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A Framework for Cross-layer Design of Energy-efficient Communication with QoS Provisioning in Multi-hop Wireless Networks

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Abstract— **Efficient use of energy while providing an adequate level of connection to individual sessions is of paramount importance in multi-hop wireless networks. Energy efficiency and connection quality depend on mechanisms that span several communication layers due to the existing co-channel interference among competing flows that must reuse the limited radio spectrum. Although independent consideration of these layers simplifies the system design, it is often insufficient for wireless networks when the overall system performance is examined carefully. The multi-hop wireless extensions and the need for routing users' sessions from source to the destination only intensify this point of view. In this work, we present a framework for cross-layer design towards energy-efficient communication. Our approach is characterized by a synergy between the physical and the medium access control (MAC) layers with a view towards inclusion of higher layers as well. More specifically, we address the joint problem of power control and scheduling with the objective of minimizing the total transmit power subject to the end-to-end quality of service (QoS) guarantees for sessions in terms of their bandwidth and bit error rate guarantees. Bearing to the NP-hardness of this combinatorial optimization problem, we propose our heuristic solutions that follow greedy approaches.**

I. INTRODUCTION

The issue of energy-efficient information transmission in wireless ad hoc networks has received significant attention in recent years [1]. The autonomous nature of such networks renders their lifetime highly dependable on energy consumption. Nevertheless, the primary goal of a communication network is to deliver an acceptable level of communication to users. Energy efficient multi-hop wireless network are not exempt from providing such a quality of service for their own users either. Then, the first major issue becomes the formulation of a meaningful QoS measure for both the multi-hop wireless networks and the applications running over them.

One can have different interpretations of QoS at different communication layers. At the lowest level, i.e. physical layer, QoS is synonymous to an acceptable *bit error rate (BER)* or *signal to interference and noise ratio (SINR)*, whereas at the MAC layer or higher layers, QoS is usually expressed in terms of minimum rate or maximum delay guarantees. For the multi-hop communications, network layer QoS pertains to end-

to-end provisioning of the guaranteed QoS for each session. In accordance with these different interpretations at different layers, it is natural to use a QoS policy that is explicitly based on both minimum short-term rate requirements and maximum tolerable BERs of the sessions. Such a QoS policy also helps classifying the applications as high bandwidth or low bandwidth and as error prone or error resilient. For instance, consider the case of a wireless ad hoc network for a battle-field operation. Users establish audio-visual communication with the command center while situation awareness data are exchanged among users. Though all of these applications exhibit quite different bandwidth, delay and error tolerance characteristics, they can be easily expressed in terms of minimum short-term rate and target BER requirements per session along each session's path.

Having defined the QoS policies for each session, the next issue is to satisfy each of these policies at a minimal energy expenditure. Wireless transmissions mainly suffer from channel impairments and other user interference operating in the same frequency band. Multi-hop wireless operation merely exacerbates the existing conditions. Unless a coordination spanning to multiple layers and multiple hops exists, either the session QoS requirements are not satisfied or they are probably satisfied at a significantly higher energy consumption than the necessary. Once the set of sessions with their source-destination pairs and QoS requirements are given, three layers together impact the contention for network resources: physical layer, medium access control (MAC) layer, and routing layer. For a cross-layer design that satisfactorily enhances the network performance, it is essential to highlight the interactions among these layers.

Physical layer with its key parameters- such as transmit power, modulation, coding rate, antenna beam coefficients-has a direct impact on multiple access of nodes in wireless channels through affecting the interference at receivers and susceptibility to it. Local adaptation of these parameters to achieve a target BER restraints both routing and MAC decisions by altering the directed topology graph, feasible transmission schedules, and payload transmission rates. Phys-

ical layer features -such as transceiver complexity, power required to drive the RF modules, and the transmit power- accumulatively govern the energy expenditure of transmitters, receivers, and idle nodes.

MAC layer is responsible for scheduling the transmissions and allocating the wireless channels. While the concurrent transmissions create mutual interference, the time evolution of the scheduled transmissions ultimately determines the bandwidth allocated to each transmitter and the packet delays. The interference imposed by simultaneous transmissions naturally affects the performance of the physical layer in terms of successfully separating the desired signals from the rest. On the other hand, as a result of transmission schedules, high packet delays and/or low bandwidth can occur, forcing the routing layer to change its route decisions. MAC layer affects the energy expenditure in two ways: (i) It mainly controls the interference level at any time instance that may lead to transmit power adaptation in the physical layer. (ii) Depending on the transmission schedules, nodes may switch to a power-saving mode, turning off all or some of their RF components.

Routing layer selects the wireless links that will eventually carry the data packets. Different routing decisions alter the set of links to be scheduled, and thereby influence the performance of MAC layer. For instance, if the routing protocol chooses flow paths that are closer to each other among the alternatives, the subsequently higher interference and contention levels in the network make it harder for MAC to resolve the transmission conflicts. Similarly, higher interference levels force the adaptation of physical layer parameters to achieve the target BER. However, as the number of independent sessions with distinct source-destination pairs increases, the routing criterion is expected to play a less important role in contention resolution as compared to the physical layer adaptations and MAC decisions. When QoS requirements are ignored and link costs that accurately quantify the energy consumption can be assigned, routing layer becomes the sole determinant of energy consumption. These link costs, however, depend on the transmit power, which is a function of decisions in all three layers. Therefore, the layer interactions necessitate iterative approaches to find the most energy efficient communication scenario.

In this paper, as a first step towards solving the problem, we assume that the session paths are already given and we address only the joint power control and scheduling problem with the objective of minimizing the total average transmission power while providing quality of service for individual sessions in terms of payload rate and BER guarantees.

The rest of the paper is organized as follows. Section-II presents an overview of the works that are closely related to our problem. In section-III, we lay out the detailed system model. Section-IV states the formal problem description with the objective function and constraint sets. We explain our solution framework in section-V and then provide our simulation results in Section-VI. We conclude the paper with a synopsis and a final discussion to emphasize the future work in section-VII.

II. RELATED WORKS

Power control has been the focus of single-hop multi-user wireless networks for more than a decade [3]–[13]. The popularity of the topic stems from the facts that it can be exploited in suppressing multi-user interference, increasing system user or throughput capacity, and reducing the transmission power hence extending the battery life of the wireless devices. Later on, power control has also been adopted as an efficient protocol design technique for ad hoc wireless networks in different layers as joint or isolated problems [2], [14]–[20]. Among these highly diverse works, it is essential to dwell upon two recent studies, [2] and [14], in order to elucidate our own contribution with this paper.

In [2], Elbatt and Ephremides investigate the problem of scheduling maximum number of links in the same time slot. In other words, authors try to maximize the per hop throughput of the network. They adapt the transmit powers to their minimum required levels such that all transmissions achieve a target SINR threshold. They show that this particular system model is actually equivalent to uplink power control in cellular networks and the iterative algorithms developed for cellular networks can be employed in ad hoc wireless networks. In the case where the set of links that have buffered packets cannot be scheduled in the same time slot, these solutions do not converge and authors suggest to remove one link at a time until a feasible set of links is achieved. However, the criterion for removing the link is not precisely addressed; especially in the case of varying target SINR thresholds for each link. Also, the system model does not cover a multi-hop wireless environment.

