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Lung-Wen Tsai

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
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Lung-Wen Tsai
Sr. Staff Research Engineer,
Power Systems Research Department,
General Motors Research Laboratories,
Warren, MI 48090-9055;
Presently, Department of
Mechanical Engineering,
University of Maryland,
College Park, MD 20742
Mem. ASME

In this paper, a random number technique for computing the value of a linkage characteristic polynomial is shown to be an effective method for identifying isomorphic graphs. The technique has been applied to the topological synthesis of one-degree-of-freedon, epicyclic gear trains with up to six links. All the permissible graphs of epicyclic gear trains were generated by a systematic procedure, and the isomorphic graphs were identified by comparing the values of their corresponding linkage characteristic polynomials. It is shown that there are 26 nonisomorphic rotation graphs and 80 displacement nonisomorphic graphs from which all the six-link, one-degree-of-freedom, epicyclic gear trains can be derived.

Introduction

One of the most important stages in mechanical design is the conceptual phase, i.e., the creation of a mechanism to satisfy a desired functional requirement. Traditionally, this is accomplished by the designer's intuition and experience. One attempt to help designers in this conceptual phase is to generate atlases of mechanisms grouped according to their functions. For example, references [1-4] contain a variety of mechanism configurations that can be used as an aid for the design of epicyclic gear trains. However, the design methodology does not insure the identification of all the design alternatives nor does it result in an optimum design.

Recently, however, there has been considerable interest in the creation of mechanisms in a systematic manner [5-23]. A promising approach is to separate the structure of a mechanism from its function. Then, kinematic structures of the same type, i.e., same degree of freedom, number of links, and nature of the desired function, are enumerated systematically with the aid of graph theory. Finally, each kinematic structure obtained is sketched and evaluated according to the functional requirements of a mechanism [14]. This method of synthesis has been successfully applied to the creation of variable-stroke mechanisms by Freudstein and Maki [15, 16].

The application of graph theory to the synthesis of epicyclic gear trains was first investigated by Buchsbaum [6], Buchsbaum and Freudenstein [7], and Freudenstein [11]. However, due to the complexity of the problem involved, the investigation was limited to epicyclic gear trains with up to five links. An inspection of existing automotive transmissions indicates that most of the automatic transmission gear trains belong to the six-link, epicyclic gear train family. Therefore, there is a need to extend the theory to include mechanism structures with six or more links. Most recently, Ravisankar and Mruthyunjaya computerized the synthesis procedure given by Buchsbaum and Freudenstein and were able to derive geared kinematic chains with up to six links [23].

In this paper, we shall first review some fundamental graphs theories established by Buchsbaum and Freudenstein [7] and Freudenstein [11], and then introduce a new method for identifying isomorphic graphs.[1] Finally, all the permissible six-link, one-degree-of-freedom, epicyclic gear trains will be generated in graphical form using an alternative systematic procedure.

Assumptions

Following the same guidelines of Buchsbaum and Freudenstein, we shall consider only those one-degree-of-freedom epicyclic gear trains which obey the following assumptions:

A1. The mechanism shall obey the general degree-of-freedom equation, i.e., no special proportions are required to insure the mobility of an epicyclic gear train.

A2. The mechanism shall be planar and its joints binary.

A3. The rotatability of all links shall be unlimited. Mechanisms with partial mobility or partially locked structures shall be excluded.

A4. Each gear must have a turning pair on it axis, and each link in a gear train must have at least one turning pair in order to maintain constant center distance between each gear pair.

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[1] See reference [11] for the definition of graph isomorphism, rotational isomorphism, and linear isomorphism.

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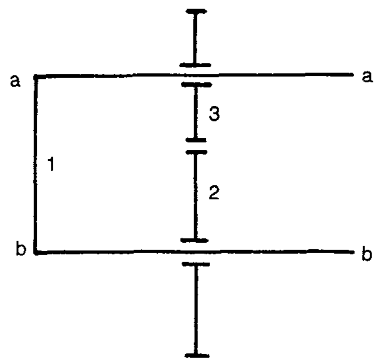
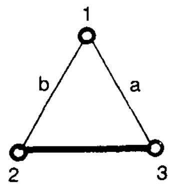
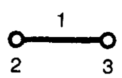


(a). Functional Representation of a Simple Epicyclic Gear Train.



(b). Graph Representation of Fig. 1(a).



(c). Rotation Graph of Fig. 1(a).

