenarios for bridge elements are used as the main factors for filtering scenarios. The costly and risky scenarios can be eliminated from the list of maintenance scenarios before analyzing the economic efficiency of the scenarios.

Highway agencies usually set budget limitations that control the maintenance cost for each year of the planning horizon. Maintenance scenarios with maintenance costs exceeding the available budget for maintenance in any year of the planning horizon are eliminated from the selection process.

The risk associated with condition of a bridge element reflects the potential loss that results from the condition state distribution of the element. High-risk values associated with an element condition results from element distributions in worse condition states with higher probabilities of failure. A target risk value can be used to control the degradation in the condition of the element during the planning horizon. Bridge maintenance scenarios with risk values more than the acceptable risk are eliminated and not included in the selection process. Benefit cost analysis is used to study the economic efficiency for each of the filtered maintenance scenarios as discussed in the following section.

3.6.1.7 Benefit-Cost Analysis for Maintenance Scenarios of Bridge Elements

The economic efficiency of the maintenance scenarios for a bridge element can be compared using benefit-cost analysis. The benefit of applying a maintenance scenario is defined as the reduction in risk due to the application of the scenario. The scenario cost is defined as the total cost of the maintenance actions applied during the analysis period of the scenario.

To study the economic efficiency of the maintenance scenarios, the following benefit-cost measures are calculated for the different maintenance scenarios of the element:

1. Benefit-cost ratio (*BCR*),
2. Net present value (*NPV*),
3. Benefit-cost reliability index ($\beta_{B/C}$) and
4. Benefit-cost failure probability ($P_{f,B/C}$).

The benefit-cost ratio of a maintenance scenario is defined as the benefit of the scenario divided by the scenario cost. The benefit of a maintenance scenario is defined herein as the present value of risk reduction associated with the element due to the application of the scenario. The cost of a maintenance scenario is the present value of the total cost of applying maintenance actions defined by the scenario in the condition states of the element during the planning horizon. The benefit-cost ratio (*BCR*) for a maintenance scenario is calculated by dividing the present value of risk reduction by the present value of total maintenance cost of the scenario as shown in Equation 3-83.

$$BCR = \frac{\Delta R_p}{C_p} \tag{3-83}$$

The present values of risk reduction ΔR_p and maintenance cost C_p for a scenario with a planning horizon of N_y years are calculated using Equations 3-72 and 3-81. Therefore, the benefit-cost ratio (BCR) for a maintenance scenario with an analysis period of N_y years can be calculated as shown in Equation 3-84.

$$BCR = \frac{\sum_{t=1}^{N_y} \left(\frac{R_{t,0}}{(1+i_d)^t} - \frac{R_t}{(1+i_d)^t} \right)}{\sum_{t=0}^{N_y - 1} \frac{C_t}{(1+i_d)^t}} \tag{3-84}$$

where

BCR = Benefit-cost ratio for the maintenance scenario,

R_t = Mitigated risk associated with element condition at year t when optimal actions are applied,

$R_{t,0}$ = Unmitigated risk associated with element condition at year t when no maintenance actions are applied,

C_t = Maintenance cost of the element at year t of the maintenance scenario,

N_y = number of years in the planning horizon of maintenance scenario, and

i_d = Annual real discount rate during the application of the maintenance scenario.

The net present value (NPV) for a maintenance scenario is defined as the scenario benefit minus the scenario cost. The scenario benefit is the present value of risk reduction and the scenario cost is the present value of the total cost of maintenance actions during the planning horizon. The net present value (NPV) for a maintenance scenario with a planning horizon of N_y years can be calculated as the present value of risk reduction ΔR_p associated with the scenario minus the present value of total cost C_p of maintenance actions applied during the analysis period of the scenario. The net present value (NPV) can be expressed as shown in Equation 3-85.

$$NPV = \sum_{t=1}^{N_y} \left(\frac{R_{t,0}}{(1+i_d)^t} - \frac{R_t}{(1+i_d)^t} \right) - \sum_{t=0}^{N_y-1} \frac{C_t}{(1+i_d)^t} \qquad (3\text{-}85)$$

A benefit-cost reliability index ($\beta_{B/C}$) can be used to account for the probabilistic characteristics of the risk-reduction and the cost for each maintenance scenario based on the reliability assessment techniques (Ayyub, 2003). The cost of the scenario can be looked at as the demand of the scenario, while the risk-reduction benefit can be looked at as the supply or offer of the scenario. The net present value (NPV) of a maintenance scenario can be considered as the performance function of what the scenario can offer minus what the scenario demands. The benefit-cost reliability index ($\beta_{B/C}$) depends on the distribution of both the risk-reduction and the cost of the scenario. $\beta_{B/C}$ is defined as the ratio between the mean and the standard deviation of the net present value for the scenario as shown in Equation 3-86.

$$\beta_{B/C} = \frac{\mu_{NPV}}{\sigma_{NPV}} \tag{3-86}$$

For normally distributed cost and risk-reduction benefit, the benefit-cost reliability index can be expressed as shown in Equation 3-87 (Ayyub, 2003),

$$\beta_{B/C} = \frac{\mu_{\Delta R} - \mu_C}{\sqrt{\sigma_{\Delta R}^{\ 2} + \sigma_C^{\ 2}}} \tag{3-87}$$

where

$\mu_{\Delta R}$ = Mean value for risk reduction of the scenario

μ_C = Mean value for maintenance cost of the scenario

$\sigma_{\Delta R}$ = Standard deviation for risk reduction of the scenario

σ_C = Standard deviation for maintenance cost of the scenario.

For log-normally distributed cost and risk-reduction benefit, the benefit-cost reliability index can be expressed as shown in Equation 3-88 (Ayyub, 2003).

$$\beta_{B/C} = \frac{\ln\left(\frac{\mu_{\Delta R}}{\mu_C}\sqrt{\frac{V_C^{\ 2}+1}{V_{\Delta R}^{\ 2}+1}}\right)}{\sqrt{\ln(V_{\Delta R}^{\ 2}+1)(V_C^{\ 2}+1)}} \tag{3-88}$$

where $V_{\Delta R}$ and V_C are the coefficients of variation for risk reduction and the maintenance cost of the scenario, respectively.

For cases with mixed distributions for cost and risk-reduction benefit, the benefit-cost index can be estimated using the advanced second moment method in reliability assessment.

The benefit-cost failure probability ($P_{f,B/C}$) represents the probability that the cost will exceed the risk-reduction benefit for a maintenance scenario. $P_{f,B/C}$ can be computed based on the cost of the maintenance scenario and the benefit of risk-reduction. The failure probability is calculated using the benefit cost index $\beta_{B/C}$ as shown in equation 3-89 (Ayyub, 2003).

$$P_{f,B/C} = P(C > \Delta R) = 1 - \Phi(\beta_{B/C}) \tag{3-89}$$

3.6.1.8 Comparison of Element Maintenance Scenarios

The different maintenance scenarios for a bridge element can be compared based on the following economic measures:

1. Cost of the maintenance actions applied in the scenario,
2. Risk associated with element condition,
3. Risk reduction associated with element maintenance,
4. Benefit-cost ratio (BCR) of risk reduction to maintenance cost,
5. Net present value (NPV) based on risk-reduction and maintenance cost, and
6. Benefit-cost reliability index ($\beta_{B/C}$) and failure probability ($P_{f,B/C}$) based on the distributions of risk-reduction and maintenance cost.

The maintenance scenarios for a bridge element can be ranked based on the abovementioned measures in the following order:

1. Low to high cost of maintenance,
2. Low to high risk associated with element condition,
3. High to low risk reduction (ΔR),
4. High to low benefit-cost ratio (BCR),
5. High to low net present value (NPV),
6. High to low benefit-cost reliability index ($\beta_{B/C}$), and
7. Low to high benefit-cost failure probability ($P_{f,B/C}$)

3.6.1.9 Risk-Based Optimal Maintenance Strategy for Bridge Elements

Decision makers may each have different preferences for selecting the best maintenance strategy for a bridge element. Some bridge engineers may select the maintenance strategy for an element based on cost only. Other engineers may consider risk and reduction in risk as the main factors for selecting a maintenance strategy. The optimal maintenance strategy for a bridge element will be based herein on the measures that include both cost of maintenance and risk-reduction benefit. The optimal strategy for a bridge element should have:

1. High benefit-cost ratio (BCR),
2. High net present value (NPV), and
3. High benefit-cost index ($\beta_{B/C}$) and low benefit-cost failure probability ($P_{f,B/C}$).

The scenarios with the highest values for BCR, NPV, $\beta_{B/C}$ and lowest $P_{f,B/C}$ are compared. The comparison result is used as a tool in the hands of the decision makers to determine the best maintenance strategy for the element.

3.6.2 Risk-Based Bridge Maintenance

Maintenance scenarios for a bridge are created based on the optimal maintenance strategies for its elements. The bridge maintenance scenarios are based on maintenance or no-maintenance options for the elements of the bridge. The decision of maintenance or no-maintenance for one or more of the bridge elements in each year of the scenario depends on the availability of resources allocated for bridge maintenance and the weight factor for the element.

Risk associated with bridge condition is calculated based on the risk associated with the maintenance strategies for the bridge elements. The risk associated with bridge condition during the planning horizon should not exceed a target risk tolerated for the bridge. The risk reduction associated with the maintenance of the bridge is calculated based on the risk reductions associated with the maintenance strategies for the bridge elements.

The cost of bridge maintenance is calculated based on the costs of the maintenance strategies for bridge elements. Maintenance cost of the bridge should not exceed the annual budget allocated for bridge maintenance. Risk reduction effectiveness of the bridge maintenance is examined using benefit-cost analysis. The benefit-cost analysis for bridge maintenance is used to select the optimal maintenance strategy for the bridge. The optimal maintenance strategy for a bridge results in efficient spending of available resources allocated for the maintenance of the bridge.

The following steps can be used to create a risk-based bridge maintenance strategy:

1. Create bridge maintenance scenarios,
2. Calculate the risk associated with bridge maintenance scenarios,
3. Calculate risk reduction due to the application of maintenance scenarios,
4. Calculate maintenance cost for the bridge maintenance scenarios,
5. Compare cost needed for the bridge maintenance scenarios with the available budget,
6. Compare the bridge maintenance scenarios using benefit-cost analysis and available resources, and
7. Select the optimal bridge maintenance strategy that best allocates available resources.

The abovementioned steps are explained in the following subsections.

3.6.2.1 Bridge Maintenance Scenarios

Bridge maintenance scenarios are created to examine the options of applying or deferring the maintenance policies for different elements of the bridge. A bridge maintenance scenario is a set of maintenance or no-maintenance options for each element in the bridge. An element maintenance option refers to applying the optimal maintenance strategy for the element. The no-maintenance option for an element refers to deferring the optimal maintenance strategy for the element and leaving the element without maintenance. The decision of applying or deferring a maintenance policy for one or more of the bridge elements can be based on the importance of the element to the bridge, the budget limitations, the cost of the maintenance policy and/or the risk associated with element conditions. If an element maintenance option in a bridge strategy was changed in any year of the planning horizon, the bridge strategy should be modified to consider this change by selecting the optimal maintenance actions corresponding to this modification. The priority for applying maintenance policies of bridge elements identified for maintenance is based on the element's importance to the bridge and the risk reduction effectiveness of the element's maintenance policy.

3.6.2.2 Risk Calculations for Bridge Maintenance Scenarios

The risk value associated with a bridge maintenance scenario is calculated based on the risk values associated with the maintenance scenarios of the elements. Risk associated with bridge maintenance for any year of the planning horizon is calculated, as shown previously in Equation 3-52, as the sum of the risks associated with the scenarios of bridge elements for that year. The mean and standard deviation for bridge risk is calculated as shown in Equations 3-53 and 3-54, respectively.

A target risk value that is acceptable for bridge maintenance is defined and risk values associated with the bridge maintenance scenarios are compared to the acceptable risk value for bridge conditions. If the maintenance options of an element are changed in any year of the planning horizon, the modification in the element strategy should be considered in the calculation of risk associated with the bridge maintenance.

3.6.2.3 Risk Reductions for Bridge Maintenance Scenarios

Risk reduction associated with bridge maintenance results from applying the optimal maintenance options in the maintenance strategies of bridge elements during the planning horizon. Risk reduction associated with the bridge maintenance for any year in the planning horizon is calculated based on the risk reductions associated with the maintenance options of bridge elements. Risk reduction associated with the bridge maintenance in any year of the planning horizon is calculated as the sum of risk reductions that result from implementing the optimal maintenance options of the bridge elements in that year.

The present value of risk reduction for bridge maintenance during the planning horizon is calculated based on risk reductions of elements. The present value of risk reduction associated with the maintenance of a bridge composed of a number of elements N_e can be calculated as the sum of the present values for the risk reductions that result from implementing the optimal maintenance options of bridge elements during the planning horizon. The mean and standard deviation of risk reduction associated with bridge maintenance can be calculated as shown in Equation 3-90 and 3-91, respectively.

$$\mu_{\Delta R_B} = \sum_{e=1}^{N_e} \mu_{\Delta R_e} \tag{3-90}$$

$$\sigma_{\Delta R_B} = \sqrt{\sum_{e=1}^{N_e} \sigma_{\Delta R_e}{}^2} \tag{3-91}$$

where $\sigma_{\Delta R_B}$ and $\sigma_{\Delta R_e}$ are the standard deviations of risk reduction for the bridge and the elements, respectively.

3.6.2.4 Costs of Bridge Maintenance Scenarios

The cost of each maintenance scenario for the bridge depends on the cost of maintenance actions applied during the maintenance policies for bridge elements. The maintenance cost in each year of a bridge maintenance scenario is calculated as the sum of costs of maintenance actions applied to bridge elements in that year of maintenance. If the optimal maintenance options in an element strategy were changed in any year of the planning horizon, the calculation of the bridge maintenance cost should consider the change in the element strategy.

Cost values for any year of bridge maintenance are discounted to present values. Present values of cost in the different years of a planning horizon are added to result in a present value for the total maintenance cost of bridge maintenance. The mean μ_{C_B} and standard deviation σ_{C_B} for total cost of bridge maintenance σ_{C_B} is calculated as shown in Equations 3-92 and 3-93.

$$\mu_{C_B} = \sum_{e=1}^{N_e} \sigma_{C_e} \tag{3-92}$$

$$\sigma_{C_B} = \sqrt{\sum_{e=1}^{N_e} \sigma_{C_e}{}^2} \tag{3-93}$$

where μ_{C_e} and σ_{C_e} are the means and standard deviations of maintenance costs associated with optimal maintenance scenarios for bridge elements, and N_e is the number of bridge elements.

3.6.2.5 Allocation of Available Resources for Maintenance of Bridges

Decision makers and planners usually have limited available resources for managing the maintenance of bridges. An annual budget is usually specified for maintenance of bridges. The cost needed for maintenance of a bridge at any year of maintenance scenario is compared to available resources allocated for the bridge maintenance. The maintenance cost should not exceed the budget allocated for bridge maintenance. The optimal strategies for some of the elements could be modified to redistribute the annual cost of bridge maintenance to meet budget limitations. This modification, if needed, should be based on benefit-cost analysis to make sure that the allocated resources available for bridge maintenance are utilized efficiently.

3.6.2.6 Benefit-Cost Analysis for Bridge Maintenance Scenarios

The goal of benefit-cost analysis for bridge maintenance is the efficient expenditure of allocated resources for bridge maintenance. This can be achieved by comparing the economic efficiency of different bridge maintenance scenarios based on a number of economic measures. The following economic measures can be used to compare the bridge maintenance scenarios:

1. Risk reduction due to the application of the scenario,
2. Benefit-cost ratio (BCR),
3. Net present value (NPV), and
4. Benefit-cost reliability index ($\beta_{B/C}$) and failure probability ($P_{f,B/C}$).

The benefit-cost ratio (BCR) and net present value (NPV) for the bridge maintenance are calculated based on the cost and risk reduction associated with bridge maintenance as shown in Equations 3-94 and 3-95, respectively.

$$BCR_B = \frac{\Delta R_B}{C_B} \tag{3-94}$$

$$NPV_B = \Delta R_B - C_B \tag{3-95}$$

where BCR_B, NPV_B, ΔR_B, and C_B are the benefit cost ratio, net present value, risk reduction, and maintenance cost associated with bridge maintenance scenario, respectively.

The benefit-cost reliability index ($\beta_{B/C}$) and benefit-cost failure probability ($P_{f,B/C}$) are calculated based on the means and standard deviations for maintenance costs and risk reductions associated with bridge maintenance as shown in Equations 3-96 and 3-97, respectively.

$$\beta_{B/C_{Bridge}} = \frac{\mu_{\Delta R_B} - \mu_{C_B}}{\sqrt{\sigma_{\Delta R_B}{}^2 + \sigma_{C_B}{}^2}} \tag{3-96}$$

$$P_{f,B/C_{Bridge}} = \Phi(-\beta_{B/C_{Bridge}}) \tag{3-97}$$

3.6.2.7 Optimal Bridge Maintenance Policy

The optimal bridge maintenance policy is the policy that utilizes resources allocated for bridge maintenance efficiently based on one or more of the abovementioned economic measures. The selection of the bridge maintenance policy depends on factors such as the availability of resources for maintenance, the attitude of the bridge manager to risk tolerance and spending of resources, the total cost and risk associated with bridge condition, and the importance and criticality of the different elements in the bridge. The availability of resources plays a vital role in the decision of which maintenance scenario to apply for a bridge. Bridge maintenance scenarios with maintenance costs exceeding available resources for maintenance are eliminated from the selection process. Bridge maintenance scenarios with risk values more than the acceptable risk are also not included in the selection process. Out of the remaining scenarios, the scenarios with the highest values for benefit-cost ratio (BCR), net present value (NPV), and benefit-cost index ($\beta_{B/C}$) are compared. The result of the comparison will help the decision makers to make informed decisions on the best maintenance strategy for the bridge.

3.6.3 Risk-Based Maintenance of Bridge Inventory

Bridge inventory refers to all of bridges managed by a highway agency. The bridge maintenance strategies result in a list of bridges in the inventory that are identified for maintenance. A risk-based maintenance strategy is established by utilizing available resource for efficient maintenance of the bridges based on a risk-based priority ranking. Priority ranking is used to schedule bridges most at risk for an efficient maintenance that best utilizes available resources allocated for the maintenance of bridges in the inventory. The priority ranking and resource allocation are described in the following subsections.

3.6.3.1 Priority Ranking of Bridges Identified for Maintenance

Bridge managers may have different tendencies towards the choice of the measure for prioritizing the maintenance of bridges. Maintenance priorities for bridges can be based on initial risks associated with conditions of bridges and cost-effectiveness of risk reductions associated with maintenance strategies for bridges identified for maintenance in the inventory.

The initial risk associated with the condition of a bridge before maintenance is calculated using initial risks associated with conditions of bridge elements as defined by Equations 3-47 and 3-52. The ranking of bridges for maintenance priorities starts by grouping bridges identified for maintenance into a number of risk groups. For demonstration purposes, it is assumed that bridges are categorized into high, medium, and low risk groups. Target risk levels R_H and R_M are defined to classify bridges into these risk groups. The high-risk group includes the bridges most at risk in the inventory that have initial risk values exceeding risk level R_H. The medium-risk group includes bridges with initial risk values between R_H and R_M. The low-risk group includes the remaining bridges identified for maintenance with initial risk values less than R_M.

The maintenance priority of the bridges in the different risk categories can be based on benefit-cost measures for risk reduction effectiveness of their maintenance strategies. Bridges in each risk category can be ranked based on the net present value, benefit-cost ratio, and benefit-cost reliability index of their maintenance strategies. It is assumed that the net present values (NPV_{Bridge}) for the maintenance strategies of bridges are used herein for setting the maintenance priority list of the bridges in each risk group. Similar maintenance priority lists can be created based on benefit-cost ratio (BCR_{Bridge}) and benefit-cost reliability index ($\beta_{B/C_{Bridge}}$).

Assuming that the initial risk values for a number N_B of bridges identified for maintenance result in N_h bridges in the high-risk group; N_m bridges in the medium-risk group; and N_l bridges in the low-risk group. The bridges in each risk group are ranked based on the NPV of their maintenance strategies starting with highest NPV and continue with decreasing NPVs. The ranking starts with the bridges in the high-risk group using the net present value (NPV) of the maintenance strategies for the bridges in this group. The bridge with the highest NPV in the high-risk group has the first maintenance priority and the bridge with the lowest NPV in the high-risk group has a maintenance priority number N_h. The bridge with the highest NPV in the medium-risk group has a maintenance priority number N_h+1 and the bridge with the lowest NPV in the medium-risk group has a maintenance priority number N_h+N_m. The bridge with the highest NPV in the low-risk group has a maintenance priority number N_h+N_m+1 and the bridge with the lowest NPV in the low-risk group will have the last priority number among the N_B bridges identified for maintenance. The proposed priority ranking method is illustrated in Figure 3-9.

The importance of the bridge in the highway network can also be considered in setting the priority ranking of bridges for maintenance. The bridge manager can make informed decisions regarding the maintenance of bridges in the inventory using the maintenance priority lists.

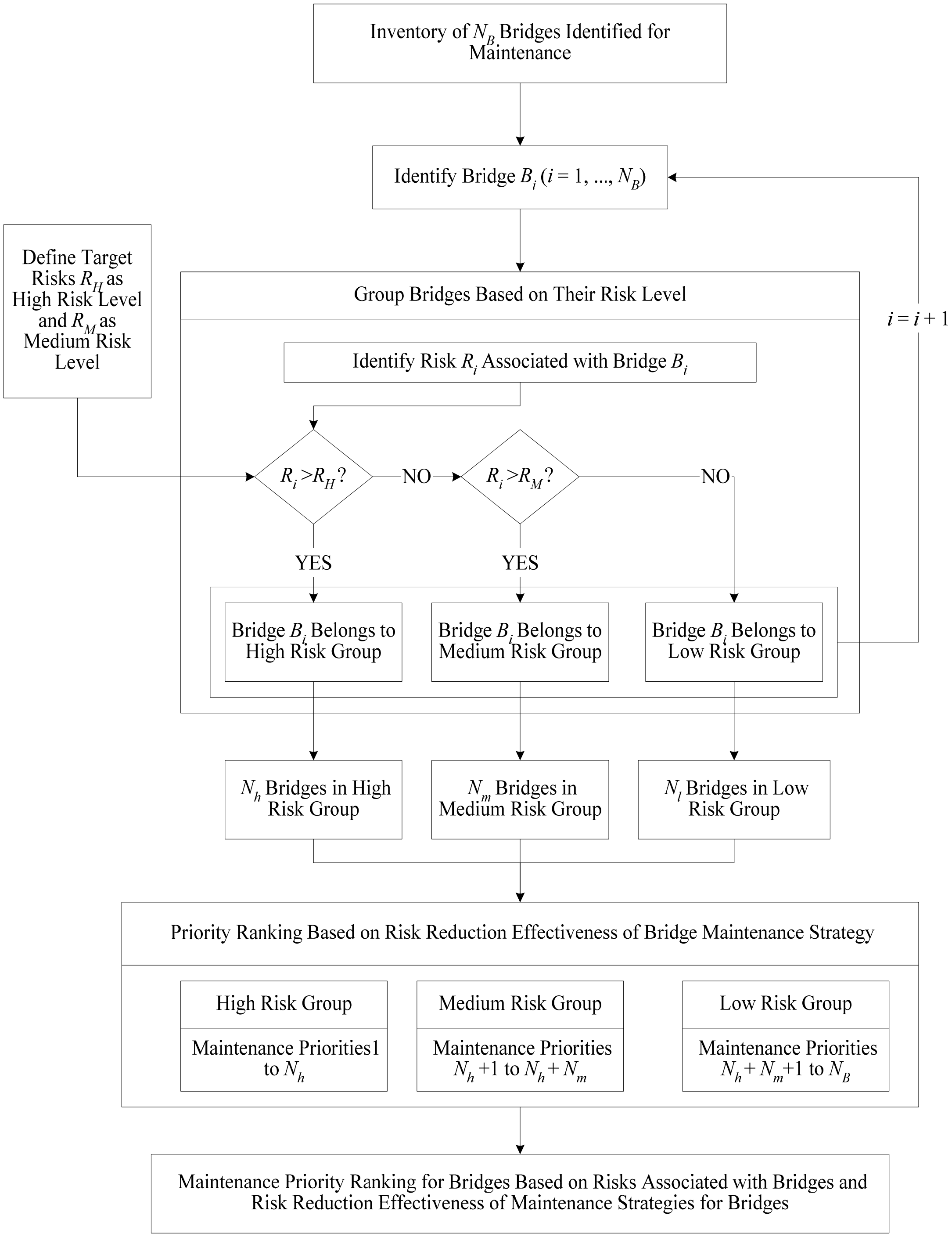


Figure 3-9. Priority Ranking for Maintenance of Bridges Based on Risk and Risk Reduction Effectiveness of Maintenance Scenarios for Bridges

3.6.3.2 Allocation of Available Resources

Highway agencies usually have limited available resources for managing the maintenance of bridges in the inventory. Bridge managers can use the priority ranking to make informed decisions for efficient allocation of available resources for maintenance of bridges. The available resources allocated for maintenance are used for bridges most at risk with cost-effective maintenance strategies.

The selection of bridges for maintenance projects follows the priority ranking order. When a bridge is selected for maintenance, the budget is reduced by the maintenance cost for that bridge. The same applies for every bridge selected for maintenance until the allocated budget for bridge maintenance is used. The allocation of available budget for maintenance of bridges is shown in Figure 3-10.

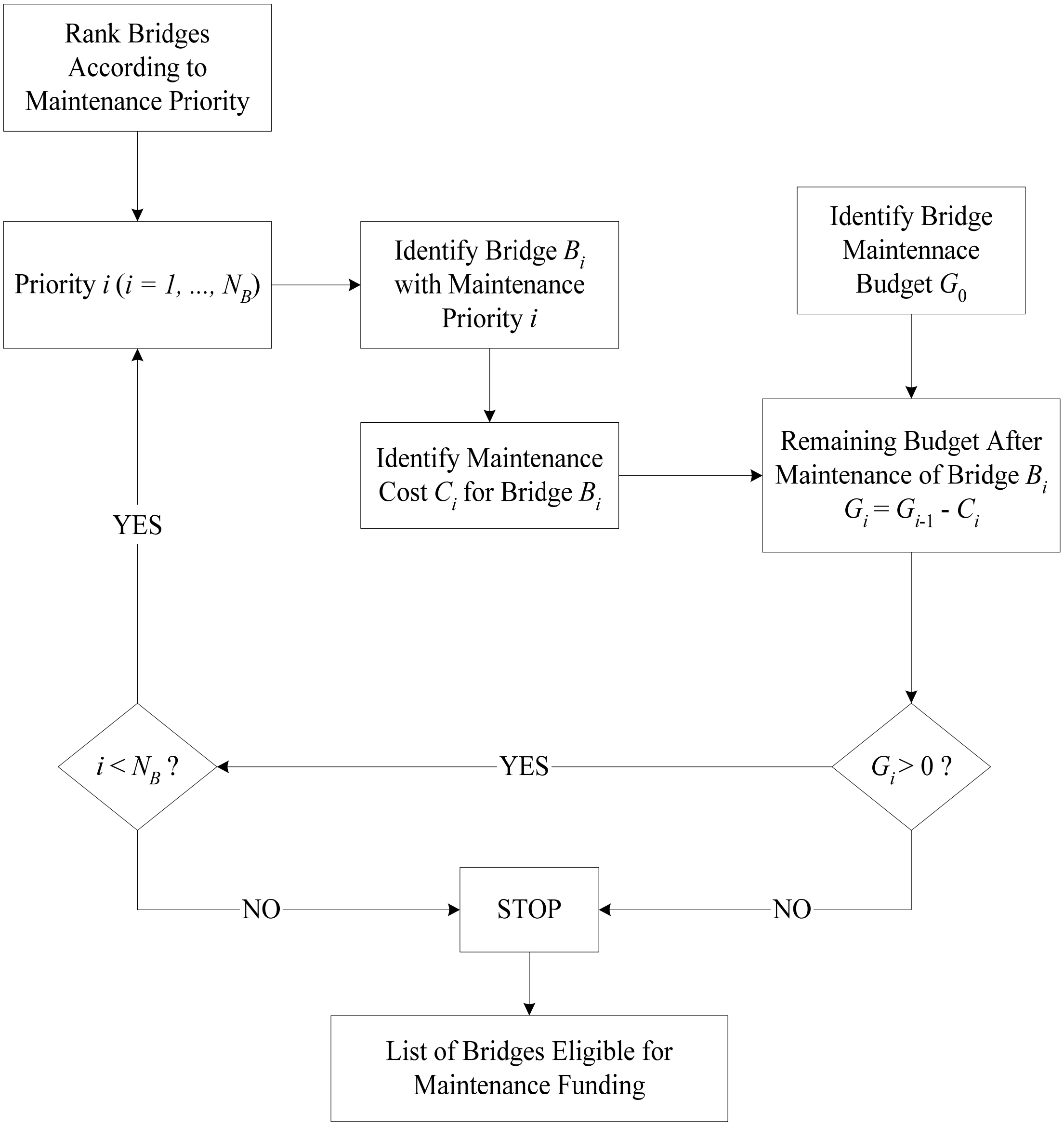


Figure 3-10. Budget Allocation for Maintenance of Bridges Based on the Priority Ranking of Bridges

Chapter 4. Case Study

4.1 Background

The methodology for risk-based bridge maintenance strategies (RBBMS), presented in Chapter 3, is demonstrated in Chapter 4 with a case study. The input required to apply the proposed methodology includes condition state distributions for bridge elements, unit costs of maintenance actions, deterioration and repair rates between the condition states of the elements, and a discount rate. A five-year planning horizon is assumed. A discount rate of 2.6% is used in the case study. This discount rate is the five years real discount rate from which the inflation premium has been removed (OMB, 2007).

4.2 Tool Used

Microsoft Excel was used for performing the needed calculations in the methodology. Spreadsheet formulas and macros using excel functions were created to develop the results in the different steps of the methodology. Examples of Excel functions used in developing the spreadsheet formulas and macros include: VLOOKUP and RANK functions for selecting the optimal actions with the minimum sum of risk and maintenance costs, SUMPRODUCT, VALUE, MID, LEN, and CONCATENATE functions for calculating the cost and risk associated with maintenance scenarios, STDEV, SQRT and SUMSQ functions for calculating standard deviations, NORMSDIST function for calculating failure probability from the reliability index, and GOAL SEEK tool for calculating the unknown parameters in the reliability index equation.

4.3 Analysis Objective

The analysis objective is to find risk-based bridge maintenance strategies that can best utilize limited resources allocated for maintenance. This objective can be achieved by performing the following steps:

1. Defining the system for risk analysis and management,
2. Modeling the element-level data needed for developing the methodology,
3. Evaluating the probabilities of failure in the condition states of the elements,
4. Estimating the failure consequences for the elements,
5. Estimating the risk associated with the condition states of the elements based on the probabilities and consequences of failure,
6. Determining the optimal maintenance actions in the condition states based on the maintenance cost and risk associated with the elements,
7. Identifying the element maintenance scenarios of implementing and/or delaying the optimal maintenance actions within the planning horizon,
8. Evaluating the cost and risk associated with the element maintenance scenarios,
9. Filtering the maintenance scenarios based on budget constraints and target risk,

10. Ranking the element maintenance scenarios based on their effectiveness of risk reduction,
11. Selecting the optimal maintenance scenario for each bridge element,
12. Selecting the best maintenance strategy for a bridge based on the available resources allocated for bridge maintenance, and
13. Prioritizing the bridges for maintenance.

These steps for achieving the analysis objective are used in developing the rest of the case study.

4.4 System Definition and Breakdown

Bridges B_1, B_2, …, B_n are assumed as the bridges identified for maintenance purposes in the case study. Each bridge is broken down into its main components. The main components are the decks, superstructures and substructures. Each main component is composed of one or more structural elements. The elements of bridges are modeled following the definition of AASHTO commonly recognized structural elements (AASHTO, 1997). The system is modeled as the set of commonly recognized elements of the bridges.

4.5 Element-Level Modeling

Bridge B_1 is selected to illustrate the risk-based methodology at the element-level. Bridge B_1 is composed mainly of five commonly recognized elements: a bare concrete deck element, a painted steel girder element, a reinforced concrete column element, a reinforced concrete cap element, and a reinforced concrete abutment element. The breakdown of Bridge B_1 into its major components and structural elements is shown in Figure 4-1.

The damage categories for the elements of bridge B_1 are damage category I for the concrete deck; damage category V for the painted steel girder; and damage category VII for the reinforced concrete column, cap, and abutment. The damage scale is measured using five condition states for the elements of damage categories I and V, and four condition states for the elements of damage category VII. The condition states are defined based on the condition state definition for the AASHTO CoRe elements.

The elements are measured using quantities and measurement units. The deck quantity is measured using the square feet (SF) of its area; the quantities of the steel girder, reinforced concrete abutment, and reinforced concrete cap are measured using the linear feet (LF) of their length; the reinforced concrete column element is measured by the number of columns so that each (EA) column is considered a unit. The quantities and measurement units for the elements of Bridge B_1 are shown in Table 4-1 (Jehelka, 1996).

The steps of the methodology in the element level will be explained for two elements of bridge B_1; the concrete deck element and the steel girder element. The other elements of bridge B_1 will be used to explain the methodology for the bridge level. The bridges in the inventory identified for maintenance will be used for setting maintenance priority ranking for bridges.

The condition of each bridge element is described by a distribution of the element quantity among a number of condition states. The quantity distributions for the concrete deck and steel girder elements are defined using assumed estimates by five bridge inspectors as shown in Tables 4-2 and 4-3, respectively. Assuming equal weights for the five bridge inspectors, the mean μ_{q_j} and standard deviation σ_{q_j} for the element quantity q_j in a particular condition state j are calculated as shown in Equations 4-1 and 4-2.

$$\mu_{q_j} = \frac{\sum_{i=1}^{5} q_{i,j}}{5} \tag{4-1}$$

$$\sigma_{q_j} = \sqrt{\frac{\sum_{i=1}^{5} (q_{i,j} - \mu_{q_j})^2}{4}} \tag{4-2}$$

where $q_{i,j}$ is the element quantity in condition state j inspected by bridge inspector i.

The elements are assumed to deteriorate one condition state at a time when no maintenance actions are applied based on the Markov chain process as modeled in

Pontis bridge management system (Golabi, et al. 1993). Mean and standard deviation values are assumed for the deterioration rates $\lambda_{i,i+1}$ from condition states i to condition states i+1 of the elements. $\lambda_{i,i+1}$ represent the transition probabilities $P_{i,i+1}$ between condition states i and i+1 of the elements. Transition probabilities $P_{i,i}$ represent the remaining percentages of the element in condition states i after deterioration. Transition probabilities $P_{i,j}$ from condition state i to any condition state other than i or i+1 are assumed as zero. Tables 4-4 and 4-5 show the assumed means and standard deviations for the deterioration rates and the transition probabilities of the concrete deck and steel girder elements of bridge B_1. The standard deviations for $P_{i,i}$ and $P_{i,i+1}$ are equal to the standard deviation for $\lambda_{i,i+1}$.

The feasible maintenance actions for the concrete deck element and the steel girder element are assumed according to definition in Pontis element-level data as shown in Tables 4-6 and 4-7. Means and standard deviations are calculated based on assumed values for the unit cost of each feasible action by five experts. The repair rates $\mu_{i,j}$ from condition states i to better condition states j as a result of applying the maintenance actions for the concrete deck and the steel girder are assumed as shown in Tables 4-8 and 4-9. These repair rates are assumed without considering the deterioration of the element in the condition states. When the deterioration rates are combined with the repair rates, the transition probabilities due to the maintenance actions are calculated for the concrete deck element and the steel girder element as shown in Tables 4-10 and 4-11.

The above-mentioned data are needed for developing the risk-based maintenance strategies for bridge elements. The transition probabilities are used for calculating the condition state distribution over the years of the planning horizon. The unit costs of the maintenance actions and the quantity distribution of the element in any particular year can be used to calculate the maintenance cost of the element in that year. The condition state distribution of the element is also used for evaluating the risks associated with the element conditions during the maintenance planning horizon.

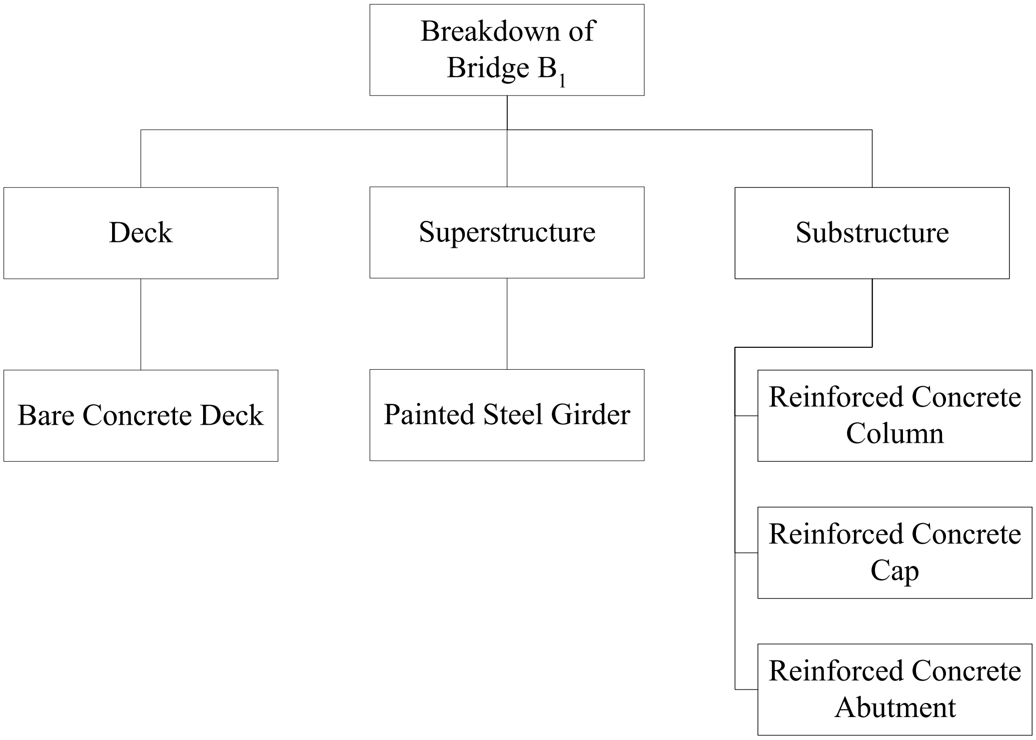


Figure 4-1. Break Down of Bridge B_1 in the Case Study into Elements

Table 4-1. Elements of Bridge B_1 Used in the Case Study

Element ID	Element Description	Quantity	Unit
12	Bare Concrete Deck	14,080	Square Foot (SF)
107	Painted Steel Open Girder	2,240	Linear Foot (LF)
205	Reinforced Concrete Column	21	Each (EA)
215	Reinforced Concrete Abutment	97	Linear Foot (LF)
234	Reinforced Concrete Cap	309	Linear Foot (LF)

Table 4-2. Quantity Distributions in Square Feet (SM) of the Concrete Deck Element Estimated by Five Bridge Inspectors

Bridge Inspector	Condition State				
	1	2	3	4	5
1	1,450	12,000	750	600	70
2	1,350	10,000	650	700	75
3	1,400	10,500	640	650	65
4	1,300	11,500	700	650	80
5	1,500	12,500	660	550	60
Mean	1,400	11,300	680	630	70
Standard Deviation	79.06	1,036.82	45.28	57.01	7.91

Table 4-3. Quantity Distributions in Linear Feet (LF) of the Steel Girder Elements Estimated by Five Bridge Inspectors

Bridge Inspector	Condition State				
	1	2	3	4	5
1	550	1,300	300	120	35
2	600	1,400	250	80	45
3	650	1,100	350	100	50
4	700	1,050	320	110	30
5	500	1,150	280	90	40
Mean	600	1,200	300	100	40
Standard Deviation	79.06	145.77	38.08	15.81	7.91

Table 4-4. Deterioration Rates of Concrete Deck Element and Steel Girder Element of Bridge B_1

Element	Statistical Parameter	Deterioration Rate ($\lambda_{i,i+1}$)			
		$\lambda_{1,2}$	$\lambda_{2,3}$	$\lambda_{3,4}$	$\lambda_{4,5}$
12	Mean	5%	10%	15%	20%
	Standard Deviation	0.2%	0.5%	1.0%	1.5%
107	Mean	3%	6%	9%	12%
	Standard Deviation	0.1%	0.3%	0.5%	0.7%

Table 4-5. Deterioration Transition Probabilities $P_{i,i}$ and $P_{i,i+1}$ for Concrete Deck Element and Steel Girder Element of Bridge B_1

Element	Statistical Parameter	Condition State								
		1		2		3		4		5
		P_{11}	P_{12}	P_{22}	P_{23}	P_{33}	P_{34}	P_{44}	P_{45}	P_{55}
12	Mean	95%	5%	90%	10%	85%	15%	80%	20%	100%
	Standard Deviation	0.2%	0.2%	0.5%	0.5%	1.0%	1.0%	1.5%	1.5%	0%
107	Mean	97%	3%	94%	6%	91%	9%	88%	12%	100%
	Standard Deviation	0.1%	0.1%	0.3%	0.3%	0.5%	0.5%	0.7%	0.7%	0%

Table 4-6. Feasible Maintenance Actions for Concrete Deck Element of Bridge B_1

Condition State	Maintenance Action		Unit Cost ($/SF)	
			Mean	Standard Deviation
1	$a(1,0)$	Do Nothing	0.00	0.00
	$a(1,1)$	Add a protective system	9.00	0.71
2	$a(2,0)$	Do Nothing	0.00	0.00
	$a(2,1)$	Repair spalls and delaminations	5.00	0.71
	$a(2,2)$	Add a protective system	10.00	0.95
3	$a(3,0)$	Do Nothing	0.00	0.00
	$a(3,1)$	Repair spalls and delaminations	6.00	0.71
	$a(3,2)$	Repair spalls and delaminations and add a protective system	12.00	1.30
4	$a(4,0)$	Do Nothing	0.00	0.00
	$a(4,1)$	Repair spalls and delaminations	7.00	0.71
	$a(4,2)$	Repair spalls and delaminations and add a protective system	15.00	1.22
5	$a(5,0)$	Do Nothing	0.00	0.00
	$a(5,1)$	Repair spalls and delaminations and add a protective system	20.00	2.02
	$a(5,2)$	Replace deck	30.00	3.41

Table 4-7. Feasible Maintenance Actions for Painted Steel Girder Element of Bridge B_1

Condition State	Maintenance Action		Unit Cost ($/SF)	
			Mean	Standard Deviation
1	$a(1,0)$	Do Nothing	0.00	0.00
	$a(1,1)$	Surface clean	10.00	1.41
2	$a(2,0)$	Do Nothing	0.00	0.00
	$a(2,1)$	Surface clean	15.00	2.00
	$a(2,2)$	Clean & paint	40.00	3.58
3	$a(3,0)$	Do Nothing	0.00	0.00
	$a(3,1)$	Spot blast, clean & paint	55.00	7.07
4	$a(4,0)$	Do Nothing	0.00	0.00
	$a(4,1)$	Spot blast, clean & paint	65.00	8.94
	$a(4,2)$	Replace paint system	75.00	7.07
5	$a(5,0)$	Do Nothing	0.00	0.00
	$a(5,1)$	Major rehab unit	200.00	34.06
	$a(5,2)$	Replace unit	350.00	54.86

Table 4-8. Repair Rates for Concrete Deck Element of Bridge B_1

Condition State	Action	Repair Rate	Mean	Standard Deviation
CS1	*a*(1,1)	μ_{21}	5%	1%
CS2	*a*(2,1)	μ_{21}	90%	5%
		μ_{32}	10%	2%
	a(2,2)	μ_{21}	100%	5%
		μ_{32}	10%	2%
CS3	*a*(3,1)	μ_{31}	80%	4%
		μ_{32}	10%	2%
		μ_{43}	15%	3%
	a(3,2)	μ_{31}	100%	5%
		μ_{43}	15%	3%
CS4	*a*(4,1)	μ_{41}	70%	4%
		μ_{42}	10%	2%
		μ_{43}	10%	2%
		μ_{54}	20%	3%
	a(4,2)	μ_{41}	100%	5%
		μ_{54}	20%	3%
CS5	*a*(5,1)	μ_{51}	95%	5%
		μ_{52}	5%	1%
	a(5,2)	μ_{51}	100%	5%

Table 4-9. Repair Rates of Steel Girder Element of Bridge B_1

Condition State	Maintenance Action	Repair Rate	Mean	Standard Deviation
CS1	*a*(1,1)	μ_{21}	3%	0.2%
CS2	*a*(2,1)	μ_{21}	10%	1.0%
		μ_{32}	6%	0.5%
	a(2,2)	μ_{21}	95%	5.0%
		μ_{32}	6%	0.5%
CS3	*a*(3,1)	μ_{31}	90%	4.0%
		μ_{32}	10%	0.8%
		μ_{43}	9%	0.6%
CS4	*a*(4,1)	μ_{41}	40%	2.0%
		μ_{42}	20%	1.5%
		μ_{43}	10%	0.5%
		μ_{54}	12%	1.0%
	a(4,2)	μ_{41}	90%	5.0%
		μ_{42}	5%	0.2%
		μ_{43}	5%	0.2%
		μ_{54}	12%	1.0%
CS5	*a*(5,1)	μ_{51}	40%	3.0%
		μ_{52}	30%	1.5%
		μ_{53}	20%	1.0%
		μ_{54}	10%	1.0%
	a(5,2)	μ_{51}	100%	5.0%

Table 4-10. Repair Transition Probabilities for Concrete Deck Element of Bridge B_1

Condition State	Maintenance Action	Statistical Parameter	Transition Condition State				
			CS1	CS2	CS3	CS4	CS5
CS1	$a(1,1)$	Mean	100%	0%			
		Standard Deviation	1.0%	1.0%			
CS2	$a(2,1)$	Mean	90%	10%	0%		
		Standard Deviation	5.0%	5.4%	2.1%		
	$a(2,2)$	Mean	100%	0%	0%		
		Standard Deviation	5.0%	5.4%	2.1%		
CS3	$a(3,1)$	Mean	80%	10%	10%	0%	
		Standard Deviation	4.0%	2.0%	5.5%	3.2%	
	$a(3,2)$	Mean	100%	0%	0%	0%	
		Standard Deviation	5.0%	0.0%	5.9%	3.2%	
CS4	$a(4,1)$	Mean	70%	10%	10%	10%	0%
		Standard Deviation	4.0%	2.0%	2.0%	5.9%	3.4%
	$a(4,2)$	Mean	100%	0%	0%	0%	0%
		Standard Deviation	5.0%	0.0%	0.0%	6.0%	3.4%
CS5	$a(5,1)$	Mean	95%	5%	0%	0%	0%
		Standard Deviation	5.0%	1.0%	0.0%	0.0%	5.1%
	$a(5,2)$	Mean	100%	0%	0%	0%	0%
		Standard Deviation	5.0%	0.0%	0.0%	0.0%	5.0%

Table 4-11. Repair Transition Probabilities for Steel Girder Element of Bridge B_1

Condition State	Maintenance Action	Statistical Parameter	Transition Condition State				
			CS1	CS2	CS3	CS4	CS5
CS1	*a*(1,1)	Mean	100%	0%			
		Standard Deviation	0.2%	0.2%			
CS2	*a*(2,1)	Mean	10%	90%	0%		
		Standard Deviation	1.0%	1.2%	0.6%		
	a(2,2)	Mean	95%	5%	0%		
		Standard Deviation	5.0%	5.0%	0.6%		
CS3	*a*(3,1)	Mean	90%	10%	0%	0%	
		Standard Deviation	4.0%	0.8%	4.2%	0.8%	
	a(3,2)	Mean	40%	20%	10%	30%	0%
		Standard Deviation	2.0%	1.5%	0.5%	2.8%	1.2%
CS4	*a*(4,1)	Mean	90%	5%	5%	0%	0%
		Standard Deviation	5.0%	0.2%	0.2%	5.2%	1.2%
	a(4,2)	Mean	40%	30%	20%	10%	0%
		Standard Deviation	3.0%	1.5%	1.0%	1.0%	3.6%
CS5	*a*(5,1)	Mean	100%	0%	0%	0%	0%
		Standard Deviation	5.0%	0.0%	0.0%	0.0%	5.0%
	a(5,2)	Mean	100%	0%			
		Standard Deviation	0.2%	0.2%			

4.6 Failure Probabilities in Condition States of Bridge Elements

For demonstration purposes, only flexural and shear failure modes are used for calculating the failure probabilities in the case study. The steps described in Chapter 3 will be followed to calculate the failure probabilities in the condition states of the Element 107 that defines a painted steel girder and Element 12 that defines a bare concrete deck. Section loss of the steel girder element and cracking of the concrete deck element can be mapped to the condition states of these elements (Estes, et al. 2003a).

4.6.1 Failure Probabilities for Steel Girder Element

The probabilities of failure in the five condition states of the painted steel girder are calculated in flexural and shear failure modes. The cross-sectional dimensions of the steel girder element are shown in Figure 4-2. The steel yield stress F_y is assumed as 50 *ksi*. The cross sectional area and the plastic section modulus for the girder element are shown in Table 4-12.

The failure probabilities are calculated in the five condition states of the element based on percentages of section loss corresponding to the condition states. The section loss is assumed to increase by worsening the condition state of the element. An average section loss of 1%, 5%, 10%, 15%, and 20% are assumed in Condition States 1, 2, 3, 4, and 5, respectively. The assumed percentages of section loss (Δ) in the different condition states of the element and the dimensions of the steel section are used to calculate the reduction in thickness (δ) in the surface of the steel element as shown in Equation 4-3.

$$\Delta = \frac{2\delta(d_{w0} - t_{w0} + 2b_{f0} + 2t_{f0}) - 4\delta^2}{d_{w0}t_{w0} + 2b_{f0}t_{f0}} \tag{4-3}$$

where

Δ = percentage of section loss in the element,

δ = reduction in thickness of the element surface,

d_{w0}, t_{w0} = web depth and thickness, and

b_{f0}, t_{f0} = flange width and thickness

For a given section loss Δ of a steel element with web and flange dimensions, the reduction in thickness δ due to corrosion can be calculated assuming equal reduction around the whole perimeter of the section. The corrosion depth is used to calculate the reduced dimensions of the element in the different condition states.

The design condition is assumed to have no section loss as shown in Figure 4-3. The reduced dimensions of the element in condition state 1, 2, 3, 4, and 5 corresponding to 1%, 5%, 10%, 15%, and 20% section loss are illustrated in Figures 4-4, 4-5, 4-6, 4-7 and 4-8 respectively. A summary of the reduced cross-sectional dimensions of the element in the five condition states is shown in Table 4-13.

The probability of failure is calculated based on the change in resistance of the element in the five condition states. In flexure failure mode, the resistance depends on the plastic section modulus, while in shear failure mode the resistance depends on the web area. The reduced plastic section modulus Z and web area A_w of the element are calculated as shown in Equations 4-4 and 4-5. The reduced section modulus and cross sectional areas of the web and the flanges in the five condition states of the element are shown in Table 4-14.

$$Z = 2(t_w d_w^2/8 + b_f t_f(d-t_f)/2 + t_w \delta(d_w+\delta)/2) \qquad (4\text{-}4)$$

$$A_w = t_w d_w \qquad (4\text{-}5)$$

The steps for calculating the failure probabilities due to flexural failure mode are:

1. Calculating the nominal moment resistance of the element with no section loss,
2. Calculating the reduced moment resistance of the element in the condition states based on the reduced section modulus,
3. Calculating the moment load effect based on a target reliability index of 3.5 and the nominal moment resistance calculated in step 1,
4. Calculating the mean and standard deviation of the moment load effect,
5. Calculating the mean and standard deviation for the element moment resistance in the condition states,
6. Calculating the reliability index in the condition states of the element, and

7. Calculating the probability of failure in the condition states of the element due to flexural failure mode.

The nominal moment resistance M_n is calculated in the design state with no section loss and in the five condition states of the element based on the section modulus Z and the steel yield stress F_y as shown in Equation 4-6 (LRFD, A-F1-1).

$$M_n = F_y Z \tag{4-6}$$

The mean and standard deviation for the moment resistance of the element are calculated using the bias factor λ_R and the coefficient of variation V_R for the moment resistance (Nowak, 1999). The bias factor and coefficient of variation for the moment resistance of the steel girder are assumed as 1.12 and 0.10, respectively (Nowak, 1999). The calculation of the nominal moment resistance in the five condition states of element 107 is shown in Table 4-15.

The moment load effect on element 107 is assumed as a combination of the dead load and the live and impact load. The components of the moment load effect are assumed as unknown. The live and impact loads together are assumed as three times the dead load on the element. The bias factor and coefficient of variation for the dead load are assumed as 1.03 and 0.08, respectively (Nowak, 1999). The bias factor and coefficient of variation for both the live and impact loads are assumed as 1.2 and 0.18, respectively (Nowak, 1999). The mean and standard deviation for the dead load and the live and impact loads can be expressed in terms of a load variable (*l*) using Equations 3-19, 3-20, and 3-22. Using Equations 3-23 and 3-24, the mean and standard deviation for the moment load effect can be expressed in terms of the load variable (*l*) as shown in Equations 4-7 and 4-8.

$$\mu_L = [\lambda_{DL}(1+2V_{DL})](\lambda_{DL} l) + [\lambda_{LL+I}(1+2V_{LL+I})][\lambda_{LL+I}(3l)] \tag{4-7}$$

$$\sigma_L = \sqrt{[\lambda_{DL}(1+2V_{DL})]^2(V_{DL}\lambda_{DL} l)^2 + [\lambda_{LL+I}(1+2V_{LL+I})]^2[(V_{LL+I}\lambda_{LL+I}(3l)]^2} \tag{4-8}$$

The load variable (*l*) can be calculated by applying Equation 3-25 using a target reliability index of 3.5 and the calculated values for the mean and standard deviation of element moment resistance with no section loss. The calculation of the moment load effect for element 107 is shown in Table 4-16.