A closer approach to our own is followed by Cruz and Santhanam in [14], where authors provide long term end-to-end rate guarantees to a set of sessions at the minimum possible long term average of the total transmit powers. Their main assumption is that the system operates at significantly low SINR values and that the link rates can be approximated as linearly dependent on SINR. Hence, the transmit power is not used for giving a quality of service guarantee in bit error rate (BER) but rather directly used as a throughput guarantee constraint. Instead of solving the relatively difficult problem of minimizing the long term average transmit power sum with the constraints on the power vector and on the long term session rates, they define and solve a dual problem that does not have a *duality gap* with the primary problem [21]. Their results reveal that all the links scheduled in a particular time slot must transmit at the maximum allowed power P_{max} rather than in more number of slots at a lower power level. The solution method to determine the set of links that must be activated simultaneously as well as the existence of schedules to achieve the rate requirements are established in the paper. Under certain continuity conditions on the optimum dual objective function, authors also extend cross-layering to the routing layer, where each small increment in session rates is routed dynamically abiding by the path costs as determined by the rate of change in dual objective function. Hence, the

optimal joint routing, scheduling, and power control policy is obtained.

Our system model differs from Cruz and Santhanam in several respects. First of all, we want to satisfy the rate requirements of the sessions not only in the long term but also in the short term within a well-defined frame duration. This prevents the sessions with low jitter or bounded delay requirement suffering from the ambiguity of the *long term* guarantees. Secondly, the end-to-end rate constraint used in [14] is actually the end-to-end throughput constraint, i.e. the number of bits that are successfully reached to the destination. We instead decouple the end-to-end throughput constraints into the transmission rate and the BER constraints, which better differentiate the applications and which are more amenable to actual system implementation. In this way, we also avoid the artificial assumptions such as approximating the rate as a linear function of SINR values.

In the following sections, we describe our system model and solution framework in detail.

III. SYSTEM MODEL

We consider a wireless network of N nodes. Each node is capable of transmitting at a power value less than or equal to P_{max}. A *directed link* exists between nodes i and j if the signal to noise ratio (SNR) at receiver j, when i transmits at this maximum power, is above a threshold γ_{ij}, i.e. $G_{ij}P_{max}/\sigma_j^2 \geq \gamma_{ij}$, where G_{ij} represents the path gain from i to j and σ_j^2 is the ambient noise at receiver j. Furthermore, we have S sessions and each session i is characterized by:

- A {source,destination} pair.
- A set of directed links that constitute the session path.
- A minimum short-term end-to-end rate requirement in bits/sec.
- Maximum BER requirement for each directed link along the session path.

The end-to-end rate requirement for a session dictates that the designated session rate must be supported across *all* links that constitute the session path. BER requirements are derived as a link budget estimation using the information on the total error tolerance of the session and its path length. In the rest of this section, we delineate the specific details of how the session requirements are satisfied.

A. Channel Model

The data packets are transmitted over the same wireless channel, which refers to the same frequency band in this paper. To prevent self-interference, half-duplex operation is enforced, i.e. a node cannot transmit and receive at the same time. We also limit ourselves only to point-to-point transmissions and no node is permitted to send multiple packets (for the same receiver or not) at the same time. The payload rate R of link l over the data channel is given by

$$R(l) = \frac{b_{sym}^l \times R_c^l}{T_{sym}^l} ,$$
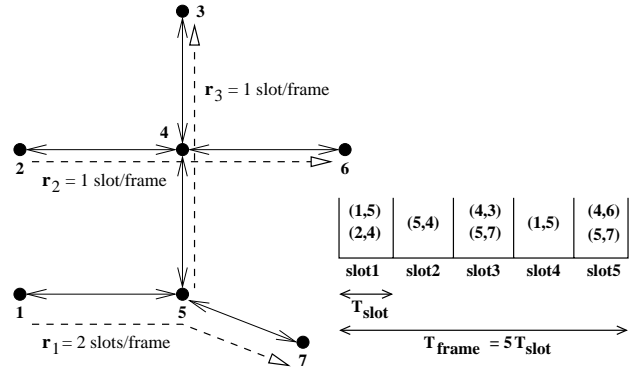

Fig. 1.   Sample topology and scheduling for concurrent multi-hop sessions.

where b_{sym}^l is the number of bits per symbol, R_c^l is the coding rate, and T_{sym}^l is the symbol duration for the transmissions over l. Time domain is divided into slots of length T_{slot} and time slots are further grouped into frames of L slots. We do not have control over the physical layer parameters b_{sym}^l, R_c^l, and T_{sym}^l, but we assume that they can be altered only before the start of each frame and that they are kept fixed throughout the frame. Hence, for link l, each slot has a constant payload rate, i.e.

$$r_l = \frac{b_{sym}^l \times R_c^l}{L \times T_{sym}^l} .$$

The scheduling is performed per frame basis and each link is assigned to a number of slots in a given frame. More precisely, the *short-term* rate requirement r_i of each session i, which traverses directed link l, necessitates allocating

$$k_i^l = \left\lceil \frac{r_i}{r_l} \right\rceil$$

time slots for link l. Here, $\lceil \cdot \rceil$ stands for the ceiling operation. Note that, in reality, we assign the time slots to the transmitter of a link and different links may have the same transmitter. As it will be clear later on, transmitters can utilize the same time slot assigned to them for different sessions only if the sessions have the same BER constraint and they traverse the same directed link. Therefore, the actual number of time slots k^l assigned to a directed link l can be bounded as;

$$\left\lceil \frac{1}{r_l} \sum_i I(l \in \mathcal{P}_i) r_i \right\rceil \leq k^l \leq \sum_i I(l \in \mathcal{P}_i) \left\lceil \frac{r_i}{r_l} \right\rceil ,$$

where $\mathcal{P}_i$ represents the flow path of session i and $I(\cdot)$ is the indicator function that is equal to one if its argument is true and zero otherwise. k^l satisfies the left hand side of the above expression when all sessions traversing link l have the same BER requirement and are able to be multiplexed together onto the same slot. In other cases, the upper-bound on the right hand side becomes valid. Obviously, the lower and upper bounds become the same when r_i's are integer multiples of r_l. Here on, without loss of generality, we restrict our attention to the session rates that are integer multiples of r_l.

Let us examine our system model as described so far on Fig.1. In the figure, bidirectional arrows show the existence of directed links between node pairs they connect. The frame length is set to 5 slots. There are three sessions initiated at nodes 1, 2, and 5 with flow paths depicted by dashed directional arrows. Session 1 has a bandwidth requirement of 2 slots per frame, whereas sessions 2 and 3 both require 1 slot per frame. Thus, total end to end bandwidth requirement becomes 8 slots per frame. Since the total figure is above the frame length, different links have to be activated at the same time. Also, the BER requirements at each receiver must be satisfied in all time slots. A sample link scheduling is given in the figure. Due to the bandwidth requirements or overlapping flows, the same link can be activated more than once during a frame period. For instance, ordered vertex pairs (1,5) and (5,7) must both be scheduled twice in the sample scenario.

Next, we elaborate on how the BER constraints of the concurrent transmissions can be satisfied using proper schedules and transmission powers.

B. SINR threshold and Feasibility of Concurrent Transmissions

In this subsection, we dwell upon the relation among the modulation level, coding rate, BER, and SINR. We will assume that BER is a one-to-one monotonically decreasing function of SINR. Therefore, a maximum tolerable BER can be mapped onto a minimum SINR threshold for a successful reception. In general, transceiver pairs may support multiple modulation levels (e.g. M-QAM with $M \in \{1, 2, \ldots, M_0\}$) and code rates (e.g. $R_c = 1/2, 3/4, 7/8, 1$). In the presence of time-varying link quality, the objective of modulation and coding rate adaptations is to increase transmission rate and to maintain an acceptable BER at the receivers. Lower modulation levels and coding rate can sustain more interference or equivalently assist in lowering average transmitted signal power at the same interference level.