Fig. 1 Representations of an epicyclic gear train

Representations of an Epicyclic Gear Train

Functional Representation. Functional representation of an epicyclic gear train refers to the conventional schematic drawing of the mechanism. Figure 1(*a*) shows the functional representation of a simple epicyclic gear train where the two meshing gears, numbered 2 and 3, rotate about the two parallel offset shafts, labeled *a-a* and *b-b*, respectively, that are supported by the carrier, numbered 1.

Graph Representation. In a graph representation, links are denoted by vertices and joints by edges. The edge connection between vertices corresponds to the pair connection between links. In order to distinguish a turning-pair connection from a gear-pair connection, turning pairs are represented by thin edges and gear pairs by heavy edges. For this reason, the thin edge is also called the turning-pair edge and the heavy edge the geared edge. Furthermore, the thin edges are labeled according to their axis locations. A graph made of thin and heavy edges is sometimes referred to as a bicolored graph since the two different edges can also be represented by two different color codes. A bicolored graph without labeling its thin edges is called an unlabeled graph. Figure 1(*b*) shows the graph representation of the mechanism shown in Fig. 1(*a*), where vertices 1, 2 and 3 correspond to links 1, 2, and 3; the thin edges 1-2 and 1-3 correspond to the turning pairs connecting links 1 and 2, and 1 and 3; the edge labels *a* and *b* correspond to the joint-axis locations $a-a$ and $b-b$; and the heavy edge 2-3 corresponds to the gear-pair connection between links 2 and 3, respectively.

Rotation Graph Representation. A rotation graph is defined for each graph by deleting the turning-pair edges and the transfer vertices and labeling each geared edge with the symbol for the associated transfer vertex [11]. Figure 1(*c*) shows the rotation graph of the mechanism shown in Fig. 1(*a*). Note that Ravisankar and Mruthyunjaya [23] used a slightly different definition for the rotation graph.

Fundamental Rules of a Graph

The following fundamental rules have been established for the graphs of one-degree-of-freedom; epicyclic gear trains which obey the aforementioned assumptions [7, 11]:

F1. For an n-link, one-degree-of-freedom, epicyclic gear train, there are n vertices, $n-1$ turning-pair edges, and n-2 geared edges.

F2. The subgraph formed by deleting all the geared edges is a tree.

F3. Any geared edge added to the tree forms a fundamental circuit (f-circuit) having one geared edge and several turning-pair edges.

F4. The number of f-circuits equals the number of geared edges.

F5. There can be no circuit formed exclusively by turning-pair edges. Otherwise, either the circuit is locked or the rotatability of the links is limited.

F6. All vertices must have at least one incident edge which represents a turning pair.

F7. Each turning-pair edge can be characterized by a level which identifies the location of its axis in space.

F8. The differential degree-of-freedom of any circuit must be at least one; for an f-circuit, it is equal to the number of vertices in the circuit minus two.

F9. In each f-circuit there is one vertex, called the transfer vertex, such that all edges on one side of the transfer vertex are at the same level and edges on the opposite side of the transfer vertex are at a different level.

F10. We also restrict ourself to graphs for which two turning-pair edges having a common level must intersect at a common vertex.

Linkage Characteristic Polynomial

Linkage Adjacency Matrix. The linkage adjacency matrix of a kinematic chain was first defined for planar linkages by Uicker and Raicu [19]. In this investigation, the definition was modified to include kinematic chains with gear pairs. The vertices of a graph are numbered sequentially from 1 to n, and the adjacency matrix, A, is defined as a square matrix of order n with its elements, $A(i, j)$, defined as follows

$$A(i,j)=\begin{cases} 1 & \text{if vertex } i \text{ is connected to vertex } j \\ & \text{by a turning-pair edge,} \\ \mathbf{g} & \text{if vertex } i \text{ is connected to vertex } j \\ & \text{by a geared edge,} \\ 0 & \text{otherwise (including } i=j). \end{cases} \tag{1}$$

For example, the adjacency matrix of the graph shown in Fig. 1(*b*) is given by

$$A=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & g \\ 1 & g & 0 \end{bmatrix} \tag{2}$$

The adjacency matrix is symmetric, i.e., $A(i, j) = A(j, i)$. From fundamental rule F1, it follows that an $n \times n$ adjacency matrix contains $2(2n-3)$ nonzero elements of which $2(n-1)$ elements have the value of "1" and $2(n-2)$ elements have the value of "g".