The moment load effect for a particular year is assumed to be the same in the five condition states of the element. The mean and standard deviation for the moment load effect and reduced moment resistance are used to calculate the reliability index, defined by Equation 3-28, in the five condition states of the element as shown in Table 4-17. The results show that the reliability index β for the element in condition states 1, 2, 3, 4 and 5 corresponding to section losses of 1%, 5%, 10%, 15% and 20% are 3.44, 3.17, 2.82, 2.46, and 2.07, respectively.

The probabilities of flexural failure in the five condition states of the steel girder element are calculated using Equation 3-30 and compared to the target probability of failure with no section loss as shown in Table 4-18. The results show that a 1% section loss of the element in condition state 1 results in a probability of failure of 1.3 times the target probability of failure while a 20% section loss of the element in condition state 5 results in a probability of failure of 82 times the target probability of failure.

Similar steps are used for calculating the failure probabilities due to shear failure mode. The nominal shear resistance V_n of the element in the five condition states is calculated based on the web area A_w and the steel yield stress F_y as shown in Equation 4-9 (LRFD, A-F2-1).

$$V_n = 0.6F_yA_w \tag{4-9}$$

The mean and standard deviation for the nominal shear resistance of the element are calculated using the bias factor and the coefficient of variation for the moment resistance (Nowak, 1999). The bias factor and coefficient of variation for the shear resistance of the steel girder are assumed to be equal to 1.14 and 0.105 respectively (Nowak, 1999). The calculation of the nominal shear resistance in the five condition states of element 107 is shown in Table 4-19.

The shear load effect on element 107 is assumed similar to the moment load effect as a combination of the dead load and the live and impact load. The mean and standard deviation for the dead load and live and impact load can be expressed in terms of a load variable (*l*). The load variable (*l*) can be calculated by applying Equation 3-13 using a target reliability index of 3.5 and the calculated values for the mean and standard deviation of element shear resistance with no section loss. The calculation of the shear load effect for element 107 is shown in Table 4-20.

The shear load effect for a particular year is assumed to be the same in the five condition states of the element. The mean and standard deviation for the shear load effect and reduced shear resistance are used in Equation 3-28 to calculate the reliability index in the five condition states of the element as shown in Table 4-21. The results show that the reliability indices for the element in condition states 1, 2, 3, 4 and 5 corresponding to section losses of 1%, 5%, 10%, 15% and 20% are 3.43, 3.12, 2.72, 2.29, and 1.83, respectively.

The probability of failure in shear is calculated in the five condition states of the element using Equation 3-30. The calculated values are compared to the target probability of failure with no section loss. Table 4-22 shows a summary calculation for the probabilities of shear failure in the five condition states of the steel girder element. The results show that a 1% section loss of the element in condition state 1 results in a probability of its failure of 1.3 times the target probability of failure while a 20% section loss of the element in condition state 5 results in a probability of its failure close to 144 times the target probability of failure. The probabilities of failure due to flexural and shear failure modes in the condition states of the painted steel girder element are shown in Table 4-23.

The probability of failure of the steel girder element for future years is calculated based on predicted element conditions and loading. This involves the prediction of the maximum moment and shear load effects on the element during the planning horizon.

For any year in the planning horizon, the cumulative distribution function (CDF) of the maximum moment and shear load effects can be derived from the cumulative distribution function of the initial load effects using Equation 3-32 (Ang and Tang, 1984).

The distributions of the initial moment and shear load effects on the steel girder element are assumed as normal. The moment and shear load effects due to the deal load component are assumed to be constant. The maximum moment and shear load effects due to the live and impact load components are assumed to increase based on Equation 3-32.

The CDFs of the initial moment and shear load effects are generated using the means and standard deviations of the initial load effects derived earlier from the reliability index equation (Tables 4-16 and 4-20). The CDFs of the maximum moment and shear load effects for two, three, four, and five years are generated from the CDFs of the initial moment and shear load effects, respectively, based on Equation 3-32. The CDFs for the maximum moment and shear load effects due to live and impact load components are shown in Figures 4-9 and 4-10, respectively.

The probability density functions (PDFs) of the maximum moment and shear load effects are generated from the CDFs of the maximum load effects using numerical differentiation. The PDFs for the maximum moment and shear load effects due to live and impact load components are shown in Figures 4-11 and 4-12, respectively.

The means and standard deviations for the maximum moment and shear load effects for two, three, four, and five years are calculated using numerical integration. The first moments of the PDFs around the origin are used to calculate the means of the maximum moment and shear load effects due to live and impact load components. Figure 4-13 and 4-14 show the means of the maximum moment and shear load effects, respectively, for a planning horizon of twenty years. The second moments of the PDFs around the mean are used to calculate the standard deviations of the maximum moment and shear load effects due to live and impact load components.

The calculated means and standard deviations for the maximum load effects due to live and impact load components are combined with the means and standard deviations of the dead load component to calculate the means and standard deviations for the total moment and shear load effects on the steel girder element as shown in

Tables 4-24 and 4-25. The increased load effects in flexure and shear over the years result in reducing the reliability in the condition states of the steel girder element, and therefore, increase the probabilities of failure. Tables 4-24 and 4-25 illustrate the calculation of the reliability index in flexure and shear in the condition states of the steel girder element for five years. The probabilities of flexure and shear failure in the condition states of the element over five years of maintenance are shown in Tables 4-26.

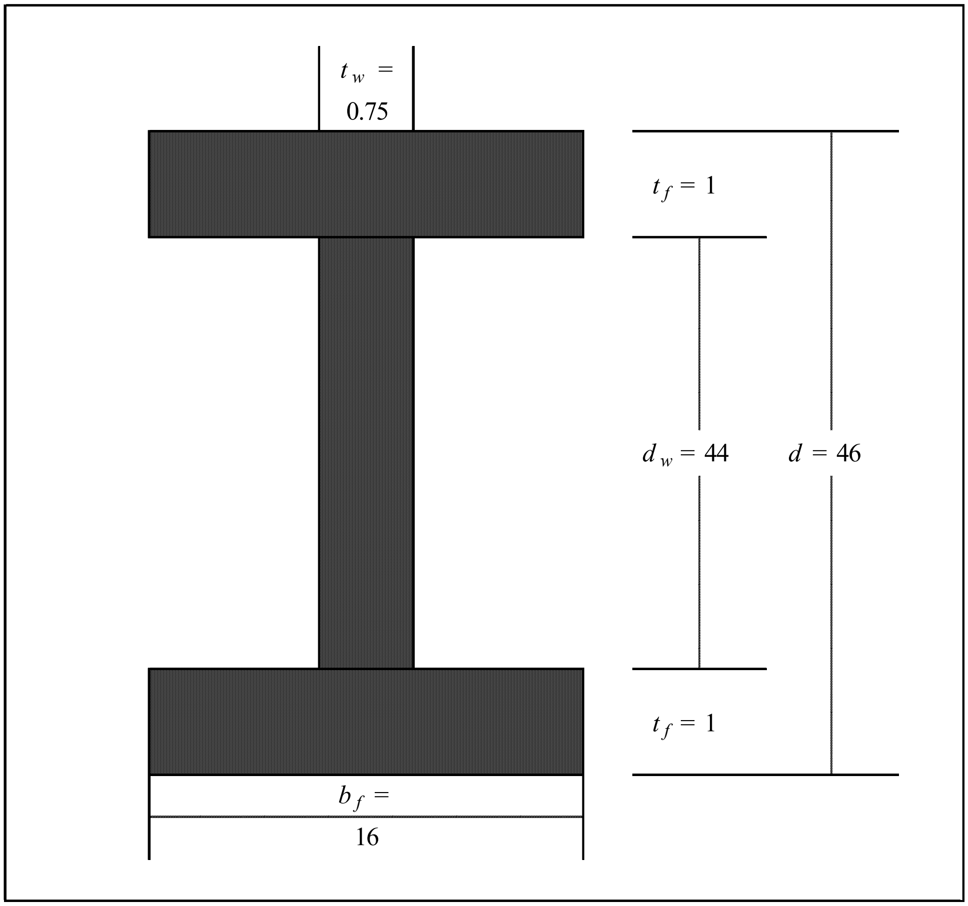


Figure 4-2. Cross Sectional Dimensions (inch) for Painted Steel Girder Element of Bridge B_1.

Table 4-12. Section Properties for Steel Girder Element of Bridge B_1.

Cross-Sectional Area, inch^2		Plastic Section Modulus (Z), inch^3
Web Area (A_w), inch^2	33	1,083
Flange Area (A_w), inch^2	16	
Total Area (A_w), inch^2	65	

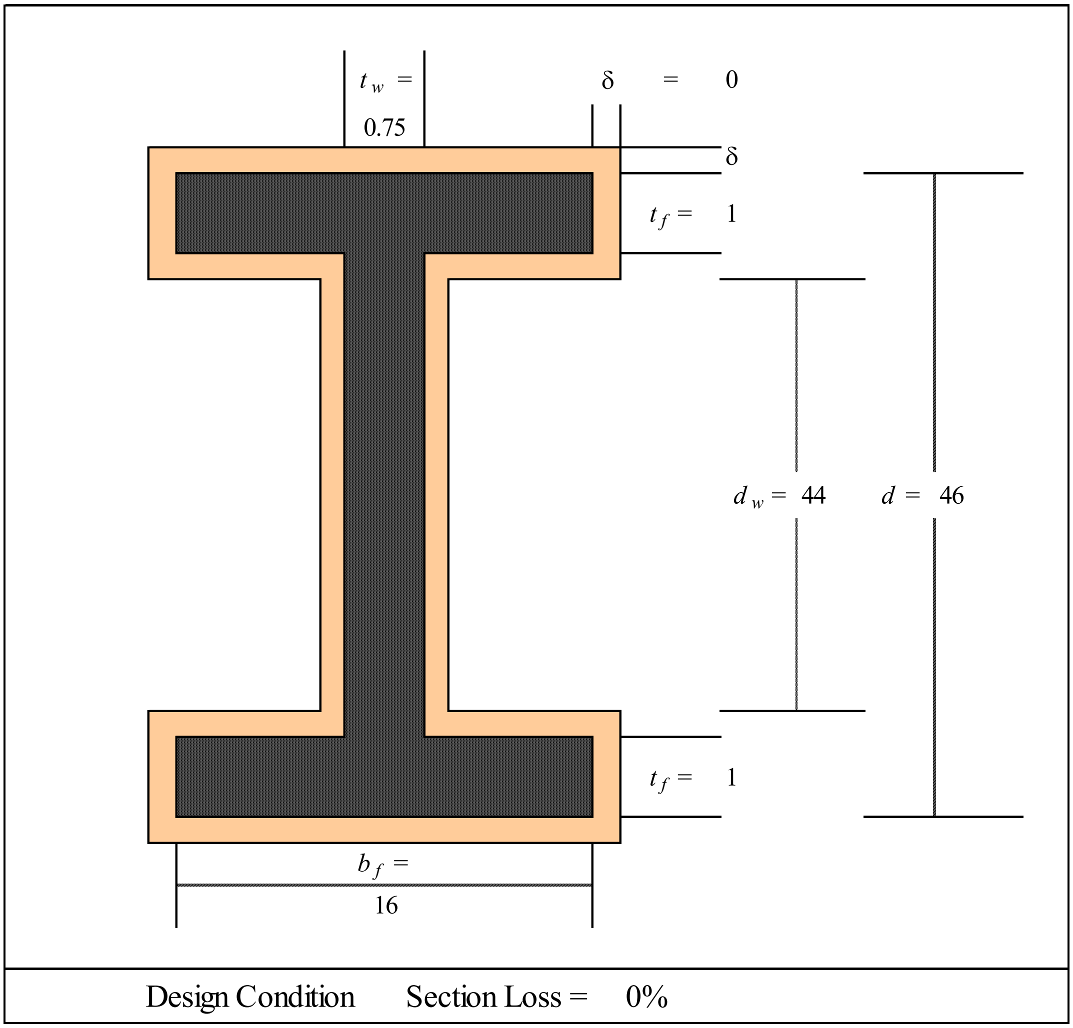


Figure 4-3. Dimensions of Steel Girder Cross-Section with No Section Loss

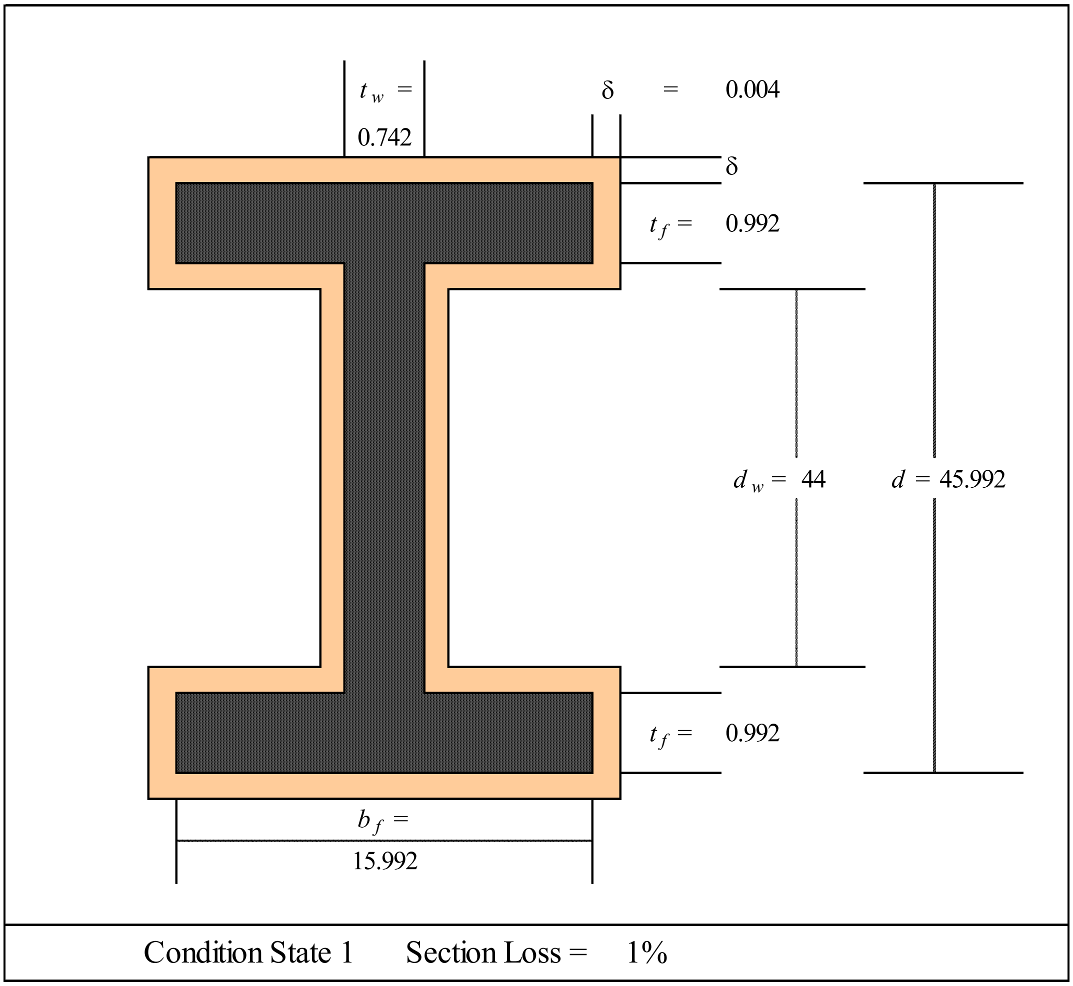


Figure 4-4. Dimensions of Steel Girder Cross-Section in Condition State 1

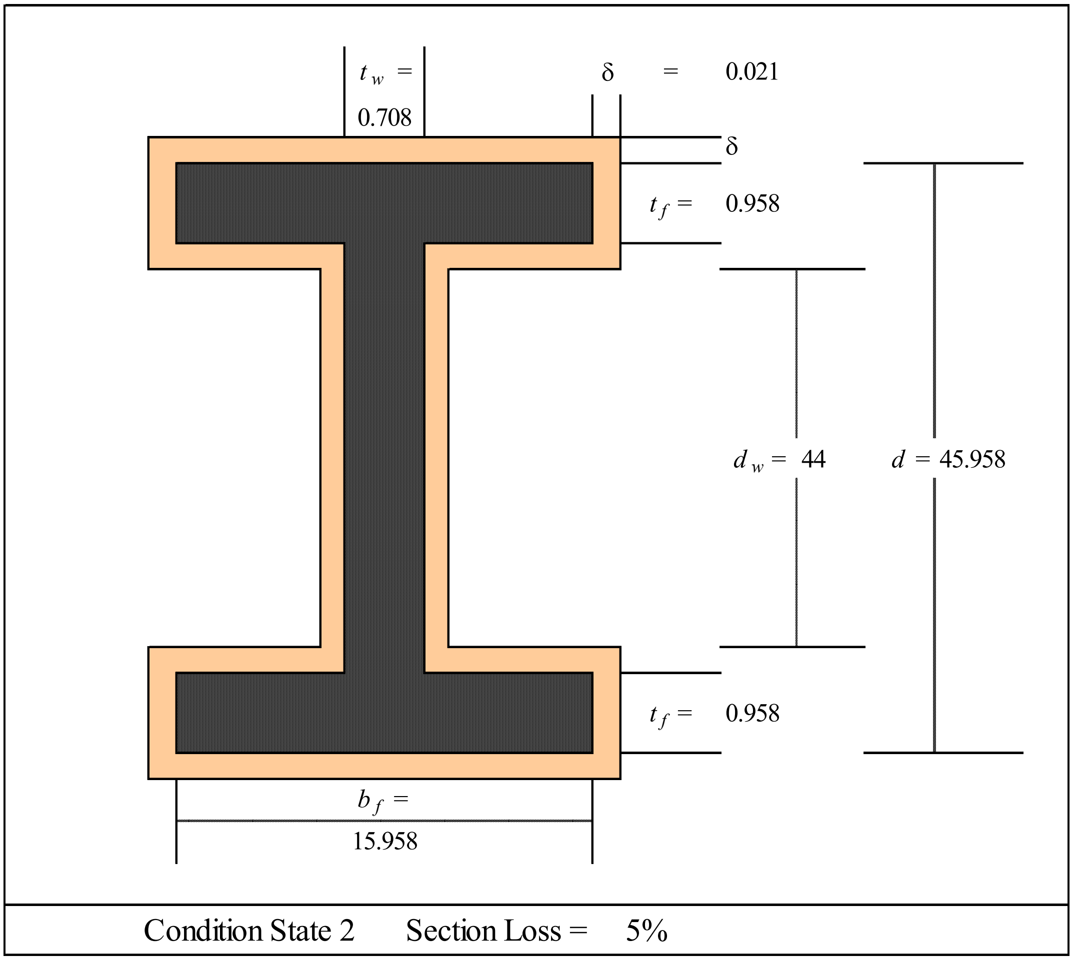


Figure 4-5. Dimensions of Steel Girder Cross-Section in Condition State 2

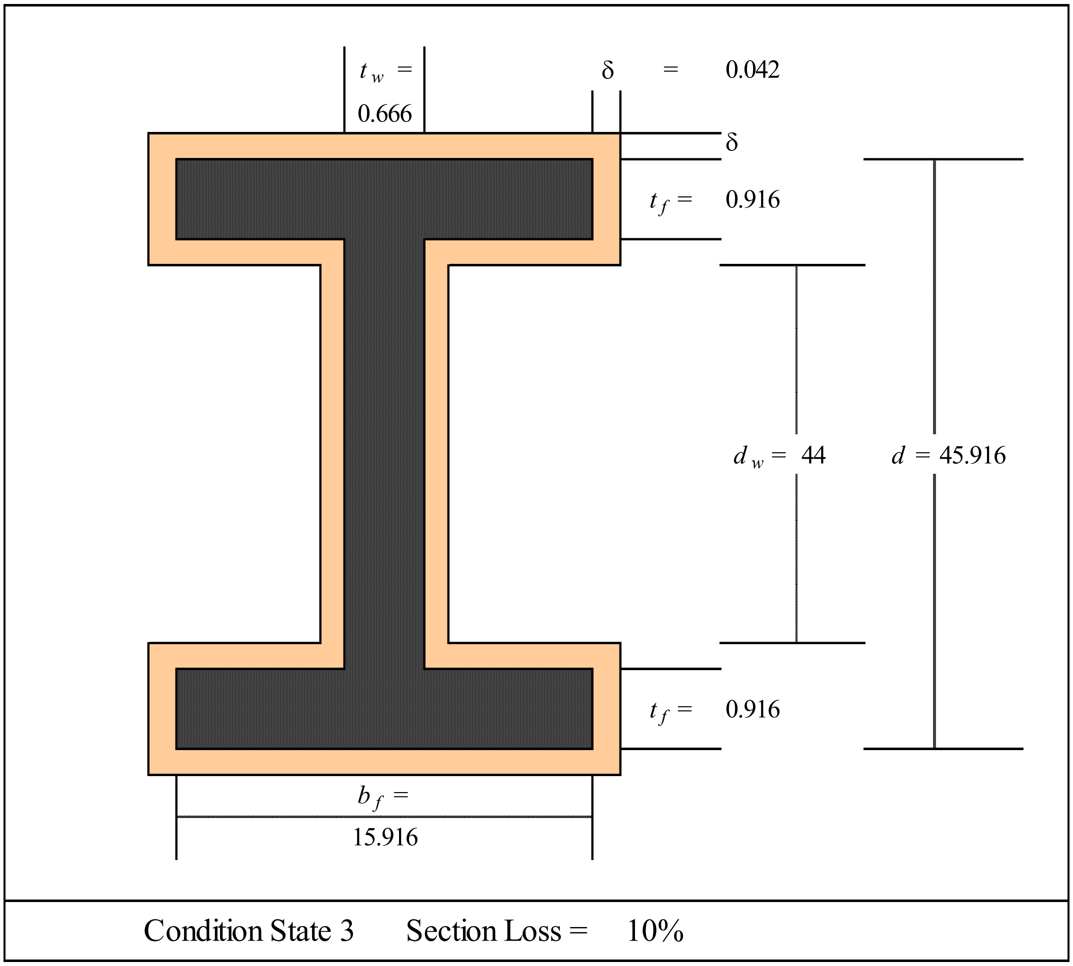


Figure 4-6. Dimensions of Steel Girder Cross-Section in Condition State 3

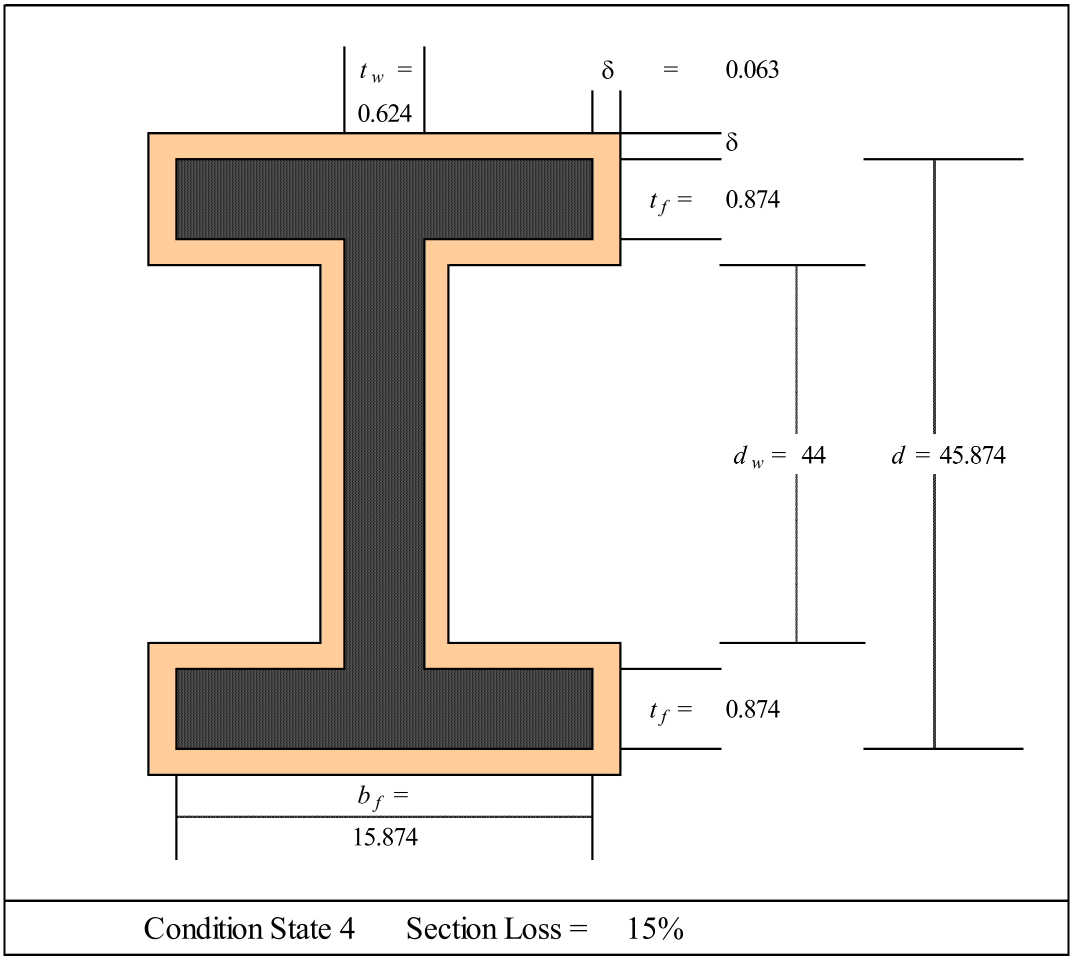


Figure 4-7. Dimensions of Steel Girder Cross-Section in Condition State 4

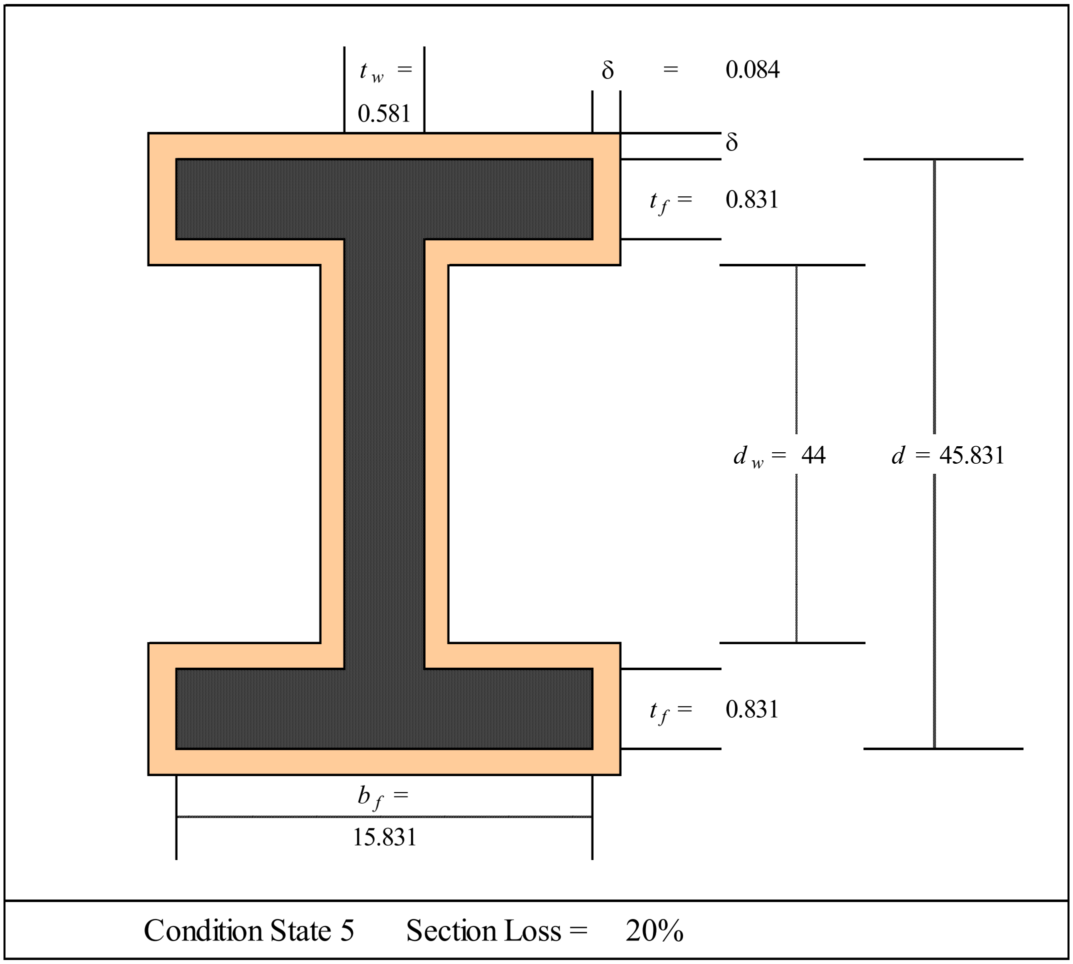


Figure 4-8. Dimensions of Element Cross-Section in Condition State 5

Table 4-13. Reduced Cross-Sectional Dimensions in Condition States of Painted Steel Girder Element

Condition State	% Section Loss	Corrosion Depth (inch)	Reduced Section Dimensions				
			Web Depth (inch)	Web Thickness (inch)	Flange Depth (inch)	Flange Thickness (inch)	Total Depth (inch)
	Δ	δ	d_w	t_w	t_f	b_f	d
Design	0%	0	44	0.75	1	16	46
CS1	1%	0.004	44	0.742	0.992	15.992	45.992
CS2	5%	0.021	44	0.708	0.958	15.958	45.958
CS3	10%	0.042	44	0.666	0.916	15.916	45.916
CS4	15%	0.063	44	0.624	0.874	15.874	45.874
CS5	20%	0.084	44	0.581	0.831	15.831	45.831

Table 4-14. Section Properties in Condition States of Painted Steel Girder Element

Condition State	% Section Loss	Corrosion Depth in inch	Reduced Sectional Areas			Reduced Sectional Modulus (inch^3)
			Web Area (inch^2)	Flange Area (inch^2)	Total Area (inch^2)	
	Δ	δ	A_w	A_f	A	Z
Design	0%	0	33	16	65	1083
CS1	1%	0.004	32.63	15.86	64.35	1072.63
CS2	5%	0.021	31.15	15.3	61.75	1031.16
CS3	10%	0.042	29.29	14.6	58.5	979.35
CS4	15%	0.063	27.44	13.91	55.25	927.57
CS5	20%	0.084	25.58	13.21	52.00	875.79

Table 4-15. Calculation of Nominal Moment Resistance in Condition States of Steel Girder Element

Condition State	Section Loss (%)	Section Modulus, inch^3	Fy, ksi	λ_R	V_R
			50	1.12	0.1
			Nominal Moment Resistance, ft-kips		
	Δ	Z	M_n	μ_R	σ_R
Design	0%	1,083.0	4,512.5	5,054.0	505.4
CS1	1%	1,072.6	4,469.3	5,005.6	500.6
CS2	5%	1,031.2	4,296.5	4,812.1	481.2
CS3	10%	979.4	4,080.6	4,570.3	457.0
CS4	15%	927.6	3,864.9	4,328.6	432.9
CS5	20%	875.8	3,649.1	4,087.0	408.7

Table 4-16. Calculation of Moment Load Effects for Steel Girder Element

Moment Resistance		Moment Load Effect		
		Load Component	DL	LL+I
λ_R	1.12	λ_{Lq}	1.03	1.2
V_R	0.1	V_{Lq}	0.08	0.18
		γ_{Lq}	1.2	1.6
		a	1	3
		L_q, ft-kips	405.1	1,215.4
		μ_{Lq}, ft-kips	417.3	1,458.5
		σ_{Lq}, ft-kips	33.4	262.5
Mean and Standard Deviation of Moment Resistance		Mean and Standard Deviation of Moment Load Effect		
M_n, ft-kips	4,512.5	L, ft-kips	2,467.6	
μ_R, ft-kips	5,054.0	μ_L, ft-kips	2,878.8	
σ_R, ft-kips	505.4	σ_L, ft-kips	430.3	
Calculating the Load Variable l From the Reliability Index Equation				
Target Reliability Index (β)	3.5	Load Variable (l), ft-kips	405.14	

Table 4-17. Calculation of Reliability Index Due to Flexural Failure Mode in Condition States of Steel Girder Element

Condition State	Section Loss (Δ)	Moment Load Effect, ft-kips		Reliability Index (β)
		μ_L	σ_L	
		2,878.8	430.3	
		Moment Resistance, ft-kips		
		$V_R = 0.1$		
		μ_R	σ_R	
Design	0%	5,054.0	505.4	3.50
CS1	1%	5,005.6	500.6	3.44
CS2	5%	4,812.1	481.2	3.17
CS3	10%	4,570.3	457.0	2.82
CS4	15%	4,328.6	432.9	2.46
CS5	20%	4,087.0	408.7	2.07

Table 4-18. Calculation of Probability of Flexural Failure in Condition States of Steel Girder Element

Condition State	Section Loss	Reliability Index	Failure Probability	Relative Failure Probability
	Δ	β	P_f	P_f/P_{f0}
Design	0%	3.50	0.023%	1.0
CS1	1%	3.44	0.030%	1.3
CS2	5%	3.17	0.076%	3.3
CS3	10%	2.82	0.239%	10.3
CS4	15%	2.46	0.702%	30.2
CS5	20%	2.07	1.908%	82.0

Table 4-19. Calculation of Nominal Shear Resistance in Condition States of Steel Girder Element

Condition State	Section Loss (%)	Web Area, inch^2	Fy, ksi	λ_R	V_R
			50	1.14	0.105
			Shear Resistance, kips		
	Δ	A_w	V_n	μ_R	σ_R
Design	0%	33	990.0	1128.6	118.5
CS1	1%	32.63	978.9	1115.9	117.2
CS2	5%	31.15	934.4	1065.3	111.9
CS3	10%	29.29	878.8	1001.8	105.2
CS4	15%	27.44	823.1	938.4	98.5
CS5	20%	25.58	767.4	874.8	91.9

Table 4-20. Calculation of Shear Load Effects for Steel Girder Element

Shear Resistance		Shear Load Effect		
		Load Component	DL	LL+I
λ_R	1.14	λ_{Lq}	1.03	1.2
ν_R	0.105	V_{Lq}	0.08	0.18
		γ_{Lq}	1.2	1.6
		a	1	3
		L_q, kips	89.4	268.1
		μ_{Lq}, kips	92.1	321.8
		σ_{Lq}, kips	7.4	57.9
Mean and Standard Deviation of Moment Resistance		Mean and Standard Deviation of Shear Load Effect		
V_n, kips	990.0	L, kips	544.4	
μ_L, kips	1,128.6	μ_L, kips	635.1	
σ_L, kips	118.5	σ_L, kips	94.9	
Calculating the Load Variable l From the Reliability Index Equation				
Target Reliability Index (β)	3.5	Load Variable (l), kips	89.38	

Table 4-21. Reliability Index Calculation Due to Shear Failure Mode in Condition States of Steel Girder Element

Condition State	Section Loss (Δ)	Shear Load Effect, kips		Reliability Index (β)
		μ_L	σ_L	
		635.1	94.9	
		Shear Resistance, kips		
		$v_R = 0.105$		
		μ_R	σ_R	
Design	0%	1128.6	118.5	3.50
CS1	1%	1115.9	117.2	3.43
CS2	5%	1065.3	111.9	3.12
CS3	10%	1001.8	105.2	2.72
CS4	15%	938.4	98.5	2.29
CS5	20%	874.8	91.9	1.83

Table 4-22. Calculation of Probabilities of Shear Failure in Condition States of Steel Girder Element

Condition State	Section Loss	Reliability Index	Failure Probability	Relative Failure Probability
	Δ	β	P_f	P_f/P_{f0}
Design	0%	3.50	0.023%	1.0
CS1	1%	3.43	0.031%	1.3
CS2	5%	3.12	0.090%	3.9
CS3	10%	2.72	0.329%	14.1
CS4	15%	2.29	1.105%	47.5
CS5	20%	1.83	3.348%	143.9

Table 4-23. Probabilities of Failure in Condition States of Steel Girder Element

Failure Mode	Condition State of the Painted Steel Girder Element				
	CS1	CS2	CS3	CS4	CS5
Flexure	0.030%	0.076%	0.239%	0.702%	1.908%
Shear	0.031%	0.090%	0.329%	1.105%	3.348%

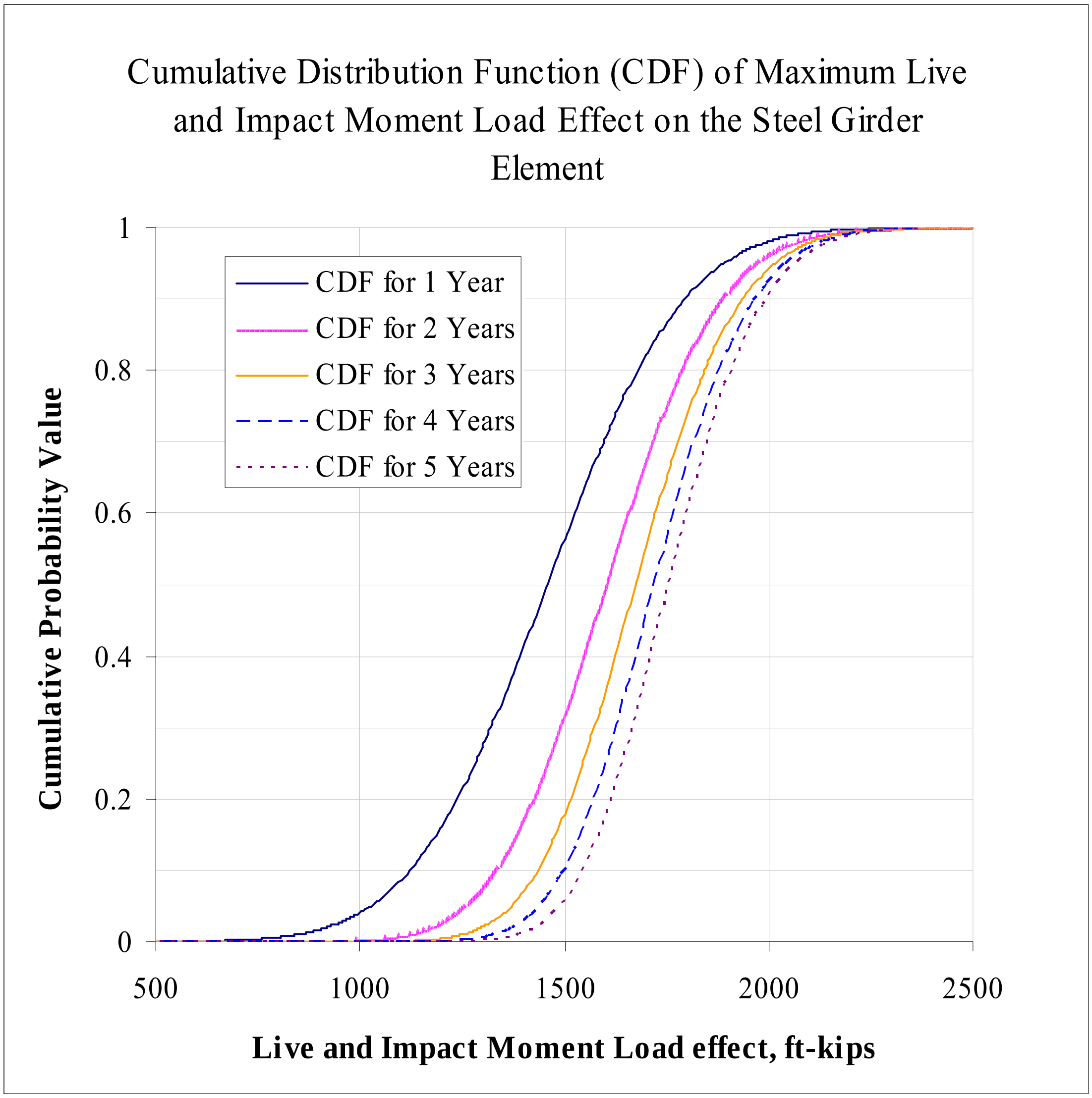


Figure 4-9. Cumulative Distribution Functions for Maximum Moment Load Effects on Steel Girder Element

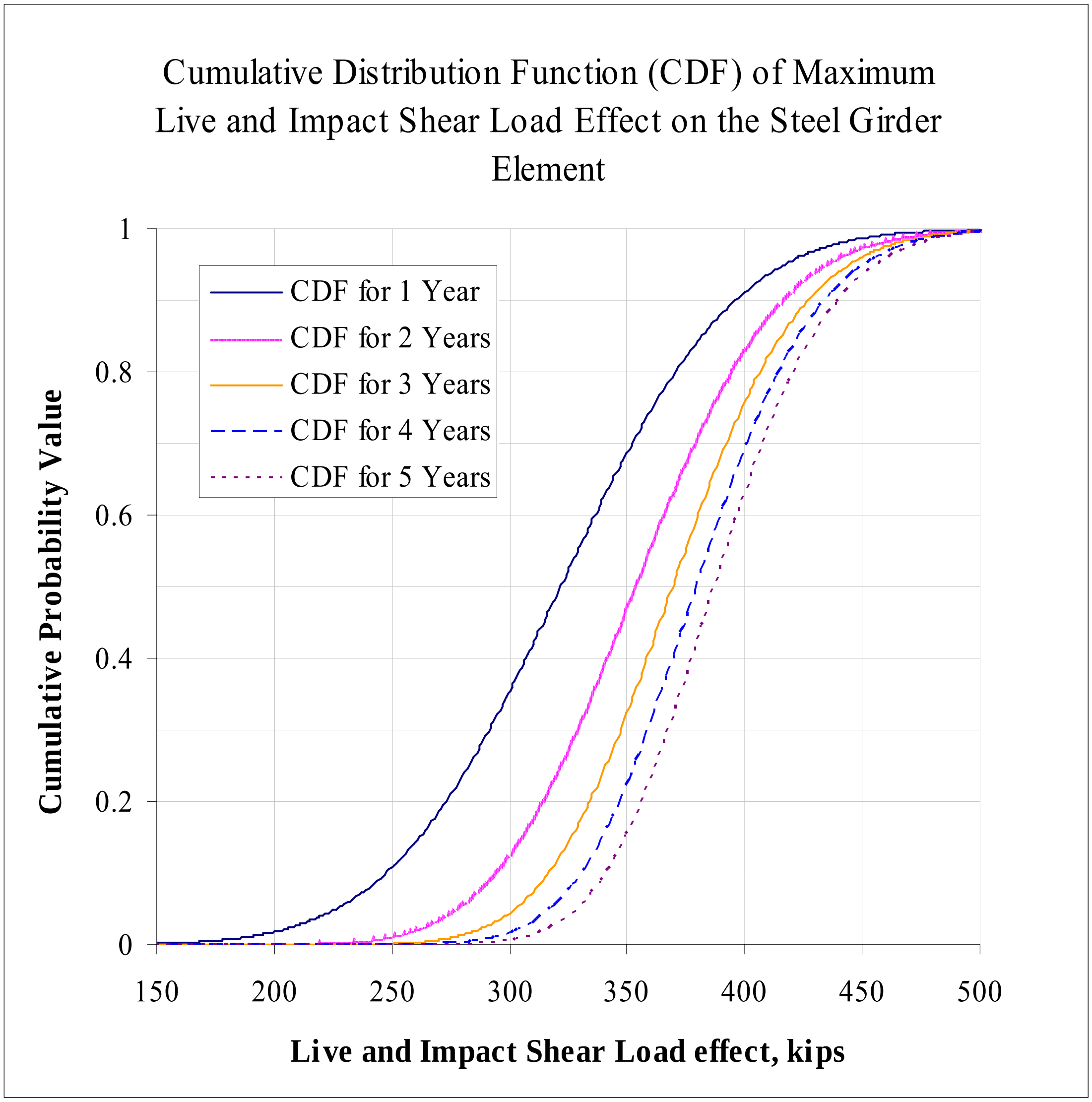


Figure 4-10. Cumulative Distribution Functions for Maximum Shear Load Effects on Steel Girder Element

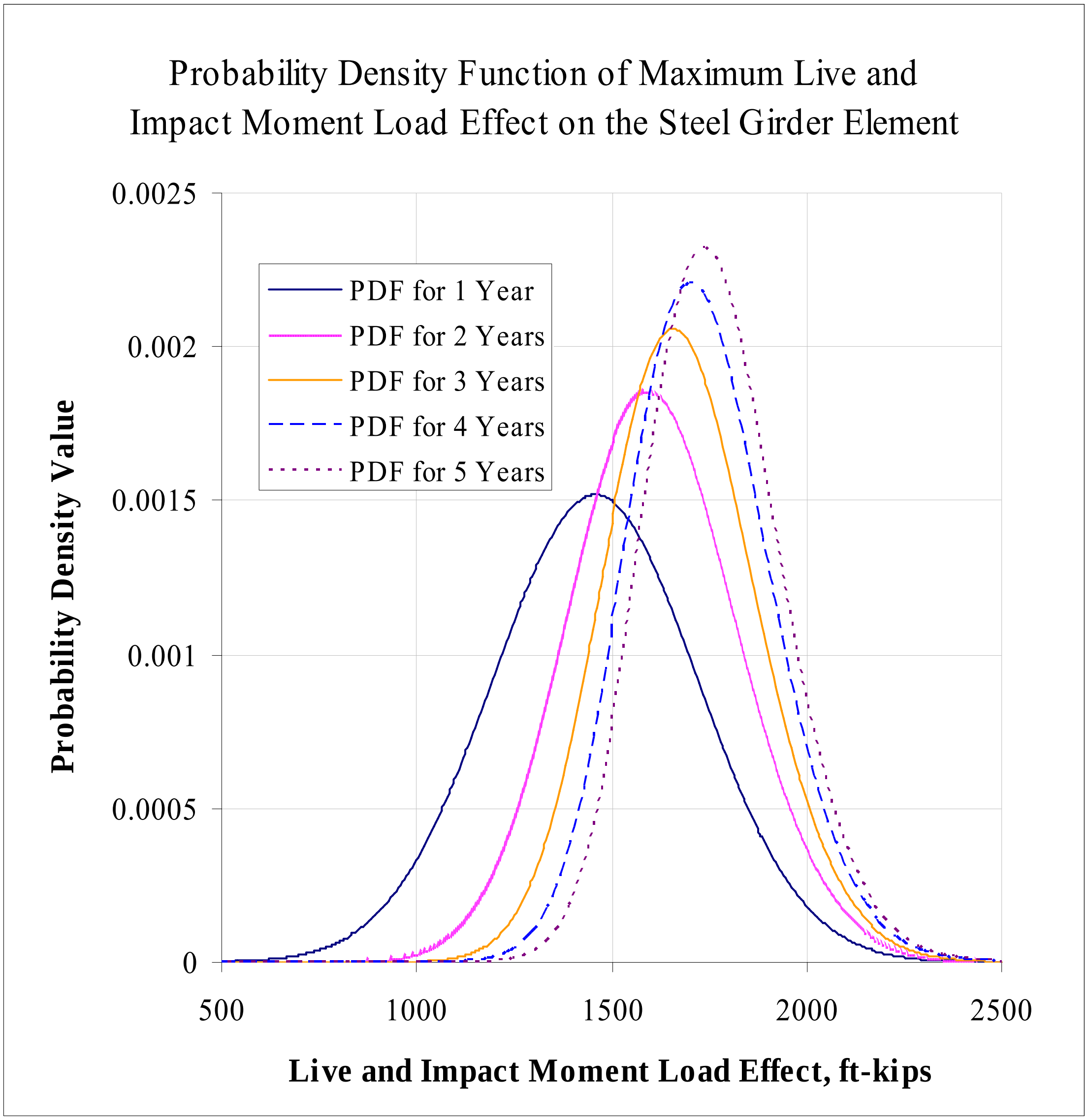


Figure 4-11. Probability Density Functions for Maximum Moment Load Effects on Steel Girder Element

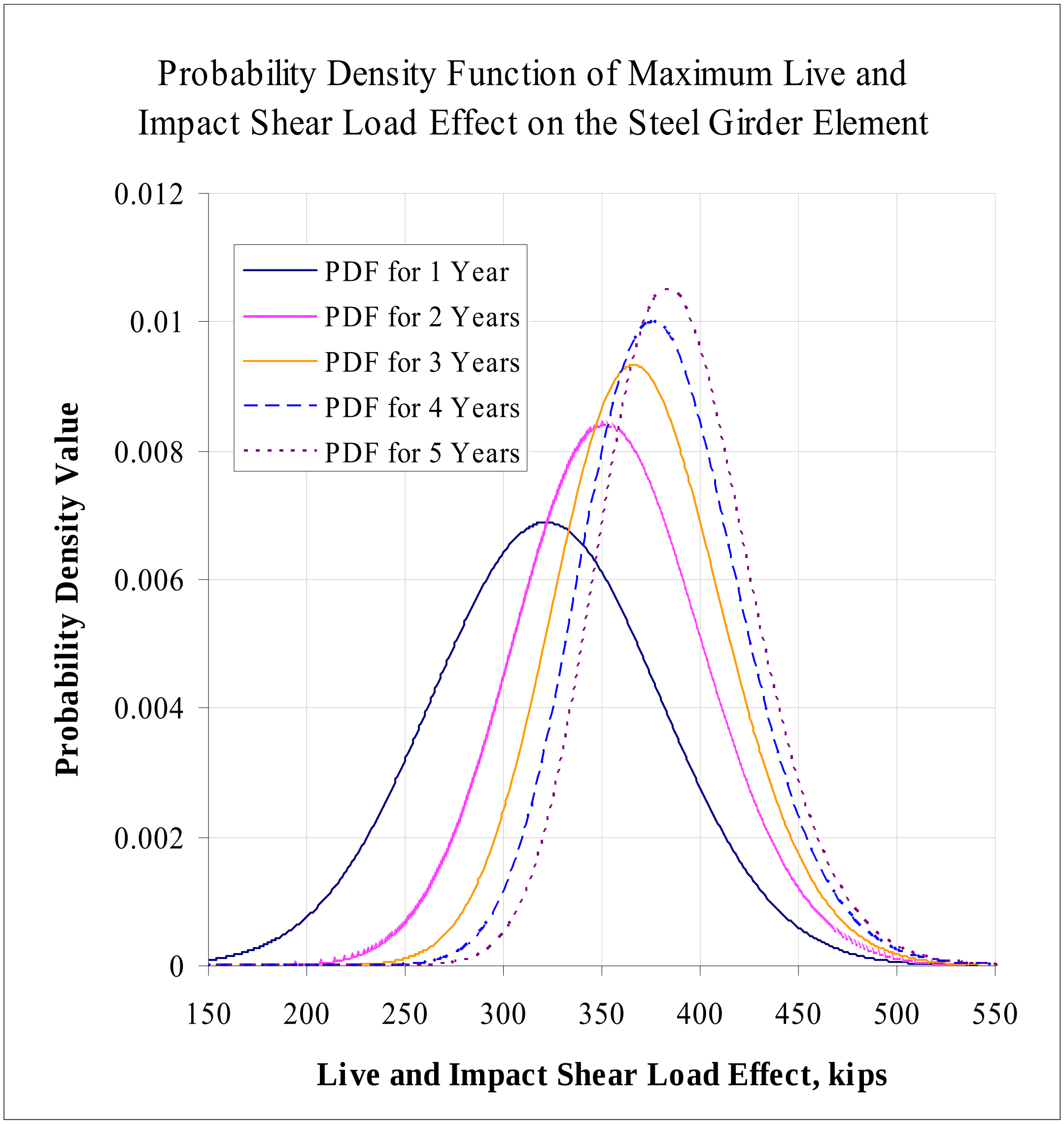


Figure 4-12. Probability Density Functions for Maximum Shear Load Effects on Steel Girder Element

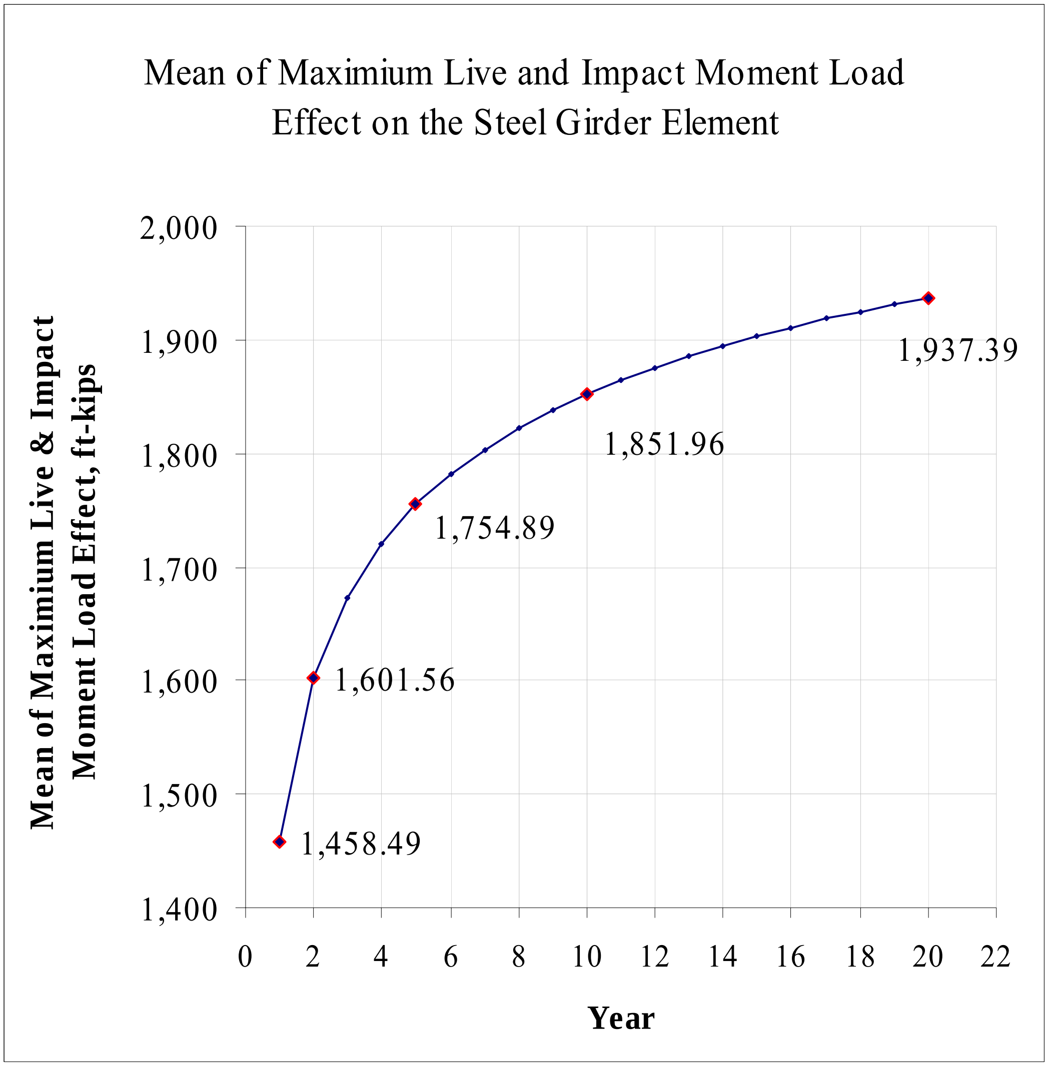


Figure 4-13. Change of Maximum Moment Load Effects on Steel Girder Element for a Planning Horizon of Twenty Years

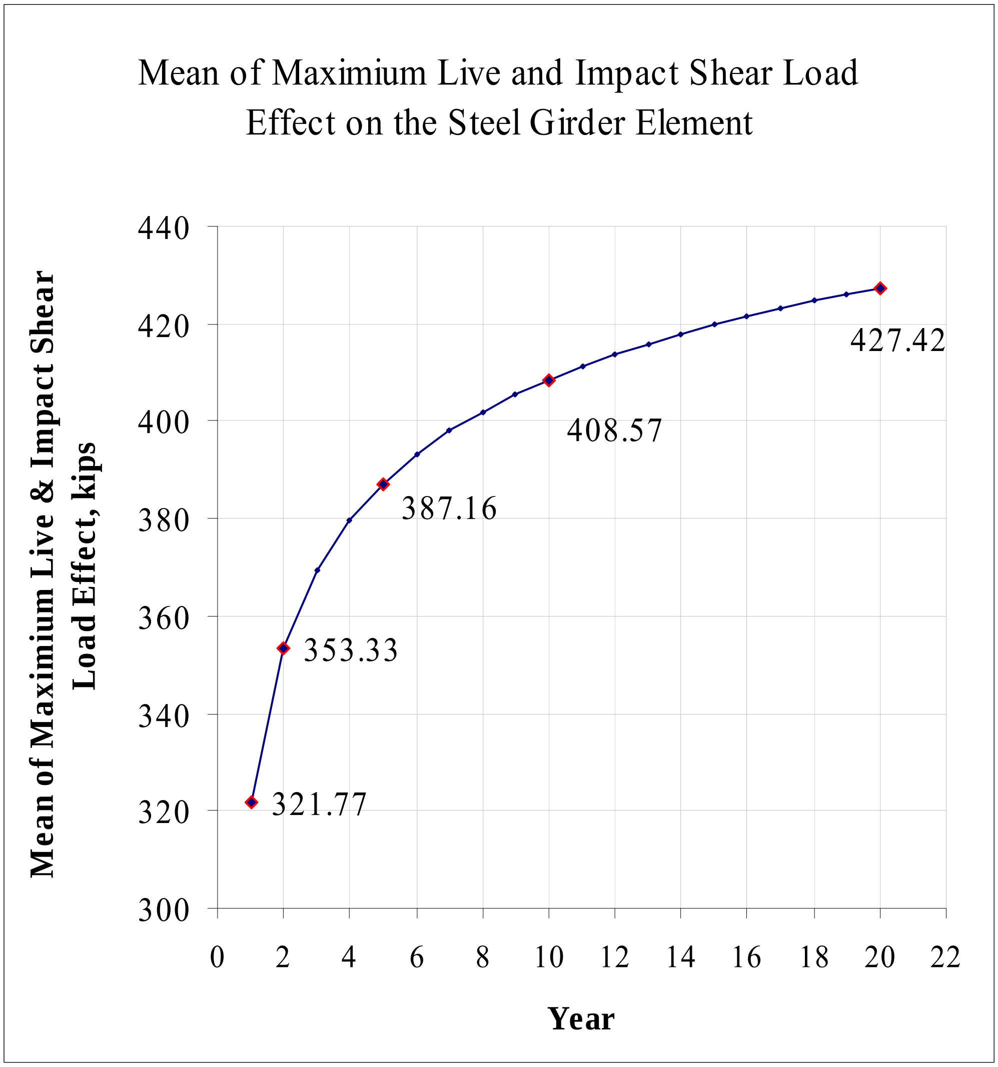


Figure 4-14. Change of Maximum Shear Load Effects on Steel Girder Element for a Planning Horizon of Twenty Years

Table 4-24. Reliability Index in Condition States of Steel Girder Element due to Flexural Failure Mode for Five Years

Year	1	2	3	4	5
$\mu_{L_{L+I}}$, ft-kips	1,458.49	1,601.56	1,673.59	1,720.53	1,754.89
$\sigma_{L_{L+I}}$ ft-kips	262.53	216.67	196.28	184.00	175.53
μ_L, ft-kips	2,878.84	3,112.32	3,229.88	3,306.49	3,362.55
σ_L, ft-kips	430.30	355.84	322.81	302.93	289.23
Condition State	Reliability Index				
CS1	3.44	3.33	3.24	3.17	3.11
CS2	3.17	3.05	2.94	2.86	2.80
CS3	2.82	2.67	2.55	2.45	2.38
CS4	2.46	2.26	2.12	2.02	1.94
CS5	2.07	1.84	1.67	1.55	1.46

Table 4-25. Reliability Index in Condition States of Steel Girder Element Due to Shear Failure Mode for Five Years

Year	1	2	3	4	5
μ_{DL+I}, kips	321.77	353.33	369.22	379.58	387.16
σ_{DL+I}, kips	57.92	47.83	43.35	40.65	38.79
μ_L, kips	635.12	686.63	712.56	729.47	741.83
σ_L, kips	94.93	78.56	71.29	66.92	63.92
Condition State	Reliability Index				
CS1	3.43	3.32	3.23	3.15	3.09
CS2	3.12	2.99	2.88	2.80	2.73
CS3	2.72	2.55	2.42	2.33	2.25
CS4	2.29	2.07	1.93	1.82	1.73
CS5	1.83	1.57	1.39	1.26	1.16

Table 4-26. Probabilities of Failure in Condition States of Steel Girder Element Due to Flexural and Shear Failure Modes for Five Years

Year	Failure Mode	Probabilities of Failure in Condition States				
		CS1	CS2	CS3	CS4	CS5
1	Flexure	0.030%	0.076%	0.239%	0.702%	1.908%
	Shear	0.031%	0.090%	0.328%	1.105%	3.348%
2	Flexure	0.043%	0.116%	0.385%	1.184%	3.323%
	Shear	0.045%	0.140%	0.541%	1.901%	5.877%
3	Flexure	0.059%	0.162%	0.543%	1.681%	4.702%
	Shear	0.063%	0.197%	0.766%	2.693%	8.219%
4	Flexure	0.076%	0.210%	0.705%	2.172%	6.007%
	Shear	0.081%	0.255%	0.992%	3.455%	10.354%
5	Flexure	0.094%	0.259%	0.865%	2.649%	7.235%
	Shear	0.100%	0.313%	1.213%	4.183%	12.302%

4.6.2 Failure Probability for Concrete Deck Element

The probabilities of failure in the five condition states of the concrete deck element of Bridge B_1 is calculated in flexural and shear failure modes. The dimensions of the cross section for the concrete deck and the reinforcing steel are shown in Table 4-27. The concrete strength is assumed as $f'c$ = 3 ksi and the reinforcing steel yield stress is assumed as Fy = 50 ksi.