For instance, when M-QAM modulation is used for the transmissions over link l, e.g. $b_{sym}^l = \log_2 M$, the BER is approximated as BER $\approx 0.2 \exp[-1.5(\text{SINR})/M - 1]$ [22]. For a maximum acceptable BER of ϵ, the SINR should satisfy

$$SINR \geq \frac{-\ln(5\epsilon)}{1.5}(M - 1) . \tag{1}$$

Thus, we map each modulation level b_{sym} and maximum acceptable BER to the SINR threshold γ_l, which is equal to the right-hand side of (1).[1] Clearly, decreasing b_{sym} or R_c also reduces the SINR threshold. On the other hand, the left-hand side of (1) is determined by channel gains, noise power, and the transmit powers of the links assigned to the same time slot. We allow the adaptation of transmit powers between consecutive time slots. Since we have assumed that the coding rate and modulation level are kept fixed throughout the frame,

transmit powers and slot assignments are the only controls we have to satisfy the BER constraints.

Suppose that $\mathcal{C}(n)$ denotes the set of links that are assigned to slot n; $T(l)$ and $R(l)$ are the transmitter and receiver end points of directed link l; P_l is the transmission power at node $T(l)$; and transmissions in slot n over link l are dedicated to packets of session $s_{l,n}$. Then, for each link $l \in \mathcal{C}(n)$, at the given modulation level and coding rate, BER requirements of $s_{l,n}$'s are mapped onto the following set of constraints:

$$\frac{G_{T(l)R(l)}P_l}{\sum\limits_{\substack{j \neq l \\ j \in \mathcal{C}(n)}} G_{T(j)R(l)}P_j + \sigma_{R(l)}^2} \geq \gamma_l \; ; \; \forall l \in \mathcal{C}(n) . \tag{2}$$

Constraints in (2) can be put into matrix form by defining $|\mathcal{C}(n)|$ by $|\mathcal{C}(n)|$ matrix $\tilde{\mathbf{G}}$ and the column vector β with entries:

$$\tilde{G}_{ij} = \frac{\gamma_j}{1 + \gamma_j} \frac{G_{T(i)R(j)}}{G_{T(j)R(j)}} \; ; \; \beta_i = \frac{\gamma_i}{1 + \gamma_i} \frac{\sigma_{R(i)}^2}{G_{T(i)R(i)}} . \tag{3}$$

Then, we obtain:

$$P \geq \tilde{G}P + \beta . \tag{4}$$

Here, $\mathbf{P}$ is simply the transmit power vector for the links assigned to slot n. $\mathcal{C}(n)$ is a *feasible* assignment for slot n if (4) is satisfied for a non-negative and finite $\mathbf{P}$.

Matrix $\tilde{\mathbf{G}}$ is non-negative and irreducible. From Perron-Frobenius theorem, $\tilde{\mathbf{G}}$ has exactly one positive real eigenvalue ρ with $\rho = \max\{|\lambda_i|\}_{i=1}^M$, where $\{\lambda_i\}_{i=1}^M$ are the eigenvalues of $\tilde{\mathbf{G}}$. ρ is called the Perron-Frobenius eigenvalue of $\tilde{\mathbf{G}}$. It is well-established that (4) is satisfied for a non-negative and finite $\mathbf{P}$ if and only if $\rho < 1$ [11]. Hence, the feasibility of $\mathcal{C}(n)$ is solely determined by the maximum eigenvalue of $\tilde{\mathbf{G}}$, which is contingent upon the channel gains and the sessions' BER requirements.

It is important to note that, in our model, link gains of different links remain constant within a time frame. Thus, our approach applies primarily to quasi-stationary or fully stationary wireless networks, when the link gain G_{ij} of each link (i, j) captures mainly path loss and shadowing effects. A more general approach would be to consider a model for link gain variation from slot to slot, where link gains are available for the entire network at the beginning of a slot.

Next, we present the notion of *virtual links* to simplify our system model.

C. Notion of Virtual Links

At this point, it is useful to introduce the notion of *virtual link* to avoid dealing with the bandwidth and BER requirements of the sessions explicitly. Let's denote the index set of active links[2] with $\Lambda^a = \{1, 2, \ldots, E\}$. As the same link can be scheduled more than once (in different slots), we index each instance of such links separately and denote them as virtual links, because they physically constitute the same link. Thus, we have a populated index set $\Lambda^v = \{1, 2, \ldots, M\}$ for virtual

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[1]In general, the SINR thresholds for each transmission (even over the same link) differ from each other; because either different modulation or coding schemes are used for different links of sessions or each session is characterized by its own BER requirement.

[2]This is the set of links which carry payload traffic as a result of routing decisions.

links where $M = \sum_{i=1}^{S} h_i(r_i/r_l)$ and h_i is the number of hops that ith session traverses. We continue to use $T(i)$ and $R(i)$ notation to denote the actual transmitting and receiving end points of the virtual link i. We formally define our problem in the next section over these virtual links, but we first need to elaborate on one more subtle point.

Our channel model restricts us to half duplex operation and point-to-point communication with one packet transmission at a time. The former condition is violated if two virtual links i and j that are scheduled in the same slot have the property of $T(i) = R(j)$ and the latter is violated if $T(i) = T(j)$. These properties suggest that the set of links scheduled for the same time slot must be a *matching set* in the corresponding topology graph. Nonetheless, we can simply absorb the *matching set* constraint into the SINR constraints by setting $G_{T(i)T(i)} = \infty$ and letting the γ_i's to be high enough. In other words, when node i is scheduled to receive and to transmit at the same time, the SINR at node i is driven to zero, violating its positive SINR requirement as a receiver. In a similar way, if two virtual links with the same transmitter are simultaneously scheduled, they will be strong interferers for each other, and hence lead to unsatisfied SINR constraints

Having comprehensively described our system model, we are now ready to formally state our problem in the next section.

IV. Joint Power Allocation and Schedule Assignment Problem

A. Formal Problem Statement

We want to minimize the total transmit power as summed over all time slots and links while satisfying the minimum rate and SINR constraints of the sessions. Since the rate requirements are expressed in number of slots per frame, assigning more slots to each link than the minimum dictated by the rate requirements and the routing paths unnecessarily increases the power consumption. Therefore, assigning one slot to each virtual link satisfies the end-to-end session rate requirements while achieving minimal power consumption. This fact motivates the formulation of the problem in terms of the virtual links. Then, our objective becomes:

$$\min_{\mathcal{A},\underline{P}} \sum_{i=1}^{M} P_i \ , \tag{5}$$

subject to

$$\frac{G_{T(i)R(i)}P_i}{\sum_{\substack{j \neq i \\ c(j)=c(i)}} G_{T(j)R(i)}P_j + \sigma_{R(i)}^2} \geq \gamma_i \ ; \ \forall i = 1,\ldots,M \ , \tag{6}$$

$$c(i) \in \mathcal{F} \triangleq \{1,2,\ldots,L\} \ ; \ \forall i = 1,\ldots,M \ , \tag{7}$$

$$P_{max} \geq P_i \geq 0 \ ; \ \forall i = 1,\ldots,M \ , \tag{8}$$

where P_i is the transmit power of node $T(i)$ and $c(i)$ is the time slot virtual link i is assigned to. Inequalities (6), (7), and (8) are the SINR, frame length, and power requirements respectively. Together they define the constraint set

$$\Omega = \{A,\underline{P} : 0 \geq \underline{P} \geq P_{max} \ and \ \underline{P} \geq \Gamma H \underline{P} + \underline{\beta}\} \ . \tag{9}$$

Here, $A : \Lambda^v \to \{1,\ldots,L\}$ is the time slot assignment of virtual links; $\underline{P}$ is the $M \times 1$ column vector with ith entry P_i; Γ is the $M \times M$ diagonal matrix with diagonal entries $\Gamma_{i,i} = \gamma_i$; H is the $M \times M$ interference matrix with entries $H_{i,j} = \delta_{ij}^A \times G_{T(i)R(j)}/G_{T(i)R(i)}$ for $i \neq j$ and $H_{i,i} = 0 \ \forall i$; $\underline{\beta}$ is the $M \times 1$ column vector with ith entry $\gamma_i \sigma_{R(i)}^2/G_{T(i)R(i)}$; and δ_{ij}^A is the assignment function that equals to 1 if $c(i) = c(j)$, otherwise it is 0. Whenever we have a pair $(A, \underline{P}) \in \Omega$, we will refer to them as *jointly feasible* allocation. Among all such pairs, we search for the ones that minimize (5), which we will call *jointly optimal* allocations.