Linkage Characteristic Polynomial. The linkage characteristic polynomial of a kinematic chain was first investigated by Uicker and Raicu [19] and subsequently more fully explored by Yan and Hall [20, 21] and Yan and Hwang [22]. In this paper, the linkage characteristic polynomial, $p(x, g)$, of an epicyclic gear train is defined to be the determinant of the matrix $(xI - A)$, where x is a dummy variable and I is a unit matrix of the same order as the adjacency matrix, A, i.e.

$$p(x, g) = \det\,(xI - A) \tag{3}$$

For example, the linkage characteristic polynomial of the mechanism shown in Fig. 1(a) is given by

$$p(x, g) = \begin{vmatrix} x & -1 & -1 \\ -1 & x & -g \\ -1 & -g & x \end{vmatrix} = x^3 - (2 + g^2)x - 2g \tag{4}$$

It has been shown that a necessary condition for two kinematic chains to be isomorphic to each other is for the two mechanisms to have identical linkage characteristic polynomials. Uicker and Raicu [19] presented a computational method, and recently, Yan and Hall [20] developed an inspection technique for the derivation of the coefficients of characteristic polynomials. Both methods can be used to derive the characteristic polynomial of a planar linkage. However, the process of identifying isomorphic graphs becomes very tedious and slow when there are a large number of complicated linkages to be identified. First, we have to find coefficients of the polynomials and then compare the polynomials term by term.

In this paper, a more efficient method was applied. The method assigns a random noninteger number (real or complex) to each of the x's and g's and then evaluates the value of the determinant of the matrix $(xI - A)$. Since the probability for two different polynomials to yield identical values with random values of x and g is nearly zero, we conclude that this random number technique can be used to identify isomorphic graphs.

Systematic Synthesis Procedure

Based on the foregoing fundamentals, it is possible to systematically synthesize kinematic structures of epicyclic gear trains for any given number of links. Buchsbaum and Freudenstein [7] and Freudenstein [11] enumerated all the permissible noncolored graphs; i.e., no distinction between the thin and heavy edges, for geared kinematic chains with up to six links. For each of the noncolored graphs enumerated, they tried to find structurally distinct ways of coloring the graphs; i.e., of choosing the heavy edges. They were able to develop bicolored graphs for epicyclic gear trains with up to five links. A methodology was outlined and an attempt was made to develop kinematic structures for epicyclic gear trains with six or more links, but it was not successful due to the complexity of the problem. An alternate approach is described next.

Instead, we start with known unlabeled graphs called the generic graphs. In view of the fundamental rule F1, we conclude that starting from a given unlabeled graph, each time we increase the number of vertices by one, both the numbers of turning-pair edges and geared edges also increase by one. The new turning-pair edge can be connected between the added vertex and any one of the existing vertices of the generic graph; the geared edge can be connected between the added vertex and any one of the remaining vertices. Hence, potentially, $n(n-1)$ unlabeled graphs of $n+1$ vertices can be derived from a known unlabeled graph of n vertices. However, some of these graphs may be rejected due to violation of the previously mentioned fundamental rules, and others may be isomorphic to one another and should be eliminated to avoid duplication. So the number of nonisomorphic unlabeled graphs is usually less than $n(n-1)$.

This procedure can be automated by a computer program using the notations of linkage adjacency matrix and linkage characteristic polynomial defined by equations (1) and (3). We start with a given linkage adjacency matrix of order n. Each time we increase the number of vertices by one, we add a row and a corresponding column to the given adjacency matrix. First, we initialize all the added elements in the new row and column to zero. Second, we set an element in the new row, excluding the diagonal element, and the corresponding element in the new column to 1. Third, we set another element in the new row, excluding the previous chosen element and the diagonal element, and the corresponding element in the new column to g. This results in a linkage adjacency matrix of order $n+1$ which represents the graph of a new gear train. The process is repeated until all possible combinations are exhausted. Finally, graph isomorphism is checked by evaluating the determinant of the linkage characteristic polynomial defined by equation (3). In this paper, the values of x and g were chosen to be 0.7682415 and 1.8152379, respectively. All the isomorphic graphs identified by using this random number technique were checked manually and not a single contradiction was found.

The process is similar to the conventional method of designing gear trains where the designer begins with a rather simple gear train and increases the complexity by adding one gear at a time. Each time he adds a new gear, he adds not only a gear mesh to the mechanism but also a journal bearing to support the added gear.

Using this method to enumerate unlabeled graphs, fundamental rules F1 to F6 are automatically satisfied. Hence, a substantial number of redundant graphs encountered by Buchsbaum and Freudenstein were avoided [7]. We may summarize the procedure for the systematic synthesis of epicyclic gear trains as follows:

1 Enumerate all the permissible unlabeled graphs of $n+1$ vertices for each known unlabeled graph of n vertices. There are $n(n\text{-}1)$ possible combinations.