For flexural failure mode, the probability of failure is calculated in the condition states of the element based on the reduction of the element capacity in flexure. The reduction of the element capacity can be calculated based on the section loss in the reinforcement of the concrete element. A method developed by Thoft-Christensen relates the reduction in reinforcement diameter ΔD to the crack widths Δw on the surface of the element based on the reinforcing bar diameter D, the concrete cover c, and a density ratio α_d of the corrosion rust product to the reinforcing steel as shown in Equation 4-10 (Thoft-Christensen, 2001).

$$\Delta D = \frac{\left(\frac{D/2}{D/2+c}+1\right)c}{(\alpha_d - 1)\pi D}\Delta w \tag{4-10}$$

This method can be used to study the reliability of reinforced concrete bridge elements based on the cracking of the element (Estes, et al. 2003a). According to this method, different levels of crack width will result in different levels of section loss in the reinforcing steel. This method can be applied to the reinforced concrete deck to calculate the flexural probability of failure in the condition states of the element.

A cross section representing the reinforced concrete deck element is assumed as shown in Figure 4-15. It is assumed that the crack width in the deck element increases by worsening the element condition state. Average crack widths of 0.01, 0.03, 0.05, 0.07 and 0.10 inch are assumed when the element is in condition states 1, 2, 3, 4, and 5, respectively. The nominal moment resistance M_{nj} of the concrete deck in condition state j is calculated as the product of the steel tensile force $A_{sj}F_y$ and the lever arm d-$a_j/2$ as shown in Equation 4-11.

$$Mn_j = A_{sj}F_y\,(d - a_j/2) \tag{4-11}$$

The bias factor and coefficient of variation are used to calculate the mean and standard deviation for the moment resistance of the element in the five condition states as shown in Table 4-28. The bias factor and coefficient of variation for the moment resistance of the concrete deck are assumed as 1.14 and 0.13, respectively (Nowak, 1999).

The moment load effect on the deck element is assumed as a combination of unknown dead load, and live and impact loads with the live and impact loads together three times the dead load. The mean and standard deviation for the dead load and live and impact load can be expressed in terms of a load variable (l). The load variable (l) can be calculated by applying Equation 3-25 using a target reliability index of 3.5 and the mean and standard deviation of moment resistance corresponding to the target reliability index. The calculation of the moment load effect for the deck element is shown in Table 4-29.

The mean and standard deviation for the moment load effect and the reduced moment resistance are used to calculate the reliability index due to flexural failure mode in the condition states of the element as illustrated in Table 4-30. The results show that the reliability index for the element in condition states 1, 2, 3, 4 and 5 are 3.41, 3.22, 3.03, 2.83 and 2.53, respectively.

The probability of flexural failure is calculated in the five condition states of the deck element using Equation 3-30. The calculated values are compared to the target probability of failure. Table 4-31 shows a summary calculation for the probabilities of flexural failure in the five condition states of the concrete deck element. The results show that the flexural failure probability of the element in condition state 1 is 1.4 times the target probability of failure compared to around 25 times when the element is in condition state 5.

The failure probabilities due to shear failure mode can be calculated based on the percentage of the distress in the deck area in each condition state. It is assumed that any percentage of distress in the deck area will result in a similar percentage reduction in the width of the deck element. The average distress in the deck area is assumed as 1%, 5%, 10%, 20% and 30% in condition states 1, 2, 3, 4 and 5 respectively. Therefore, the width *b* of the deck element will be reduced by 1%, 5%, 10%, 20% and 30% in condition states 1, 2, 3, 4 and 5, respectively.

The nominal shear resistance V_n of the element is calculated in the condition states of the element based on the concrete compressive strength f_c', the effective depth *d*, and the reduced width *b* of the element section as shown in Equation 4-12 (ACI 11-3).

$$V_n = 2\sqrt{f_c'}bd \tag{4-12}$$

The mean and standard deviation for the nominal shear resistance of the element is calculated using the bias factor and the coefficient of variation for the moment resistance. The bias factor and coefficient of variation for the shear resistance are assumed as 1.4 and 0.17, respectively (Nowak, 1999). The calculation of the nominal shear resistance in the five condition states of the concrete deck element is shown in Table 4-32.

The shear load effect on the deck element is assumed similar to the moment load effect as a combination of the dead load and the live and impact load. The mean and standard deviation for the dead load and live and impact load can be expressed in terms of a load variable (l). The load variable (l) can be calculated by applying Equation 3-25 using a target reliability index of 3.5 and the calculated values for the mean and standard deviation of the element shear resistance with no section loss. The calculation of the shear load effect for the deck element is shown in Table 4-33.

The shear load effect for a particular year is assumed to be the same in the five condition states of the element. The mean and standard deviation for the shear load effect and the reduced shear resistance are used to calculate the reliability index in the five condition states of the element as shown in Table 4-34. The results show that the reliability index for the element in condition states 1, 2, 3, 4 and 5 corresponding to deck area distress of 1%, 5%, 10%, 20% and 30% are 3.45, 3.26, 3.01, 2.43, and 1.74, respectively.

The probability of failure in shear is calculated in the five condition states of the concrete deck element using Equation 3-30 and compared to the target probability of failure as shown in Table 4-35. The results show that the shear failure probability of the element in condition state 1 is 1.19 times the target probability of failure compared to more than 175 times in condition state 5. The probabilities of failure due to flexural and shear failure modes are summarized in Table 4-36.

The probability of failure of the concrete deck element for future years can be calculated, as shown earlier in the steel girder example, based on future prediction of the largest moment and shear load effects on the element. The CDFs and PDFs of the extreme load effects in the future can be derived from the cumulative distribution function of the initial load effects as shown in Equations 3-32 and 3-33, respectively (Ang and Tang, 1984). The CDFs and PDFs for the extreme load effects will shift to the right with increasing number of years in the future (Ang and Tang, 1984).

The distributions of the initial moment and shear load effects on the concrete deck element are assumed as normal. The moment and shear load effects due to the deal load component are assumed to be constant. The maximum moment and shear load effects due to live and impact load components are assumed to increase based on Equation 3-32.

The CDFs of the initial moment and shear load effects are generated using the means and standard deviations of the initial load effects derived earlier from the reliability index equation (Tables 4-29 and 4-33). The CDFs of the maximum moment and shear load effects for two, three, four, and five years are generated from the CDFs of the initial moment and shear load effects, respectively, based on Equation 3-32. The

CDFs for the maximum moment and shear load effects due to live and impact load components are shown in Figures 4-16 and 4-17, respectively.

The PDFs of the maximum moment and shear load effects are generated from the CDFs of the maximum load effects using numerical differentiation. The PDFs for the maximum moment and shear load effects due to live and impact load components are shown in Figures 4-18 and 4-19, respectively.

The means and standard deviations for the maximum moment and shear load effects for two, three, four, and five years are calculated using numerical integration. The first moments of the PDFs around the origin are used to calculate the means of the maximum moment and shear load effects due to live and impact load components. Figures 4-20 and 4-21 show the means of the maximum moment and shear load effects, respectively, for a planning horizon of twenty years. The second moments of the PDFs around the mean are used to calculate the standard deviations of the maximum moment and shear load effects due to live and impact load components.

The calculated means and standard deviations for the maximum load effects due to live and impact load components are combined with the means and standard deviations of the dead load component to calculate the means and standard deviation for the total moment and shear load effects on the concrete deck element as shown in Tables 4-37 and 4-38. The increased load effects in flexure and shear over the years result in reducing the reliability in the condition states of the concrete deck element.

Tables 4-37 and 4-38 illustrate the calculation of the reliability index in flexure and shear in the condition states of the concrete deck element for five years. The probabilities of flexure and shear failure in the condition states of the deck element over five years are shown in Tables 4-39.

Table 4-27. Cross Sectional Properties of Concrete Deck Element

Section Properties			
Concrete		Reinforcing Steel	
f'c, ksi	3	*Fy*, ksi	50
b, inch	40		
h, inch	10		
c, inch	1.2	*n*	4
d, inch	8.425	*D*, inch	0.75

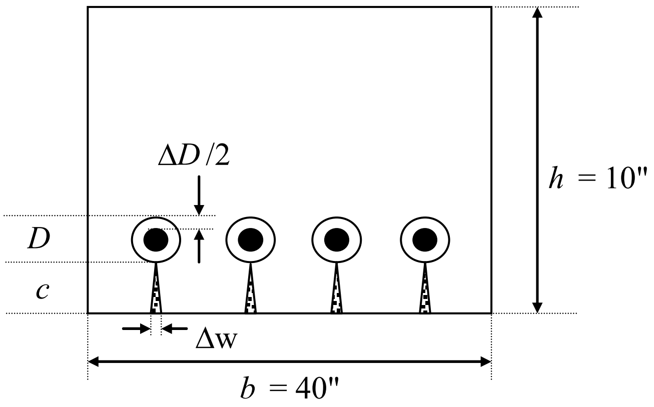


Figure 4-15. Cross Section for Concrete Deck Element

Table 4-28. Calculation of Nominal Moment Resistance in Condition States of Concrete Deck Element

Condition State	Crack Width, inch	Rebar Diameter Reduction, inch	Reduced Rebar Diameter, inch	Steel Area, inch^2	Fy, ksi	λ_R	V_R
					50.00	1.14	0.13
					Nominal Moment Resistance, ft-kips		
	Δw	ΔD	D-ΔD	A_s	M_n	μ_R	σ_R
Design	0.00	0.000	0.75	1.77	70.44	80.30	10.44
1	0.01	0.006	0.74	1.74	69.31	79.02	10.27
2	0.03	0.019	0.73	1.68	67.08	76.48	9.94
3	0.05	0.032	0.72	1.62	64.88	73.97	9.62
4	0.07	0.044	0.71	1.57	62.72	71.50	9.29
5	0.10	0.063	0.69	1.48	59.53	67.86	8.82

Table 4-29. Calculation of Moment Load Effects for Deck Element

Moment Resistance		Moment Load Effect		
		Load Component	DL	LL+I
λ_R	1.14	λ_{Lq}	1.03	1.2
V_R	0.13	V_{Lq}	0.08	0.18
		γ_{Lq}	1.2	1.6
		a	1	3
		L_q, ft-kips	5.96	17.88
		μ_{Lq}, ft-kips	6.14	21.46
		σ_{Lq}, ft-kips	0.49	3.86
Mean and Standard Deviation of Moment Resistance		Mean and Standard Deviation of Load Effect		
M_n, ft-kips	70.44	L, ft-kips	36.31	
μ_R, ft-kips	80.30	μ_L, ft-kips	42.36	
σ_R, ft-kips	10.44	σ_L, ft-kips	6.33	
Calculating the Load Variable *l* From the Reliability Index Equation				
Target Reliability Index (β)	3.5	Load Variable (*l*), ft-kips	5.96	

Table 4-30. Reliability Index Calculation Due to Flexural Failure Mode in Condition States of Deck Element

Condition State	Crack Width (Δ), inch	Moment Load Effect, ft-kips		Reliability Index (β)
		μ_L	σ_L	
		42.36	6.33	
		Moment Resistance, ft-kips		
		$V_R = 0.13$		
		μ_R	σ_R	
Design	0.00	80.30	10.44	3.50
1	0.01	79.02	10.27	3.41
2	0.03	76.48	9.94	3.22
3	0.05	73.97	9.62	3.03
4	0.07	71.50	9.29	2.83
5	0.10	67.86	8.82	2.53

Table 4-31. Calculation of Probabilities of Flexural Failure in Condition States of Concrete Deck Element

Condition State	Crack Width, inch	Reliability Index	Failure Probability	Relative Failure Probability
	Δw	β	P_f	P_f/P_{f0}
Design	0.00	3.50	0.023%	1.00
1	0.01	3.41	0.033%	1.40
2	0.03	3.22	0.064%	2.74
3	0.05	3.03	0.122%	5.26
4	0.07	2.83	0.231%	9.92
5	0.10	2.53	0.576%	24.75

Table 4-32. Calculation of the Nominal Shear Resistance in Condition States of Concrete Deck Element

Condition State	Distress in Deck Area, %	Reduced Deck Width, inch	d, inch	$f'c$, psi	λ_R	V_R
			8.425	3000	1.40	0.17
			Nominal Shear Resistance, kips			
		b	Vn		μ_R	σ_R
Design	0%	40.00	36.92		51.68	8.79
1	1%	39.60	36.55		51.17	8.70
2	5%	38.00	35.07		49.10	8.35
3	10%	36.00	33.22		46.51	7.91
4	20%	32.00	29.53		41.35	7.03
5	30%	28.00	25.84		36.18	6.15

Table 4-33. Calculation of Shear Load Effects for Concrete Deck Element

Shear Resistance		Shear Load Effect		
		Load Component	DL	LL+I
λ_R	1.4	λ_{Lq}	1.03	1.2
V_R	0.17	V_{Lq}	0.08	0.18
		γ_{Lq}	1.2	1.6
		a	1	3
		L_q, kips	3.42	10.27
		μ_{Lq}, kips	3.53	12.33
		σ_{Lq}, kips	0.28	2.22
Mean and Standard Deviation of Shear Resistance		Mean and Standard Deviation of Shear Load Effect		
V_n, kips	36.92	L, kips	20.85	
μ_R, kips	51.68	μ_L, kips	24.33	
σ_R, kips	8.79	σ_L, kips	3.64	
Calculating the Load Variable *l* From the Reliability Index Equation				
Target Reliability Index (β)	3.5	Load Variable (l), kips	3.42	

Table 4-34. Reliability Index Calculation Due to Shear Failure Mode in Condition States of Concrete Deck Element

Condition State	% Distress in Deck Area	Shear Load Effect, kips		Reliability Index (β)
		μ_L	σ_L	
		24.33	3.64	
		Shear Resistance, kips		
		$V_R = 0.17$		
		μ_R	σ_R	
Design	0%	51.68	8.79	3.50
CS1	1%	51.17	8.70	3.45
CS2	5%	49.10	8.35	3.26
CS3	10%	46.51	7.91	3.01
CS4	20%	41.35	7.03	2.43
CS5	30%	36.18	6.15	1.74

Table 4-35. Calculation of Probabilities of Shear Failure in Condition States of Concrete Deck Element

Condition State	% Distress in Deck Area	Reliability Index	Failure Probability	Relative Failure Probability
		β	P_f	P_f/P_{f0}
Design	0%	3.50	0.023%	1.00
1	1%	3.45	0.028%	1.19
2	5%	3.26	0.055%	2.36
3	10%	3.01	0.132%	5.67
4	20%	2.43	0.763%	32.81
5	30%	1.74	4.112%	176.71

Table 4-36. Probabilities of Failure in the Condition States of Concrete Deck Element Due to Flexural and Shear Failure Modes

Failure Mode	Condition State of the Concrete Deck Element				
	CS1	CS2	CS3	CS4	CS5
Flexure	0.033%	0.064%	0.122%	0.231%	0.576%
Shear	0.028%	0.055%	0.132%	0.763%	4.112%

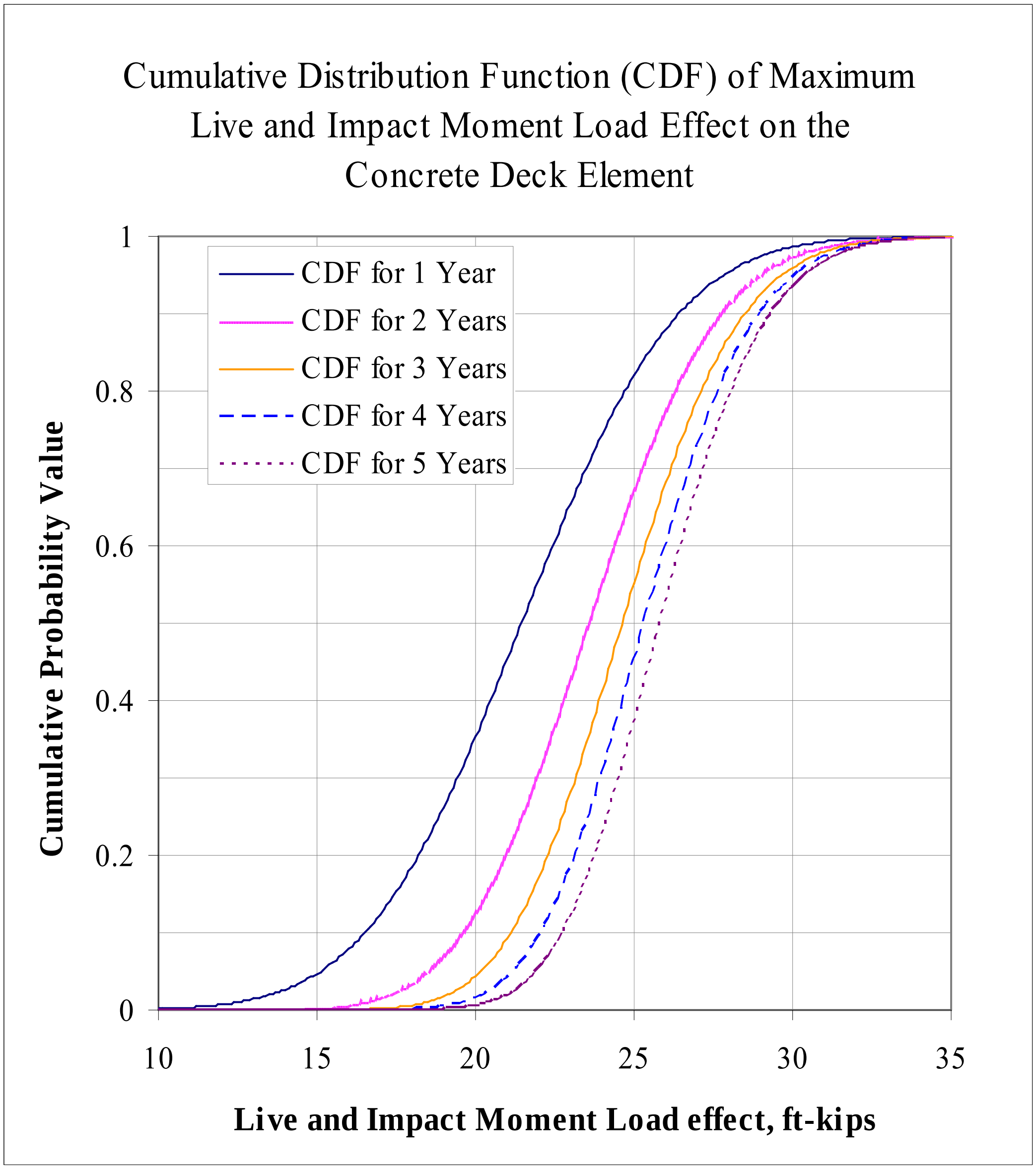


Figure 4-16. Cumulative Distribution Functions for Maximum Moment Load Effects on Concrete Deck Element

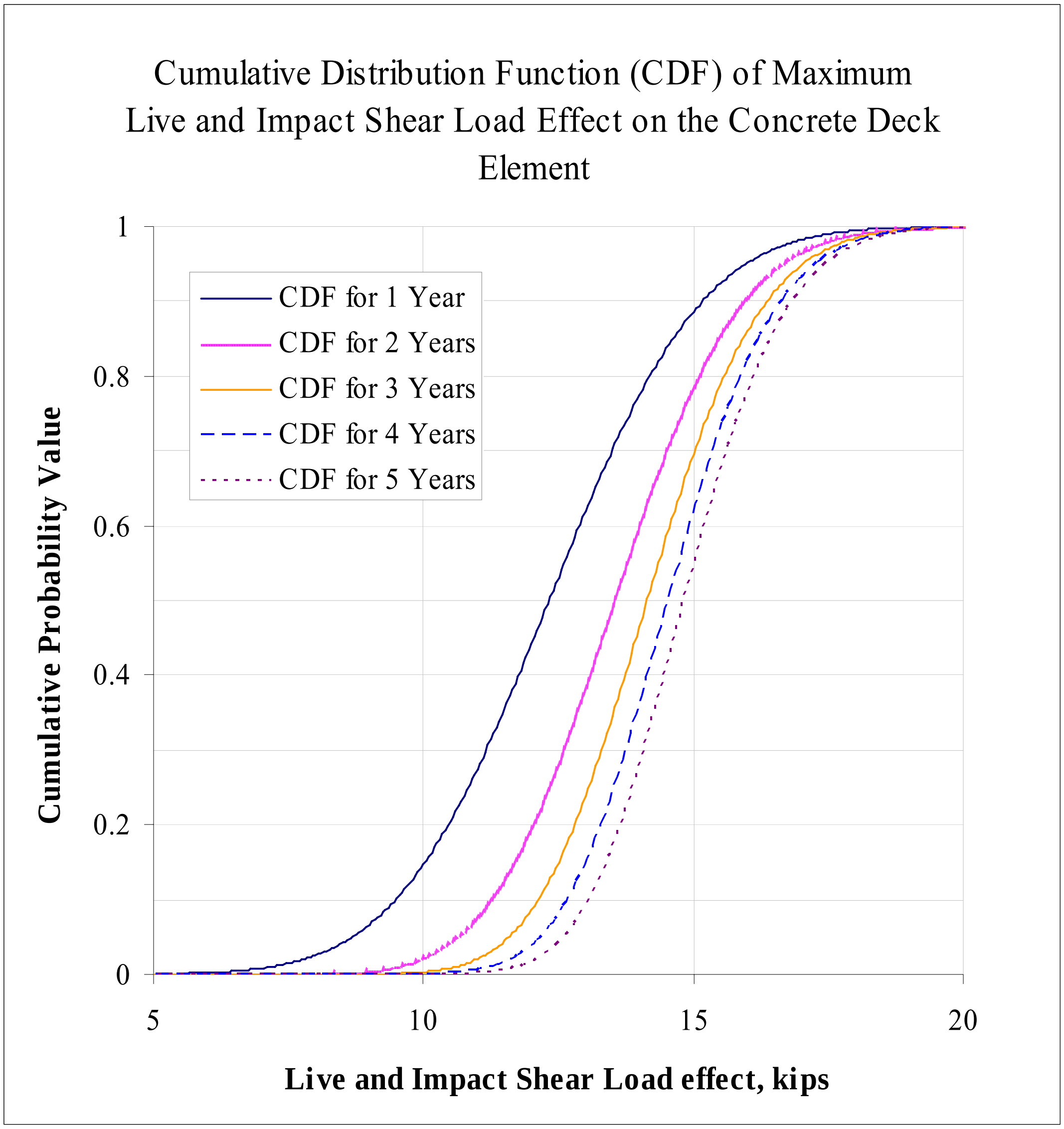


Figure 4-17. Cumulative Distribution Functions for Maximum Moment Load Effects on Concrete Deck Element

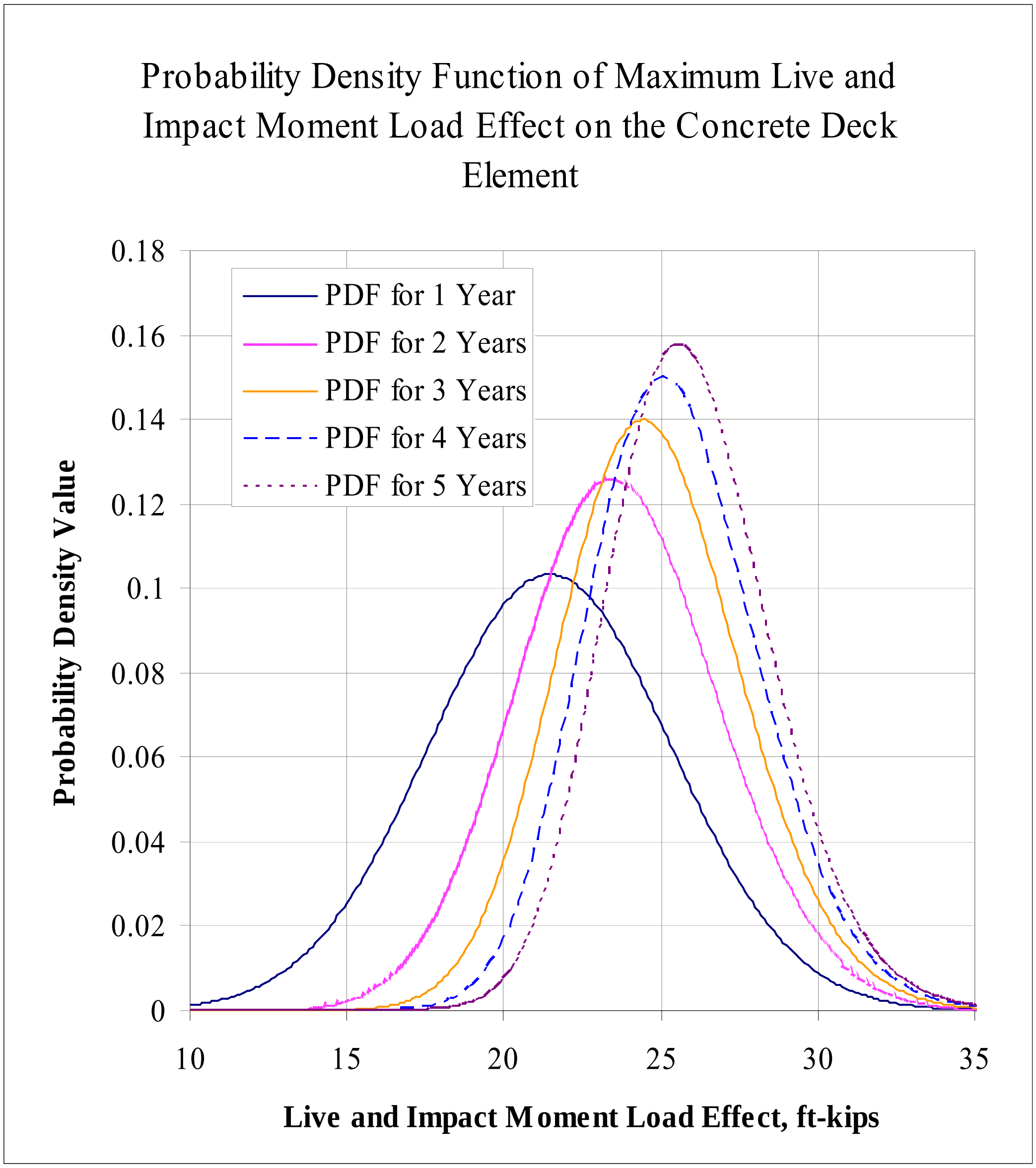


Figure 4-18. Probability Density Functions for Maximum Moment Load Effects on Concrete Deck Element

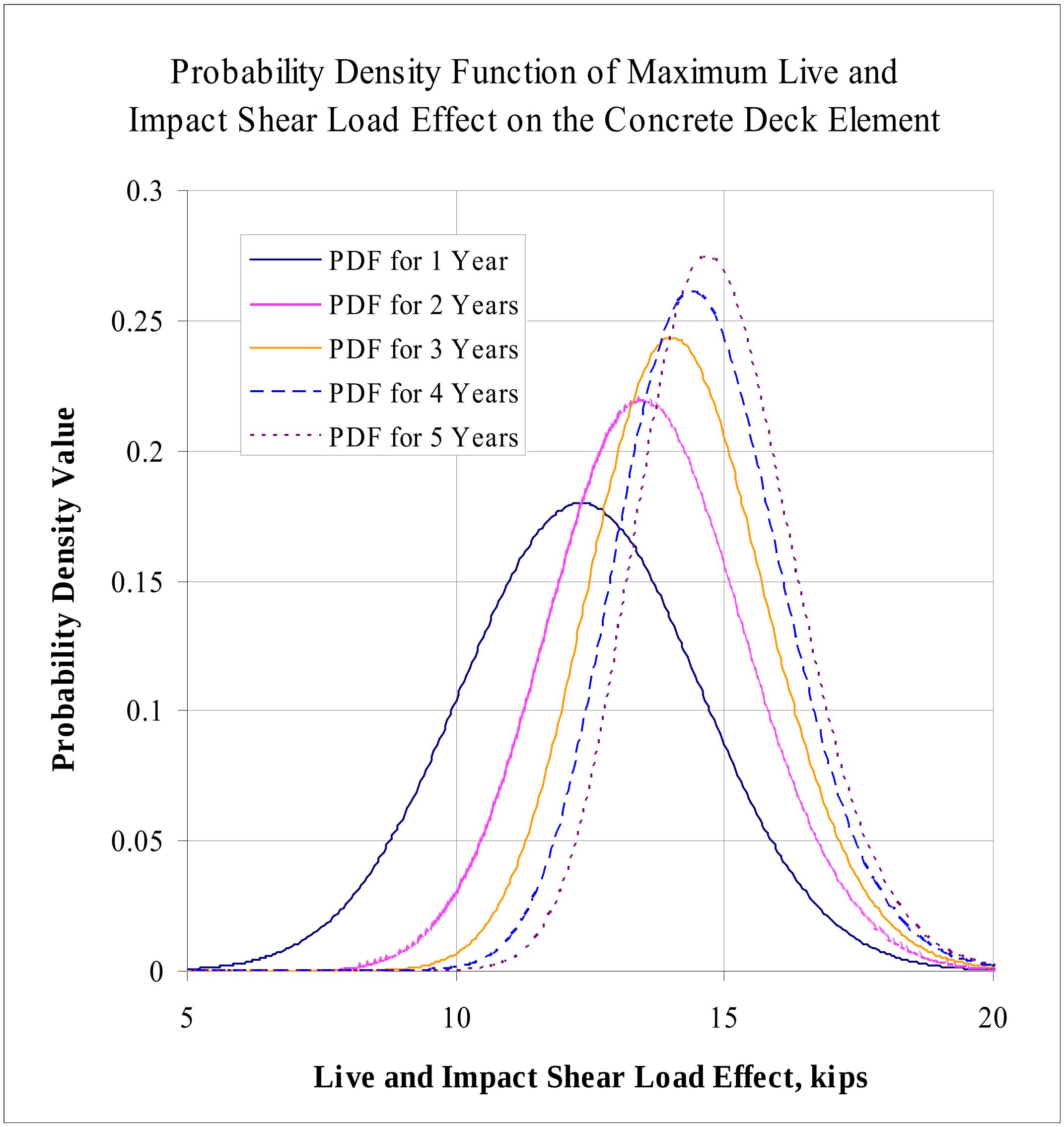


Figure 4-19. Probability Density Functions for Maximum Shear Load Effects on Concrete Deck Element

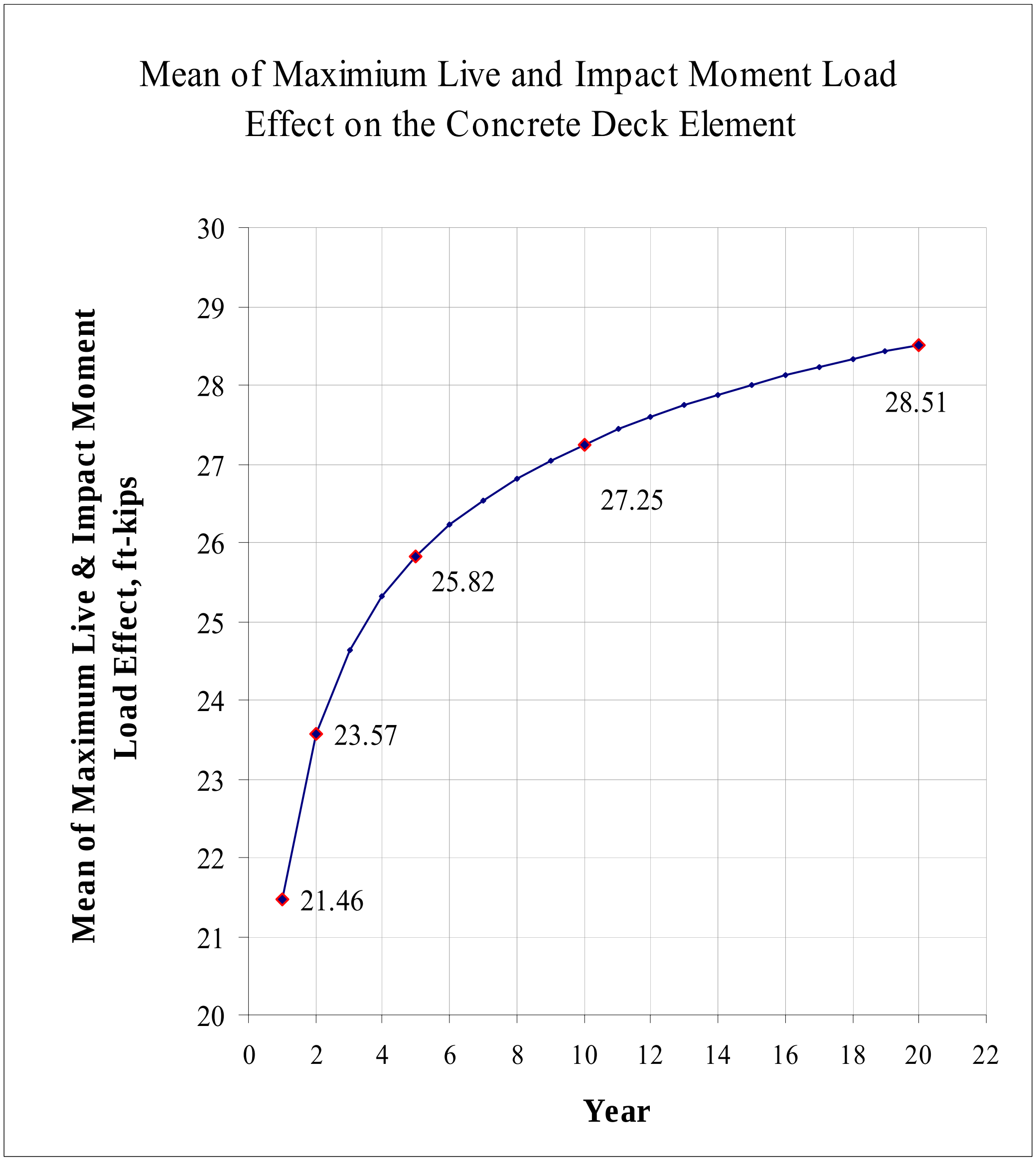


Figure 4-20. Change of the Maximum Moment Load Effects on Concrete Deck Element for a Planning Horizon of Twenty Years

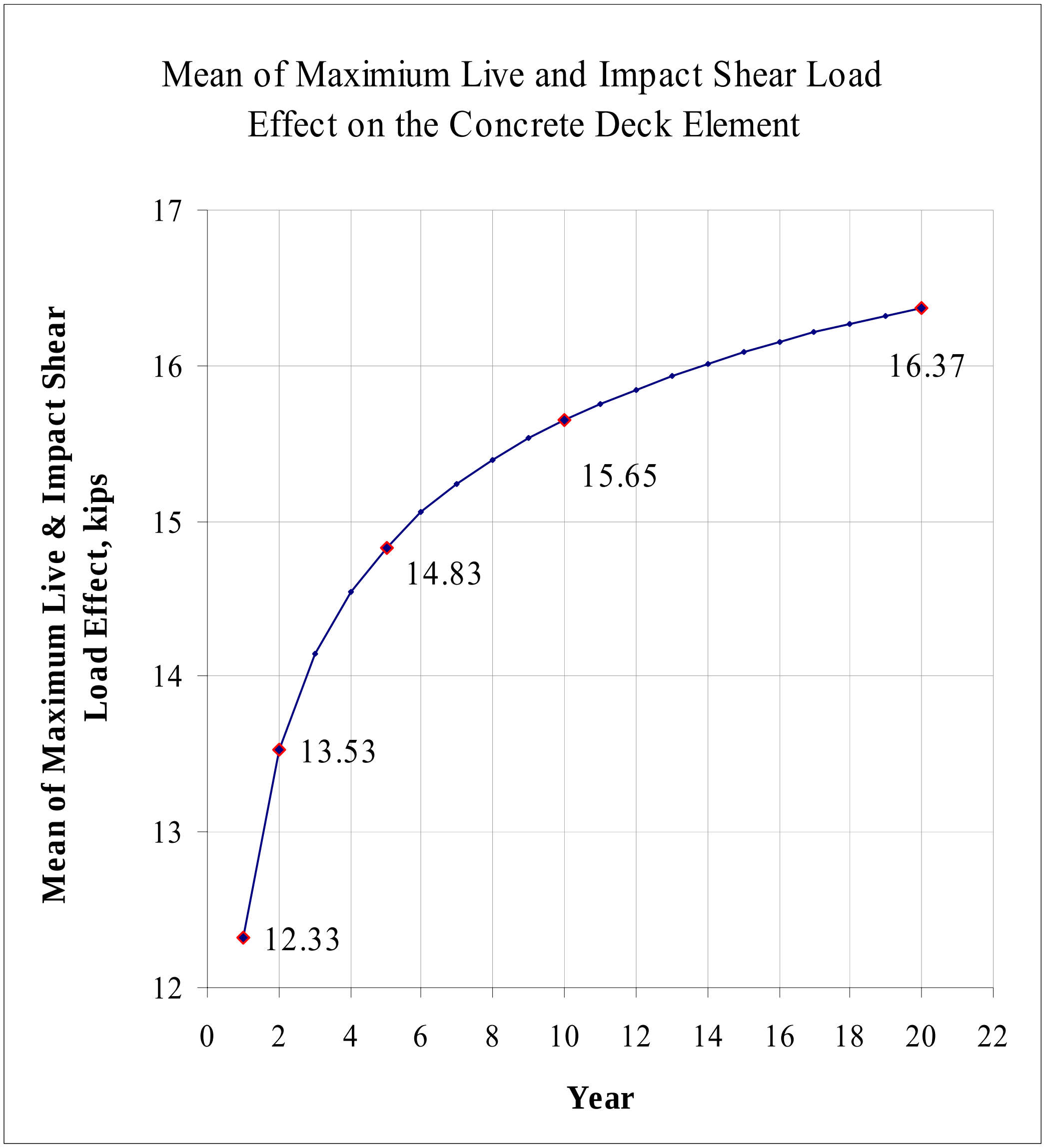


Figure 4-21. Change of the Maximum Shear Load Effects on Steel Concrete Deck Element for a Planning Horizon of Twenty Years

Table 4-37. Reliability Index in Condition States of Concrete Deck Element Due to Flexural Failure Mode for Five Years

Year	1	2	3	4	5
$\mu_{L_{L+I}}$, ft-kips	21.46	23.57	24.63	25.32	25.82
$\sigma_{L_{L+I}}$ ft-kips	3.86	3.19	2.89	2.71	2.59
μ_L, ft-kips	42.36	45.79	47.52	48.65	49.48
σ_L, ft-kips	6.33	5.24	4.76	4.46	4.26
Condition State	Reliability Index				
CS1	3.41	3.28	3.18	3.11	3.05
CS2	3.22	3.08	2.98	2.90	2.84
CS3	3.03	2.88	2.77	2.69	2.62
CS4	2.83	2.67	2.55	2.46	2.39
CS5	2.53	2.33	2.21	2.11	2.03

Table 4-38. Reliability Index in Condition States of the Deck Element Due to Shear Failure Mode for Five Years

Year	1	2	3	4	5
μ_{L+I}, kips	12.33	13.53	14.14	14.54	14.83
σ_{L+I}, kips	2.22	1.83	1.66	1.56	1.49
μ_L, kips	24.33	26.30	27.29	27.94	28.42
σ_L, kips	3.64	3.01	2.73	2.56	2.45
Condition State	Reliability Index				
CS1	3.45	3.32	3.23	3.16	3.11
CS2	3.26	3.12	3.02	2.95	2.90
CS3	3.01	2.85	2.74	2.67	2.61
CS4	2.43	2.23	2.11	2.02	1.95
CS5	1.74	1.48	1.33	1.22	1.14

Table 4-39. Probabilities of Failure in Condition States of Concrete Deck Element Due to Flexural and Shear Failure Modes for Five Years

Year	Failure Mode	Condition State				
		CS1	CS2	CS3	CS4	CS5
1	Flexure	0.033%	0.064%	0.122%	0.231%	0.576%
	Shear	0.028%	0.055%	0.132%	0.763%	4.112%
2	Flexure	0.052%	0.103%	0.201%	0.385%	0.979%
	Shear	0.045%	0.091%	0.220%	1.290%	6.892%
3	Flexure	0.072%	0.144%	0.282%	0.540%	1.371%
	Shear	0.063%	0.126%	0.304%	1.763%	9.183%
4	Flexure	0.093%	0.185%	0.361%	0.691%	1.744%
	Shear	0.079%	0.159%	0.381%	2.190%	11.124%
5	Flexure	0.114%	0.225%	0.439%	0.836%	2.098%
	Shear	0.095%	0.189%	0.454%	2.578%	12.807%

4.7 Estimation of Failure Consequences

The consequences of failure for a bridge element include agency cost of the element damage (C_e), agency cost to repair the damage of other elements in the bridge (C_b), user cost (C_u) due to traffic congestion, cost of traffic accidents (C_a), health and safety cost (C_h) of injuries and/or deaths, cost of environmental damage (C_{env}), cost to the nearby businesses (C_{nb}) due to loss of revenue, and cost to the general public (C_p).

The agency cost of the element damage (C_e) is assumed as the cost to replace the element plus 10% of the cost for inspection and removal of the element. The cost of the element replacement is equal to the unit replacement cost (c_R) multiplied by the total quantity (Q) of the element. Since the unit replacement cost may vary between the different highway agencies, a range between lower and upper limits is used in the calculation. The cost of other types of consequences can be estimated using expert opinions in different fields. For demonstration purposes, ranges between lower and upper limits are assumed for the costs of the different consequence types due to the flexural and shear failures of the concrete deck and steel girder elements as shown in Tables 4-40 and 4-41. Assuming a uniform probability distribution for the consequences of failure, the means and standard deviations of the costs in each type of consequences can be calculated for the steel girder and concrete deck elements as shown in Tables 4-42 and 4-43. The means and standard deviations for the total consequences of flexural and shear failures for the deck and girder elements are calculated using Equations 3-35 and 3-36 as shown in Tables 4-42 and 4-43.

Table 4-40. Upper and Lower Limits for Consequences of Flexural and Shear Failure of Steel Girder Element

Consequence Type	Cost ($)	Consequences of Flexural Failure		Consequences of Shear Failure	
		Lower Limit	Upper Limit	Lower Limit	Upper Limit
Element Damage	C_e	739,200	985,600	887,040	1,182,720
Property Damage of the Bridge	C_b	1,000,000	1,200,000	1,200,000	1,440,000
Traffic Delay of Bridge Users	C_u	1,500,000	2,000,000	1,800,000	2,400,000
Damage due to Accidents	C_a	1,200,000	1,500,000	1,440,000	1,800,000
Health and Safety damage (injuries and/or deaths)	C_h	3,000,000	3,500,000	6,000,000	7,000,000
Environmental Damage	C_{env}	60,000	90,000	72,000	108,000
Damage to Nearby Businesses	C_{nb}	100,000	150,000	120,000	180,000
Impact on the General Public	C_p	80,000	120,000	96,000	144,000

Table 4-41. Upper and Lower Limits for Consequences of Flexural and Shear Failure of Concrete Deck Element

Consequence Type	Cost ($)	Consequences of Flexural Failure		Consequences of Shear Failure	
		Lower Limit	Upper Limit	Lower Limit	Upper Limit
Element Damage	C_e	387,200	542,080	464,640	650,496
Property Damage of the Bridge	C_b	100,000	110,000	120,000	132,000
Traffic Delay of Bridge Users	C_u	1,000,000	1,200,000	1,200,000	1,440,000
Damage due to Accidents	C_a	800,000	1,000,000	960,000	1,200,000
Health and Safety damage (injuries and/or deaths)	C_h	2,800,000	3,000,000	3,360,000	3,600,000
Environmental Damage	C_{env}	50,000	60,000	60,000	72,000
Damage to Nearby Businesses	C_{nb}	80,000	100,000	96,000	120,000
Impact on the General Public	C_p	60,000	80,000	72,000	96,000

Table 4-42. Means and Standard Deviations for Consequences of Flexural and Shear Failure of Steel Girder Element

Consequence Type	Cost ($)	Consequences due to Flexural Failure		Consequences due to Shear Failure	
		Mean	Standard Deviation	Mean	Standard Deviation
Element Damage	C_e	862,400	71,130	1,034,880	85,355
Property Damage of the Bridge	C_b	1,100,000	57,735	1,320,000	69,282
Traffic Delay of Bridge Users	C_u	1,750,000	144,338	2,100,000	173,205
Damage due to Accidents	C_a	1,350,000	86,603	1,620,000	103,923
Health and Safety damage (injuries and/or deaths)	C_h	3,250,000	144,338	6,500,000	288,675
Environmental Damage	C_{env}	75,000	8,660	90,000	10,392
Damage to Nearby Businesses	C_{nb}	125,000	14,434	150,000	17,321
Impact on the General Public	C_p	100,000	11,547	120,000	13,856
Total Consequences of the Element Failure	C_F	8,612,400	240,782	12,934,880	369,890

Table 4-43. Means and Standard Deviations for Consequences of Flexural and Shear Failure of Concrete Deck Element

Consequence Type	Cost ($)	Consequences due to Flexural Failure		Consequences due to Shear Failure	
		Mean	Standard Deviation	Mean	Standard Deviation
Element Damage	C_e	464,640	22,355	557,568	53,652
Property Damage of the Bridge	C_b	105,000	2,887	126,000	3,464
Traffic Delay of Bridge Users	C_u	1,100,000	57,735	1,320,000	69,282
Damage due to Accidents	C_a	900,000	57,735	1,080,000	69,282
Health and Safety damage (injuries and/or deaths)	C_h	2,900,000	57,735	3,480,000	69,282
Environmental Damage	C_{env}	55,000	2,887	66,000	3,464
Damage to Nearby Businesses	C_{nb}	90,000	5,774	108,000	6,928
Impact on the General Public	C_p	70,000	5,774	84,000	6,928
Total Consequences of the Element Failure	C_F	5,684,640	102,874	6,821,568	131,904

4.8 Estimating Risk in Condition States of Bridge Elements

Risk in each condition state is calculated as the product of the probability of failure and the consequences of failure in that condition state. The probabilities and consequences of failure for the steel girder element and the concrete deck element are shown in Tables 4-44 and Table 4-45. Risks due to flexural and shear failure modes are added in each condition state. The risks associated with the five condition states of the steel girder element and the deck element are calculated as shown in Tables 4-46 and 4-47. A unit cost for the risk in each condition state is calculated by dividing the total risk in the condition state by the total quantity of the element.

The increased probability of failure due to the increased moment and shear load effects in future years increases the risk associated with condition states of the steel girder and concrete deck elements. Tables 4-48 and 4-49 show the increases in the risk unit costs associated with the condition states of the steel girder element and concrete deck element over five years.