Given the assignment instance, our problem reduces to classical power control problem in cellular networks and we can check if there exists a feasible solution [11]. Moreover, we can find the optimum power allocation at each slot centrally or iteratively. In fact, the optimum power allocation is *Pareto optimal*, i.e. all the links transmit at their minimum feasible power, and the constraint (6) is satisfied with equality [8]. However, finding the jointly optimum transmit power and time slot allocation is not straight-forward extension to the continuous transmission scheme as in the cellular voice services [13]. Since our constraint set does not satisfy the necessary monotonicity feature of the standard function [10], the existing iterative solutions cannot solve our problem. Besides, the constraint set is not a convex set in general and we cannot also apply standard techniques that minimizes a linear function over a convex set. Only under special conditions -such as when frame length L is larger than the number of virtual links M- the optimum joint allocation is trivial, e.g. each virtual link is placed on a distinct time slot and transmission power is set to the value just enough for combating the ambient noise. Then, the main question is whether we can find a jointly feasible allocation and an efficient optimal solution under general circumstances. Next, we will prove that the feasibility problem is indeed NP-complete [23], which leads us to device suboptimal approximation algorithms.

B. Intractability of the Jointly Feasible Allocation

Let us first define the following problem.

P1(feasibility problem): Given the gain matrix G, frame length L, session rate and SINR constraints, is there a schedule and power assignment that satisfy both the rate and SINR constraints? ∎

To show the NP-completeness of P1, we provide an alternative formulation of our optimization problem and its corresponding feasibility question. In this formulation, we assume that each session has the same BER (or SINR) requirement and the virtual link notion is put aside. Naturally, the gain matrix G and the SINR constraint define a super-set $\mathcal{X}$ of activation vectors $X_1, X_2, \ldots, X_\kappa$ where each X_i have exactly E entries from the binary set $\{0,1\}$. The entries with value 1 correspond to the indices of simultaneously transmitting active links while each transmission satisfies the given SINR constraint. Clearly, the vectors majorized by any X_i are also the members of $\mathcal{X}$. Suppose that we also know the power vectors that achieve the minimum total transmit power $P_{X_i}^*$ for each X_i. Then, our

objective becomes:

$$\min_{\underline{m}=[m_1 \ldots m_\kappa]^T} \sum_{i=1}^{\kappa} m_i P_{X_i}^* \ , \qquad (10)$$

subject to

$$[X_1 X_2 \ldots X_\kappa] \underline{m} \ \geq \ [\rho_1 \ldots \rho_E]^T \qquad (11)$$

and to

$$[1 \ldots 1] \underline{m} \leq L \ . \qquad (12)$$

Here, ρ_i is the total flow rate through link i as determined by routing decisions and session rate requirements, m_i is the number of slots that activation set X_i is used, and L is the number of slots in a frame as before. (11) is the short-hand representation of the rate requirements, whereas (12) simply states the total number of slots cannot be larger than the frame length. Objective function (10) and constraints (11)-(12) constitute an integer programming problem. Hence, its feasibility problem given below is NP-complete [23].

P2(alternative feasibility problem): Given the finite set of $\mathcal{X}$ with the associated minimizing power assignments and SINR threshold, is there a E-tuple $\underline{m}$ of integers such that constraint (12) is satisfied for fixed L and the rate constraints in (11) hold? ∎

Now, we can easily prove that P1 is also NP-complete.

Lemma 1: P1 is NP-complete.

Proof: Given any schedule and power allocation, e.g. an instance for P1, it takes $O(M^2)$ time steps to check if the session rates and SINR constraints are satisfied. Therefore, P1 is in NP.

Consider the following mapping: (i) Since we know the SINR threshold γ of P2, set γ_i in P1 as γ. (ii) Starting from the members of $\mathcal{X}$ that has the least number of active link, compute the elements of gain matrix using γ and minimizing power vectors. (iii) For the entries of G that cannot be computed, enter ∞. (iv) Create virtual links for the physical links that have a rate more than 1 slot/frame. (v) Keep the frame length same.

This procedure takes $O(\kappa^2)$ time steps and an instance of P2 is mapped onto an instance of P1 in polynomial time. Since P1 solves exactly the same problem, P2 reduces to P1 in polynomial time. However, P2 is NP-complete and P1 is in NP, which makes P1 NP-complete. ∎

This intractability result demands sup-optimal but efficient algorithms to perform the joint scheduling and power allocation. Before proceeding with our algorithmic proposals, we will derive some useful upper and lower bounds in the next sub-section.

C. Performance Bounds

The main interest of this section is to derive bounds on the total transmit power in a specific slot n in terms of path gains given that the virtual link assignment is feasible. Then, the transmit power of link i in slot n satisfies the inequality:

$$P_i \geq \frac{\gamma_i \sigma_{R(i)}^2}{G_{T(i)R(i)}} + \sum_{\substack{j \neq i \\ c(j)=c(i)}} \gamma_i \frac{G_{T(j)R(i)}}{G_{T(i)R(i)}} P_j \ . \qquad (13)$$

Summing up both sides of inequality (13) over all links in slot n and rearranging the terms in the second summation, we obtain:

$$\sum_{i \in s_n} P_i \geq \sum_{i \in s_n} \frac{\gamma_i \sigma_{R(i)}^2}{G_{T(i)R(i)}} + \sum_{i \in s_n} \sum_{\substack{j \neq i \\ j \in s_n}} \gamma_j \frac{G_{T(i)R(j)}}{G_{T(j)R(j)}} P_i \ , \quad (14)$$

where s_n denotes the set of virtual links in slot n. Further, we define the following:

$$\Theta_i(n) \triangleq \sum_{\substack{j \neq i \\ j \in s_n}} \gamma_j \frac{G_{T(i)R(j)}}{G_{T(j)R(j)}} \ ; \quad \alpha(n) = \sum_{i \in s_n} \frac{\gamma_i \sigma_{R(i)}^2}{G_{T(i)R(i)}} \ .$$

Note that $\Theta_i(n)$ can be understood as the *effective* interference of virtual link i on other users in the same slot and $\alpha(n)$ represents the capability of slot n to combat the noise term. Thus, we have the inequality:

$$\sum_{i \in s_n} (1 - \Theta_i(n)) P_i \geq \alpha(n) > 0 \ . \qquad (15)$$

It follows from (15) that

$$\max_{i \in s_n}(1 - \Theta_i(n)) \sum_{i \in s_n} P_i \geq \alpha(n) \ . \qquad (16)$$

