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Control Synthesis and Adaptation for an Underactuated Autonomous Underwater Vehicle

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Control Synthesis and Adaptation for an Underactuated Autonomous Underwater Vehicle[1]

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Abstract

The motion of an autonomous underwater vehicle (AUV) is controllable even with reduced control authority such as in the event of an actuator failure. In this paper we describe a technique for synthesizing controls for underactuated AUV's and show how to use this technique to provide adaptation to changes in control authority. Our framework is a motion control system architecture which includes both feedforward control as well as feedback control. We confine ourselves to kinematic models and exploit model nonlinearities to synthesize controls. Our results are illustrated for two examples, the first a yaw maneuver of an AUV using only roll and pitch actuation, and the second a "parking maneuver" for an AUV. Experimental results for the yaw maneuver example are described.

Keywords: Autonomous Underwater Vehicles, Adaptive Control, Actuator Failure, Nonlinear Control Synthesis

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1 Introduction

Autonomous underwater vehicles (AUV's) are expected to play an increasingly larger role in underwater missions since they can be sent into ocean environments too risky for manned vehicles and too deep for tethered vehicles. Critical to their success is their capability for accurate and reliable autonomous motion control. In particular, an AUV should be able to control its own motion, adapting both to external dynamical disturbances as well as to configuration related changes such as the loss of an actuator.

Adaptation is a critical feature of an AUV since there is a great deal of uncertainty associated with maneuvering underwater, particularly at great depths. In one respect it is necessary for the AUV to adapt to bifurcations, i.e., to the onset of wave-induced or self-induced oscillations, to avoid gross errors in motion. These oscillations are dynamical phenomena that have been studied in some detail [1]. Typically, adaptation in this setting means gain parameter adaptation wherein control gains are reduced when necessary to avoid exciting dynamical modes of the vehicle-plus-fluid medium system.

On the other hand, adaptation of an AUV to configuration related changes has not been well understood. That is, in the event of a failure which reduces control authority of the AUV, one would like the AUV to be able to maintain complete control of its motion so that it can successfully complete its mission. In this paper we discuss a systematic way for an AUV to adapt to changes in control authority.

Toward this end we address the control synthesis problem for an underactuated AUV, i.e., an AUV with fewer control actuators than number of desired degrees of freedom. For example, we study the three-dimensional attitude control problem for an AUV with only two actuators, illustrating how to perform a yaw maneuver with only roll and pitch actuators. Our objective is to provide a controller that permits an AUV to control its motion with reduced control authority. A reduction in control authority might be the result of a failure of an actuator or a deliberate decision to limit the number and choice of actuators in use, e.g., for cost effectiveness.

Our strategy for control synthesis is based on our recent results in nonlinear constructive controllability applied to AUV motion [2]. From a well-known theorem in nonlinear control theory one can show whether or not an underactuated AUV is controllable, i.e., whether or not there *exists* a control law to drive the AUV into any position and orientation. Given that the underactuated AUV is controllable, the nonlinear constructive controllability problem is then one of *finding* a control law that translates and orients the AUV as desired. The fact that an underactuated AUV is controllable is a nonlinear effect, and, thus, it is the nonlinearities in the system model that we exploit. Indeed, a linearization of our underactuated AUV model will in general not be controllable implying that control techniques based on linearization will not be useful.

In our solution of the nonlinear constructive controllability problem, we derive algorithms that generate *open-loop controls* for AUV motion. Open-loop control is control in the feedforward path of the control system. It is constructed to drive the AUV as desired based on an understanding of how the AUV responds to certain types of control input. In its direct approach to control, open-loop control can lead to improved performance accuracy and reduced control effort. We refer to the use of open-loop control as *strategic planning* and a set of open-loop controls for an AUV as a *motion script* or *motion plan.*

Reactive control or *feedback control*, on the other hand, is control based on sensory feedback. A feedback control law converts a measured error in motion into a corrective actuation signal. As such, feedback control provides robustness to disturbances. Traditionally, one would consider controlling an AUV with purely feedback control. However, because of our ability to synthesize open-loop control for AUV motion, we propose a motion control system architecture that combines both open-loop and feedback control in an effort to realize the benefits of each. This architecture has the additional advantage of including adaptation to changes in control authority.

We focus in this paper on the kinematics of the AUV as a first step towards introducing strategic planning and incorporating configurational adaptation into the control strategy. Thus, the control inputs we specify are to be interpreted as rotational and translational velocities. The next step will then be to reintroduce the AUV dynamics into the overall control system design.

In Section 2 we describe our model for the AUV. In Section 3 we define control authority and controllability. The condition for determining controllability is described and illustrated for several choices of AUV control authority. Open-loop control synthesis for underactuated AUV's is presented in Section 4. Motion scripts are illustrated for two examples: a yaw maneuver of an AUV using only roll and pitch actuation and a "parking maneuver" for an AUV with original and reduced control authority. In Section 5 we describe the proposed motion control architecture which includes strategic planning, reactive control and adaptation to changes in control authority. In Section 6 we describe an experiment that was run to test our controls on an AUV in a neutral buoyancy tank at the University of Maryland. We give final remarks in Section 7.

2 Model

In this section we describe a kinematic model of the motion of an AUV. That is, we describe the position and orientation of the AUV as a function of its translational and rotational velocities. To do so in a global way, we make use of the space of matrices referred to as the Special Euclidean group of matrices $SE(3)$. $SE(3)$ is a matrix Lie group consisting of matrices that describe rotations and translations in $\Re^3$. Define

$$\begin{aligned} SE(3) \triangleq \{ & \left[\begin{array}{c|c} \mathcal{A} & b \\ \hline 0 & 1 \end{array}\right] \in \Re^{4\times 4} \mid \mathcal{A} \in \Re^{3\times 3}, \\ & \mathcal{A}^T\mathcal{A} = I,\ \det(\mathcal{A}) = 1,\ b \in \Re^3\}. \end{aligned}$$

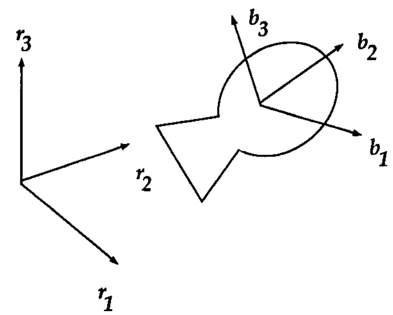

Figure 1: Autonomous Underwater Vehicle.

Then $\mathcal{A}$ is an orthogonal (rotation) matrix and b is a vector. Suppose that X is an element of $SE(3)$ and $y \in \Re^3$. Then X maps y into $\mathcal{A}y + b$ by multiplication:

$$X \begin{bmatrix} y \\ 1 \end{bmatrix} = \left[\begin{array}{c|c} \mathcal{A} & b \\ \hline 0 & 1 \end{array}\right] \begin{bmatrix} y \\ 1 \end{bmatrix} = \begin{bmatrix} \mathcal{A}y + b \\ 1 \end{bmatrix}.$$

The new vector $\mathcal{A}y + b$ can be interpreted as y rotated by $\mathcal{A}$ and translated by b.