Table 4-44. Probabilities and Consequences of Flexural and Shear Failure Modes in Condition States of Steel Girder Element

Failure Mode	Consequences (C_F) ($)		Probabilities of Failure ($P_{f,j}$)				
			CS1	CS2	CS3	CS4	CS5
Flexural	Mean	8,612,400	0.030%	0.076%	0.239%	0.702%	1.908%
	Standard Deviation	240,782					
Shear	Mean	12,934,880	0.031%	0.090%	0.329%	1.105%	3.348%
	Standard Deviation	369,890					

Table 4-45. Consequences and Probabilities of Flexural and Shear Failure Modes in Condition States of Concrete Deck Element

Failure Mode	Consequences (C_F) ($)		Probabilities of Failure ($P_{f,j}$)				
			CS1	CS2	CS3	CS4	CS5
Flexural	Mean	5,684,640	0.033%	0.064%	0.122%	0.231%	0.576%
	Standard Deviation	102,874					
Shear	Mean	6,821,568	0.028%	0.055%	0.132%	0.763%	4.112%
	Standard Deviation	131,904					

Table 4-46. Risks to the Condition States of the Steel Girder Element

Failure Mode	Risk in the Condition States of Element (R_j), $					
	Condition State	CS1	CS2	CS3	CS4	CS5
Flexural	Mean	2,549.92	6,579.70	20,572.87	60,419.03	164,312.1
	Standard Deviation	71.29	183.95	575.17	1,689.17	4,593.78
Shear	Mean	3,958.95	11,643.28	42,488.73	142,988.6	433,101.5
	Standard Deviation	113.21	332.96	1,215.02	4,088.95	12,385.12
Total Risk ($)	Mean	6,508.86	18,222.99	63,061.59	203,407.6	597,413.6
	Standard Deviation	133.79	380.39	1,344.28	4,424.12	13,209.62
Risk Unit Cost ($/ft)	Mean	2.91	8.14	28.15	90.81	266.70
	Standard Deviation	0.06	0.17	0.60	1.98	5.90

Table 4-47. Risks to the Condition States of Concrete Deck Element

Failure Mode	Risk in the Condition States of the Deck Element (R_j), $					
	Condition State	CS1	CS2	CS3	CS4	CS5
Flexural	Mean	1,855.47	3,618.37	6,952.20	13,124.35	32,731.11
	Standard Deviation	33.58	65.48	125.81	237.51	592.33
Shear	Mean	1,882.41	3,752.75	8,995.22	52,078.66	280,479.17
	Standard Deviation	36.40	72.56	173.93	1,007.01	5,423.41
Total Risk ($)	Mean	3,737.87	7,371.13	15,947.42	65,203.01	313,210.29
	Standard Deviation	49.52	97.74	214.67	1,034.64	5,455.66
Risk Unit Cost ($/ft^2)	Mean	0.27	0.52	1.13	4.63	22.25
	Standard Deviation	0.00	0.01	0.02	0.07	0.39

Table 4-48. Risks Unit Costs in Condition States of Steel Girder Element for Five Years

Year	Risk Unit Cost ($/ft) in the Condition States of the Steel Girder Element					
	Condition State	CS1	CS2	CS3	CS4	CS5
1	Mean	2.91	8.14	28.15	90.81	266.70
	Standard Deviation	0.06	0.17	0.60	1.98	5.90
2	Mean	4.27	12.57	46.05	155.29	467.15
	Standard Deviation	0.09	0.26	0.99	3.39	10.34
3	Mean	5.90	17.60	65.15	220.16	655.39
	Standard Deviation	0.12	0.37	1.39	4.80	14.48
4	Mean	7.63	22.80	84.35	283.04	828.83
	Standard Deviation	0.16	0.48	1.80	6.17	18.28
5	Mean	9.40	28.06	103.35	343.41	988.56
	Standard Deviation	0.19	0.59	2.21	7.47	21.75

Table 4-49. Risks Unit Costs in Condition States of Concrete Deck Element for Five Years

Year	Risk Unit Cost $(\$/\text{ft}^2)$ in Condition States of Deck Element					
	Condition State	CS1	CS2	CS3	CS4	CS5
1	Mean	0.27	0.52	1.13	4.63	22.25
	Standard Deviation	0.00	0.01	0.02	0.07	0.39
2	Mean	0.43	0.86	1.88	7.80	37.34
	Standard Deviation	0.01	0.01	0.03	0.12	0.65
3	Mean	0.60	1.19	2.61	10.72	50.03
	Standard Deviation	0.01	0.02	0.04	0.17	0.87
4	Mean	0.76	1.51	3.31	13.40	60.94
	Standard Deviation	0.01	0.02	0.04	0.21	1.05
5	Mean	0.92	1.83	3.97	15.87	70.51
	Standard Deviation	0.01	0.02	0.05	0.25	1.21

4.9 Risk Estimation at the Element Level

The risk value in each condition state of the element represents the risk associated with the element condition if the element is totally in that condition state. The probability of the element being in a particular condition state is equal to the percentage of the element quantity in that condition state. The initial quantity distribution among the possible condition states of the element is used to calculate the initial risk associated with the element condition. The calculation of the initial risks associated with conditions of steel girder element and concrete deck element is illustrated in Tables 4-50 and 4-51. The quantity distribution of the element at any time can be used to calculate the risk associated with the element condition at that time.

Table 4-50. Calculation of the Initial Risk Associated with Condition of Steel Girder Element

Initial Risk Associated with Condition of Steel Girder Element						
Condition State		CS1	CS2	CS3	CS4	CS5
Risk Unit Cost ($/ft)	Mean	2.91	8.14	28.15	90.81	266.70
	Standard Deviation	0.06	0.17	0.60	1.98	5.90
Quantity (ft)	Mean	600	12,00	300	100	40
	Standard Deviation	79.06	145.77	38.08	15.81	7.91
Risk Distribution ($)	Mean	1,743	9,762	8,446	9,081	10,668
	Standard Deviation	232	1,203	1,087	1,449	2,122
Total Initial Risk ($) Associated with Element Condition	Mean	39,700				
	Standard Deviation	3,047				

Table 4-51. Calculation of the Initial Risk Associated with Condition of the Concrete Deck Element

Initial Risk Associated with Condition of the Concrete Deck Element						
Condition State		CS1	CS2	CS3	CS4	CS5
Risk Unit Cost ($\$/ft^2$)	Mean	0.27	0.52	1.13	4.63	22.25
	Standard Deviation	0.00	0.01	0.02	0.07	0.39
Quantity (ft^2)	Mean	1400	11300	680	630	70
	Standard Deviation	79.06	1,036.82	45.28	57.01	7.91
Risk Distribution ($)	Mean	372	5,916	770	2,917	1,557
	Standard Deviation	22	548	52	268	178
Total Initial Risk ($) Associated with Element Condition	Mean	11,532				
	Standard Deviation	638				

4.10 Risk Estimation at the Bridge Level

The initial risks associated with conditions of the concrete deck element, the steel girder elements, and the other elements of bridge B_1 are used to calculate the initial risk associated with condition of bridge B_1. The risks associated with conditions of elements 205, 215, and 234 are calculated using the same method used for elements 12 and 107. Assuming that conditions of elements 205, 215, and 234 are associated with risk values as shown in Table 4-52, the mean value for the initial risk associated with bridge B1 condition is calculated as the sum of the mean values for the initial risks associated with conditions of the five elements of the bridge. The standard deviation for the initial risk associated with the bridge condition is calculated as the square root of the sum of the squares of the standard deviations for the risks associated with elements conditions as shown in Table 4-52.

Table 4-52. Initial Risk Associated with Condition of Bridge B_1

Element ID	Element Description	Initial Risk ($) Associated with Element Condition	
		Mean	Standard Deviation
12	Bare Concrete Deck	11,532	638
107	Painted Steel Open Girder/Beam	39,700	3,047
205	Reinforced Concrete Column or Pile Extension	100,000	5,000
215	Reinforced Concrete Abutment	50,000	2,000
234	Reinforced Concrete Cap	80,000	2,500
Initial Risk ($) Associated with Condition of Bridge B_1		281,232	6,704

4.11 Element-Level Risk-Based Maintenance

The risk-based maintenance methodology for each element includes determining the optimal maintenance actions in the condition states of the element; identifying the maintenance scenarios for the element; evaluating costs and risks associated with the maintenance scenarios; filtering the maintenance scenarios; ranking the maintenance scenarios based on their economic efficiency, and selecting the optimal maintenance scenario for the element. The steps used in the risk-based methodology for the maintenance of the elements are demonstrated for both the concrete deck element (Element 12) and the steel girder element (Element 107) of bridge B_1 in the following sections.

4.12 Risk-Based Maintenance for Concrete Deck Element

The steps for the risk-based maintenance for the concrete deck element are explained in the following sections.

4.12.1 Optimal Maintenance Actions for Concrete Deck Element

The concrete deck element has two feasible actions in condition state 1 and three feasible actions in condition states 2, 3, 4 and 5 as shown in Table 4-53. A total of 162 scenarios for the possible set of maintenance actions in the different condition states were created. Each scenario is identified by the letter A followed by five digits, where each digit represents the number of the action used in the condition state. For example, if actions number 0, 1, 1, 2, 2 were used in condition states 1, 2, 3, 4, and 5, respectively, these actions are denoted $a(1,0)$, $a(2,1)$, $a(3,1)$, $a(4,2)$, and $a(5,2)$. The scenario of using these actions in the condition states is referred to as scenario A01122.

The cost of applying the maintenance actions and the risk associated with element conditions after applying the actions are calculated and added for each scenario of actions. A discount rate of 2.6% is used to convert the cost values at different times (OMB, 2007). The Markovian behavior assumed for condition states of the element implies that if maintenance actions are applied at a particular year, the change in the condition states happens after one year. Therefore, risks associated with element conditions are evaluated one year after applying the maintenance actions. The cost of maintenance actions after one year is increased by 102.6% to account for the value of time.

The scenario of actions in the condition states of the deck element that results in the lowest summation of maintenance cost and risk associated with the element condition is selected as the optimal scenario of possible actions. The set of maintenance actions in the five condition states represented by this scenario are considered optimal. Different years may result in different optimal actions; therefore, different scenarios of maintenance actions are created for each year of the planning period. The selection of optimal actions can be based on both the mean and the standard deviation for the summation of maintenance cost and risk associated with element conditions. If considered in the selection, the standard deviation for the sum of maintenance cost and risk can be calculated using the standard deviations for risk and maintenance cost shown in Equations 3-50 and 3-77. For demonstration purposes in the case study, it is assumed that the selection of the optimal actions is based on the mean value for the summation of the maintenance cost of actions and the risk associated with element conditions.

For the first year of maintenance, the 162 scenarios of actions in the condition states include scenario A00000 with minimum maintenance cost of \$0 and a risk value of \$14,815, and scenario A12222 with minimum risk of \$3,738 and a maintenance cost of \$152,828. Neither of the two scenarios is considered optimal. The best five scenarios that result in the lowest summation of risk and cost for the concrete deck are shown in Figure 4-22. . The application of the actions in these scenarios will result in new condition state distributions for the deck element. The condition state distribution of the deck element for these scenarios is shown in Figure 4-23. Scenario A00011 is associated with the least summation of maintenance cost and risk for the first year of maintenance. The optimal scenario of actions for each year of maintenance will be selected in this methodology as the one with the least summation of maintenance cost and risk. Therefore, scenario A00011 is selected as the scenario of optimal actions for the first year of maintenance.

Scenario A00011 implies that actions $a(1,0)$, $a(2,0)$, $a(3,0)$, $a(4,1)$ and $a(5,1)$ are applied in condition states 1,2,3, 4 and 5, respectively, for the first year of maintenance. Therefore, these actions are the optimal actions in the condition states of the concrete deck element. That means, out of the 162 possible scenarios of actions in the first year, the optimal scenario is to do nothing in condition states 1, 2 and 3, to repair deck spalls and cracks in condition state 4, and to repair deck spalls and cracks and add a protective system in condition state 5. The first-year optimal maintenance actions in the condition states of the concrete deck element are shown in Table 4-54.

The application of the optimal actions in the first year will result in a new condition state distribution for the deck element based on the transition probabilities associated with each action. The calculation of the condition state distribution for the deck element after applying the first-year optimal maintenance actions is shown in Table 4-55.

The calculation of the maintenance cost and risk associated with the deck element for scenario A00011 is shown in Table 4-56. The maintenance cost is calculated by multiplying the unit costs of actions by the element quantities in the corresponding condition states before applying the actions. The risk associated with the element condition after applying the actions is calculated by multiplying the risk unit costs by the element quantities in the corresponding condition states.

If the optimal actions of scenario A00011 were implemented in the first year, scenario A00010 will be associated with the lowest sum of risk and cost in the second year of maintenance. If the first-year optimal actions were deferred and the do-nothing scenario (A00000) was applied instead, scenario A00011 will result in the lowest sum of risk and cost in the second year. Therefore, the scenario with the optimal actions in the second year is scenario A00010 if the first-year optimal actions are applied, and scenario A00011 if the first-year optimal actions are delayed. The second-year scenarios of optimal actions are shown in Figure 4-24. The condition state distributions of the deck element associated with the second-year optimal scenarios are shown in Figure 4-25. Consequently, the optimal action in condition states 1, 2, and 3 is to do nothing in the second year of the deck maintenance. The optimal action in condition state 4 is to repair deck spalls and cracks. The optimal action in condition state 5 depends on the application of the first-year optimal action in condition state 5. If the first-year optimal action is applied in condition state 5, the optimal action is to do nothing. If the first-year optimal action in condition state 5 is delayed, the second-year optimal action is to repair deck cracks and add a protective system. The second-year optimal maintenance actions for the concrete deck element are shown in Table 4-57.

For the third year of maintenance, four cases of applying or delaying the optimal actions of previous years are considered. Scenario A00010 is associated with the lowest sum of cost and risk in two cases where the second-year optimal actions were applied. In the other two cases where second-year optimal actions were delayed, scenario A00011 is associated with the lowest sum of cost and risk. Therefore, the scenario with the optimal actions will be A00010 if the second-year optimal actions were applied and A00011 if the second-year optimal actions were delayed. The third-year scenarios of optimal actions are shown in Figure 4-26. The condition state distributions of the deck element associated with the third-year scenarios of optimal actions are shown in Figure 4-27.

It is clear that the selection of the third-year optimal actions depend on the application or delay of the second-year optimal actions only. The application or delay of the first-year optimal actions does not affect the selection of the third-year optimal action. Therefore, the optimal actions in the third year of maintenance are: to do nothing in condition states 1, 2, 3 and 5, and to repair deck spalls and cracks in condition states 4 if the optimal actions were applied in the second year (and first year). If the optimal actions were delayed in the second year (and the first year), the third-year optimal actions are: to do nothing in condition states 1, 2 and 3, to repair deck spalls and cracks in condition state 4, and to repair deck cracks and add a protective system in condition state 5. The third-year optimal maintenance actions for the concrete deck element are shown in Table 4-58.

For the fourth year of maintenance, eight cases of applying or delaying optimal actions of previous years are considered. Scenario A00010 is associated with the lowest sum of risk and maintenance costs in four cases where the third-year optimal actions were applied. In the other four cases, where the third-year optimal actions were delayed, scenario A00011 is associated with the lowest sum of risk and maintenance cost. Therefore, the optimal actions for the concrete deck maintenance are the set of actions in scenario A00010 if the optimal actions in the third year were applied and the set of actions in scenario A00011 if the optimal actions in the third year were delayed.

The four cases where third-year optimal actions were applied and scenario A00010 is selected as the scenario of the fourth-year optimal actions are shown in Figure 4-28. The four cases where third-year optimal actions were delayed and scenario A00011 is selected as the scenario of the fourth-year optimal actions are shown in Figure 4-29. The condition state distribution of the deck element associated with the fourth-year scenarios of optimal actions where third-year optimal actions were applied is shown in Figure 4-30. The condition state distribution of the deck element associated with the fourth-year scenarios of optimal actions where third-year optimal actions were delayed is shown in Figure 4-31.

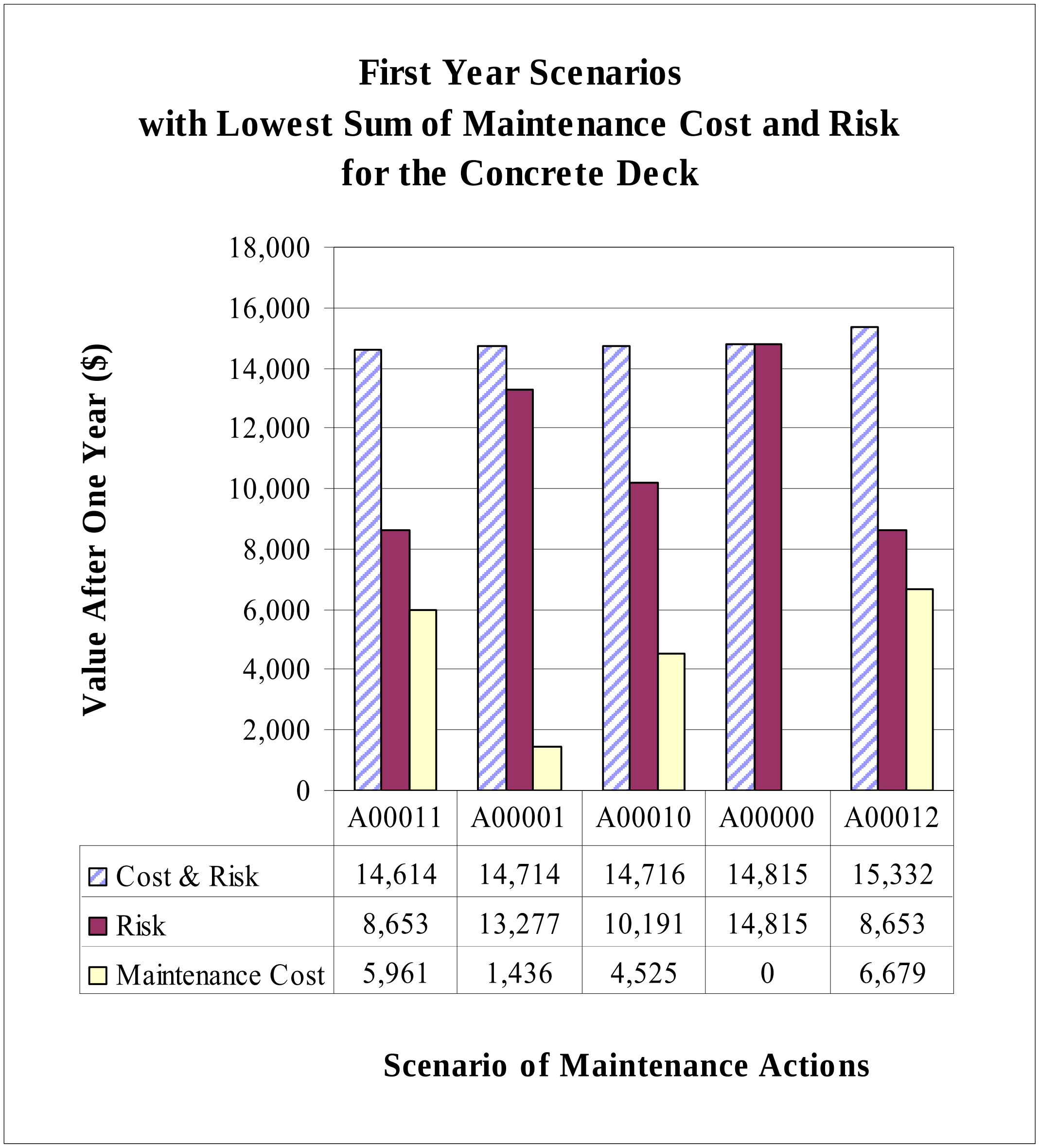
For the fifth year of maintenance, sixteen cases of applying or delaying the optimal actions of previous years are considered. Scenario A00010 is associated with the lowest sum of risk and maintenance cost in eight cases where the fourth-year optimal actions were applied. In the other eight cases where the fourth -year optimal actions were delayed, scenario A00011 is associated with the lowest sum of risk and maintenance cost. Therefore, the optimal actions for the concrete deck maintenance are the set of actions in scenario A00010 if the optimal actions in the fourth year were applied and the set of actions in scenario A00011 if the optimal actions in the fourth year were delayed.

The sixteen cases of the fifth-year optimal actions are shown in Table 4-59. The condition state distribution of the deck element associated with the fifth-year scenarios of optimal actions where fourth-year optimal actions were applied is shown in Figure 4-32. The condition state distribution of the deck element associated with the fifth-year scenarios of optimal actions where fourth-year optimal actions were delayed is shown in Figure 4-33.

Therefore, the optimal actions for the deck maintenance in each year of the five-year planning horizon are to repair the deck spalls and cracks in condition states 4 and to do nothing in condition states 1, 2 and 3. The optimal action in condition state 5 is to do nothing if the previous year optimal actions were applied, and to repair deck spalls and cracks and add a protective system if the previous year optimal actions were delayed. The risk-based optimal actions for the maintenance of the concrete deck element are shown in Table 4-60.

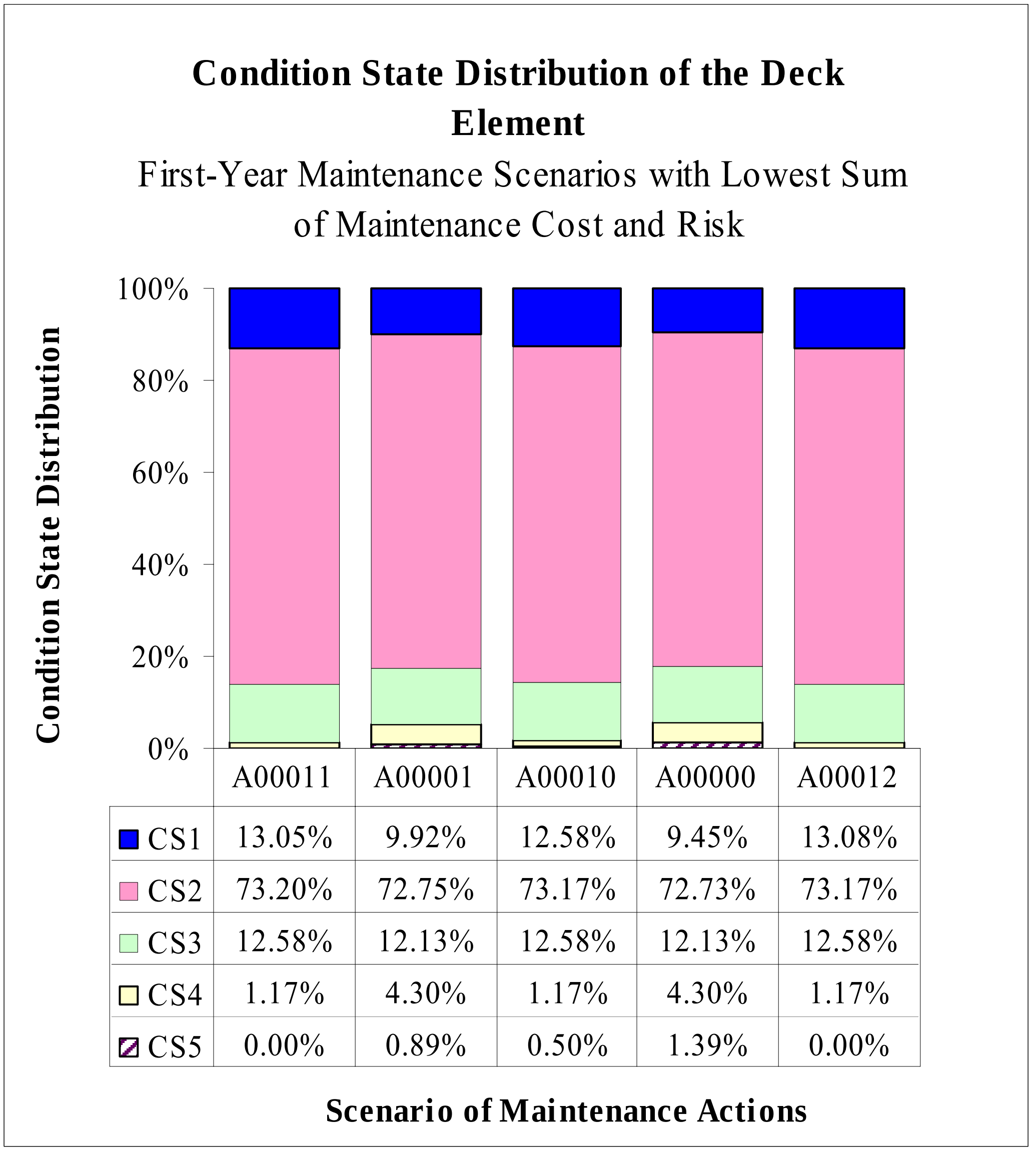
Table 4-53. Feasible Maintenance Actions for the Concrete Deck Element

Condition State	Maintenance Action		
	Action Number	Symbol	Description
1	0	$a(1,0)$	Do Nothing
	1	$a(1,1)$	Add a protective system
2	0	$a(2,0)$	Do Nothing
	1	$a(2,1)$	Repair spalls and cracks
	2	$a(2,2)$	Add a protective system
3	0	$a(3,0)$	Do Nothing
	1	$a(3,1)$	Repair spalls and cracks
	2	$a(3,2)$	Repair spalls and cracks and add a protective system
4	0	$a(4,0)$	Do Nothing
	1	$a(4,1)$	Repair spalls and cracks
	2	$a(4,2)$	Repair spalls and cracks and add a protective system
5	0	$a(5,0)$	Do Nothing
	1	$a(5,1)$	Repair spalls and cracks and add a protective system
	2	$a(5,2)$	Replace deck



	A00011	A00001	A00010	A00000	A00012
▨ Cost & Risk	14,614	14,714	14,716	14,815	15,332
■ Risk	8,653	13,277	10,191	14,815	8,653
□ Maintenance Cost	5,961	1,436	4,525	0	6,679

Figure 4-22. Scenarios of Actions with Lowest Summation of Maintenance Cost and Risk in the First Year of Maintenance for the Concrete Deck Element



	A00011	A00001	A00010	A00000	A00012
■ CS1	13.05%	9.92%	12.58%	9.45%	13.08%
■ CS2	73.20%	72.75%	73.17%	72.73%	73.17%
■ CS3	12.58%	12.13%	12.58%	12.13%	12.58%
□ CS4	1.17%	4.30%	1.17%	4.30%	1.17%
▨ CS5	0.00%	0.89%	0.50%	1.39%	0.00%

Figure 4-23. Condition State Distribution for the Concrete Deck Element for the Five Scenarios with Lowest Summation of Maintenance Cost and Risk in the First Year of Maintenance

Table 4-54. First-Year Optimal Maintenance Actions for the Concrete Deck Element

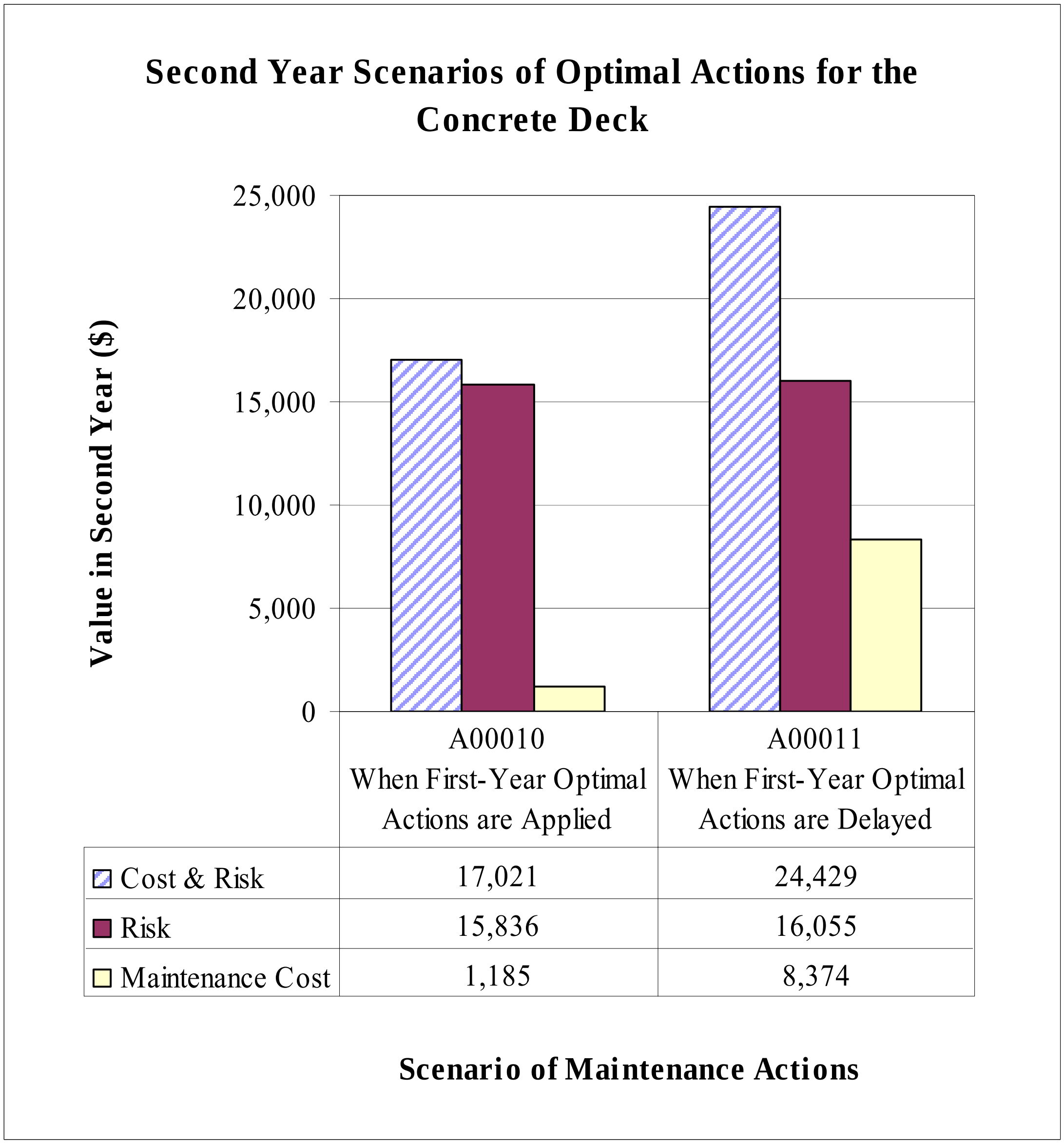
Condition State	Maintenance Action		
	Action Number	Symbol	Description
1	0	*a*(1,0)	Do Nothing
2	0	*a*(2,0)	Do Nothing
3	0	*a*(3,0)	Do Nothing
4	1	*a*(4,1)	Repair spalls and cracks
5	1	*a*(5,1)	Repair spalls and cracks and add a protective system

Table 4-55. Condition State Distribution of the Concrete Deck Element after Applying First-Year Optimal Maintenance Actions

Quantity Distribution Before Applying the Actions (ft^2)		Maintenance Action	Transition of the Deck Quantity				
Condition State	Quantity Distribution		1	2	3	4	5
1	1,400	*a*(1,0)	95%	5%	0%	0%	0%
			1,330	70	0	0	0
2	11,300	*a*(2,0)	0%	90%	10%	0%	0%
			0	10,170	1,130	0	0
3	680	*a*(3,1)	0%	0%	85%	15%	0%
			0	0	578	102	0
4	630	*a*(4,1)	70%	10%	10%	10%	0%
			441	63	63	63	0
5	70	*a*(5,1)	95%	5%	0%	0%	0%
			66.5	3.5	0	0	0
Quantity Distribution After Applying the Actions (ft^2)			1,837.5	10,306.5	1,771	165	0

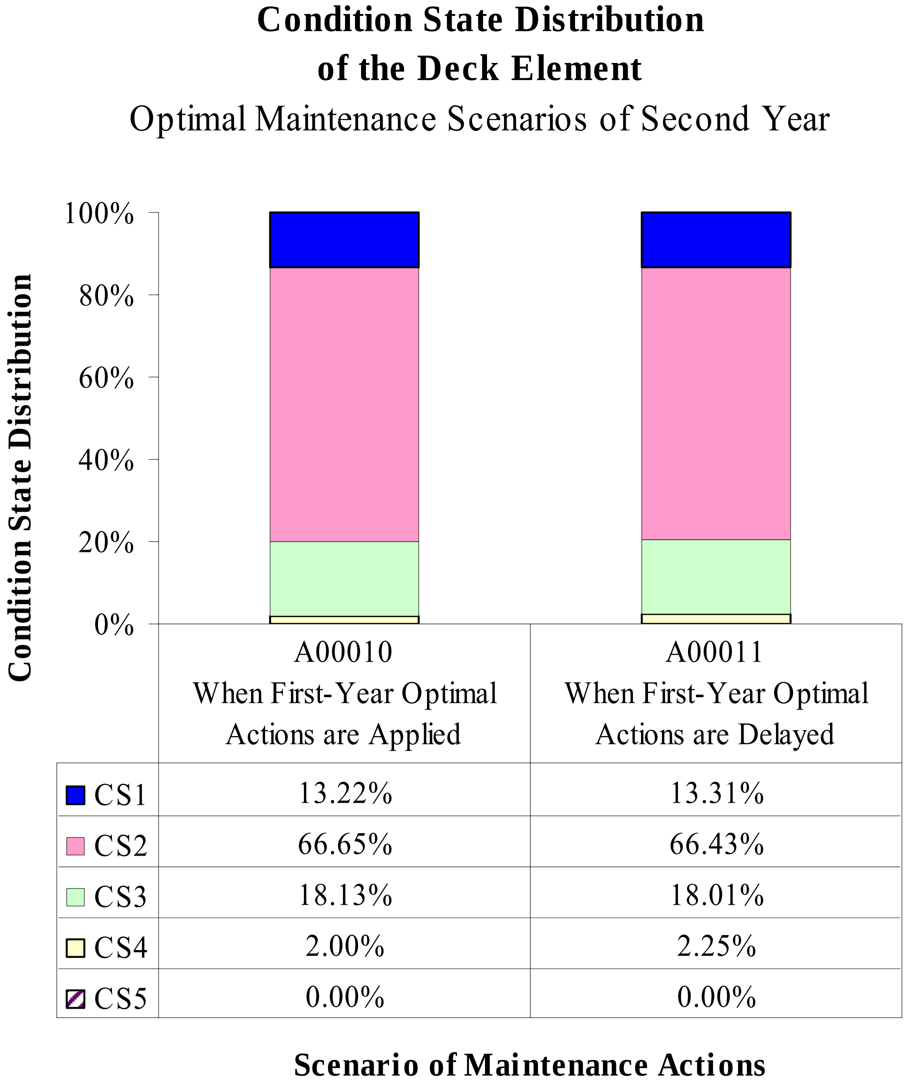
Table 4-56. Calculation of Maintenance Cost and Risk Associated with the First-Year Optimal Actions of the Concrete Deck Element

Discount Rate = 2.6%							Total
Condition State		CS1	CS2	CS3	CS4	CS5	
Maintenance Action		$a(1,0)$	$a(2,0)$	$a(3,0)$	$a(4,1)$	$a(5,1)$	
Unit Cost of Maintenance Action $(\$/\text{ft}^2)$ (Table 4-6)		0.00	0.00	0.00	7.00	20.00	
Unit Cost of Risk $(\$/\text{ft}^2)$ (Table 4-47)		0.27	0.52	1.13	4.63	22.25	
Quantity (ft^2)	Before Actions (Table 4-2)	1,400	11,300	680	630	70	14,080
	After Actions (Table 4-55)	1,837.5	10,306.5	1,771	165	0.0	14,080
Maintenance Cost (\$) When Actions are Applied		0.0	0.0	0.0	4,410	1,400	5,810
Maintenance Cost Value (\$) One Year After Applying the Actions		0.0	0.0	0.0	4,524.7	1,436.4	5,961.1
Risk (\$) Associated with the Deck Condition After Applying the Actions		487.8	5,395.6	2,005.9	764.1	0	8,653.4
Sum of Maintenance Cost and Associated Risk After Applying the Actions (\$)		487.8	5,395.6	2,005.9	5,288.8	1,436.4	14,614.5



	A00010 When First-Year Optimal Actions are Applied	A00011 When First-Year Optimal Actions are Delayed
▨ Cost & Risk	17,021	24,429
■ Risk	15,836	16,055
□ Maintenance Cost	1,185	8,374

Figure 4-24. Scenarios of Optimal Actions with Lowest Summation of Maintenance Cost and Risk in the Second Year of Maintenance for the Concrete Deck Element

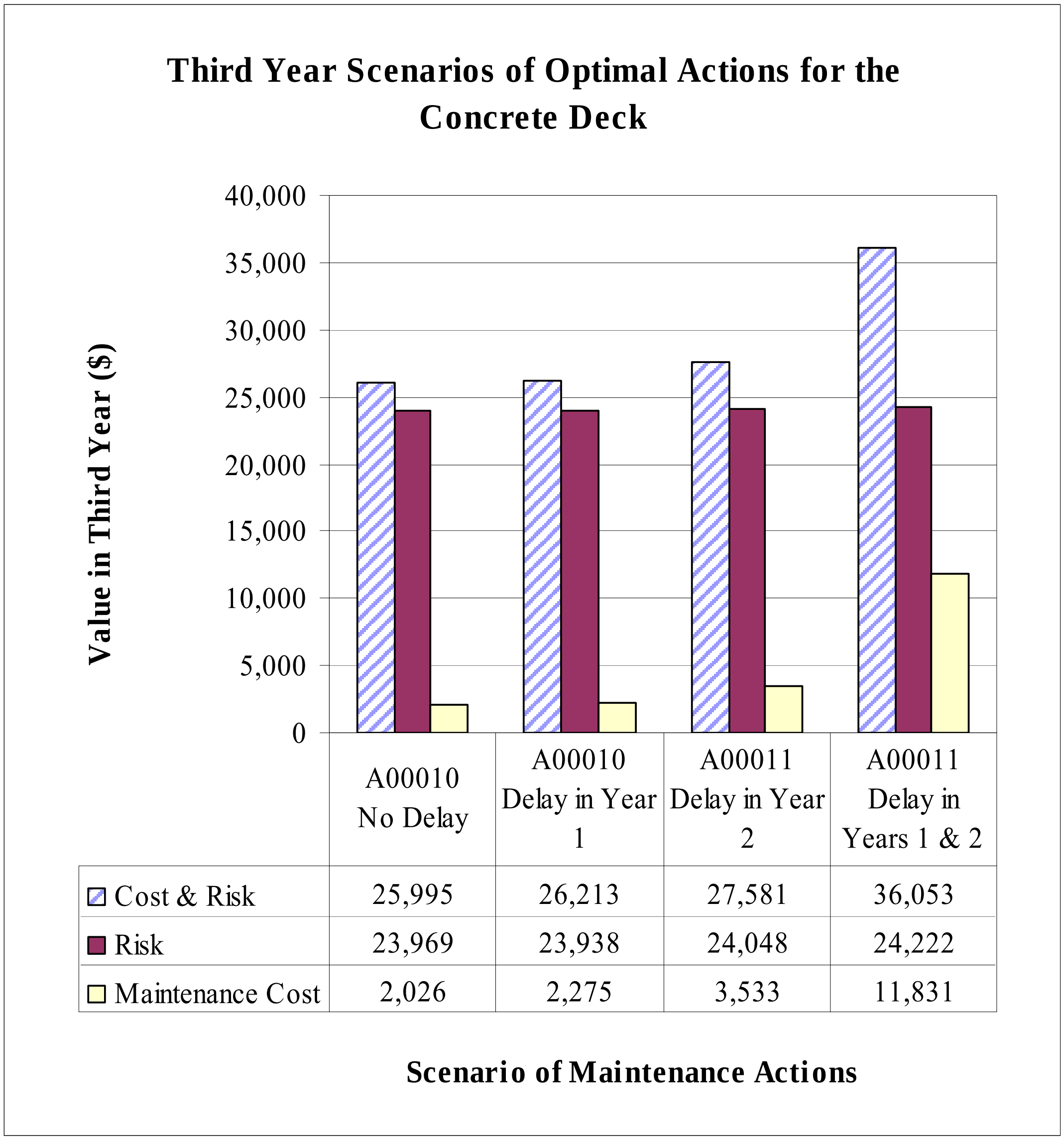


	A00010 When First-Year Optimal Actions are Applied	A00011 When First-Year Optimal Actions are Delayed
■ CS1	13.22%	13.31%
■ CS2	66.65%	66.43%
■ CS3	18.13%	18.01%
□ CS4	2.00%	2.25%
▨ CS5	0.00%	0.00%

Figure 4-25. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios with Lowest Summation of Maintenance Cost and Risk in the Second Year of Maintenance

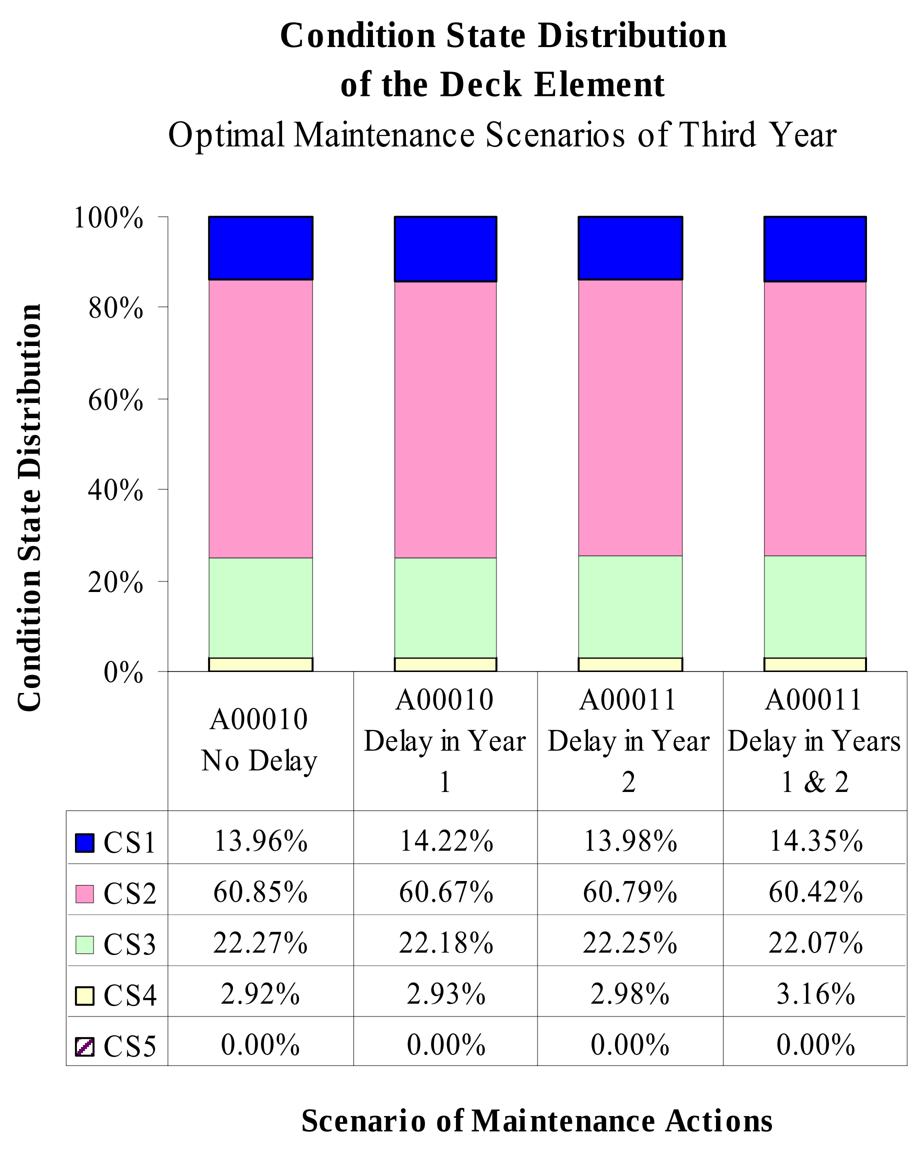
Table 4-57. Second-Year Optimal Maintenance Actions for the Concrete Deck Element

Condition State		Second-Year Optimal Maintenance Action		
		Action Number	Symbol	Description
1		0	$a(1,0)$	Do Nothing
2		0	$a(2,0)$	Do Nothing
3		0	$a(3,0)$	Do Nothing
4		1	$a(4,1)$	Repair spalls and cracks
5	If Firs-Year Optimal Maintenance Actions were Applied	0	$a(5,0)$	Do Nothing
	If Firs-Year Optimal Maintenance Actions were Delayed	1	$a(5,1)$	Repair spalls and cracks and add a protective system



	A00010 No Delay	A00010 Delay in Year 1	A00011 Delay in Year 2	A00011 Delay in Years 1 & 2
▨ Cost & Risk	25,995	26,213	27,581	36,053
■ Risk	23,969	23,938	24,048	24,222
☐ Maintenance Cost	2,026	2,275	3,533	11,831

Figure 4-26. Scenarios of Optimal Actions with Lowest Summation of Maintenance Cost and Risk in the Third Year of Maintenance for the Concrete Deck Element

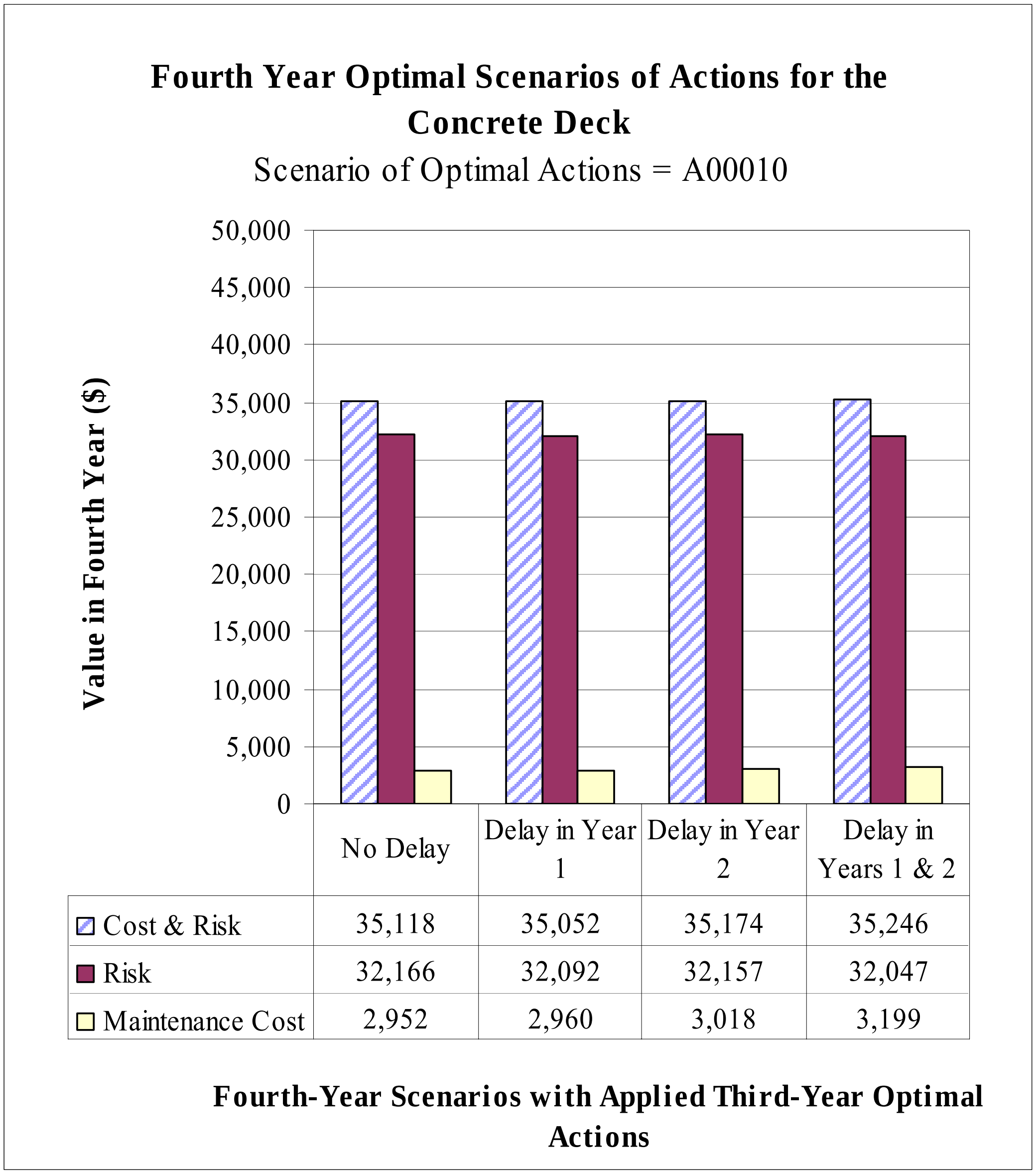


	A00010 No Delay	A00010 Delay in Year 1	A00011 Delay in Year 2	A00011 Delay in Years 1 & 2
■ CS1	13.96%	14.22%	13.98%	14.35%
■ CS2	60.85%	60.67%	60.79%	60.42%
■ CS3	22.27%	22.18%	22.25%	22.07%
□ CS4	2.92%	2.93%	2.98%	3.16%
▨ CS5	0.00%	0.00%	0.00%	0.00%

Figure 4-27. Condition State Distribution for the Concrete Deck Element Associated with Optimal Scenarios of Lowest Summation of Maintenance Cost and Risk in the Third Year of Maintenance

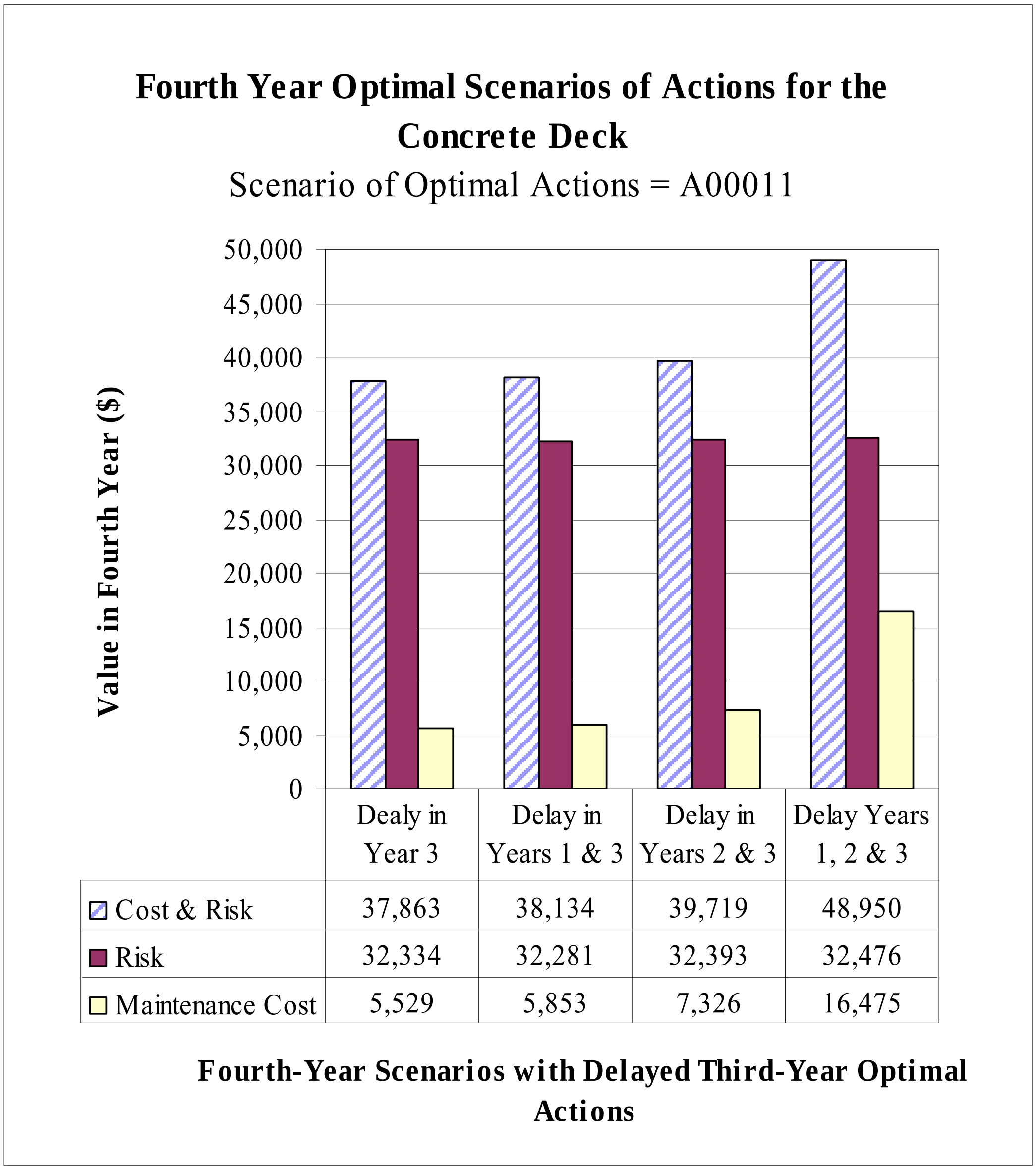
Table 4-58. Third-Year Optimal Maintenance Actions for the Concrete Deck Element

Condition State		Second-Year Optimal Maintenance Action		
		Action Number	Symbol	Description
1		0	$a(1,0)$	Do Nothing
2		0	$a(2,0)$	Do Nothing
3		0	$a(3,0)$	Do Nothing
4		1	$a(4,1)$	Repair spalls and cracks
5	If Second-Year Optimal Maintenance Actions were Applied	0	$a(5,0)$	Do Nothing
	If Second-Year Optimal Maintenance Actions were Delayed	1	$a(5,1)$	Repair spalls and cracks and add a protective system