2 Check for graph isomorphism by comparing the linkage characteristic polynomials to obtain a set of nonisomorphic unlabeled graphs. Remember that some of them could be rotationally isomorphic [11].

3 Identify the transfer vertices by inspection or by using the Boolean algebra technique developed by Freudenstein [11].

4 Eliminate graphs that violate the fundamental rules F7–F9.

5 Transform the remaining graphs into rotation graphs and check for rotational isomorphism. Among two rotationally isomorphic graphs, the one with fewer ways of labeling its edges is considered as a subset of the other and, therefore, is eliminated from further consideration. This results in a set of rotationally nonisomorphic graphs.

6 Repeat Steps 1–5 again for each rotationally nonisomorphic graph of $n+1$ vertices enumerated.

7 Label each of the rotationally nonisomorphic graphs in as many ways as possible to obtain a set of displacement nonisomorphic graphs [11].

Graph Enumeration

Using the aforementioned procedure, rotationally nonisomorphic graphs were enumerated for epicyclic gear trains with up to six links. The following describes the sequence of enumeration.

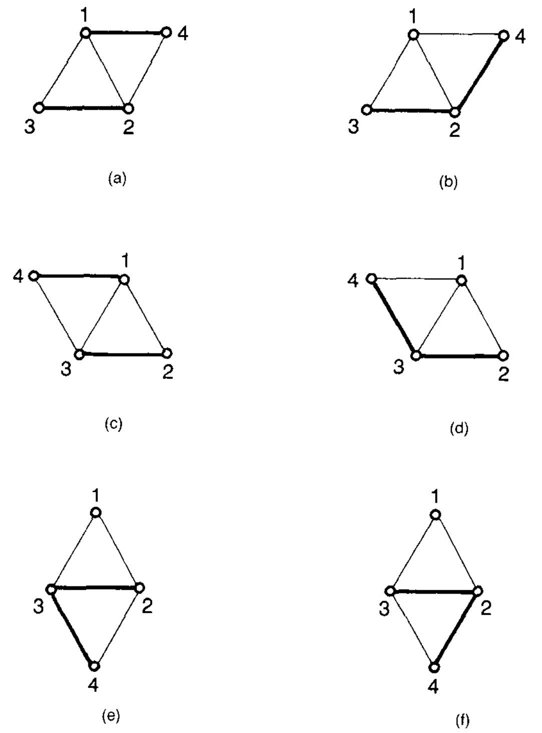
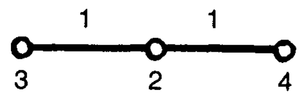
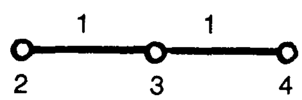


Fig. 2 Six possible graphs derived from Fig. 1(*b*), but only three are nonisomorphic

(a). Rotation Graph of Fig. 2(a).



(b). Rotation Graph of Fig. 2(b).



(c). Rotation Graph of Fig. 2(e).

Fig. 3 Rotation graphs of Figs. 2 (*a*, *b*, *e*)

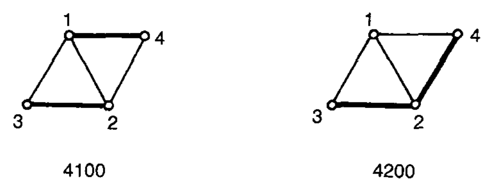


Fig. 4 Rotationally nonisomorphic graphs of four-link chains

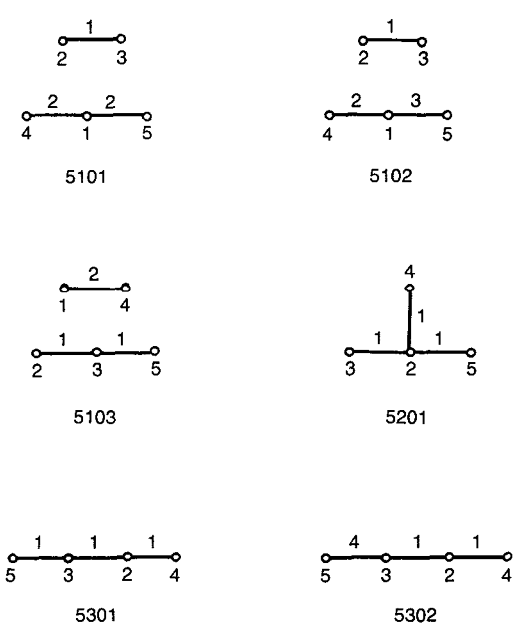


Fig. 5 Nonisomorphic rotation graphs of five-link chains

Three-Link Chains. We begin with the most fundamental three-link chain. The only possible three-link epicyclic gear train is shown in Fig. 1 where the three vertices are connected together by two turning-pair edges and one geared edge.