An element from $SE(3)$ can similarly be used to describe the position and orientation of an AUV at time t. We identify the position and orientation of the AUV at time t with that element in $SE(3)$ which maps at time t each of the three axes of an orthonormal frame fixed on the AUV into the corresponding axes of an inertial frame. For example, consider an AUV as in Figure 1 with $(\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3})$ an orthonormal frame fixed on the vehicle and $(\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3})$ an inertial frame. Then $X(t) \in SE(3)$, defined so that

$$\begin{bmatrix} \mathbf{r_i} \\ 1 \end{bmatrix} = X(t) \begin{bmatrix} \mathbf{b_i} \\ 1 \end{bmatrix},$$

determines the orientation and position of the AUV at time t. Because every position and orientation of the AUV corresponds to a distinct element in $SE(3)$, $SE(3)$ provides a global representation of the configuration space of the AUV. This is not true for local parametrizations such as those based on Euler angles or quaternions.

To describe the velocity of the AUV we define $se(3)$, the Lie algebra associated to $SE(3)$. $se(3)$ is a vector space with matrix elements that describe infinitesimal rotations and translations in $\Re^3$. Define

$$se(3) \triangleq \{ \begin{pmatrix} A & x \\ 0 & 0 \end{pmatrix} \in \Re^{4\times 4} | A \in \Re^{3\times 3}, \\ A + A^T = 0,\ x \in \Re^3 \}.$$

Then A is a skew symmetric matrix and x is a vector. The vector space $se(3)$ becomes a Lie algebra by defining a binary operation referred to as the Lie bracket, denoted by $[\cdot,\cdot]$, and defined by

$$\begin{aligned} [\cdot,\cdot] : se(3) \times se(3) &\longrightarrow se(3) \\ [A, B] &\mapsto AB - BA. \end{aligned}$$

This operation is also known as the commutator of matrices. If $[A, B] = AB - BA = 0$ for some $A, B \in se(3)$, we say that A and B commute.

Now let $\Omega = (\Omega_1, \Omega_2, \Omega_3)^T$ be the angular velocity of the vehicle and let $v = (v_1, v_2, v_3)^T$ be the vehicle translational velocity, all with respect to $(\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3})$. To associate angular and translational velocity with an element in $se(3)$ we define the following basis $\{A_1, \ldots, A_6\}$ for $se(3)$:

$$A_1 = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right], \quad A_4 = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right],$$

$$A_2 = \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right], \quad A_5 = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right], \tag{1}$$

$$A_3 = \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right], \quad A_6 = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 \end{array}\right].$$

A_1, A_2, A_3 correspond to infinitesimal rotations about the $\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}$ axes, i.e., in the roll, pitch and yaw directions, respectively. A_4, A_5, A_6 correspond to infinitesimal translations along the $\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}$ axes, respectively. We can then express the velocity of the AUV as $(\sum_{i=1}^{3} \Omega_i(t)A_i + \sum_{i=4}^{6} v_{i-3}(t)A_i)$ which is an element in $se(3)$. Further, $X(t)$ satisfies

$$\dot{X} = X(\sum_{i=1}^{3} \Omega_i(t)A_i + \sum_{i=4}^{6} v_{i-3}(t)A_i). \tag{2}$$

Equation (2) provides a global kinematic model of the motion of the AUV by describing X, the orientation and postion of the vehicle, as a function of the vehicle velocities Ω and v.

As described in Section 1 above, we confine ourselves in this paper to a kinematic model of the AUV motion. Accordingly, we specify control inputs that can be interpreted as rotational and translational velocities. Let us identify

$$\epsilon u_i(t) = \begin{cases} \Omega_i(t), & i = 1,2,3 \\ v_{i-3}(t), & i = 4,5,6 \end{cases} \tag{3}$$

where ϵ is assumed to be a small parameter and each $\epsilon u_i(t)$, $i = 1, \ldots, 6$, is interpreted as a small-amplitude control input. Further, suppose that we can directly actuate only $m \leq 6$ of these control inputs, i.e., with appropriate reordering of indices if necessary, $u_{m+1} = \ldots = u_6 = 0$. Then the kinematic model of (2) can be rewritten as the control system

$$\dot{X} = \epsilon X U, \quad U(t) = \sum_{i=1}^{m} u_i(t)A_i, \quad m \leq 6, \tag{4}$$

where $X(t) \in SE(3)$ and $U(t) \in se(3)$.

The formulation (4) is actually more general than as derived since the matrices $\{A_1, \ldots, A_6\}$ can be chosen to be any basis for $se(3)$ with the coefficients $u_1(t), \ldots, u_6(t)$ interpreted accordingly. For example, consider an AUV with two actuators. Suppose that one actuator controls angular velocity (denoted by ϵu_1) about an axis that lies in the $\mathbf{b_1}$-$\mathbf{b_2}$ plane. For example, suppose that it controls angular velocity about the $\mathbf{b_1}$ and $\mathbf{b_2}$ axes equally. Suppose the other actuator controls angular and translational velocity about the $\mathbf{b_2}$ axis, simultaneously. This second actuator provides a screw-like velocity that we denote by ϵu_2. Define $B_1 = A_1 + A_2$ and $B_2 = A_2 + A_5$ where A_1, A_2, A_5 are define by (1). Then $X(t)$ satisfies

$$\dot{X} = \epsilon X(u_1 B_1 + u_2 B_2). \tag{5}$$

In this paper when we refer to A_i, $i = 1, \ldots, 6$, we will mean as defined by (1). We will consider this to be the standard basis for $se(3)$.

Equation (4) is referred to as a *drift-free, left-invariant system* on the matrix Lie group $SE(3)$. By drift-free it is meant that if the control is set equal to zero, i.e., $u(t) = (u_1(t), \ldots, u_6(t)) = 0$, then the rate of change of the state is zero, i.e., $\dot{X} = 0$. A system with drift would have a nonzero rate of change of state under these conditions. The fact that the AUV kinematics can be described by a drift-free system is advantageous for our purposes because it simplifies the determination of system controllability (see Section 3).

Left-invariance refers to the fact that the kinematic description (4) is independent of where we place the body-fixed orthonormal frame $(\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3})$ and inertial frame $(\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3})$. This implies that if we know the solution of (4) with initial condition $X(0) = I \in SE(3)$ to be $X_I(t)$, then we know the solution of (4) with any initial condition $X(0) = X_0$ as $X(t) = X_0 X_I(t)$.

3 Control Authority and Controllability

Control authority of the AUV is defined to be the m available control inputs, i.e., the set $\{A_1, \ldots, A_m\} \subset se(3)$ such that $U(t) = \sum_{i=1}^{m} u_i(t) A_i$ and $u_1(t), \ldots, u_m(t)$ can be actuated independently. For example, control authority represented by the set $\{A_1, A_2, A_3, A_4\}$ is interpreted to mean that the AUV is equipped with independent actuators for roll, pitch, yaw and translation along the $\mathbf{b_1}$ axis. If the yaw actuator is unavailable in this example then the control authority is represented by the set

$\{A_1, A_2, A_4\}$.