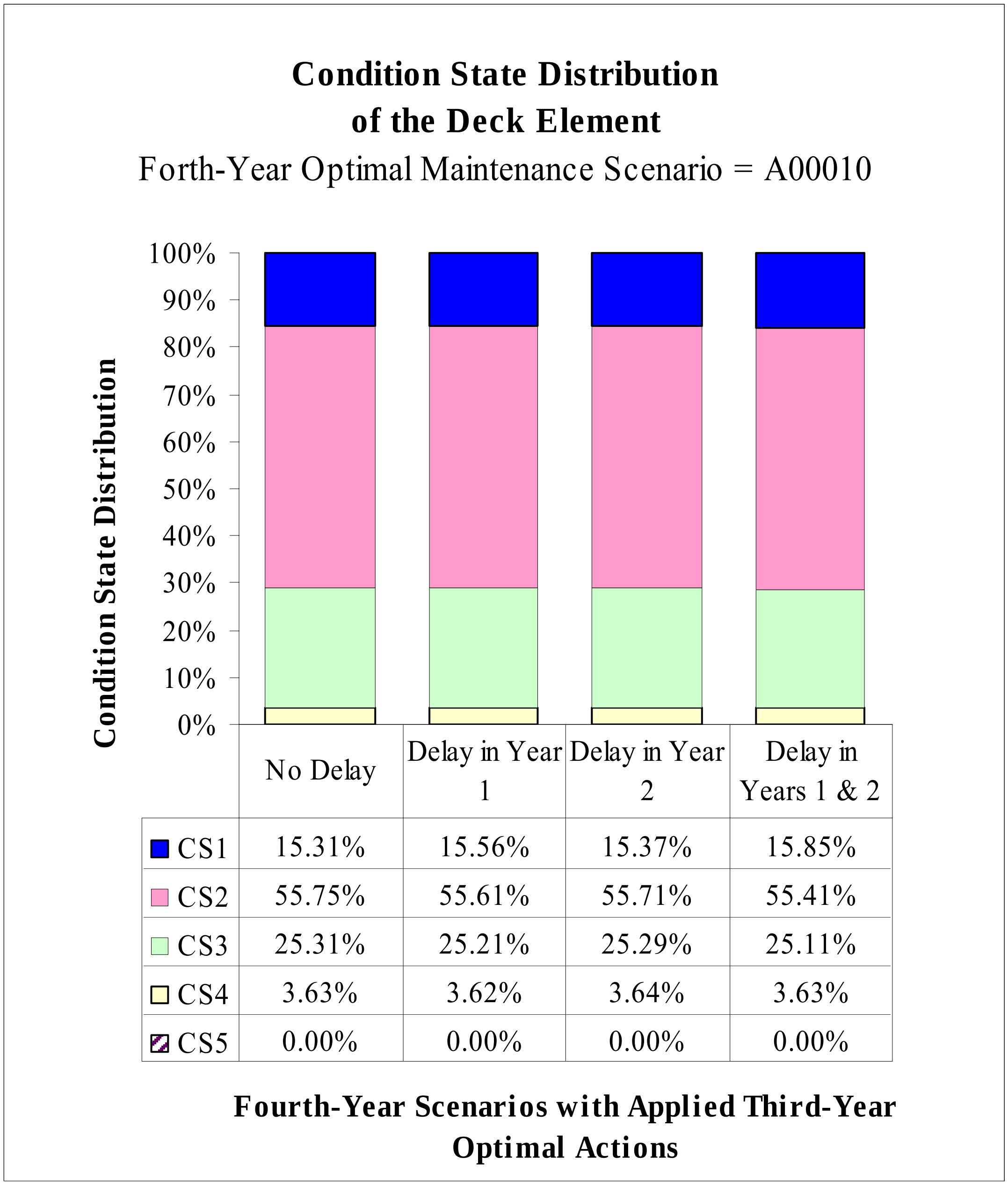
	No Delay	Delay in Year 1	Delay in Year 2	Delay in Years 1 & 2
▨ Cost & Risk	35,118	35,052	35,174	35,246
■ Risk	32,166	32,092	32,157	32,047
☐ Maintenance Cost	2,952	2,960	3,018	3,199

Figure 4-28. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Concrete Deck Maintenance Where Third-Year Optimal Actions Were Applied



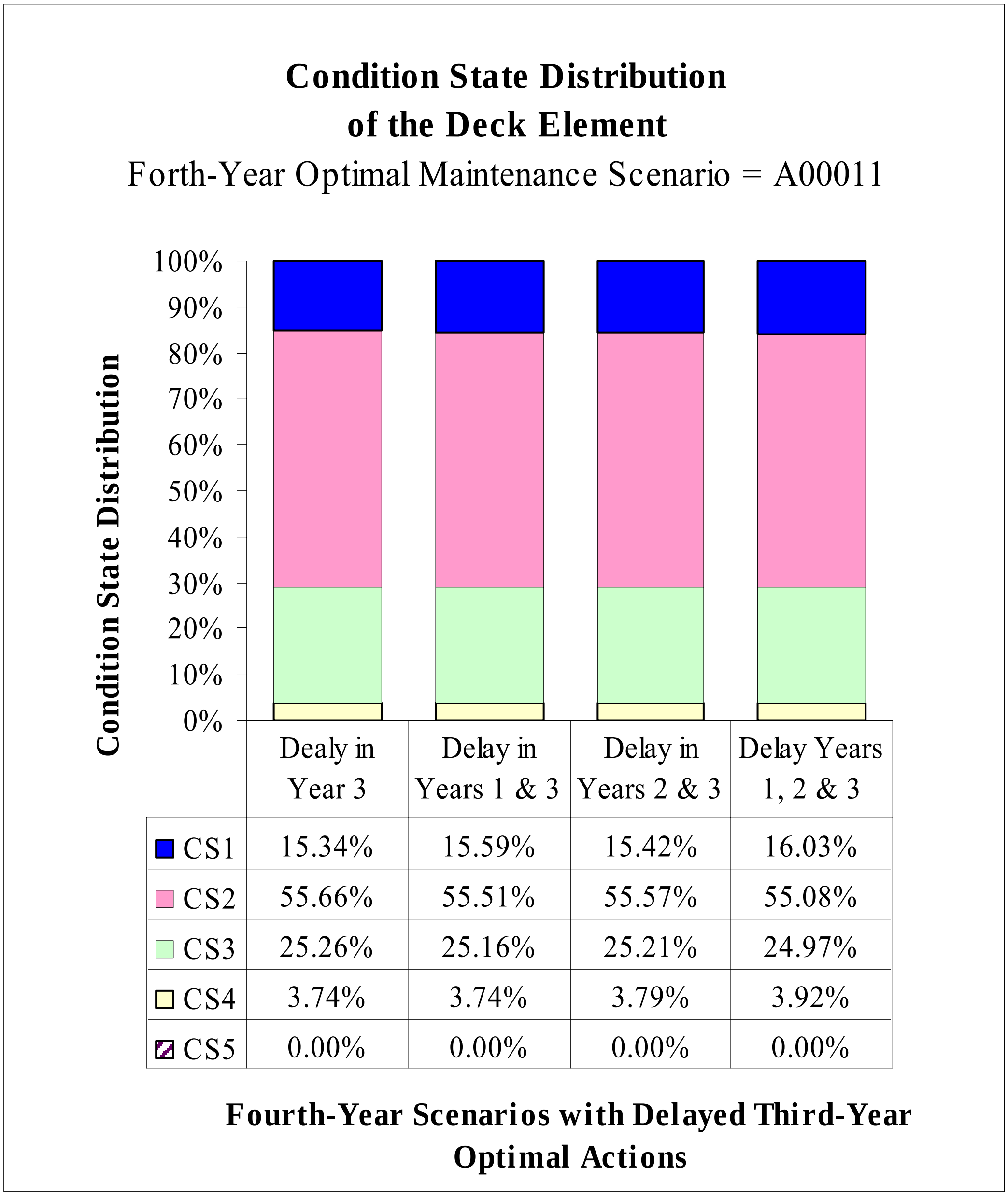
	Dealy in Year 3	Delay in Years 1 & 3	Delay in Years 2 & 3	Delay Years 1, 2 & 3
▨ Cost & Risk	37,863	38,134	39,719	48,950
■ Risk	32,334	32,281	32,393	32,476
□ Maintenance Cost	5,529	5,853	7,326	16,475

Figure 4-29. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Concrete Deck Maintenance Where Third-Year Optimal Actions Were Delayed



	No Delay	Delay in Year 1	Delay in Year 2	Delay in Years 1 & 2
■ CS1	15.31%	15.56%	15.37%	15.85%
■ CS2	55.75%	55.61%	55.71%	55.41%
■ CS3	25.31%	25.21%	25.29%	25.11%
□ CS4	3.63%	3.62%	3.64%	3.63%
▨ CS5	0.00%	0.00%	0.00%	0.00%

Figure 4-30. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of Maintenance with Applied Third-Year Optimal Actions

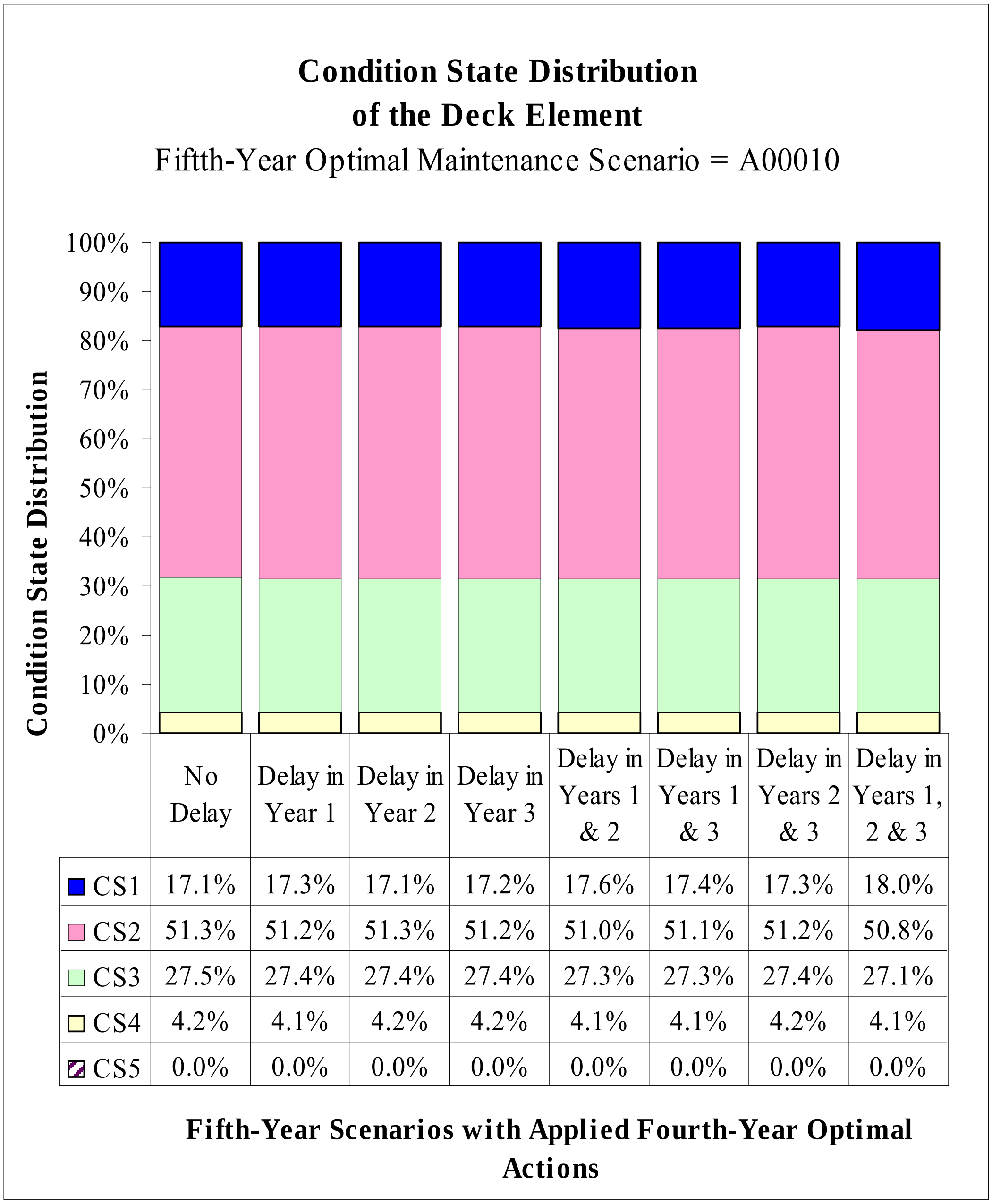


	Dealy in Year 3	Delay in Years 1 & 3	Delay in Years 2 & 3	Delay Years 1, 2 & 3
🟦 CS1	15.34%	15.59%	15.42%	16.03%
🟪 CS2	55.66%	55.51%	55.57%	55.08%
🟩 CS3	25.26%	25.16%	25.21%	24.97%
🟨 CS4	3.74%	3.74%	3.79%	3.92%
▨ CS5	0.00%	0.00%	0.00%	0.00%

Figure 4-31. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of Maintenance with Delayed Third-Year Optimal Actions

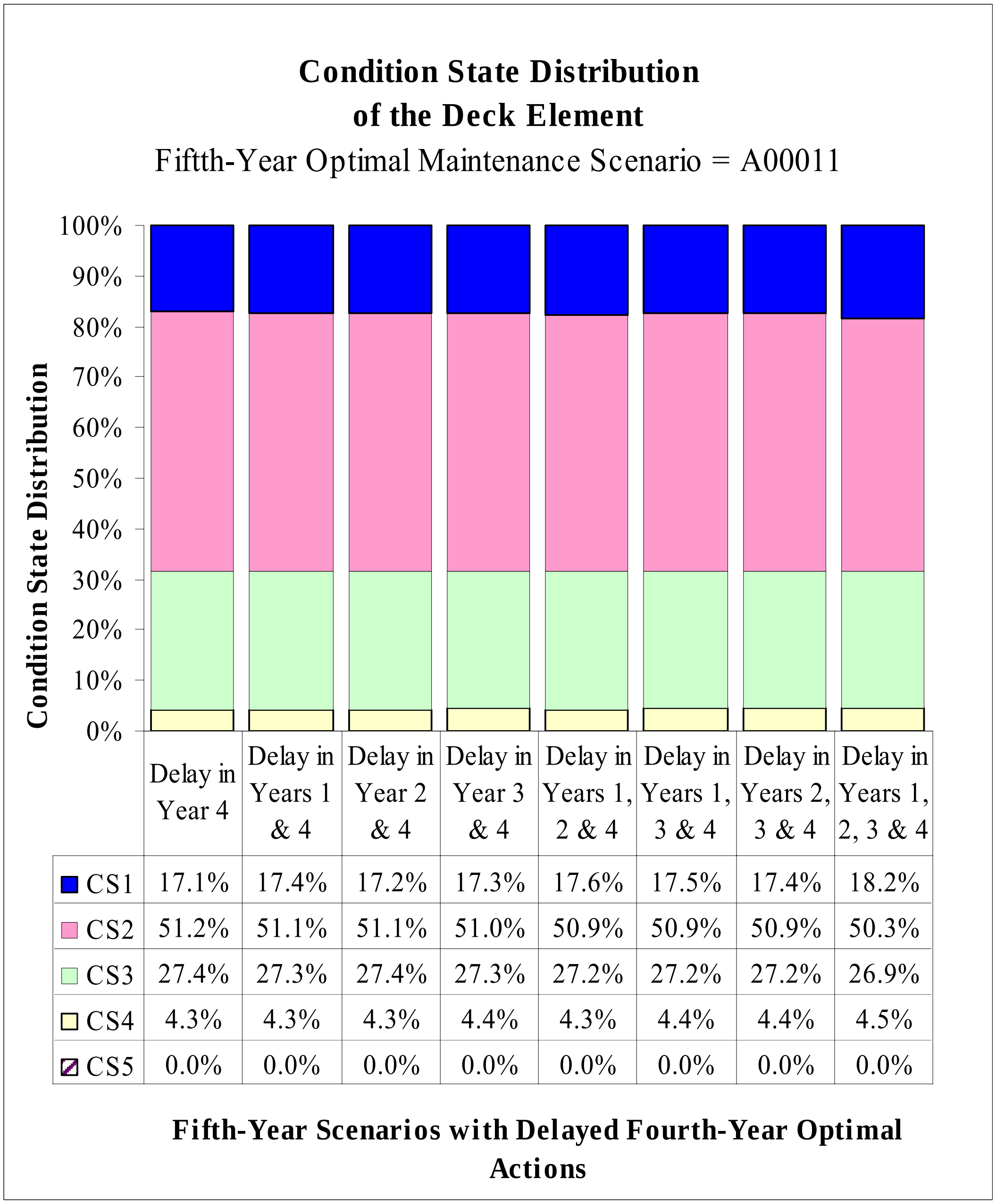
Table 4-59. Fifth-Year Optimal Maintenance Actions for the Concrete Deck Element

Years of Delayed Optimal Actions	Risk	Maintenance Cost	Cost & Risk	Fifth-Year Scenario of Optimal Actions for the Deck
None	40,041	3,674	43,715	A00010
Year 1	39,951	3,660	43,612	A00010
Year 2	40,020	3,676	43,696	A00010
Year 3	40,023	3,785	43,808	A00010
Years 1 & 2	39,856	3,667	43,523	A00010
Years 1 & 3	39,930	3,785	43,716	A00010
Years 2 & 3	39,991	3,833	43,824	A00010
Years 1, 2 & 3	39,785	3,960	43,744	A00010
Year 4	40,331	7,428	47,759	A00011
Years 1 & 4	40,242	7,424	47,666	A00011
Year 2 & 4	40,316	7,513	47,829	A00011
Year 3 & 4	40,449	10,501	50,949	A00011
Years 1, 2 & 4	40,170	7,735	47,905	A00011
Years 1, 3 & 4	40,374	10,874	51,248	A00011
Years 2, 3 & 4	40,477	12,521	52,998	A00011
Years 1, 2, 3 & 4	40,439	22,314	62,753	A00011



	No Delay	Delay in Year 1	Delay in Year 2	Delay in Year 3	Delay in Years 1 & 2	Delay in Years 1 & 3	Delay in Years 2 & 3	Delay in Years 1, 2 & 3
■ CS1	17.1%	17.3%	17.1%	17.2%	17.6%	17.4%	17.3%	18.0%
■ CS2	51.3%	51.2%	51.3%	51.2%	51.0%	51.1%	51.2%	50.8%
■ CS3	27.5%	27.4%	27.4%	27.4%	27.3%	27.3%	27.4%	27.1%
□ CS4	4.2%	4.1%	4.2%	4.2%	4.1%	4.1%	4.2%	4.1%
▨ CS5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Figure 4-32. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios of Lowest Summation of Maintenance Cost and Risk in the Fifth Year of Maintenance with Applied Fourth-Year Optimal Actions



	Delay in Year 4	Delay in Years 1 & 4	Delay in Year 2 & 4	Delay in Year 3 & 4	Delay in Years 1, 2 & 4	Delay in Years 1, 3 & 4	Delay in Years 2, 3 & 4	Delay in Years 1, 2, 3 & 4
CS1	17.1%	17.4%	17.2%	17.3%	17.6%	17.5%	17.4%	18.2%
CS2	51.2%	51.1%	51.1%	51.0%	50.9%	50.9%	50.9%	50.3%
CS3	27.4%	27.3%	27.4%	27.3%	27.2%	27.2%	27.2%	26.9%
CS4	4.3%	4.3%	4.3%	4.4%	4.3%	4.4%	4.4%	4.5%
CS5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Figure 4-33. Condition State Distribution for the Concrete Deck Element Associated with the Optimal Scenarios of Lowest Summation of Maintenance Cost and Risk in the Fifth Year of Maintenance with Delayed Fourth-Year Optimal Actions

Table 4-60. Risk-Based Optimal Maintenance Actions for the Concrete Deck Element in Each of the Five Years of Maintenance

Condition State		Optimal Maintenance Action		
		Action Number	Symbol	Description
1		0	$a(1,0)$	Do Nothing
2		0	$a(2,0)$	Do Nothing
3		0	$a(3,0)$	Do Nothing
4		1	$a(4,1)$	Repair spalls and cracks
5	If Previous-Year Optimal Maintenance Actions were Applied	0	$a(5,0)$	Do Nothing
	If Previous-Year Optimal Maintenance Actions were Delayed	1	$a(5,1)$	Repair spalls and cracks and add a protective system

4.12.2 Maintenance Scenarios of Implementing/Deferring Optimal Maintenance Actions for Concrete Deck Element

To study the economic efficiency of applying or delaying the optimal actions for the maintenance of the concrete deck element, 32 scenarios were created during the five years of maintenance. Each scenario is represented by five digits of ones and zeros to represent the application or delay of the optimal actions in the five years of maintenance. For example, scenario 01011 means delaying the first and third-year optimal actions and applying the second, fourth and fifth-year optimal actions during the five years maintenance period.

Each scenario is composed of five actions in the five years of maintenance. The action in each year is either to apply the set of optimal actions for that year or delay the optimal actions and choose the no-maintenance option, A00000. The optimal actions in each year of maintenance are selected using the method described in the previous section.

The deck maintenance scenarios and their corresponding optimal actions during the five years planning horizon are shown in Table 4-61. Table 4-61 shows that, for example, scenario 01011 is composed of the actions A00000, A00011, A00000, A00011 and A00010 in years 1, 2, 3, 4, and 5, respectively. Therefore, scenario 01011 refers to doing nothing in the first and third year, applying optimal actions $a(1,0)$, $a(2,0)$, $a(3,0)$, $a(4,1)$, $a(5,1)$ in condition states 1, 2, 3, 4, 5 in the first and fourth year, and applying optimal actions $a(1,0)$, $a(2,0)$, $a(3,0)$, $a(4,1)$, $a(5,0)$ in condition states 1, 2, 3, 4, 5 in the fifth year.

Table 4-61. Five Years Maintenance Actions for the Concrete Deck Element for Different Cases of Delay and Application of Optimal Actions

Years of Delayed Optimal Actions	Scenario	Set of Maintenance Actions				
		Year 1	Year 2	Year 3	Year 4	Year 5
No Delay	11111	A00011	A00010	A00010	A00010	A00010
Year 1	01111	A00000	A00011	A00010	A00010	A00010
Year 2	10111	A00011	A00000	A00011	A00010	A00010
Year 3	11011	A00011	A00010	A00000	A00011	A00010
Year 4	11101	A00011	A00010	A00010	A00000	A00011
Years 1 & 2	00111	A00000	A00000	A00011	A00010	A00010
Years 1 & 3	01011	A00000	A00011	A00000	A00011	A00010
Years 1 & 4	01101	A00000	A00011	A00010	A00000	A00011
Years 2 & 3	10011	A00011	A00000	A00000	A00011	A00010
Year 2 & 4	10101	A00011	A00000	A00011	A00000	A00011
Year 3 & 4	11001	A00011	A00010	A00000	A00000	A00011
Years 1, 2 & 3	00011	A00000	A00000	A00000	A00011	A00010
Years 1, 2 & 4	00101	A00000	A00000	A00011	A00000	A00011
Years 1, 3 & 4	01001	A00000	A00011	A00000	A00000	A00011
Years 2, 3 & 4	10001	A00011	A00000	A00000	A00000	A00011
Years 1, 2, 3 & 4	00001	A00000	A00000	A00000	A00000	A00011
Year 5	11110	A00011	A00010	A00010	A00010	A00000
Year 1 & 5	01110	A00000	A00011	A00010	A00010	A00000
Year 2 & 5	10110	A00011	A00000	A00011	A00010	A00000
Year 3 & 5	11010	A00011	A00010	A00000	A00011	A00000
Year 4 & 5	11100	A00011	A00010	A00010	A00000	A00000
Years 1, 2 & 5	00110	A00000	A00000	A00011	A00010	A00000
Years 1, 3 & 5	01010	A00000	A00011	A00000	A00011	A00000
Years 1, 4 & 5	01100	A00000	A00011	A00010	A00000	A00000
Years 2, 3 & 5	10010	A00011	A00000	A00000	A00011	A00000
Year 2, 4 & 5	10100	A00011	A00000	A00011	A00000	A00000
Year 3, 4 & 5	11000	A00011	A00010	A00000	A00000	A00000
Years 1, 2, 3 & 5	00010	A00000	A00000	A00000	A00011	A00000
Years 1, 2, 4 & 5	00100	A00000	A00000	A00011	A00000	A00000
Years 1, 3, 4 & 5	01000	A00000	A00011	A00000	A00000	A00000
Years 2, 3, 4 & 5	10000	A00011	A00000	A00000	A00000	A00000
Years 1, 2, 3, 4 & 5	00000	A00000	A00000	A00000	A00000	A00000

4.12.3 Cost and Risk Associated with Maintenance Scenarios of Concrete Deck Element

The maintenance cost is calculated in each of the scenarios based on the unit cost of actions and the condition state distribution of the deck element. The present values for the maintenance costs in the five years are evaluated based on a real discount rate of 2.6% (OMB, 2007). The total present value of the maintenance cost for each scenario is calculated as shown in Table 4-62. Table 4-62 shows, for example, that the scenario of delaying the first, second, and third-year optimal actions, and applying the fourth and fifth-year optimal actions requires a maintenance cost of \$16,057 in the fourth year and \$3,859 in the fifth year, while the scenario of applying the first, second, and third-year optimal actions, and delaying the fourth and fifth-year optimal actions requires a maintenance cost of \$5,810 in the first year, \$1,155 in the second year, and \$1,975 in the third year. The total present value of maintenance cost is \$18,350 for the first scenario with delay in the first three years and \$8,812 for the second scenario with delay in the last two years.

For each year of a maintenance scenario, the risk associated with the deck condition is calculated based on the risk unit costs in the condition states and the condition state distribution of the element after applying the maintenance actions. The risk values during the five years of the maintenance scenarios are shown in Table 4-63. Table 4-63 shows, for example, that applying the first-year optimal actions (A00011) will result in a risk value of \$8,653, while delaying the first-year optimal actions and choosing the no-maintenance option (A00000) will result in a risk value of \$14,815.

Table 4-62. Maintenance Cost for the Different Maintenance Scenarios of Concrete Deck Element

Years of Delayed Optimal Actions	Maintenance Cost of the Concrete Deck ($)					Total Present Cost ($)
	Year 1	Year 2	Year 3	Year 4	Year 5	
No Delay	5,810	1,155	1,975	2,878	3,581	14,708
Year 1	0	8,162	2,218	2,885	3,568	15,952
Year 2	5,810	0	3,444	2,941	3,583	15,038
Year 3	5,810	1,155	0	5,389	3,689	15,255
Year 4	5,810	1,155	1,975	0	7,240	15,345
Years 1 & 2	0	0	11,531	3,118	3,574	17,066
Years 1 & 3	0	8,162	0	5,705	3,690	16,566
Years 1 & 4	0	8,162	2,218	0	7,236	16,591
Years 2 & 3	5,810	0	0	7,140	3,736	15,792
Year 2 & 4	5,810	0	3,444	0	7,322	15,689
Year 3 & 4	5,810	1,155	0	0	10,235	16,172
Years 1, 2 & 3	0	0	0	16,057	3,859	18,350
Years 1, 2 & 4	0	0	11,531	0	7,539	17,757
Years 1, 3 & 4	0	8,162	0	0	10,598	17,519
Years 2, 3 & 4	5,810	0	0	0	12,203	16,823
Years 1, 2, 3 & 4	0	0	0	0	21,748	19,626
Year 5	5,810	1,155	1,975	2,878	0	11,476
Year 1 & 5	0	8,162	2,218	2,885	0	12,733
Year 2 & 5	5,810	0	3,444	2,941	0	11,804
Year 3 & 5	5,810	1,155	0	5,389	0	11,925
Year 4 & 5	5,810	1,155	1,975	0	0	8,812
Years 1, 2 & 5	0	0	11,531	3,118	0	13,841
Years 1, 3 & 5	0	8,162	0	5,705	0	13,237
Years 1, 4 & 5	0	8,162	2,218	0	0	10,062
Years 2, 3 & 5	5,810	0	0	7,140	0	12,421
Year 2, 4 & 5	5,810	0	3,444	0	0	9,081
Year 3, 4 & 5	5,810	1,155	0	0	0	6,936
Years 1, 2, 3 & 5	0	0	0	16,057	0	14,867
Years 1, 2, 4 & 5	0	0	11,531	0	0	10,954
Years 1, 3, 4 & 5	0	8,162	0	0	0	7,955
Years 2, 3, 4 & 5	5,810	0	0	0	0	5,810
Years 1, 2, 3, 4 & 5	0	0	0	0	0	0

Table 4-63. Risk Associated with the Concrete Deck Condition during the Five Years of Maintenance

Years of Delayed Optimal Actions	Annual Risk Associated with the Concrete Deck Condition($)				
	Year 1	Year 2	Year 3	Year 4	Year 5
No Delay	8,653	15,836	23,969	32,166	40,041
Year 1	14,815	16,055	23,938	32,092	39,951
Year 2	8,653	17,875	24,048	32,157	40,020
Year 3	8,653	15,836	28,685	32,334	40,023
Year 4	8,653	15,836	23,969	40,615	40,331
Years 1 & 2	14,815	30,773	24,222	32,047	39,856
Years 1 & 3	14,815	16,055	29,234	32,281	39,930
Years 1 & 4	14,815	16,055	23,938	40,563	40,242
Years 2 & 3	8,653	17,875	32,325	32,393	39,991
Year 2 & 4	8,653	17,875	24,048	40,792	40,316
Year 3 & 4	8,653	15,836	28,685	48,237	40,449
Years 1, 2 & 3	14,815	30,773	52,278	32,476	39,785
Years 1, 2 & 4	14,815	30,773	24,222	41,203	40,170
Years 1, 3 & 4	14,815	16,055	29,234	49,120	40,374
Years 2, 3 & 4	8,653	17,875	32,325	53,517	40,477
Years 1, 2, 3 & 4	14,815	30,773	52,278	80,281	40,439
Year 5	8,653	15,836	23,969	32,166	52,313
Year 1 & 5	14,815	16,055	23,938	32,092	52,177
Year 2 & 5	8,653	17,875	24,048	32,157	52,299
Year 3 & 5	8,653	15,836	28,685	32,334	52,667
Year 4 & 5	8,653	15,836	23,969	40,615	65,225
Years 1, 2 & 5	14,815	30,773	24,222	32,047	52,104
Years 1, 3 & 5	14,815	16,055	29,234	32,281	52,574
Years 1, 4 & 5	14,815	16,055	23,938	40,563	65,122
Years 2, 3 & 5	8,653	17,875	32,325	32,393	52,794
Year 2, 4 & 5	8,653	17,875	24,048	40,792	65,495
Year 3, 4 & 5	8,653	15,836	28,685	48,237	75,703
Years 1, 2, 3 & 5	14,815	30,773	52,278	32,476	53,010
Years 1, 2, 4 & 5	14,815	30,773	24,222	41,203	66,096
Years 1, 3, 4 & 5	14,815	16,055	29,234	49,120	76,887
Years 2, 3, 4 & 5	8,653	17,875	32,325	53,517	82,554
Years 1, 2, 3, 4 & 5	14,815	30,773	52,278	80,281	115,637

4.12.4 Filtering Maintenance Scenarios for Concrete Deck Element

Risk and cost limitations are assumed for filtering the maintenance scenario of the concrete deck element. The maximum risk value allowed for the deck condition in any year of the maintenance period is assumed to be \$50,000. The annual budget for the maintenance cost of the deck is assumed not to exceed \$10,000.

Based on the above-mentioned criteria, 23 scenarios out of the 32 deck maintenance scenarios will be eliminated. Table 4-64 shows the maintenance scenarios that are eliminated because they exceed the maximum risk allowed for the deck condition. Table 4-65 shows the maintenance scenarios that are eliminated because they exceed the annual budget for the deck maintenance.

Table 4-64. Deck Maintenance Scenarios Exceeding Maximum Risk Value Allowed for the Concrete Deck Condition

Years of Delayed Optimal Actions	Risk Associated with the Deck Element Condition($)				
	Year 1	Year 2	Year 3	Year 4	Year 5
Year 5	8,653	15,836	23,969	32,166	52,313
Year 1 & 5	14,815	16,055	23,938	32,092	52,177
Year 2 & 5	8,653	17,875	24,048	32,157	52,299
Year 3 & 5	8,653	15,836	28,685	32,334	52,667
Year 4 & 5	8,653	15,836	23,969	40,615	65,225
Years 1, 2 & 3	14,815	30,773	52,278	32,476	39,785
Years 1, 2 & 5	14,815	30,773	24,222	32,047	52,104
Years 1, 3 & 5	14,815	16,055	29,234	32,281	52,574
Years 1, 4 & 5	14,815	16,055	23,938	40,563	65,122
Years 2, 3 & 4	8,653	17,875	32,325	53,517	40,477
Years 2, 3 & 5	8,653	17,875	32,325	32,393	52,794
Year 2, 4 & 5	8,653	17,875	24,048	40,792	65,495
Year 3, 4 & 5	8,653	15,836	28,685	48,237	75,703
Years 1, 2, 3 & 4	14,815	30,773	52,278	80,281	40,439
Years 1, 2, 3 & 5	14,815	30,773	52,278	32,476	53,010
Years 1, 2, 4 & 5	14,815	30,773	24,222	41,203	66,096
Years 1, 3, 4 & 5	14,815	16,055	29,234	49,120	76,887
Years 2, 3, 4 & 5	8,653	17,875	32,325	53,517	82,554
Years 1, 2, 3, 4 & 5	14,815	30,773	52,278	80,281	115,637

Table 4-65. Deck Maintenance Scenarios Exceeding the Annual Budget for the Concrete Deck Maintenance

Years of Delayed Optimal Actions	Annual Maintenance Cost ($) of the Deck Element				
	Year 1	Year 2	Year 3	Year 4	Year 5
Years 1 & 2	0	0	11,531	3,118	3,574
Year 3 & 4	5,810	1,155	0	0	10,235
Years 1, 2 & 3	0	0	0	16,057	3,859
Years 1, 2 & 4	0	0	11,531	0	7,539
Years 1, 3 & 4	0	8,162	0	0	10,598
Years 2, 3 & 4	5,810	0	0	0	12,203
Years 1, 2 & 5	0	0	11,531	3,118	0
Years 1, 2, 3 & 4	0	0	0	0	21,748
Years 1, 2, 3 & 5	0	0	0	16,057	0
Years 1, 2, 4 & 5	0	0	11,531	0	0

4.12.5 Ranking Maintenance Scenarios for Concrete Deck Element

The risk reduction in each year of a maintenance scenario is calculated as the non-mitigated risk minus the mitigated risk. The non-mitigated risk in any particular year results from delaying the optimal actions and applying the no-maintenance option (A00000) to the element in that year. The mitigated risk results from applying the optimal actions to the element in that year. The risk reduction in each of the five years is calculated for the different scenarios of the deck maintenance. The present values for the risk reduction in the five years are evaluated based on a discount rate of 2.6%. The total present value of risk reduction associated with each maintenance scenario is calculated as shown in Table 4-66.

Benefit-cost analysis is used to study the economic efficiency of the filtered maintenance scenarios. The following benefit-cost measures are calculated and compared for the different maintenance scenarios of the concrete deck element:

1. Benefit-cost ratio (BCR) and net present value (NPV) based on the mean values of risk reduction and maintenance cost,
2. Benefit-cost reliability index ($\beta_{B/C}$) and benefit-cost failure probability ($P_{f,B/C}$) based on the mean and standard deviations of risk reduction and maintenance cost.

The benefit-cost ratio (BCR) for each maintenance scenario is calculated by dividing the present value of risk reduction by the present value of maintenance cost. The net present value (NPV) of a maintenance scenario is calculated as the present value of risk reduction minus the present value of maintenance cost. The benefit-cost ratio and net present value are calculated and compared for the filtered maintenance scenarios of the deck element after eliminating the scenarios that exceed the risk and cost limitations as shown in Table 4-67. Table 4-67 shows that scenario 01101 with delayed first and fourth-year optimal actions has the highest BCR and NPV among the filtered maintenance scenarios of the concrete deck followed by scenarios 01011 with delayed first and third-year optimal actions. Scenario 11111 with no delay in any of the optimal actions ranks ninth in terms of benefit-cost ratio and net present value.

The benefit-cost reliability index is calculated using the means and standard deviations for maintenance cost and risk reduction of each scenario. The standard deviation σ_C of the total cost in present value for a maintenance scenario is calculated, as shown in Equation 4-13, as the square root of the sum of the squares of the standard deviations for the maintenance costs $\sigma_{C_{t,p}}$ in the five years of the scenario.

$$\sigma_C = \sqrt{\sum_{t=1}^{5} \sigma_{C_t}{}^2} \tag{4-13}$$

The standard deviation for the maintenance cost σ_{C_t} in any year *t* of the maintenance scenario is calculated based on the means and standard deviations of the unit costs of actions and the element quantities in the condition states as shown in Equation 4-14.

$$\sigma_{C_t} = \sqrt{\sum_{i=1}^{5} (\sigma_{C_i}{}^2 \mu_{q_i(t-1)}{}^2 + \mu_{C_i}{}^2 \sigma_{q_i(t-1)}{}^2)} \tag{4-14}$$

where,

σ_{C_t} =	Standard deviation for the maintenance cost at year *t* of the maintenance scenario,
μ_{c_i}, σ_{c_i} =	Mean and standard deviation for the unit cost of the optimal maintenance action in condition state *i*, and
$\mu_{q_i(t\text{-}1)}$, $\sigma_{q_i(t\text{-}1)}$ =	Mean and standard deviation for the element quantity in condition state *i* at year *t*-1.

The standard deviation of the total risk reduction in present value $\sigma_{\Delta R_p}$ for a maintenance scenario is calculated as the square root of the sum of the squares of the standard deviations for the risk reductions $\sigma_{\Delta R_t}$ in the five years of the scenario as shown in Equation 4-17.

$$\sigma_{\Delta R} = \sqrt{\sum_{t=1}^{5} \sigma_{\Delta R_t}{}^2} \tag{4-15}$$

The standard deviation for the risk reduction $\sigma_{\Delta R_t}$ at any year *t* of the maintenance scenario is calculated based on the standard deviations for the mitigated and unmitigated risks associated with the element condition in year *t* as shown in Equation 4-18.

$$\sigma_{\Delta R_t} = \sqrt{\sigma_{R_t}{}^2 + \sigma_{R_{t,0}}{}^2} \tag{4-16}$$

where,

$\sigma_{\Delta R_t}$ = Standard deviation for the risk reduction at year *t* of the maintenance scenario

σ_{R_t} = Standard deviation for the mitigated risk at year *t* of the maintenance scenario

$\sigma_{R_{t,0}}$ = Standard deviation for the unmitigated risk at year *t* of the maintenance scenario

The standard deviation σ_{R_t} for the risk associated with the element conditions at a particular year *t* of the maintenance scenario is calculated based on the means and standard deviations of the risk unit costs and the element quantities in the condition states as shown in Equation 4-19.

$$\sigma_{R_t} = \sqrt{\sum_{j=1}^{5} (\sigma_{R_j(t)}{}^2 \mu_{q_j(t)}{}^2 + \sigma_{q_j(t)}{}^2 \mu_{R_j(t)}{}^2)} \qquad (4\text{-}17)$$

where,

σ_{R_t} = Standard deviation for the risk associated with the element condition at year *t* of the maintenance scenario,

$\mu_{R_j(t)}$, $\sigma_{R_j(t)}$ = Mean and standard deviation for the risk unit cost when the element is in condition state *j*, and

$\mu_{q_j(t)}$, $\sigma_{q_j(t)}$ = Mean and standard deviation for the element quantity in condition state *j* at year *t*.

The standard deviations for the element quantities in the condition states at a particular year *t* are calculated based on the means and the standard deviations for the element quantities in the condition states at year *t*-1 and the transition probabilities among the condition states as shown in Equation 4-20.

$$\sigma_{q_j(t)} = \sqrt{\sum_{i=1}^{5} (\sigma_{q_i(t-1)}{}^2 \mu_{P_{ij}}{}^2 + \sigma_{P_{ij}}{}^2 \mu_{q_i(t-1)}{}^2)} \qquad (4\text{-}18)$$

where,

$\sigma_{q_j(t)}$ = Standard deviation for the element quantity in condition state *j* at year *t* of the maintenance scenario.

$\mu_{R_j(t)}$, $\sigma_{R_j(t)}$ = Mean and standard deviation for the risk unit cost when the element is in condition state *j*.

$\mu_{q_j(t)}$, $\sigma_{q_j(t)}$ = Mean and standard deviation for the element quantity in condition state *j* at year *t*.

Equations 4-15 and 4-16 are used to calculate the standard deviations for the total present value of maintenance cost for each scenario. Equations 4-17, 4-18, 4-19, and 4-20 are used to calculate the standard deviation for the total present value of risk reduction associated with each scenario. Table 4-68 shows the mean and standard deviation values for the risk reductions and the maintenance costs associated with the filtered maintenance scenarios for the concrete deck element.

The means and standard deviations of the maintenance costs and risk reductions are used to calculate the benefit-cost reliability index ($\beta_{B/C}$) for each scenario. Assuming normal distributions for the maintenance costs and risk reductions, the benefit-cost reliability index ($\beta_{B/C}$) and benefit-cost probability of failure ($P_{f,B/C}$) associated with each maintenance scenario can be calculated using Equations 4-19 and 4-20.

$$\beta_{B/C} = \frac{\mu_{\Delta R} - \mu_C}{\sqrt{{\sigma_{\Delta R}}^2 + {\sigma_C}^2}} \tag{4-19}$$

$$P_{f,B/C} = \Phi(-\beta_{B/C}) \tag{4-20}$$

Table 4-69 shows the calculated values for the benefit-cost reliability index and benefit-cost probability of failure. According to Table 4-69, Scenario 01101 with delayed first and fourth-year optimal actions has the highest benefit-cost reliability index ($\beta_{B/C}$) and lowest benefit-cost failure probability ($P_{f,B/C}$). Scenario 11111 with no delay in any of the optimal actions ranks ninth in terms of benefit-cost reliability index and probability of failure.

Table 4-66. Risk Reduction Associated with Maintenance Scenarios of the Concrete Deck Element

Years of Delayed Optimal Actions	Risk Reduction Associated with the Deck Maintenance ($)					Total Present Value of Risk Reduction ($)
	Year 1	Year 2	Year 3	Year 4	Year 5	
No Delay	6,161	2,039	4,716	8,449	12,272	30,727
Year 1	0	14,719	5,295	8,471	12,226	37,283
Year 2	6,161	0	8,277	8,636	12,279	32,262
Year 3	6,161	2,039	0	15,902	12,644	33,414
Year 4	6,161	2,039	4,716	0	24,893	34,204
Years 1 & 2	0	0	28,056	9,156	12,248	45,012
Years 1 & 3	0	14,719	0	16,839	12,644	40,300
Years 1 & 4	0	14,719	5,295	0	24,880	40,769
Years 2 & 3	6,161	0	0	21,124	12,803	36,330
Year 2 & 4	6,161	0	8,277	0	25,179	35,815
Year 3 & 4	6,161	2,039	0	0	35,254	38,950
Years 1, 2 & 3	0	0	0	47,805	13,226	54,774
Years 1, 2 & 4	0	0	28,056	0	25,925	48,780
Years 1, 3 & 4	0	14,719	0	0	36,513	46,098
Years 2, 3 & 4	6,161	0	0	0	42,076	43,014
Years 1, 2, 3 & 4	0	0	0	0	75,198	66,140
Year 5	6,161	2,039	4,716	8,449	0	19,934
Year 1 & 5	0	14,719	5,295	8,471	0	26,530
Year 2 & 5	6,161	0	8,277	8,636	0	21,462
Year 3 & 5	6,161	2,039	0	15,902	0	22,293
Year 4 & 5	6,161	2,039	4,716	0	0	12,309
Years 1, 2 & 5	0	0	28,056	9,156	0	34,239
Years 1, 3 & 5	0	14,719	0	16,839	0	29,178
Years 1, 4 & 5	0	14,719	5,295	0	0	18,885
Years 2, 3 & 5	6,161	0	0	21,124	0	25,068
Year 2, 4 & 5	6,161	0	8,277	0	0	13,669
Year 3, 4 & 5	6,161	2,039	0	0	0	7,942
Years 1, 2, 3 & 5	0	0	0	47,805	0	43,141
Years 1, 2, 4 & 5	0	0	28,056	0	0	25,977
Years 1, 3, 4 & 5	0	14,719	0	0	0	13,982
Years 2, 3, 4 & 5	6,161	0	0	0	0	6,005
Years 1, 2, 3, 4 & 5	0	0	0	0	0	0

Table 4-67. Benefit-Cost Ratio (BCR) and Net Present Value (NPV) for the Filtered Maintenance Scenarios of the Concrete Deck Element

Years of Delayed Optimal Actions	Scenario	BCR	NPV ($)	Scenario Ranking Based on	
				BCR	NPV
Years 1 & 4	01101	2.457	24,177	1	1
Years 1 & 3	01011	2.433	23,733	2	2
Year 1	01111	2.337	21,331	3	3
Years 2 & 3	10011	2.300	20,537	4	4
Year 2 & 4	10101	2.283	20,126	5	5
Year 4	11101	2.229	18,859	6	6
Year 3	11011	2.190	18,159	7	7
Year 2	10111	2.145	17,224	8	8
No Delay	11111	2.089	16,019	9	9

Table 4-68. Means and Standard Deviations for Risk Reduction and Maintenance Cost Associated with Filtered Maintenance Scenarios for the Concrete Deck

Years of Delayed Optimal Actions	Scenario	Risk Reduction ($)		Maintenance Cost ($)	
		Mean	Standard Deviation	Mean	Standard Deviation
Years 1 & 4	01101	40,769	4,771	16,591	1,214
Years 1 & 3	01011	40,300	4,909	16,566	1,197
Year 1	01111	37,283	5,795	15,952	1,053
Years 2 & 3	10011	36,330	4,916	15,792	1,168
Year 2 & 4	10101	35,815	4,217	15,689	1,175
Year 4	11101	34,204	4,799	15,345	1,145
Year 3	11011	33,414	4,893	15,255	1,126
Year 2	10111	32,262	4,804	15,038	1,091
No Delay	11111	30,727	5,768	14,708	981

Table 4-69. Benefit-Cost Reliability Index and Probability of Failure Associated with the Maintenance Scenarios for the Concrete Deck Element

Years of Delayed Optimal Actions	Scenario	NPV ($)		Benefit-Cost Reliability Index ($\beta_{B/C}$)	Benefit-Cost Probability of Failure ($P_{f,B/C}$)	Rank Based on Benefit-Cost Index Reliability
		Mean	Standard Deviation			
Years 1 & 4	01101	24,177	4,923	4.911	4.53E-07	1
Years 1 & 3	01011	23,733	5,053	4.697	1.32E-06	2
Year 1	01111	21,331	5,890	3.622	1.46E-04	6
Years 2 & 3	10011	20,537	5,053	4.064	2.41E-05	4
Year 2 & 4	10101	20,126	4,377	4.598	2.14E-06	3
Year 4	11101	18,859	4,933	3.823	6.60E-05	5
Year 3	11011	18,159	5,021	3.617	1.49E-04	7
Year 2	10111	17,224	4,927	3.496	2.36E-04	8
No Delay	11111	16,019	5,851	2.738	3.09E-03	9

4.12.6 Selecting the Optimal Maintenance Scenario for Concrete Deck Element

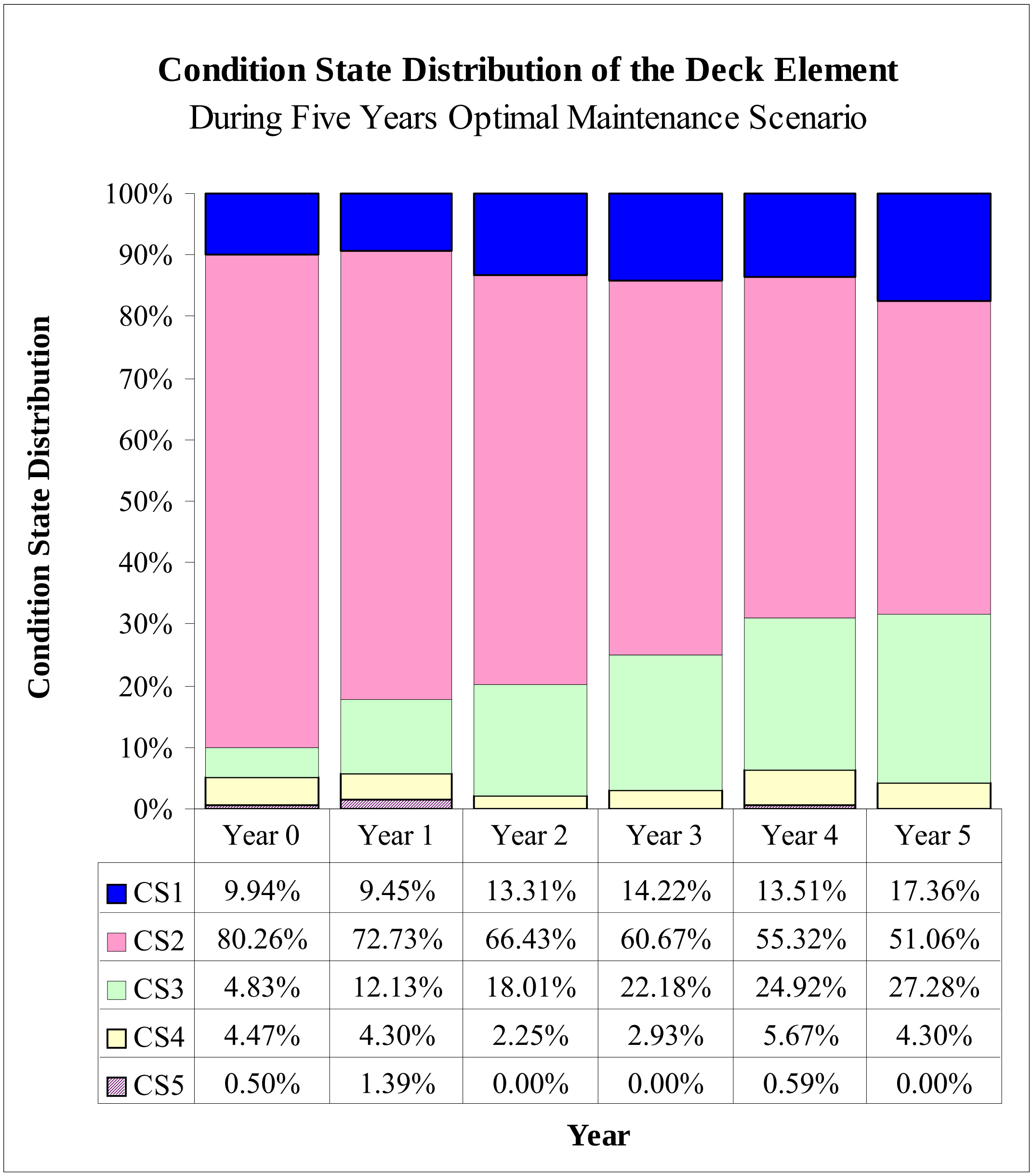
Comparisons between the different scenarios for the concrete deck element in terms of BCR, NPV, $\beta_{B/C}$ and $P_{f,B/C}$ are shown in Table 4-70. The rankings of the maintenance scenarios based on the above-mentioned economic measures are shown in Table 4-71. Based on Table 4-71, the best risk-based cost-efficient maintenance strategy for the concrete deck element is scenario 01101 in which the first and fourth-year optimal actions are delayed and the second, third, and fifth-year optima actions are applied. Based on the assumptions made in developing the risk and maintenance cost for the concrete deck, scenario 01101 has a mean net present value of $24,177 and a standard deviation of $4,923. The benefit-cost ratio for this scenario is 2.457 and the benefit-cost reliability index is 4.911. This implies that there is a 4.53E-7 probability that the maintenance cost will exceed the risk reduction associated with this scenario. The optimal sets of actions in years 1, 2, 3, 4, and 5 for scenario 01101 are A00000, A00011, A00010, A00000, and A00011, respectively, as previously shown in Table 4-61. A00000 represent doing nothing in the five condition states of the element. A00011 represents doing nothing in condition states 1, 2, and 3, and applying action $a(4,1)$ in condition state 4 and action $a(5,1)$ in condition state 5. A00010 represents doing nothing in condition states 1, 2, 3, and 5, and applying action $a(4,1)$ in condition state 4. The condition state distribution of the deck element when the risk-based optimal maintenance strategy is followed is shown in Figure 4-34.

Table 4-70. Comparison Between Maintenance Scenarios for the Concrete Deck in Terms of BCR, NPV, $\beta_{B/C}$, and $P_{f,B/C}$

Years of Delayed Optimal Actions	Scenario	Benefit-Cost Ratio (BCR)	Net Present Value (NPV), $	Benefit-cost Reliability Index ($\beta_{B/C}$)	Benefit-Cost Probability of Failure ($P_{f,B/C}$)
Years 1 & 4	01101	2.457	24,177	4.911	4.53E-07
Years 1 & 3	01011	2.433	23,733	4.697	1.32E-06
Year 1	01111	2.337	21,331	3.622	1.46E-04
Years 2 & 3	10011	2.300	20,537	4.064	2.41E-05
Year 2 & 4	10101	2.283	20,126	4.598	2.14E-06
Year 4	11101	2.229	18,859	3.823	6.60E-05
Year 3	11011	2.190	18,159	3.617	1.49E-04
Year 2	10111	2.145	17,224	3.496	2.36E-04
No Delay	11111	2.089	16,019	2.738	3.09E-03

Table 4-71. Ranking of Maintenance Scenarios for the Concrete Deck Based on Benefit-Cost Ratio (BCR), Net Present Value (NPV), and Benefit-Cost Reliability Index ($\beta_{B/C}$) and Probability of Failure ($P_{f,B/C}$)

Years of Delayed Optimal Actions	Scenario	Scenario Ranking Based on		
		BCR	NPV	$\beta_{B/C}$ & $P_{f,B/C}$
Years 1 & 4	01101	1	1	1
Years 1 & 3	01011	2	2	2
Year 1	01111	3	3	6
Years 2 & 3	10011	4	4	4
Year 2 & 4	10101	5	5	3
Year 4	11101	6	6	5
Year 3	11011	7	7	7
Year 2	10111	8	8	8
No Delay	11111	9	9	9



	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
CS1	9.94%	9.45%	13.31%	14.22%	13.51%	17.36%
CS2	80.26%	72.73%	66.43%	60.67%	55.32%	51.06%
CS3	4.83%	12.13%	18.01%	22.18%	24.92%	27.28%
CS4	4.47%	4.30%	2.25%	2.93%	5.67%	4.30%
CS5	0.50%	1.39%	0.00%	0.00%	0.59%	0.00%

Figure 4-34. Condition State Distribution Associated with Five Years Optimal Maintenance Strategy

4.13 Risk-Based Maintenance for Steel Girder Element

The steps for the risk-based maintenance for the steel girder element are explained in the following subsections.