Remember that $\alpha(n) > 0$ and this implies $\max_{i \in s_n}(1 - \Theta_i(n)) > 0$ or $\min_{i \in s_n} \Theta_i(n) < 1$. In other words, in order to have a feasible power allocation, the minimum *effective interference* in a slot must be strictly less than one. We will refer the link which has the minimum effective interference on other links as *minimum interferer*. Hence, we obtained the following lower bound on the total transmit power of a specific slot assignment:

$$\Sigma_n \geq \frac{\alpha(n)}{1 - \min_{i \in s_n}[\Theta_i(n)]} \ , \qquad (17)$$

where $\Sigma_n = \sum_{i \in s_n} P_i$. When we consider the trivial upper bound for Σ_n using the feasibility constraint $P_i \leq P_{max}$, minimum effective interference must satisfy the additional necessary condition of

$$\min_{i \in s_n}[\Theta_i(n)] \leq 1 - \frac{\alpha(n)}{|s_n| P_{max}} \ . \qquad (18)$$

In (18), $|s_n|$ is the number of links assigned to time slot n. It is also straight-forward to see that the inequalities (13)-(15) are satisfied with equality at the optimum power allocation. Let Σ_n^* be the optimum total transmit power of slot n, then it must satisfy the following upper bound provided that $\max_{i \in s_n}[\Theta_i(n)] < 1$:

$$\Sigma_n^* \leq \frac{\alpha(n)}{\min_{i \in s_n}(1 - \Theta_i(n))} = \frac{\alpha(n)}{1 - \max_{i \in s_n}[\Theta_i(n)]} \ . \qquad (19)$$

We can infer from (19) that minimizing $\max_{i \in s_n}[\Theta_i(n)]$ both decrements the upper bound and traps the total transmit power within tighter intervals. In addition, if the variation between maximum and minimum effective interference is
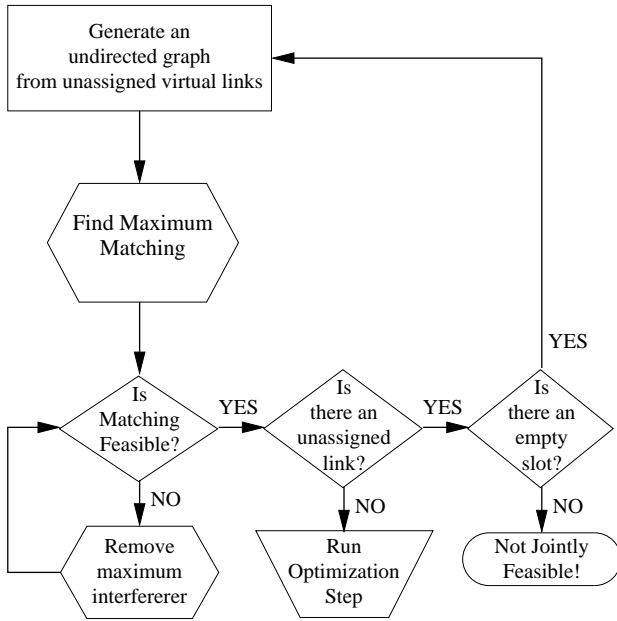
Fig. 2. Block Diagram for Algorithm A.

these directional edges connecting the same vertex pair to the same slot. Hence, we can replace directional edges with undirectional edges and prune the extra edges connecting same vertex pairs. In this way, we obtain an undirected graph. The same assumptions further render only simultaneous scheduling of *matching* edges[3] possible. Then, putting as many links as possible in the same slot becomes *maximum matching* problem, which is solvable in polynomial time [23].

Next step in the algorithm involves (i) one-to-one mapping of the maximum matching back to virtual links and (ii) checking if we have a feasible power allocation for this set of virtual links. When an undirected link in the matching set corresponds to the same directed link, we pick the one that has a smaller SINR threshold, which obviously has a better chance to satisfy the slot feasibility. In the case where the undirected link corresponds to the links with opposite polarities, we pick any of them. If the maximum matching fails to be feasible, we remove the link with maximum interference on the matching set. This process continues until the matching set is reduced to a feasible one. The matching set is infeasible provided that: (1) Perron-Frobenius eigenvalue ρ is larger than or equal to one or (2) ρ is smaller than one[4], but any of the links fails to satisfy maximum power constraint. Removal of the maximum interferer is beneficial not only in limiting the total transmit power of the matching set (see (19)), but also for avoiding the ambiguity in case, where successive removals lead to infeasibility as a result of having $\rho \geq 1$. The virtual links in the resulting matching are pruned from the directed graph and we continue with the next time slot until all virtual links are assigned to a feasible slot.

If we cannot assign all the virtual links for a given frame length L, we declare the problem instance as *not jointly feasible*. In the situations, where all the links are assigned to a number of slots less than L, we run an optimization step to shift the links to non-utilized/under-utilized slots. A greedy approach would be as follows. For a link reassignment a that involves reassignment of link i from slot s to slot s', we compute the factor $\Delta P(a) = P(before) - P(after)$, where P(before) is the total power consumption before the reassignment and P(after) is the total power consumption after the reassignment. The link that is selected for reassignment is the one that causes the maximal power consumption decrease $\Delta P(a)$. The algorithm terminates when no further link reassignments can cause power consumption decrease, i.e., when $\Delta P(a) < 0$ for all reassignments a of links from slots s to slots s'. Evidently, we restrict the re-assignments to the ones that ensure the joint feasibility.

sufficiently small, the upper bound also becomes a tight one. Quite intuitively, both the upper and lower bounds suggest that we should minimize $\alpha(n)$, i.e. choose a set of links, in which each link has a good channel gain or low SINR requirements. These observations are the main ingredients in the design of our heuristic algorithms which are revealed in the next section.

V. INTERFERER-BASED SUB-OPTIMAL HEURISTIC ALGORITHMS

We explore two greedy sub-optimal algorithms to solve the joint power allocation and schedule assignment problem that we refer to as algorithms A and B respectively.

A. Algorithm A

The first approach follows a top-down design strategy. It starts with the feasibility problem **P1** and searches for the minimum frame length L^* to satisfy the rate and SINR requirements. Clearly, **P1** is solvable if and only if $L^* \leq L$. Once the problem instance is identified to be feasible, the links from congested time slots are shifted to the empty or less congested ones to further reduce the transmit powers of the virtual links. The decision criterion on *which link to be shifted to which time slot* is explained in details below.

Block diagram in Fig.2 summarizes Algorithm A. We are given a set of virtual links, each of which has to be scheduled for once throughout the frame duration. Initially, all the time slots are empty. Starting from the first slot, we want to pack as many virtual links among the unscheduled links as possible into a single slot. The unassigned virtual links with their transmitters and receivers form a directed graph that possibly has multiple directional edges between the same vertex pair. Because of the point-to-point and half duplex communication assumptions, we cannot assign any two of

B. Algorithm B

The second strategy on the other hand follows a bottom-up approach. The iteration is performed over the unassigned links. Initially, we assign exactly one link to each slot until no empty slot remains. We choose the maximum interfering link

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[3]These are the edges that do not share a common vertex.