Assuming that we are interested in controlling the full six dimensions of AUV motion, i.e., position and orientation in $\Re^3$, then an AUV with control authority comprised of $m < 6$ independent control inputs is an *underactuated* AUV. In this underactuated condition, it is often still possible to maintain complete control of the AUV motion. To demonstrate this we consider the problem of specifying controls for the AUV modelled by (4) that drive the AUV from its initial orientation and position to some final desired orientation and position in a given fixed amount of time. We state this problem formally as

(P) Given an initial position and orientation $X_i \in SE(3)$, a final position and orientation $X_f \in SE(3)$ and a time $t_f > 0$, find controls $u(t) = (u_1(t), \ldots, u_m(t))$, $t \in [0, t_f]$, such that $X(0) = X_i$ and $X(t_f) = X_f$.

System (4) is said to be *controllable* if there exists a solution to **(P)**, i.e., if for the given control authority the AUV can be translated and rotated into any desired position and orientation. There are well-known results from nonlinear systems theory for determining controllability of systems of the form (4) on Lie groups [3, 4]. Let

$$\begin{aligned} \mathcal{C} &= \{C \mid C = [C_k, [C_{k-1}, [\cdots, [C_1, C_0] \cdots]]], \\ &\quad C_i \in \{A_1, \ldots, A_m\},\ i = 0, \ldots, k\}. \end{aligned} \tag{6}$$

Then

$$\text{System (4) is controllable} \iff \operatorname{span}(\mathcal{C}) = se(3). \tag{7}$$

The set $\mathcal{C}$ consists of the matrices $\{A_1, \ldots, A_m\}$ as well as matrices of the form $[A_i, A_j]$, $[A_i, [A_j, A_k]]$, $[A_i, [A_j, [A_k, A_l]]]$, etc. We refer to $\operatorname{span}(\mathcal{C})$ as the Lie algebra generated by $\{A_1, \ldots, A_m\}$. The controllability condition (7) then implies that the AUV is controllable if and only if the Lie algebra generated by $\{A_1, \ldots, A_m\}$ is $se(3)$, the six-dimensional space of infinitesimal rotations and translations.

Intuitively, the controllability condition tells us that an AUV can be controlled not only in directions that are directly actuated but also in directions corresponding to the Lie brackets of directly actuated directions. The fact that an underactuated AUV is controllable results from the fact that rotations do not commute with one another and rotations do not commute with translations, i.e., Lie brackets of the associated matrices produce new nonzero matrices. This is best illustrated by example.

Example (a). First consider an AUV with control authority $\{A_1, A_2, A_3, A_4\}$, i.e., with actuators for roll, pitch, yaw and translation along the $\mathbf{b_1}$ axis. Performing the matrix multiplication in the Lie bracket operation one finds that

$$[A_3, A_4] = A_5, \quad [A_4, A_2] = A_6.$$

Thus, $\{A_1, \ldots, A_6\}$ are all contained in $\mathcal{C}$ and so $\text{span}(\mathcal{C}) = \text{span}\{A_1, \ldots, A_6\} = se(3)$. By (7) the AUV is controllable. The bracket $[A_3, A_4] = A_5$ expresses the fact that yaw rotation (motion in the A_3 direction) and translation along the $\mathbf{b_1}$ axis (motion in the A_4 direction) do not commute. Specifically, if the AUV first translates and then yaws it does not end up in the same place as it would have had it first yawed and then translated. This is shown in an overhead view of the AUV in Figure 2(a). Further, as shown in Figure 2(b) if the AUV yaws in the clockwise direction, translates in the positive $\mathbf{b_1}$ direction, yaws in the counterclockwise direction and then translates in the negative $\mathbf{b_1}$ direction, the AUV will experience a net translation along the $\mathbf{b_2}$ axis (the A_5 direction). A similar interpretation can be made for the bracket $[A_4, A_2] = A_6$ which expresses the fact that pitch and translation do not commute and can be used to produce net vertical translation (motion in the A_6 direction). We refer to the AUV in this example as a single-bracket system since only single iterations of Lie bracketing are needed to satisfy the controllability condition.

Example (b). Now consider an AUV with control authority $\{A_1, A_2, A_4\}$. This is the AUV of Example (a) without a yaw actuator (perhaps due to an actuator failure). In

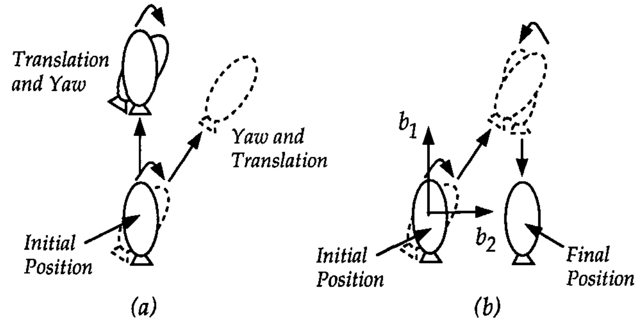

Figure 2: Overhead View of AUV Performing Yaws and Translations.

this case we find that

$$[A_1, A_2] = A_3, \quad [[A_1, A_2], A_4] = A_5, \quad [A_4, A_2] = A_6.$$

Thus, $A_1, \ldots, A_6$ are all contained in $\mathcal{C}$. So $\text{span}(\mathcal{C}) = se(3)$ and the AUV is controllable. The bracket $[A_1, A_2] = A_3$ expresses the fact that roll (motion in the A_1 direction) and pitch (motion in the A_2 direction) do not commute. Specifically, if the AUV rolls and then pitches it is not in the same orientation as it would have been had it first pitched then rolled. Further, if the AUV rolls in the positive direction, pitches in the positive direction, rolls in the negative direction and then pitches in the negative direction, the AUV will experience a net yaw motion (motion in the A_3 direction). We refer to the AUV in this example as a double-bracket system since we needed to use a bracket of brackets (double bracket) to satisfy the controllability condition.

Example (c). The AUV with two actuators described by (5) is controllable since $(B_1, B_2, [B_1, B_2], [B_1, [B_1, B_2]], [B_2, [B_1, B_2]], [B_1, [B_1, [B_1, B_2]]])$ span $se(3)$. In this case the AUV is a triple-bracket system.

The controllability condition for linear systems involves checking the rank of a matrix referred to as the controllability Grammian. If the linear system is controllable a control law can be constructed using the controllability Grammian. For nonlinear systems such as the AUV described by system (4), the nonlinear controllability condition (7) does not lead immediately to an explicit procedure for constructing controls. Finding the control law that solves the problem **(P)** is referred to as the *constructive controllability problem.*

This problem is addressed in the next section.

4 Control Synthesis

The computation of open-loop controls for nonlinear constructive controllability has been explored carefully in the context of robotics and more abstract problems [2, 5, 6, 7, 8, 9, 10, 11]. In this setting, the use of periodic controls (outputs of coupled oscillators) has played an important role. In this paper we also use periodically time-varying controls, i.e., we let $u_i(t)$, $i = 1, \ldots, m$, be periodic in t of period T. In particular, we present motion scripts based on small-amplitude, sinusoidal control inputs.

Part of the motivation for using these controls comes from noting that sinusoidal controls that are out of phase will generate motion in single Lie bracket directions. For instance, consider the AUV of Example (a) above and suppose that $u_3(t) = \cos t$ and $u_4(t) = \sin t$. Then, the AUV will be continuously performing a motion roughly like that illustrated in Figure 2(b) and will experience a net translation along the $\mathbf{b_2}$ axis. Small-amplitude, low-frequency sinusoidal controls are also justified from a practical point of view. Using these small, gentle control inputs, we can avoid both exciting vibrational modes of the AUV and making large off-course excursions in the vehicle's orientation and position.