4.13.1 Optimal Maintenance Actions for Steel Girder Element

The steel girder element has two feasible maintenance actions in condition states 1 and 3, and three feasible actions in condition states 2, 4 and 5 as shown in Table 4-72. A total of 108 scenarios for the possible set of maintenance actions in the condition states of the element were created. The scenarios were identified based on the sets of actions in the condition states as done earlier in the concrete deck element example.

The maintenance costs and the risks associated with the element conditions were calculated for the possible scenarios of actions. The summation of the maintenance costs and risks, taking into consideration a discount rate of 2.6% when adding the two values, is used to select the optimal set of actions in the condition states of the steel girder element. The maintenance actions that result in the lowest sum of maintenance cost and risk in each year of maintenance are considered optimal.

For the first year of maintenance, the 108 scenarios of actions in the condition states include scenario A00000 with minimum maintenance cost of \$0 and a risk value of \$85,076, and scenario A12122 with minimum risk of \$10,617 and a maintenance cost of \$97,460. Neither of the two scenarios is considered optimal. The best five scenarios that result in the lowest summation of risk and cost for the steel girder element are shown in Figure 4-35. The condition state distributions of the steel girder element associated with these scenarios are shown in Figure 4-36. Figure 4-35 shows that scenario A00021 has the lowest sum of risk and maintenance cost. Therefore, out of the 108 possible scenarios of actions in the first year, scenario A00021 represents the set of optimal actions $a(1,0)$, $a(2,0)$, $a(3,0)$, $a(4,2)$, $a(5,1)$ in condition states 1,2,3, 4, and 5 for the first year of maintenance. The first-year optimal actions are: to do nothing in condition states 1, 2 and 3; to replace paint system in condition state 4; and to do a major rehab unit in condition state 5.

If the optimal actions of scenario A00021 were applied in the first year, scenario A00020 will be associated with the lowest sum of risk and cost in the second year of maintenance; therefore, the second-year optimal actions are selected from scenario A00020. If the first-year optimal actions were delayed, scenario A00021 will result in the lowest sum of risks and costs in the second year; therefore, the second-year optimal actions are selected from scenario A00021. The second-year optimal actions are to do nothing in condition states 1, 2 and 3; and to replace the paint system in condition states 4. The second-year optimal action in condition state 5 is to do nothing if the first-year optimal actions were applied, and to do a major rehab unit in condition state 5 if the first-year optimal actions were delayed. The second-year scenarios of optimal actions are shown in Figure 4-37. The condition state distribution of the steel girder element associated with the second-year optimal scenarios is shown in Figure 4-38.

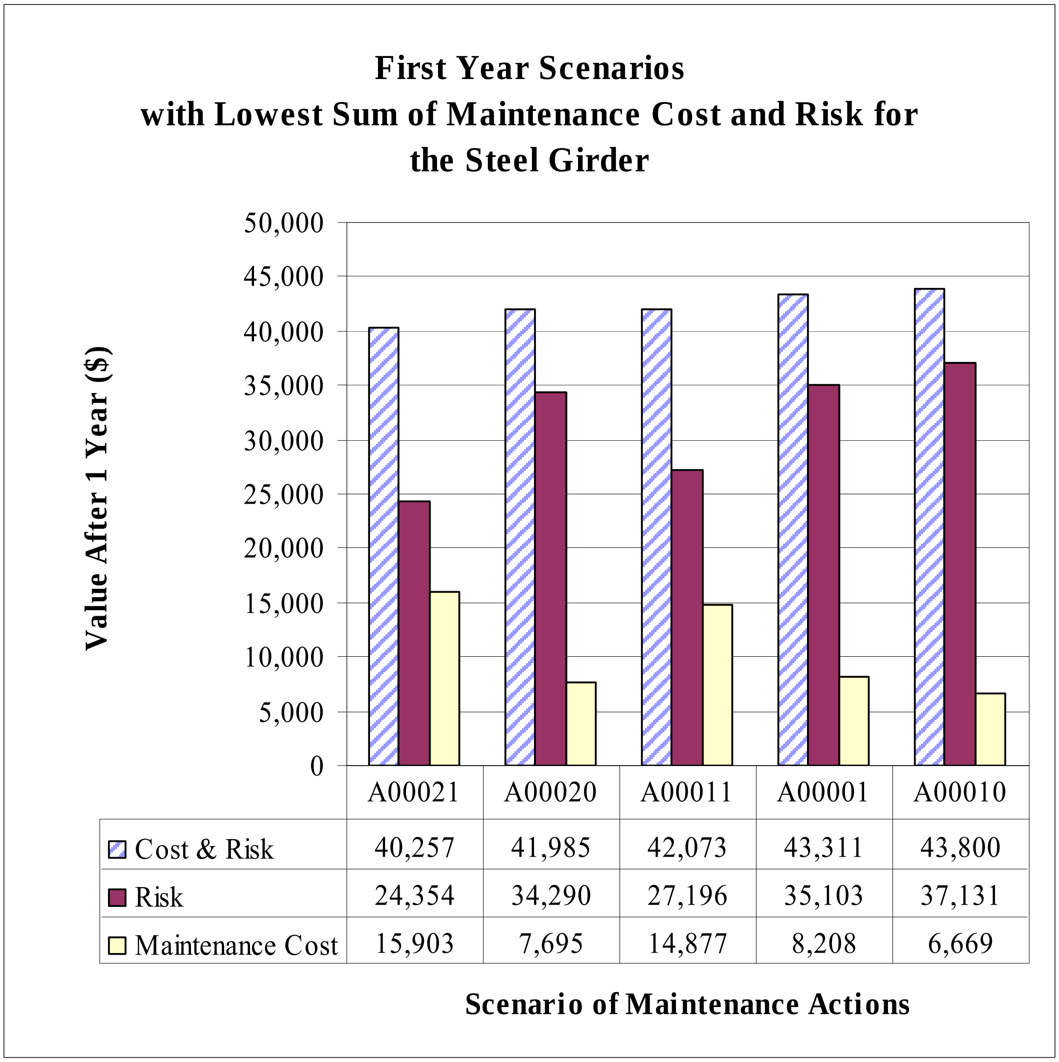
For the third-year of maintenance, four cases of applying or delaying the optimal actions of previous years are considered. Scenario A00120 is selected as the scenario with the optimal actions if the second-year (and first-year) optimal actions were applied. If the optimal actions for the second (and first) year were delayed, the third-year optimal actions are selected from scenario A00121. The third-year scenarios of optimal actions are shown in Figure 4-39. The condition state distributions of the steel girder associated with the third-year scenarios of optimal actions are shown in Figure 4-40. Therefore, the optimal actions in the third year of maintenance are: to do nothing in condition states 1 and 2; to spot blast, clean and paint in condition state 3; and to replace the paint system in condition state 4. The optimal action in condition state 5 is to do nothing if the second-year optimal actions were applied, and to do a major rehabilitation if the second-year optimal actions were delayed regardless of applying or delaying the first-year optimal actions. The third-year optimal maintenance actions for the steel girder element are shown in Table 4-73.

For the fourth year of maintenance, the scenario of optimal actions is scenario A00100 if the optimal actions in the second and third year were both applied regardless of applying or delaying first-year optimal actions. If the optimal actions in the second (and first) year were delayed while the third-year optimal actions were applied, the fourth-year scenario of optimal actions scenario A00120. If the optimal actions in the third year (and the years before) were delayed, the fourth-year optimal actions are selected from scenario A00121. The cases where the fourth-year optimal actions are selected from scenario A00100, A00120, and A00121 are shown in Figures 4-41, 4-42, and 4-43, respectively. The condition state distributions of the steel girder element when these optimal scenarios are applied in the fourth year are shown in Figures 4-44, 4-45, and 4-46.

For the fifth year of the steel girder maintenance, the scenario of optimal actions is scenario A00100 if the optimal actions were applied in all previous years, delayed in the first and/or second year. If the optimal actions were delayed in the third year (and previous years) and applied in the fourth year, or delayed in the fourth (and first) year only, the fifth-year optimal actions are selected from scenario A00120. If the optimal actions were delayed in the forth year and the second or the third year, the fifth-year optimal actions are selected from scenario A00121. The fifth-year scenarios of optimal actions for the different cases of delays in previous years of optimal actions are shown in Figures 4-47, 4-48, and 4-49. The fifth-year optimal maintenance actions in the condition states of the steel girder element for the different cases of delays in previous years are shown in Table 4-74. The condition state distribution of the steel girder element when the scenarios of optimal actions (A00100, A00120, A00121) are applied in the fifth year is shown in Figures 4-50, 4-51, and 4-52.

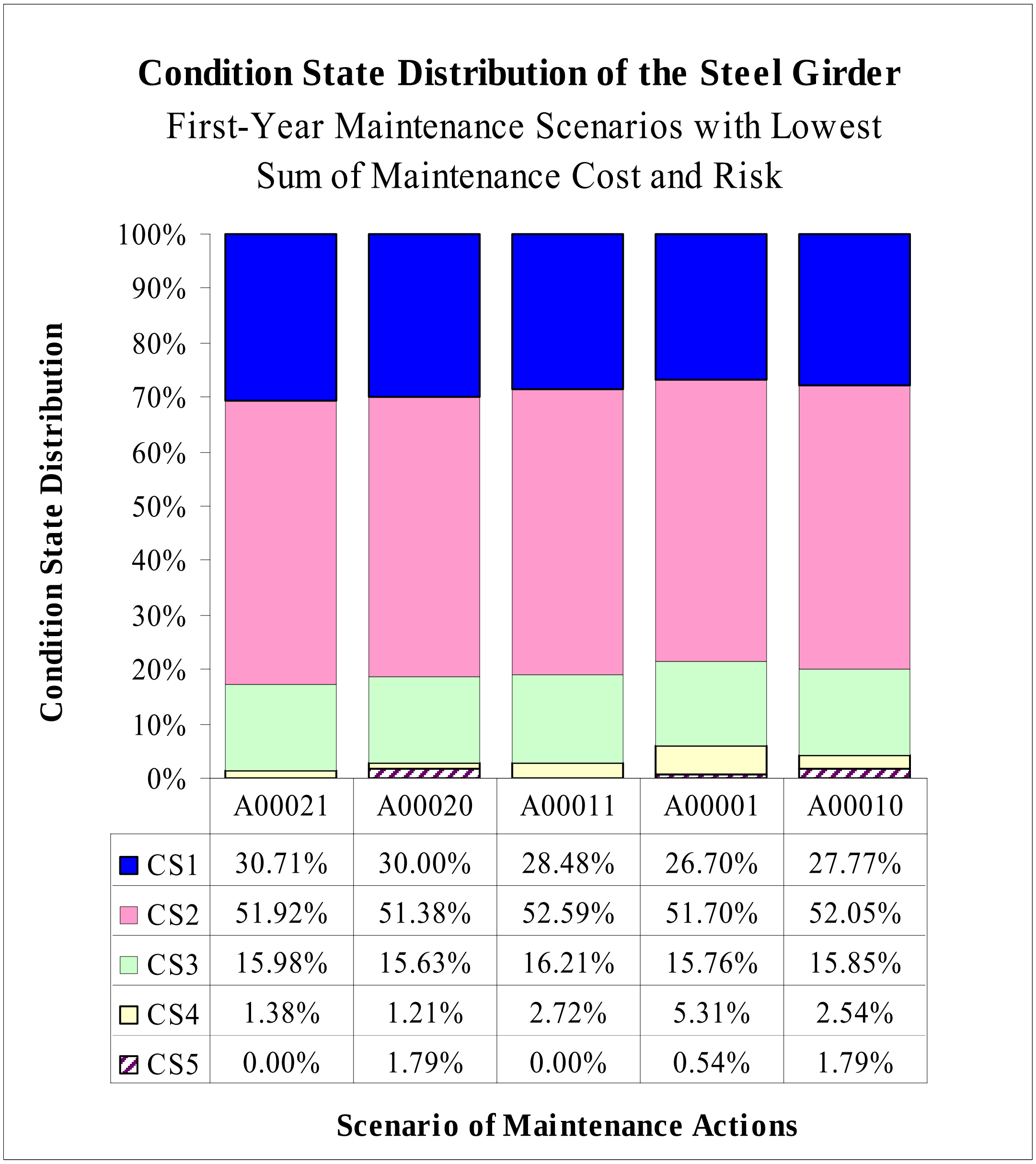
Table 4-72. Feasible Maintenance Actions for the Steel Girder Element

Condition State	Maintenance Action		
	Action Number	Symbol	Description
1	0	$a(1,0)$	Do Nothing
	1	$a(1,1)$	Surface clean
2	0	$a(2,0)$	Do Nothing
	1	$a(2,1)$	Surface clean
	2	$a(2,2)$	Surface clean and restore top coat
3	0	$a(3,0)$	Do Nothing
	1	$a(3,1)$	Spot blast, clean and paint
4	0	$a(4,0)$	Do Nothing
	1	$a(4,1)$	Spot blast, clean and paint
	2	$a(4,2)$	Replace paint system
5	0	$a(5,0)$	Do Nothing
	1	$a(5,1)$	Major rehab unit
	2	$a(5,2)$	Replace unit



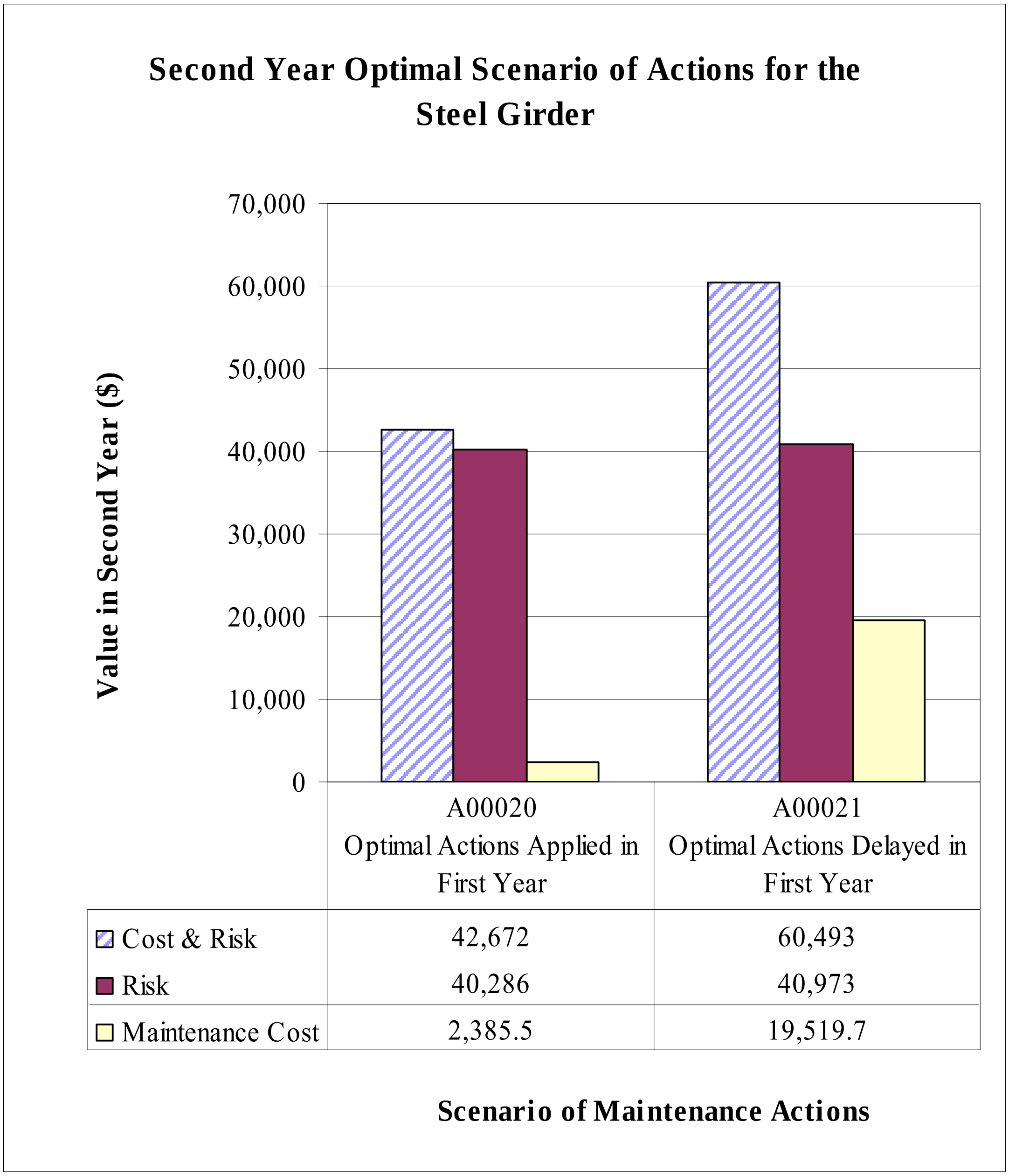
	A00021	A00020	A00011	A00001	A00010
▨ Cost & Risk	40,257	41,985	42,073	43,311	43,800
■ Risk	24,354	34,290	27,196	35,103	37,131
☐ Maintenance Cost	15,903	7,695	14,877	8,208	6,669

Figure 4-35. Scenarios with Lowest Summation of Maintenance Cost and Risk in First Year of Maintenance for the Steel Girder Element



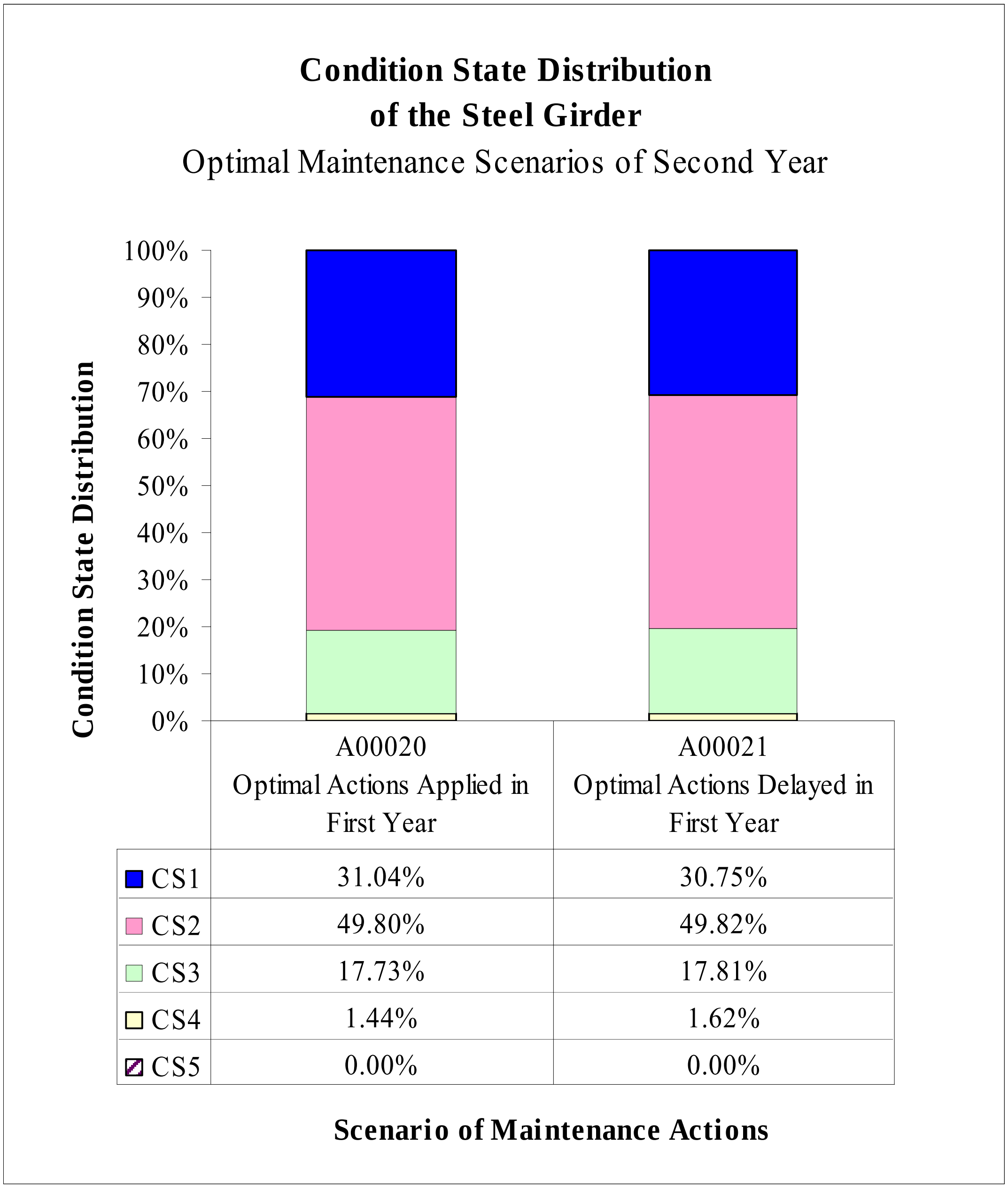
	A00021	A00020	A00011	A00001	A00010
■ CS1	30.71%	30.00%	28.48%	26.70%	27.77%
■ CS2	51.92%	51.38%	52.59%	51.70%	52.05%
■ CS3	15.98%	15.63%	16.21%	15.76%	15.85%
□ CS4	1.38%	1.21%	2.72%	5.31%	2.54%
▨ CS5	0.00%	1.79%	0.00%	0.54%	1.79%

Figure 4-36. Condition State Distribution for the Steel Girder Element for the Five Scenarios with Lowest Summation of Maintenance Cost and Risk in the First Year of Maintenance



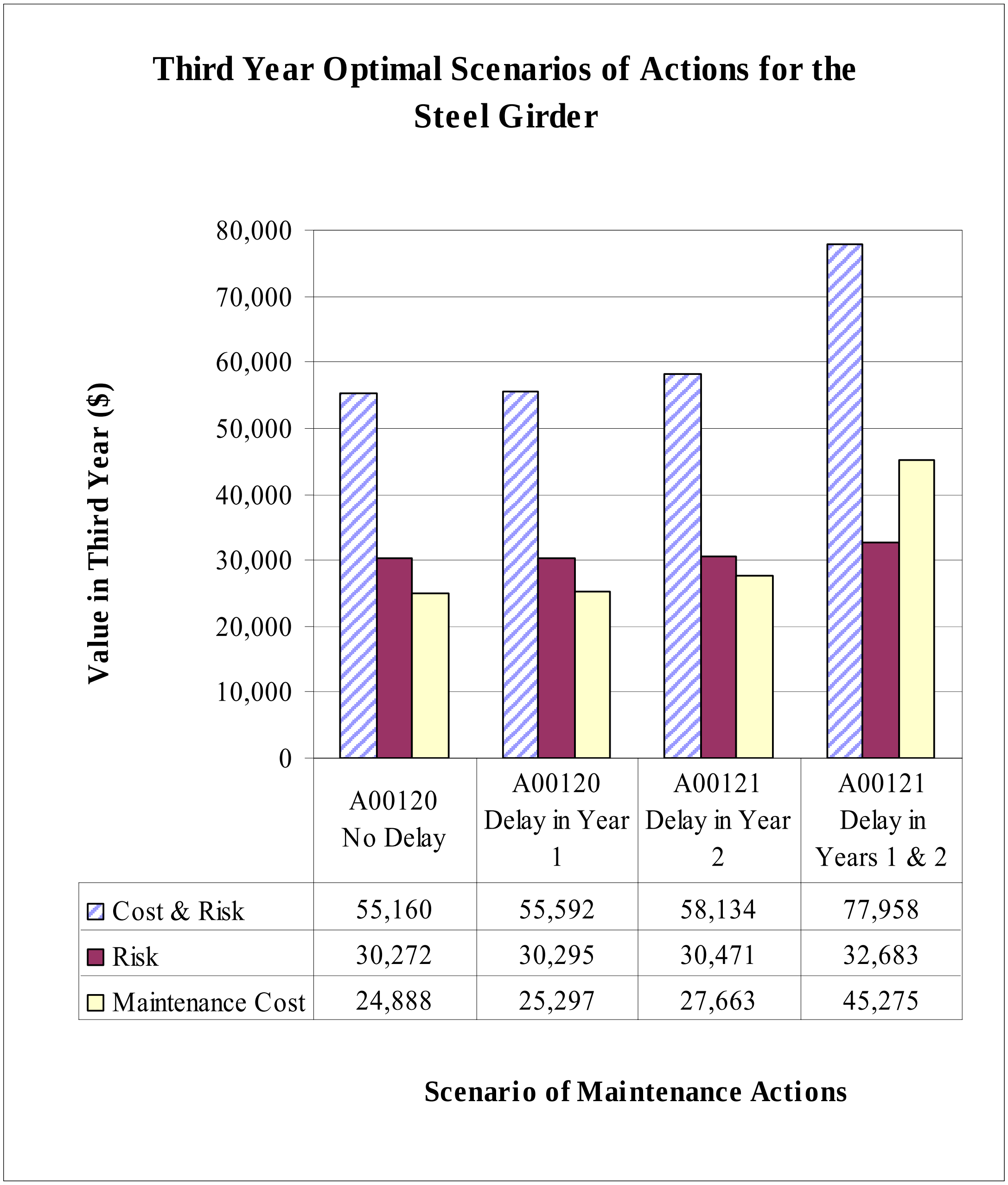
	A00020 Optimal Actions Applied in First Year	A00021 Optimal Actions Delayed in First Year
▨ Cost & Risk	42,672	60,493
■ Risk	40,286	40,973
□ Maintenance Cost	2,385.5	19,519.7

Figure 4-37. Scenarios of Optimal Actions with Lowest Summation of Maintenance Cost and Risk in the Second Year of Maintenance for the Steel Girder Element



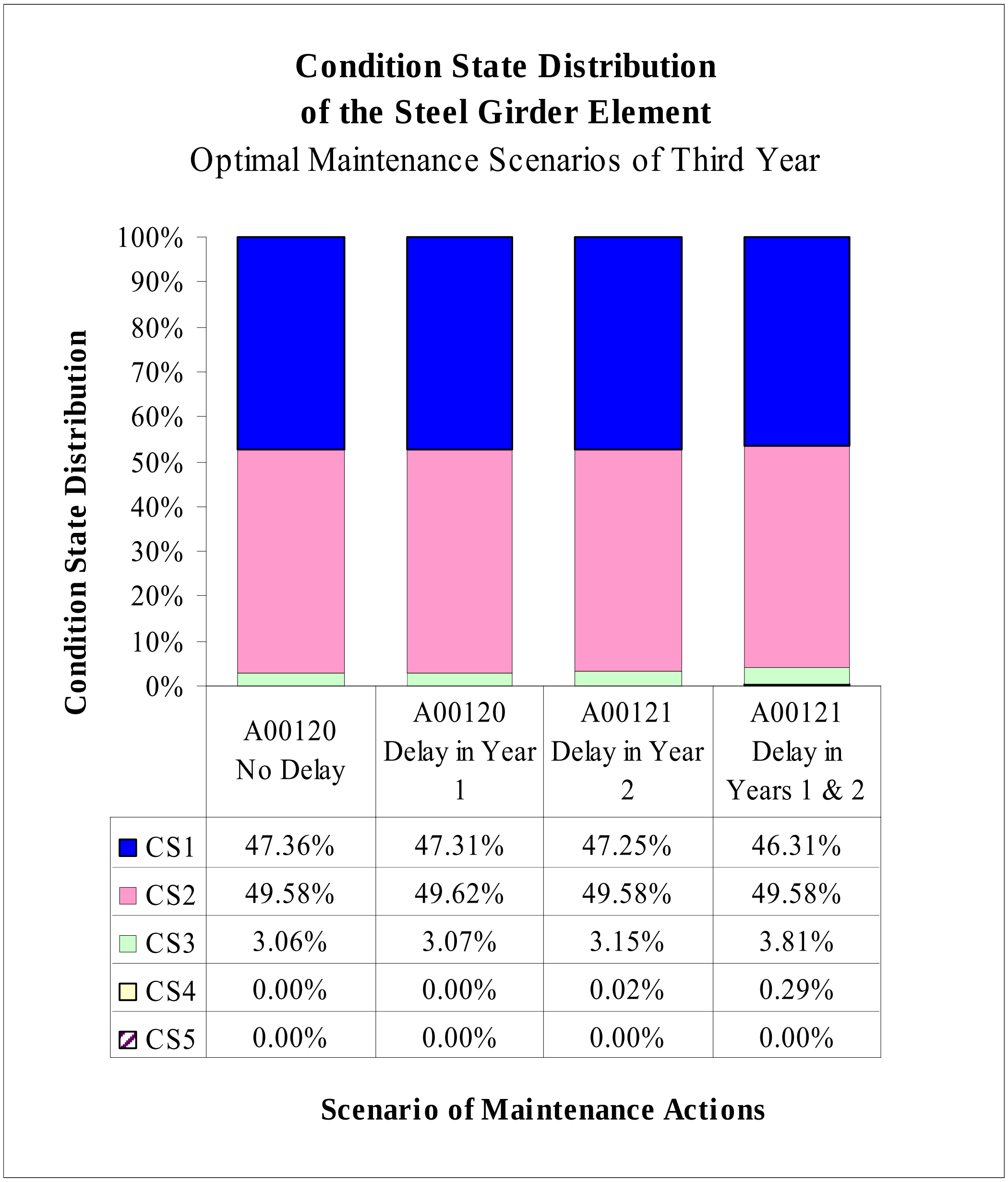
	A00020 Optimal Actions Applied in First Year	A00021 Optimal Actions Delayed in First Year
🟦 CS1	31.04%	30.75%
🟪 CS2	49.80%	49.82%
🟩 CS3	17.73%	17.81%
🟨 CS4	1.44%	1.62%
▨ CS5	0.00%	0.00%

Figure 4-38. Condition State Distribution for the Steel Girder Element Associated with the Optimal Scenarios with Lowest Summation of Maintenance Cost and Risk in the Second Year of Maintenance



	A00120 No Delay	A00120 Delay in Year 1	A00121 Delay in Year 2	A00121 Delay in Years 1 & 2
▨ Cost & Risk	55,160	55,592	58,134	77,958
■ Risk	30,272	30,295	30,471	32,683
□ Maintenance Cost	24,888	25,297	27,663	45,275

Figure 4-39. Scenarios of Optimal Actions with Lowest Summation of Maintenance Cost and Risk in the Third Year of Maintenance for the Steel Girder Element

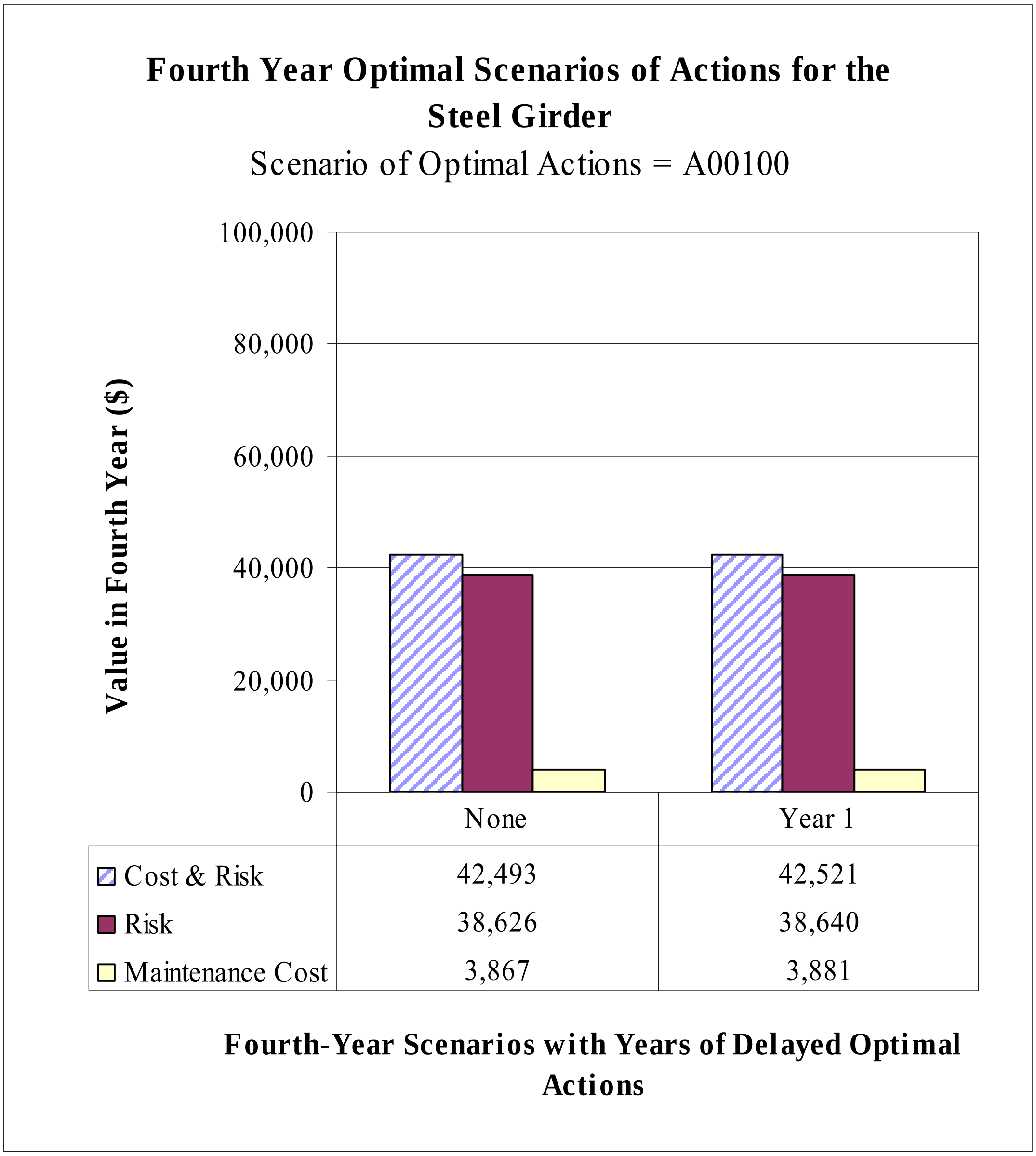


	A00120 No Delay	A00120 Delay in Year 1	A00121 Delay in Year 2	A00121 Delay in Years 1 & 2
🟦 CS1	47.36%	47.31%	47.25%	46.31%
🟪 CS2	49.58%	49.62%	49.58%	49.58%
🟩 CS3	3.06%	3.07%	3.15%	3.81%
🟨 CS4	0.00%	0.00%	0.02%	0.29%
▨ CS5	0.00%	0.00%	0.00%	0.00%

Figure 4-40. Condition State Distribution for the Steel Girder Element Associated with Optimal Scenarios of Lowest Summation of Maintenance Cost and Risk in the Third Year of Maintenance

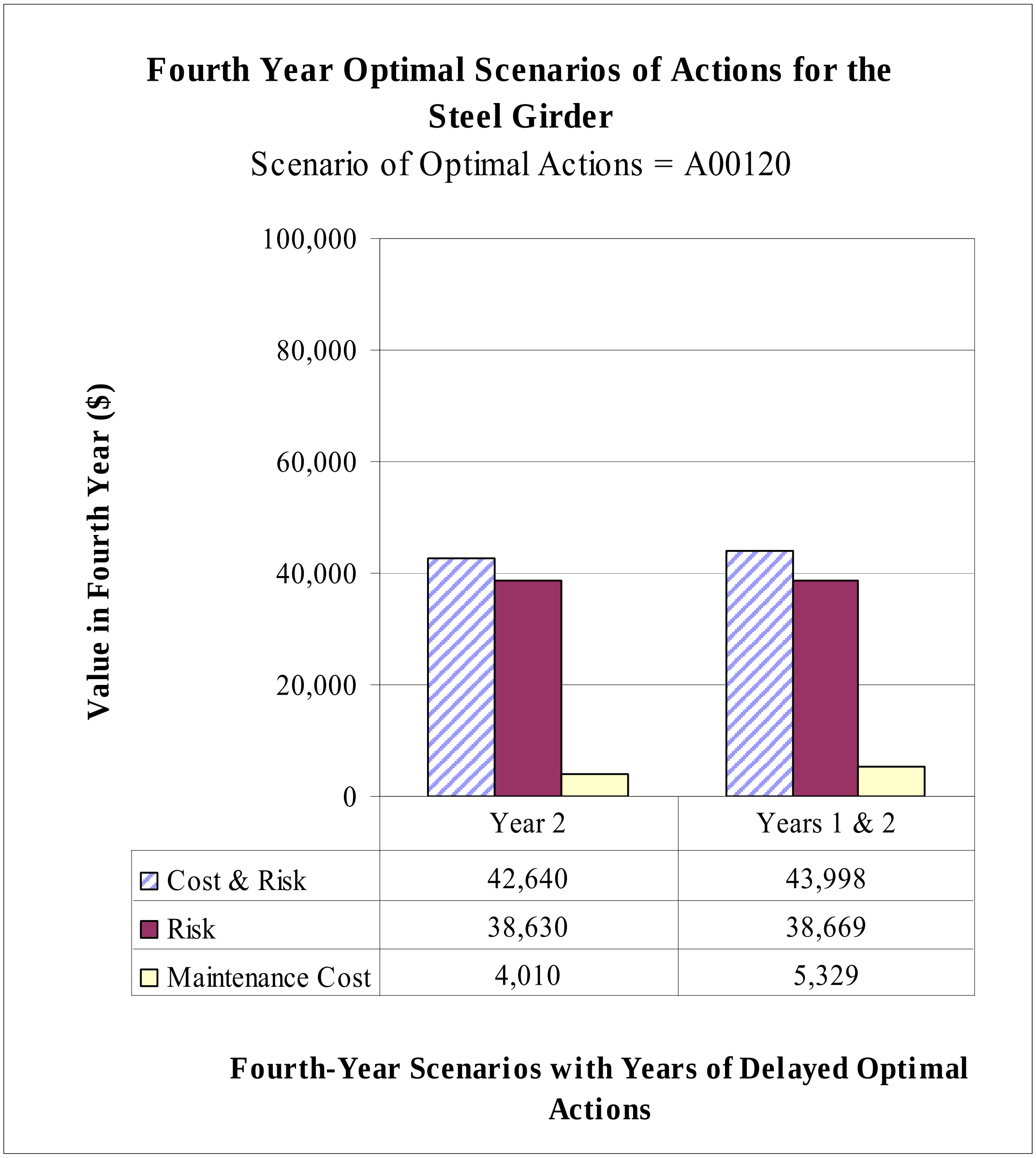
Table 4-73. Third-Year Optimal Maintenance Actions for the Steel Girder

Years of Delayed Optimal Actions	Scenario of Maintenance Actions for the Steel Girder	Third-Year Optimal Maintenance Actions for the Steel Girder				
		CS1	CS2	CS3	CS4	CS5
None	A00120	$a(1,0)$	$a(2,0)$	$a(3,1)$	$a(4,2)$	$a(5,0)$
Year 1	A00120	$a(1,0)$	$a(2,0)$	$a(3,1)$	$a(4,2)$	$a(5,0)$
Year 2	A00121	$a(1,0)$	$a(2,0)$	$a(3,1)$	$a(4,2)$	$a(5,1)$
Years 1 and 2	A00121	$a(1,0)$	$a(2,0)$	$a(3,1)$	$a(4,2)$	$a(5,1)$



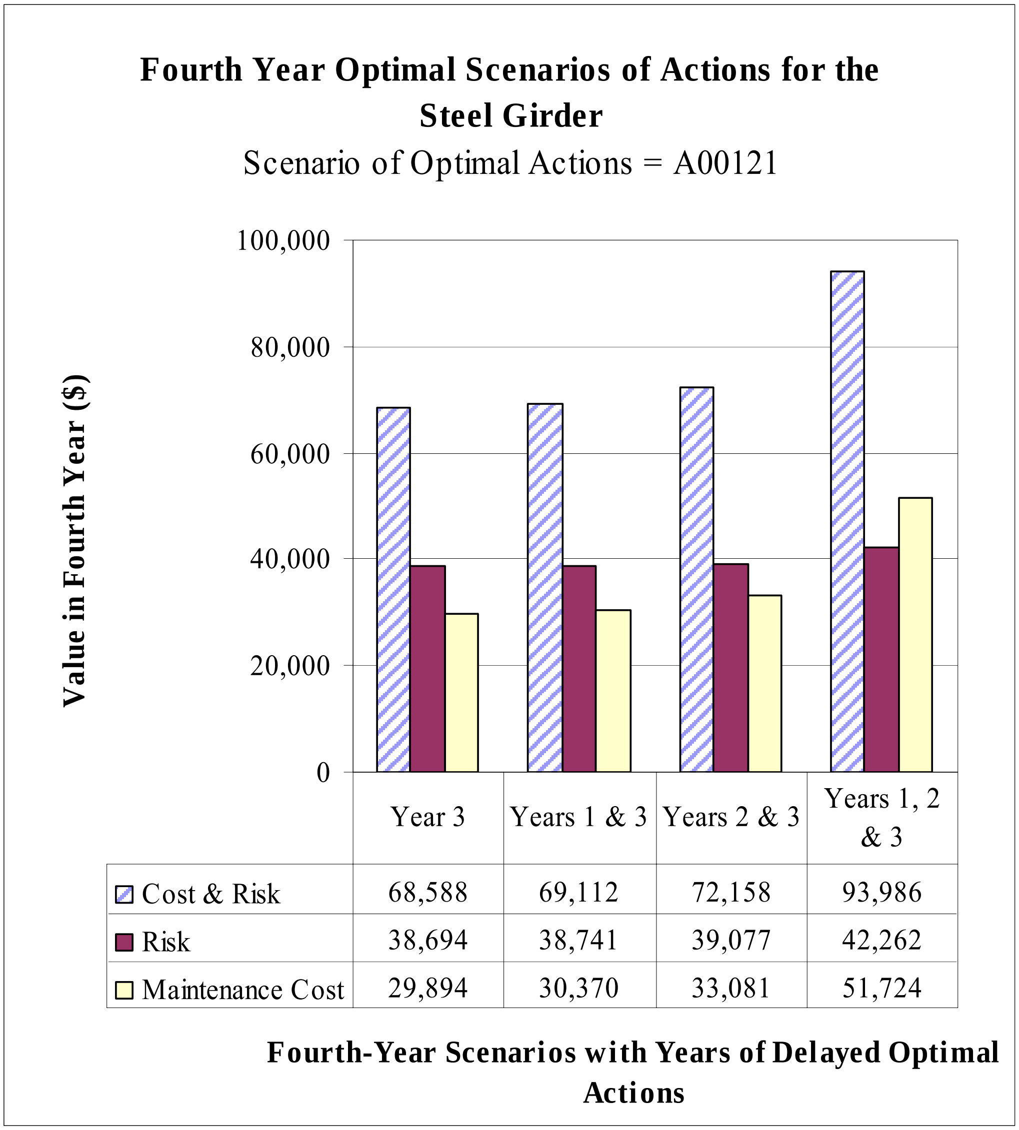
	None	Year 1
▨ Cost & Risk	42,493	42,521
■ Risk	38,626	38,640
☐ Maintenance Cost	3,867	3,881

Figure 4-41. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Steel Girder Maintenance with No Delay or Delay in First-Year Optimal Actions



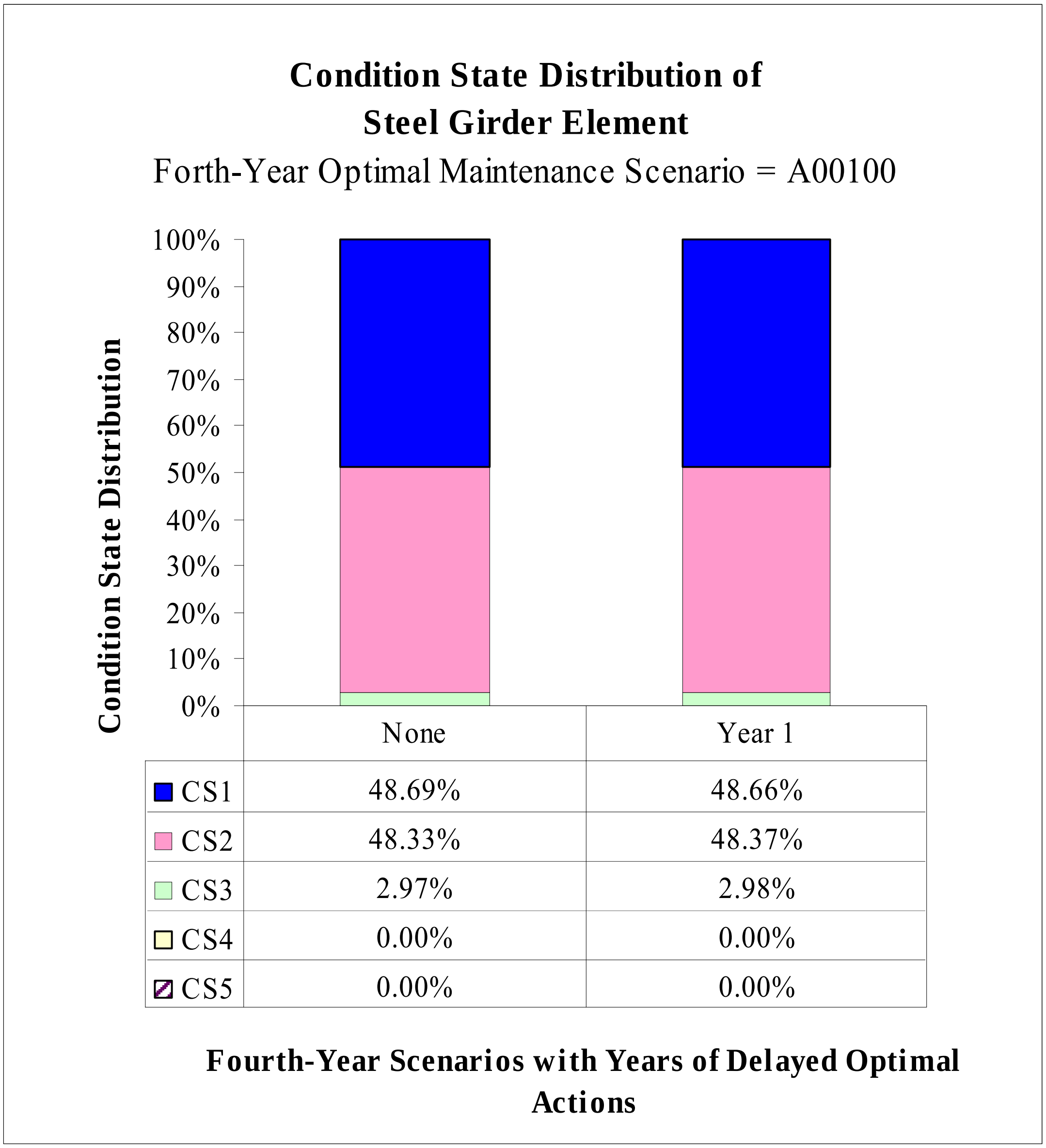
	Year 2	Years 1 & 2
▨ Cost & Risk	42,640	43,998
■ Risk	38,630	38,669
☐ Maintenance Cost	4,010	5,329

Figure 4-42. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Steel Girder Maintenance with Delay in the Optimal Actions of the Second (and First) Year



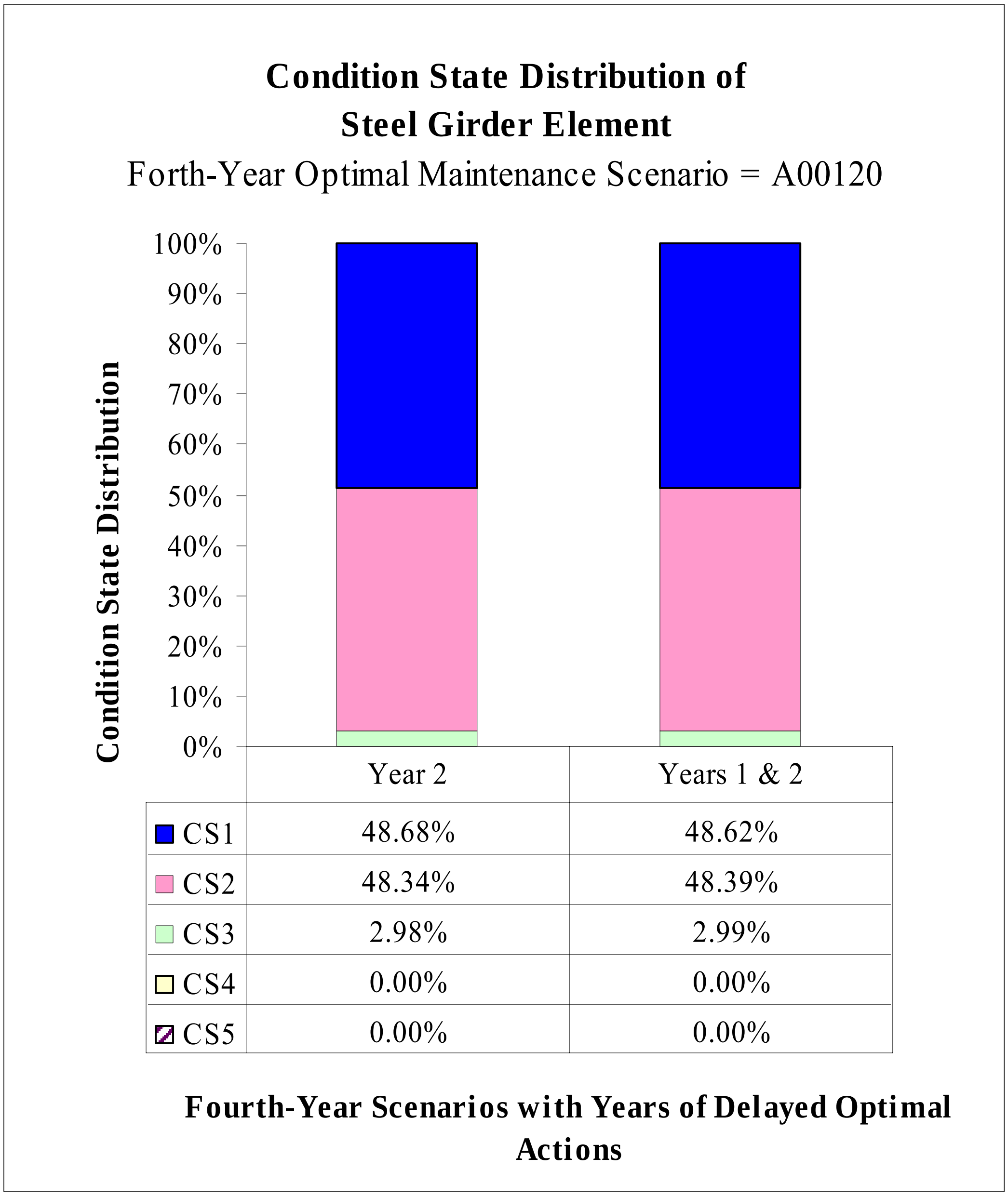
	Year 3	Years 1 & 3	Years 2 & 3	Years 1, 2 & 3
▨ Cost & Risk	68,588	69,112	72,158	93,986
■ Risk	38,694	38,741	39,077	42,262
□ Maintenance Cost	29,894	30,370	33,081	51,724

Figure 4-43. Scenarios with Lowest Summation of Maintenance Cost and Risk in the Fourth Year of the Steel Girder Maintenance with Delay in the Optimal Actions of the Third Year and One or Two Years Before



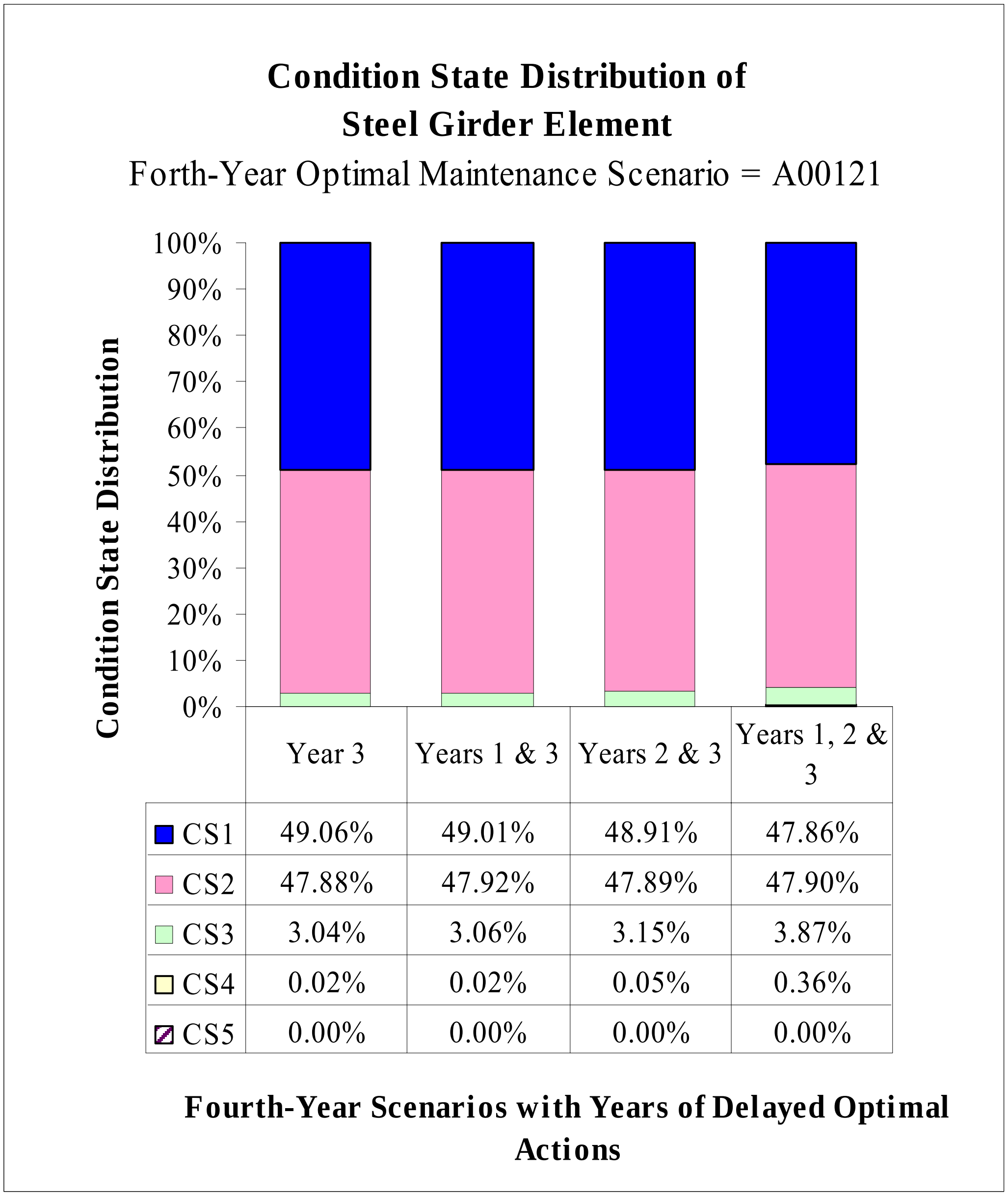
	None	Year 1
■ CS1	48.69%	48.66%
■ CS2	48.33%	48.37%
■ CS3	2.97%	2.98%
☐ CS4	0.00%	0.00%
▨ CS5	0.00%	0.00%