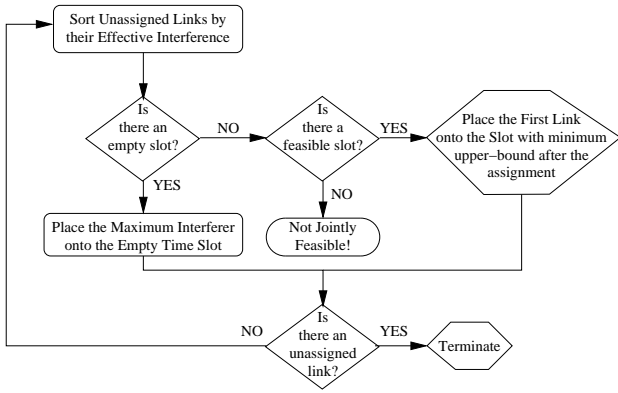[4]Then, we can compute the optimal power allocation by matrix inversion.
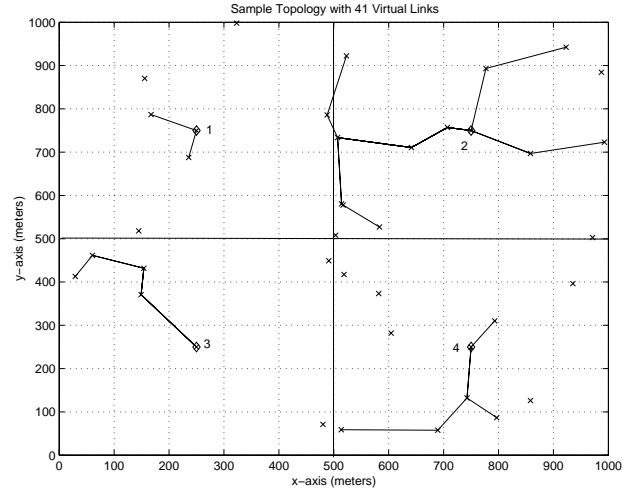
Fig. 3. Block Diagram for Algorithm B.

Fig. 4. Multi-hop cellular topology instance with fixed base-station locations at the center of square cells and randomly distributed relay/source nodes over a 1000mx1000m topology.

among all the unassigned links to place in the next empty slot. In this respect, we distribute the transmitters that are within close proximity of each other onto different time slots. In the second stage, algorithm performs the assignment using a water-filling argument: We assign a link i to slot s only if this assignment is feasible and s has the minimum upper-bound after the assignment as computed by the expression (19) among all feasible assignments. Algorithm terminates when all the links are exhausted or no feasible assignment can be found for an unassigned link. In the latter case, the problem instance is declared as not jointly feasible. The block diagram in Fig.(3) summarizes these steps. Evidently, Algorithm B tries to balance the power consumption in each time slot. An alternative scheme would be to directly rely on the actual total transmit power levels rather than on the upper-bounds. We could therefore directly assign the cost of an assignment a that maps link i onto slot s by looking at the increase in the total power consumption and at each iteration the assignment with the minimum cost would be chosen. However, this approach may potentially fail to distribute the transmitters in close proximity effectively leading to more costly slot assignments later on.

In the next section, we evaluate Algorithms A and B over a multi-hop enabled cellular network topology under different routing decisions. We believe that such topologies are well-suited for actual implementation of our centralized heuristic solutions.

VI. SIMULATION RESULTS

We have investigated the performance of our heuristic proposals on a $1000m \times 1000m$ square topology. The network is partitioned into four square cells and four nodes are positioned at the center of each cell. These nodes at the cell centers can be viewed as cluster heads that concentrate traffic in each ad hoc domain to relay to the other domains or access points/base-stations of an infrastructure/overlay network. For the convenience of expression, we refer to them as base-stations. The remaining wireless nodes are randomly distributed over the whole topology. Source nodes are randomly picked and their destination points are set as the closest base-stations from their

own position in the Euclidean sense. Each session as identified by its source node has a fixed rate requirement of 1 slot/frame as well as an SINR requirement uniformly picked from the set $\{4, 5, 6, 7, 8\}$. The noise power is assumed to be same at each receiver and transmit powers are normalized with respect to the noise power. The channel gains are computed by only taking the path loss factor into account with the path loss exponent of two for transceiver pairs close than 100 meters and four otherwise. We limit the maximum normalized transmission power to be 31.25, which corresponds to a transmission range of 250 meters at the highest SINR requirement. We have considered two different shortest path routing schemes for each given scenario with link costs equal to a unit value (i.e. minimum-hop routing) and transmission power just enough to combat the noise for the specific session (i.e. minimum-power routing). A sample topology instance with the set of links as determined by the session source nodes and minimum-power routing is depicted in Fig.4.

We have compared our heuristic proposals by (i) their success in identifying problem instances as feasible or not and (ii) their total transmit powers as averaged over all feasible problem instances. As the variable system parameter, we use the frame length in number of slots while keeping the session requirements fixed. Note that this is essentially equivalent to keeping the frame duration fixed and altering the traffic load in terms of the session rate requirements. For the few scenario settings where problem instance sizes turn out to be manageable, we have also computed the performance of optimal solutions.

Figures 5 and 6 show the average performance for the scenarios where we limit the number of sessions to seven and use the minimum-hop routing. In the plot legends, when there is an *upper-bound* label next to the algorithms A and B, it indicates that the upper-bound in (19) is used in the heuristics instead of the actual total transmission powers of
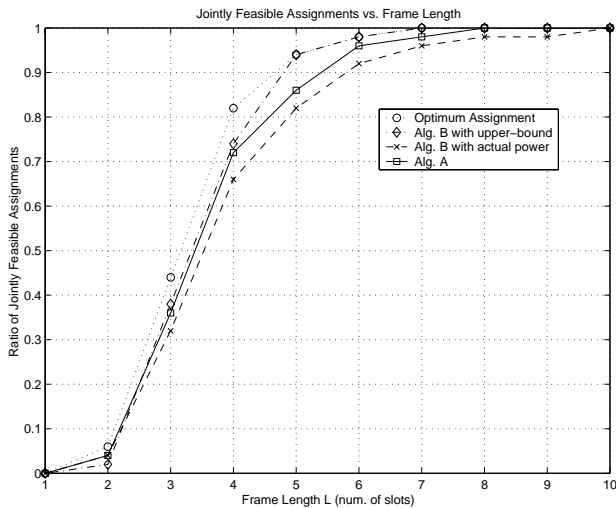
Fig. 5. Ratio of jointly feasible scenarios for 7 sessions and minimum-hop routing.

Fig. 6. Total transmit power averaged over the jointly feasible scenarios for 7 sessions and minimum-hop routing.

the slots. Similarly, the *actual power* label corresponds to the utilization of the actual power levels in the greedy heuristics. Quite interestingly, we observe that Algorithm A, which is specifically designed for first finding a feasible solution, is actually outperformed by Algorithm B, which relies on the upper-bound formulation we derived. In other words, a water-filling argument with a proper cost function can actually be more successful than a top-down design strategy as Algorithm A does by computing maximal matching sets and then by proceeding with link pruning. However, an inadequate cost function that does not assist in distributing the links, which exhibit high interference to each other, onto different slots results with a degraded performance as seen in Fig.5. Algorithm B also successfully matches the performance of optimal solution in finding the feasible solutions provided that they exist. On the other hand, when the total power consumption as summed over all the virtual links is observed, Algorithm A executes much better than the other heuristics. Algorithm B also performs comparable to optimal solution when the actual power values are computed to decide on the next assignment. However, due to the lower success rate in identifying the feasible solutions, Algorithm B with the actual power heuristic can resolve only the less constrained scenarios and the lower power consumption figures should not be misleading. Surely, the optimal solution performs better at each problem instance. The overall suggestion of the power consumption results is that greedy approaches, which directly operate on the objective function, have an advantage in minimizing the objective function.