Averaging theory is the main analysis tool we use to derive a systematic means of synthesizing controls. The details of our work on averaging for a class of nonlinear systems similar to the AUV described by (4) can be found in [2]. As applied to the AUV problem, the basic idea is to find an average approximation $\bar{X}(t)$ to $X(t)$ such that the "distance" between $\bar{X}(t)$ and $X(t)$ is "small" on a sufficiently long time interval. The formulas for $\bar{X}(t)$ are revealing. In particular, they expose the behavior of the AUV in terms of elements of $\mathcal{C}$, i.e., in terms of the control authority and the associated Lie bracket directions.

From the average formulas we have derived algorithms for open-loop control synthesis. If the AUV is controllable, given a specification of control authority and X_i, X_f and t_f, the algorithms produce small (ϵ) amplitude sinusoidal controls that drive the AUV such that $X(t_f)$ is close to X_f with accuracy of order ϵ^q. Here, q is an integer that can be no smaller than one more than the number of bracket iterations used to satisfy the controllability condition (e.g., $q \geq 2$ for a single-bracket system and $q \geq 3$ for a double-bracket system). The controls are synthesized by driving $\bar{X}(t)$ exactly and using the averaging results to ensure that $X(t)$ will stay close to $\bar{X}(t)$. The fact that the algorithms synthesize controls as a function of a simple specification of control authority is central to our scheme for adaptation to changes in control authority described in the next section.

In what follows we present the motion scripts generated by our algorithms for the control authority specified in Examples (a) and (b). We illustrate these motion scripts for two example maneuvers, one a yaw maneuver using only roll and pitch actuation and the second a "parking maneuver". The details of the algorithms can be found in [2].

While (4) has well-defined, global-in-time solutions for all piecewise continuous controls, in general there are no simple global representations of these solutions to (4). However, there are representations by products of exponentials [12] or exponentials of sums [13] that are valid locally in time. For example, according to Wei and Norman [12] the solution to (4) with $X(0) = I$ can be expressed as

$$X(t) = e^{\gamma_1(t)A_1} e^{\gamma_2(t)A_2} \cdots e^{\gamma_6(t)A_6}$$

for $|t| < t_0$, some $t_0 > 0$. $\gamma(t) = (\gamma_1(t), \ldots, \gamma_6(t))^T \in \Re^n$ is the solution to the ordinary differential equation

$$\begin{bmatrix} \dot\gamma_1 \\ \dot\gamma_2 \\ \dot\gamma_3 \\ \dot\gamma_4 \\ \dot\gamma_5 \\ \dot\gamma_6 \end{bmatrix} = \begin{bmatrix} \sec\gamma_2\cos\gamma_3 & -\sec\gamma_2\sin\gamma_3 & 0 & 0 & 0 & 0 \\ \sin\gamma_3 & \cos\gamma_3 & 0 & 0 & 0 & 0 \\ -\tan\gamma_2\cos\gamma_3 & \tan\gamma_2\sin\gamma_3 & 1 & 0 & 0 & 0 \\ 0 & -\gamma_6 & \gamma_5 & 1 & 0 & 0 \\ \gamma_6 & 0 & -\gamma_4 & 0 & 1 & 0 \\ -\gamma_5 & \gamma_4 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} \tag{8}$$

where the unavailable control inputs are set to zero. The parameters γ_1, γ_2, γ_3 can be interpreted as Euler angles and describe the orientation of the vehicle. The parameters γ_4, γ_5, γ_6 parametrize the position of the vehicle.

Due to the left invariance of (4) we can, without loss of generality, assume that $X(0) = X_i = I$. Let $\gamma_f = (\gamma_{f1}, \ldots, \gamma_{f6})^T$ be the local representation of the desired final position and orientation of the AUV defined by

$$X_f = e^{\gamma_{f1}A_1} e^{\gamma_{f2}A_2} \cdots e^{\gamma_{f6}A_6}.$$

Here we have assumed that X_f is sufficiently close to the identity, i.e., we consider a small AUV maneuver. To perform a large AUV maneuver, we would choose intermediate target points between X_i and X_f and use the motion scripts below repeatedly to meet each intermediate target point successively.

Motion Script for Example (a)

Assume that X_f, t_f and ϵ are given and X_f is close enough to the identity I such that $\gamma_f = O(\epsilon)$, i.e., γ_f has magnitude of order ϵ. The motion script for the AUV with control authority $\{A_1, A_2, A_3, A_4\}$ of Example (a) is given below. These controls drive the AUV from $X_i = I$ to X_f with $O(\epsilon^2)$ accuracy. To begin we choose an integer $M \geq 1/\pi\epsilon$ and compute the period, frequency and amplitudes of our control inputs as

$$T = \frac{t_f}{M + 3/2}, \quad \omega = \frac{2\pi}{T}.$$

$$\alpha_4 = \left(\frac{\gamma_5^2+\gamma_6^2}{\pi^2 M^2}\right)^{1/4}, \quad \alpha_2 = \frac{\gamma_6}{\alpha_4 \pi M}, \quad \alpha_3 = -\frac{\gamma_5}{\alpha_4 \pi M}.$$

Let $t_1 = \frac{T}{4}$, $t_2 = t_1 + MT$, $t_3 = t_2 + \frac{3T}{4}$, $t_4 = t_f = t_3 + \frac{T}{2}$. The controls are then defined as follows:

$$\epsilon u_1(t) = \begin{cases} 0 & 0 \le t \le t_3 \\ \frac{1}{2}\gamma_{f1}\omega \sin(\omega(t-t_3)) & t_3 \le t \le t_4 \end{cases}$$

$$\epsilon u_2(t) = \begin{cases} 0 & 0 \le t \le t_1 \\ \alpha_2 \omega \sin(\omega(t-t_1)) & t_1 \le t \le t_2 \\ 0 & t_2 \le t \le t_3 \\ \frac{1}{2}\gamma_{f2}\omega \sin(\omega(t-t_3)) & t_3 \le t \le t_4 \end{cases}$$

$$\epsilon u_3(t) = \begin{cases} 0 & 0 \le t \le t_1 \\ \alpha_3 \omega \sin(\omega(t-t_1)) & t_1 \le t \le t_2 \\ 0 & t_2 \le t \le t_3 \\ \frac{1}{2}\gamma_{f3}\omega \sin(\omega(t-t_3)) & t_3 \le t \le t_4 \end{cases} \qquad (9)$$

$$\epsilon u_4(t) = \begin{cases} \alpha_4 \omega \sin \omega t & 0 \le t \le t_3 \\ \frac{1}{2}\gamma_{f4}\omega \sin(\omega(t-t_3)) & t_3 \le t \le t_4 \end{cases}$$

In this motion plan, sinusoids are used out of phase in 1-1 resonance to achieve motion in the single Lie bracket directions. Given a fixed t_f, the choice of M determines the frequency of the sinusoids. If there are practical limitations on the frequencies that can be used, M can be chosen accordingly. The amplitudes $\alpha_2, \alpha_3, \alpha_4$ have been chosen to minimize a measure of control energy. However, other practical considerations, such as exceeding minimum actuator signal levels and avoiding actuator saturation levels, can be accounted for in the choice of the amplitudes.