Figure 4-44. Condition State Distribution for the Steel Girder Element Associated with Fourth-Year Optimal Maintenance Scenarios for Cases with No Delay or Delay in First-Year Optimal Actions



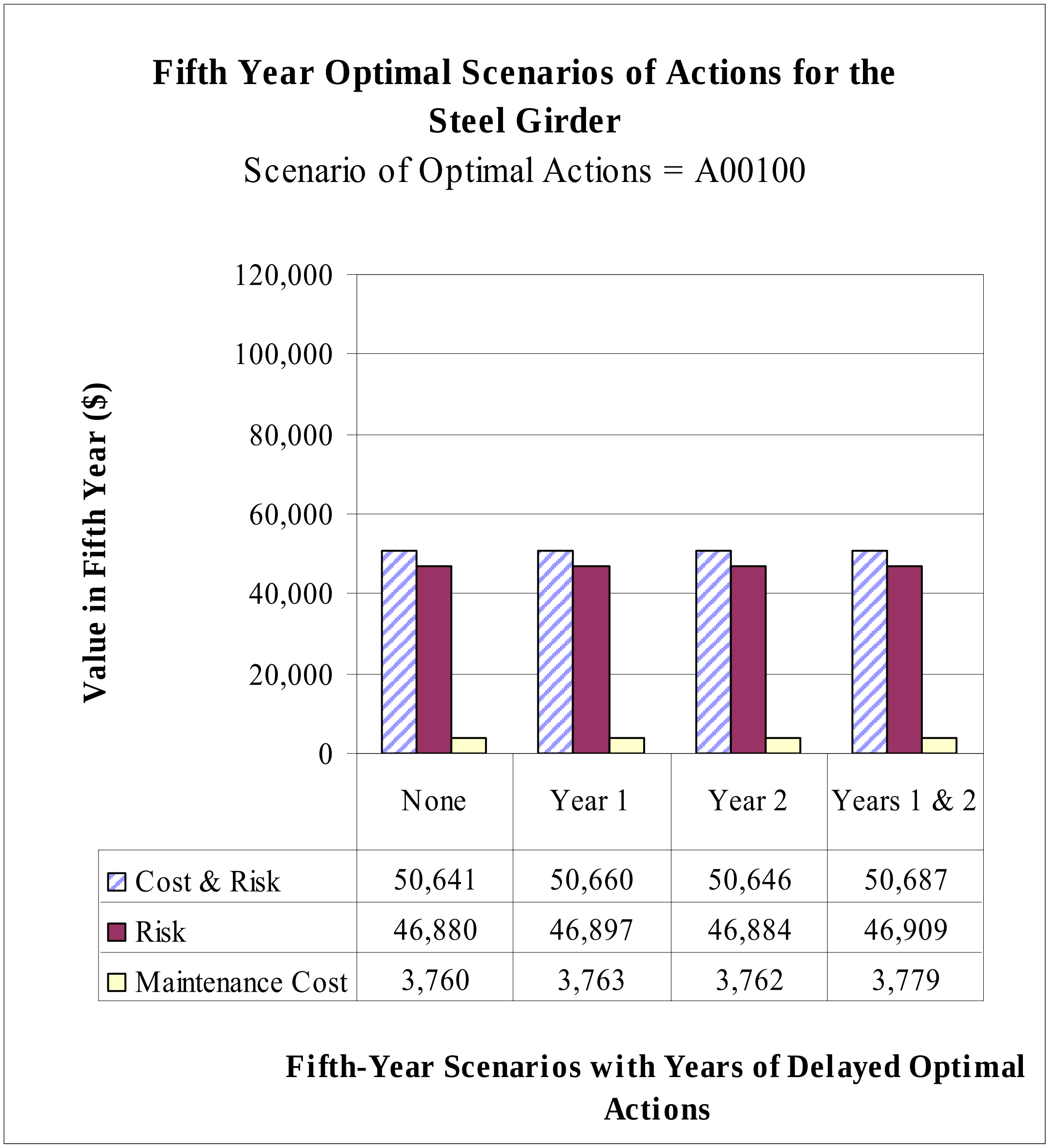
	Year 2	Years 1 & 2
■ CS1	48.68%	48.62%
■ CS2	48.34%	48.39%
■ CS3	2.98%	2.99%
□ CS4	0.00%	0.00%
▨ CS5	0.00%	0.00%

Figure 4-45. Condition State Distribution for the Steel Girder Element Associated with Fourth-Year Optimal Maintenance Scenarios for Cases with Delay in the Optimal Actions of the Second (and First) Year



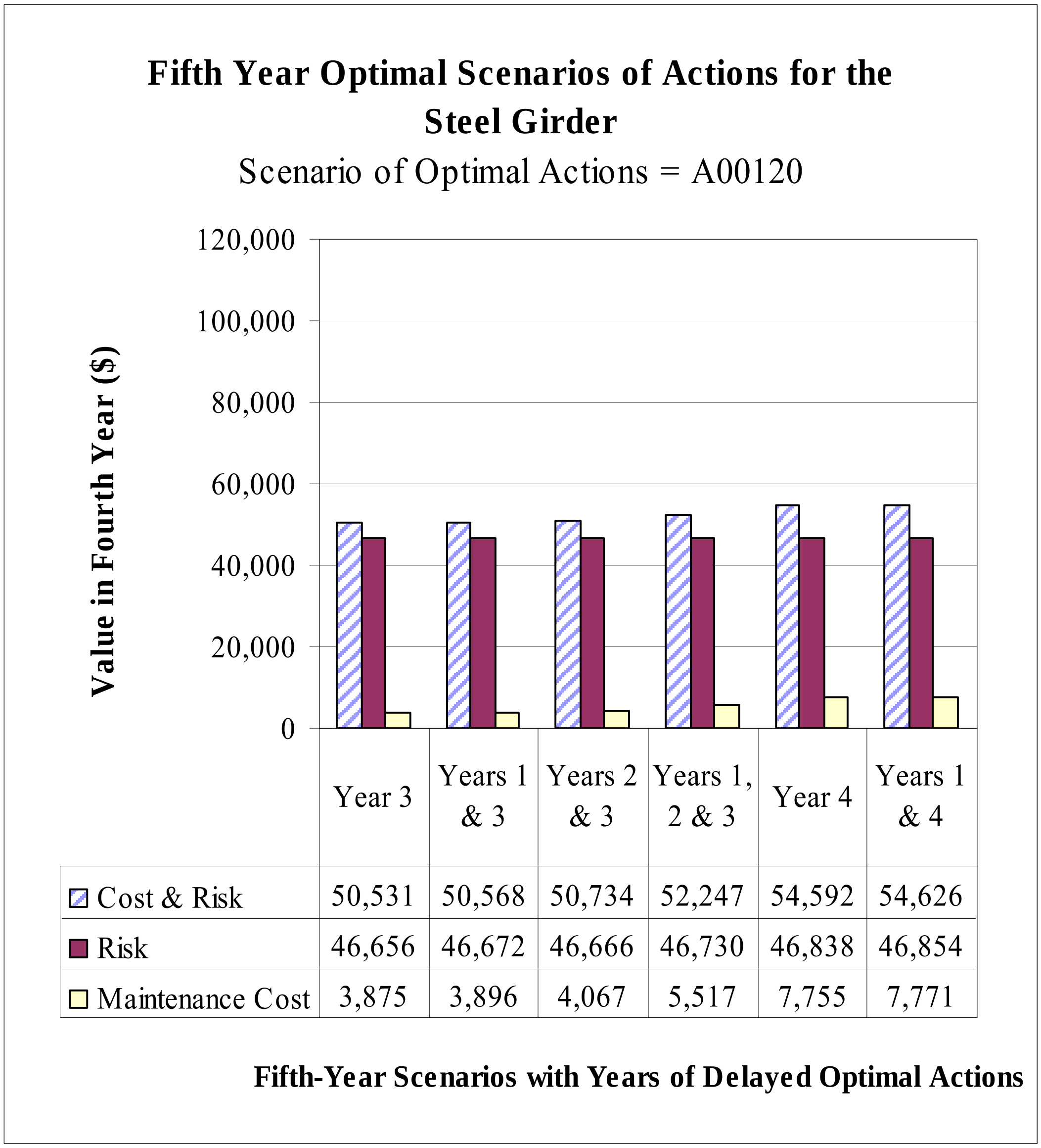
	Year 3	Years 1 & 3	Years 2 & 3	Years 1, 2 & 3
🟦 CS1	49.06%	49.01%	48.91%	47.86%
🟪 CS2	47.88%	47.92%	47.89%	47.90%
🟩 CS3	3.04%	3.06%	3.15%	3.87%
🟨 CS4	0.02%	0.02%	0.05%	0.36%
▨ CS5	0.00%	0.00%	0.00%	0.00%

Figure 4-46. Condition State Distribution for the Steel Girder Element Associated with Fourth-Year Optimal Maintenance Scenarios for Cases with Delay in the Optimal Actions of the Third Year and One or Two Years Before



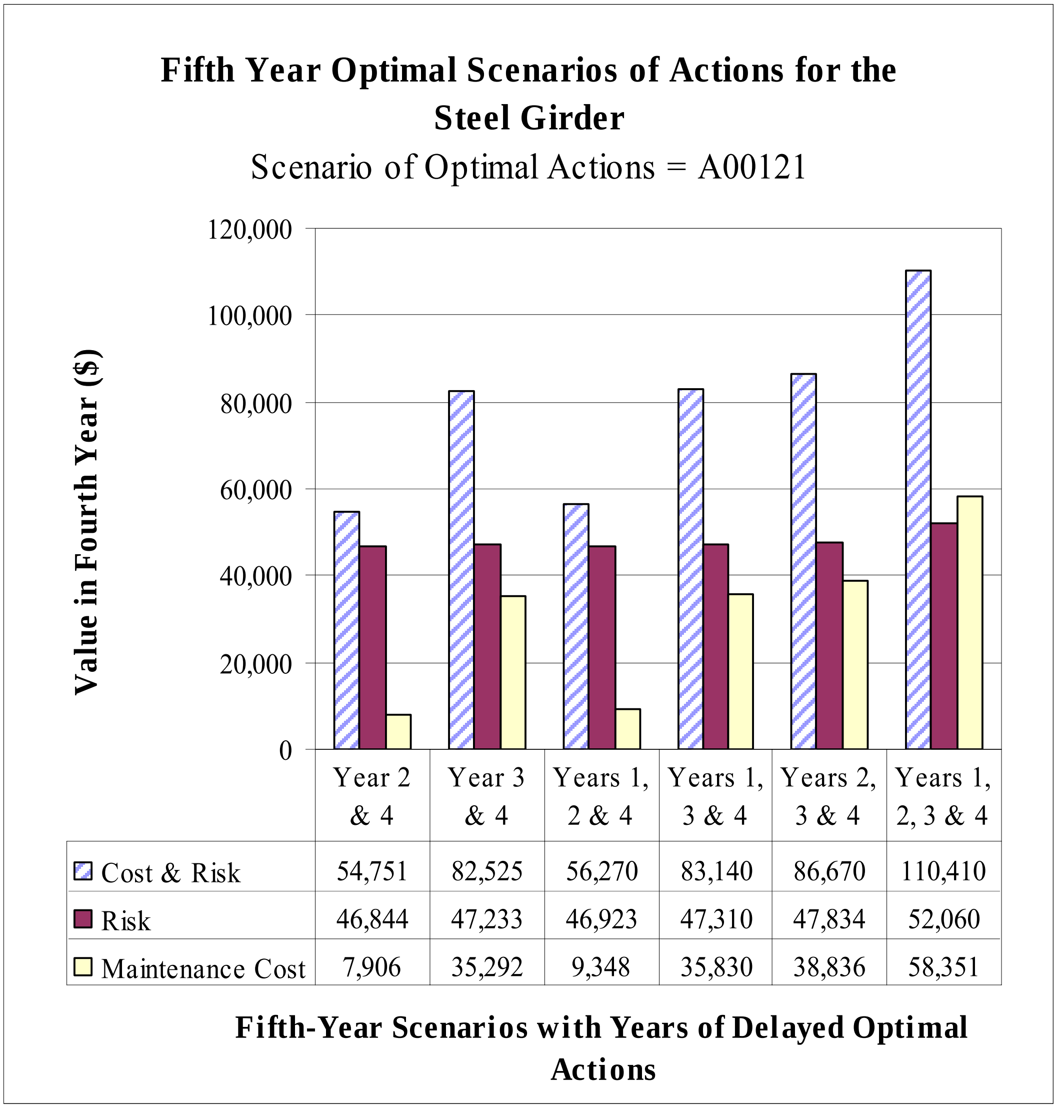
	None	Year 1	Year 2	Years 1 & 2
▨ Cost & Risk	50,641	50,660	50,646	50,687
■ Risk	46,880	46,897	46,884	46,909
□ Maintenance Cost	3,760	3,763	3,762	3,779

Figure 4-47. Scenarios of Optimal Actions (A00100) in the Fifth Year of the Steel Girder Maintenance with No Delay and Delay of First and/or Second-Year Optimal Actions



	Year 3	Years 1 & 3	Years 2 & 3	Years 1, 2 & 3	Year 4	Years 1 & 4
▨ Cost & Risk	50,531	50,568	50,734	52,247	54,592	54,626
■ Risk	46,656	46,672	46,666	46,730	46,838	46,854
□ Maintenance Cost	3,875	3,896	4,067	5,517	7,755	7,771

Figure 4-48. Scenarios of Optimal Actions (A00120) in the Fifth Year of the Steel Girder Maintenance with Corresponding Cases of Delayed Optimal Actions

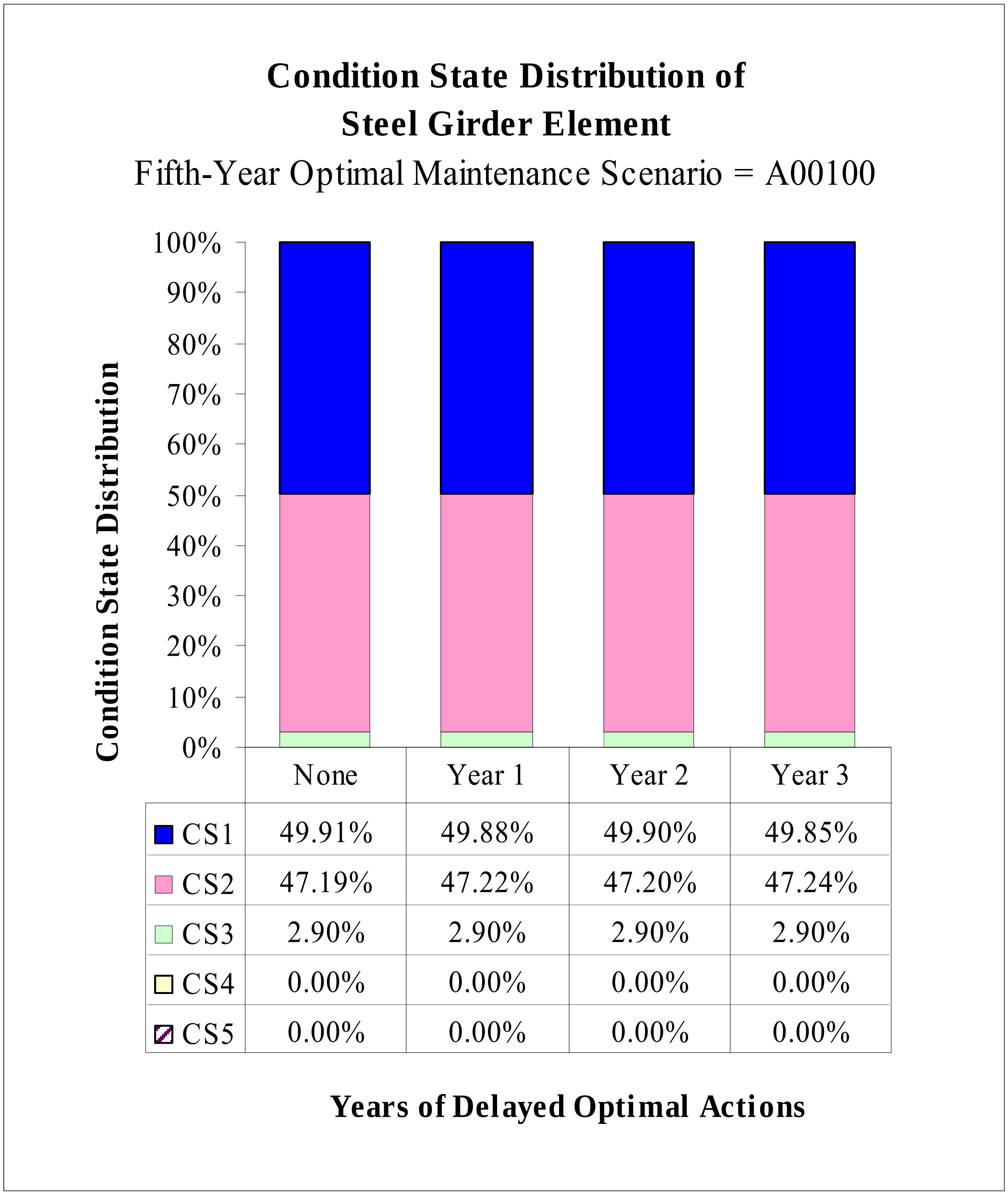


	Year 2 & 4	Year 3 & 4	Years 1, 2 & 4	Years 1, 3 & 4	Years 2, 3 & 4	Years 1, 2, 3 & 4
▨ Cost & Risk	54,751	82,525	56,270	83,140	86,670	110,410
■ Risk	46,844	47,233	46,923	47,310	47,834	52,060
□ Maintenance Cost	7,906	35,292	9,348	35,830	38,836	58,351

Figure 4-49. Scenarios of Optimal Actions (A00121) in Fifth Year of Maintenance with Corresponding Cases of Delayed Optimal Actions

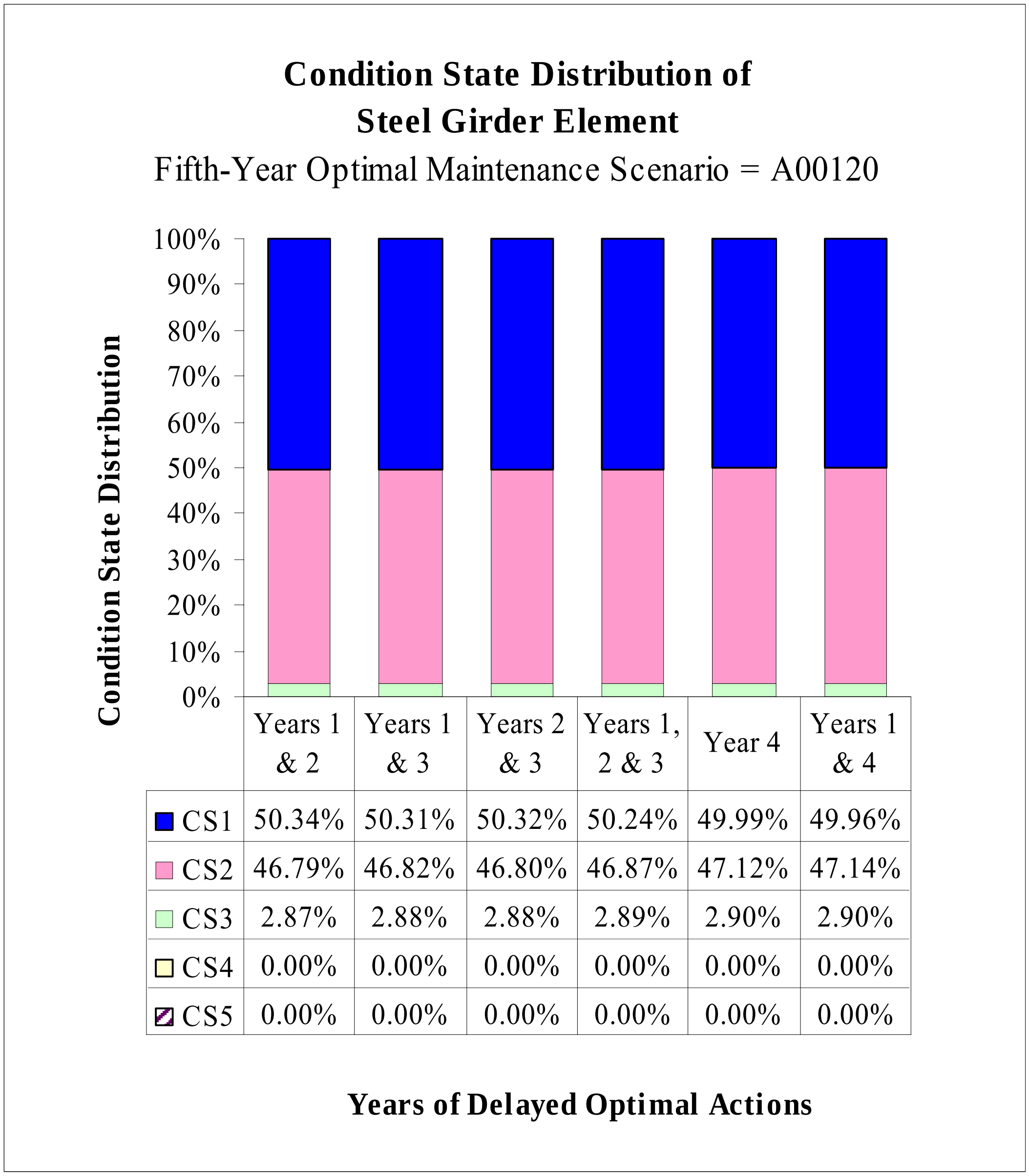
Table 4-74. Fifth-Year Optimal Maintenance Actions in the Condition States of the Steel Girder Element in the Different Cases of Delay of Previous-Years Optimal Actions

Years of Delayed Optimal Actions	Fifth-Year Optimal Maintenance Actions for the Steel Girder				
	CS1	CS2	CS3	CS4	CS5
None	$a(1,0)$	$a(2,0)$	$a(3,1)$	$a(4,0)$	$a(5,0)$
Year 1					
Year 2					
Years 1 & 2					
Year 3	$a(1,0)$	$a(2,0)$	$a(3,1)$	$a(4,2)$	$a(5,0)$
Years 1 & 3					
Years 2 & 3					
Years 1, 2 & 3					
Year 4					
Years 1 & 4					
Year 2 & 4	$a(1,0)$	$a(2,0)$	$a(3,1)$	$a(4,2)$	$a(5,1)$
Year 3 & 4					
Years 1, 2 & 4					
Years 1, 3 & 4					
Years 2, 3 & 4					
Years 1, 2, 3 & 4					



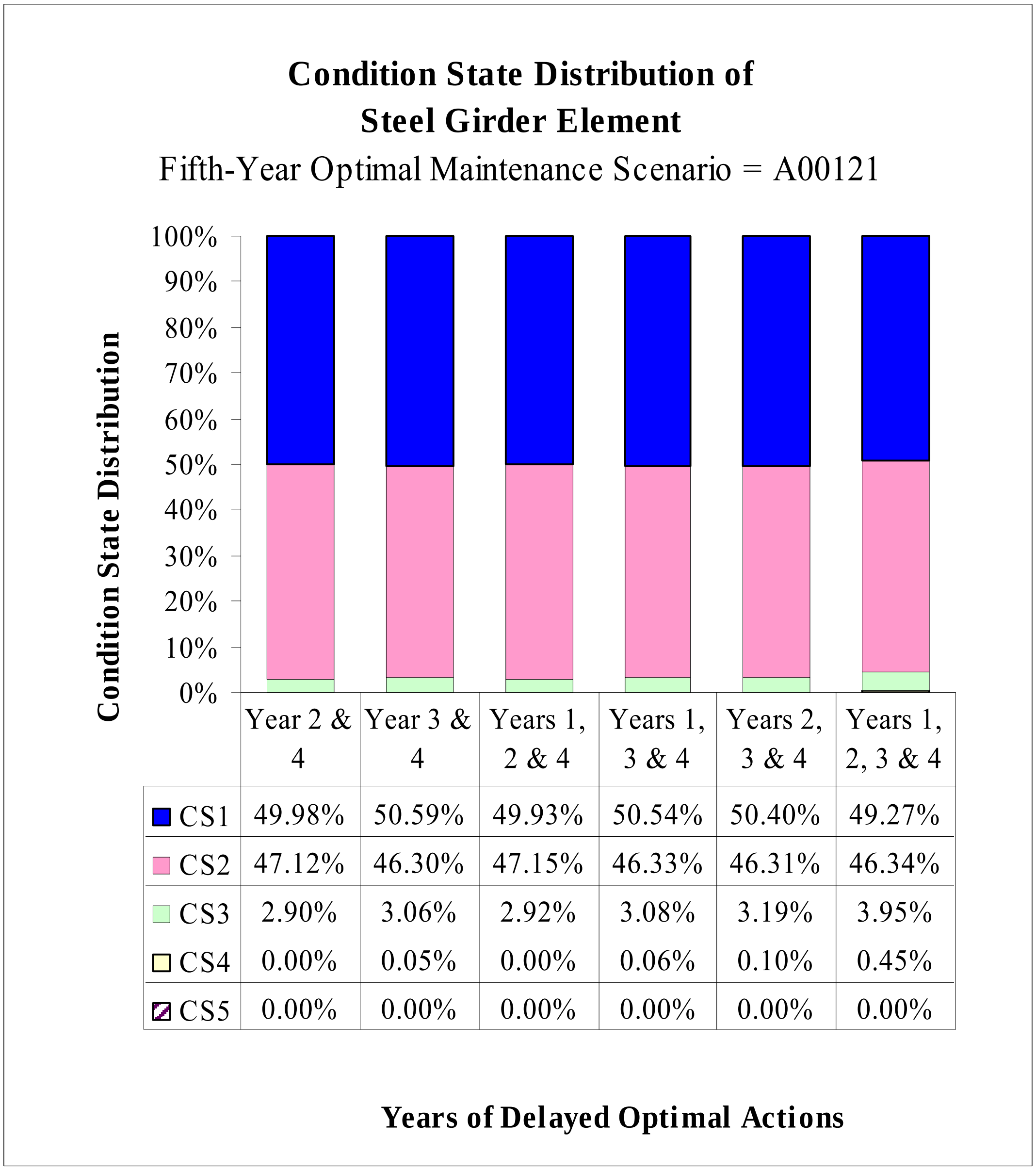
	None	Year 1	Year 2	Year 3
■ CS1	49.91%	49.88%	49.90%	49.85%
■ CS2	47.19%	47.22%	47.20%	47.24%
■ CS3	2.90%	2.90%	2.90%	2.90%
□ CS4	0.00%	0.00%	0.00%	0.00%
▨ CS5	0.00%	0.00%	0.00%	0.00%

Figure 4-50. Condition State Distribution for the Steel Girder Element for Cases That Use Scenario A00100 for Fifth-Year Optimal Maintenance Actions



	Years 1 & 2	Years 1 & 3	Years 2 & 3	Years 1, 2 & 3	Year 4	Years 1 & 4
🟦 CS1	50.34%	50.31%	50.32%	50.24%	49.99%	49.96%
🟪 CS2	46.79%	46.82%	46.80%	46.87%	47.12%	47.14%
🟩 CS3	2.87%	2.88%	2.88%	2.89%	2.90%	2.90%
🟨 CS4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
▨ CS5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Figure 4-51. Condition State Distribution for the Steel Girder Element for Cases That Use Scenario A00120 for Fifth-Year Optimal Maintenance Actions



	Year 2 & 4	Year 3 & 4	Years 1, 2 & 4	Years 1, 3 & 4	Years 2, 3 & 4	Years 1, 2, 3 & 4
■ CS1	49.98%	50.59%	49.93%	50.54%	50.40%	49.27%
■ CS2	47.12%	46.30%	47.15%	46.33%	46.31%	46.34%
■ CS3	2.90%	3.06%	2.92%	3.08%	3.19%	3.95%
□ CS4	0.00%	0.05%	0.00%	0.06%	0.10%	0.45%
▨ CS5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Figure 4-52. Condition State Distribution for the Steel Girder Element for Cases That Use Scenario A00121 for Fifth-Year Optimal Maintenance Actions

4.13.2 Maintenance Scenarios of Implementing/Deferring the Optimal Maintenance Actions for Steel Girder Element

To study the economic efficiency of applying or delaying the optimal actions for the steel girder maintenance, 32 scenarios were created during the five years of maintenance similar to the example of the concrete deck element. The maintenance scenarios and the optimal actions during the five years of maintenance are shown in Table 4-75. In Table 4-75, for example, it is shown that scenario 10011 of delayed second and third-year optimal actions is composed of the actions A00021, A00000, A00000, A00121 and A00120 in years 1, 2, 3, 4 and 5, respectively. Therefore, scenario 10011 refers to applying actions $a(1,0)$, $a(2,0)$, $a(3,0)$, $a(4,2)$ and $a(5,1)$ in condition states 1, 2, 3, 4, and 5 in the first year, doing nothing in the second and third year, applying actions $a(1,0)$, $a(2,0)$, $a(3,1)$, $a(4,2)$ and $a(5,1)$ in condition states 1, 2, 3, 4 and 5 in the fourth year, and applying actions $a(1,0)$, $a(2,0)$, $a(3,1)$, $a(4,2)$ and $a(5,0)$ in condition states 1, 2, 3, 4 and 5 in the fifth year of maintenance.

Table 4-75. Maintenance Actions for Different Scenarios of Delayed and Applied Optimal Actions for the Steel Girder Element

Years of Delayed Optimal Actions	Scenario	Maintenance Options				
		Year 1	Year 2	Year 3	Year 4	Year 5
No Delay	11111	A00021	A00020	A00120	A00100	A00100
Year 1	01111	A00000	A00021	A00120	A00100	A00100
Year 2	10111	A00021	A00000	A00121	A00120	A00100
Year 3	11011	A00021	A00020	A00000	A00121	A00120
Year 4	11101	A00021	A00020	A00120	A00000	A00120
Years 1 & 2	00111	A00000	A00000	A00121	A00120	A00100
Years 1 & 3	01011	A00000	A00021	A00000	A00121	A00120
Years 1 & 4	01101	A00000	A00021	A00120	A00000	A00120
Years 2 & 3	10011	A00021	A00000	A00000	A00121	A00120
Year 2 & 4	10101	A00021	A00000	A00121	A00000	A00121
Year 3 & 4	11001	A00021	A00020	A00000	A00000	A00121
Years 1, 2 & 3	00011	A00000	A00000	A00000	A00121	A00120
Years 1, 2 & 4	00101	A00000	A00000	A00121	A00000	A00121
Years 1, 3 & 4	01001	A00000	A00021	A00000	A00000	A00121
Years 2, 3 & 4	10001	A00021	A00000	A00000	A00000	A00121
Years 1, 2, 3 & 4	00001	A00000	A00000	A00000	A00000	A00121
Year 5	11110	A00021	A00020	A00120	A00100	A00000
Year 1 & 5	01110	A00000	A00021	A00120	A00100	A00000
Year 2 & 5	10110	A00021	A00000	A00121	A00120	A00000
Year 3 & 5	11010	A00021	A00020	A00000	A00121	A00000
Year 4 & 5	11100	A00021	A00020	A00120	A00000	A00000
Years 1, 2 & 5	00110	A00000	A00000	A00121	A00120	A00000
Years 1, 3 & 5	01010	A00000	A00021	A00000	A00121	A00000
Years 1, 4 & 5	01100	A00000	A00021	A00120	A00000	A00000
Years 2, 3 & 5	10010	A00021	A00000	A00000	A00121	A00000
Year 2, 4 & 5	10100	A00021	A00000	A00121	A00000	A00000
Year 3, 4 & 5	11000	A00021	A00020	A00000	A00000	A00000
Years 1, 2, 3 & 5	00010	A00000	A00000	A00000	A00121	A00000
Years 1, 2, 4 & 5	00100	A00000	A00000	A00121	A00000	A00000
Years 1, 3, 4 & 5	01000	A00000	A00021	A00000	A00000	A00000
Years 2, 3, 4 & 5	10000	A00021	A00000	A00000	A00000	A00000
Years 1, 2, 3, 4 & 5	00000	A00000	A00000	A00000	A00000	A00000

4.13.3 Cost and Risk Associated with Maintenance Scenarios of Steel Girder Element

The maintenance cost for the steel girder is calculated for the five years of the planning horizon for maintenance. The present values for the maintenance cost are evaluated using a discount rate of 2.6% (OMB, 2007). The total present value of the maintenance cost for each scenario is calculated as shown in Table 4-76. The risk associated with the different scenarios of the steel girder is calculated for each year of maintenance in the planning horizon as shown in Table 4-77. Table 4-77 shows that, for example, applying the first and second-year optimal actions (using actions A00021 and A00020 from Table 4-75) will result in a risk value of $24,354 in the first year and $40,286 in the second year, while delaying the first and second-year optimal actions and choosing the no-maintenance option (A00000) in both years will result in a risk value of $45,038 in the first year and $85,076 in the second year.

Table 4-76. Maintenance Costs for the Different Maintenance Scenarios of Steel Girder Element

Years of Delayed Optimal Actions for	Maintenance Cost Per Year ($)					Total Present Cost ($)
	Year 1	Year 2	Year 3	Year 4	Year 5	
No Delay	15,500	2,325	24,258	3,769	3,665	47,607
Year 1	0	19,025	24,656	3,783	3,668	48,777
Year 2	15,500	0	26,962	3,908	3,666	48,040
Year 3	15,500	2,325	0	29,137	3,777	48,151
Year 4	15,500	2,325	24,258	0	7,558	47,630
Years 1 & 2	0	0	44,128	5,193	3,683	50,052
Years 1 & 3	0	19,025	0	29,601	3,797	49,376
Years 1 & 4	0	19,025	24,656	0	7,574	48,800
Years 2 & 3	15,500	0	0	32,243	3,964	48,930
Year 2 & 4	15,500	0	26,962	0	7,706	48,067
Year 3 & 4	15,500	2,325	0	0	34,398	48,807
Years 1, 2 & 3	0	0	0	50,413	5,377	51,529
Years 1, 2 & 4	0	0	44,128	0	9,111	50,142
Years 1, 3 & 4	0	19,025	0	0	34,922	50,057
Years 2, 3 & 4	15,500	0	0	0	37,851	49,658
Years 1, 2, 3 & 4	0	0	0	0	56,872	51,323
Year 5	15,500	2,325	24,258	3,769	0	44,300
Year 1 & 5	0	19,025	24,656	3,783	0	45,467
Year 2 & 5	15,500	0	26,962	3,908	0	44,732
Year 3 & 5	15,500	2,325	0	29,137	0	44,743
Year 4 & 5	15,500	2,325	24,258	0	0	40,810
Years 1, 2 & 5	0	0	44,128	5,193	0	46,728
Years 1, 3 & 5	0	19,025	0	29,601	0	45,950
Years 1, 4 & 5	0	19,025	24,656	0	0	41,965
Years 2, 3 & 5	15,500	0	0	32,243	0	45,353
Year 2, 4 & 5	15,500	0	26,962	0	0	41,113
Year 3, 4 & 5	15,500	2,325	0	0	0	17,766
Years 1, 2, 3 & 5	0	0	0	50,413	0	46,677
Years 1, 2, 4 & 5	0	0	44,128	0	0	41,920
Years 1, 3, 4 & 5	0	19,025	0	0	0	18,543
Years 2, 3, 4 & 5	15,500	0	0	0	0	15,500
Years 1, 2, 3, 4 & 5	0	0	0	0	0	0

Table 4-77. Risk Associated with Maintenance Scenarios of the Steel Girder Element During Five Years of Maintenance

Years of Delayed Optimal Actions	Risk Associated with Steel Girder Maintenance ($)				
	Year 1	Year 2	Year 3	Year 4	Year 5
No Delay	24,354	40,286	30,272	38,626	46,880
Year 1	45,038	40,973	30,295	38,640	46,897
Year 2	24,354	46,050	30,471	38,630	46,884
Year 3	24,354	40,286	67,345	38,694	46,656
Year 4	24,354	40,286	30,272	45,005	46,838
Years 1 & 2	45,038	85,076	32,683	38,669	46,909
Years 1 & 3	45,038	40,973	68,554	38,741	46,672
Years 1 & 4	45,038	40,973	30,295	45,042	46,854
Years 2 & 3	24,354	46,050	76,885	39,077	46,666
Year 2 & 4	24,354	46,050	30,471	45,322	46,844
Year 3 & 4	24,354	40,286	67,345	103,109	47,233
Years 1, 2 & 3	45,038	85,076	135,336	42,262	46,730
Years 1, 2 & 4	45,038	85,076	32,683	48,837	46,923
Years 1, 3 & 4	45,038	40,973	68,554	104,928	47,310
Years 2, 3 & 4	24,354	46,050	76,885	116,790	47,834
Years 1, 2, 3 & 4	45,038	85,076	135,336	194,712	52,060
Year 5	24,354	40,286	30,272	38,626	54,456
Year 1 & 5	45,038	40,973	30,295	38,640	54,478
Year 2 & 5	24,354	46,050	30,471	38,630	54,463
Year 3 & 5	24,354	40,286	67,345	38,694	54,559
Year 4 & 5	24,354	40,286	30,272	45,005	64,007
Years 1, 2 & 5	45,038	85,076	32,683	38,669	54,521
Years 1, 3 & 5	45,038	40,973	68,554	38,741	54,630
Years 1, 4 & 5	45,038	40,973	30,295	45,042	64,063
Years 2, 3 & 5	24,354	46,050	76,885	39,077	55,133
Year 2, 4 & 5	24,354	46,050	30,471	45,322	64,470
Year 3, 4 & 5	24,354	40,286	67,345	103,109	148,015
Years 1, 2, 3 & 5	45,038	85,076	135,336	42,262	59,893
Years 1, 2, 4 & 5	45,038	85,076	32,683	48,837	69,537
Years 1, 3, 4 & 5	45,038	40,973	68,554	104,928	150,501
Years 2, 3, 4 & 5	24,354	46,050	76,885	116,790	165,996
Years 1, 2, 3, 4 & 5	45,038	85,076	135,336	194,712	262,921

4.13.4 Filtering the Maintenance Scenarios for Steel Girder Element

Limitations for maintenance costs and risks are assumed for filtering the maintenance scenarios of the steel girder element. It is assumed that the maximum value allowed for the risk associated with the steel girder condition in any year of the maintenance period is $80,000. It is also assumed that the annual budget for the maintenance cost of the steel girder should not exceed $40,000.

Based on the above-mentioned criteria, 14 scenarios out of the 32 maintenance scenarios for the steel girder element will be eliminated. Table 4-79 shows the maintenance scenarios that were eliminated because they exceed the maximum risk allowed for the girder condition or the annual budget for maintenance.

Table 4-78. Eliminated Scenarios Exceeding the Maximum Risk and/or the Annual Budget for the Steel Girder Maintenance

Scenario	Years of Delayed Optimal Actions	Reason for Scenario Elimination	
		Exceeding Maximum Risk	Exceeding Maintenance Budget
00111	Years 1 & 2	X	X
11001	Year 3 & 4	X	
00011	Years 1, 2 & 3	X	X
00101	Years 1, 2 & 4	X	X
00110	Years 1, 2 & 5	X	X
01001	Years 1, 3 & 4	X	
10001	Years 2, 3 & 4	X	
11000	Year 3, 4 & 5	X	
00001	Years 1, 2, 3 & 4	X	X
00010	Years 1, 2, 3 & 5	X	X
00100	Years 1, 2, 4 & 5	X	X
01000	Years 1, 3, 4 & 5	X	
10000	Years 2, 3, 4 & 5	X	
00000	Years 1, 2, 3, 4 & 5	X	

4.13.5 Ranking Maintenance Scenarios of Steel Girder Element

The risk reduction associated with each year of maintenance for the steel girder is calculated as the non-mitigated risk due to the delay of the optimal actions minus the mitigated risk due to the application of the optimal actions in that year. The present values for the risk reductions in the five years are evaluated using a discount rate of 2.6%. The total present value of risk reduction associated with each maintenance scenario for the steel girder element is calculated as shown in Table 4-79.

Benefit-cost analysis is used to study the economic efficiency of the maintenance scenarios. The benefit-cost ratios (BCR) and net present values (NPV) for the eighteen filtered scenarios are calculated based on the mean values for risk reduction and maintenance cost. The benefit-cost ratio (BCR) for each maintenance scenario of the steel girder is calculated by dividing the mean present value of the risk reduction by the mean present value of maintenance cost. The mean net present value (NPV) for each maintenance scenario is calculated as the mean present value of risk reduction minus the mean present value of maintenance cost.

The benefit-cost ratios and net present values are calculated and compared for the filtered scenarios of the girder maintenance after eliminating the scenarios that exceed the risk and cost limits as shown in Table 4-80. It is shown in Table 4-80, for example, that scenario 01010 with delayed first, third and fifth-year optimal actions have the highest benefit-cost ratio (BCR) among the filtered maintenance scenarios of the steel girder, while scenarios 01011 with delayed first- and third-year optimal actions have the highest net present value (NPV). Scenario 01010 ranked second in terms of NPV, and scenario 01011 ranked second in terms of BCR. Scenario 11111 with no delay of optimal actions ranked sixteenth in terms of BCR and fourteenth in terms of NPV. The ranking of the maintenance scenarios for the steel girder element based on the BCR and NPV are shown in Table 4-81 with the scenario in the top row as the most preferable and the scenario in the bottom as the least preferable.

The benefit-cost reliability index ($\beta_{B/C}$) and benefit-cost failure probability ($P_{f,B/C}$) are also used to measure the probability of the risk reduction exceeding the maintenance cost for the maintenance scenarios of the steel girder. The benefit-cost reliability index is calculated using the means and standard deviations for the maintenance cost and risk reduction associated with each scenario.

The standard deviations of risk reduction and maintenance cost are calculated as described earlier in the concrete deck example. Equations 4-15 and 4-16 are used to calculate the standard deviations for the total present value of maintenance cost for each scenario. Equations 4-17, 4-18, 4-19, and 4-20 are used to calculate the standard deviation for the total present value of risk reduction associated with each scenario. The means and standard deviations for the risk reductions and the maintenance costs were calculated for the filtered maintenance scenarios of the steel girder element and summarized in Table 4-82.

The means and standard deviations of the maintenance cost and risk reduction are used to calculate the benefit-cost reliability index ($\beta_{B/C}$) associated with the filtered scenarios for the steel girder maintenance. Assuming normal distributions for the maintenance costs and risk reductions, the benefit-cost reliability index ($\beta_{B/C}$) and benefit-cost probability of failure ($P_{f,B/C}$) associated with each maintenance scenario are calculated using Equations 4-21 and 4-22. Table 4-83 shows a summary of the calculated values for $\beta_{B/C}$ and $P_{f,B/C}$ for the filtered scenarios of the steel girder maintenance. According to Table 4-83, scenario 01010 with delayed first, third and fifth-year optimal actions has the highest $\beta_{B/C}$ value and lowest $P_{f,B/C}$ value. Scenario 11111 with no delay in any of the optimal actions ranks eighteenth in terms of $\beta_{B/C}$ and $P_{f,B/C}$.

Table 4-79. Risk Reduction Associated with Maintenance Scenarios of the Steel Girder Element

Years of Delayed Optimal Actions	Risk Reduction Per Year ($)					Total Present Value of Risk Reduction ($)
	Year 1	Year 2	Year 3	Year 4	Year 5	
No Delay	20,684	5,764	37,073	6,380	7,576	72,382
Year 1	0	44,103	38,259	6,402	7,581	89,765
Year 2	20,684	0	46,414	6,692	7,578	75,838
Year 3	20,684	5,764	0	64,415	7,903	90,716
Year 4	20,684	5,764	37,073	0	17,169	75,062
Years 1 & 2	0	0	102,653	10,167	7,613	110,916
Years 1 & 3	0	44,103	0	66,186	7,958	108,623
Years 1 & 4	0	44,103	38,259	0	17,208	92,455
Years 2 & 3	20,684	0	0	77,713	8,466	97,736
Year 2 & 4	20,684	0	46,414	0	17,626	78,636
Year 3 & 4	20,684	5,764	0	0	100,782	114,279
Years 1, 2 & 3	0	0	0	152,449	13,163	149,151
Years 1, 2 & 4	0	0	102,653	0	22,614	114,936
Years 1, 3 & 4	0	44,103	0	0	103,191	132,658
Years 2, 3 & 4	20,684	0	0	0	118,161	124,089
Years 1, 2, 3 & 4	0	0	0	0	210,862	185,465
Year 5	20,684	5,764	37,073	6,380	0	65,718
Year 1 & 5	0	44,103	38,259	6,402	0	83,096
Year 2 & 5	20,684	0	46,414	6,692	0	69,173
Year 3 & 5	20,684	5,764	0	64,415	0	83,765
Year 4 & 5	20,684	5,764	37,073	0	0	59,961
Years 1, 2 & 5	0	0	102,653	10,167	0	104,220
Years 1, 3 & 5	0	44,103	0	66,186	0	101,624
Years 1, 4 & 5	0	44,103	38,259	0	0	77,319
Years 2, 3 & 5	20,684	0	0	77,713	0	90,290
Year 2, 4 & 5	20,684	0	46,414	0	0	63,134
Year 3, 4 & 5	20,684	5,764	0	0	0	25,635
Years 1, 2, 3 & 5	0	0	0	152,449	0	137,574
Years 1, 2, 4 & 5	0	0	102,653	0	0	95,045
Years 1, 3, 4 & 5	0	44,103	0	0	0	41,896
Years 2, 3, 4 & 5	20,684	0	0	0	0	20,160
Years 1, 2, 3, 4 & 5	0	0	0	0	0	0

Table 4-80. Benefit-Cost Ratio (BCR) and Net Present Value (NPV) for Filtered Maintenance Scenarios of Steel Girder Element

Scenario	Years of Delayed Optimal Actions	BCR	NPV ($)	Scenario Ranking Based on	
				BCR	NPV
11111	Years 1, 2 & 4	1.520	24,774	16	14
11110	Year 2 & 4	1.483	21,418	17	17
11101	Years 1, 3 & 4	1.576	27,432	13	13
11100	Year 2 & 5	1.469	19,151	18	18
11011	Years 1, 2, 4 & 5	1.884	42,565	6	6
11010	Year 3	1.872	39,022	7	8
10111	Years 1, 2 & 5	1.579	27,798	12	12
10110	Year 4	1.546	24,441	14	15
10101	Years 2, 3 & 4	1.636	30,569	11	11
10100	Year 4 & 5	1.536	22,020	15	16
10011	Year 1	1.997	48,806	3	3
10010	Year 5	1.991	44,937	4	4
01111	Years 1 & 2	1.840	40,987	9	7
01110	Year 2	1.828	37,629	10	9
01101	Years 1, 3 & 5	1.895	43,654	5	5
01100	Year 2, 4 & 5	1.842	35,354	8	10
01011	Years 1 & 4	2.200	59,247	2	1
01010	Year 3 & 5	2.212	55,674	1	2

Table 4-81. Ranking of Filtered Maintenance Scenarios of Steel Girder Element based on Benefit-Cost Ratio (BCR) and Net Present Value (NPV)

Ranking Based on BCR			Ranking Based on NPV		
Scenario	Years of Delayed Optimal Actions	BCR	Scenario	Years of Delayed Optimal Actions	NPV ($)
01010	Years 1, 3 & 5	2.212	01011	Years 1 & 3	59,247
01011	Years 1 & 3	2.200	01010	Years 1, 3 & 5	55,674
10011	Years 2 & 3	1.997	10011	Years 2 & 3	48,806
10010	Years 2, 3 & 5	1.991	10010	Years 2, 3 & 5	44,937
01101	Years 1 & 4	1.895	01101	Years 1 & 4	43,654
11011	Year 3	1.884	11011	Year 3	42,565
11010	Year 3 & 5	1.872	01111	Year 1	40,987
01100	Years 1, 4 & 5	1.842	11010	Year 3 & 5	39,022
01111	Year 1	1.840	01110	Year 1 & 5	37,629
01110	Year 1 & 5	1.828	01100	Years 1, 4 & 5	35,354
10101	Year 2 & 4	1.636	10101	Year 2 & 4	30,569
10111	Year 2	1.579	10111	Year 2	27,798
11101	Year 4	1.576	11101	Year 4	27,432
10110	Year 2 & 5	1.546	11111	No Delay	24,774
10100	Year 2, 4 & 5	1.536	10110	Year 2 & 5	24,441
11111	No Delay	1.520	10100	Year 2, 4 & 5	22,020
11110	Year 5	1.483	11110	Year 5	21,418
11100	Year 4 & 5	1.469	11100	Year 4 & 5	19,151

Table 4-82. Means and Standard Deviations for Risk Reductions and Maintenance Costs for Filtered Maintenance Scenarios for Steel Girder

Scenario	Delay of Optimal Actions for Years	Risk Reduction ($)		Maintenance Cost ($)	
		Mean	Standard Deviation	Mean	Standard Deviation
11111	No Delay	72,382	10,263	47,607	4,448
11110	Year 5	65,718	8,543	44,300	4,398
11101	Year 4	75,062	9,074	47,630	4,475
11100	Year 4 & 5	59,961	6,901	40,810	4,251
11011	Year 3	90,716	9,388	48,151	4,569
11010	Year 3 & 5	83,765	7,700	44,743	4,401
10111	Year 2	75,838	8,817	48,040	4,476
10110	Year 2 & 5	69,173	7,370	44,732	4,426
10101	Year 2 & 4	78,636	7,879	48,067	4,499
10100	Year 2, 4 & 5	63,134	5,831	41,113	4,267
10011	Years 2 & 3	97,736	8,961	48,930	4,590
10010	Years 2, 3 & 5	90,290	7,009	45,353	4,415
01111	Year 1	89,765	11,052	48,777	4,599
01110	Year 1 & 5	83,096	9,276	45,467	4,551
01101	Years 1 & 4	92,455	9,821	48,800	4,626
01100	Years 1, 4 & 5	77,319	7,617	41,965	4,408
01011	Years 1 & 3	108,623	9,972	49,376	4,724
01010	Years 1, 3 & 5	101,624	8,383	45,950	4,560

Table 4-83. Benefit-Cost Reliability Index ($\beta_{B/C}$)and Probability of Failure ($P_{f,B/C}$) Associated with Filtered Maintenance Scenarios for Steel Girder Element

Scenario	Delay of Optimal Actions for Years	Net Present Value ($)		Benefit-cost Reliability Index ($\beta_{B/C}$)	Benefit-cost Probability of Failure ($P_{f,B/C}$)	Rank Based on $\beta_{B/C}$ and $P_{f,B/C}$
		Mean	Standard Deviation			
11111	No Delay	24,774	11,185	2.21	1.34E-02	18
11110	Year 5	21,418	9,608	2.23	1.29E-02	17
11101	Year 4	27,432	10,118	2.71	3.35E-03	15
11100	Year 4 & 5	19,151	8,105	2.36	9.07E-03	16
11011	Year 3	42,565	10,441	4.08	2.28E-05	6
11010	Year 3 & 5	39,022	8,869	4.40	5.42E-06	5
10111	Year 2	27,798	9,888	2.81	2.47E-03	14
10110	Year 2 & 5	24,441	8,597	2.84	2.23E-03	13
10101	Year 2 & 4	30,569	9,073	3.37	3.77E-04	11
10100	Year 2, 4 & 5	22,020	7,225	3.05	1.15E-03	12
10011	Years 2 & 3	48,806	10,068	4.85	6.24E-07	4
10010	Years 2, 3 & 5	44,937	8,284	5.42	2.90E-08	2
01111	Year 1	40,987	11,971	3.42	3.09E-04	10
01110	Year 1 & 5	37,629	10,332	3.64	1.35E-04	9
01101	Years 1 & 4	43,654	10,856	4.02	2.89E-05	7
01100	Years 1, 4 & 5	35,354	8,800	4.02	2.94E-05	8
01011	Years 1 & 3	59,247	11,034	5.37	3.95E-08	3
01010	Years 1, 3 & 5	55,674	9,543	5.83	2.70E-09	1

4.13.6 Selecting the Optimal Maintenance Scenario for Steel Girder Element

Comparisons between the different scenarios for the steel girder element in terms of BCR, NPV, $\beta_{B/C}$ and $P_{f,B/C}$ are shown in Table 4-84. The ranking of the maintenance scenarios based on the above-mentioned economic measures is shown in Table 4-85. The ranking of the maintenance scenarios according to different economic measures can help the decision makers to make informed maintenance decisions.