Four-Link Chains. There are 3 × 2 = 6 possible ways to add an additional vertex to the graph of three-link chain shown in Fig. 1(*b*). This can be accomplished by connecting the additional vertex to any one of the three existing vertices by a turning-pair edge and to any one of the remaining two with a geared edge. The six possible graphs are shown in Fig. 2 where vertices 1, 2, and 3 designate the original vertices of the three-link chain and 4 the added vertex. However, Fig. 2(*c*) is isomorphic with Fig. 2(*a*), Fig. 2(*d*) is isomorphic with Fig. 2(*b*), and Fig. 2(*f*) is isomorphic with Fig. 2(*e*). Therefore, there are only three nonisomorphic unlabeled graphs.

After identifying the transfer vertices, rotation graphs of Figs. 2(*a*), 2(*b*), and 2(*e*) were derived as shown in Figs. 3 (*a*–*c*), respectively. It can be shown that Fig. 3(*b*) is isomorphic with Fig. 3(*c*). Hence, Fig. 2(*b*) is rotationally isomorphic with Fig. 2(*e*). Because Fig. 2(*e*) has fewer ways of labeling its edges than Fig. 2(*b*), it was eliminated from further consideration. Hence, we obtained only two rotationally nonisomorphic graphs as shown in Fig. 4. Note that we are not concerned with the labeling of the edges at this point since labeling the edges simply affects the axis-location of the mechanism.

Five-Link Chains. There are 4 × 3 = 12 possible ways to add an additional vertex to each of the rotationally nonisomorphic graphs shown in Fig. 4. Since there are two rotationally nonisomorphic graphs of four-link chains, a total of 2 × 12 = 24 possible graphs can be generated for five-link chains.

A computer program was designed to enumerate all these possible graphs systematically. The program also applied the linkage characteristic polynomial to check for graph isomorphism. Eleven out of the 24 graphs were initially found to be nonisomorphic, one of which was rejected because of the violation of fundamental rule F9. The remaining ten were then transformed into rotation graphs and checked again for rota-