Motion Script for Example (b)

Assume that X_f, t_f and ϵ are given and X_f is close enough to the identity I such that $\gamma_f = O(\epsilon^2)$. The motion script for the AUV with control authority $\{A_1, A_2, A_4\}$ of Example (b) is given below. These controls drive the AUV from $X_i = I$ to X_f with

$O(\epsilon^3)$ accuracy. To begin we choose an integer $M \geq 1/\pi\epsilon$ and compute the period, frequency and amplitudes of our control inputs as

$$T = \frac{t_f}{3(M+1)+1/2}, \qquad \omega = \frac{2\pi}{T},$$

$$\rho_2 = \left(\frac{\gamma_{f5}}{6\pi M}\right)^{1/3}, \quad \rho_1 = \left(\left|\frac{\gamma_{f5}}{\rho_2 \pi M}\right|\right)^{1/2}, \quad \rho_4 = \frac{\gamma_{f5}}{\rho_1 \rho_2 \pi M},$$

$$\alpha_2 = \left(\frac{\gamma_{f3}^2 + \gamma_{f6}^2}{\pi^2 M^2}\right)^{1/4} \alpha_1 = -\frac{\gamma_{f3}}{\alpha_2 \pi M}, \quad \alpha_4 = -\frac{\gamma_{f6}}{\alpha_2 \pi M}.$$

Let $t_1 = \frac{T}{4}$, $t_2 = t_1 + MT$, $t_3 = t_2 + \frac{3T}{4}$, $t_4 = t_3 + \frac{T}{4}$, $t_5 = t_4 + MT$, $t_6 = t_5 + \frac{3T}{4}$, $t_7 = t_6 + \frac{T}{4}$, $t_8 = t_7 + MT$, $t_9 = t_8 + \frac{3T}{4}$, $t_{10} = t_9 + \frac{T}{2}$. The controls are then defined as follows:

$$\epsilon u_1(t) = \begin{cases} \rho_1 \omega \sin \omega t & 0 \leq t \leq t_6 \\ 0 & t_6 \leq t \leq t_7 \\ \alpha_1 \omega \sin(\omega(t - t_7)) & t_7 \leq t \leq t_8 \\ 0 & t_8 \leq t \leq t_9 \\ \frac{1}{2}\gamma_{f1} \omega \sin(\omega(t - t_9)) & t_9 \leq t \leq t_{10} \end{cases}$$

$$\epsilon u_2(t) = \begin{cases} 2\rho_2 \omega \sin \omega t & 0 \leq t \leq t_1 \\ 2\rho_2 \omega \cos(2\omega(t - t_1)) & t_1 \leq t \leq t_2 \\ 2\rho_2 \omega \cos(\omega(t - t_2)) & t_2 \leq t \leq t_3 \\ -2\rho_2 \omega \sin(\omega(t - t_3)) & t_3 \leq t \leq t_4 \\ -2\rho_2 \omega \cos(2\omega(t - t_4)) & t_4 \leq t \leq t_5 \\ -2\rho_2 \omega \cos(\omega(t - t_5)) & t_5 \leq t \leq t_6 \\ \alpha_2 \sin(\omega(t - t_6)) & t_6 \leq t \leq t_9 \\ \frac{1}{2}\gamma_{f2} \omega \sin(\omega(t - t_9)) & t_9 \leq t \leq t_{10} \end{cases} \tag{10}$$

$$\epsilon u_4(t) = \begin{cases} \rho_4 \omega \sin \omega t & 0 \leq t \leq t_3 \\ 0 & t_3 \leq t \leq t_7 \\ \alpha_4 \omega \sin(\omega(t - t_7)) & t_7 \leq t \leq t_8 \\ 0 & t_8 \leq t \leq t_9 \\ \frac{1}{2}\gamma_{f4} \omega \sin(\omega(t - t_9)) & t_9 \leq t \leq t_{10} \end{cases}$$

In this motion plan, sinusoids are used in 1-1 resonance to achieve motion in the single Lie brackets directions and in 1-2 resonance to achieve motion in the double Lie bracket direction. Choices of frequency and amplitudes can be made to meet practical considerations as discussed above for Example (a).

Yaw Maneuver Example

Consider an AUV with control authority $\{A_1, A_2, A_4\}$, i.e., with actuators for roll, pitch and translation along $\mathbf{b_1}$ as in Example (b). Suppose the AUV is to perform a pure yaw maneuver. In local coordinates this implies that $\gamma_{f_i} = 0,\ i = 1, 2, 4, 5, 6$, and $\gamma_{f3} \neq 0$. For these values, according to the motion plan for Example (b)

$$\alpha_2 = \sqrt{|\gamma_{f3}|/\pi M}, \quad \alpha_1 = -\alpha_2 \operatorname{sgn}(\gamma_{f3})$$

and $\alpha_4 = \rho_1 = \rho_2 = \rho_4 = 0$. So $u_4(t) = 0$ for this maneuver, i.e., we use only roll and pitch to get a net yaw motion. Also, u_1 and u_2 are only nonzero during the time interval $[t_6, t_9]$. That is the total time required for the maneuver is $t_9 - t_6 = (M+1)T$. So we can compute

$$T = \frac{t_f}{M+1}, \quad \omega = \frac{2\pi}{T}.$$

The controls reduce to

$$\epsilon u_1(t) = \begin{cases} 0 & 0 \leq t \leq \frac{T}{4} = s_1 \\ \alpha_1 \omega \sin(\omega(t - s_1)) & s_1 \leq t \leq s_1 + MT = s_2 \\ 0 & s_2 \leq t \leq s_2 + \frac{3T}{4} = t_f \end{cases}$$

$$\epsilon u_2(t) = \alpha_2 \sin(\omega t) \quad 0 \leq t \leq t_f \tag{11}$$

One can show further, since yaw motion is motion in a single bracket direction, given that $\gamma_{f3} = O(\epsilon)$ these controls will achieve the yaw motion with $O(\epsilon^2)$ accuracy.

For numerical illustration, let $\epsilon = 0.1$, $\gamma_{f3} = 0.2$ and $t_f = 22$. Choose $M = 10$, then $T = 2$, $\omega = \pi$, $-\alpha_1 = \alpha_2 = 0.08$. Figure 3 shows plots of the active controls ϵu_1 and ϵu_2 as a function of time. Figure 4 shows plots of a simulation of the response of the local parameters $\gamma(t)$ as a function of time. The simulation was produced using MATLAB.

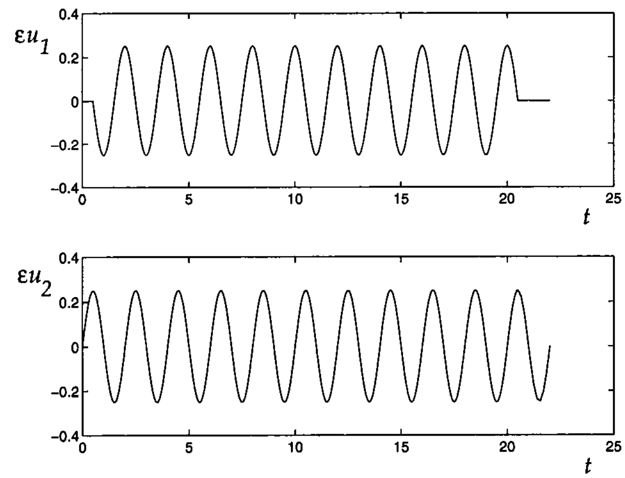

Figure 3: Control Input Signals for Yaw Maneuver.