Figures 7 and 8 present the same topology and session requirements, but at a different routing strategy, i.e. minimum-power routing. Since the problem size is quite large, we did not compute the optimal values. Nevertheless, we know that the optimum strategy when L gets large enough is to schedule one link at a time. Therefore, we show this asymptotic result in total power consumption figures. The relative performances
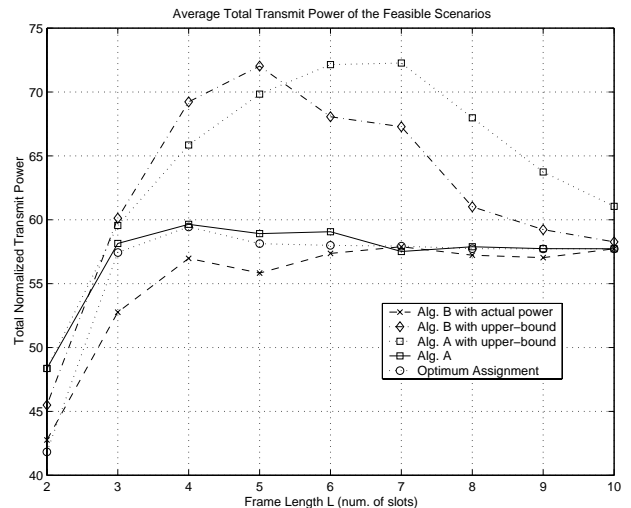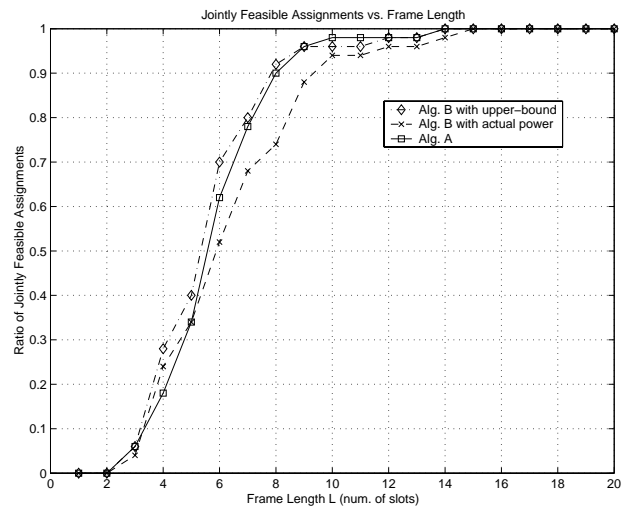

Fig. 7. Ratio of jointly feasible scenarios for 7 sessions and minimum-power routing.

have similar tendencies as in the minimum-hop routing except for the following points: (1) Using power as an explicit factor in link costs for routing protocols significantly ameliorates the overall power consumption. (2) Higher number of active links forces the system to use longer frame lengths to satisfy the session requirements. Thus, reducing the power consumption in the routing layer often fails to satisfy the session QoS requirements even at moderate frame lengths.

Figures 9 and 10 give more insights when the network load is increased by changing the number of sessions from 7 to 15. The relative performances remain same with wider performance gaps and the nominal values of the operating points get worse both in terms of the required frame length to satisfy the session requirements in majority of the scenarios and the settled down total power consumption.

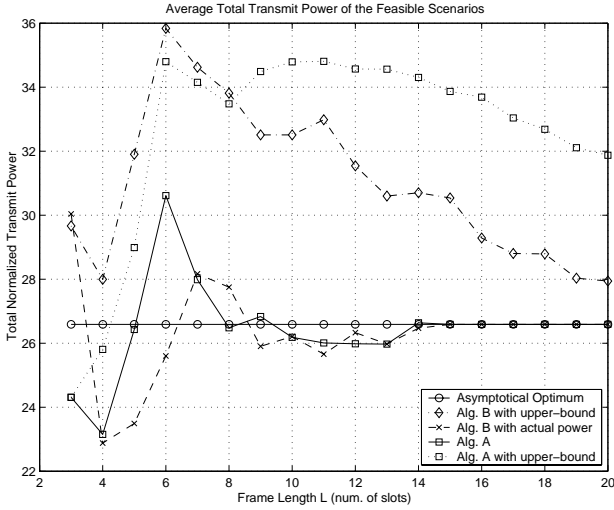This concludes our main results, which make a strong

Fig. 8. Total transmit power averaged over the jointly feasible scenarios for 7 sessions and minimum-power routing.

Fig. 10. Total transmit power averaged over the jointly feasible scenarios for 15 sessions and minimum-hop routing.
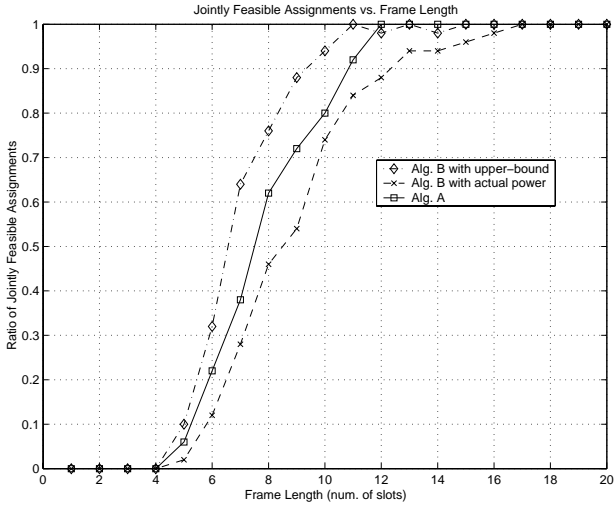

Fig. 9. Ratio of jointly feasible scenarios for 15 sessions and minimum-hop routing.

connection to our arguments as discussed in detail in the introduction of this paper. We now briefly summarize and draw the future direction of our work in the next section.

VII. SUMMARY AND FUTURE DIRECTION

We considered the problem of energy-efficient communication in wireless multi-hop networks with the objective of providing the end-to-end QoS guarantees to a set of sessions. We have started with first formulating a QoS framework that is able to capture both the different definitions of QoS from network layer to physical layer and the general requirements of the individual sessions. We stated the close inter-action between these layers and pointed out the fact that independent decisions on different layers for achieving a local objective would deteriorate the performance of other layers leading to a failure in achieving the main goal. With an open view towards including more layers and system parameters into the picture,
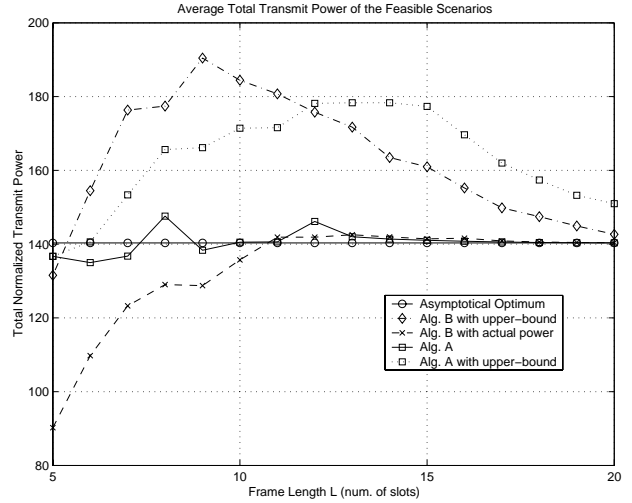
our focus has been on addressing the joint power control and scheduling problem. By introducing the notion of virtual links and assuming a one-to-one mapping between BER and SINR requirements for each wireless transmission, we decoupled this joint optimization problem from the underlying session based requirements. We proved the NP-completeness of the problem and provided the performance bounds for justifying the later proposed heuristic solutions.