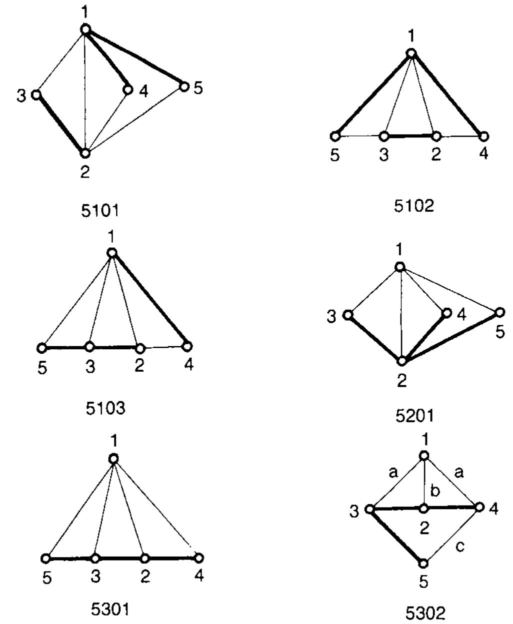


Fig. 6 Rotationally nonisomorphic graphs of five-link chains

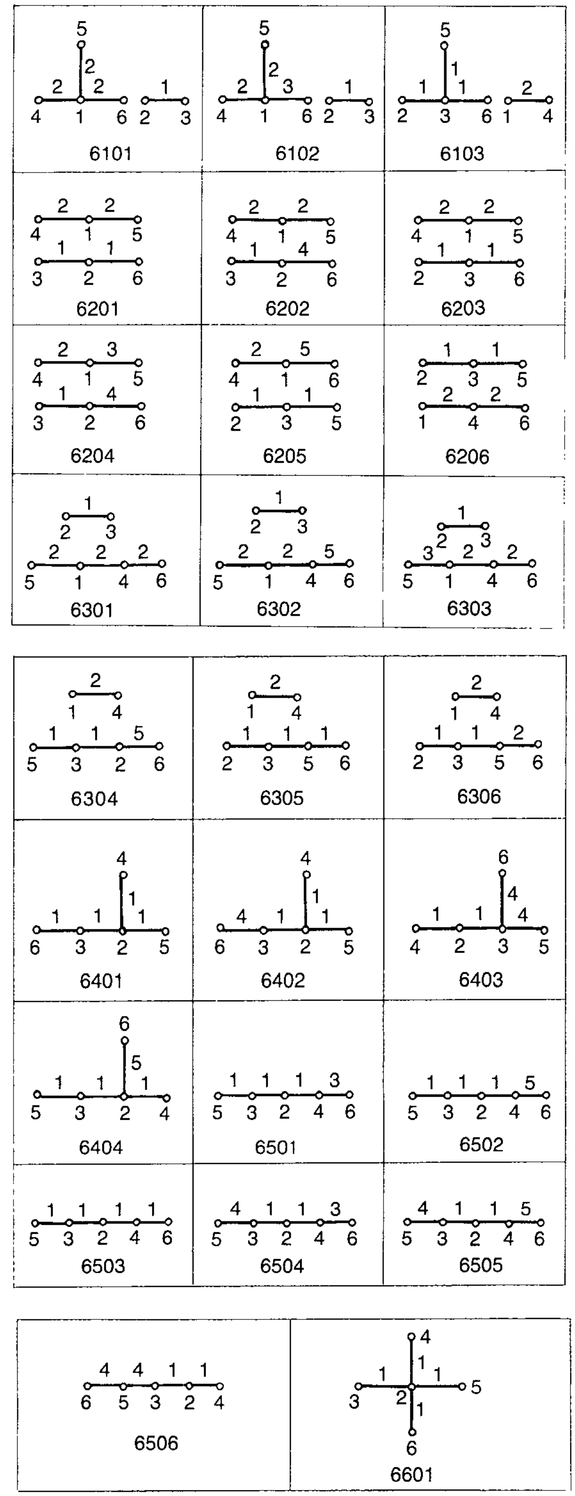


Fig. 7 Nonisomorphic rotation graphs of six-link chains

tional isomorphism. Finally, six nonisomorphic rotation graphs were obtained as shown in Fig. 5. The corresponding rotationally nonisomorphic graphs are shown in Fig. 6. The result is in complete agreement with that of Freudenstein [11]. In Fig. 6, graph ID 5302 can be labeled in two different ways which results in two rotation graphs, one of which is identical to that of graph ID 5301 and the other results in a rotationally nonisomorphic graph which can only be differentiated by labeling the edges. For this reason, graph ID 5302 is labeled as shown in Fig. 6.

All the graphs shown in Figs. 5 and 6 were identified by a four-digit number. The first digit denotes the number of vertices, or links, contained in the graph. The second denotes the gear-pair connection topology. For example, graphs ID 5101, 5102, and 5103 all have two interconnected geared edges and a separated geared edge. The last two digits are used as a serial number to identify the graphs. Obviously, two rotation graphs must have the same type of gear-pair connection in order to be isomorphic.

Six-Link Chains. There are $5 \times 4 = 20$ possible ways to add one additional vertex to each of the rotationally nonisomorphic graphs shown in Fig. 6. Since there are six rotationally nonisomorphic graphs of five-link chains, a total of $6 \times 20 = 120$ possible graphs can be generated. Again, all the 120 possible graphs were generated systematically and checked for isomorphism by the computer program. Sixty-three graphs were initially found to be nonisomorphic. Seven were rejected because of the violation of fundamental rule F9. The remaining graphs were then transformed into rotation graphs and checked for rotational isomorphism. As a result, we obtained 26 nonisomorphic rotation graphs as shown in Fig. 7. Each rotation graph in Fig. 7 was given an ID number similar to that of five-link chains. Finally, the corresponding rotationally nonisomorphic graphs were labeled in as many ways as possible. As a result, 80 displacement nonisomorphic graphs were obtained as shown in Fig. 8. Note that the present method excludes all the graphs that have several geared edges forming a loop.[2]

The conversion of labeled graphs into mechanisms was discussed in detail in references [6, 7, 11]. Each geared edge in Fig. 8 can be made of three different gear meshes; namely, external to external, external to internal, and internal to external gear meshes. Since there are four geared edges in each graph of the six-link chains, every labeled graph can generate $3 \times 3 \times 3 \times 3 = 81$ functional mechanisms. Potentially, $80 \times 81 = 6480$ mechanisms can be generated from the graphs shown in Fig. 8. However, many of the gear trains may be isomorphic to one another due to the symmetry. So the actual number of nonisomorphic gear trains is much less than 6480. Although it

[2] Ravisankar and Mruthyunjaya obtained 27 nonisomorphic rotation graphs, one of which has four geared edges forming a loop and does not produce any labeled graph.