The horizontal lines of Figure 4 represent the desired final parameter values γ_f. From Figure 4 it can be observed that the yaw motion is achieved with $O(\epsilon^2)$ accuracy.

Parking Maneuver Example

Consider an AUV with control authority $\{A_1, A_2, A_3, A_4\}$, i.e., with actuators for roll, pitch, yaw and translation along $\mathbf{b_1}$ as in Example (a). Suppose the AUV is to perform a translation in the A_5 direction, i.e., a sideways motion or a "parking maneuver". In local coordinates this implies that $\gamma_{f_i} = 0$, $i = 1, 2, 3, 4, 6$, and $\gamma_{f_5} \neq 0$. For these values, according to the motion plan for Example (a), $u_1(t) = u_2(t) = 0$ throughout and all controls are zero during the time interval $[t_3, t_4]$. Thus, we can compute

$$T = \frac{t_f}{M+1}, \quad \omega = \frac{2\pi}{T}$$

and use the controls defined during the time interval $[0, t_3]$.

For numerical illustration, let $\epsilon = 0.05$, $\gamma_{f_5} = 0.05$ and $t_f = 16$. Choose $M = 7$, then $T = 2$, $\omega = \pi$, $-\alpha_3 = \alpha_4 = 0.048$. Figure 5 shows the two active controls ϵu_3 and ϵu_4 as a function of time. Figure 6 shows a simulation of the response of the AUV in terms of local parameters with the horizontal lines indicating the desired final parameters values. It is clear that the vehicle has been translated with $O(\epsilon^2) = O(0.0025)$ accuracy. It is

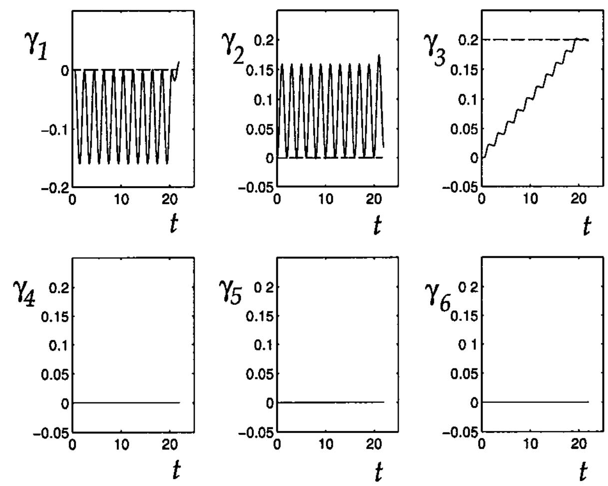

Figure 4: AUV Yaw Maneuver.

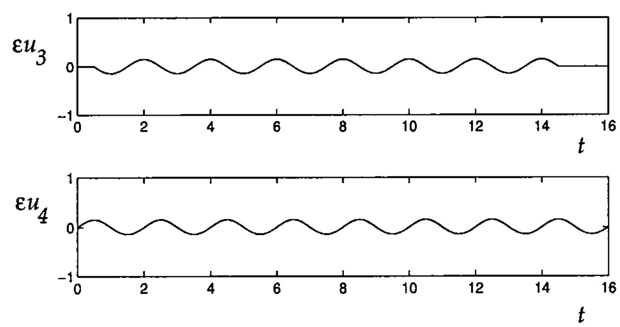

Figure 5: Control Input Signals for Park Maneuver with Example (a) Control Authority.

also interesting to note that the controls for this maneuver are of the same form as for the yaw maneuver. This is appropriate since in both cases the vehicle is being driven in a single Lie bracket direction.

Next we consider the same parking maneuver, but we assume that the control authority of the AUV has been reduced to $\{A_1, A_2, A_4\}$ as in Example (b). In this case the desired parking maneuver corresponds to motion in a double-bracket direction. Plugging γ_f for this maneuver in the motion plan for Example (b), we see that we need to use all three available controls to park the AUV. During the time interval $[t_6, t_{10}]$ all controls

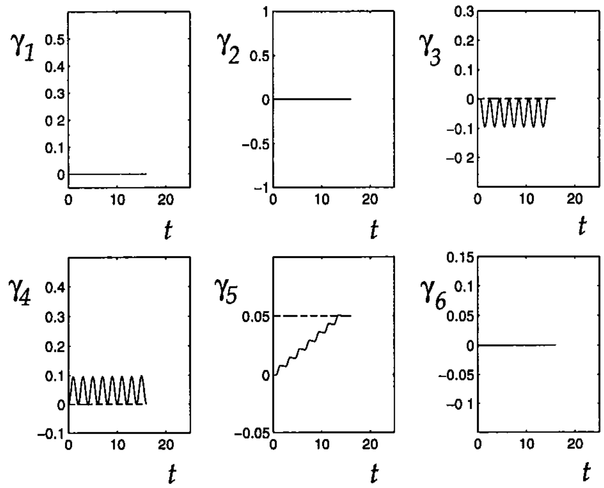

Figure 6: AUV Park Maneuver with Example (a) Control Authority.

are zero so we can compute

$$T = \frac{t_f}{2(M+1)}, \quad \omega = \frac{2\pi}{T}$$

and use the controls defined during the time interval $[0, t_6]$ only.

For numerical illustration, let $\epsilon = 0.14$, $\gamma_{f_5} = 0.05$ and $t_f = 16$. Choose $M = 3$, then $T = 2$, $\omega = \pi$. Figure 7 shows the three controls ϵu_1, ϵu_2 and ϵu_4, as a function of time. Figure 8 shows a simulation of the response of the AUV in terms of local parameters. The horizontal lines represent the desired final values of the parameters. It is clear, from Figure 8, that at the end of the simulation, the vehicle has been moved as desired with $O(\epsilon^3) = O(0.0027)$ accuracy. This is the same order of accuracy as was achieved with the controls in the previous section for the same desired motion during the same amount of time. The fact that the motion is more difficult to achieve without the third rotational control u_3 is reflected in the greater control effort used in this example (see Figure 7) versus the previous example (see Figure 5).

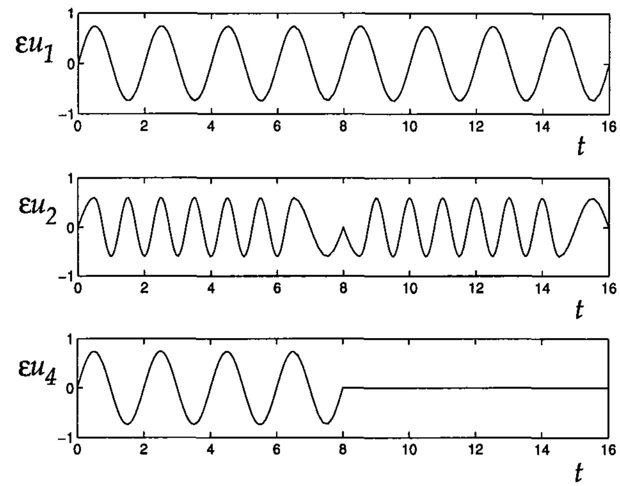

Figure 7: Control Input Signals for Park Maneuver with Example (b) Control Authority.