Our comparison of proposed heuristics has revealed the following observations: (1) A top-down design strategy such as first solving the feasibility problem, then minimizing the power consumption performs better in terms of the objective function. This is contrary to the general expectation that it should also perform better to find a feasible solution. (2) Water-filling argument outperforms the top-down design strategy in finding a feasible solution provided that a proper cost function is defined. In this respect, the upper-bound expression as found in (19) serves well as a cost function. (3) Routing layer plays a dominant role in reducing power consumption, but it happens at the expense of QoS provisioning.

It is worth to mention that we have not distinguished the order of virtual links within a frame neither in our problem definition nor in our proposed solutions. This assumes that every link has a packet to transmit in its own turn. However, session packets follow their routing paths and the links closer to the source point must be scheduled before the ones that are further away to guarantee that there is actually a packet to transmit.[5] Having noted that, it is not hard to modify our problem's constraint set as well as our solutions to reflect this requirement. The following rule must be applied to narrow down the feasibility constraints in the problem statement and the search space in the heuristics:

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[5]This assumes that no packets are lost due to channel errors, which is a common problem for all reservation based schemes. In such situations, the links may use the slots for other traffic without violating the BER requirements of other transmissions or the node can transit into the power save mode.

"A virtual link l that is labelled by $(s; h)$ -where s is the session source node and $h \geq 0$ is the hop distance of $T(l)$ from this source point- can be assigned to slot k if and only if the total number of virtual links that are labelled by $(s; h-1)$ from slot 1 to $(k-1)$ is more than the total number of virtual links that are labelled by $(s; h)$."

This work is one of the initial steps for a proper treatment of cross-layer design in multi-hop wireless networks. The following research directions are not partially or fully addressed in this paper and remain as the major problems to be investigated in future works:

a) Our algorithms are centralized in the sense that they are executed by a central agent that has global network knowledge. This definitely limits the application towards ad hoc networks that have some level of infrastructure support. An interesting issue would be to devise partially or fully distributed algorithms based on only local node information. Such algorithms would be executed independently at each node, yet the transmission schedules and transmit powers should converge to an optimal or near-optimal solutions.

b) A more general case involves the issue of routing as well. Then, the routes of each session from source to destination are not given a priori to the central controller, but instead they need to be identified. In that case, the problem also involves an interaction with the network layer. At a first stage, we can assume the existence of several alternative routes for each session, which are provided by the routing protocol. The routing decision then determines the route that each session will follow. A meaningful objective would be to consider those routes whose paths are maximally disjoint, so that corresponding links are relatively far from each other and do not much interfere with each other. With this rule, scheduling decisions are also facilitated, in the sense that co-channel link sets will be formed where links are in general far from each other. Then, smaller amounts of transmission power is consumed to serve all links. The most general (and difficult to treat) is the case when the routes are completely unspecified, except for the source and the destination of each session. Hence, a cross-layer mechanism that jointly performs routing, scheduling and power control should be designed. Routes from each source to each destination can be found either proactively (routes ready to use, which is more suitable for our centralized framework) or reactively (routes on demand).

c) Anycasting services, in which we have a freedom of selecting the destination point for each source node among a set of equivalent nodes, will possibly find a vast class of applications especially in overlaid ad hoc networks. For instance, consider the cases where the primary goal of ad hoc nodes is to reach an infrastructure network that supplies a set of access points. Even when the routes are fixed, the selection of access point plays a dominant role to determine the set of links to be scheduled, altering the attainable performance.

d) Finding the performance limits of wireless multi-hop networks that constitute a finite field is still an open problem. It is also equally important to find the performance gaps of the heuristic solutions with these limits.

REFERENCES

[1] Anthony Ephremides, "Energy concerns in wireless networks," *IEEE Wireless Communications*, pp. 48–59, August 2002.

[2] T. Elbatt and A. Ephremides, "Joint Scheduling and Power Control for Wireless Ad Hoc Networks," in *Proc. Infocom-2002*, June 2002.

[3] J. Zander, "Distributed co-channel interference control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, no. 3, pp. 305–311, August 1992.

[4] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 57–62, February 1992.

[5] S.A. Grandhi, R. Vijayan, and D.J. Goodman, "Distributed algorithm for power control in cellular radio systems," in *Proc. 30th Annual Allerton Conf. Comm., Control Computing, Urbana, Illinois*, September 1992.

[6] S.A. Grandhi, R. Vijayan, D.J. Goodman, and J. Zander, "Centralized power control in cellular radio systems," *IEEE Trans. veh. Technol.*, vol. 42, pp. 466–468, November 1993.

[7] G.J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. veh. Technol.*, vol. 42, no. 4, pp. 641–646, November 1993.

[8] R.D. Yates, "A framework for uplink power control and base station assignment," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1341–1347, September 1995.

[9] N. Bambos, S. Chen and G. Pottie, "Radio Link Admission Algorithms for Wireless Networks with Power Control and Active Link Quality Protection," in *Proc. Infocom-1995*, 1995, pp. 97–104.

[10] C. Huang and R.D. Yates, "Rate of convergence for minimum power assignment algorithm in cellular radio systems," *Wireless Networks*, vol. 4, pp. 223–231, 1998.

[11] Sennur Ulukus and R.D. Yates, "Stochastic Power Control for Cellular Radio Systems," *IEEE Trans. on Commun.*, vol. 46, no. 6, pp. 784–798, June 1998.

[12] S. Hanly and D. Tse, "Power control and capacity of spread-spectrum wireless networks," *Automatica*, vol. 35, pp. 1987–2012, 1999.

[13] Fredrik Berggren, Seong-Lyun Kim, and Jens Zander, "Joint Power Control and Intracell Scheduling of DS-CDMA Nonreal Time Data," *IEEE JSAC*, vol. 19, no. 10, pp. 1860–1870, October 2001.

[14] R.L. Cruz and A.V. Santhanam, "Optimal Routing, Link Scheduling and Power Control in Multi-hop Wireless Networks," in *Proc. Infocom-2003*, April 2003.

[15] ——, "Optimal Link Scheduling and Power Control in CDMA Multihop Wireless Networks," in *Proc. Globecom-2002*, November 2002.

[16] R. Wattenhofer, Li Li, Paramvir Bahl, and Yi-Min Wang, "Distributed Topology Control for Power Efficient Operation in Multihop Wireless Ad Hoc Networks," in *Proc. Infocom-2001*, 2001.

[17] R. Ramanathan and R. Rosales-Hain, "Topology Control of Multi-hop Wireless Networks using Transmit Power Adjustment," in *Proc. Infocom-2000*, 2000.

[18] J. Monks, V. Barghavan and W. Hwu, "A Power Controlled Multiple Access Protocol for Wireless Packet Networks," in *Proc. Infocom-2001*, April 2001.

[19] V. Rodoplu and Teresa Meng, "Minimum energy mobile wireless networks," *IEEE JSAC*, vol. 17, no. 8, pp. 1333–1344, 1999.

[20] S. Singh, M. Woo and C.S. Raghavendra, "Power-aware routing in mobile ad hoc networks," in *Proc. of Fourth Annual ACM/IEEE Conference on Mobile Computing and Networking*, October 1998, pp. 181–190.

[21] D. P. Bertsekas, *Nonlinear Programming*. Athena Scientific, 1995.

[22] A.J. Goldsmith and S.-G. Chua, "Variable-rate variable-power MQAM for fading channels," *IEEE Trans. on Commun.*, vol. 45, no. 10, pp. 1218–1230, October 1997.

[23] Michael R. Garey and David S. Johnson, *Computers and Intractability A Guide to the Theory of NP-Completeness*. W.H. Freeman and Company, 1979.