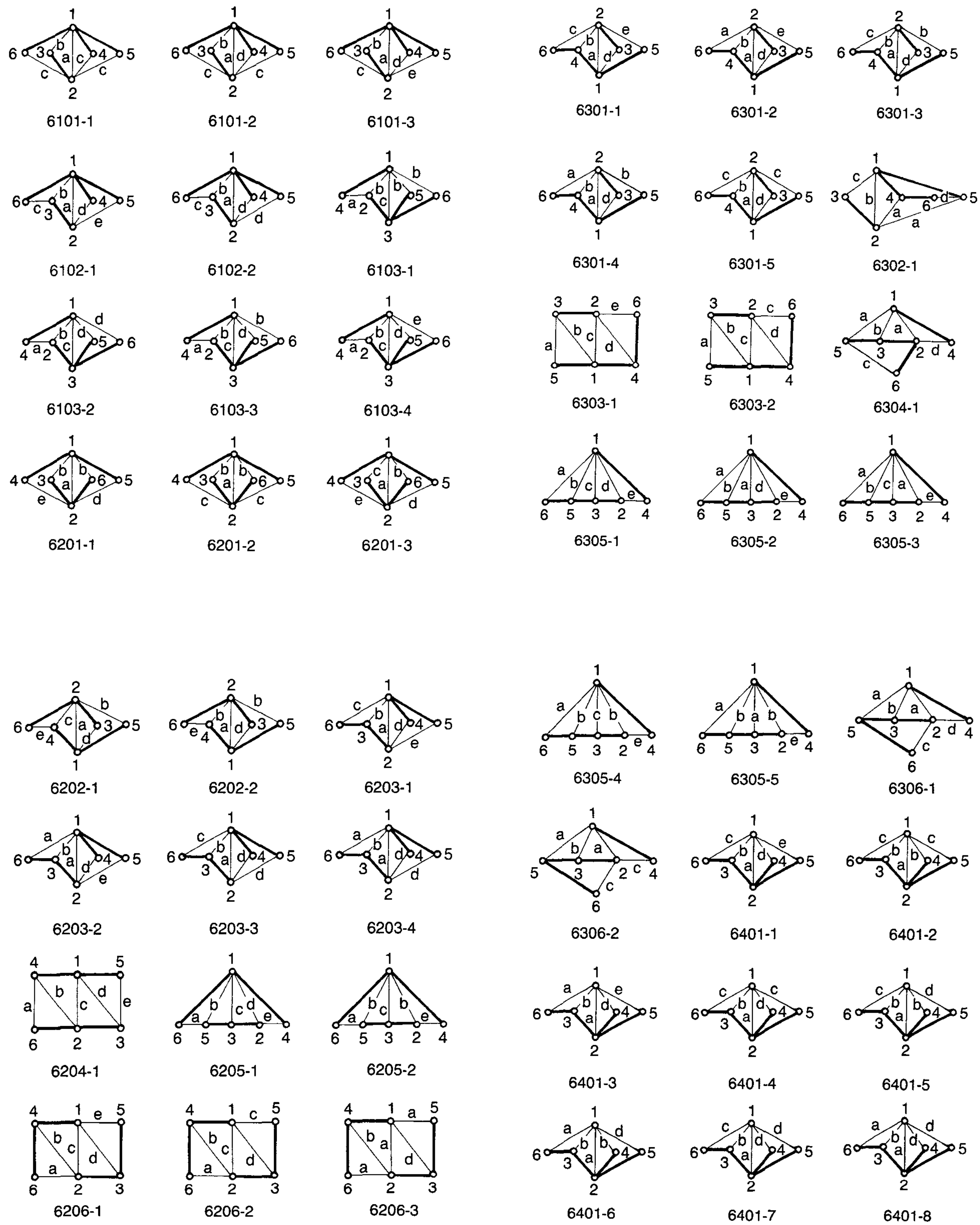


Fig. 8 Rotationally nonisomorphic graphs of six-link chains

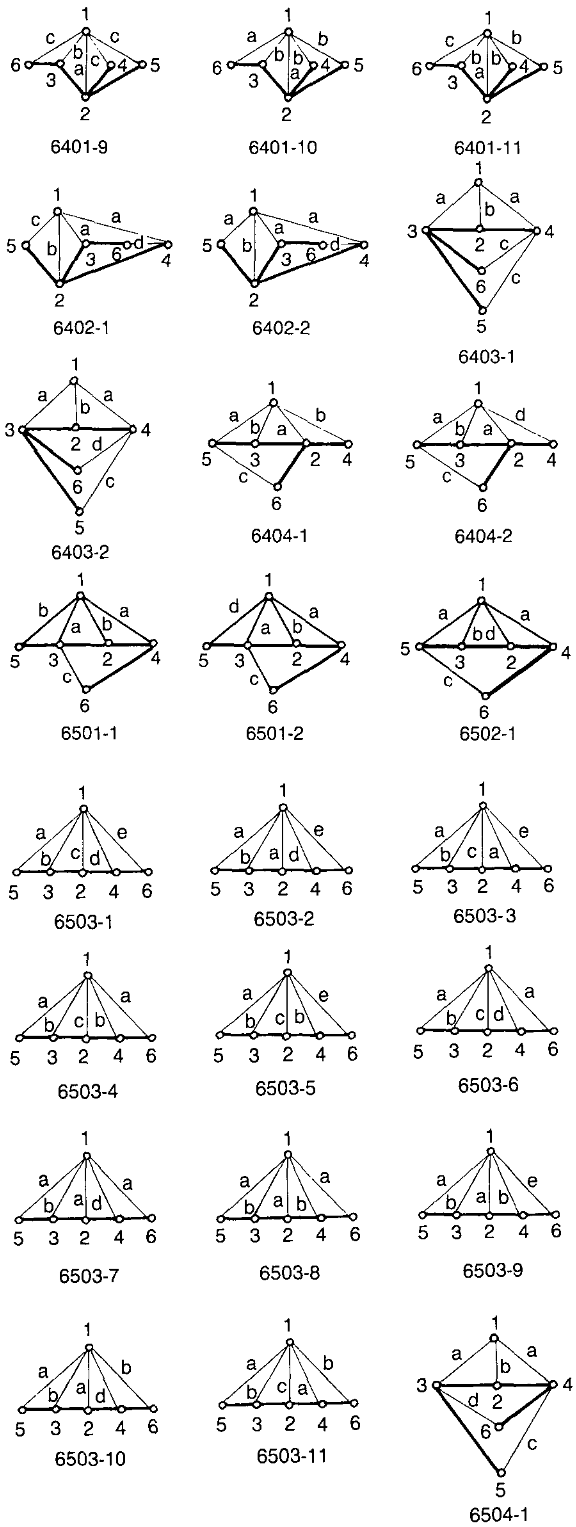


Fig. 8 Continued

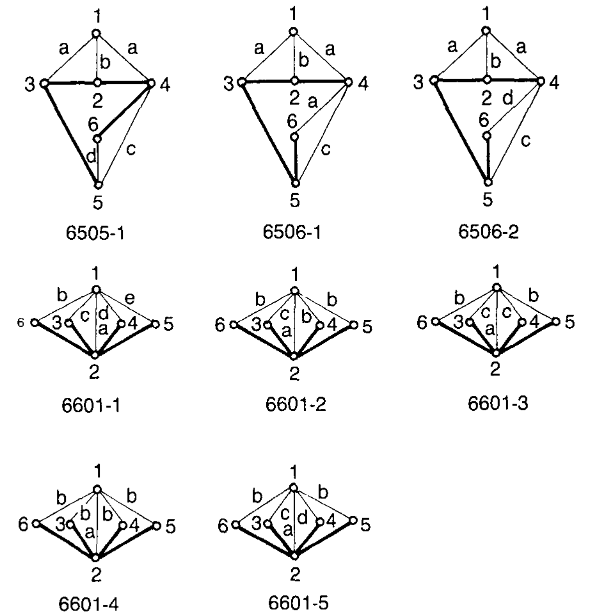


Fig. 8 Continued

is beyond the scope of this investigation, it is anticipated that many new, novel gear trains can be revealed from these graphs.

Summary

It has been shown that the random number technique for computing the value of the linkage characteristic polynomial is an effective method for identifying isomorphic graphs. All the permissible graphs of one-degree-of-freedom, epicyclic gear trains with up to six links were generated by an alternative systematic procedure, and the isomorphic graphs were identified using the random number technique. It is shown that there are 26 nonisomorphic rotation graphs and 80 displacement nonisomorphic graphs from which all the six-link, one-degree-of-freedom, epicyclic gear trains can be constructed. The results are in complete agreement with those of Ravisankar and Mruthyunjaya.

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