5 Motion Control Architecture with Adaptation

Figure 9 represents our proposed AUV motion control system architecture. This architecture incorporates open-loop control, i.e., strategic planning, and feedback control along with a second level of feedback to allow for adaptation to changes in control authority. The strategic planner represents the algorithms that produce motion scripts such as those described in Section 4 for motion with original or reduced control authority. Level 1 reactive control is intended to complement the open-loop control by adding (possibly intermittent) feedback to make the motion control more robust. Level 2 reactive control is used for adaptation to changes in control authority such as an actuator failure.

As an example of how open-loop and feedback control could be used together, consider the problem of driving the AUV from X_i to X_f where the motion from X_i to X_f is relatively large. As discussed in Section 4, to use the motion scripts for a large maneuver, we first choose intermediate target points $X_1, \ldots, X_r$ between X_i and X_f. Then we apply the appropriate motion script to drive the AUV from X_i to the first target point X_1. At this point we could then use feedback control to correct for disturbances

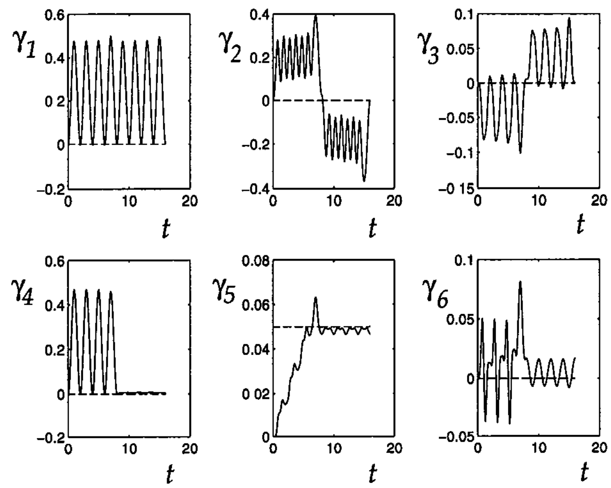

Figure 8: AUV Park Maneuver with Example (b) Control Authority.

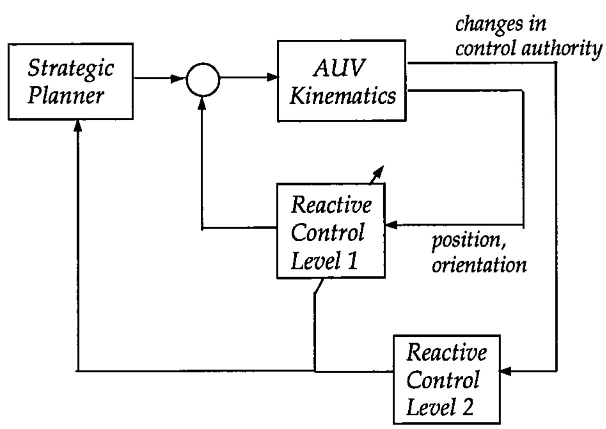

Figure 9: AUV Control Architecture

to the motion control. For instance, based on measurements of the current position and orientation or the AUV we could modify the selection of remaining target points. Alternatively, we could apply corrective actuation according to a local feedback control law to ensure that the AUV reaches X_1 independently of disturbances. After feedback is used the AUV would then proceed to each successive target point again using the open-loop controls of the motion script with feedback applied intermittently, i.e., after each or every few target points.

Level 2 reactive control is activated when a change in control authority is sensed.

This change in control authority could be an actuator failure or even an actuator coming back on line. The sensed change is sent to the strategic planner, which based on the new representation of control authority switches motion scripts. For example, suppose the AUV originally has control authority $\{A_1, A_2, A_3, A_4\}$ of Example (a). If the yaw actuator fails and the failure is sensed, the Level 2 reactive controller tells the strategic planner to switch from the motion script for Example(a) to the motion script for Example (b). The change in open-loop control could be effected on the fly by turning off all active controls withough affecting the current state of the AUV and then initiating the new set of open-loop controls.

6 AUV Experiment

In this section we describe an experimental implementation of the yaw maneuver described in Section 4 on an AUV built and operated in the Space Systems Laboratory (SSL) at the University of Maryland. The SSL designs, builds and evaluates integrated telerobotic systems, including free-flying vehicles and modular manipulators for space operations such as space structure assembly and satellite servicing [14]. Testing is done in the SSL's Neutral Buoyancy Research Facility, a water tank 50 feet in diameter and 25 feet deep, located at the University of Maryland.

SSL's Supplemental Camera and Maneuvering Platform (SCAMP), illustrated in Figure 10, moves freely and carries a video camera on board [15]. SCAMP is a 28-inch diameter icosahexahedron (26-sided) object weighing 167 pounds in air. About each of its three axes, SCAMP has a pair of ducted fan propellers, shown in Figure 10. A pair of propellers run in the same direction provides translation and run in opposing directions provides rotation about its associated axis. There is an on-board, closed-loop motor controller for each propeller that linearly converts an 8-bit (-128 to +127) command sent to the motor into a propeller speed. All on-board processing is done using a Motorola 68HC11 microcomputer. The Motorola 68HC11 communicates with

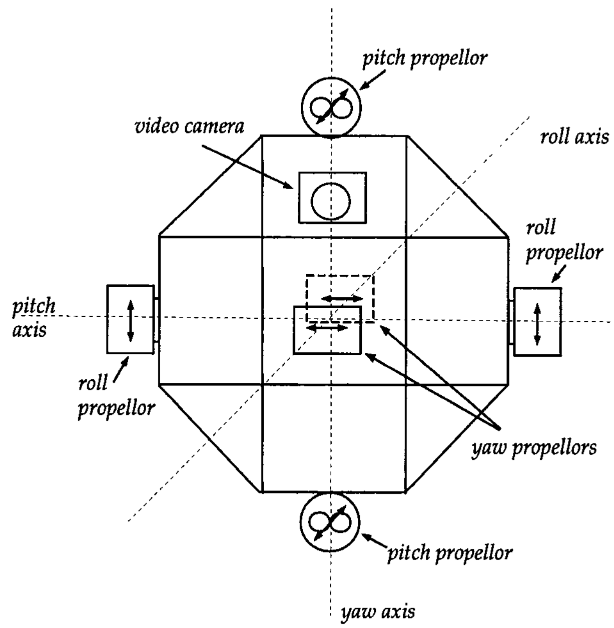

Figure 10: Front View of SCAMP.

the control station by means of a message-based serial protocol developed at the SSL. Data is transmitted over a fiber optic link. The control station consists of a MacIntosh IIfx computer and two hand controllers (joysticks). One hand controller is used for translation and the other for rotation. SCAMP also has a 7 lbs lead-weight pendulum that is located below the center of SCAMP along the yaw axis. The pendulum remains fixed with respect to the coordinate axes fixed on the vehicle unless it is actuated to control pitch. But for the pendulum, video camera, and the internal electronics, SCAMP is essentially a symmetric rigid body.