For demonstration purposes, the optimal maintenance scenario for the steel girder element will be selected as the one with the highest net present value. Therefore, the strategy for the steel girder element is to apply scenario 01011 in which the second, fourth and fifth-year optimal actions are applied and the first and third-year optimal actions are delayed. Based on the assumptions made in developing the risk and maintenance cost for the steel girder, scenario 01011 has a mean net present value of \$59,247 and a standard deviation of \$11,034. The benefit-cost ratio for this scenario is 2.20 and the benefit-cost reliability index is 5.37. This implies that there is a 3.95E-8 probability that the maintenance cost will exceed the risk reduction associated with this scenario. The optimal actions in year 1, 2, 3, 4, and 5 for maintenance scenario 01011 are A00000, A00021, A00000, A00121, and A00120, respectively, as shown earlier in Table 4-75. A00000 refers to doing nothing in all condition states of the steel girder. A00021 refers to doing nothing in condition states

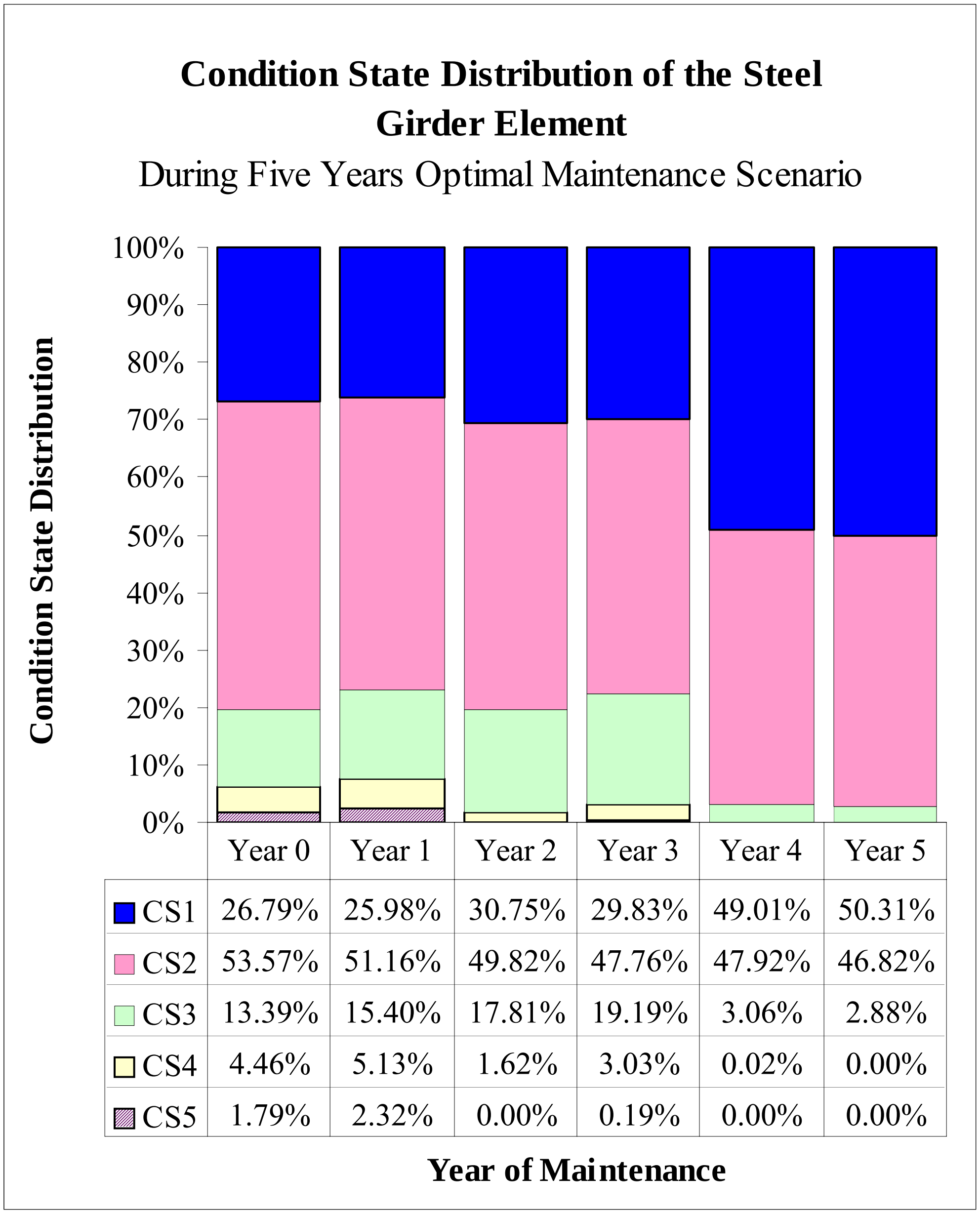
1, 2, and 3, and applying action *a*(4,2) in condition state 4 and action *a*(5,1) in condition state 5. A00121 refers to doing nothing in condition states 1, and 2, and applying actions *a*(3,1), *a*(4,2), and *a*(5,1) in condition states 3, 4 and 5, respectively. A00120 refers to doing nothing in condition states 1, 2, and 5, and applying action *a*(3,1) in condition state 3 and action *a*(4,2) in condition state 4. The condition state distribution of the steel girder element when the optimal maintenance strategy is followed is shown in Figure 4-53.

Table 4-84. Comparison Between the Maintenance Scenarios for the Steel Girder in Terms of BCR, NPV, $\beta_{B/C}$ and $P_{f,B/C}$

Scenario	Years of Delayed Optimal Actions	Benefit-Cost Ratio (BCR)	Net Present Value (NPV), $	Benefit-cost Reliability Index ($\beta_{B/C}$)	Benefit-cost Probability of Failure ($P_{f,B/C}$)
01011	Years 1 & 3	2.20	59,247	5.37	3.95E-08
01010	Years 1, 3 & 5	2.21	55,674	5.83	2.70E-09
10011	Years 2 & 3	2.00	48,806	4.85	6.24E-07
10010	Years 2, 3 & 5	1.99	44,937	5.42	2.90E-08
01101	Years 1 & 4	1.89	43,654	4.02	2.89E-05
11011	Year 3	1.88	42,565	4.08	2.28E-05
01111	Year 1	1.84	40,987	3.42	3.09E-04
11010	Year 3 & 5	1.87	39,022	4.40	5.42E-06
01110	Year 1 & 5	1.83	37,629	3.64	1.35E-04
01100	Years 1, 4 & 5	1.84	35,354	4.02	2.94E-05
10101	Year 2 & 4	1.64	30,569	3.37	3.77E-04
10111	Year 2	1.58	27,798	2.81	2.47E-03
11101	Year 4	1.58	27,432	2.71	3.35E-03
11111	No Delay	1.52	24,774	2.21	1.34E-02
10110	Year 2 & 5	1.55	24,441	2.84	2.23E-03
10100	Year 2, 4 & 5	1.54	22,020	3.05	1.15E-03
11110	Year 5	1.48	21,418	2.23	1.29E-02
11100	Year 4 & 5	1.47	19,151	2.36	9.07E-03

Table 4-85. Ranking of the Maintenance Scenarios for the Steel Girder Element Based on BCR, NPV, $\beta_{B/C}$ and $P_{f,B/C}$

Scenario	Years of Delayed Optimal Actions	Scenario Ranking Based on		
		BCR	NPV	$\beta_{B/C}$ & $P_{f,B/C}$
01011	Years 1 & 3	2	1	3
01010	Years 1, 3 & 5	1	2	1
10011	Years 2 & 3	3	3	4
10010	Years 2, 3 & 5	4	4	2
01101	Years 1 & 4	5	5	7
11011	Year 3	6	6	6
01111	Year 1	9	7	10
11010	Year 3 & 5	7	8	5
01110	Year 1 & 5	10	9	9
01100	Years 1, 4 & 5	8	10	8
10101	Year 2 & 4	11	11	11
10111	Year 2	12	12	14
11101	Year 4	13	13	15
11111	No Delay	16	14	18
10110	Year 2 & 5	14	15	13
10100	Year 2, 4 & 5	15	16	12
11110	Year 5	17	17	17
11100	Year 4 & 5	18	18	16



	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
■ CS1	26.79%	25.98%	30.75%	29.83%	49.01%	50.31%
□ CS2	53.57%	51.16%	49.82%	47.76%	47.92%	46.82%
□ CS3	13.39%	15.40%	17.81%	19.19%	3.06%	2.88%
□ CS4	4.46%	5.13%	1.62%	3.03%	0.02%	0.00%
▨ CS5	1.79%	2.32%	0.00%	0.19%	0.00%	0.00%

Figure 4-53. Condition State Distribution Associated with the Five Years Risk-Based Optimal Maintenance Strategy for the Steel Girder Element

4.14 Bridge-Level Risk-Based Maintenance

The optimal maintenance scenarios for all elements of the bridge are determined following the same steps used for the concrete deck and steel girder elements. The cost of the bridge maintenance is the sum of the maintenance costs for the optimal scenarios of the elements. The risk associated with the bridge maintenance is the sum of the risks associated with the optimal maintenance scenarios of the elements.

The maintenance cost for the bridge in each year of maintenance is compared to the annual budget allocated for the bridge maintenance. Also the risk associated with the bridge is compared to a maximum risk value for the bridge. If the total maintenance cost exceeds the annual budget, or the total risk exceeds the maximum risk in a particular year, the optimal actions for one or more of the elements will be delayed. The selection of delaying the optimal maintenance actions for an element depends on the weight factor for the element. For example, the weight factors for the concrete deck and steel girder are 6 and 10, respectively; therefore, the steel girder has a higher priority of maintenance than the concrete deck, and delay, if needed, will be considered for the deck.

If the optimal maintenance actions in a particular year were delayed for an element, the optimal scenario for the element is modified to consider that year of delay. For example, the optimal maintenance scenario for the concrete deck is scenario 01101 with delay of the first and fourth-year optimal actions. If the annual budget or risk limitations for the bridge in the third year required that the optimal maintenance actions for the deck should be delayed, the optimal maintenance scenario for the deck is modified to become scenario 01011 instead of 01101. Scenario 01011 with delay of first- and third-year optimal actions is the most efficient maintenance scenario with third-year delay. This implies that the optimal actions for the fourth and the fifth year will be A00011 and A00010 instead of A00000 and A00011, respectively, as previously shown in Table 4-61.

The risk reduction associated with the bridge maintenance is the sum of the risk reductions that result from implementing the optimal maintenance scenarios of the bridge elements. The benefit-cost ratio (BCR) for the bridge maintenance is equal to the present value of the risk reduction for the bridge maintenance divided by the present value of the total cost for the bridge maintenance during the five years. The net present value (NPV) for the bridge maintenance is equal to the present value of the risk reduction for the bridge maintenance minus the present value of the total maintenance cost for the bridge during the five years.

If an optimal scenario of an element is modified due to the delay of the optimal actions in a particular year, the risk reduction and maintenance cost associated with the modified scenario are used in calculating the BCR and the NPV for the bridge. For example, if scenario 01011 for the deck maintenance with delay of first- and third-year optimal actions is used instead of scenario 01101 with delay of first- and fourth-year optimal actions, the present values of the maintenance cost and risk-reduction will be \$16,566 and \$40,300 instead of \$16,591 and \$40,769 as shown previously in Table 4-62 and 4-66.

The benefit-cost reliability index for the bridge maintenance is calculated using the means and standard deviations for the total maintenance costs and risk reductions associated with the optimal maintenance scenarios for bridge elements. The standard deviation for the total cost of the bridge maintenance σ_{C_B} is calculated, as shown in Equation 4-21, as the square root of the sum of squares of the standard deviations of the maintenance costs σ_{C_e} associated with the optimal maintenance scenarios for the bridge elements.

$$\sigma_{C_B} = \sqrt{\sum_{e=1}^{N_e} \sigma_{C_e}{}^2} \tag{4-21}$$

where N_e is the number of the bridge elements.

The standard deviation for the risk reduction associated with bridge maintenance $\sigma_{\Delta R_B}$ is calculated as the square root of the sum of the squares of the standard deviations for the risk reductions $\sigma_{\Delta R_e}$ associated with the optimal maintenance scenarios for bridge elements as shown in Equation 4-22.

$$\sigma_{\Delta R_B} = \sqrt{\sum_{t=1}^{N_e} \sigma_{\Delta R_e}{}^2} \tag{4-22}$$

Assuming normal distributions for the maintenance cost and risk reduction, the benefit-cost reliability index ($\beta_{B/C_{Bridge}}$) and benefit-cost probability of failure ($P_{f,B/C_{Bridge}}$) associated with the bridge maintenance can be calculated using Equations 4-23 and 4-24.

$$\beta_{B/C_{Bridge}} = \frac{\mu_{\Delta R_B} - \mu_{C_B}}{\sqrt{{\sigma_{\Delta R_B}}^2 + {\sigma_{C_B}}^2}} \tag{4-23}$$

$$P_{f,B/C_{Bridge}} = \Phi(-\beta_{B/C_{Bridge}}) \tag{4-24}$$

For example, we assume that budget limitations required the delay of the deck optimal actions in the third year so that scenario 01011 is used instead of 01101, while the optimal scenario for the steel girder is kept as 01011. We also assume that the means and standard deviations for the optimal maintenance scenarios of the reinforced concrete column, abutment and cap were calculated as shown in Table 4-86. The mean and standard deviation for the risk reduction and maintenance cost associated with the bridge maintenance is calculated using Equations 4-21 and 4-22. Using the mean values for the total risk reduction and maintenance cost associated with the bridge maintenance, the benefit-cost ratio and net present value for the bridge maintenance are calculated as 2.275 and \$287,981, respectively. Using both the mean and standard deviation for the risk reduction and maintenance cost associated with the bridge maintenance, the benefit-cost reliability index and probability of failure are calculated as 4.358 and 6.56E-6, respectively.

Table 4-86. Maintenance Cost and Risk Reduction Associated with Bridge Maintenance

Element ID	Element Description	Present Value of Total Risk Reduction ($)		Present Value of Total Maintenance Cost ($)	
		Mean	Standard Deviation	Mean	Standard Deviation
12	Bare Concrete Deck	40,300	4,909	16,566	1,197
107	Painted Steel Open Girder/Beam	108,623	9,972	49,376	4,724
205	Reinforced Concrete Column or Pile Extension	180,000	50,000	80,000	15,000
215	Reinforced Concrete Abutment	65,000	12,000	30,000	5,000
234	Reinforced Concrete Cap	120,000	35,000	50,000	10,000
Present Values Associated with Bridge Maintenance		513,923	63,187	225,942	19,333
Benefit-Cost Ratio (BCR) Associated with Bridge Maintenance		2.275			
Net Present Value (NPV) Associated with Bridge Maintenance		$287,981			
Benefit-cost Reliability Index ($\beta_{B/C}$) Associated with Bridge Maintenance		4.358			
Benefit-cost Probability of Failure ($P_{f,B/C}$) Associated with Bridge Maintenance		6.56E-06			

4.15 Risk-Based Maintenance of the Bridge Inventory

The bridge inventory is composed of the group of bridges identified for maintenance by the highway agency. The initial risks associated with the conditions of bridges before maintenance is calculated using the initial risks associated with conditions of the elements before maintenance. The bridges are ranked based on the risk associated with their condition before maintenance and classified into high, medium, and low risk groups. The high-risk group includes the bridges most at risk within the inventory. The medium and low risk groups include bridges with medium and low initial risks based on defined values between the high, medium and low risks.

Bridge managers may have different tendencies towards the choice of the measure for prioritizing the maintenance of bridges. The maintenance priority for the bridges can be based on benefit-cost measures in the different risk categories. The bridges in the risk categories can be ranked based on the net present value, benefit-cost ratio, and benefit-cost reliability index. The net present values (NPV_{Bridge}) for the maintenance strategies of bridges are used for setting the maintenance priority list of the bridges in each risk group. Similar maintenance priority lists can be created based on the benefit-cost ratio (BCR_{Bridge}) and the benefit-cost reliability index ($\beta_{B/C_{Bridge}}$). Also the importance of the bridge in the highway network can be considered in setting the priority ranking of bridges for maintenance. The bridge manager can make informed decisions regarding the maintenance of the bridges in the inventory using the maintenance priority lists.

It is assumed that the inventory of bridges includes N_B bridges that are identified for maintenance; of which N_h bridges are in the high-risk group; N_m bridges are in the medium-risk group; and N_l bridges are in the low-risk group. The ranking starts with the bridges in the high-risk group followed by bridges in the medium-risk group and ends with bridges in the low-risk group. The net present value (NPV) is used for ranking the bridges for maintenance in each risk group. The bridges in each risk group are ranked based on the NPV of their maintenance strategies starting with highest NPV and continue with decreasing NPVs. The bridge with the largest NPV in the high-risk group has the first maintenance priority and the bridge with the lowest NPV in the high-risk group has a maintenance priority number N_h. The bridge with the highest NPV in the medium-risk group has a maintenance priority number N_h+1 and the bridge with the lowest NPV in the medium-risk group has a maintenance priority number N_h+N_m. The bridge with the highest NPV in the low-risk group has a maintenance priority number N_h+N_m+1 and the bridge with the lowest NPV in the low-risk group will have the last priority number among the N_B bridges identified for maintenance.

The selection of the bridges for maintenance projects follows the priority ranking order and is constrained by the budget limitations. When the first bridge is selected, the budget is reduced by the maintenance cost for that bridge. The same applies for every bridge selected for maintenance until the allocated budget for bridge maintenance is used.

For example, It is assumed that there are 1,000 bridges identified for maintenance in the inventory, and 100 of them are classified in the high-risk group, 300 in the medium-risk group and 600 in the low-risk group. The first maintenance priority will be for the bridge with the highest NPV among the 100 bridges in the high-risk group. The bridge with the lowest NPV in the high-risk group will be ranked 100 for maintenance priority. The bridges in the medium-risk grouped will have priorities 101 to 400 and in the low-risk group will have priorities 401 to 1000, ranked by decreasing NPVs. The bridge with the last priority for maintenance is the one with lowest NPV of maintenance in the low-risk group.

It is assumed that the budget allocated for bridge maintenance can cover only the first 70 bridges in the high-risk group. Bridges are scheduled for maintenance starting with the bridge that has first priority and ending with the bridge that has priority number 70 in the high-risk group. The maintenance strategies for the remaining 30 bridges in the high-risk group as well as the 700 bridges in the medium and low-risk groups will be delayed. This maintenance strategy will assist the bridge manager to best utilize the available budget allocated for maintenance for selecting the bridges that are most at risk for maintenance activities that are economically efficient.

Chapter 5. Conclusions and Recommendations

5.1 Research Contributions

The study presented in this dissertation provides a methodology for maintenance of bridges based on risk analysis and management. The overall goal of this study was to develop risk-based bridge maintenance strategies that best utilize limited available resources. The methodology for developing the maintenance strategies includes two main goals: 1) Estimating risks associated with bridge conditions; and 2) managing risks efficiently to best utilize limited resources. The goal was achieved based on the following primary contributions:

1. Reliability analysis of bridge elements in different condition states,
2. Risk assessment,
3. Selection of optimal maintenance actions,
4. Selection of best timings for applying optimal actions,
5. Priority ranking of bridges for maintenance

The results from the reliability analysis are failure probabilities of bridge elements in different condition states. Probabilities of failure increase by worsening the condition states of bridge elements due to reductions in their resistance. Load-increase with time increases the probabilities of failure in the condition states of bridge elements.

The risk associated with a bridge element in any condition state is evaluated based on the probabilities and consequences of failure. Different types of failure consequences for bridge elements were identified. The total risk to the element at any particular time is evaluated based on the condition state distribution of the element and the risk in the different condition states. The risk associated with the bridge condition is estimated based on the risks associated with conditions of the bridge elements. The outcomes herein are risk estimates for the condition states of bridge elements in each year of maintenance, risk estimates for the bridge elements, and risk estimates for the bridges that are identified for maintenance.

The risks associated with conditions of bridge elements are used in the decision analysis for selecting the optimal maintenance actions in the different condition states of bridge elements. The optimal maintenance actions for bridge elements are defined as the actions that result in the lowest summation of maintenance cost and risk associated with element maintenance. This method is innovative and different from the method used in Pontis bridge management system and other approaches in the bridge management application. In this new method, optimal maintenance actions may differ from one year to another. Also, optimal actions of a particular year may differ based on the implementation or delay of optimal actions for previous years. The outcomes herein are optimal maintenance actions in the condition states of bridge elements for each year of maintenance and for each case of delays or implementations of optimal actions for previous years.

The possible scenarios of implementing or delaying optimal maintenance actions for bridge elements are created for a maintenance planning horizon. Risk reductions and maintenance costs are compared for each scenario of actions. Budget and risk constraints are used to filter the maintenance scenarios by eliminating the scenarios that exceed the risk or cost limits. Economic measures are used to compare risk reductions with maintenance costs for the filtered maintenance scenarios. Maintenance scenarios are ranked based on the cost effectiveness of risk reduction. The outcomes herein are scenarios of optimal maintenance actions for bridge elements with best time implementation that result in the most efficient risk reduction.

Bridges identified for maintenance are grouped into risk categories based on their risk values. Priority ranking is established for bridges based on their risk groups and efficiency of their maintenance scenarios. Bridges most at risk are given higher priorities for maintenance than bridges in the medium or low-risk groups. Bridges in each risk group are ranked for maintenance based on the risk-reduction effectiveness of their maintenance scenarios. The bridge with the most efficient maintenance scenarios in the high-risk group has the highest priority for maintenance. The limited resources available for maintenance are allocated for the bridges based on their priority ranking. The outcome is a risk-based priority-ranking list of bridges with efficient maintenance strategies that best utilize the limited resources.

This dissertation successfully achieved the stated goal. The method developed in Chapter 3 was applied to the case study in Chapter 4. The case study illustrated the proposed methodology and demonstrated that it is practically feasible. The results of the methodology are influenced by the discount rate, the deterioration and repair rates of bridge elements, the unit costs of maintenance actions, and the probabilities and consequences of failure.

The proposed methodology allows decision makers to identify bridges most at risk and allocate limited resources for efficient maintenance strategies. Although the proposed methodology was developed for bridge maintenance strategies, it can be applied to other fields that include aging systems of elements.

5.2 Main Conclusions

The following conclusions can be drawn from this study:

- Risk analysis is a viable tool for managing the maintenance of bridges based on probabilities and consequences of failure,
- The proposed procedure for estimating and managing the risks associated with conditions of bridge elements is feasible and practical and applicable to the element-level data used by most state highway agencies in the United States,
- The proposed risk-based methodology differs from the current method used in the current Pontis bridge management system by using the risk for selecting the optimal maintenance policies of bridge elements,

- Selection of the optimal maintenance policies for bridge elements depends on the failure probabilities in the condition states of elements and the consequences of elements failures,
- Optimal maintenance policies for bridge elements may change from one year to another,
- Optimal timing of maintenance policies for bridge elements depends upon risk reductions and maintenance costs of optimal policies,
- Delaying the optimal maintenance actions for bridge elements results in increasing risks on elements,
- Implementing the optimal maintenance actions every year of maintenance is not cost effective, and
- Ranking of bridges for maintenance depends on conditions of bridge elements, and cost effectiveness of maintenance policies.

5.3 Recommendation for Future Research

Several areas for future research are needed to build on or extend the methodology presented in this dissertation. Some of the subjects that are significant for future research include:

- Identifying the different failure modes for commonly recognized bridge elements. Failure modes should include not only ultimate strength failure modes but also serviceability and extreme event failure modes,
- Evaluating the probabilities of failure for commonly recognized bridge elements due to the different failure modes,

- Developing a methodology to quantify different types of failure consequences for commonly recognized bridge elements,
- Applying the proposed methodology using different deterioration models for condition prediction,
- Integrating the proposed methodology in the preservation model of currently used Pontis bridge management system to account for risk in selecting optimal maintenance policies of elements and priority ranking of bridges for maintenance.
- Applying the risk-based methodology using different short-term and long-term planning horizons and comparing the optimal maintenance polices with the method used in currently used Pontis bridge management system,
- Applying the risk-based methodology using modified condition states based on quantified and measurable defects of bridge elements, and
- Applying the proposed methodology on other systems composed of aging elements.

Appendix A Failure Consequences for Bridge Elements

A.1 Classification of Failure Consequences for Bridge Elements

Failure consequences of bridge elements may include consequences on the highway agency related to the element and the bridge, consequences on the bridge users, consequences on health and safety, consequences that increase the traffic accident rates, consequences on the environment, consequences to the nearby businesses, and consequences on the public. Each type of these consequences includes a number of factors that contribute to the cost of the consequence. The element consequences are classified into the following types.

Agency Consequences Related to the Element

The consequences on the highway agency related to the bridge element C_e as a result of its failure include:

1. Cost of special inspection of the element (C_{e1}),
2. Cost of the element demolition and removal of the debris (C_{e2}), and
3. Cost of the element replacement (C_{e3}).

The inspection cost (C_{e1}) can be calculated by multiplying the cost data for the hourly rates of inspectors and the equipments needed by the number of hours needed for the inspection. The removal cost (C_{e2}) can be calculated based on the hourly rates of labors, technicians and equipments used in the removal, and the number of hours needed for the removal of the debris of the element. The replacement cost of the element (C_{e3}) can be calculated using the unit replacement cost available in the database of the bridge management system for the highway agency and the quantity of the bridge element.

Agency Consequences Related to the Bridge

Property damage of other elements of the bridge due to the element failure depends on the mode of failure, type of bridge element, and hazards that caused failure. The failure of some elements may cause partial or complete damage of other elements of the bridge and other consequences related to the bridge. These consequences affect the highway agency that owns and manages the bridge.

The agency consequences (C_b) related to the bridge may include the following:

1. Cost of special inspection (C_{b1}) to determine levels of damage of other components of the bridge as a result of the element failure,
2. Cost of maintenance and repair (C_{b2}) of other elements in the bridge that need repair as a result of the element failure,
3. Cost of demolition and replacement (C_{b3}) of other damaged elements in the bridge as a result of the element failure,
4. Cost of bridge strengthening (C_{b4}) that may be needed as a result of the element failure,

5. Cost (C_{b5}) of fixing or replacing utilities, lightings, and traffic signals on the bridge,
6. Cost of traffic management (C_{b6}) due to detouring that result from the element failure,
7. Consequences (C_{b7}) that may result in critical scour conditions due to the failure of some bridge elements,
8. Consequences (C_{b8}) related to load posting, and user-request permitting,
9. Consequences (C_{b9}) on the highway network due to congestion resulting from the element failure.

The abovementioned consequences can be calculated in monetary terms. The property loss can be assessed as the actual value of the damaged property of the bridge. Replacement costs of the damaged parts of the bridge can also be used to assess the loss. The consequences can be estimated based on the inventory cost data of the highway agency for inspection, maintenance, repair, replacement, and traffic management. The inventory agency cost data may include cost components of force account work (work done by agency's own crew) such as crew labor and equipment operating costs; contract work performance by the contractor; site costs such as warning signs and worker protection barriers; and project support costs for project supervision and inspection (Thompson, et al. 1996).

Consequences to Bridge Users

The failure of a bridge element has consequences C_u on the travelers who use the bridge in their travel. Consequences on the users may include the following:

1. Cost of travel time delay (C_{u1}) of bridge users and other travelers in the highway network as a result of the element failure. Time delay could be due to traffic congestion, detours, or closures of bridge lanes. Travel time cost includes costs of time lost due to rerouted or diverted traffic travel, congestion that create queuing delays or stopping, or traffic delays that result from lane closures or work zones,
2. Cost of operating the vehicles (C_{u2}) of bridge users and other travelers in the highway network due to detours and traffic delays that result from the element failure. Vehicle operating cost includes increased gas or fuel consumption, maintenance, and depreciation of vehicles, and
3. Cost of damage to the vehicles (C_{u3}) of the bridge users as a result of the bridge element failure.

Consequences related to the bridge users can be estimated based on the duration of time delay, the traffic volume, the percentage of trucks in the traffic stream, the numbers of vehicles likely to be delayed, the lengths of delay, and the reduction in travel speed (Hawk, 2003). Time costs depend on factors such as the mean hourly wage in the region and population income. Time cost lost by bridge users due to traffic delay is usually greater than the operating cost of their vehicles. Estimate of the detour costs may be calculated as twice the distance between the bridge under analysis and the closest alternate crossing. Cost of highway vehicle damage may be estimated as proportional to traffic levels (Hawk, 2003).

Consequences Related to Traffic Accidents

The failure of a bridge element may have consequences (C_a) that result in increasing accident rates on the bridge and the highway network. Accident cost includes cost of accidents on the bridge structure and cost of accidents under the bridge. The consequences related to accidents that could result from the failure of a bridge element include the following costs:

1. Cost of traffic accidents due to vehicle collisions with the bridge (C_{a1}) as a result of the element failure,
2. Cost for load tests (C_{a2}) to determine the damage due to accidents that result from the element failure,
3. Cost of specific actions (C_{a3}) needed to repair or replace damaged components of the bridge due to accidents that result from the element failure,
4. Cost of damages to vehicles and other properties (C_{a4}) due to accidents resulting from the element failure,
5. Cost of removal of damaged cars and debris (C_{a5}) due to accidents resulting from the element failure,
6. Cost of emergency services and police officers (C_{a6}) needed for accidents resulting from the element failure,
7. Cost of insurance (C_{a7}) for accidents resulting from the element failure,

8. Cost of travel time delay (C_{a8}) due to the emergency services for accidents resulting from the element failure, and
9. Cost of congestion in the highway network (C_{a9}) due to accidents resulting from the element failure.

Consequences Relating to Health and Safety

Failure of a bridge element may have consequences (C_h) that affect the health and safety of drivers or pedestrians over or under the bridge, or other people close to the bridge. These health and safety consequences due to failure include the following:

1. Possible losses of human lives (C_{h1}),
2. Possible body injuries (C_{h2}),
3. Medical care expenses (C_{h3}),
4. Legal expenses (C_{h4}),
5. Insurance expenses (C_{h5}),
6. Lost of productivity of affected people (C_{h6}),
7. Cost of pain and suffering (C_{h7}),
8. Cost of loss of enjoyment of life (C_{h8}),
9. Loss of future earnings (C_{h9}), and
10. Cost of emergency services (C_{h10}).

Human consequences in terms of loss of life or injury are affected by the number of vehicles and pedestrians passing over or under the bridge, the failure type and extent, the warning of failure (ductile or brittle), and the likely effects on road accidents due to failure (Shetty, 1997). Human injuries and loss of lives can be converted into monetary values based on the type and severity of injuries, and expected value of human lives. Different methods were used to assess the health consequences of accidents or exposure to risk such as the value of human life, the injury expenses, and the loss of earnings due to death or injury. The value of life (VOL) and the statistical value of life (SVOL) are among the measures that were used for assessing the value of human life (Ayyub, 2003).

The value of life (VOL) is based on analytical methods such as the willingness-to-pay (WTP) method. WTP method depends on the willing of a group of people to pay for an increase in safety to reduce the probability of death or injury, or to be compensated for an increase in risk of a given type. The statistical value of life (SVOL) is based on assigning monetary values to human injuries and fatalities using data and statistics. SVOL reported in transportation ranged from \$50k to \$29M, with a median of \$312k. Transportation studies have used \$1.4M in 1990 dollars (Ayyub, 2003).

The human capital method (HC) was used to assess the discounted loss of a person's future earnings as a result of injury or death due to the incident. The Abbreviated Injury Scale (AIS) was used to measure the severity of injury. Injuries are ranked on a non-arithmetic scale of 1 to 6, with 1 being minor, 5 severe, and 6 un-survivable. Relationship between the AIS and a fraction of the WTP value (for example \$3,000,000) based on FAA guidance documents in 2001 dollars) are shown in Table A-1 (Ayyub, 2003). These percentages reflect the loss of quality and quantity of life resulting from an injury typical of that level. The Office of Aviation Policy and Plans (APO) – suggested values for medical and legal costs per-victim associated with injuries in 2001 dollars as shown in Table A-2 (Ayyub, 2003).

The abovementioned methods can be used to assess the loss of the different health consequences that affect the travelers on and under the bridges. Severity of health consequences may vary significantly depending on the failure type. For example, health consequences are expected to be more severe in shear failure than in flexural failure.

Table A-1. Relationship between the AIS and a fraction of the WTP value (based on FAA guidance documents in 2001 dollars)

AIS Scale Code	Injury Severity	Multiplier	WTP Value
1	Minor	0.20%	$6,000
2	Moderate	1.55%	$46,500
3	Serious	5.75%	$172,500
4	Severe	18.75%	$562,500
5	Critical	76.25%	$2,287,500
6	Fatal	100.00%	$3,000,000

Table A-2. Per-Victim Medical and Legal Costs Associated with Injuries (2001 Dollars)

AIS Scale Code	Injury Severity	Emergency/ Medical	Legal/ Court	Total Direct Cost
1	Minor	$600	$1,900	$2,500
2	Moderate	$4,600	$3,100	$7,700
3	Serious	$16,500	$4,700	$21,200
4	Severe	$72,500	$39,100	$111,600
5	Critical	$219,900	$80,100	$300,000
6	Fatal	$52,600	$80,100	$132,700

Consequences Relating to the Environment

Failure of bridge elements may have consequences C_{env} that result in environmental damage surrounding the bridge. Environmental consequences are affected by many factors such as proportion of vehicles carrying hazardous substances, nature of the crossing (road, rail or river), nature of the adjacent environment (urban, industrial, rural) that can influence the amount of pollution damage and resulting in clean-up costs (Shetty, 1997), likelihood of potential extreme events such as hurricanes or earthquakes (Ryall, 2001), and air pollution that results from the fuel consumption of vehicles on the bridge.

Environmental damages due to the element failure may be classified into the following types:

1. Increased air pollution (C_{env1}) due to fuel emissions of delayed or diverted vehicles in traffic congestion that result from the element failure (Hawk, 2003),
2. Impacts on water quality (C_{env2}) in flowing streams or rivers under or adjacent to the bridge due to pollutants or waste products that result from the element failure (Hawk, 2003),
3. Disturbance to the agricultural land (C_{env3}),
4. Impacts on the plants, trees, and forests (C_{env4}),
5. Disposal of waste material and debris (C_{env5}) that result from the element failure,
6. Noise and dust (C_{env6}) due to the element failure,
7. Environmental damage (C_{env7}) caused by spillage of hazardous material from vehicles on and under the bridge as a result of the element failure, and
8. Environmental damage (C_{env8}) caused by fire and chemical spills resulting from traffic collisions with bridges as a result of the element failure.

Environmental studies about the abovementioned impacts could be used for estimating the environmental consequences that result from the failure of bridge elements. Assumptions can be made based on the type of the environmental impact. For example the cost of environmental damage due to increased air pollution as a result of traffic congestion can be assumed based on the number of delayed vehicles, the pollution per vehicle, health impact due to air pollution, the area affected by the pollution, and the population of the area around the bridge (Hawk, 2003). The consequences of environmental damage to the surface water, the land use, the trees, and disposal material can be assumed in a similar way.

Consequences to Nearby Businesses

Failure of some bridge elements may result in consequences (C_{nb}) on normal activity of nearby businesses due to disruptions of services that result in increased production costs and losing customers due to congestion delays (Hawk, 2003). The consequences on the nearby businesses could be classified in different forms that include:

1. Loss of revenue (C_{nb1}) to the nearby businesses due to the element failure,
2. Loss of productivity (C_{nb2}) to the nearby businesses as a result of the element failure,
3. Cost of possible damages of surrounding properties (C_{nb3}) due to the element failure,
4. Cost of delay of services (C_{nb4}) to the nearby businesses due to the element failure, and
5. Cost of business travel (C_{nb5}) due to detours that result from the element failure.

These business losses due to the element failure can be measured in monetary terms as the increase in production cost and loss in revenue.

Consequences to the General Public

The traffic congestion that results from the failure of bridge elements may have consequences (C_p) that result in the disruption of the normal life of the public. This disruption may affect the access to schools, health-care providers, and community facilities. It may also have effect on the government and public in general. Examples of consequences on the general public due to the failure of a bridge element may include the following:

1. Consequences on the public due to possible closure of the bridge (C_{p1}) as a result of the element failure,
2. Consequences on the public due to congestion in the highway network (C_{p2}) as a result of the element failure,
3. Damages to the society and general public due to public relation costs (C_{p3}),
4. Disturbance of emergency services (C_{p4}), and
5. Consequences on access to schools, libraries, health care facilities and governmental agencies (C_{p5})

When consequences of failure are not available, the severity of the consequences can be classified using qualitative approaches. Appendix A.2 explains a scheme for classifying consequences of failure for bridge elements based on their criticality and importance of that element to the bridge.

A.2 Qualitative Consequences for Bridge Elements

The consequence severity of failure consequences for bridge elements can be categorized based on their importance to the bridge. A severity classification is defined based on the weighting factor of the commonly recognized bridge elements and the status of the element as fracture critical.

Severity of consequences of failure for commonly recognized bridge elements can be classified into the following categories:

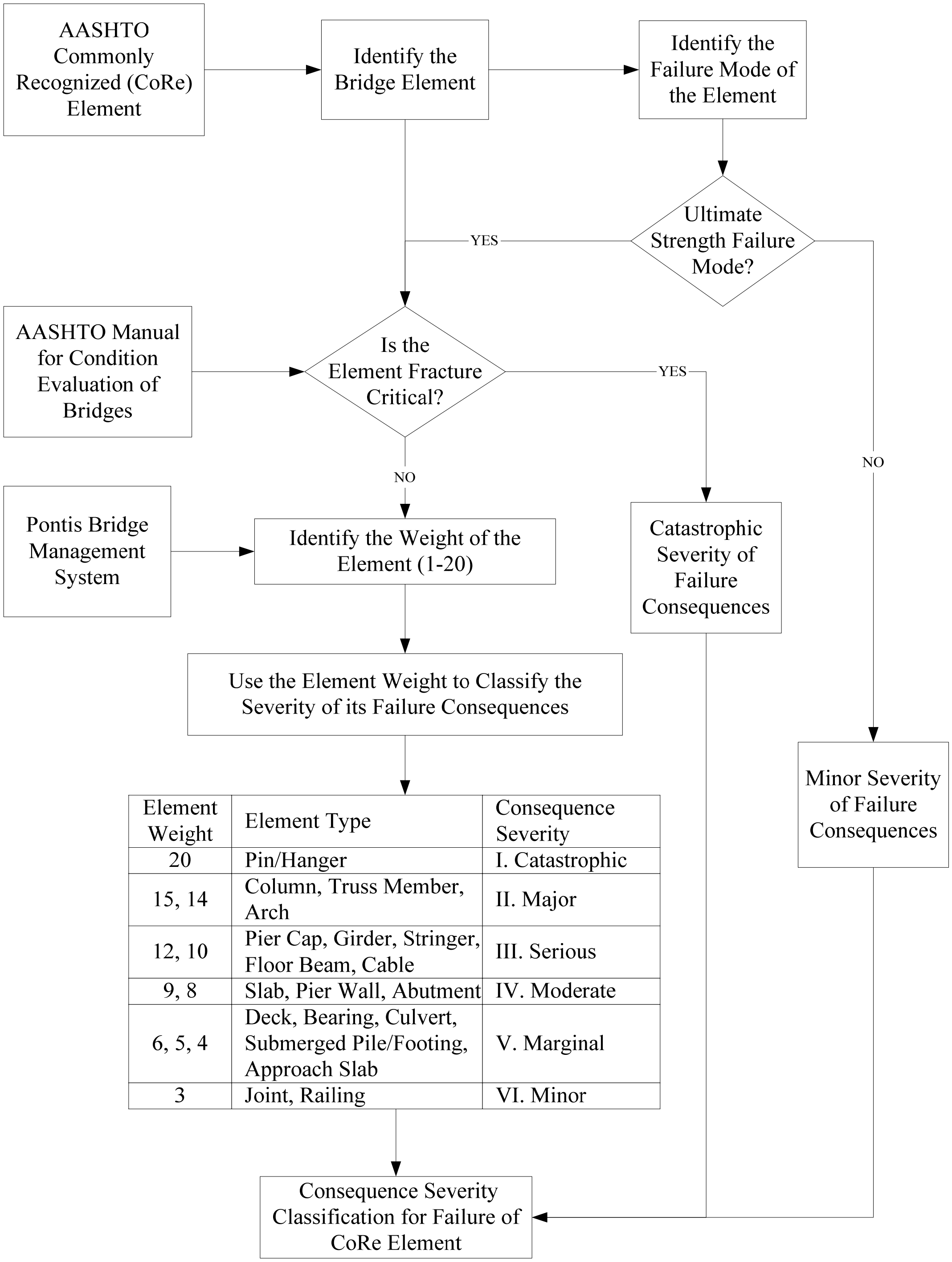
1. Category I - Catastrophic consequence severity - is assumed for fracture critical elements based on the AASHTO Manual for Condition Evaluation of Bridges (AASHTO, 1994) and elements with a weighting factor of 20 on a scale of 1-20 that defines the element importance,
2. Category II - Major consequence severity - is assumed for elements with weighting factors of 14 or 15,
3. Category III - Serious consequence severity - is assumed for elements with weighting factors of 10 or 12,
4. Category IV - Moderate consequence severity - is assumed for elements with weighting factors of 8 or 9,
5. Category V - Marginal consequence severity - is assumed for elements with weighting factors of 4, 5 or 6, and
6. Category VI - Minor consequence severity - is assumed for elements with a weighting factor of 3.

Table A-3 shows the assumed severity levels of the failure consequences for commonly recognized elements in the absence of failure consequences data. The approach for estimating the severity of bridge elements based on their weight factors in the absence of any data about the failure consequences is illustrated in Figure A-1. Table A-4 shows an example for the extent of the consequences that result from each category of the elements.

This approach is presented here as a guide only for the severity of consequences that may result from the failure of commonly recognized bridge elements. Although the classification based on this approach could be useful for prioritizing element criticalities, it is not used in the quantitative methodology for risk-based maintenance for setting maintenance strategies for the bridge elements proposed in this dissertation.

Table A-3. Assumed Severity Levels of Consequence for CoRe Elements Failure

Is the Element Fracture Critical?	Element		Severity of Failure Consequences	
	Weight	Type	Category	Description
Yes	Any		I	Catastrophic
No	20	Pin and/or Pin and Hanger Assembly	I	Catastrophic
	14,15	Column or Pile Extension, Arch, Truss	II	Major
	10,12	Pier Cap, Cable, Girder, Stringer, Floor Beam	III	Serious
	8, 9	Slab, Pier Wall, Abutment	IV	Moderate
	4, 5, 6	Deck, Bearing, Culvert, Submerged Pile /Footing, Approach Slab	V	Marginal
	3	Joint/Seal, Railing	VI	Minor



Element Weight	Element Type	Consequence Severity
20	Pin/Hanger	I. Catastrophic
15, 14	Column, Truss Member, Arch	II. Major
12, 10	Pier Cap, Girder, Stringer, Floor Beam, Cable	III. Serious
9, 8	Slab, Pier Wall, Abutment	IV. Moderate
6, 5, 4	Deck, Bearing, Culvert, Submerged Pile/Footing, Approach Slab	V. Marginal
3	Joint, Railing	VI. Minor

Figure A-1. Methodology for Estimating Consequence Severity of Bridge Elements Failure Based on Importance When No Failure Consequences Data is Available

Table A-4. Severity Classification of Commonly Recognized Bridge Elements

Category	Description	Level of Consequences
I	Catastrophic	May cause the collapse of the bridge and result in injury or death
II	Major	May cause severe injury, major bridge damage, or major damage in the traffic system
III	Serious	May cause serious injury, serious bridge damage, or serious damage in the traffic system.
IV	Moderate	May cause moderate injury, moderate bridge damage, or moderate damage in the traffic system that will result in dissatisfaction and considerable delay or loss of system availability or degradation.
V	Marginal	May cause marginal injury, marginal bridge damage, or marginal damage in the traffic system that will result in some delay or loss of bridge functionality.
VI	Minor	Not serious consequences enough to cause injury, bridge damage or traffic system delay, but will result in unscheduled maintenance or repair.
VII	Insignificant	No significant consequence.

Bibliography

Al-Wazeer, A., Harris, B., and Nutakor, C. (2007a). "CoRe Concerns." Public Roads, January/February Issue, Vol. 70, No. 4, pp 20-23, FHWA, U.S. Department of Transportation.

Al-Wazeer, A., Nutakor, C., and Harris, B. (2007b). "Comparison of Neural Network Method Versus National Bridge Inventory Translator in Predicting Bridge Condition Ratings." 86th Annual Meeting of the Transportation Research Board, Paper No.(07-0572), National Research Council, Washington, D.C.

Al-Wazeer, A., Harris, B., and Nutakor, C. (2005). "Applying LCC to Bridges." Public Roads, November/December Issue, Vol. 69, No. 3, pp 66-71, FHWA, U.S. Department of Transportation.

Al-Wazeer, A. (2001). "Development of a Methodology to Calculate the Minimum Failure Cost in Pontis BMS." Presentation, Pontis Task Force Meeting, Denver, CO. March.

American Association of State Highway and Transportation Officials (AASHTO). (2004). "AASHTO LRFD Bridge Design Specifications." Washington, D.C.

American Association of State Highway and Transportation Officials (AASHTO). (2001). "Pontis Technical Manual." AASHTO, Washington, D.C.

American Association of State Highway and Transportation Officials (AASHTO). (2000). “Manual for Condition Evaluation of Bridges.”, 2nd Edition, Washington, D.C.

American Association of State Highway and Transportation Officials (AASHTO). (1997). “AASHTO Guide for Commonly Recognized (CoRe) Structural Elements, Washington, D.C.

American Institute of Steel Construction, Inc. (1995). Manual of Steel Construction, Load & Resistance Factor Design.” Vol. I, Structure Members, Specifications, & Codes, Second Edition.

American Institute of Steel Construction, Inc. (1995). Manual of Steel Construction, Load & Resistance Factor Design.” Vol. II, Connections, Second Edition.

Ang, A.H-S., and W.H. Tang. (1984). “Probability concepts in engineering planning and design: Vol. II, Decision, Risk, and Reliability.” Wiley, New York.

Ayyub, B.M. (2003). “Risk Analysis in Engineering and Economics.” Chapman and Hall/SRC Press, FL.

Ayyub, B.M. and McCuen, R.H. (1997). “Probability, statistics, & Reliability for Engineers.” CRC Press, FL.

Ayyub, B.M. and McCuen, R.H. (1996). “Numerical Methods for Engineers.” Prentice Hall, NJ.

Basoz, N.I. “Risk Assessment for Highway Transportation Systems.” (1996). PhD Dissertation, Stanford University.

Cambridge Systematics, Inc. (2003). “Pontis Release 4.2 Technical Manual.” AASHTO, Washington, D.C.

Cambridge Systematics, Inc. (2003). “Pontis Release 4.2 User’s Manual.” AASHTO, Washington, D.C.

Chase, S.B., Small, E.P, and Nutakor, C. (1999). “An In-Depth Analysis of the National Bridge Inventory Database Utilizing Data Mining, GIS and Advanced Statistical Methods.” 8th international Bridge Management Conference, Transportation Research Board, Washington, D.C.

Dunker, K.F., and Rabbat, B.G., (1993). “Why America’s Bridges Are Crumbling.” SCIENTIFIC AMERICAN, March, p 66-72.

Estes, A. C., and D. M. Frangopol. (2003a). “Updating Time-Dependent Reliability of Highway Bridges with Pontis Bridge Management System Inspection Results.” 9th international Bridge Management Conference, Transportation Research Board, Washington, D.C.

Estes, A. C., and D. M. Frangopol. (2003b). “Updating Bridge Reliability Based on Bridge Management Systems Visual Inspection Results.” Journal of Bridge Engineering, Vol. 8, No. 6, pp. 374-382.

Estes, A. C., and D. M. Frangopol. (1999). “Repair Optimization of Highway Bridges Using System Reliability Approach.” Journal of Structural Engineering, Vol. 125, No. 7, pp. 766–775.

Estes, A. C. (1997). “A System Reliability Approach to the Lifetime Optimization of Inspection and Repair of Highway Bridges.” Ph.D. Thesis, University of Colorado, Boulder, CO.

Frangopol, D. M., and A. C. Estes. (1997). “Lifetime Bridge Maintenance Strategies Based on System Reliability.” Structural Engineering International, Vol. 7, No. 3, pp. 193–198.

Federal Highway Administration. (1995). “Recording and Coding Guide for the structure Inventory and Appraisal of the Nation’s Bridges.” Report No. FHWA-PD-96-001, U.S. Department of Transportation, Washington, D.C.

Federal Highway Administration. (1987). “Bridge Management Systems.” Report No. FHWA-DP-71-01, Washington, D.C.

Golabi, K., Thompson, P.D., and Hyman, W.A. (1993). “Pontis Version 2.0 Technical Manual: A Network Optimization System for Bridge Improvements and Maintenance.” Report No. FHWA-SA-94-031, Federal Highway Administration by Optima, Inc. and Cambridge Systematic, Inc, Cambridge, MA.

Gurenich, D. “Guidelines for Evaluation of Minimum Failure Cost”, Pontis Technical Notes web site, http://aashtoware.camsys.com/support.htm , Cambridge Systematics, Inc., February 7, 2002.

Hawk, H. (2003). “Bridge Life-Cycle Cost Analysis.” NCHRP Report 483, National Cooperative Highway Research Program, Transportation Research Board, Washington, D.C.

Hawk, H. and Small, E. P. (1998). “The BRIDGIT Bridge Management System.” Structural Engineering International, IABSE, 8(4), 309-314.

Hundson, S.W., Carmichael, R.F., Moser, L.O., Hundson, W.R., and Wilkes, W.J. (1987). “Bridge Management Systems.” NCHRP Report 300, Project No. 12-28(2), Transportation Research Board, National Research Council, Washington, D.C.

Hearn, G. (1999). “NDE in Bridge Management Systems.” 8th international Bridge Management Conference, Vol. I, Transportation Research Board, Washington, D.C.

Hearn, G., Cavallin, J., and Frangopol, D.M. (1997). “Generation of NBI Condition Ratings from Commonly Recognized (CoRe) element data.” Proceedings of the Specialty Conference on Infrastructure Condition Assessment: Art, Science, and Practice, pp. 41-50, Boston, MA, August 25-27

Jahelka, B. (1996). “Pontis Bridge Inspection Training Manual.” Kansas Department of Transportation, Bureau of Design, Bridge Management Section.

Jiang, Y., and Sinha, K.C. (1992). “Simulation Approach to Prediction of Highway Structure Conditions.” Transportation Research Record, No. 1347, pp11-17, Transportation Research Board, National Research Council, Washington, D.C.

Kumamoto, H., and Henley, E.J. (1996). "Probabilistic Risk Assessment and Management for Engineers and Scientists." Second Edition, IEEE Press, New York, NY.

MacGregor, J.G. (1997). "Reinforced Concrete Mechanics and Design." Third Edition, Prentice Hall, NJ.

Menzies, J.B. (1997). "Bridge failures, Hazards and Societal Risk." Symposium on the Safety of Bridges, (Ed.) P.G. Das, pp.36-41, ICE, London.

National Engineering Technology Corporation. (1994). "BRIDGIT: Description of System Features." (1994). NCHRP Project No. 12-28(2)A, Report for National Cooperative Highway Research Program, Washington, D.C.

Nowak, A.S. (1999). "Calibration of LRFD Bridge Design Code." NCHRP Report 368, Transportation Research Board, Washington, D.C.

Nowak, A.S., (1995). "Calibration of LRFD Bridge Design Code." Journal of Structural Engineering, Vol.121, No.8, pp.1245-1251.

O'Connor, D.S. and Hyman, W.A. (1989). "Bridge Management Systems." Final Report, Report FHWA-DP-71-01R, Demonstration Project 71, Demonstration Projects Division, Federal Highway Administration, Washington, D.C.

Office of Management and Budget (OMB). (2007). "Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs." OMB Circular A-94, Appendix

C. (01/02/2007). Office of Management and Budget, White House Website. http://www.whitehouse.gov/omb/circulars/a094/a94_appx-c.html.

Rayall, M.J. (2001). “Bridge Management.” Butterworth-Heinmann, Oxford.

Robert, W.E., Shepard, R.W., and Aldayuz, J. (2003). “Pontis Bridge Management System: State of the Practice in Implementation and Development.” 9th international Bridge Management Conference, Transportation Research Board, Washington, D.C.

Roelfstra G., Adey, B., Hajdin, R., and Bruhwiler, E. (1999). “The Condition Evaluation of Concrete Bridges Based on Segmental Approach, Nondestructive test Methods, and Deterioration Models.” 8the International Bridge Management Conference, Vol. I, Transportation Research Board, Washington, D.C.

Salmon, C.G. and Johnson, J.E. “Steel Structures, Design and Behavior.” Fourth Edition, Harper Collins College Publishers, NY.

Shepard, R.W., and M.B. Johnson. (1999). “California Bridge Health Index.” 8th International Bridge Management Conference, Vol. II, Transportation Research Board, National Research Council, Washington, D.C.

Shetty, N.K., Chubb, M.S. and Haldin D. (1997) “An Overall Risk-Based Assessment Procedure For Substandard Bridges.” Symposium on the Safety of Bridges, (Ed.) P.G. Das, pp.225-235, ICE, London.

Small, E.P., Philbin, T., Fraher, M., and Romack, G. (1999). "Current Status of Bridge Management System Implementation in the United States," 8th International Bridge Management Conference, Vol. II, Transportation Research Board, National Research Council, Washington, D.C.

Small, E.P., and Cooper, J. (1998). "Condition of the Nation's Highway Bridges, A Look at the Past, Present, and Future." TR News, Transportation Research Board, 194, January-February, Washington, D.C., 3-8.

Thoft-Christensen, P. (2001)."Corrosion Crack Based Assessment of the Life-Cycle Reliability of Concrete Structures." In Proc., 8th International Conference on Structural Safety and Reliability June 17–22, 2001 (R. B. Corotis, G. I. Schueller, and M. Shinozuka, eds.), A. A. Balkema Publishers, Newport Beach, CA.

Thompson, P., Sobanjo, J., and Kerr, R. (2003). "Florida DOT Project-Level Bridge Management Models." Journal of Bridge Engineering, Vol. 8, No. 6, pp 345-352.

Thompson, P., Small E., Johnson, M., and Marshal, A. (1998). "The Pontis Bridge Management System." Structural Engineering International, IABSE, Vol. 8,No.4, pp. 303-308.

Thompson, P. and Markow, M. J. (1996) "Collecting and Managing Cost Data for Bridge Management Systems." Synthesis of Highway Practice 227, National Cooperative Highway Research Program, Transportation Research Board, National Research Council, Washington, D.C.

US Department of Transportation. (1995). “Bridge Inspectors Field Guide – Commonly Recognized (CoRe) Elements.” National Highway Institute, Federal Highway Administration, Washington, D.C.