In the experiments described here, the hand controllers were bypassed. Instead open-loop control signals were computed on the MacIntosh computer according to the motion script for the yaw maneuver above and sent directly to the propeller motors. The main objective of the experiments was to illustrate our motion scripts for an AUV with reduced control authority. In particular, we assumed that the yaw propellers on SCAMP were inactive and that a pure yaw maneuver was desired.

For our experiments we used the fact that due to the drag of the water, a constant

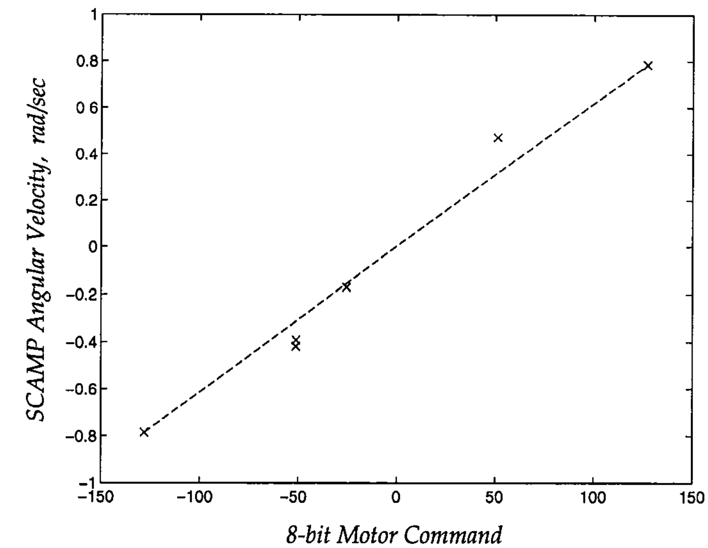

Figure 11: Calibration Data for SCAMP.

propeller speed corresponds to a constant vehicle speed. As a result, a constant 8-bit command sent to the motor corresponds to a constant vehicle speed. Calibration data to determine the relationship between the 8-bit motor command units and vehicle angular velocity is plotted in Figure 11. The data for the 8-bit commands of +127 and -128 was provided from previous testing. The other data points were obtained at the time of the experiments described here. A constant 8-bit signal was sent to the yaw propellers for a period of 60 seconds. Angular velocity was measured by counting the number and direction of rotations during the 60 second period. For the purposes of the experiments we assumed that vehicle angular velocity is linearly related to the 8-bit motor command signal. Based on the data in Figure 11, we assumed a proportionality constant of 163 motor command units per rad/sec.

We note that there is a time delay for the vehicle speed to follow the motor command related to the natural frequency of the vehicle. Because of the symmetry of the vehicle, we assumed that this delay is the same for roll, pitch and yaw. The open-loop controls for the yaw maneuver are sinusoidal roll and pitch control signals, and the net yaw motion is a function of the phase difference between the sinusoids. For our experiments we used sinusoids with frequencies less than the natural frequency of the vehicle, and we counted on the fact that equal time delays in all of the sinusoidal control signals should not upset their phase relationships.

The controls used in the experiment are given by (11) with the roles of u_1 and u_2 exchanged (for convenience) as follows:

$$\epsilon u_1(t) = \alpha_1 \omega \sin \omega t \quad 0 \le t \le t_3$$

$$\epsilon u_2(t) = \begin{cases} 0 & 0 \le t \le t_1 \\ \alpha_2 \omega \sin(\omega(t - t_1)) & t_1 \le t \le t_2 \\ 0 & t_2 \le t \le t_3 \end{cases}$$

with α_1, α_2 computed according to

$$\alpha_1 = \sqrt{\left|\frac{\gamma_{f3}}{\pi M}\right|}, \qquad \alpha_2 = \frac{\gamma_{f3}}{\alpha_1 \pi M}. \tag{12}$$

We chose $\epsilon = 0.4$, $|\alpha_1| = 0.4$, $|\alpha_2| = 0.4$, $\omega = \pi/4$ rad/s and $M = 6.25$. From this we can compute $T = 8$ s, $t_1 = 2$ s, $t_2 = 52$ s, $t_3 = 58$ s. Further, from (12) $\gamma_{f_3} = \alpha_1 \alpha_2 \pi M = \pm\pi$. Thus, we expected to see a net yaw rotation of ± 180 degrees within $O(\epsilon^2)$ accuracy or within about ± 10 degrees accuracy. ϵ was chosen to be relatively large so that the oscillations of the vehicle would be large enough to observe and the test would not be too time consuming, i.e., $t_f = t_3$ relatively small. The fact that M was not chosen to be an integer implies that there should be a small net rotation about the roll and pitch axes at $t = t_f$. However, the passive effect of the pendulum was expected to quickly remove these rotations.

Several repetitions of this experiment were run in May and June 1994. Sensors were not available to take velocity or position measurements during the experiments. However, many of the experiments were recorded on videotape. During these experiments SCAMP was observed to make a net yaw rotation consistently as expected. Gentle oscillations about the roll and pitch axes were clearly visible throughout the experiments with no significant final net roll or pitch rotation. When $\alpha_1 \alpha_2 > 0$, SCAMP was observed to rotate about the yaw axis in the counter clockwise direction (looking from above). When $\alpha_1\alpha_2 < 0$, SCAMP was observed to rotate about the yaw axis in the clockwise direction (looking from above). The net yaw rotation was typically slightly less than 180 degrees but within the accuracy of the predicted motion.

For one series of the experiments with $\alpha_1\alpha_2 < 0$ (i.e., clockwise yaw motion) towards the end of the experimentation, the net yaw rotation was not quite as high as expected (closer to 90 degrees than 180 degrees). This may have been due to error in the approximation of the motor command to velocity conversion or to asymmetry in the roll and pitch propellers in this direction such that the phase difference between these signals was affected. Additionally, there was some difficulty keeping SCAMP neutrally buoyant throughout the test, but this did not seem to have an effect on the attitude motion.

7 Final Remarks

We have demonstrated that it is possible to control an AUV with reduced control authority and have described a control synthesis technique for underactuated AUV's. The synthesis technique provides motion scripts, i.e., open-loop controls, to drive an AUV as desired based on the available control authority. Two example motion scripts were provided in full, one for an AUV with four control actuators and one for an AUV with three control actuators. These motion scripts were illustrated for a yaw maneuver and a parking maneuver.

Exploiting our ability to synthesize open-loop controls for an underactuated AUV, we proposed a motion control architecture which incorporates strategic planning, reactive control and adaptation to changes in control authority. The adaptation involves changing motion scripts at run-time in response to system configuration changes. We note that due to the discrete nature of the changes to which the adaptation responds, we have deviated from what is typically considered adaptive control theory.

Our open-loop controls were tested on an AUV in a neutral buoyancy tank at the

University of Maryland. The experiment demonstrated an AUV yaw maneuver using only roll and pitch actuation. In particular, the experiment illustrated Lie bracket effects through sinusoidal controls (in this case, roll and pitch propeller thrust reversals). The experiment did not involve precision measurements; however, one could do so in future experimentation.

The control synthesis and adaptation in this paper was based on a kinematic model of an AUV. Future work will reintroduce the AUV dynamics into the control system design.

Acknowledgements

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